Final Report on AFOSR GRANT
AFOSR-76-2997
July 1977
"UTILIZATION OF SOURCE IMPEDANCE TO DECREASE THE
WEIGHT OF D.C. POWER SUPPLY FILTERS"

by

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"UTILIZATION OF SOURCE IMPEDANCE TO DECREASE THE WEIGHT OF D.C. POWER SUPPLY FILTERS"

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### REPORT DOCUMENTATION PAGE

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<th>1. REPORT NUMBER</th>
<th>AFOSR-TR-77-0985</th>
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<td>2. GOVT ACCESSION NO.</td>
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<td>3. RECIPIENT'S CATALOG NUMBER</td>
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<th>4. TITLE (and Subtitle)</th>
<th>UTILIZATION OF SOURCE IMPEDANCE TO DECREASE THE WEIGHT OF D.C. POWER SUPPLY FILTERS</th>
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<td>5. TYPE OF REPORT &amp; PERIOD COVERED</td>
<td>FINAL REPORT</td>
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<td>6. PERFORMING ORG. REPORT NUMBER</td>
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<tr>
<th>7. AUTHOR(s)</th>
<th>Stuart, Thomas A.</th>
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<td>8. CONTRACT OR GRANT NUMBER(s)</td>
<td>AFOSR-76-2997</td>
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<tr>
<th>9. PERFORMING ORGANIZATION NAME AND ADDRESS</th>
<th>University of Toledo</th>
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<tr>
<td></td>
<td>Department of Electrical Engineering</td>
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<td>Toledo, Ohio 43606</td>
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<tr>
<th>10. PROGRAM ELEMENT, PROJECT, TASK AREA &amp; WORK UNIT NUMBERS</th>
<th>61102F</th>
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<td>Bldg 410, Bolling AFB, DC 20332</td>
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<tr>
<th>12. REPORT DATE</th>
<th>18 July 1977</th>
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<tr>
<td>13. NUMBER OF PAGES</td>
<td>153</td>
</tr>
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<th>14. MONITORING AGENCY NAME &amp; ADDRESS (IF different from Controlling Office)</th>
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<tr>
<td>15. SECURITY CLASS. (of this report)</td>
<td>UNCLASS</td>
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</table>

| 16. DISTRIBUTION STATEMENT (of this Report) | Approved for public release; distribution unlimited. |

| 17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, IF different from Report) | |

| 18. SUPPLEMENTARY NOTES | |

| 19. KEY WORDS (Continue on reverse side IF necessary and identify by block number) | Power Supply  Superconducting  
Filter  Alternator  
Source Impedance  |

| 20. ABSTRACT (Continue on reverse side IF necessary and identify by block number) | This report presents a study of how the source impedance can be used to reduce the weight of the output filter of a rectified superconducting alternator power supply. |
**Title:** Utilization of Source Impedance to Decrease the Weight of D.C. Power Supply Filters.

**Abstract:**
This report presents a study of how the source impedance can be used to reduce the weight of the output filter of a rectified superconducting alternator power supply.
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SUMMARY

This report presents an analysis of a DC power supply consisting of a superconducting alternator, a rectifier bridge, and an LC output filter. The main purpose of this research was to determine if changes in the size of the alternator inductances would allow the use of a smaller filter. To perform this study it was necessary to examine the behavior of the filter and to determine how its operation was affected by the alternator parameters.

Basically, the filter performs two functions:
1. It attenuates the output ripple voltage.
2. It limits the initial fault current when a short circuit occurs at the load.

Both of these functions also depend upon the values of the alternator inductances.

Since the first function refers to the steady state behavior, it was necessary to develop a model for this operating mode. This was done first for a system with an uncontrolled rectifier bridge and then these results were extended to a controlled rectifier bridge system. The second function is a transient phenomenon, so it was also necessary to develop a second model to describe the transient behavior.

Once the system models were complete, a study was performed where the unfiltered ripple voltage was calculated for various values of the alternator inductances. It was found that under certain conditions the ripple voltage can be decreased by increasing the armature self
inductance \( L_a \). A program was then written which calculated the weight of the LC filter that was required for a given set of specifications and alternator parameters. This program indicated that an increase in \( L_a \) could decrease the required filter weight by as much as 22%.

Other investigations included a sensitivity analysis of the alternator inductances and the design and testing of a phase controlled voltage regulator with current overload protection.
1. ALTERNATOR WITH UNCONTROLLED RECTIFIER BRIDGE

1.1 Introduction

Recent advancements in superconducting alternators have created a strong interest in using these machines for airborne electric power supplies. The predominant advantage of this power source is its relatively low weight for applications requiring multi-megawatt outputs at several kV. This low weight characteristic occurs because of two factors:

1. Even with the required cryogenic equipment, the superconducting alternator system weighs much less than a conventional alternator.

2. The higher armature voltages of the superconducting machine may eliminate the need for heavy output inverters and transformers.

These attributes are discussed in further detail in such references as [1] - [12], and a very recent example of such a machine is described by McCabria, et al. in [13,14]. This particular machine develops 10MVA at 5 kV and weighs approximately 1,000 pounds (alternator weight only). This same reference also includes projected estimates for a 25MVA machine weighing between 1882 and 2160 pounds, depending on rated output voltage (again, these figures only include the weight of the alternator).

The potential advantages of superconducting alternators have prompted extensive research in this area, most of which has concentrated on ac loads (again see [1] - [14]). Applications for these machines also exist in high power dc systems however, where the alternator is connected to a rectifier bridge followed by a large filter choke. This mode of operation
has been studied in detail for conventional alternators, (see [15] through [21]), but until now no such analysis has been presented for the superconducting machine.

One of the more rigorous analyses of conventional rectified alternators is that presented by Franklin [17,18] for salient pole machines. By assuming constant flux linkages for the rotor windings, this study derives a set of nonlinear equations in terms of the electrical variables of interest. Certain approximations then lead to a linearization involving a constant K factor, and an explicit solution is obtained. The advantages of this approach are readily apparent since it provides a closed form expression for each of the variables, once the proper K factor has been found. The determination of K is somewhat distracting however, since it is load dependent and requires the use of numerical methods. In the following section it will be shown that this K factor can actually be eliminated from the final solution if a Newton-Raphson algorithm is used. This new approach appears to have certain advantages since it is somewhat less complicated and does not depend on any linearization factors.

The essence of the work presented here is:

1. Franklin's basic analysis methods are extended to the superconducting machine.

2. The dependence on the previously mentioned K factor is eliminated. As stated above, this is accomplished by using a Newton-Raphson algorithm where a K=1 is used only to find a starting point.

3. A numerical example predicting the rectified characteristics of the machine described by McCabria, et al. in [13,14] is included.
The overall intent is to provide an analytical model of the steady state behavior of the superconducting alternator with a rectified output. This analysis is regarded as a preliminary step to the eventual design and testing of these machines for D.C. loads.

1.2 Steady State Alternator-Rectifier Model

The armature of the superconducting machine is assumed to be $Y$ connected as indicated for the basic two pole machine in Figure 1. The $d$ and $q$ windings shown in this figure are equivalent windings that account for the effect of the cylindrical damper shield located between the rotor and the stator (see [5], [13] or [14] for example). Output voltage and current waveforms are shown in Figure 2, where the indicated $\theta$ corresponds to Figure 1. Formulation of this problem proceeds in much the same manner as in [17,18], but there are some important differences in the machine parameters. It also should be noted that the method of solution is quite different from these earlier references, and certain equations are employed in a different manner.

The following approximations are utilized:

1. All winding and diode resistances are quite small and can be ignored.
2. All diode voltage drops are negligible.
3. The load inductance, $L_L$, is sufficiently large to maintain a constant $L_L$, i.e., the effect of load current variations is ignored.
4. Each armature winding is assumed to have a perfect sinusoidal distribution about the stator.
Figure 1. Equivalent Circuit for the Superconducting Alternator with Uncontrolled Rectifier Bridge.
Figure 2. Output Voltage and Armature Currents with Uncontrolled Rectifier Bridge.
5. The effect of the damper shield is modelled by equivalent direct axis and quadrature axis windings on the rotor (d and q).

6. The rotor speed is assumed constant.

Since the superconducting alternator is an air core machine there are no saturation or saliency effects.

Although the line to line voltages in Figure 2 are shown as perfect sinusoids, it should be noted that they are actually distorted somewhat. Thus commutation actually starts at some $\beta > 90^\circ$ and not at $\theta = \beta = 90^\circ$ as indicated in the figure. Another interesting characteristic is the fact that the stator MMF is constant in magnitude and direction during the conduction interval and abruptly shifts to a new direction during the commutation interval. This phenomenon is termed "MMF jump" as is described further in such references as Stepina [16] and Franklin [17].

The waveforms shown in Figure 2 indicate that $\mu < \pi/3$. Franklin points out that it is also possible to reach a mode where $\mu = \frac{\pi}{3}$. Although a large number of simulations were conducted in this present study, the $\mu = \pi/3$ mode was never reached for the machine used in the numerical example. The conclusion drawn was that this appeared to be an unlikely operating mode for this application, so it was not included in the analysis.

1.3 Steady State Equations

The primary goal of this section is to derive five equations that are expressed in terms of the following variables:
\[ \beta = \text{angle at which commutation starts} \]
\[ \mu = \text{commutation angle} \]
\[ I_f = \text{average field current} \]
\[ W = \text{variable defined by equation (20.)} \]
\[ V = \text{variable defined by equation (21.)} \]

These equations turn out to be nonlinear with respect to \( \beta \) and \( \mu \), but they can be solved by some numerical method such as the Newton-Raphson algorithm. Once these variables have been found it is possible to determine the time dependent expressions for the output voltage and the current in each winding.

It is assumed that the field current consists of the constant component, \( I_f \), and a time varying component, \( i_f \),

\[ i_{f(tot)} = I_f + i_f \]  \hspace{1cm} (1.)

The winding currents during conduction and commutation are indicated as follows,

Conduction (Interval 1-2 in Figure 2), \( \beta + \mu - \pi/3 \leq \theta < \beta \)

\[
\begin{bmatrix}
i_a \\
i_b \\
i_c \\
i_{f(tot)} \\
i_d \\
i_q
\end{bmatrix} =
\begin{bmatrix}
I_L \\
-I_L \\
0 \\
(I_f + i_f) \\
i_d \\
i_q
\end{bmatrix}
\]  \hspace{1cm} (2.)
Commutation (Interval 2-3 in Figure 2), $\beta \leq \theta < \beta + \mu$

$$\begin{bmatrix}
I_L \\
(-I_L + i_k) \\
-i_k \\
(I_f + i_f) \\
id \\
i_q
\end{bmatrix}$$

(3.)

The flux linkages are given by the following expression,

$$\lambda = \begin{bmatrix}
\lambda_a \\
\lambda_b \\
\lambda_c \\
\lambda_f \\
\lambda_d \\
\lambda_q
\end{bmatrix} = [L] i$$

(4.)
\[
\begin{bmatrix}
L_a & -M_a & -M_a & M_d \cos \theta & -M_d \sin \theta \\
-M_a & L_a & -M_a & M_d \cos \left( \theta - \frac{2\pi}{3} \right) & -M_d \sin \left( \theta - \frac{2\pi}{3} \right) \\
-M_a & -M_a & L_a & M_d \cos \left( \theta - \frac{4\pi}{3} \right) & -M_d \sin \left( \theta - \frac{4\pi}{3} \right) \\
M_d \cos \theta & M_d \cos \left( \theta - \frac{2\pi}{3} \right) & M_d \cos \left( \theta - \frac{4\pi}{3} \right) & L_d & 0 \\
-M_d \sin \theta & -M_d \sin \left( \theta - \frac{2\pi}{3} \right) & -M_d \sin \left( \theta - \frac{4\pi}{3} \right) & 0 & L_d \\
\end{bmatrix}
\]

where, \([L]\) represents the inductance matrix for a superconducting alternator, as given by Kirtley in [5,6]. Note that \(L_d\) and \(M_d\) are the same for the equivalent direct and quadrature axis damper "winding" (as pointed out in the above references, these equivalents account for the damper shield and are not actual windings).

The d and q windings are short circuited, implying that \(\lambda_d\) and \(\lambda_q\) are constant since the winding resistances are assumed to be negligible. Likewise, \(\lambda_f\) may be assumed constant if \(I_f + i_f\) is supplied by a low impedance source. Therefore,
\[ v = \begin{bmatrix} v_a \\ v_b \\ v_c \\ v_f \\ v_d \\ v_q \end{bmatrix} = -\frac{d\lambda}{dt} \begin{bmatrix} 0 \\ -\omega \frac{d\lambda}{d\theta} \end{bmatrix} \] (6.)

Figure 2 indicates that,

\[ v_o = v_{ab} = -\omega \frac{d}{d\theta} [\lambda_a - \lambda_b], \beta + \mu - \pi/3 < \omega t < \beta + \mu \] (7.)

The average voltage, \( V_L \), at the output of the rectifier bridge is,

\[ V_L = \frac{3}{\pi} \int_{\beta+\mu-\pi/3}^{\beta} v_{012} d\theta + \int_{\beta}^{\beta+\mu} v_{023} d\theta \] (8.)

where \( v_{012} \) and \( v_{023} \) represent the output voltage functions over 1-2 and 2-3 respectively.

\[ V_L = -\frac{3\omega}{\pi} \left[ \int_{\beta+\mu-\pi/3}^{\beta} \frac{d\lambda_a}{d\theta} - \frac{d\lambda_b}{d\theta} \right]_{12} d\theta + \int_{\beta}^{\beta+\mu} \left[ \frac{d\lambda_a}{d\theta} - \frac{d\lambda_b}{d\theta} \right]_{23} d\theta \] (9.)

Figure 2 indicates

\[ v_{012} = v_{023} \quad \theta = \beta \] (10.)

\[ V_L = -\frac{3\omega}{\pi} \left[ (\lambda_a - \lambda_b)_{23} (\beta+\mu) - (\lambda_a - \lambda_b)_{12} (\beta+\mu-\pi/3) \right] \] (11.)
(2.) and (3.) become the same when \( i_k = 0 \), therefore using (3.) and (5.),

\[
\lambda_q = \sqrt{3} I_q M_d \sin(\theta + \pi/6) + \sqrt{3} i_k M_d \cos \theta + i L_d \tag{12.}
\]

where \( i_k = 0 \) over the conduction interval.

\( \lambda_q \) is assumed to be constant over 1-3. At \( \theta = \beta + \mu - \pi/3 \) we have \( i_q = i_{q0} \).

\( i_k = 0 \)

\[
\lambda_q (\beta + \mu - \pi/3) = -\sqrt{3} I_q M_d \sin(\beta + \mu - \pi/6) + i_{q0} L_d \tag{13.}
\]

\[
i_q = -\sqrt{3} I_q K \left[ \sin(\beta + \mu - \pi/6) - \sin(\theta + \pi/6) \right] - \sqrt{3} i_k q \cos \theta i_{q0} \tag{14.}
\]

where \( K_q = M_d / L_d \tag{15.} \)

Using a similar procedure, expressions for \( \lambda_f \) and \( \lambda_d \) may be determined.

The two simultaneous equations for \( \lambda_f \) and \( \lambda_d \) may then be solved for \( i_f \) and \( i_d \).

\[
i_d = \sqrt{3} L_d K [\cos(\beta + \mu - \pi/6) - \cos(\theta + \pi/6)] - \sqrt{3} i_k d \sin \theta + i_{d0} \tag{16.}
\]

\[
i_f = \sqrt{3} L_f K [\cos(\beta + \mu - \pi/6) - \cos(\theta + \pi/6)] - \sqrt{3} i_k f \sin \theta + i_{f0} \tag{17.}
\]

where \( K_f = \frac{M_f L_d - M_d M_f d}{L_f L_d - (M_f d)^2} \), \( K_d = \frac{M_d L_f - M_f M_d d}{L_f L_d - (M_f d)^2} \tag{18.} \)

As pointed out by Shilling [15], the rotor currents are periodic with respect to the 6th harmonic; therefore,

\[
\int_{\beta + \mu}^{\beta + \mu - \pi/3} i_f d\theta = \int_{\beta + \mu - \pi/3}^{\beta + \mu - \pi/3} i_d d\theta = \int_{\beta + \mu}^{\beta + \mu - \pi/3} i_q d\theta = 0 \tag{19.}
\]

Since \( i_k = 0 \) for \( \beta + \mu - \frac{\pi}{3} < \theta \leq \beta \), we may define the following constants

\[
W = \int_{\beta}^{\beta + \mu} i_k \sin \theta d\theta \tag{20.}
\]
Integrating (14.), (16.) and (17.) over $\beta+\mu-\frac{\pi}{3} \leq \theta \leq \beta+\mu$ and solving for $i_{q_0}$, $i_{d_0}$ and $i_{f_0}$ produces,

\[ i_{q_0} = \sqrt{3} I_L K_q \left[ \sin (\beta+\mu-\pi/6) - \frac{3}{\pi} \sin (\beta+\mu) \right] + \frac{3\sqrt{3}}{\pi} K_q V \]  
\[ (22.) \]

\[ i_{d_0} = -\sqrt{3} I_L K_d \left[ \cos (\beta+\mu-\pi/6) - \frac{3}{\pi} \cos (\beta+\mu) \right] + \frac{3\sqrt{3}}{\pi} K_d W \]  
\[ (23.) \]

\[ i_{f_0} = i_{d_0} \left( \frac{K_f}{K_d} \right) \]  
\[ (24.) \]

Therefore, substituting into (14.), (16.) and (17.),

\[ i_q = \sqrt{3} K_q \left[ \frac{3}{\pi} \left[ V - I_L \sin (\beta+\mu) \right] + [I_L \sin (\theta+\pi/6) - i_k \cos (\theta)] \right] \]  
\[ (25.) \]

\[ i_d = \sqrt{3} K_d \left[ \frac{3}{\pi} \left[ W + I_L \cos (\beta+\mu) \right] - [I_L \cos (\theta+\pi/6) + i_k \sin (\theta)] \right] \]  
\[ (26.) \]

\[ i_f = i_d \left( \frac{K_f}{K_d} \right) \]  
\[ (27.) \]

From (4.), we have,

\[ \lambda_a = I_L \left( L_a + M_a \right) + \left[ (I_f + i_f) M_f + i_d M_d \right] \cos \theta - i q_d \sin \theta \]  
\[ (28.) \]

\[ \lambda_b = -I_L \left( L_a + M_a \right) + i_k \left( L_a + M_a \right) + \left[ (I_f + i_f) M_f + i_d M_d \right] \cos \left( \theta - \frac{2\pi}{3} \right) - i q_d \sin \left( \theta - \frac{2\pi}{3} \right) \]  
\[ (29.) \]

\[ \lambda_c = -i_k \left( L_a + M_a \right) + \left[ (I_f + i_f) M_f + i_d M_d \right] \cos \left( \theta - \frac{4\pi}{3} \right) - i q_d \sin \left( \theta - \frac{4\pi}{3} \right) \]  
\[ (30.) \]

Using the results of (25.) - (27.),

\[ (I_f + i_f) M_f + i_d M_d = I_f M_f + \frac{3\sqrt{3}}{\pi} M_o W - \sqrt{3} I_L M_o \left[ \cos (\theta+\pi/6) \right] \]  

\[ - \frac{3}{\pi} \cos (\beta+\mu) \]  
\[ - \sqrt{3} i_k M_o \sin \theta \]  
\[ (31.) \]
\[ i_{k} = \frac{3\sqrt{3}}{\pi} M_{\infty} V \sqrt{3} I_{L} M_{\infty} \left[ \sin(\theta+\pi/6) - \frac{3}{\pi} \sin(\beta+\omega) \right] \]
\[ - \sqrt{3} i_{K_{\infty}} \cos \theta \] (32.)

where, \( M_{\infty} = (K_{f} M_{f} + K_{d} M_{d}) = \frac{M_{f}^{2} L_{d} + M_{d}^{2} L_{f} - 2 M_{d} M_{f} M_{fd}}{L_{d} L_{f} - (M_{fd})^{2}} \) (33.)

\[ M_{\infty} = K_{f} M_{f} d = \frac{M_{d}^{2}}{L_{d}} \] (34.)

(11.) can be used to find \( V_{L} \), by noting that

\[ i_{k} = I_{L} \cos \theta = \beta + \omega \]
\[ i_{k} = 0 \cos \theta = \beta + \omega - \pi/3, \]

\[ V_{L} = \frac{3\omega}{\pi} \left[ \frac{3}{4} I_{L} \Lambda_{o} + \frac{3}{2} I_{L} \Lambda_{d} \cos(2\beta+2\omega+\pi/3) + \frac{\sqrt{3}}{\pi} I_{f} M_{f} \sin(\beta+\omega) \right. \]
\[ + \frac{9}{2\pi} I_{L} \Lambda_{d} \sin(2\beta+2\omega) + \frac{9}{\pi} [M_{o} W \sin(\beta+\omega) + M_{\infty} V \cos(\beta+\omega)] \] (35.)

where, \( \Lambda_{f} = (M_{o} + M_{\infty}) \), \( \Lambda_{d} = (M_{o} - M_{\infty}) \)
\[ \Lambda_{o} = \frac{4}{3} (L_{a} + M_{a}) - \Lambda_{f} \] (36.)

