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AN OPERATIONAL APPROACH TO INTEGRATED WORKING CAPITAL PLANNING

by

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Abstract

Working capital management involves the integration of three separate activities -- marketing, production, and financial -- into a single planning system. Although this integration can best be achieved by an optimization process, most successful industrial applications and some academic working capital models are based on non-optimizing financial statement simulators or budget compilers. This lack of real-world application of optimization models is primarily a function of the excessive solution time traditionally required for large problems, the necessity to solve the models iteratively, and the lack of understanding on the part of decision-makers of the algebraic equation systems used in the models. We describe an interactive working capital planning system that experience shows can overcome these three problems. The system surmounts these limitations by coupling recent advances in graphical modeling with network solution procedures.
INTRODUCTION

Working capital management involves the intricate balancing of three separate activities -- marketing, production, and financial. The integration of these activities into a single planning system is, therefore, very important in practice, but has proven to be a difficult task. A number of published models have made major contributions toward technical integration, but at the expense of large, mathematically complex models that are generally both operationally and computationally infeasible. The planning model presented in this paper overcomes many of the conceptual and operational difficulties by reformulating the working capital problem in a structure that captures the complex reality of the environment in a visual format that managers can understand and is amenable to remarkably efficient solution and sensitivity analysis.

The first section of the paper outlines the various approaches suggested in the academic literature for solving the working capital planning problem. Strengths and limitations of these approaches are pointed out, and the characteristics of an operationally effective and meaningful planning structure are identified. In the second section, a graphical modeling format is described. In other industrial applications, this approach has been shown to possess these
required characteristics, and it has successfully overcome the implementation problems that have plagued most financial optimization models. We then utilize this modeling format to structure the working capital planning problem for a firm with geographically separate production facilities and centralized financial activities. The concluding section of the paper concerns the interaction of the marketing, production, and financial planning components and discusses the viability of using the model in practice.

Approaches to Working Capital Planning

In an operating company, all current asset investments and the related financing support are tied together by a complex set of flow linkages. The interactions in this short-term funds system, both cross-sectional and inter-temporal, make clear that planning of the individual asset or credit element in isolation can result in severe suboptimization. Moreover, recognizing that the firm's liquidity is controlled by developments within this system, extensive planning efforts are clearly warranted.

One planning approach is to investigate the various tradeoffs within the structure of the financial statement simulator or computerized budget compiler. This is the approach taken in many corporate models (see for example, [4], [8], and [13], and the critique in [1]). A "satisfactory" short-term strategy may be determined with these models -- satisfactory in the sense that the resulting balance sheet and income statement have "reasonable" values that should not cause an adverse market reaction. With this technique, however, management cannot investigate all possible strategies and there is no way to verify the absence of another combination of inputs that the decision maker
would consider to dominate the chosen set. This means it is almost impossible to validate such models. In addition, simulations often require substantial computer time. A novel but telling example is that it requires two CPU minutes of computer time to simulate each single minute of a computer's operation.

Another approach is to determine the optimal strategy for short-term funds management -- optimal in the sense that the solution is the best plan for the assumed set of policy constraints. Alternative (suboptimal) strategies can then be investigated and a dollar cost assigned to departures from the optimum. Also, the sensitivity of the optimum to changes in input parameters can be ascertained and used as the basis for contingency planning.

Optimization models have been criticized both within the business community and by some academicians as inferior to "simulation" models when used in a planning context. The inability to solve large, realistic models both in a reasonable amount of time and at a low cost on even the largest computer systems lends credence to this argument. However, the most severe criticism of optimization techniques focuses on the need to set forth explicitly an "objective" or criterion by which alternative strategies can be compared and their relative desirability ascertained. With simulation models (both deterministic and probabilistic), no objective need be supplied external to the decision maker. The models simply translate input data and assumptions into the logical output consequences based on the structural characteristics of the decision process. The decision maker can investigate as many alternate courses of action as desired and then select, based on intuition or other subjective factors, the "best" of the alternatives. Thus, simulation is not free of objective specification since there exists an implicit objective (or objectives) within the decision maker by which the alternatives are judged.
It is our contention that in any decision process, requiring an explicit objective or criterion to be set forth from the start of the analysis has several advantages. First, explicit objectives lead to greater consistency. For many reasons such as personnel transfers or decentralization of responsibility, different people are called upon to make essentially similar or on-going decisions. As each individual is different, each may be inclined to rely on different and even inconsistent implicit criteria for choosing among alternatives. Explicit objectives can mitigate this problem somewhat; or, since people and not models make decisions, explicit objectives consciously invoke the exception principle if the decision maker decides to "override" the model.