The current \( i_{k} \) exists only during the commutation period where the "b" and "c" phases are shorted together, i.e.,

\[ v_{bc} = 0, \beta - \theta < \beta + \omega \]

\[ \therefore \left( \lambda_{b} - \lambda_{c} \right) = \text{constant}, \beta - \theta < \beta + \omega \]

For \( \theta = \omega, i_{k} = 0 \), therefore setting \( \left( \lambda_{b} - \lambda_{c} \right)_{\theta} = \left( \lambda_{b} - \lambda_{c} \right)_{\beta} \) one obtains,

\[ i_{k} = \frac{1}{(\Lambda_{o} + \Lambda_{d} \cos(2\theta))} \left[ \frac{2MfI_{f}}{\sqrt{3}} (\sin\beta - \sin\theta) + \frac{6}{\pi} M_{o} W (\sin\beta - \sin\theta) \right] \]
\[ + \frac{\beta}{3} \left( \frac{\Lambda_L}{\pi} \right)^2 \left[ \sin(28+\mu) - \sin(8+\mu) \right] \]

\[ - \frac{3}{\pi} \left( \frac{\Lambda_f}{\pi} \right)^2 \left[ \sin(\theta - \phi - \psi) \right] - I_L \Lambda_d \left[ \cos(28 - \pi/3) - \cos(28 - \pi/3) \right] \]

(37.)

Again utilizing,

\[ v_{bc} = 0, \quad \beta \leq \theta \leq \beta + \nu \]

we have for \( i_k = 0 \) \( @ \theta = \beta, \)

\[ (v_b - v_c)_b = -\omega \frac{d}{d\theta} (\lambda - \lambda_c)_b = 0 \]

(38.)

which leads to,

\[ -\Lambda_d \sin(2\beta - \pi/3) = \frac{I_f}{\sqrt{3}} M_f \cos \beta + \frac{3}{\pi I_L} (M W \cos \beta - M_{\infty} V \sin \beta) \]

\[ + \frac{3}{2\pi} \left( \Lambda_d \cos(2\beta + \mu) + \Lambda_f \cos \mu \right) \]

(39.)

Utilizing \( i_k = I_L \) \( @ \theta = \beta + \nu \) in (37.) leads to,

\[ \Delta_o + 2\Lambda_d \cos(2\beta + \mu) \cos(\mu/3) = -\frac{\mu}{\sqrt{3}} \frac{I_f}{I_L} M_f \left[ \cos(\beta + \frac{\mu}{2}) \sin(\frac{\mu}{2}) \right] \]

\[ + \frac{6}{\pi I_L} \left[ M W (\sin \beta - \sin(\beta + \mu)) + M_{\infty} V (\cos \beta - \cos(\beta + \mu)) \right] \]

\[ - \frac{3}{\pi} \left[ 2\Lambda_d \cos(2\beta + \frac{3\mu}{2}) \sin(\frac{\mu}{2}) + \Lambda_f \sin(\mu) \right] \]

(40.)

One could substitute (37.) into (20.) and (21.) and integrate to find two more equations, which along with (35.), (39.) and (40.) would yield five nonlinear equations for the five unknowns, \( I_f, \beta, \mu, V \) and \( W \).

This process is simplified considerably by use of the following approximation,

\[ \Lambda_d = 0, \quad (i.e., M_o = M_{\infty}) \]

(41.)
Equations (35.), (37.), (39.) and (40.) indicate that \( \Delta_d \) always appears in conjunction with \( \Delta_f \) or \( \Delta_o \). Therefore, (41.) is acceptable if \( \Delta_d \) is small in comparison to \( \Delta_f \) and \( \Delta_o \).

The superconducting alternator considered in this study (the same machine described by McCabria, et al., in [13,14] has the following parameters\(^1\):

\[
\begin{align*}
L_f &= 1.2 \text{ H.} \\
M_f &= 7.9 \times 10^{-3} \text{ H.} \\
L_d &= 8.2 \times 10^{-8} \text{ H.} \\
M_{fd} &= 1.9 \times 10^{-4} \text{ H.} \\
L_a &= 3.0 \times 10^{-4} \text{ H.} \\
M_d &= 3.8 \times 10^{-6} \text{ H.} \\
M_a &= 1.5 \times 10^{-4} \text{ H.}
\end{align*}
\]

\( \Delta_d = 0.01 \times 10^{-4} \text{ H.}, \Delta_f = 3.5 \times 10^{-4} \text{ H.}, \Delta_o = 2.5 \times 10^{-4} \text{ H.} \) (43.)

Therefore the approximation given by (41.) appears to be acceptable, at least for this particular example.

Using (41.), equations (20.), (21.), (35.), (39.) and (40.) reduce to,

\[
\begin{align*}
0 &= \Delta_0 \omega + A (\cos (\beta + \mu) - \cos \beta) + \frac{B}{4} (2\mu \sin(2\beta + 2\mu) + \sin 2\beta) \\
&+ \frac{C}{2} \sin^2(\beta + \mu) - \sin^2 \beta \\
0 &= \Delta_0 V + A(\sin \beta - \sin(\beta + \mu)) + \frac{B}{4} \sin^2(\beta + \mu) - \sin^2 \beta \\
&+ \frac{C}{2} (2\mu \sin(2\beta + 2\mu) - \sin 2\beta) \\
0 &= -\frac{3\omega}{\pi} \left[ \frac{3}{4} L_o \Delta_0 + \sqrt{3} I_f M_f \sin (\beta + \mu) \\
&+ \frac{9M}{\pi} \right] \left( W \sin (\beta + \mu) + V \cos (\mu + \mu) \right)
\end{align*}
\]

\(^1\) These parameters were supplied by H. Southall of the U.S. Air Force Aero Propulsion Laboratory.
\[ 0 = \frac{I_f}{\sqrt{3} I_L} M \cos \beta + \frac{3 M}{\pi I_L} \left( W \cos \beta - V \sin \beta \right) + \frac{3}{\pi} M_{\infty} \cos \omega \]  

\[ 0 = -\frac{4}{\sqrt{3}} \frac{I_f}{I_L} M \cos \left( \beta + \frac{\omega}{2} \right) \sin \left( \frac{\omega}{2} \right) - \frac{6}{\pi} \frac{M_{\infty}}{I_L} \sin \omega + \frac{6}{\pi} \frac{M_{\infty}}{I_L} \left[ W \left( \sin \beta - \sin \left( \beta + \omega \right) \right) + V \left( \cos \beta - \cos \left( \beta + \omega \right) \right) \right] \]  

where

\[ A = \frac{2}{\sqrt{3}} \frac{I_f}{I_L} M \cos \beta + \frac{6}{\pi} \frac{M_{\infty}}{I_L} \left( W \sin \beta + V \cos \beta - I_L \sin \omega \right) \]  

\[ B = \frac{2}{\sqrt{3}} \frac{I_f}{I_L} M + \frac{6}{\pi} \frac{M_{\infty}}{I_L} \left( W + I_L \cos \left( \beta + \omega \right) \right) \]  

\[ C = \frac{6}{\pi} \frac{M_{\infty}}{I_L} \left( V - I_L \sin \left( \beta + \omega \right) \right) \]

(44.) - (48.) provide five nonlinear equations which are functions of the variables \( I_f, \beta, \mu, V \) and \( W \). Actually, these equations are linear with respect to \( I_f, V \) and \( W \) so it would be possible to eliminate these variables and have a set of two equations which are functions of \( \beta \) and \( \mu \). The equations involved in this reduction are quite cumbersome however, so one might as well work directly with (44.) - (48.).

1.4 Solution for \( I_f, \beta, \mu, V \) and \( W \)

Rewriting the variables and equations in matrix form,

\[
X = \begin{bmatrix}
\beta \\
\mu \\
I_f \\
V \\
W
\end{bmatrix} \quad (52.)
\]
Jacobian matrix is 
\[ f(x) \approx \text{R.H.S. of} \]

\[
F(x) = \frac{\partial f(x)}{\partial x}
\]

(54.)

(52.) - (54.) can be used to form the standard Newton-Raphson equation,

\[ F(x_o) (x - x_o) = f(x) - f(x_o) \]

(55.)

where \( x_o \) is some initial starting point which must be reasonably close to the desired \( x \). In this case \( f(x) = 0 \), so we have,

\[ F(x_o) (x - x_o) = -f(x_o) \]

(56.)

It remains to find a satisfactory value for \( x_o \). This is accomplished by making use of the linearization suggested by Franklin$^1$,

\[ i_k = (\theta - \beta) \frac{I_l}{\mu} \]

(57.)

Substituting (57.) into (20.) and (21.) gives the result,

\[ W = -I_l \cos (\beta + \mu) + \frac{2 I_l \sin(\mu/2) \cos(\beta + \mu/2)}{\mu} \]

(58.)

---

$^1$ Actually, Franklin uses a "K factor" as mentioned in the Introduction, where 0.5 \( \leq K \leq 0.9 \). This produces the approximation, \( i_k = K(\theta - \beta) \frac{I_l}{\mu} \). Since (57.) is only used to find a starting point for the Newton-Raphson algorithm, \( K \) is not critical, and \( K = 1.0 \) is used.
After substituting (57.) - (59.) into (35.), (39.) and (40.) and performing some rather laborious calculations, one obtains,

\[ \mu = \cos^{-1} \left[ \frac{4\pi V_L - 3\omega I_{\Delta o}}{4\pi V_L + 3\omega I_{\Delta o}} \right] \]  
(60.)

\[ \beta = \tan^{-1} \left[ \frac{\pi A_o + 12 M_{\infty} (\sin \frac{\mu}{2}) \left(1 - \cos \mu \right)}{12 M_{\infty} \sin \left(\frac{\mu}{2}\right) \cos \left(\frac{\mu}{2}\right) \left(1 - \cos \mu \right)} \right] \]  
(61.)

\[ I_f = \frac{1}{\sqrt{3} M_f \sin (\beta + \mu)} \left[ \frac{\pi V_L}{3\omega} - \frac{3}{4} I_{\Delta o} - \frac{18 I_{\Delta o}}{\pi \mu} \left(\sin \frac{\mu}{2}\right)^2 \right] \]  
(62.)

(58.) - (62.) provide a value of \( \alpha_0 \) which produces convergence within three or four iterations, depending on the convergence tolerance.

1.5 Numerical Results

This example is based on the same 4 pole, 400 Hz, 10 MVA/5kV superconducting alternator described by McCabria, et al. in [13,14]. When operating into a bridge rectifier at full load with a large filter inductance, this system will have the output values (see [24]),

\[ V_L = 6760 \text{ V. dc} \]  
(64.)

\[ I_L = 1420 \text{ A. dc} \]

The following results assume that the system is operating with a closed loop controller, i.e., \( I_f \) is varied to maintain a constant \( V_L \).
The inductance parameters for this machine are indicated in (42.). All data is presented in terms of the actual magnitudes, but a per unit system would serve just as well.

$I_f$ vs. $I_L$:

Figure 3. shows $I_f$ vs. $I_L$, where $I_f$ is varied to maintain a constant $V_L$. As seen from the curve, the variation in $I_f$ is approximately linear up to 200% of the full load value of $I_L$.

$\beta$ and $\mu$ vs. $I_L$:

Figures 4. and 5. indicate the variation in $\beta$ and $\mu$ respectively with respect to $I_L$. Figure 5. indicates that $\mu$ remains less than 60° as mentioned earlier.

Harmonic Content of $v_o$ vs. $I_L$:

One of the more important problems in this type of power system is the weight of the output filter ($L_o$ and $C_o$ as shown in Figure 1.). In order to minimize the combined weight of these components it is necessary to know the harmonic content of $v_o$ under all load conditions. The output voltage is obtained from (7.) by substitution:

Conduction period ($i_k = 0$), $\beta + \mu - \pi/3 < \theta \leq \beta$,

$$v_{012} = \omega (\sqrt{3} I_f M_f + \frac{9}{\pi} M_\infty W) \sin (\theta + \pi/6) + \frac{9\omega}{\pi} M_\infty V \cos (\theta + \pi/6)$$

$$+ \frac{9\omega}{\pi} L M_\infty \sin (\theta + \pi/6 - \beta - \mu)$$

(65.)

Commutation period ($i_k \neq 0$), $\beta < \theta \leq \beta + \mu$,
Figure 3. $I_f$ versus $I_L$ with Uncontrolled Rectifier Bridge.
Figure 4. $\beta$ versus $I_L$ with Uncontrolled Rectifier Bridge.
Figure 5. \( \mu \) versus \( I_L \) with Uncontrolled Rectifier Bridge.
\[ v_{023} = \sum_{n=6}^{\infty} a_n \cos(n\omega t) + \sum_{n=6}^{\infty} b_n \sin(n\omega t) \quad n = 6, 12, 18, \ldots \]
Figure 6. First Three Harmonics of $v_o$ versus $I_L$ with Uncontrolled Rectifier Bridge.
of this winding is beyond the scope of this report, but it is certainly related to the variations in $i_f$. The instantaneous value of $i_f$ is given by (27.) and plotted as a function of $\theta$ in Figure 7. The rms value of $i_f$ was obtained by numerical integration and is shown as a function of $I_L$ in Figure 8.

Since laboratory data on an actual rectified superconducting alternator was not available at the time of this report, it is difficult to say how these calculated variables will eventually compare with experimental results. However, an analytical review of the $i_f$ calculations does indicate that the $i_f$'s shown in Figures 7 and 8 may be higher than the actual values. It is believed that the reason for this is that the inductance matrix in (5.) does not fully account for the high frequency attenuation provided by the damper shield. This effect is still under investigation.
Figure 7. $i_f$ versus $\theta$ at Full Load with Uncontrolled Rectifier Bridge. (See text for discussion of $i_f$ calculations.)
Figure 8. RMS value of $i_f$ versus $I_L$ with uncontrolled Rectifier Bridge. (See text for discussion of $i_f$ calculations.)
Armature Current vs $\theta$:

The variation in $i_c$ during conduction and commutation is shown in Figure 9. Note that the commutation value is simply $-i_k$ as given by (37.).
Figure 9. $i_c$ versus $\theta$ at Full Load.
2. ALTERNATOR WITH CONTROLLED RECTIFIER BRIDGE

2.1 Introduction

It is probable that most rectified alternator applications will require some type of closed loop voltage regulation. A good indication of this is provided by McCabria, et. al. [13], which describes a 3 phase 10 MVA/5kV machine that has an open loop voltage regulation of 26.5%. The previous analysis is applicable for the most obvious means of control, which is to vary the current in the superconducting field winding. This section considers another possibility, which is to control the turn-on angle of the output rectifier bridge (in this case composed of thyristors).

Although phase-control is widely used when an A.C. bus is the source, this method is usually not employed with a dedicated rectified alternator. In the case of a conventional alternator it is much better to regulate the voltage by means of field control since this technique is relatively simple and produces the minimum ripple at the output. However, in the case of a superconducting alternator there are some potential problems associated with the use of field control for voltage regulation (see [10]). One area of concern is the fact that abrupt variations in field current may cause this winding to leave the superconducting mode. Another problem is the long time constant of the field circuit which produces a very slow step response.

The use of a phase controlled rectifier bridge on the output should provide relatively lower field current variations and a faster step re-
response. It also should be noted that gated devices may be required for the rectifier bridge in order to provide short circuit protection. Thus these same devices can be used to provide voltage regulation.

2.2 Steady State Model

The schematic diagram for the alternator with a controlled rectifier bridge is shown in Figure 10, and the output voltage and current waveforms are shown in Figure 11. The approximations used in this section are identical to those used for the analysis of the uncontrolled rectifier bridge.

2.3 Steady State Equations

Equations (44.) - (48.) of the previous section provide five expressions which can be solved numerically to find the variables $I_f$, $\beta$, $\mu$, $V$ and $W$. However, with the thyristor bridge, commutation does not start when $v_c = v_b$ but at some later time when $v_c > v_b$, i.e., $\beta$ is controlled externally to produce the desired $V_L$. This means that (47.) no longer applies since it is based on $v_c = v_b$ at $\theta = \beta$.

Therefore, there are only four equations to work with, (44.), (45.), (46.) and (48.), and $I_f$ is held constant slightly above the minimum value required for maximum $I_L$.

Let, $x' = \begin{bmatrix} \beta \\ \mu \\ V \\ W \end{bmatrix}$  (72.)
Figure 10. Equivalent Circuit for the Superconducting Alternator with Controlled Rectifier Bridge.

Figure 11. Output voltage and armature currents for the alternator with controlled rectifier bridge.
A solution for $x'$ can be obtained from the standard Newton-Raphson equation,

$$F(x') (x' - x') = f'(x') - f'(x')$$

where $x'$ is some initial starting point sufficiently close to $x$ and,

$$F(x') = \text{the Jacobian matrix} = \frac{\partial f(x')}{\partial x'}$$

A satisfactory value for $x'$ is obtained by using the previous diode bridge solution, $x$, where $I_f$ is a variable, or by using the approximate method reported by Franklin in [17,18].

2.4 Numerical Results

This numerical example is based on the same 4 pole, 400 Hz., 10 MVA/5kV superconducting machine used in the previous example. The same full load conditions are assumed,

$$V_L = 6760 \text{ V.D.C.}$$

$$I_L = 1420 \text{ A.D.C.}$$

The minimum value of $I_f$ that will produce $I_L = 1420$A corresponds to the solution for the uncontrolled rectifier bridge (i.e., minimum $\beta$). This current is found from the first section,

$$I_f(\text{min.}) = 250\text{A.}$$
In an actual system it will be desirable to set $I_f$ at some value above that given by (77.). This will insure against low voltage if $I_f$ decreases for any reason. For this example, the high value, $I_{f(\text{max.})}$, was arbitrarily taken to be,

$$I_{f(\text{max.})} = 1.1 I_{f(\text{min.})} \quad (78.)$$

A plot of $I_f$ vs. $I_L$ for both the controlled and uncontrolled rectifier bridge is shown in Figure 12.

$\beta$, $\mu$, the first three harmonics of $v_o$, and the rms value of $i_f$ are plotted vs. $I_L$ in Figures 13 through 18 respectively. Note that the corresponding values for the diode bridge case are included for reference purposes.

As would be expected, Figure 13 indicates that the thyristor bridge $\beta$ must exceed the diode bridge $\beta$ to compensate for the higher $I_f$. The thyristor bridge $\beta$ also drops as $I_L$ increases in order to compensate for the higher voltage drop across the armature windings.

Figure 14 reveals an interesting characteristic in that the $\mu$ for the thyristor bridge is considerably less than the $\mu$ for the diode bridge. This is not too surprising when Figures 10 and 11 are considered. Referring to these figures, it is observed that because of the delayed $\beta_1$ commutation from b to c does not start until $v_{bc} > 0$. Therefore, more voltage is present to force the commutation process than in the diode case where commutation begins at $v_{bc} = 0$. Because of this higher $v_{bc}$, $i_b$ will be driven to zero in less time, thus producing a smaller $\mu$ for the thyristor case.
Figure 12. $I_f$ vs. $I_L$ for both the thyristor bridge and the diode bridge.
Figure 13. $\beta$ vs. $I_L$ for both the thyristor bridge and the diode bridge.
Figure 14. $\mu$ vs. $I_L$ for both the thyristor bridge and the diode bridge.
The voltage harmonics shown in Figure 15 to 17 are obtained by calculating the Fourier coefficients of the output voltage, $v_0$, which can be obtained from (65.) - (71.). Again as expected, those figures generally indicate higher $v_0$ harmonics for the thyristor case than for the diode case. It is also noted that the harmonics for the thyristor bridge tend to be higher for the lower values of $I_L$. This characteristic is caused by the higher $\beta$ values under light loading conditions.

The rms value of $i_f$ can be found by numerical integration of (27.). Figure 18 indicates that the higher thyristor $\beta$ will lead to higher values of $i_f$ (rms) over the load range. As was noted in the discussion for the uncontrolled rectifier bridge, these $i_f$ (rms) calculations may be higher than the actual values due to a failure to account for the high frequency attenuation of the damper shield.
Figure 15. 6th harmonic of \( v_o \) vs. \( I_L \) for both the thyristor bridge and the diode bridge.
Figure 16. 12th harmonic of $v_o$ vs. $I_L$ for both the thyristor bridge and the diode bridge.
Figure 17. 18th harmonic of $v_o$ vs. $I_L$ for both the thyristor bridge and the diode bridge.
Figure 18. $i_f (\text{rms})$ vs. $I_L$ for both the thyristor bridge and the diode bridge.
3. FAULT CURRENT CALCULATIONS

3.1 Introduction

This discussion is based only on the controlled rectifier bridge configuration shown in Figure 10. The reason for this choice is that this circuit has the capability of fast turn off in the event of a fault. Fast turn off can be achieved by the system in Figure 1 only if some type of series switch is added to the circuit.

In addition to filtering the output voltage, the inductor, \( L_o \), in Figure 10 must be capable of limiting \( I_L \) in the event of a short across the load. The length of time that \( L_o \) must perform this function is limited however, since the bridge can be turned off on the next cycle after the fault is detected. It is also common to select \( L_o \) to limit the \( C_o \) charging current when the system is initially turned on. This can also be accomplished by a further delay in \( \beta \) however, so charging current will not be used as a constraint in this analysis.

To determine if \( L_o \) is of adequate size, it will be necessary to calculate the transient load current, \( i_{LF} \), that occurs after the fault. Since the differential equations involved have time varying coefficients, a numerical solution will be required. In this particular study it is assumed that the fault occurs at the beginning of a conduction period and that the bridge will not be turned off until this conduction period, the next commutation period, and a final conduction period are complete. This corresponds to the interval, \( AE \), shown in Figure 19. The rationale behind this choice is that some time is required for \( i_{LF} \) to exceed \( I_{L\text{(max.)}} \), at which time a current overload sensing circuit is enabled to
A: Fault occurs at the start of a conduction period
B: On-coming thyristors fire and commutation begins
C: Commutation interval ends
D: $I_{L(\text{max})}$ is exceeded, next trigger signal is blanked
E: $i_{LF}$ decays to 0

Figure 19. Transient Behavior for the Controlled Rectifier Bridge.
blank all the thyristor gate signals. Other choices are certainly possible, such as assuming that the fault occurs at the start of a commutation period and that the thyristor gates are blanked before the next firing pulse.

3.2 Circuit Model and Equations

Figure 20 represents the equivalent circuit with phase "a" conducting and a short across the load. Note that the armature resistance $R_a$ and the parasitic resistance of the filter choke, $R_p$, have been included since they aid in limiting the fault current. The periods AB, BC, etc. in Figure 19 will correspond to the following thyristors being on:

<table>
<thead>
<tr>
<th>Period</th>
<th>Thyristors Conducting</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB, conduction</td>
<td>Q1, Q6</td>
</tr>
<tr>
<td>BC, commutation</td>
<td>Q1, Q6, Q2</td>
</tr>
<tr>
<td>CE, conduction</td>
<td>Q1, Q2</td>
</tr>
</tbody>
</table>

The steady state analysis has assumed that the winding resistances are negligible. That practice will be continued here for all windings except those on the armature; as before, it implies that $\lambda_f$, $\lambda_d$ and $\lambda_q$ are approximately constant over the relatively short transient period, AE, and that the voltages across the closed f, d and q windings are approximately zero.

The equations for the commutation period, BC, are found to be,

$$\frac{di}{dt} = [B]i$$

(79.)

where
Figure 20. Equivalent circuit for conduction and commutation periods while fault is present.

\[
\begin{bmatrix}
    i_L(t) \\
    i_{f(tot)} \\
    i_d \\
    i_q \\
    i_{kF}
\end{bmatrix} = \begin{bmatrix}
    i' \\
    \cdots \\
    \cdots \\
    i_{kF}
\end{bmatrix}
\] (80)

and the [A] and [B] matrices are defined in Appendix II.
The equations for the conduction period, AB, (Q2 off) can be expressed in a similar manner,

\[
[A_{11}] \frac{di'}{dt} = [B_{11}]i'
\]

(81.)

where \([A_{11}], [B_{11}]\) and \(i'\) are submatrices of \([A], [B]\) and \(i\). These submatrices are also defined in Appendix II.

Since the fault is assumed to occur at A in Figure 19, (81.) will be solved first, then (79.). It is unnecessary to formulate equations specifically for the second conduction period, CE, since this period can be analyzed by using (81.).

As predicted earlier, \([A]\) and \([B]\) have time dependent elements. This implies that \(i\) must be found by some numerical integration technique. The modified Euler method was chosen for this particular study, but other techniques could also be employed.

Starting with the conduction period, AB, (81.) can be solved for \(\frac{di'}{dt}\) at each time increment, \(\Delta t\), and the next value of \(i'\) can then be found by using the standard modified Euler equations (see [27.]). This process is simplified however, if the approximation of constant \(\lambda_f\), \(\lambda_d\), and \(\lambda_q\) is again utilized. Writing the flux linkage equations for the most general case (the commutation period, BC), one obtains,

\[
\begin{bmatrix}
\lambda_f \\
\lambda_d \\
\lambda_q
\end{bmatrix} = \begin{bmatrix}
\sqrt{3}M_f \cos (\omega t + \pi/6) \\
\sqrt{3}M_d \cos (\omega t + \pi/6) \\
\sqrt{3}M_d \sin (\omega t + \pi/6)
\end{bmatrix} i_{LF} + \begin{bmatrix}
\sqrt{3}M_f \sin (\omega t) \\
\sqrt{3}M_d \sin (\omega t) \\
\sqrt{3}M_d \cos (\omega t)
\end{bmatrix} i_{KF}
\]

(cont.)

47.
or in vector notation, \( \lambda_{fdq} = \frac{Xi_{LF}}{2} + \frac{Y}{kF} + \frac{i}{[C]} \frac{i_{fdq}}{i_{fdq}}. \) \( (83. \)

\( \lambda_{fdq} \) can be found initially by substituting the steady state values for \( i_{fdq}, \) \( i_{LF} \), and \( i_{kF} \) at \( \omega t = \beta + \mu = \pi/3 \) (i.e., \( i_{fdq} \) is found from (25.), (26.), (27.) and \( i_{LF} = I_L \cdot i_{kF} = 0 \)). At each time increment \( i_{LF} \) and \( i_{kF} \) can be found from the modified Euler equations while \( i_{fdq} \) can be found from (83.) (using the constant value of \( \lambda_{fdq} \)).

Figure 19 indicates that the firing angle is delayed before the fault, but not afterwards (point B). The reason for this is that once the output is shorted the voltage regulator will call for the minimum firing angle, and the oncoming thyristors will conduct as soon as possible.

Since the load current is no longer constant, the steady state equations that yield \( \beta(\text{min.}) \) (i.e., the firing angle for an uncontrolled rectifier bridge) do not apply, and the firing angle (point B) must be found by determining the first point at which \( v_{bc} \geq 0 \). As \( v_{bc} \) becomes positive, Q2 will start to conduct and the commutation of Q6 will commence. \( v_{bc} \) for the AB conduction state can be readily found from the bc loop,

\[
\begin{align*}
\frac{dv_{bc}}{dt} &= a_{LF} \frac{di}{dt} + (L + M) \frac{di}{d\tau} - \sqrt{3} \sin (\omega t) \left( M_f \frac{di_{f(tot)}}{dt} + M_d \frac{di}{d\tau} \right) \\
&\quad - \sqrt{3} \cos (\omega t) \left( \frac{di}{dt} + \sqrt{3} \omega \cos (\omega t) (M_f i_{f(tot)} + M_d i) \right) \\
&\quad + \sqrt{3} \omega M_d \sin (\omega t) \frac{di}{dt} \frac{d\tau}{d\tau} \frac{d}{d\tau} (84.)
\end{align*}
\]
$v_{bc}$ is tested at each time increment of the AB interval; at the first point where $v_{bc} > 0$ the computer program branches to the equations for the BC commutation interval. It will also be necessary to perform some test to determine when the commutation period ends at point C. This is done by comparing $i_{iF}$ and $i_{kF}$ at each time increment of the BC interval; the commutation period ends at the first point where $i_{kF} > i_{iF}$.

It should be noted that it may take several machine cycles to commutate the thyristors after the firing signals have been blanked. This can be illustrated conceptually by the simplified model shown in Figure 21 (this model is of little quantitative use however, since it does not account for the winding resistances and inductances of the machine). If $e_s(t) = v_{ab}$ and the thyristors start to conduct at $\omega t = \pi/3$ (approximate), $i(t)$ will have the form,

$$i(t) = I_L + \frac{\sqrt{2}v}{\omega L_0} (1/2 - \cos (\omega t)) \quad (85.)$$

where $I_L$ = load current at $\omega t = \pi/3$.

At $\omega t = \frac{2\pi}{3}$ (approximate), conduction switches from phase b to phase c so the source voltage becomes $e_s(t) = v_{ac}$. The load current now has the form,

$$i(t) = I_L + \frac{\sqrt{2}v}{\omega L_0} \left( \frac{3}{2} - \cos (\omega t - 2\pi/3) \right) \quad (86.)$$

Even if the bridge is blanked during the $v_{ac}$ cycle (to prevent the next commutation) (86.) never goes negative, meaning that the conducting thyristors will not turn off.
\( e_s(t) = \sqrt{2} V \sin(\omega t) \)

(Refer to Figure 19 for \( v_{ab} \) and \( v_{bc} \) waveforms)

(a.) Simplified equivalent circuit during fault.

(b.) Fault current.

Note: Actual waveform will be damped due to presence of \( 2R_a + R_p \)

Figure 21. Effect of \( L_o \) in limiting fault current.
A similar phenomenon can occur in the actual physical circuit, except that $i(t)$ will eventually decay to zero due to resistive damping. This may require several cycles however, and more cycles will be required for larger values of $L_o$. Thus if $L_o$ is large, several cycles may be required to complete turn off; however, if it is too small the peak fault current will be excessive.

3.3 Numerical Results

The transient analysis algorithm can be used to plot post fault current waveforms for various values of $L_o$ and $R_p$. Two parametric studies of $i_{LF}$ for variations in $L_o$ and $R_p$ are shown in Figures 22 and 23 respectively. Figure 24 shows a plot of $i_{LF}$ that requires three cycles for $i_{LF}$ to reach zero. Presumably commutation would occur on the fourth cycle where $i_{LF}$ would attempt to go negative.

This algorithm could also be used to plot $i_{f(tot)}$, $i_d$, and $i_q$ during a fault condition. However, as stated earlier for the steady state calculations, the $i_{f(tot)}$ values may be too high since the model does not include the high frequency attenuation of the damper shield.
Figure 22. Fault current vs. $\theta$ for different values of $L_0$. $R_p = 0.05\Omega$. 

$+ L_0 = 1.0 \text{ mH.}$

$\circ L_0 = 3.0 \text{ mH.}$

$\ast L_0 = 9.0 \text{ mH.}$
Figure 23. Fault current vs. $\theta$ for different values of $R_p$. $L_o = 3.0$ mH.
Figure 24. Exponential decay of $i_{LF}$.
4. VARIATION OF THE ALTERNATOR PARAMETERS TO DECREASE OUTPUT RIPPLE VOLTAGE

4.1 Introduction

As noted previously, there are two basic methods for regulating the dc output voltage, \( V_L \), of the rectified alternator:

1. Use an uncontrolled rectifier bridge, and regulate \( V_L \) by controlling the average field current, \( I_f \).

2. Hold \( I_f \) constant, and regulate the voltage by means of a controlled rectifier bridge.

The first method provides the minimum ripple voltage, but it tends to have a slow response time due to the long time constant of the field winding. Therefore the analysis of this section is based on the controlled rectifier bridge.

If the alternator is modeled by an ideal ac voltage in series with an inductor, it is well known that an increase in this inductance will increase the commutation angle, \( \mu \), shown in Figure 11. For constant \( V_L \) and \( I_L \) this effect can also lead to a reduction in output ripple voltage, as illustrated in Figure 25. Comparing parts (a.) and (b.) of the figure it is seen that the same average output voltage, \( V_{L'} \), is achieved by different combinations of the angle, \( \beta \) and \( \mu \). However, the deviation of \( v_o \) is less in part (b.). This indicates than an increase in \( \mu \) requires the bridge firing angle to decrease, resulting in a more level \( v_o \). Therefore, the \( \beta_2, \mu_2 \) combination in (b.) produces a lower ripple voltage than the \( \beta_1, \mu_1 \) combination in (a.). This implies that at least over a limited range of \( \mu \) values it is possible to decrease the ripple voltage by increasing \( \mu \). Thus for a fixed load, it is possible to decrease
\[ V_L = \text{average output voltage} \]

(source inductance for (b.) is higher than that for (a.))

Figure 25. Effect of \( \mu \) upon output ripple voltage for fixed \( V_L \) and \( I_L \).
the ripple by increasing the source inductance since this causes an increase in $u$.

The model used in this study was considerably more complicated than the one just described, but it was found that for a constant load, the ripple could be decreased if the armature inductance, $L_a$, was increased beyond its specified value of 0.3 mH. This implies that a lighter weight filter could be used if $L_a$ were increased. A word of caution is in order however, since these gains may be offset by an increase in alternator weight (due to the larger $L_a$). Ultimately, it would be desirable to develop a procedure that would minimize the combined weight of the alternator and filter. This would require a detailed weight analysis of the superconducting alternator however, and such an effort would be beyond the scope of this present study.

4.2 Effect of $L_a$ on the Output Voltage Harmonics

As discussed in the previous section, it is possible to reduce the full load ripple voltage by changing the output impedance of the alternator. The model used in this study cannot be reduced to a single ac source in series with such an impedance, but a similar effect will occur if the armature self inductance, $L_a$, is varied. Changes in $L_a$ will, of course, change the armature mutual inductances. To account for this, it is assumed that the coefficient of coupling between $L_a$ and each of the other windings, remains constant while $L_a$ is varied, i.e.,

$$k_{af} = \frac{M^2}{\sqrt{L_a L_f}} = \text{constant} \quad (87.)$$


\[ k_{aa} = \frac{M_a}{L_a} = \text{constant} \quad (88.) \]

\[ k_{ad} = \frac{M_d}{\sqrt{L_a L_d}} = \text{constant} \quad (89.) \]

\((L_f \text{ and } L_d \text{ also remain constant.})\)

4.3 Numerical Results

Since \(k_{af}\) and \(L_f\) are assumed constant, (87.) indicates that an increase in \(L_a\) will also increase \(M_f\). Thus an increase in \(L_a\) implies that the same magnetic flux linkage from field to armature, \(M_f I_f\), can be achieved with a lower \(I_f\) (since a thyristor bridge is used for voltage regulation it is assumed that \(I_f\) will be held constant at 110% of the minimum allowable value for a given \(L_a\), as discussed in the section on controlled rectifiers). Figure 26 indicates the decrease in the required \(I_f\) as \(L_a\) is increased for \(I_L = 1420\) A dc. This decrease in \(I_f\) might allow the use of smaller superconducting wire for the rotor winding, thus decreasing the rotor size. An alternate approach would be to hold \(M_f\) constant and allow \(L_f\) to decrease as \(L_a\) increased; thus the field winding would have fewer turns (both effects appear small).

Figure 27 indicates that the 6th harmonic of \(v_o\) reaches a minimum at \(L_a = 0.72\) mH. This leads to a decrease in the size of the output filter, \(L_o C_o\), since less attenuation is required.

\[ \text{Break frequency} = f_b = \frac{1}{\sqrt{L_o C_o}} \quad (90.) \]

Figure 27 also indicates that the 12th and 18th harmonics generally continue to increase with \(L_a\), but their effect is less important since
filter attenuation is much greater at these frequencies. Figure 28 shows that the 6th harmonic continues to decrease as $L_a$ increases, indicating that the optimum $L_a$ at 40% of full load lies somewhere above 0.9 mH. Thus the optimum $L_a$ is different for different values of $I_L$. 
Figure 26. $I_f$ vs. $L_3$ for the controlled rectifier bridge. $I_L = 1420$ A.d.c.
Figure 27. 6th, 12th and 18th harmonics of $v_o$ vs. $L_a$ for the controlled rectifier bridge. $I_L = 1420$ A.d.c.
Figure 28. 6th, 12th and 18th harmonics of $v_o$ vs. $L_a$ for the controlled rectifier bridge.

$I_L = 568$ A.d.c.

62.
5. MINIMIZATION OF $L_o C_o$ FILTER WEIGHT

5.1 Introduction

The previous sections have considered the following topics:

1. Steady state behavior with an uncontrolled rectifier bridge.
2. Steady state behavior with a controlled rectifier bridge.
3. Transient currents that occur when a short circuit is placed across the output.
4. Reducing the full load output ripple voltage by increasing $L_a$.

The results of these studies can now be used in designing an $L_o C_o$ output filter for minimum weight. This analysis assumes the use of a controlled rectifier bridge for voltage regulation. The maximum allowable ripple voltage will be based on the size of the sixth harmonic that is present at full load. It should be noted that this harmonic will actually be greater at minimum load since the firing angle of the thyristors will be greater. This study assumes that the load will be fairly constant however, and that the presence of ripple voltage will be more important at full load than at lighter loads. Hence the filter optimization is based on full load conditions.

5.2 Calculation of $L_o$ and $C_o$ for Minimum Total Filter Weight

In the weight minimization algorithm, $L_o$ and $C_o$ are calculated to provide a given amount of ripple attenuation at full load. This calculation is based only on the sixth harmonic and ignores the harmonic attenuation provided by the load in conjunction with $L_o$. Therefore,
\[
\left| \frac{1}{1-L_C \omega_6^2} \right| \leq k_1
\]

where \( k_1 \) = specified magnitude of the 6th harmonic attenuation

and \( \omega_6 = 15079.64 \) radians/sec.

\[
L_C \geq \frac{k_1+1}{k_1 \omega_6^2}
\]

Due to the high value of the magnetic field and the low weight requirement, it is assumed that \( L_o \) will be an air core reactor. Aluminum was chosen for the conductor due to its low weight/conductance ratio. The physical configuration of the inductor is shown in Figure 29. This particular design is chosen to produce the minimum loss for a given amount of material (see [25,26]). The inductance is,

\[
L_o = (24.5 \times 10^{-7}) N^2 a \quad \text{H.}
\]

and the weight of \( L_o \) is given by

\[
L_o \text{ wt.} = 3 \pi f D_w a^3 \quad \text{lbs.}
\]

where

\[
N = \text{number of turns} \\
a = \text{thickness of the coil (m.)} \\
D_w = \text{density of Al (5837.8 lbs/m.}^3) \\
f = \text{filling factor of the conductors (assumed to be 0.7)}
\]

The filling factor can be expressed,
Figure 29. $L_0$ dimensions.
\[
f = \frac{NF_w}{a^2}
\]  \hspace{1cm} (95.)

where \( F_w \) = cross sectional area of one winding (m.\(^2\)).

The common method of specifying capacitor weight is in terms of joules/lb. Therefore the total weight of the \( L \) \( C \) filter can be expressed,

\[
T_{wt} = C_w \text{ weight} + L_w \text{ weight}
\]

\[
= \frac{C_w V^2}{2D_c} + 3 \pi f D_w a^3 \text{ lbs.} \quad (96.)
\]

where \( D_c \) = energy density of \( C_w \) (joules/lb.).

Therefore substituting (92.), (93.) and (95.) into (96.)

\[
T_{wt} = k_2 a^5 + k_3 a^3
\]  \hspace{1cm} (97.)

where \( k_2 = \left( \frac{1}{51.1 \times 10^{-7}} \right) \left( \frac{F_w V}{f} \right)^2 \left( \frac{k_1 + 1}{D_c k_1 \omega^2} \right) \)

\( k_3 = 3 \pi f D_w \)

To find the minimum value of \( T_{wt} \), set

\[
\frac{dT_{wt}}{da} = -\frac{5k_2}{a^6} + 3k_3 a^2 = 0
\]  \hspace{1cm} (98.)

\[ a = \left( \frac{5k_2}{3k_3} \right)^{1/8} \]  \hspace{1cm} (99.)

Once \( a \) is determined, \( N \) can be found from (95.) , \( L \) \( C \) from (93.) and \( C_w \) from (92.).
5.3 \(L_C\) Design Algorithm

The flow chart for the \(L_C\) filter design program is shown in Figure 30. The following discussion refers to the seven blocks indicated in the figure.

1. Read input data. This includes the following information: \(C_o\), energy density \(\left( D_c \right)\), \(L_o\), current density, maximum 6th harmonic ripple voltage at full load, and maximum short circuit current, \(I_{L(max)}\). This particular program assumes that the following quantities are constant:

\[
V_L = 6760 \text{ V}\text{.dc}
\]

\[
I_L = 1420 \text{ A}\text{.dc}
\]

\[
\omega_6 = 15079.6 \text{ rad./sec.} \text{ (line frequency} = 400 \text{ Hz.)}
\]

\(V_L, I_L\) and \(\omega_6\) could be varied if desired, by making a few minor changes in the program.

2. Set \(L_a = 0.3 \text{ mH}\), the normal design value specified in [13,14].

3. Assuming that a controlled rectifier bridge is used, the minimum weight \(L_o\) and \(C_o\) that will meet the 6th harmonic ripple specification are calculated.

4. A transient analysis subroutine to called to determine if the peak short circuit current will exceed the specified value of \(I_{L(max)}\). The details of this analysis are given in section 3.

5. If \(I_{L(max)}\) is exceeded, \(L_o\) is increased by 10\%, and step 3 is repeated \((C_o\) is simultaneously decreased to maintain a constant \(L_oC_o\) product.) If \(I_{L(max)}\) is not exceeded, the \(L_o\) and \(C_o\) design data is printed.

6. The program calculates \(L_o\) and \(C_o\), first for the normal \(L_a\) and then for the optimum values of \(L_a\). If the calculation for the optimum \(L_a\) has
Figure 30. $L_o C_o$ weight minimization flow chart.
been completed the program ends. If not, the program branches to step 7.

7. Set \( L_a = 0.72 \) mH, the optimum value for minimum ripple at full load calculated in Section 4. Steps 3 through 6 are then repeated.

5.4 Numerical Results

A sample of the computer results for the optimization program are shown in the following example. Note that use of the optimum \( L_a \) decreases the total filter weight by approximately 22%.