Another advantage of requiring explicit objectives is that they are exposed for all to see and examine. This should lead to a more critical examination of the rationale behind the objectives which, in turn, should result in more logical decisions.

Many times, it is argued, a single objective is totally inappropriate as the decision must be made in light of numerous trade-offs. We are in complete agreement with this contention, but again believe that the multiple goals must be made explicit, as must also the relative importance (weights) attached to each goal. This might be a difficult requirement to place on a manager, but nevertheless, forcing decision makers to crystallize their thought process and to enumerate explicitly their subjective beliefs should ultimately lead to better decisions.

None of the foregoing should be interpreted as advocating rigidity of objective function specification. In order for an optimization model to be useful for planning, it must be possible (and relatively simple) to test the impact of various objectives (or weights assigned to multiple objectives). This form of sensitivity
analysis adds an entirely new dimension of richness to the planning process and permits managers to glean cost-benefit information that is impossible to obtain with traditional "simulation" models.

In this regard, we view optimization as a powerful subset of simulation modeling. It is always possible to constrain all or most input parameters in an optimization model. If this is done, the model can be used to perform a standard "simulation" analysis, but an objective function value can be obtained. This value, when compared to the objective function value of the original specification, will reveal the cost of the action in terms of the reduction of the objective function. Assuming a correct specification of the model, the "optimization" technique would reveal the same solution as the "simulation", but would also provide the decision maker with a very important extra piece of information -- how much it would cost. Therefore, it is our opinion that optimization is a potentially richer modeling methodology than "simulation" and that this potential can be tapped if optimization algorithms are available for efficiently solving large-scale models which capture the essence of the situation.

A number of optimization models have appeared in the finance literature in the past decade (e.g. [9], [10], and [12]) that address the problem of working capital management. Some of these models provide for the simultaneous optimization of many critical marketing, production, and financial decisions. Further, these models have a high degree of environmental disaggregation. Unfortunately, in attempting to combine simultaneously the relevant range of operating decisions with the financial decisions, the models become mathematically incomprehensible to the manager and computationally infeasible for even the largest computers.

What management needs is (1) an understandable optimization planning structure, (2) usable on a real-time computer (interactive) basis, that will (3) assist deci-
sion makers in their full range of working capital decisions. Recent developments in network algorithms and computer codes (as discussed in [3], [5]) now make it possible to circumvent most of the implementation problems associated with such a model.

A Network Modeling Format

A network can be thought of as consisting of m reservoir nodes which are connected pairwise by n directed cash flow arcs, although it is not necessary for all pairs of nodes to be connected. Let $b_i$ represent the amount of supply or demand at node $i$ (where supply is denoted as a positive quantity and demand as a negative quantity). Each admissible arc in the network can be described in terms of five parameters. $L_{ij}$ and $U_{ij}$ respectively denote the lower and upper bounds on the amount of flow on the arc from node $i$ to node $j$ (henceforth denoted by arc $(i,j)$, $U_{ij}$ need not be finite). $x_{ij}$ denotes the actual flow leaving node $i$, $p_{ij}x_{ij}$ denotes the actual flow entering node $j$, and $c_{ij}$ is the unit cost of the flow, $x_{ij}$, from node $i$ to node $j$. Letting $N$ denote the set of admissible arcs, these parameters are related in a programming framework as follows:

$$\text{MINIMIZE: } \sum_{(i,j) \in N} c_{ij}x_{ij}$$

$$\text{SUBJECT TO: } + \sum_{(i,j) \in N} x_{ij} - \sum_{(j,i) \in N} p_{ji}x_{ji} = b_i \quad i = 1, 2, \ldots, m$$

$$L_{ij} \leq x_{ij} \leq U_{ij} \quad (i,j) \in N$$

The flow across the arc $(i,j)$ can be viewed as subject to magnification or reduction by the factor $p_{ij}$. That is, for every unit of flow on arc $(i,j)$ that
leaves node $i$, $p_{ij}$ units enter node $j$. Thus, if $p_{ij} < 1$ the flow is attenuated, if $p_{ij} > 1$ the flow is amplified and if $p_{ij}$ equals one the flow is unaltered. A network is designated as "pure" if all $p_{ij}$ equal one, otherwise it is generalized network or transshipment problem. In Figure 1 the various conventions for drawing and interpreting network graphs are shown.