The total filter weight will obviously decrease if a higher energy density \( (D_e) \) is used for \( C_o \) and/or a higher current density is used for \( L_o \). Plots of filter weight vs. energy density and current density are shown in Figures 31 and 32 respectively. The filter weight will also be affected if the allowable \( I_{L(max.)} \) is changed. A plot of this is shown in Figure 33.
WRITE THE FOLLOWING PARAMETERS FOR THE FILTER

ALL INPUTS HAVE FORMAT = F7.2 UNLESS OTHERWISE SPECIFIED

CAP. ENERGY DENSITY (JJ/ULE/ES) = 50.0

CURRENT DENSITY FOR L WIRE (CUR MIL/AW1) = 90.0

MAX. RMS VALUE OF 6TH HARM. OF VO (VOLTS) = 20.0

ALLOWABLE PEAK FAULT CURRENT (AMPS) = 2500.0

THE FOLLOWING VALUES ARE BASED ON NORMAL LA = 0.300E-03 H.
IF= 0.275E+03 BETA= 0.212E+01 MU= 0.307E+00
IL= 0.142E+04 VL= 0.375E+04 VI= 0.120E+04

OPT. VALUES BEFORE FAULT TEST ARE LO= 0.422E-02 H., CO= 0.606E-04 F.D. H.
ES= 0.547E-01
MAX LOAD CURRENT FROM FAULT = 0.250E+04

FAULT CURRENT T03 LARGE, LO INCREASED
MAX LOAD CURRENT FROM FAULT = 0.250E+04
MAX LOAD CURRENT FROM FAULT = 0.245E+04

LO= 0.511E-02, CO= 0.523E-04, ILF= 0.245E+04, RES= 0.615E-01
L'T= 0.542E+02 CM= 0.240E+02, TOTAL V= 0.731E+02

NJ. TURNS = 134 L RADIUS = 0.223E+00 L LENGTH = 0.111E+00

THE FOLLOWING VALUES ARE BASED ON OPTIMUM LA = 0.720E-03 H.
IF= 0.205E+03 BETA= 0.223E+01 MU= 0.604E+00
IL= 0.142E+04 VL= 0.675E+04 VI= 0.620E+03

OPT. VALUES BEFORE FAULT TEST ARE LO= 0.232E-02 H., CO= 0.499E-04 F.D. H.
ES= 0.423E-01
MAX LOAD CURRENT FROM FAULT = 0.250E+04

FAULT CURRENT T03 LARGE, LO INCREASED
MAX LOAD CURRENT FROM FAULT = 0.250E+04
MAX LOAD CURRENT FROM FAULT = 0.250E+04

LO= 0.341E-02, CO= 0.412E-04, ILF= 0.250E+04, RES= 0.432E-01
L'T= 0.425E+02 CM= 0.133E+02, TOTAL V= 0.613E+02

NJ. TURNS = 114 L RADIUS = 0.206E+00 L LENGTH = 0.103E+00

WRITE "0" TO END, OR "1" FOR ANOTHER RUN

FORMAT=12

STOP --
Figure 31. Filter weight vs. $C_o$ energy density. Current density = 100 cir. mile/amp, all other variables are the same as in the example run.
Figure 32. Filter weight vs. $L_0$ current density. All other variables are the same as in the example run.
Figure 33. Filter weight vs. maximum allowable short
circuit current. Current density = 100
cir. mils/amp, all other variables are
the same as in the example run.
6. SENSITIVITY ANALYSIS

6.1 Introduction

Since the values of the alternator inductances, \( L_a, L_f, L_d, M_a, M_f, M_d \) and \( M_{fd} \), are subject to numerical error, it is of interest to see how errors in these parameters will affect the calculations for \( I_f, \beta, \mu, V \) and \( W \). As noted in Sections 1 and 2, the calculations for the uncontrolled and controlled rectifier bridges are quite similar. This implies that the effect of a given parameter error should be about the same for both types of systems. Therefore it was decided to limit the sensitivity analysis to the uncontrolled rectifier case. For convenience we define the following,

\[
X = \begin{bmatrix} I_f \\ \beta \\ \mu \\ V \\ W \end{bmatrix}
\]

\[
Y = \begin{bmatrix} L_a \\ L_f \\ L_d \\ M_a \\ M_f \\ M_d \\ M_{fd} \end{bmatrix}
\]

(100.)

(101.)
\[
\begin{align*}
\begin{bmatrix}
(44.) \\
(45.) \\
(46.) \\
(47.) \\
(48.)
\end{bmatrix}
\end{align*}
\]

Theoretically, this analysis could be performed by either of two methods:

1. Use (102.) to find \( \frac{3x}{3y} \) and solve for \( \Delta x \) for a given \( \Delta y \), i.e., \( \Delta x = \frac{3x}{3y} \Delta y \). This will be referred to as the differential method.

2. Simply replace \( y \) by \( y + \Delta y \) and use the Newton Raphson method to find the resulting \( x + \Delta x \). This will be referred to as the deliberate error method.

The differential method is certainly the more elegant of the two, so this was investigated first. Unfortunately this approach depends on solving sets of simultaneous equations that have ill-conditioned coefficient matrices. Two algorithms were used for solving these equations, but both failed due to excessive round-off errors. Therefore it was necessary to resort to the deliberate error method. This second approach worked satisfactorily even though it is rather inefficient in terms of computation time. Both methods will be discussed for completeness, even though the first did not produce satisfactory results.

6.2 Differential Method

One usually does not bother to describe methods that do not work, but this analysis is interesting from a conceptual standpoint, so it is
included for that reason. It is also possible that the problems with this method may eventually be solved, even though it was unsuccessful in this present research.

Taking the partial derivative of (102.) produces an equation of the form,

\[
\frac{\partial f(x,y)}{\partial y_i} = [C] \frac{\partial x}{\partial y_i} + r = 0
\]  

(103.)

where [C] is a (5x5) coefficient matrix, and r is a (5x1) vector.

\[
\frac{\partial x}{\partial y_i} = -[C]^{-1} r
\]  

(104.)

which is the ith column of \(\frac{\partial x}{\partial y}\), a (5x7) matrix. Therefore it is conceptually possible to use (104.) for all seven elements of \(y\) to find \(\frac{\partial x}{\partial y}\). \(\Delta x\) for a given \(\Delta y\) is then,

\[
\Delta x = \frac{\partial x}{\partial y} \Delta y
\]  

(105.)

Unfortunately, the [C] matrices indicated by (103.) are very ill conditioned in this application, and this prevented finding a solution for \(\frac{\partial x}{\partial y}\). Two methods of solution were attempted, the first being the DGE LG double precision subroutine from the IBM Scientific Subroutine Package and the second being a Shipley-Coleman inversion algorithm to find \([C]^{-1}\). Both of these programs use pivoting for size, but they were still incapable of finding the correct solution. Therefore this approach was abandoned in favor of the deliberate error method.

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6.3 Deliberate Error Method

In this method a given error, $\Delta y_i$, is added to $y_i$ and the resulting $\Delta x$ is calculated by the equations described in section 1. The terms of $\Delta y$ are not independent however, since mutual inductance terms are present, and this must be accounted for in the analysis. Therefore, the approach used in this particular study was to assume that the following terms can be varied independently of one another: $L_a$, $L_f$, $L_d$, $k_{aa}$, $k_{af}$, $k_{ad}$, and $k_{fd}$, where the last four terms are the coefficients of coupling, i.e.,

\[ k_{aa} = \frac{M_a}{L_a}, \quad k_{af} = \frac{M_f}{\sqrt{L_a L_f}}, \quad k_{ad} = \frac{M_d}{\sqrt{L_a L_d}}, \quad k_{fd} = \frac{M_{fd}}{\sqrt{L_f L_d}} \] (106)

The independent and dependent parameters are listed as follows:

<table>
<thead>
<tr>
<th>Independent</th>
<th>Dependent</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_a$</td>
<td>$M_a$, $M_f$, $M_d$</td>
</tr>
<tr>
<td>$L_f$</td>
<td>$M_f$, $M_{fd}$</td>
</tr>
<tr>
<td>$L_d$</td>
<td>$M_d$, $M_{fd}$</td>
</tr>
<tr>
<td>$k_{aa}$</td>
<td>$M_a$</td>
</tr>
<tr>
<td>$k_{af}$</td>
<td>$M_f$</td>
</tr>
<tr>
<td>$k_{ad}$</td>
<td>$M_d$</td>
</tr>
<tr>
<td>$k_{fd}$</td>
<td>$M_{fd}$</td>
</tr>
</tbody>
</table>

For example, a 10% increase in $L_a$ implies (new value = 1.1 $L_a$).
\[ M_a = 1.1 k_{a_a} L_a, \quad M_f = k_{a_f} \sqrt{1.1 L_f L_d}, \quad M_d = k_{a_d} \sqrt{1.1 L_f L_d}, \]

whereas a 10% increase in \( k_{a_f} \) implies (new value = 1.1 \( k_{a_f} \)),

\[ M_{a_f} = 1.1 k_{a_f} \sqrt{L_f L_d} \]

The effect of these errors will be described in the next section on numerical results.

6.4 Numerical Results

The following paragraphs discuss the effects of varying each of the machine inductances, i.e., the effect of a deliberate error. Note that since the algorithm used in Section 1 depends upon the approximation given by (41.), it is necessary to restrict the parameter variations to the range where (41.) is valid. It is assumed that (41.) is satisfied as long as the following condition is met:

\[ \left| M_o - M_{oo} \right| < 0.1 M_{oo} \]

\( \Delta L_a \): Results are shown in Figures 34 and 35. These figures indicate that all of the \( x \) variables are quite sensitive with respect to \( \Delta L_a \).

\( \Delta L_f \): Results are shown in Figures 36 and 37. It is noted that \( \beta, \mu, V \) and \( W \) do not vary with respect to \( L_f \). The reason for this is that the algorithm automatically adjusts \( I_f \) to compensate for any \( L_f \) changes, so that the \( M_f I_f \) flux linkages remain constant (note that \( M_f \) is dependent on \( L_f \)). Compare with the \( M_f \) results shown in Figures 42 and 43.
Figure 34. $\beta$ and \( \mu \) variation vs. \( L_a \).
Figure 35. $I_f$, $W$, and $V$ variation vs. $L_a$. 
Figure 36. $\beta$ and $\mu$ variation vs. $L_f$. 
Figure 37. $I_f$, $W$ and $V$ variation vs. $L_f$. 

82.
$\Delta L_d$: Results are shown in Figures 38 and 39. These figures indicate that the calculations are completely insensitive to $L_d$ variations. The reason for this stems from the previously mentioned approximation,

$$M_0 \approx M_{oo}$$ (107.)

which is given by (41.) in Section 1. Once this approximation is made, $M_0$ is replaced by $M_{oo}$ in all subsequent calculations. The value of $M_{oo}$ is,

$$M_{oo} = \frac{m_2}{L_d} = k_{ad}^2 L_a$$ (108.)

The result of this is that $L_d$ does not actually appear in (44.)-(51.), which are the equations used to find $x$.

$\Delta k_{aa}$ ($\Delta M_a$): Results are shown in Figures 40 and 41. The figures indicate that $u$ is quite sensitive to $\Delta M_a$ variations, while $I_f$ and $B$ are less sensitive. $W$ and $V$ also vary considerably with respect to $M_a$.

$\Delta k_{af}$ ($\Delta M_f$): Results are shown in Figures 42 and 43. For an explanation of these results, refer to the discussion for $\Delta L_f$.

$\Delta k_{ad}$ ($\Delta M_d$): Results are shown in Figures 44 and 45.

$\Delta k_{fd}$ ($\Delta M_{fd}$): Results are shown in Figures 46 and 47. These figures indicate that the calculations are insensitive to $M_{fd}$ variations. The reason for this is much the same as for $L_d$, i.e., once the approximation (107.) is made, $M_{fd}$ does not appear in any of the subsequent equations.
Figure 38. $\theta$ and $\mu$ variation vs. $L_d$. 
Figure 39. $I_f$, $W$ and $V$ variation vs. $L_d$. 

85.
Figure 40. $\beta$ and $\mu$ variations vs. $M_a$. 
Figure 41. $I_f$, $W$ and $V$ variation vs. $M_a$. 

87.
Figure 42. $\beta$ and $\mu$ variation vs. $M_f$. 
UTILIZATION OF SOURCE IMPEDANCE TO DECREASE THE WEIGHT OF D.C.-ETC. (U)

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Figure 43. $\beta$ and $\mu$ variation vs. $M_f$. 
Figure 44. $\beta$ and $\mu$ variation vs. $M_d$. 
MD VARIATION

Figure 45. $I_f$, $W$ and $V$ variation vs. $M_d$. 

91.
Figure 46. $\beta$ and $\mu$ variation vs. $M_{fd}$.
Figure 47. $I_f$, $W$ and $V$ variation vs. $M_{fd}$.
To summarize, it can be seen that $I_f$, $\beta$, $\nu$, $V$ and $W$ as a group are most sensitive to errors in $L_a$, $M_a$ and $M_d$. $I_f$ is also quite sensitive to errors in $L_f$ and $M_f$, but since the algorithm adjusts to maintain a constant $M_f I_f$ product (mutual flux linkages) errors in $L_f$ and $M_f$ have virtually no effect on $\beta$, $\nu$, $V$ and $W$. 
7. VOLTAGE REGULATOR AND CURRENT OVERLOAD PROTECTION CIRCUITS

7.1 Introduction

The experimental portion of this study consisted of designing and building a phase controlled voltage regulator for eventual testing with an alternator. This circuit was designed and tested early in the project when it was thought that the alternator could be modelled by a voltage source in series with single inductance. Testing the regulator with this type of a source would have been a fairly simple matter, but such a test now appears to have limited value since a more detailed machine model was employed. Therefore it was decided to concentrate more effort on the analytical study and postpone this testing until a conventional alternator with known inductances could be obtained. Schematics of the complete design are shown in Figures 48 and 49. Operation of the voltage regulator circuit in Figure 48 is described in [23], and the operation of the current overload circuit in Figure 49 is described in [22]. The parts list is shown in Table I.

7.2 Experimental Results

As stated above, the experimental results were limited to building and testing a phase controlled voltage regulator circuit with a current overload. This circuit has the following characteristics:

1. The output voltage can be varied continuously from 0 to 290 V.d.c. with a 100 ohm load.
2. The overload circuit turns the regulator off at a load current of approximately 3.5 A.d.c.
Figure 48. Voltage regulator circuit.

Note: 1. $\pm$ = power ground
   (must be separate from ground in Figure. 49.)
2. Ref. Fig. 49 for $\circ$, $\bullet$
Note: 1. j must be separate from power ground
2. Ref. Fig. 48 for...
<table>
<thead>
<tr>
<th>Part Number</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>R15</td>
<td>10KΩ</td>
</tr>
<tr>
<td>R25</td>
<td>250KΩ trimpot</td>
</tr>
<tr>
<td>R27</td>
<td>4.7KΩ</td>
</tr>
<tr>
<td>R26</td>
<td>3.76KΩ</td>
</tr>
<tr>
<td>R1</td>
<td>11KΩ - 2 watt</td>
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<tr>
<td>R28</td>
<td>330Ω</td>
</tr>
<tr>
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<td>1.5KΩ</td>
</tr>
<tr>
<td>R10</td>
<td>1.5KΩ</td>
</tr>
<tr>
<td>R11</td>
<td>10KΩ</td>
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<tr>
<td>R12</td>
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</tr>
<tr>
<td>R13</td>
<td>4.7KΩ</td>
</tr>
<tr>
<td>R14</td>
<td>3.76KΩ</td>
</tr>
<tr>
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<tr>
<td>R9</td>
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</tr>
<tr>
<td>R17</td>
<td>5.6KΩ</td>
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<tr>
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<tr>
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</tr>
<tr>
<td>R6</td>
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<tr>
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</tr>
<tr>
<td>R33</td>
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</tr>
<tr>
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<tr>
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<tr>
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</tr>
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</tr>
<tr>
<td>R63</td>
<td>10KΩ</td>
</tr>
<tr>
<td>R64</td>
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<td>10KΩ trimpot</td>
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<td>20KΩ</td>
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<tr>
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<td>Reed Relay SPST N/O</td>
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<td>Telefunken UAA145</td>
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<tr>
<td>A4-A5</td>
<td>Fairchild 9601</td>
</tr>
<tr>
<td>A6</td>
<td>Signetics 7493</td>
</tr>
<tr>
<td>A7</td>
<td>National Semiconductor N7408</td>
</tr>
<tr>
<td>SCR1-SCRG</td>
<td>2N1849</td>
</tr>
</tbody>
</table>
3. No misfiring problems were observed once the final design was complete.

Certain waveforms of interest are shown in Figures 50 through 53.
Figure 50. AC output voltage across the load for a delay angle of 15°. (Scale = 15°/div.)

Figure 51. (Top) Line to neutral input voltage. (Bottom) Thyristor firing pulses for a delay angle of 15°. (Note: 0° delay angle corresponds to 30° on the line to neutral voltage waveform. (Scale = 30°/div.)
Figure 52. (Top) Ramp voltage at pin 7 of the UAA145. (Bottom) Thyristor firing pulses for a delay angle of 15°. (Scale = 30°/div.)

Figure 53. (Top) Pulse formation control signal at pin 11 of UAA145. (Bottom) Thyristor firing pulses for a delay angle of 15°. (Scale = 30°/div.)
8. CONCLUSIONS

This study indicates that it is possible to utilize $L_a$ to help perform some of the functions normally assigned to the $L_oC_o$ output filter. This implies that a smaller filter can be used, thus decreasing the weight of the $L_o$ and $C_o$ components. For the 10 MVA/5kV example alternator with a controlled rectifier bridge it was shown that an increase in $L_a$ from 0.3 mH to 0.72 mH decreases the filter weight by about 17 lbs., a 22% reduction. This example also indicated 0.72 mH to be an optimum value, i.e., filter weight increased for $L_a > 0.72$ mH. Naturally, this savings may be offset by an increase in alternator weight due to the larger $L_a$. Therefore any final weight optimization study should consider the alternator and filter as a combined system.

In the course of developing the filter weight minimization program it was necessary to derive both steady state and transient models for the alternator and rectifier bridge. Because of the large amounts of information provided by these models, it appears they may be useful for simulating the system during the design stage. Once experimental data becomes available for comparison, these models may be refined as necessary in order to accurately predict the various winding currents, commutation angles, etc. It is stressed that this experimental verification is necessary, and plans have been made to proceed with this for a system with a conventional alternator.
9. RECOMMENDATIONS

This study indicates that if $L_a$ is increased up to a certain optimum point, it is possible to significantly reduce the size of the output filter. Information of this type should be brought to the attention of machine designers, but it may or may not influence the design of future alternators due to the many other factors which govern the size of $L_a$. Ultimately the alternator and filter should be considered together in future weight minimization studies.

Perhaps the most pressing need at this point is to obtain some experimental data to compare with the predicted results. Eventually this must be done using a superconducting alternator; however, it is unlikely that such a machine will be available for this purpose for quite some time. In the interim, it is proposed that tests should be conducted on a conventional alternator-rectifier system in order to evaluate the models.
10. REFERENCES


22. T. A. Stuart, "Overload Protection and Filtering Requirements for Phase Controlled Voltage Regulators," ASEE-USAF Summer Faculty Research Program Report (sponsored by Auburn University), Wright-Patterson AFB, Ohio, August 15, 1975.


11. RESEARCH PUBLICATIONS


APPENDIX I: GLOSSARY OF TERMS

A, B, C = constants defined by (49.), (50.) and (51.)

a = thickness of $L_o$

$C_o$ = output filter capacitor

$D_c$ = energy density of $C_o$

$D_w$ = density of aluminum

f = fill-in factor for $L_o$

$f_b$ = break frequency of output filter

$F_w$ = cross sectional area of one winding of $L_o$

$i_a, i_b, i_c$ = line currents

$i_d, i_q$ = currents in the equivalent direct and quadrature windings

\[ i_{fo}, i_{do}, i_{q0} \] = field and damper currents at $\theta = \beta + \mu - \pi/3$

$K_q, K_f, K_d$ = constants defined by (15.) and (18.)

$k_{aa}$ = coefficient of coupling between armature phase windings

$k_{ed}$ = coefficient of coupling between armature and equivalent damper windings

$k_{af}$ = coefficient of coupling between field and armature

$k_{fd}$ = coefficient of coupling between field and equivalent direct axis damper windings

$k_1$ = specified harmonic attenuation factor of output filter
\( k_2, k_3 \) = constants defined just below (97.)

\( L_L \) = load current

\( L_a \) = self inductance of each armature winding

\( L_d \) = self inductance of the direct and quadrature axis windings

\( L_f \) = self inductance of the field winding

\( L_o \) = output filter inductor

\( M_a \) = magnitude of mutual inductance between armature windings

\( M_d \) = magnitude of mutual inductance between damper and armature windings

\( M_f \) = magnitude mutual inductance between field and armature windings

\( M_{fd} \) = mutual inductance between field and direct axis damper windings

\( M_0, M_{\infty} \) = constants defined by (33.) and (34.)

\( N \) = number of turns for \( L_o \)

\( R_a \) = resistance of one armature winding

\( R_o \) = resistance of inductor, \( L_o \)

\( v_f, v_d, v_q \) = voltages across rotor windings

\( v_{ab}, v_{bc}, v_{ca} \) = phase to phase armature voltages

\( v_o \) = instantaneous rectifier output voltage

\( V_L \) = average output voltage

\( V \) = variable defined by (21.)

\( W \) = variable defined by (20.)

\( \beta \) = angle at which commutation starts

\( \Lambda_o \) = constant defined by (36.)

\( \Lambda_f, \Lambda_d \) = constants defined by (36.)

110.
\[ \lambda_a, \lambda_b, \lambda_c, \lambda_f, \lambda_d, \lambda_q = \text{flux linkages} \]

\[ \theta = \text{time angle} \]

\[ \mu = \text{commutation angle} \]

\[ \omega = \text{electrical angular velocity} \]
### APPENDIX II: TRANSIENT MATRICES

\[
[A] = \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
(L_0 + 2L_a + 2M_a) & \sqrt{3} M_f \cos (\omega t + \frac{\pi}{6}) & \sqrt{3} M_d \cos (\omega t + \frac{\pi}{6}) & -\sqrt{3} M_d \sin (\omega t + \frac{\pi}{6}) & -(L_a + M_a) \\
\sqrt{3} K_f \cos (\omega t + \frac{\pi}{6}) & 1 & 0 & 0 & \sqrt{3} K_f \sin (\omega t) \\
\sqrt{3} K_d \cos (\omega t + \frac{\pi}{6}) & 0 & 1 & 0 & \sqrt{3} K_d \sin (\omega t) \\
-\sqrt{3} K_q \sin (\omega t + \frac{\pi}{6}) & 0 & 0 & 1 & \sqrt{3} K_q \cos (\omega t) \\
-(L_a + M_a) & \sqrt{3} M_f \sin (\omega t) & \sqrt{3} M_d \sin (\omega t) & \sqrt{3} M_d \cos (\omega t) & 2(L_a + M_a)
\end{bmatrix}
\]

\[
[B] = \begin{bmatrix}
B_{11} & B_{12} \\
B_{21} & B_{22}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
-(R_p + 2R_a) \sqrt{3} M_f \omega \sin (\omega t+\pi/6) & \sqrt{3} M_d \omega \sin (\omega t+\pi/6) & \sqrt{3} M_d \omega \cos (\omega t+\pi/6) & R_a \\
\sqrt{3} K_f \omega \sin (\omega t+\pi/6) & 0 & 0 & 0 & -\sqrt{3} K_f \omega \cos (\omega t) \\
\sqrt{3} K_d \omega \sin (\omega t+\pi/6) & 0 & 0 & 0 & -\sqrt{3} K_d \omega \cos (\omega t) \\
\sqrt{3} K_q \omega \cos (\omega t+\pi/6) & 0 & 0 & 0 & \sqrt{3} K_q \omega \sin (\omega t) \\
R_a & -\sqrt{3} M_f \omega \cos (\omega t) & -\sqrt{3} M_d \omega \cos (\omega t) & \sqrt{3} M_d \omega \sin (\omega t) & -2R_a
\end{bmatrix}
\]
APPENDIX III: MAIN PROGRAMS

The following programs are listed in alphabetical order. All sub-routines except GELG and ARCSIN are listed in APPENDIX IV. It should be noted that the notation in the programs occasionally differs from that in the text:

<table>
<thead>
<tr>
<th>Text</th>
<th>Program</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_a$</td>
<td>$L_o$</td>
</tr>
<tr>
<td>$M_a$</td>
<td>$L_{ab}$</td>
</tr>
</tbody>
</table>

1. **CONT**: Finds the solution for the controlled rectifier bridge case.
2. **MACH2**: Finds the value of $L_a$ which produces the minimum value of the 6th harmonic of $v_o$.
3. **MAST**: Finds the minimum filter weight for a given set of specifications.
4. **PLTDAT**: General purpose program that includes various simulations for both the controlled and uncontrolled rectifier bridge.
5. **SENSI3**: Sensitivity analysis program.
6. **TABLE**: Determines the harmonics of $v_o$ and $i_f(rms)$ for various values of $L_a$.
7. **TEST2**: Calling program for fault current simulation.
8. **UNCONT**: Finds the solution for the uncontrolled rectifier bridge case.
MAIN PLOTTER PROGRAM

C DIMENSION FD(5), F(5,1), FF(5,5), MUS(50), RMSIF(50), 
1 RMS(50), ALIF(21), TBETA(21), TMU(21), FF1(4,4), F1(4,1), 
1 FD1(4), ILS(50), BETAS(50), IFS(50), APIL(50), APBETA(50), 
1 IAPMU(50), APIF(50), AHAR6(50), AHAR12(50), AHAR18(50), 
1 AHAR24(50), AHAR30(50), APRMS(50), HAR6(50), HAR12(50), 
1 HAR18(50), HAR24(50), HAR30(50), AL0(21), XHAR6(21), XHAR12(50), 
1 XHAR18(21), XHAR24(21), XHAR30(21), AIR(60), AID(60), AIF(60), 
1 AIIK(60), THETA(60) 

REAL*4 ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

C INPUT MACHINE PARAMETERS

WRITE(7,50) 

50 FORMAT( '0',2X, 'THIS IS THE DATA FOR THE PHASE CONTROLLED 
1 BRIDGE RECTIFIER')

LF=0.12E 01 
LD=0.82E-07 
MF=0.79E-02 
MD=0.38E-05 
MF=0.19E-03 
LAB=0.15E-03 
L0=0.3E-03 

C L0=LA AND LAB=MA 
K1=1.0 
VL=6760.0 
IL=1420.0 
OMEGA=2513.27 
FREQ=400.0 
KF=(MF*LD-MD*MFD)/(LF*LD-(MFD)**2) 
KD=(1*LD*MF-MF*MFD)/(LF*LD-(MFD)**2) 
MO=MD/LD 
M00=MD**2/LD 
M0=M00 
DELTAF=M0+M00 
DELTAO=(1,333*(L0+LAB))-DELTAF 
DIL=1420.0/15.0 
KF=MF/SDRT(LF*L0) 
LAB=LAB/L0 
KOD=KD/SDRT(L0*LD) 
DLO=0.0 
IL=1420.0 
K3=0 
MF=KFO*SDRT(LF*L0) 
LAB=KAB*L0 
MD=KOD*SDRT(L0*LD) 
M00=MD**2/LD 
M0=M00 
DELTAF=M0+M00 
DELTAO=(1,333*(L0+LAB))-DELTAF 
KF=(MF*LD-MD*MFD)/(LF*LD-(MFD)**2) 
CALL NEWTON(MU,BETA,IF,FLV,FREQ,DELTAO,IL,VL,M00,K1, 
IF,M0,OMEGA,DELTAF,ZETA) 
IL=0.0 
IF=1.1*IF 
D0 100 LLL=1,17 
K3=0
CALL PHACON(IF,MF,BETA,M0,W,V,IL,MU,DELTAO,OMEGA,VL,M00)
APIL(LLL)=IL
APBETA(LLL)=BETA*180.0/3.1416
APMU(LLL)=MU*180.0/3.1416
CALL FS(MU,BETA,OMEGA,IL,IF,M0,DELTAO,MF,W,V,L0,LAB,
1DELTAF,LLL,AHAR6,AHAR12,AHAR18,AHAR24,AHAR30)
CALL RMS(BETA,LLL,MU,APRMS,IL,DELTAO,MF,M0,W,V,M00,IF,
1KF,DELTAF)
IL=IL+DIL
100 CONTINUE
WRITE(7,300)I0,IF
300 FORMAT(’0’,5X,’0=’,E10.3,5X,’IF=’,E10.3)
WRITE(7,203)
203 FORMAT(’0’,5X,’IL,IFRMS,6TH,12TH,18TH,BETA,MU’)
DO 201 KK=1,17
WRITE(7,202)APIL(KK),APRMS(KK),AHAR6(KK),AHAR12(KK),
1AHAR18(KK),APBETA(KK),APMU(KK)
201 CONTINUE
202 FORMAT(’’,2X,F7.1,2X,F7.3,2X,E10.3,2X,E10.3,2X,
1E10.3,2X,F7.1,2X,F5.2)
STOP
END
DIMENSION FD(5),F(5,1),FF(5,5),MUS(50),RMSIF(50),
1RMS(50),ALIF(21),TBETA(21),MU(21),FFI(4,4),F1(4,1),
1FD(1,4),ILS(50),BETAS(50),IFS(50),APIL(50),APBETA(50),
1APMU(50),APIF(50),AHAR6(50),AHAR12(50),AHAR18(50),
1AHAR24(50),AHAR30(50),APRMS(50),HAR6(50),HAR12(50),
1HAR18(50),HAR24(50),HAR30(50),ALO(21),XHAR6(21),XHAR12(50),
1XHAR18(21),XHAR24(21),XHAR30(21),AIQ(60),AILD(60),AILF(60),
1AIK(60),THETA(60)
REAL*4 M0,M00,MU,IF,IL,K1,LD,LF,MF,MFD,L0,MD,K0,
1NN1,NN2,OMEGA,LA,MUP,MUF,KFO,KAB,KOD,ILS,IFS,MUS,KF,KD
C INPUT MACHINE PARAMETERS
LF=0.12E+01
LD=0.82E-07
MF=0.79E-02
MD=0.38E-05
MFD=0.19E-03
LAB=0.15E-03
L0=0.3E+03
C L0=LA AND LAB=MA
K1=1.0
VL=6760.0
IL=1420.0
OMEGA=2513.27
FREQ=400.0
KF=(MF*LD-MD*MFD)/(LF*LD-(MFD)**2)
KD=(MD*LF-MF*MFD)/(LF*LD-(MFD)**2)
K0=MD/LD
M00=MD**2/LD
M0=M00
DELTA=MO+M00
DELT0=(1.333*(L0+LAB))-DELTAF
KFO=MF/SORT(LF*L0)
KAB=LAB/L0
KOD=MD/SORT(L0*LD)
DLO=0.0
H&M=9999.0
DO 200 L=1,21
L0=L0+DLO
KK=0
MF=KFO*SORT(LF*L0)
LAB=KAB*L0
MD=KOD*SORT(L0*LD)
M00=MD**2/LD
M0=M00
DELTA=MO+M00
DELT0=(1.333*(L0+LAB))-DELTAF
KF=(MF*LD-MD*MFD)/(LF*LD-(MFD)**2)
CALL NEWTON(MU,BETA,IF,W,V,FREQ,DELTAF,IL,MU,DELTA0,OMEGA,VL,M00,K1,
1MF,M0,OMEGA,DELTA,ZETA)
DLO=0.03E-03
IF=1.1*IF
CALL PHACON(IF,MF,BETA,M0,W,V,IL,MU,DELTAF0,OMEGA,VL,M00)
CALL FS(MU,BETA,OMEGA,IL,IF,M0,DELTAF,MF,W,V,L0,LAB,
1DELTAF,L,MU,AHAR6,AHAR12,AHAR18,AHAR24,AHAR30)
CALL RMS(BETA,L,MU,APRMS,IL,DELTAF,MF,M0,W,V,M00,IF,
1KF,DELTAF)
WRITE(7,302)L0,AHAR6(L),APRMS(L)
116.
302 FORMAT('0',2X,'LA = ',1E10.3,5X,'6 TH HARMONIC OF VO = ',
1E10.3,5X,'IF RMS = ',1E10.3)
IF(HEM,LT,AHAR6(L)) GO TO 200
HEM=AHAR6(L)
BLA=LO
BAPRMS=APRMS(L)
200 CONTINUE
WRITE(7,301)
301 FORMAT('0',5X,'THIS IS THE OPTIMUM ARMATURE INDUCTANCE')
WRITE(7,300)BLA,HEM,BAPRMS
300 FORMAT('0',2X,'BEST LA = ',1E10.3,5X,'PEAK 6TH HARM. OF VO = ',
1E10.3,1X,'PEAK IF RMS = ',1E10.3)
LO=BLA
MF=KFO*SQRT(LF*LO)
LAB=KAB*LO
MD=KOD*SQRT(LO*LD)
LAB=KAB*LO
MD=KOD*SQRT(LO*LD)
MOO=MD*MD/LD
MO=MOO
DELTAF=MO+MOO
DELTAO=(1.3333*(LO+LAB))-DELTAF
KF=(MF*LD-MD*MF)/(LF*LD-MF*FD*FD)
RETURN
END
C MASTER OPTIMIZATION OF L-C FILTER DESIGN
REAL*4 K1,K2,ILFMAX,IF,MU,LA,LAB,LF,LD,MD,MF,MFD,
1IL,M0,LWT,LEN,N1,L0
21 CONTINUE
WRITE(7,1)
WRITE(7,104)
104 FORMAT('O',5X,'ALL INPUTS HAVE FORMAT = F7.2 UNLESS
1 OTHERWISE SPECIFIED')
1 FORMAT('O',1X,'WRITE THE FOLLOWING PARAMETERS FOR THE FILTER')
WRITE(7,2)
2 FORMAT('O',1X,'CAP. ENERGY DENSITY (JOULES/LB.) =')
100 FORMAT(F7.2)
103 FORMAT('O',5X,'FORMAT=I2')
101 FORMAT(I2)
READ(5,100)DC
WRITE(7,4)
4 FORMAT('O',1X,'CURRENT DENSITY FOR L WIRE (CIR MIL/AMP) =')
READ(5,100)CMA
WRITE(7,6)
6 FORMAT('O',1X,'MAX. RMS VALUE OF 6TH HARM. OF VO (VOLTS) =')
READ(5,100)V2
WRITE(7,7)
7 FORMAT('O',1X,'ALLOWABLE PEAK FAULT CURRENT (AMPS) =')
READ(5,100)ILFMAX
IL=1420.0
VL=6760.0
LF=1.2
LD=0.82E-07
MFD=0.19E-03
OMEGA=2513.27
DO 50 IZ=1,2
C IZ=1 IS FOR LA NORMAL
C IZ=2 IS FOR LA OPTIMUM
IF(IZ.EQ.2) GO TO 51
IF=275.0
BETA=2.12
MU=0.307
LA=0.300E-03
LAB=0.15E-03
C LAB=MU
MD=0.38E-05
MF=0.79E-02
W=144.0
V=-138.0
DELTAD=0.248E-03
M0=0.176E-03
V1=0.120E-04
WRITE(7,53)LA
53 FORMAT('O',2X,'THE FOLLOWING VALUES ARE BASED ON NORMAL
1LA =''E10.3,2X,'H. ')
GO TO 52
51 CONTINUE
IL=1420.0
IF=239.0
BETA=2.28
MU=0.604
LA=0.720E-03
LAB=0.360E-03
MD=0.589E-05
MF=0.122E-01
W=164.0
V=-340.0
DELTAO=0.595E-03
MO=0.423E-03
V1=620.0
WRITE(7,54)LA
54  FORMAT(’0’,2X,’THE FOLLOWING VALUES ARE BASED ON’)
      10  OPTIMUM LA = ’E10.3’,2X,’H.’)
WRITE(7,300)IF,BETA,MU
300  FORMAT(’2X,’IF=’,E10.3,5X,’BETA=’,E10.3,5X,’MU=’,E10.3)
WRITE(7,305)IL,VL,V1
305  FORMAT(’2X,’IL=’,E10.3,5X,’VL=’,E10.3,5X,’V1=’,E10.3)
K1=V2/V1
C  FIND CONDUCTOR AREA IN CIR MILS & SQ. CM.
CH=CMA*IL
A16=(1+K1)/(K1*36*OMEGA**2)
AM=CM*5.07E-10
FW=AM
F=0.7
D=(5937.8)
A2=2*(VL*FW)**2*(1+1)/(DC*51.1E-07*K1*36*(OMEGA**2)*F*F)
A3=3.1416*FW
A=(5*A2/(3*A3))**0.125
N=A**2*FW
LO=(25.5E-07)**(F/FW)**2)***(A)**5
CO=A16/LO
      RES=N**3.1416*A*(2.83E-08)/FW
WRITE(7,55)L0,CO,RES
55  FORMAT(’0’,2X,’OPT. VALUES BEFORE FAULT TEST’)
      1 ARE LO=’,E10.3,1X,’H,’ CO=’,E10.3,1X,’FD,’1X,’RES=’,E10.3)
K=1
15  RES=N**3.1416*A*(2.83E-08)/FW
CALL FAULT2(IF,BETA,MU,LA,LAB,LF,LD,MD,MF,MFD,W,V,IL,
1OMEGA,DELTAO,MO,VL,LO,ILFMAX,RES)
WRITE(7,345)IL
345  FORMAT(’2X,’MAX LOAD CURRENT FROM FAULT =’,E10.3)
      IF(IL.LT.ILFMAX) GO TO 14
K=K+1
L0=LO*1.1
CO=A16/LO
      A=((LO*(FW/F)**2)/25.5E-07)**0.2
N=(A**2)*FW
IF(K.GT.100) GO TO 14
IL=1420.0
IF(K.GT.2) GO TO 15
WRITE(7,16)
16  FORMAT(’0’,2X,’FAULT CURRENT TOO LARGE, LO INCREASED’)
      GO TO 15
14  CONTINUE
LWT=A3*A**3
CWT=A2/(A**5)
WT=LWT+CWT
WRITE(7,17)L0,CO,IL,RES
119.
17 FORMAT('O',1X,'LO= ',E10.3,5X,'C0= ',E10.3,5X,'ILF= ',E10.3,1X,
     1 RES=' ',E10.3)
WRITE(7,18)LWT,CWT,WT
18 FORMAT('O',1X,'LWT= ',E10.3,5X,'CWT= ',E10.3,5X,'TOTAL WT = ',E10.3)
RAD=2*A
WRITE(7,19)N,RAD,A
19 FORMAT('O',1X,'NO. TURNS = ',I4,5X,'L RADIUS = ',
     1E10.3,'M',5X,'L LENGTH = ',E10.3,'M')
50 CONTINUE
WRITE(7,20)
20 FORMAT('O',1X,'WRITE "0" TO END, OR "1" FOR ANOTHER RUN')
WRITE(7,103)
READ(5,101)KEY
IF(KEY.GT.0) GO TO 21
STOP
END

120.
C MAIN PLOTTER PROGRAM
DIMENSION FD(5),F(5,1),FF(5,5),MUS(50),RMSIF(50),
1XRMS(50),ALIF(21),TBETA(21),TMU(21),FF1(4,4),F1(4,1),
1FD1(4),ILS(50),BETAS(50),IFS(50),APIL(50),APBETA(50),
1APMU(50),APIF(50),AHAR6(50),AHAR12(50),AHAR18(50),
1AHAR24(50),AHAR30(50),APRMS(50),HAR6(50),HAR12(50),
1HAR18(50),HAR24(50),HAR30(50),AL0(21),XHAR6(21),XHAR12(50),
1XHAR18(21),XHAR24(21),XHAR30(21),AIQ(60),AI6(60),
1AIK(60),THETA(60).
REAL*4 MO,M00,MU,IF,IL,K1,LD,LF,MF,MFD,LAB,LO,MD,K0,
1NN1,NP2,OMEGA,LA,MUF,MOF,KF0,KAB,KOD,IFS,APIL,FD1,
1FD1,ILS,IFS,APIL,MU,M0,M00,APBETA,ALIF.