Figure 1. GRAPHICAL REPRESENTATION OF NETWORK PARAMETERS

Graphical presentations will overcome one of the major deterrents to increased utilization of programming models by decision makers. Since most line managers do not think in terms of mathematical equations, they have a difficult time communicating their thoughts and ideas to the operations researchers who often cast relationships into a series of equations. The net result is too often mutual suspicion and distrust, leading to planning models that do not fit reality and remain unused (see for example, [7]). Because of this, practitioners very early began to develop special schematic modeling procedures both for representing input problem data and for exhibiting solutions. For example, Shell Oil Company developed a graphical problem generation system referred to as AMBUSH for mathematical optimization problems involving their refinery operations. As the use of these
schematic procedures became widespread in industry, an important observation was made: many problems that were previously expressed as complex algebraic models could be given a pictorial, network-related formulation which is mathematically equivalent to the algebraic statement of the problem. This observation, derived from applications and not theory, has given birth to the NETFORM (network formulation) technology.

When problems can be formulated as networks or NETFORMS (and there is reason to believe that up to 70 percent of all mathematical programming applications can be structured as networks, ([6], p. 1), they not only enhance the important communications between models and managers, but they also gain the advantage of dramatically increased solution efficiency. Large-scale network problems can be solved quickly and efficiently using highly specialized solution algorithms that exploit the mathematical structure inherent in network formulations. For example, Glover and Klingman ([6], p. 5) discuss a manpower planning model involving 2294 equations and 450,000 variables that only requires 26 minutes of central processing time to optimize on an IBM 360-65. Problems of this size are computationally infeasible using general purpose linear problemming algorithms such as the CDC-APEX III code or the IBM-MPSX code. As another example, an integer U. S. Air Force pilot training model with 730 equations and 460 zero-one variables was transformed into a mixed integer generalized network and solved in 10 seconds on a CDC 6600. The original integer formulation did not solve in one hour using a state-of-the-art integer programming algorithm.

The Short-Term Funds System Model

A mixed integer generalized network formulation is particularly useful in modeling the short-term funds system. The magnification factors, $p_{ij}$, allow the
analyst to convert cash flows to product flows, to model the cash impact of product or security earnings or costs, and to differentiate various asset configurations. For example, if the "supply" available at node $i$ was denominated in terms of product, $p_{ij}$ could be specified as the sales price per unit so that for every unit of product sold (that is, leaves node i), the sales revenue in dollars generated by the sale would enter node $j$. Thus, product would be converted to dollars by the $p_{ij}$. The availability of upper and lower limits on each flow permits management to impose liquidity requirements, maximum and minimum production and inventory levels, and ending conditions that insure systematic balances at the conclusion of the planning period. Integer variables allow the fixed charge problem to be incorporated where required for realism.

The short-term funds system model outlined in this section is initiated with a reservoir of liquid assets including cash, marketable securities of varying maturities, accounts receivable, and various classes of inventory -- raw materials, work-in-process, and finished goods. These assets are financed by accounts payable, short-term borrowing and long-term financing. Over the planning period, credit is extended by each marketing unit, production is undertaken to meet the anticipated demand, existing financial obligations are settled and the activity level is supported by the cash generated from operations, the initiating liquid assets, and additional short- and long-term credit. Also during the planning period, the exogenously determined magnitude and timing of capital requirements for fixed asset investments, the payment of cash dividends and other cash or financing needs are programmed into the model. Funds are drawn through the system according to the optimal allocation pattern to a liquidity reservoir at the planning horizon.
For ease of presentation, the model is outlined in its three interrelated planning components. In the market planning component (MPC), credit terms are determined and product from various production sites is allocated to regional markets. Units of finished goods tie the MPC to the production planning component (PPC) where production schedules and inventory policies are set. The financial planning component (FPC) -- the only component included explicitly in most models appearing in the finance literature -- is linked to both of the other components and provides the interface with sources and uses of funds external to the system. For optimization, the flows within and between these components are set to maximize net revenues (total revenue minus total cost, where the required return on equity is included explicitly as a cost).

The Marketing Planning Component (MPC)

Market planning is indicated in Figure 2. In this example, market demand is parameterized for a single-product serving two geographical markets over a three-segment planning period. A single product is included to simplify the discussion; however, since finished goods inventories are product specific, it is easy to extend the model to the multiproduct case by including a market planning component for each product line. Units of finished goods are made available by each subsidiary in each time period to satisfy this geographic demand. The finished goods inventory nodes (the six lined nodes in Figure 2) provide the linkage with the production planning component, and the six cash sink nodes (the six shaded nodes in Figure 2) provide the link with the financial planning component.