C INPUT MACHINE PARAMETERS
LF=0,12E-01
LD=0,82E-07
MF=0,79E-02
MD=0,38E-05
MFD=0,19E-03
LAB=0,15E-03
LO=3E-03

C LO=LA AND LAB=MA
K1=1,0
VL=6760,0
IL=1420,0
OMEGA=2513,27
FREQ=400,0
KF=(MF*LD-MD*MFD)/(LF*LD-(MFD)**2)
KD=(MD*LF-MF*MFD)/(LF*LD-(MFD)**2)
KD=MD/LD
M00=MD**2/LD
M0=M00
DELTAF=M0+M00
DELTAF=(1.333*(LO+LAB))-DELTAF
DIL=2130,0/50,0
CALL NEWTON(MU,BETA,IF,W,V,FREQ,DELTAF,IL,VL,M00,K1,
1MF,M0,OMEGA,DELTAF,ZETA)
CALL LITLI(DELTAF,MF,IF,BETA,M0,W,
1M00,V,IL,DELTAF,MU,AID,AIG,AIF,AIK,THETA,K0,KF,KD)
KF=M0/SQR(LF/L0)
KAB=LAB/L0
KOD=MD/SQR(LO*LD)
DLO=0,0
IL=1420,0
DD=200 L=1,21
LO=LO+DLO
KKK=0
MF=K0*SQR(LF/L0)
LAB=KAB,0
MD=KOD*SQR(LO*LD)
M00=MD**2/LD
M0=M00
DELTAF=M0+M00
DELTAF=(1.333*(LO+LAB))-DELTAF
KF=(MF*LD-MD*MFD)/(LF*LD-(MFD)**2)
CALL NEWTON(MU,BETA,IF,W,V,FREQ,DELTAF,IL,VL,M00,K1,
1MF,M0,OMEGA,DELTAF,ZETA)
IF=1,1*IF
CALL PHACON(IF,MF,BETA,M0,W,V,IL,MU,DELTAF,OMEGA,VL,M00)
AL0(L)=L0
CALL FS(MU,BETA,OMEGA,IL,IF,MO,DELTA0,MF,W,V,L0,LAB,
1DELTAFL,L,XHAR6,XHAR12,XHAR18,XHAR24,XHAR30)
CALL RMS(BETA,L,MU,XRMS,IL,DELTA0,MF,MO,W,V,MOO,IF,
1KF,DELTAF)
ALIF(L)=IF
TBETA(L)=BETA*180./3.1416
TMU(L)=MU*180./3.1416
DL0=0.03E-03
CONTINUE
WRITE(7,201)
DO 202 L=1,21
WRITE(7,203)AL0(L),XRMS(L),XHAR6(L),XHAR12(L),XHAR18(L),
1TBETA(L),TMU(L)
202 CONTINUE
201 FORMAT(’ ’,5X,’L0’,’10X,’IF’,’10X,’6TH’,’10X,’12TH’,’10X,’18TH’,
112X,’BETA’,’12X,’MU’)
203 FORMAT(’ ’,1X,7E13.4)
L0=0.3E-03
LAB=0.15E-03
MD=0.38E-05
MOO=MD*MD/LD
MO=MO0
MF=0.79E-02
KF=(MF*LD-MD*MFD)/(LF*LD-MFD*MFD)
DELTAF=M0+M00
1DELTAF=(1.3333*(L0+LAB))-DELTAF
IL=0.0
WRITE(7,2800)
DO 100 LLL=1,50
KKK=0
CALL NEWTON(MU,BETA,IF,W,V,FREQ,DELTAO,IL,VL,MO0,K1,
1MF,MO,OMEGA,DELTAF,ZETA)
ILS(LLL)=IL
BETAS(LLL)=BETA*180.0/3.1416
MUS(LLL)=MU*180.0/3.1416
IFS(LLL)=IF
2800 FORMAT(’ ’,4X,’IL’,”BETA’,”7X,’MU’,”8X,’IF’)
CALL FS(MU,BETA,OMEGA,IL,IF,MO,DELTA0,MF,W,V,L0,LAB,
1DELTAFL,L,XHAR6,XHAR12,XHAR18,XHAR24,XHAR30)
CALL RMS(BETA,L,L,MU,XRMS,IF,DELTA0,MF,MO,W,V,MOO,IF,
1KF,DELTAF)
IF=1.1*IF
CALL PHACON(IF,MF,BETA,MO,W,V,IL,MO,DELTAO,OMEGA,VL,MO0)
APIL(LLL)=IL
APBETA(LLL)=BETA
APMU(LLL)=MU
APIF(LLL)=IF
CALL FS(MU,BETA,OMEGA,IL,IF,MO,DELTAO,MF,W,V,L0,LAB,
1DELTAFL,L,XHAR6,XHAR12,XHAR18,XHAR24,XHAR30)
CALL RMS(BETA,L,L,MU,XRMS,IL,DELTA0,MF,MO,W,V,MOO,IF,
1KF,DELTAF)
IL=IL+DIL
IF(LLL.EQ.1) IL=35.0
CONTINUE
DO 101 LLL=1.50
WRITE(7,2801) ILS(LLL), BETAS(LLL), MUS(LLL), IFS(LLL)

101 CONTINUE

DO 102 LLL=1,50
WRITE(7,2802) LLL, HAR6(LLL), HAR12(LLL), HAR18(LLL), 
HAR24(LLL),
HAR30(LLL)
2802 FORMAT(3E13.4)

102 CONTINUE

DO 103 LLL=1,50
WRITE(7,2803) LLL, RMSIF(LLL)
2803 FORMAT(3E20.5)

103 CONTINUE

110 104 L=1,50
WRITE(7,2802) L, AHAR6(L), AHAR12(L), AHAR18(L), 
AHAR24(L), AHAR30(L)

104 CONTINUE

DO 105 L=1,50
WRITE(7,2803) L, APRMS(L)
105 CONTINUE

WRITE(7,110)
110 FORMAT(3E13.4)

110 106 L=1,50
WRITE(7,107) APIL(L), APBETA(L), APMU(L), APIF(L)

107 FORMAT(3E13.4)

106 CONTINUE

STOP

END
C TEST PROGRAM FOR NEWTON SENSITIVITY ANALYSIS

DIMENSION FIt(S), F(5, 1), RFF(5, 5), AIF(11), ABETA(11), AMU(11),
1AW(11), AV(11), A0(11), A00(11), ALAMD(11)
REAL*4 M0, M00, MU, IF, IL, K1, LD, LF, MF, MFD, LAB, LO, MD, KD, ILMAX,
1LA, KFO, KAD, K0, KD, IK, MA, IFT, IQ, LAMQ, LAMD, ILO, ILC, IKO, IKC
DO 34 11=1,7
LF=0.12E 01
LD=0.82E-07
MF=0.79E-02
MD=0.38E-05
MFD=0.19E-03
LAB=0.15E-03
LO=0.3E-03
C LO=LA AND LAB=MA
CAF=PIF/SORT (LO*LF)
CAB=LAB/LO
CAD=MD/SORT (LO*LD)
CFD=MFD/SORT (LF*LD)
DO 35 12=1,11
IF(I1.GT .6) GO TO 36
IF(I1.GT .5) GO TO 37
IF(I1.GT .4) GO TO 38
IF(I1.GT .3 ) GO TO 39
IF(I1.GT .2) GO TO 40
IF(I1.GT .1) GO TO 41
LF=0.6+0.12*(I2-1)
MF=CAF*SORT (LO*LF)
MFD=CFD*SORT (LF*LD)
A1(I2)=LF
GO TO 42
35 L1'=0.41E-07+(0.082E-07)*(I2-1)
LD=CAF*SORT (LO*LD)
MFD=CFD*SORT (LF*LD)
A1(I2)=LD
GO TO 42
36 LD=0.41E-07+(0.082E-07)*(I2-1)
MF=0.395E-02+(0.079E-02)*(I2-1)
A1(I2)=MF
GO TO 42
37 MF=0.095E-03+(0.019E-03)*(I2-1)
A1(I2)=MFD
GO TO 42
38 MF=0.095E-03+(0.019E-03)*(I2-1)
A1(I2)=HD
GO TO 42
39 MF=0.095E-03+(0.019E-03)*(I2-1)
A1(I2)=LAB
GO TO 42
40 MF=0.095E-03+(0.019E-03)*(I2-1)
A1(I2)=LO
11=1.0
VL=6760.0
IL=1420.0
OMEGA=2513.27
FREQ=400.0  
KF=(MF*LD-MD*MFD)/(LF*LD-MFD*MFD)  
KD=(MD*LF-MF*MFD)/(LF*LD-MFD*MFD)  
KQ=MD/LD  
MOO=MD**2/LD  
AMOO(I2)=MOO  
AMO(I2)=KF*MF+KD*MD  
ALAMO(I2)=AMO(I2)-MOO  
MO=MOO  
DELTA=MO+MOO  
DELTA0=(1.3333*(LO+LAB))-DELTA  
CONTINUE  
CALL NEW3DN(MU,BETA,IF,W,V,FREQ,DELTA0,IL,VL,MOO,K1,  
MF,M0,OMEGA,DELTA,F,KEI)  
IF(K,LT.200) GO TO 80  
AIF(I2)=0.0  
ABETA(I2)=0.0  
AMU(I2)=0.0  
AW(I2)=0.0  
AV(I2)=0.0  
GO TO 35  
CONTINUE  
AIF(I2)=IF  
ABETA(I2)=BETA  
AMU(I2)=MU  
AW(I2)=W  
AV(I2)=V  
CONTINUE  
IF(I1.GT.6) GO TO 46  
IF(I1.GT.5) GO TO 47  
IF(I1.GT.4) GO TO 48  
IF(I1.GT.3) GO TO 49  
IF(I1.GT.2) GO TO 50  
IF(I1.GT.1) GO TO 51  
WRITE(7,100)  
10 FORMAT('0',5X,'LF VARIATION')  
GO TO 52  
WRITE(7,101)  
1 FORMAT('0',5X,'LD VARIATION')  
GO TO 52  
WRITE(7,102)  
2 FORMAT('0',5X,'MF VARIATION')  
GO TO 52  
WRITE(7,103)  
3 FORMAT('O',5X,'MFD VARIATION')  
GO TO 52  
WRITE(7,104)  
4 FORMAT('0',5X,'MD VARIATION')  
GO TO 52  
WRITE(7,105)  
5 FORMAT('0',5X,'LAB VARIATION')  
GO TO 52  
WRITE(7,106)  
6 FORMAT('0',5X,'LO VARIATION')  
CONTINUE  
WRITE(7,120)  
20 FORMAT('0',7X,'PARAMETER'8X,'IF',11X,'BETA',11X,'MU')
DO 55 L=1, 11
WRITE (7, 110) A1 (L), AIF (L), ABETA (L), AMU (L), AW (L), AV (L),
  1AM00 (L), AM0 (L), AALAM (L)
110 FORMAT (' ', 2X, 9E14.4)
55 CONTINUE
34 CONTINUE
STOP
END
C PROGRAM TABLE.FOR
C MAIN PLOTTER PROGRAM
DIMENSION FD(5), F(5,1), FF(5,5), MUS(50), RMSIF(50),
1XRS(50), ALIF(21), TBETA(21), TMU(21), FF1(4,4), F1(4,1),
1FD(4), ILS(50), BETAS(50), IFS(50), APIL(50), APBETA(50),
1APMU(50), APIF(50), AHAR6(50), AHAR12(50), AHAR24(50),
1AHAR4(50), AHAR30(50), APRMS(50), HAR6(50), HAR12(50),
1HAR24(50), HAR30(50), ALO(21), XHAR6(21), XHAR12(50),
1XHAR24(21), XHAR30(21), ALO(60), AID(60), AIF(60),
1AIL(60), THETA(60)
REAL*4 MO, MOO, MU, IF, IL, K1, LD, LF, MF, MFD, LAB, LO, MD, KQ,
1NN, NF2, OMEGA, LA, MUP, MUF, KFO, KAB, KOD, ILS, IFS, MUS, KF, KD
C INPUT MACHINE PARAMETERS
LF=0.12E01
LD=0.82E-07
MF=0.79E-02
MD=0.38E-05
MFD=0.19E-03
LAB=0.15E-03
LO=0.3E-03
K1=1.0
OMEGA=2513.27
FREQ=400.0
KF=(MF*LD-MD*MFD)/(LF*LD-(MFD)**2)
KD=(MD*LF-MF*MFD)/(LF*LD-(MFD)**2)
KQ=MD/LD
MOO=MD**2/LD
MO=MOO
DELTAF=M0+MOO
DELTAO=(1.1333*(LO+LAB)) - DELTAF
DIL=1420.0/15.0
KFO=MF/SQRT(LF*LO)
KAB=LAB/LO
KOD=MD/SQRT(LO*LD)
DL=0.0
DO 200 L=1, 21
IL=1420.0
LO=LO+DL0
KKK=0
MF=KFO*SQRT(LF*LO)
LAB=KAB*LO
MD=KOD*SQRT(LO*LD)
MOO=MD**2/LD
MO=MOO
DELTAF=M0+MOO
DELTAO=(1.3333*(LO+LAB)) - DELTAF
KF=(MF*LD-MD*MFD)/(LF*LD-(MFD)**2)
CALL NEWTON(MU, BETA, IF, W, V, FREQ, DELTAO, IL, VL, MOO, K1,
1MF, MO, OMEGA, DELTAF, ZETA)
DL=0.03E-03
IL=0.0
IF=1.1*IF
DO 100 LLL=1, 17
KKK=0
CALL PHACON(IF, MF, BETA, MO, W, V, IL, MU, DELTAO, OMEGA, VL, MOO)

127.
APIL(LLL)=IL
APBETA(LLL)=BETA*180.0/3.1416
APMU(LLL)=MU*180.0/3.1416
CALL FS(MU,BETA,OMEGA,IL,IF,MO,DELTA0,MF,W,V,LO,LAB,
1DELTAF,LLL,AHAR6,AHAR12,AHAR18,AHAR24,AHAR30)
CALL RMS(BETA,LLL,MU,APRMS,IL,DELTA0,MF,MO,W,V,M00,IF,
1KF,DELTAF)
IL=IL+DIL
100 CONTINUE
WRITE(7,300)LO,IF
300 FORMAT(0,5X,LO=,E10.3,5X,IF=,E10.3)
WRITE(7,203)
203 FORMAT(0,5X,IL,IFRMS,6TH,12TH,18TH,BETA,MU)
DO 201 KK=1,17
   WRITE(7,202)APIL(KK),APRMS(KK),AHAR6(KK),AHAR12(KK),
1AHAR18(KK),APBETA(KK),APMU(KK)
201 CONTINUE
200 CONTINUE
202 FORMAT(0,2X,F7.1,2X,F7.2,2X,E10.3,2X,E10.3,2X,
1E10.3,2X,F7.1,2X,F5.2)
STOP
END
C TEST PROGRAM FOR FAULT

```
DIMENSION FDC5(5,1),FF(5,5),A(4,4),B(5,5),RH(5,1),CIL(300),
1CIK(300),CWT(300)
REAL*4 MO,M00,MU,IF,IL,K1,LD,LF,MF,MFD,LAB,LO,MD,KD,ILFMAX,
ILFMAX,KF,KD,KD,KF,IK,M1,IFT,IQ,LAMQ,LAMD,IL0,ILC,IK0,IKC
LF=0.12E 01
LD=0.82E-07
MF=0.79E-02
MD=0.38E-05
MFD=0.19E-03
LAB=0.15E-03
LO=0.3E-03
LA=LO
ILFMAX=0.1E 06
WRITE(7,200)
200 FORMAT('0',2X,'TYPE THE VALUE OF RP DESIRED')
READ(5,201)RP
201 FORMAT(E20.10)
K1=1.0
VL=6760.0
IL=1420.0
OMEGA=2513.27
FREQ=400.0
KF=(MF*LD-MD*MFEI)/(LF*LD-MD*I*MFD)
KD=(MD*LF-MF*MFD)/(LF*LD-MD*I*MFD)
KO=MF/LD
M00=MF*2/LD
M0=M00
DELTAF=M0+M00
DELTAO=(1.3333*(LO+LAB))-DELTAF
CALL NEUTON(MU,BETA,IF,W,V,FREQ,DELTAO,IL,VL,M00,K1,
1MF,M0,OMEGA,DELTAF,ZETA)
500 WRITE(7,299)
299 FORMAT('0',1X,'ENTER VALUE OF LO DESIRED---FORMAT E10.3')
READ(5,298)L0
298 FORMAT(E10.3)
10 FORMAT(0',1X,5E13.4)
IL=1420.0
CALL FAULT2(IF,BETA,MU,LA,LAB,LF,LD,MD,MF,MFD,W,VL,IL,
1OMEGA,DELTA0,M0,VL,LO,ILFMAX,RP)
WRITE(7,300)IL
300 FORMAT(0',3X,'ILFMAX FROM FAULT =E10.3')
WRITE(7,600)
600 FORMAT(0',3X,'TYPE '0' TO END, OR '1' FOR ANOTHER RUN')
READ(5,700)KEY
700 FORMAT(I4)
IF(KEY.GT.0) GO TO 500
STOP
END
```
C MAIN PLOTTER PROGRAM
DIMENSION FD(5),FF(5,5),MUS(50),RMSIF(50),
1XRMS(50),ALIF(21),TBETA(21),TMU(21),FF1(4,4),F1(4,1),
1FD1(4),ILS(50),BETAS(50),IFS(50),APIL(50),APBETA(50),
1APMU(50),APIF(50),AHAR6(50),AHAR12(50),AHAR18(50),
1AHAR24(50),AHAR30(50),APRMS(50),HAR6(50),HAR12(50),
1HAR18(50),HAR24(50),HAR30(50),ALO(21),XHAR6(21),XHAR12(50),
1XHAR18(21),XHAR24(21),XHAR30(21),AIQ(60),AID(60),AIF(60),
1AIK(60),THETA(60)
REAL*4 MO,M00,MU,IF,IL,K1,L1,LF,MF,MFD,LAB,LO,MD,KD,
1NN1,NN2,OMEGA,LA,MUF,MUF,K0,KOD,ILS,IFS,MUS,KF,KD
C INPUT MACHINE PARAMETERS
WRITE(7,50)
50 FORMAT('O',2X,'THIS IS THE DATA FOR THE UNCONTROLLED
1 BRIDGE RECTIFIER')
LF=0.12E 01
LD=0.82E-07
MF=0.79E-02
MD=0.38E-05
MFD=0.19E-03
LAB=0.15E-03
LO=0.3E-03
C LO=LA AND LAB=MA
K1=1.0
UL=6760.0
IL=1420.0
OMEGA=2513.27
FREQ=400.0
KF=(MF*LD-MD*MFD)/(LF*LD-(MFD)**2)
KD=(MD*LF-MF*MFD)/(LF*LD-(MFD)**2)
KO=MD/LD
M00=MD**2/LD
MO=M00
DELTAF=M0+M00
DELTAF0=(1.333*(LO+LAB))-DELTAF
DIL=1420.0/15.0
KFO=MF/SQRT(LF*LO)
KAB=LAB/LO
KOD=MD/SQRT(LO*LD)
DLO=0.0
IL=1420.0
KKK=0
MF=KFO* SQRT(LF*LO)
LAB=KAB*LO
MD=KOD* SQRT(LO*LD)
M00=MD**2/LD
MO=M00
DELTAF=M0+M00
DELTAF0=(1.3333*(LO+LAB))-DELTAF
KF=(MF*LD-MD*MFD)/(LF*LD-(MFD)**2)
IL=0.0
DO 100 LLL=1,17
KKK=0
CALL NEWTON(MU,BETA,IF,W,V,FREQ,DELTAF0,IL,UL,M00,K1,
1MF,MO,OMEGA,DELTAF,ZETA)
APIL(LLL)=IL
APIF(LLL) = IF
APBETA(LLL) = BETA * 180.0 / 3.1416
APMU(LLL) = MU * 180.0 / 3.1416
CALL FS(MU, BETA, OMEGA, IL, IF, MO, DELTAO, MF, W, V, L0, LAB, I, DELTAF, LLL, AHAR6, AHAR12, AHAR18, AHAR24, AHAR30)
CALL RMS(BETA, LLL, MU, APRMS, IL, DELTAO, MF, MO, W, V, M00, IF, I, KF, DELTAF)
IL = IL + DIL
100 CONTINUE
WRITE(7, 300) L0
300 FORMAT('0', 5X, 'L0=', , E10.3)
WRITE(7, 203)
203 FORMAT('0', 5X, 'IL, IFRMS, 6TH, 12TH, 18TH, BETA, MU, IF')
DO 201 KK = 1,17
WRITE(7, 202) APIL(KK), APRMS(KK), AHAR6(KK), AHAR12(KK), AHAR18(KK), APBETA(KK), APMU(KK), APIF(KK)
201 CONTINUE
STOP
END
APPENDIX IV. SUBROUTINES

The following subroutines for the main programs of Appendix III are listed in alphabetical order. Subroutines GELG and ARSIN are not included. GELG is a program for solving simultaneous equations that is part of the IBM Scientific Subroutine Package. ARSIN is a series for the arcsin function. It should be noted that the notation in the programs occasionally varies from that in the text:

<table>
<thead>
<tr>
<th>Text</th>
<th>Program</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_a$</td>
<td>$L_o$</td>
</tr>
<tr>
<td>$M_a$</td>
<td>$L_{ab}$</td>
</tr>
</tbody>
</table>

1. **FS**: Finds the harmonics of $v_o$.
2. **FAULT2**: Calculates the fault current.
3. **JACOB**: Calculates the Jacobian matrix for the uncontrolled rectifier bridge.
4. **JACOB4**: Calculates the Jacobian matrix for the controlled rectifier bridge.
5. **LITLI**: Calculates $i_d$, $i_q$, $i_f$ and $i_k$ vs. $\theta$.
6. **NEWTON**: Newton-Raphson algorithm for the uncontrolled rectifier bridge.
7. **NEW3ON**: Same as NEWTON except variable K is included in argument list to test for convergence. Used only with SENS13.
8. **PHACON**: Newton-Raphson algorithm for the controlled rectifier bridge.
9. **RHS**: Calculates right hand side vector for **NEWTON**.

10. **RHS4B4**: Calculates right hand side vector for **PHACON**.

11. **RMS**: Find rms value of $i_f$.