As shown in Figure 2 there are three different product disposition paths that can be chosen by the model: (1) finished goods inventory can be used to satisfy
Figure 2. THE MARKET PLANNING COMPONENT
demand in the subsidiary's market region, (2) product from one production site can be transferred to another subsidiary for sale in its market, and (3) products produced or available in one period can be carried in finished goods inventory to later periods. On these arcs the $p_{ij}$ are used for dollar-product conversion and to allow for the cash magnification or attenuation as product moves through the marketing cycle. Stochastic upper ($U_{ij}$) and lower ($L_{ij}$) bounds insure that inventory is available in the system, within warehousing constraints, to meet market demand with a given degree of confidence. All marketing costs, including transportation and spoilage where relevant, and point of sale profits are carried to the objective function through the $c_{ij}$ parameters.

The other major element in the market planning component is the collection of accounts receivable. Since all of the buyers of the firm's product do not pay cash, take discounts or even pay on time, an aging schedule (which approximate the relationship between sales and collections for a given set of credit terms) is specified for each subsidiary. The anticipated demand in each market segment is then broken down in proportion to the aging schedule and apportioned via upper bounds to the various accounts receivable arcs. There is, of course, a separate aging schedule associated with each set of possible credit terms. The model would be run with each aging schedule in order to select the most desirable alternative. This "simulation" approach to decision modeling is based on the availability of highly efficient model structures and solution speeds outlined above.

The master cash sink insures that all cash is drawn through the system. Bounds on the arcs entering the cash sink insure that desired planning horizon conditions are maintained.
The Production Planning Component (PPC)

The production planning component (PPC) serves the demand generated in the market planning component. It should be noted that the lined finished goods inventory nodes in Figure 3 that provide the connection with market planning are the same nodes as one set (market area A or B) of the finished goods nodes in Figure 2. The three cross-hatched cash source nodes are connecting links with the financial planning component.

In addition to finished goods inventory, accounts payable, raw materials and work-in-process inventories, production levels are modeled as a part of the production planning component. Starting with the purchase of raw materials, the fixed ordering cost is incorporated into the model by including a 0-1 integer NETFORM representation for each trade supplier in each period. The fixed cost allocator node signals the placement of an order. This node has a supply of +1 to be assigned as an integer value to either the trade suppliers if an order is placed (in which case the fixed ordering is incurred through the \( c_{ij} \), and \( p_{ij} \) designates the number of units available from that supplier) or to the accounts payable node (in which case it offsets the accounts payable requirement). If the demand at the accounts payable node is not satisfied by the fixed cost allocation node, the demand is actuated and the model will select the optimal payment pattern. Note that a dummy arc from the trade supplier node reduces demand for payment for all goods not actually purchased. This ability to include integer variables in network structures greatly enhances the flexibility of these models and increases the range of applications with which they can deal without requiring a major increase in solution time (as would be the case with a mixed integer programming code).
Figure 3. THE PRODUCTION PLANNING COMPONENT
The treatment of raw materials inventory (and finished goods inventory) is an example of using multiple capacitated arcs to represent an increasing cost function. Carrying costs are modeled as an increasing step function of the number of units carried in inventory. This step function is included as multiple capacitated arcs between the relevant raw materials inventory nodes. In a cost minimization model, the least cost arc will be used to capacity before the higher cost arc has any flow.

Thus, adding the production subnetwork to the marketing subnetwork allows for the full range of operating policies associated with working capital management, including inventory, production, distribution, receivables, and payables. With the incorporation of multiple arcs where relevant, the model explicitly considers the trade-off between ordering costs, production costs (regular or overtime), shipping costs and carrying costs in both raw materials and finished goods inventories, determining the optimum operating policy as a function of cost and market demand.

The Financial Planning Component (FPC)

The link between the internal cash sinks of the market planning component and the cash source nodes of the production system, as well as the critical interface with external sources and uses of funds, is provided by the financial planning component through the cash source and sink pair for each operating unit in each period. The graphical representation in Figure 4 presents the linkages for a single production-marketing subsidiary. The production and marketing functions are combined in one subsidiary for ease in presentation. For applications where these functions are in separate subsidiaries or locations, extra nodes and arcs
inserted between the source and sink pair would allow for the additional cash and cost impact of this separation.

The market planning component provides cash to each period's cash sink. This cash is transferred to the cash source node where it is available for the production component or can be transferred to the next period cash sink directly or through investments in marketable securities. Additional funds are available from beginning cash balances, bank credit and long-term capital, as well as the collection of outstanding accounts receivable.