12. **TERMA**: Performs repetitive calculation for **FS**.
SUBROUTINE FS(MU,BETA,OMEGA,IL,IF,MO,DELTAO,MF,W,V,LO,LAB,
DELTAF,LLL,HAR6,HAR12,HAR18,HAR24,HAR30)

DIMENSION CN(5),HAR6(50),HAR12(50),HAR18(50),HAR24(50),HAR30(50)
REAL*4 MU,IL,IF,MO,MF,LO,LAB

A=-OMEGA*(1.732*IF*PIF+2.865*MO*W)*1.91
B=2.865*OMEGA*MO*V*1.91
C=-2.865*OMEGA*IL*MO*1.91
DD=(OMEGA/DELTAO)*(1.5*MO-LO-LAB)
D=DD*(1.155*MF*IF+1.91*MO*W)*1.91
E=+1.91*MO*V*DD*1.91
F=0.955*IL*DELTAF*DD*1.91

DO 10 K=1,5
B1=BETA+MU-1.047
B2=B1+1.047
N=K*6
CALL TERMA(N,A,2.094,B2,AN1,BN1)
AN=AN1
BN=BN1
CALL TERMA(N,A,2.094,B1,AN1,BN1)
AN=AN-AN1
BN=BN-BN1
CALL TERMA(N,B,0.524,B2,AN1,BN1)
AN=AN+AN1
BN=BN+BN1
CALL TERMA(N,B,0.524,B1,AN1,BN1)
AN=AN-AN1
BN=BN-BN1
ANG=2.094-BETA-MU
CALL TERMA(N,C,ANG,B2,AN1,BN1)
AN=AN+AN1
BN=BN+BN1
CALL TERMA(N,C,ANG,B1,AN1,BN1)
AN=AN-AN1
BN=BN-BN1
B1=BETA
B2=BETA+MU
CALL TERMA(N,D,0.0,B2,AN1,BN1)
AN=AN+AN1
BN=BN+BN1
CALL TERMA(N,D,0.0,B1,AN1,BN1)
AN=AN-AN1
BN=BN-BN1
CALL TERMA(N,E,1.571,B2,AN1,BN1)
AN=AN+AN1
BN=BN+BN1
CALL TERMA(N,E,1.571,B1,AN1,BN1)
AN=AN-AN1
BN=BN-BN1
ANG=-BETA-MU
CALL TERMA(N,F,ANG,B2,AN1,BN1)
AN=AN+AN1
BN=BN+BN1
CALL TERMA(N,F,ANG,B1,AN1,BN1)
AN=AN-AN1
BN=BN-BN1
CN(K)=SQRT(AN**2+BN**2)
CONTINUE
HAR6(LLL)=CN(1)
HAR12(LLL)=CN(2)
HAR18(LLL)=CN(3)
HAR24(LLL)=CN(4)
HAR30(LLL)=CN(5)
RETURN
END
SUBROUTINE FAULT2(IF,BETA,MU,LA,LAB,LF,LF,LD,MD,MF,MFD,W,V,IL,
OMEGA,DELTALO,M0,VL,LO,ILFMAX,RP)
C ROTOR FLUX LINKAGES ASSUMED CONSTANT
DIMENSION A(4,4),B(5,5),CIL(300),CIK(300),CWT(300),
1ARH(4,1),BRH(5,1)
REAL*4 L0,KD,KF,KO,IK,IF,MU,LA,LAB,LF,LD,MD,MF,MFD,IL,IFT,
1ID,IQ,LAMQ,LAMF,LAMD,IL0,ILC,IK0,IKC,M0,ILFMAX
KK=O
JJ=O
C SPECIFY INITIAL CONDITIONS FOR CONDUCTION PERIOD
WT=BETA+MU-1.047
C L0=0.1
KD=(MD*LF-MF*MFD)/(LF*LD-MFD*MFD)
KF=(MF*LD-MD*MFD)/(LF*LD-MFD*MFD)
KO=MD/LD
IFT=IF+1.732*KF*((3/3.1416)*(W+IL*
1+COS(BETA+MU))-(IL*COS(WT+0.524)))
1+IL*SIN(WT+0.524))
F1=1/(LF*LD-MFD*MFD)
LAMQ=-1.732*MD*IL*SIN(WT+0.524)+LD*IQ
LAMF=1.732*IL*MF*COS(WT+0.524)+LF*IFT+MFD*ID
LAMD=1.732*IL*MD*COS(WT+0.524)+MFD*IFT+LD*ID
SF=F1*(LAMF*LD-LAMD*MFD)
SD=F1*(LAMD*LF-LAMF*MFD)
SQ=LAMQ/LD
A(1,1)=L0+2*(LA+LAB)
A(2,2)=1.0
A(3,3)=1.0
A(4,4)=1.0
A(2,3)=0.0
A(2,4)=0.0
A(3,2)=0.0
A(3,4)=0.0
A(4,2)=0.0
A(4,3)=0.0
B(1,1)=L0+2*(LA+LAB)
B(2,2)=1.0
B(3,3)=1.0
B(4,4)=1.0
B(2,3)=0.0
B(2,4)=0.0
B(3,2)=0.0
B(3,4)=0.0
B(4,2)=0.0
B(4,3)=0.0
B(5,1)=-LA-LAB
B(5,5)=2*(LA+LAB)
B(1,5)=-LA-LAB
C GO INTO CONDUCTION DO LOOP
DWT=3.1416/150
DT=DWT/2513.3
WT=WT+0.524
JUMP=O
110 WT=WT-1.047*JUMP
JUMP=JUMP+1
IF(JUMP.GE.4) GO TO 99
RA=0.0164
DO 1 K=1,3000
KEY=0
ILO=IL
42 KEY=KEY+1
IF(KEY.GT.2) GO TO 71
C FIND DY/DX AT 0
A(1,2)=1.732*MF*COS(WT)
A(1,3)=1.732*MD*COS(WT)
A(1,4)=-1.732*MD*SIN(WT)
A(2,1)=1.732*KF*COS(WT)
A(3,1)=-1.732*KD*COS(WT)
A(4,1)=-1.732*KQ*SIN(WT)
71 CONTINUE
ARH(1,1)=-(RP+2*RA)*IL+1.732*MD*OMEGA*SIN(WT)*
1IFT+1.732*MD*OMEGA*COS(WT)*ID
1+1.732*MD*OMEGA*COS(WT)*IQ
ARH(2,1)=1.732*KF*OMEGA*SIN(WT)*IL
ARH(3,1)=1.732*KD*OMEGA*SIN(WT)*IL
ARH(4,1)=1.732*KQ*OMEGA*COS(WT)*IL
ARH(1,1)=ARH(1,1)-A(1,2)*ARH(2,1)-A(1,3)*ARH(3,1)
1-A(1,4)*ARH(4,1))/(A(1,1)-A(1,2)*A(2,1)
1-A(1,3)*A(3,1)-A(1,4)*A(4,1))
ARH(2,1)=ARH(2,1)-A(2,1)*ARH(1,1)
ARH(3,1)=ARH(3,1)-A(3,1)*ARH(1,1)
ARH(4,1)=ARH(4,1)-A(4,1)*ARH(1,1)
C ARH(1,1)=DIL/DT AT 0
IF(KEY.GT.1) GO TO 2
WT=WT+DWT
IL=IL+ARH(1,1)*EI1
ILC=IL
DILDT=ARH(1,1)
GO TO 3
2 CONTINUE
IL=ILO+((DILDT+ARH(1,1))*DT)/2
3 CONTINUE
IFT=SF-1.732*IL*COS(WT)*KF
ID=SD-1.732*IL*COS(WT)*KD
IQ=SQ+1.732*KQ*IL*SIN(WT)
IF(KEY.LT.2) GO TO 12
IF(ABS(IL-ILC).LE.1.0) GO TO 10
ILC=IL
IF(KEY.GT.50) GO TO 997
GO TO 12
10 CONTINUE
KK=KK+1
CIL(KK)=IL
IF(IL.GT.ILFMAX) GO TO 997
CK(KK)=0.0
IF(KK.GE.299) GO TO 997
301 CONTINUE
C TEST FOR END OF CONDUCTION PERIOD
VBCT=aRA*IL+(LAB+LA)*ARH(1,1)-1.732*MF*SIN(WT-0.524)*ARH(2,1)
1-1.732*OMEGA*COS(WT-0.524)*(MF*IFT+MD*ID)
1-1.732*MD*SIN(WT-0.524)*ARH(3,1)+1.732*MD*COS(WT-0.524)*ARH(4,1)
1+1.732*MD*OMEGA*SIN(WT-0.524)*IQ
137.
FAULT IS PRESENT FOR 1 CONDUCTION PERIOD PLUS 1 COMMUTATION PERIOD + 1 COMMUTATION PERIOD WHERE NEXT SCRS ARE BLANKED--PROGRAM ENDS WHEN PEAD IL IS PAST

IF((JUMP .GE.2).AND.,(IL.LT.CIL(KK—1))) GO TO 99
IF(JUMP .GE.2) GO TO 1
IF(VBCT.GT.0.0) GO TO 11
CONTINUE
WRITE(7,720)
720 FORMAT(' ',5X,'CONDUCTION PERIOD DOES NOT END')
GO TO 99
11 CONTINUE
C CALCULATE COMMUTATION INTERVAL
IK=0.0
DO 26 K=1,200
KEY=0
IL=IL
IK=IK
26 KEY=KEY+1
IF(KEY.GT.2) GO TO 70
C FIND DY/DX AT 0
B(1,2)=1.732*MF*COS(WT)
B(1,3)=1.732*MD*COS(WT)
B(1,4)=—1.732*MD*SIN(WT)
B(2,1)=1.732*KF*COS(WT)
B(2,5)=1.732*KF*SIGMA(WT—0.524)
B(3,1)=1.732*KD*COS(WT)
B(3,5)=1.732*KD*SIN(WT—0.524)
B(4,1)=—1.732*KQ*COS(WT)
B(4,5)=1.732*KQ*SIGMA(WT—0.524)
B(5,2)=1.732*MF*SIN(WT)
B(5,3)=1.732*MD*SIN(WT—0.524)
B(5,4)=1.732*MD*COS(WT—0.524)
27 CONTINUE
BRH(1,1)=—(RP+2*RA)*IL+RA*IK+1.732*MF*OMEGA*SIN(WT)
1*IFT+1.732*MD*OMEGA*SIN(WT)*ID
1+1.732*MD*OMEGA*COS(WT)*IQ
BRH(2,1)=1.732*KF*OMEGA*(SIGMA(WT)*IL—IK*COS(WT—0.524))
BRH(3,1)=1.732*KD*OMEGA*(IL*COS(WT)—IK*SIN(WT—0.524))
BRH(4,1)=1.732*KQ*OMEGA*(IL*SIN(WT)+IK*COS(WT—0.524))
BRH(5,1)=RA*IL—2*RA*IK—1.732*MF*OMEGA*IFT*
1*COS(WT—0.524)—1.732*MD*OMEGA*ID*
1COS(WT—0.524)+1.732*MD*OMEGA*IQ*
1SIN(WT—0.524)
H=B(1,1)—B(1,2)*B(2,1)—B(1,3)*B(3,1)—B(1,4)*B(4,1)
C=B(1,5)—B(1,2)*B(2,5)—B(1,3)*B(3,5)—B(1,4)*B(4,5)
D=BRH(1,1)—B(1,2)*BRH(2,1)—B(1,3)*BRH(3,1)—B(1,4)*BRH(4,1)
E=B(5,1)—B(5,2)*B(2,1)—B(5,3)*B(3,1)—B(5,4)*B(4,1)
F=B(5,5)—B(2,5)*B(5,2)—B(5,3)*B(3,5)—B(5,4)*B(4,5)
G=BRH(5,1)—B(5,2)*BRH(2,1)—B(5,3)*BRH(3,1)—B(5,4)*BRH(4,1)
BRH(1,1)=(D*F-C*G)/(H*F-C*E)
BRH(5,1)=(H*G-D*E)/(H*F-C*E)
C BRH(1,1)=DIL/DT, BRH(5,1)=DIK/DT AT 0
IF(KEY.GT.1) GO TO 20
WT=WT+DWT
IL=IL+BRH(1,1)*DT
IK=IK+BRH(5,1)*DT
ILC=IL

138.
IKC=IK
DILDT=BRH(1,1)
DKD=BRH(5,1)
GO TO 30
20 CONTINUE
IL=ILO+((DILDT+BRH(1,1))*DT)/2
IK=IKO+((DIKDT+BRH(5,1))*DT)/2
30 CONTINUE
IFT=SF-1.732*KF*(IL*COS(WT)+IK*SIN(WT-0.524))
ID=SD-1.732*KD*(IL*COS(WT)+IK*SIN(WT-0.524))
IQ=SQ+1.732*KQ*(IL*SIN(WT)-IK*COS(WT-0.524))
IF(KEY.LT.2) GO TO 120
IF((ABS(IL-ILC).LE.1.0).AND.(ABS(IK-IKC).LE.1.0)) GO TO 100
ILC=IL
IKC=IK
IF(KEY.GT.50) GO TO 997
GO TO 120
100 CONTINUE
KK=KK+1
CIL(KK)=IL
CIK(KK)=IK
300 CONTINUE
IF(IL.GT.ILFMAX) GO TO 997
C TEST FOR END OF COMMUTATION PERIOD
IF(IK.0E.IL) GO TO 110
26 CONTINUE
WRITE(7,721)
721 FORMAT(' ', 'COMmutation PERIOD DOES NOT END')
GO TO 997
99 CONTINUE
DUM=BETA+MU-1.047
997 DO 86 K=1,KK
DUM=DUM+WDT
CWT(K)=DUM
86 CONTINUE
DO 144 L=1,KK
WRITE(7,145)CIL(L),CIK(L),CWT(L)
145 FORMAT(' ', 'COMmutation PERIOD DOES NOT END')
144 CONTINUE
RETURN
END
SUBROUTINE JACOB(MF,BETA,IF,W,V,IL,MO,MU,DELTAO,OMEGA,FF,A,B,C)
DIMENSION FF(5,5)
REAL*4 MO,MO,MU,IF,K1,OMEGA
PAIF=1.155*MF*SIN(BETA)
PBIIF=1.155*MF
PABETA=1.155*IF*MF*COS(BETA)+1.91*MO*(W*COS(BETA)-V*SIN(BETA))
PBETA=-1.91*MO*IL*SIN(BETA+MU)
PCBETA=-1.91*MO*IL*COS(BETA+MU)
PAW=-1.91*MO*IL*COS(MU)
PBU=0.0
PVU=0.0
PBWI=-1.91*MO*IL*SIN(BETA+MU)
PBU=1.91*MO*IL*COS(BETA+MU)
PAW=1.91*MO*SIN(BETA)
PBU=1.91*MO
PBU=1.91*MO*COS(BETA)
PCU=1.91*MO
FF(1,1)=(COS(BETA+MU)-COS(BETA))**PAIF
1+0.25*(2*MU-SIN(2*(BETA+MU)))->SIN(2*BETA))**PBIF
FF(1,2)=(COS(BETA+MU)-COS(BETA))**PAIF
1-A*(SIN(BETA+MU)-SIN(BETA))-0.25*(2*MU
1-SIN(2*(BETA+MU))+SIN(2*BETA))**PBIF
1+0.5*B*(-COS(2*(BETA+MU))+COS(2*BETA))
1+0.5*(-SIN(2*(BETA+MU))**2-(SIN(BETA))**2)
1*C*(-SIN(BETA+MU)**2-SIN(BETA)**2)*PBWI
1+0.5*(-SIN(BETA+MU)**2-SIN(BETA)**2)*PCU
FF(1,3)=PAW*(COS(BETA+MU)-COS(BETA))**A*SIN(BETA+MU)
1+0.25*(2*MU-SIN(2*(BETA+MU)))+SIN(2*BETA))**PBIF
1+0.5*(-SIN(BETA+MU)**2-SIN(BETA)**2)*PBWI
1+0.5*(-SIN(BETA+MU)**2-SIN(BETA)**2)*PCU
FF(1,4)=PAW*(COS(BETA+MU)-COS(BETA))**A*SIN(BETA+MU)
1+0.25*(2*MU-SIN(2*(BETA+MU)))+SIN(2*BETA))**PBIF
1+0.5*(-SIN(BETA+MU)**2-SIN(BETA)**2)*PBWI
1+0.5*(-SIN(BETA+MU)**2-SIN(BETA)**2)*PCU
FF(1,5)=DELTAO+PAW*(COS(BETA+MU)-COS(BETA))**A*SIN(BETA+MU)
1+0.25*(2*MU-SIN(2*(BETA+MU)))+SIN(2*BETA))**PBIF
1+0.5*(-SIN(BETA+MU)**2-SIN(BETA)**2)*PBWI
1+0.5*(-SIN(BETA+MU)**2-SIN(BETA)**2)*PCU
FF(2,1)=PAW*(COS(BETA+MU)-COS(BETA))**A*SIN(BETA+MU)
1+0.25*(2*MU-SIN(2*(BETA+MU)))+SIN(2*BETA))**PBIF
1+0.5*(-SIN(BETA+MU)**2-SIN(BETA)**2)*PBWI
1+0.5*(-SIN(BETA+MU)**2-SIN(BETA)**2)*PCU
FF(2,2)=PAW*(COS(BETA+MU)-COS(BETA))**A*SIN(BETA+MU)
1+0.25*(2*MU-SIN(2*(BETA+MU)))+SIN(2*BETA))**PBIF
1+0.5*(-SIN(BETA+MU)**2-SIN(BETA)**2)*PBWI
1+0.5*(-SIN(BETA+MU)**2-SIN(BETA)**2)*PCU
FF(2,3)=PAW*(COS(BETA+MU)-COS(BETA))**A*SIN(BETA+MU)
1+0.25*(2*MU-SIN(2*(BETA+MU)))+SIN(2*BETA))**PBIF
1+0.5*(-SIN(BETA+MU)**2-SIN(BETA)**2)*PBWI
1+0.5*(-SIN(BETA+MU)**2-SIN(BETA)**2)*PCU
FF(2,4)=PAW*(COS(BETA+MU)-COS(BETA))**A*SIN(BETA+MU)
1+0.25*(2*MU-SIN(2*(BETA+MU)))+SIN(2*BETA))**PBIF
1+0.5*(-SIN(BETA+MU)**2-SIN(BETA)**2)*PBWI
1+0.5*(-SIN(BETA+MU)**2-SIN(BETA)**2)*PCU
FF(2,5)=PAW*(COS(BETA+MU)-COS(BETA))**A*SIN(BETA+MU)
1+0.25*(2*MU-SIN(2*(BETA+MU)))+SIN(2*BETA))**PBIF
1+0.5*(-SIN(BETA+MU)**2-SIN(BETA)**2)*PBWI
1+0.5*(-SIN(BETA+MU)**2-SIN(BETA)**2)*PCU
FF(3,1)=(3.*OMEGA/3.1416)**1.732*MF*SIN(BETA+MU))
FF(3,2)=(3.*OMEGA/3.1416)**1.732*IF*MF*COS(BETA+MU))
1+(9.*MO/3.1416)**(W*COS(BETA+MU)-V*SIN(BETA+MU)))
FF(3,3)=(3.*OMEGA/3.1416)**(1.732*IF*MF*COS(BETA+MU))
1+(9.*MO/3.1416)**(W*COS(BETA+MU)-V*SIN(BETA+MU)))
FF(3,4)=27.*OMEGA*MO*COS(BETA+MU)/(3.1416)**2

140.
FF(3,5)=27.*OMEGA*M0*SIN(BETA+MU)/((3.1416)**2)
FF(4,1)=MF*COS(BETA)/1.732
FF(4,2)=(-IF*MF*SIN(BETA)/1.732)
1+((3.*M0/3.1416)*(-W*SIN(BETA)-V*COS(BETA)))
FF(4,3)=-3.*M0*IL*SIN(MU)/3.1416
FF(4,4)=-3.*M0*SIN(BETA)/3.1416
FF(4,5)=3.*M0*COS(BETA)/3.1416
FF(5,1)=-4.*MF*COS(BETA+MU/2)*SIN(MU/2)/1.732
FF(5,2)=(4.*IF*MF*SIN(BETA+MU/2)*SIN(MU/2)/1.732)
1+((6.*M0/3.1416)*(W*(COS(BETA)-COS(BETA+MU))
1-V*(SIN(BETA)-SIN(BETA+MU))))
FF(5,3)=(-4.*IF*MF/1.732)*(-0.5*SIN(BETA+MU/2)*SIN(MU/2))
1+0.5*COS(BETA+MU/2)*COS(MU/2))
1+6.*M0/3.1416*(-W*COS(BETA+MU)+V*SIN(BETA+MU))
1-6.*IL*M0*COS(MU)/3.1416
FF(5,4)=6.*M0*(COS(BETA)-COS(BETA+MU))/3.1416
FF(5,5)=6.*M0*(SIN(BETA)-SIN(BETA+MU))/3.1416
DO 2 II=1,5
FF(1,II)=FF(1,II)*1.0E 04
2 CONTINUE
DO 3 II=1,5
FF(2,II)=FF(2,II)*1.0E 04
3 CONTINUE
DO 4 II=1,5
FF(4,II)=FF(4,II)*1.0E 04
4 CONTINUE
DO 5 II=1,5
FF(5,II)=FF(5,II)*1.0E 04
5 CONTINUE
RETURN
END
SUBROUTINE JACOB4(MF, BETA, IF, W, V, IL, M0, MU, DELTAO, OMEGA, FF,
1A, B, C)
DIMENSION FF(4,4)
REAL*4 M0, M00, MU, IF, IL, K1, OMEGA
PAI=1.155*MF*Sin(BETA)
PB=1.155*MF
PAIF=1.155*MF*Cos(BETA)+1.91*M0*(W*Cos(BETA)-V*Sin(BETA))
PBIF=1.91*M0*IL*Sin(BETA+MU)
PAIF=-1.91*M0*IL*Cos(BETA+MU)
PAMU=-1.91*M0*IL*Cos(MU)
PWMU=0.0
PVMU=0.0
PBMU=-1.91*M0*IL*Sin(BETA+MU)
PCMU=-1.91*M0*IL*Cos(BETA+MU)
PAW=1.91*M0*Sin(BETA)
PBW=1.91*M0
PAV=1.91*M0*Sin(BETA)
FCV=1.91*M0
FF(1,1)=(Cos(BETA+MU)-Cos(BETA))*PAIF
1-A*(Sin(BETA+MU)-Sin(BETA))-0.25*(2.*MU
1-Sin(2.*(BETA+MU))+Sin(2.*(BETA)))*PBIF
1+0.5*(-Cos(2.*(BETA+MU))+Cos(2.*(BETA)))
1+0.5*((Sin(BETA+MU))**2-(Sin(BETA))**2)
1+PAIF*Cos(BETA+MU)-Sin(BETA)*Sin(BETA+MU)
1-A*(Sin(BETA+MU)-Sin(BETA))-A*Sin(BETA+MU)
1+0.25*(2.*MU-Sin(2.*(BETA+MU)))+Cos(2.*(BETA))
1+PBIF*(B/2.)*(-Cos(2.*(BETA+MU)))+0.5*((Sin(BETA+MU))**2
1-(Sin(BETA))**2)**2
1-(Sin(BETA))**2)**2)**2
FF(1,3)=PAW*(Cos(BETA+MU)-Cos(BETA))+0.5*((Sin(BETA+MU))**2
1-(Sin(BETA))**2)**2)
FF(1,4)=DELTAO+PAW*(Cos(BETA+MU)-Cos(BETA))
1+0.25*(2.*MU-Sin(2.*(BETA+MU)))+Sin(2.*(BETA))
FF(2,1)=PAIF*(Sin(BETA)-Sin(BETA+MU))
1+A*(Sin(BETA+MU)-Sin(BETA))
1+0.5*((Sin(BETA+MU))**2-(Sin(BETA))**2)*PBIF
1+PAIF*(Sin(BETA+MU)+Cos(BETA+MU)+Sin(BETA)*Cos(BETA))
1+0.25*PCMU*(2.*MU+Sin(2.*(BETA+MU)))+Sin(2.*(BETA))
1-(Sin(BETA))**2)**2)
FF(2,2)=PAW*(Sin(BETA)-Sin(BETA+MU))
1-A*(Cos(BETA+MU)+0.5*PBW*Cos(BETA+MU))**2
1-(Sin(BETA))**2)**2)+B*(Sin(BETA+MU)+Cos(BETA+MU))
1+0.25*PCMU*Cos(2.*(BETA+MU)+Sin(2.*(BETA+MU)))+Sin(2.*(BETA))
1+0.5*(1+Cos(2.*(BETA+MU)))
FF(2,3)=DELTAO+PAW*(Sin(BETA)-Sin(BETA+MU))
1+0.25*PCV*(2.*MU+Sin(2.*(BETA+MU)))+Sin(2.*(BETA))
FF(2,4)=PAW*(Sin(BETA)-Sin(BETA+MU))
1+0.5*PBW*((Sin(BETA+MU))**2-(Sin(BETA))**2)
FF(3,1)=(3.0*OMEGA/3.1416)*((1.732*IF*MF*Cos(BETA+MU))
1+((9.*M0/3.1416)*((W*Cos(BETA+MU)-V*Sin(BETA+MU))))
FF(3,2)=(3.0*OMEGA/3.1416)*((1.732*IF*MF*Cos(BETA+MU))
1+((9.*M0/3.1416)*((W*Cos(BETA+MU)-V*Sin(BETA+MU))))
FF(3,3)=27.0*OMEGA*M0*Cos(BETA+MU)/(3.1416)**2
FF(3,4)=27.0*OMEGA*M0*Sin(BETA+MU)/(3.1416)**2
FF(4,1)=(4.*IF*MF*Sin(BETA+MU/2.)*Sin(MU/2.)/1.732)
1+((6.*M0/3.1416)*((W*Cos(BETA)-Cos(BETA+MU))
1+V*Sin(BETA)-Sin(BETA+MU))))
FF(4,2)=(-4.*IF*MF/1.732)*(-0.5*Sin(BETA+MU/2.)*Sin(MU/2.)
1-VE(SIN(BETA)-Sin(BETA+MU))))
1+0.5*COS(BETA+MU/2.)*COS(MU/2.)
1+(6.*MO/3.1416)*(-W*COS(BETA+MU)+V*SIN(BETA+MU))
1-6.*MO*COS(MU)*IL/3.1416