The arc connecting the period k cash source with the period k+1 cash sink node represents subsidiary cash balances maintained, normally the minimum required compensating balance specified as a lower bound. An intrasubsidiary cash transfer pool is included for each period so that excess cash in one production and marketing subsidiary can be sent to or utilized by another subsidiary instead of being retained and invested in marketable securities. Depending upon the firm, a more rigid structure may be desired to represent these transfers as firm intersubsidiary loans.

Cash not used in production or required for paying creditors or other expenses can be invested in marketable securities. The marketable security investment node collects these excess funds for investment, perhaps adjusted for transactions costs by specifying $p_{ij}$ less than one, and determines the maturity of the short-term portfolio. Return on investment is included on the maturity arcs with the cash impact imbedded in the $p_{ij}$ and the revenue impact transferred to the objective function by the $c_{ij}$.

Two sources of short-term credit are included in Figure 4 for each period. The arcs connecting the bank nodes to the loan collector node have upper bounds
Figure 4. THE FINANCIAL PLANNING COMPONENT
representing the maximum borrowing from those particular sources, and an upper bound on the arc from the loan collector to the cash sink node restricts the maximum borrowing from all sources. We have included the option of borrowing short-term credit for either one or two periods, subject, of course, to the maximum borrowing constraints on each bank supply arc. The attenuation factor, $p_{ij}$, on each repayment arc equals the appropriate present value interest factor (PVIF) to include the cash impact, and the objective function cost, $c_{ij}$, equals the same PVIF times the interest rate, $r$, to incorporate the cost impact.

Two other features are shown in Figure 4 that managers may want to include to gauge the impact on working capital planning: the fixed charge sinks and the long-term capital system. The purpose of the fixed charge sinks is to capture all net requirements for cash not included explicitly elsewhere within the model. These requirements include cash dividends, expected tax payments, sinking fund requirements for existing obligations, payments for fixed assets, etc. Both the magnitude and timing of these elements can be included as data, and the working capital planning ramifications -- securing the cash to meet the obligations -- are determined by the model.

The other feature, sources of long-term capital, can be input as data, or, with the inclusion of a 0-1 integer selection network similar to the one described in [3], the model can determine the optimum magnitude and timing of bond and/or stock issues. By using multiple capacitiated arcs between the short-term loan collector nodes and the cash sink nodes, and defining the increasing cost function as management's subjectively derived required risk premium as the amount of short-term debt gets larger and larger, a crude approximation of the optimal debt maturity schedule can be obtained when the selection network is utilized.
Implementing the Model

As indicated by the model overview, a host of interrelationships act in concert to shape the optimal allocation of short-term funds. Market demand is the main driver of the model as it is by far the greatest producer of revenues. Since demand may be greater than or less than the production capacity at any given site, a procedure is included for transshipping products from one market area to another. The solution algorithm determines the ordering schedule for raw materials, where and how much of the raw materials will be carried in inventory, a production schedule for each production unit, where and how much finished goods will be carried in inventory, where and when the finished goods should be sold, and the timing and amount of cash inflows into the firm from sales.

Since the production must be financed, the model determines simultaneously the optimal financing pattern from internal cash flow, trade credit, short-term credit arrangements, and long-term capital. Any excess cash in a period can also be invested in short-term marketable securities the maturity structure of which is determined by the model.

Since many of the stock and flow variables are subject to uncertainty, there are two methods by which risk can be captured by the model. If the marginal distributions of the risky elements are known or can be estimated, they can readily be incorporated in a chance-constrained programming format. (For an excellent justification for the use of chance-constrained programming in this context see [10].) Even if the stochastic variable is imbedded in the constraint matrix in the form of a magnification factor, $p_{ij}$, instead of the more traditional case where it is restricted to the right hand side, chance constraints can be formed using the approach of [2].
The other method by which risk can be approached is through scenario analysis. Since the interactive NETFORM planning system is computationally very efficient, a series of solutions can be generated in which the variable(s) under examination can be varied throughout some reasonable range. In this way, the sensitivity of the optimum solution (translated, of course, to a financial statement presentation) can be ascertained, and some probability attached to each of the scenarios so investigated. Many large firms of which we are familiar use this scenario approach to risk analysis, although it is generally conducted with a "simulation" model rather than with an interactive optimization planning system.

We believe that the scenario approach may, at this time, be operationally more meaningful in a firm than the standard risk analysis through chance constraints because of the difficulties in accessing meaningful distributions of a host of uncertain variables (see [14], [15] for a discussion of assessment problems.)

This model, in which the relationships of all major variables impacting on working capital policy are included, is particularly suited to this type of analysis and can illuminate the sensitivity of the financial statements of the firm to environmental changes. Managerial reaction to the changes and contingency planning can best be planned when this type of information is available.

References


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