IF (4,3)=6.*MO*(COS(BETA)-COS(BETA+MU))/3.1416
FF(4,4)=6.*MO*(SIN(BETA)-SIN(BETA+MU))/3.1416

DO 2 II=1,4
FF(1,II)=FF(1,II)*1.0E 04
2 CONTINUE
DO 3 II=1,4
FF(2,II)=FF(2,II)*1.0E 04
3 CONTINUE
DO 4 II=1,4
FF(4,II)=FF(4,II)*1.0E 04
4 CONTINUE
RETURN
END
SUBROUTINE LITLICDELTA O,MF,BETA,M0,W,
1M00,V,IL,DELTAF,MU,AID,AIQ,AIF,AIK,THETA,KQ,KF,KD)
DIMENSION AIF(60),AID(60),AIQ(60),AIK(60),THETA(60)
REAL*4 MF,M0,M00,IL,MU,MU,KQ,KQ,KF
WRITE(7,10)KQ,KD
10 FORMAT('KF = ',E15.3,'KQ = ',E15.3,'KD = ',E15.3)
WRITE(7,11)IL
WRITE(7,12)BETA
WRITE(7,13)MU
WRITE(7,14)IF
WRITE(7,15)V
WRITE(7,16)W
11 FORMAT('IL = ',F10.2)
12 FORMAT('BETA = ',F10.3)
13 FORMAT('MU = ',F10.3)
14 FORMAT('IF = ',F10.3)
15 FORMAT('V = ',F10.3)
16 FORMAT('W = ',F10.3)
9 FORMAT('THETA','IK','IL','IIi','IF')
ANG=BETA+MU-1.0472
DO 100 L=1,60
X=ANG-BETA
Y=ANG-(BETA+MU)
Z=ANG-(BETA+1.0472)
IF(X) 2,2,3
3 IF(Y) 5,2,4
4 IF(Z) 2,2,5
2 AIIK(L)=0.0
GO TO 6
5 AIIK(L)=(1./DELTAF)*(1.155*MF*IF*/(SIN(BETA)-SIN(ANG))
+1.91*M0*V*/(SIN(BETA)-SIN(ANG))+1.91*MO0*V*0.955*IL*DELTAO*(SIN(MU)+SIN(ANG-BETA-MU))
1-COS(ANG))/0.955*IL*DELTAO*(SIN(MU)+SIN(ANG-BETA-MU))
6 AIQ(L)=1.732*KQ*(0.955*(V*-IL*Sin(BETA+MU))
1+IL*Sin(ANG+0.524)-AIIK(L)*Cos(ANG)))
AID(L)=1.732*KD*0.955*(W+IL*Cos(BETA+MU))
1-IL*Cos(ANG+0.524)+AIIK(L)*Sin(ANG)))
AIF(L)=AIIK(L)*KF/KD
THETA(L)=ANG*180.0/3.1416
AIIK(L)=-AIIK(L)
ANG=ANG+0.01745
WRITE(7,200)THETA(L),AIQ(L),AIIK(L),AID(L),AIF(L)
200 FORMAT(9,1X,F14.2,4E14.3)
100 CONTINUE
RETURN
END
SUBROUTINE NEWTON(MU, BETA, IF, W, V, FREQ, DELTA0, IL, VL, MO, K1, 
MF, MO, OMEGA, DELTA, ZETA) 
DIMENSION FD(5), F(5, 1), FF(5, 5) 
REAL*4 M0, MO, MU, IF, IL, K1, MF
C SLIGHT ERROR FOR IL.NE.0 BUT.LT.35
KAT=0
K=0
IF(IL.GE.35.) GO TO 50
MU=0.0
BETA=3.1416/2.
IF=(1./(MF*1.732))*(4.17E-04)*VL-0.75*IL*DELTA0)
W=0.0
V=IL
GO TO 51
50 X = SQRT((18*FREQ*DELTA0*IL)/(4*VL+18*FREQ*DELTA0*IL))
CALL ARCSIN(X)
MU = 2*X
ZETA = SIN((MU)/2)/((MU)/2)
A = (1+ZETA*K1*SIGMA(MU/2))/(1-K1)*COS(MU) + ZETA*K1*COS(MU/2))
B = ATAN(A)
IF(B.GE.0.) GO TO 2
BETA = 3.1416+B
GO TO 3
2 BETA = B
3 IF=(1./(1.732*MF*SIN(BETA+MU)))*((VL/(6*FREQ))-(0.75*IL*DELTA0)-
14.5*IL*ZETA*K1*SIGMA(MU))/((1-K1)*COS(MU)-ZETA*SIN(MU))
W = IL*K1*(-COS(BETA+MU)+ZETA*SIN(BETA+MU/2))
V = IL*K1*(SIN(BETA+MU)+ZETA*SIN(BETA+MU/2))
C WRITE(7,510)
C WRITE(7,300)K, BETA, MU, IF, W, V
51 CONTINUE
DO 70 K=1, 200
KK=0
A = 1.155*IF*M0*SIN(BETAMU)+1.91*M0*(W*SIN(BETA)+
1+V*COS(BETA))-1.91*M0*IL*SIN(MU)
B = 1.155*M0*W+1.91*IL*M0*COS(BETA+MU)
C = 1.91*M0*(V-IL*SIN(BETA+MU))
CALL RHS(IF, MF, BETA, MO, W, V, IL, VL, DELTA0, F, OMEGA, A, B, C, VL)
C WRITE(7,486)(F(1, 1), F(2, 1), F(3, 1), F(4, 1), F(5, 1)
486 FORMAT(5E20.7)
DO 71 L=1, 5
FD(L) = -F(L, 1)
Y = ABS(FD(L))
IF(Y.GT.0.001) KK=1
71 CONTINUE
IF(KK.GE.1) GO TO 72
GO TO 75
72 CONTINUE
CALL JACOB(IF, BETA, MO, W, V, IL, MO, MU, DELTA0, OMEGA, A, B, C)
IF(KAT.NE.0) GO TO 100
100 KAT=1
IEQN=5
IVEC=1
EPS=0.01
K10=0
CALL GELG(FD,FF,IEQN,IVEC, EPS,K10)
BETA=BETA+FD(2)
MU=MU+FD(3)
IF=IF+FD(1)
V=V+FD(4)
W=W+FD(5)
C WRITE(7,300)K,BETA,MU,IF,W,V
300 FORMAT( ' ',1X,'BETA=',E14.7,3X,'MU=',E14.7,
70 CONTINUE
C WRITE(7,78)
78 FORMAT( ' ',1X,'NEWTON-RHAPSON DOES NOT CONVERGE')
75 CONTINUE
IF(K.EQ.1) WRITE(7,500)
500 FORMAT( ' ',1X,'NEWTON DID NOT ITTERATE, K=1')
C WRITE(7,487)
487 FORMAT( '0',2X,'**********FINAL SOLUTION **********')
C WRITE(7,300)K,BETA,MU,IF,W,V
C WRITE(7,488)
488 FORMAT( '0',2X,'THIS IS THE RHS VECTOR FOR TEST')
C WRITE(7,489)F(1,1),F(2,1),F(3,1),F(4,1),F(5,1)
489 FORMAT(5E20.7)
RETURN
END
SUBROUTINE NEW3ON(MU,BETA,IF,W,V,FREQ,DELTAO,IL,VL,M00,K1,
MF,M0,OMEGA,DELTAF,ZETA,K)
DIMENSION FD(5),F(5,1),FF(5,5)
REAL*4 M0,M0O,MU,IF,IL,K1,MF
C SLIGHT ERROR FOR IF.NE.35 BUT LT.35
KAT=0
IF(IF.GE.35,O) GO TO 50
MU=0.0
BETA=3.1416/2.
IF=(1./MF+1.732)*((4.17E-04)*VL—0.75*IL*DELTAO)
W=0.0
V=IL
GO TO 51
50 X=((4*3.1416*VL—9*OMEGA*IL*DELTAO)/(4*3.1416*VL
+9*OMEGA*IL*DELTAO))
CALL ARCCOS(X)
MU=X
ZETA=SIN(MU/2)/(MU/2)
A=-((3.1416*DELTAO)/(6*(1-COS(MU)))*M00)+(1-K1)*SIN(MU)
+ZETA*K1*SIN(MU/2))/((1-K1)*COS(MU)+ZETA*K1*COS(MU/2))
B=ATAN(A)
IF(B.GE.0.) GO TO 2
BETA=3.1416+B
GO TO 3
2 BETA = B
3 IF=(1/(1.732*MF*SIN(BETA)+MU))**((VL/(6*FREQ))—(0.75*IL*DELTAO)—(14.5*IL*ZETA*K1*DELTAF*SIN(MU/2)/3.1416))
W = IL*K1*(-COS(BETA+MU)+ZETA*COS(BETA+MU/2))
V=IL*K1*(SIN(BETA+MU)—ZETA*SIN(BETA+MU/2))
51 CONTINUE
DO 70 K=1,200
KK=0
A=1.155*IF*MF*SIN(BETA)+1.91*M0*(W*SIN(BETA)
+V*COS(BETA))
B=1.155*IF*MF+1.91*M0*W+1.91*IL*M0*COS(BETA+MU)
C=1.91*M0*(V—IL*SIN(BETA+MU))
CALL RHS(IF,MF,BETA,M0,W,V,IL,MU,DELTAO,OMEGA,A,B,C,VL)
WRITE(7,486)F(1,1),F(2,1),F(3,1),F(4,1),F(5,1)
486 FORMAT(5E20.7)
DO 71 L=1,5
FD(L)=-F(L,1)
Y=ABS(FD(L))
IF(Y.GT.0.01) KK=1
71 CONTINUE
IF(KK.GE.1) GO TO 72
GO TO 75
72 CONTINUE
CALL JACOB(MF,BETA,IF,W,V,IL,M0,MU,DELTAO,OMEGA,FF,A,B,C)
IEQN=5
IVEC=1
EPS=0.01
K1O=0
CALL GELG(FD,FF,IEQN,IVEC,EPS,K10)
BETA=BETA+FD(2)
MU=MU+FD(3)
IF=IF+FD(1)
V=V+FD(4)
w = w + F(d)5
C WRITE (7, 300) K, BETA, MU, IF, W, V
300 FORMAT (' ', 1X, I4, 3X, 'BETA=' , E14.7, 3X, 'MU=' , E14.7, 
70 CONTINUE
WRITE (7, 78)
78 FORMAT (' ', 1X, 'NEWTON—RHAPSON DOES NOT CONVERGE')
75 CONTINUE
IF (K.EQ.1) WRITE (7, 500)
500 FORMAT (' ', 1X, 'NEWTON DID NOT ITERATE, K=1')
C WRITE (7, 487)
487 FORMAT ('0', 2X, '*********FINAL SOLUTION *********')
C WRITE (7, 300) K, BETA, MU, IF, W, V
C WRITE (7, 488)
488 FORMAT ('0', 2X, 'THIS IS THE RHS VECTOR FOR TEST')
C WRITE (7, 489) F(1, 1), F(2, 1), F(3, 1), F(4, 1), F(5, 1)
489 FORMAT (5E20.7)
RETURN
END
SUBROUTINE PHACON(IF,MF,BETA,M0,W,V,IL,MU,DELTAO,OMEGA,VL,M00)
REAL*4 IF,MF,M0,IL,MU,M00,OMEGA
DIMENSION FD1(4),F1(4,1),FF1(4,4)
KAT=0
BETA=BETA+(5.0*3.1416/180.0)
DO 100 K=1,70
KK=0
A=1.155*IF*MF*SQRT(BETA)+1.91*M0*(W*SIN(BETA)
1+V*COS(BETA))
B=1.155*IF*MF+1.91*M0*W+1.91*IL*M0*COS(BETA+MU)
C=1.91*M0*(V-IL*SIN(BETA+MU))
CALL RHS4B4(IF,MF,BETA,M0,W,V,IL,MU,DELTAO,F1,OMEGA,A,B,C,VL)
DO 101 L=1,4
FD1(L)=-F1(L,1)
Y=ABS(FD1(L))
IF(Y.GE.0.01) KK=1
101 CONTINUE
IF(KK.GE.1) GO TO 102
G0 TO 105
102 CONTINUE
CALL JACDB4(MF,BETA,IF,W,V,IL,M0,MU,DELTAO,F1,OMEGA,FF1,A,B,C)
IF(KAT.NE.0) GO TO 300
300 KAT=1
IEQN=4
IVEC=1
EPS=0.01
K10=0
CALL GELG(FD1,FF1,IEQN,IVEC,EPS,K10)
BETA=BETA+FD1(1)
MU=MU+FD1(2)
V=V+FD1(3)
W=W+FD1(4)
100 CONTINUE
WRITE(7,2000)
2000 FORMAT(' ',IX,'NEWT-RAP DOESN T CONV FOR PHASE CONTROL')
105 IF(K.EQ.1) WRITE(7,305)
305 FORMAT(' ',IX,'PHACON DID NOT ITERATE, K=1')
RETURN
END
SUBROUTINE RHS(IF,MF,BETA,M0,W,V,IL,MU,DELTAO,OMEGA,A,B,C,VL)
DIMENSION F(5,1)
REAL*4 M0,MU,MF,IL,IF,OMEGA
F(1,1)=DELTAO*W+A*(COS(BETA+MU)-COS(BETA))+(B/4.)*
1*(C/2.)*((SIN(BETA+MU))**2-(SIN(BETA))**2)
F(2,1)=DELTAO*V+A*(SIN(BETA)-SIN(BETA+MU))
F(3,1)=-VL+0.955*OMEGA*(0.75*IL*DELTAO+1.732*IF*MF*
1*V*COS(BETA+MU)+2.865*M0*(W*SIN(BETA+MU)
1+V*COS(BETA+MU))
F(4,1)=(0.5774*IF)*MF*COS(BETA)+(0.955*M0)
1*(W*COS(BETA)-V*SIN(BETA))+(0.955*M0*COS(MU)*IL
F(5,1)=DELTAO*IL-(2.3094*IF)*MF*COS(BETA+MU/2)
1+V*COS(BETA+MU)+2.865*M0*(W*SIN(BETA)
1-1.91*M0*SIN(MU)*IL
F(1,1)=F(1,1)*1.0E 04
F(2,1)=F(2,1)*1.0E 04
F(4,1)=F(4,1)*1.0E 04
F(5,1)=F(5,1)*1.0E 04
RETURN
END
SUBROUTINE RHS4 \( \text{IF}, \text{MF}, \text{RETA}, \text{M0}, \text{W}, \text{V}, \text{JL} \)

DIMENSION \( I(4,1) \)

REAL*4 \( \text{M0}, \text{MU}, \text{MF}, \text{IL}, \text{IF}, \text{OMEGA,} \)

\( \text{F}(1,1) = \text{DELTA O} \times \text{W} + \text{A} \times \text{CCOS(E4ETA+MU)—COS(ETA)} + \text{CR/4.} \times \text{SINC}^2.(\text{BETA+MU}) + \text{SINC}(2.*\text{BETA}) \)

\( \text{F}(2,1) = \text{DELTAO} \times \text{V} + \text{A} \times \text{CCOS(ETA)} — \text{SINC^2.(BETA+MU)} + \text{SINC^2.(BETA)} \times \text{COS(ETA)} + \text{C/2.} \times \text{C^2.(BETA+MU)} — \text{SINC^2.(BETA)} \)

\( \text{F}(3,1) = -\text{VL} + 0.955 \times \text{OMEGA} \times (0.75 \times \text{IL} \times \text{DELTAO} + 1.732 \times \text{IF} \times \text{MF} * \text{SIN(BETA+MU)} + 2.865 \times \text{V} \times \text{COS(BETA)} + \text{COS(BETA+MU)}) \)

\( \text{F}(4,1) = -\text{DELTAO} \times \text{IL} - (2.309 \times \text{IF} \times \text{MF} * \text{COS(BETA+MU)} + 1.91 \times \text{M0} * \text{COS(BETA+MU)} - \text{DELTAO} \times \text{IL} - (2.309 \times \text{IF} \times \text{MF} * \text{COS(BETA+MU)} + 1.91 \times \text{M0} * \text{COS(BETA+MU)} - \text{DELTAO} \times \text{IL} - (2.309 \times \text{IF} \times \text{MF} * \text{COS(BETA+MU)} + 1.91 \times \text{M0} * \text{COS(BETA+MU)}) \)

\( \text{F}(4,1) = \text{F}(4,1) \times 1.04 \)

\( \text{F}(2,1) = \text{F}(2,1) \times 1.04 \)

\( \text{F}(4,1) = \text{F}(4,1) \times 1.04 \)

RETURN

END
SUBROUTINE RMS(BETA, LLL, MU, RMSIF, IL, DELTA0, MF, MO, W, V, MO0, IF1, KF, DELTAF)
DIMENSION RMSIF(50)
REAL*4 IK, IF, IL, MU, MF, MO0, MO, KF
THETA = BETA + MU - (3.1416/3.)
DTHETA = 3.1416/300.
SIF = 0.0
DO 1 K = 1, 50
  THETA = THETA + DTHETA
  IK = (1./DELTA0)**((2./1.732)*IF*MF*(SIN(BETA) - SIN(THETA))
1 + (6./3.1416)*(MO*W*(SIN(BETA) - SIN(THETA)) + MO0*V*(COS(BETA)
1 - COS(THETA)) - (3./3.1416)*IL*DELTAF*(SIN(MU) + SIN(THETA-
1 BETA - MU)))
  IF(THETA.LT.BETA) IK = 0.0
  SIF = (1.732*KF*(3./3.1416)*(W + IL*COS(BETA + MU)) - (IL*COS
1 THETA + 3.1416/6.) + IK*SIN(THETA)))**2*SIF
  CONTINUE
RMSIF(LLL) = SQRT(SIF)/SQRT(50.)
RETURN
END
SUBROUTINE TERMA(N,Y,PHI,X,AN1,BN1)
AN1=Y*(SIN((N+1)*X)*COS(PHI)+COS((N+1)*X)*SIN(PHI))
1/(2*(N+1))
AN1=AN1+Y*(SIN((N-1)*X)*COS(PHI)-COS((N-1)*X)*SIN(PHI))
1/(2*(N-1))
BN1=Y*(-COS((N+1)*X)*COS(PHI)+SIN((N+1)*X)*SIN(PHI))
1/(2*(N+1))
BN1=BN1-Y*(COS((N-1)*X)*COS(PHI)+SIN((N-1)*X)*SIN(PHI))
1/(2*(N-1))
RETURN
END
APPENDIX V: DISTRIBUTION LIST

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