TURBULENCE MEASUREMENTS OF
A TWO-DIMENSIONAL HELIUM JET
IN A MOVING AIRSTREAM

by

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ABSTRACT

Measurements of the streamwise velocity component and the concentration of helium are made in a two-dimensional helium jet discharging into a moving airstream. The transverse distribution of the unconditioned and conditioned statistics of the velocity and helium concentration at various downstream positions are presented. In addition range conditioned point statistics provide information on the structure of turbulent zones of various durations.

The gross properties of the jet agree with previous data. Entrainment is found to occur on the leeward edges of the turbulent zones; because the turbulent fluid is moving faster than the external stream these edges are on the upstream end of the zones. The interfaces as given by the velocity and concentration cannot be distinguished and are found to be relatively thicker than previously measured temperature interfaces.

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I. INTRODUCTION

It is widely recognized that measurements of passive scalars provide valuable information on the properties of turbulent shear flows. Accordingly, it is somewhat surprising that there appear to be no data on passive scalars in the fundamentally important flow which results from a two-dimensional jet issuing into a moving stream. In this paper we present data on the streamwise velocity and helium concentration in a two-dimensional helium jet discharging into an airstream. The measurements are sufficiently downstream from the exit plane of the jet so that only small concentrations of helium are involved and so that the helium may be treated as passive.

The velocity in two-dimensional jets in a moving stream has been widely investigated. Rodi provides a valuable critical assessment of available velocity data on a variety of low-speed turbulent shear flows; with respect to the two-dimensional jet in the moving stream he cites the results of Weinstein et al., Bradbury and Riley, and Everitt but concludes that there are no measurements sufficiently far downstream so that flow similarity is achieved. Relative to a different flow, the two-dimensional jet discharging into a quiescent ambient, Kotsovinos observes recently that the discrepancy among the data for such jets can be
Rodi concludes that no available data on the two-dimensional jet in a moving stream have extended far enough downstream to apply to this third region. In attempting to reduce our measurements to similarity form, we find that in several respects our data do not agree with their well-known wake counterparts; we refer particularly to the gross behavior associated with the decay of the centerline velocity and with the growth of the half-velocity location. Other quantities which involve normalization with respect to centerline values, e.g., are comparable to far-wake results.

Despite the absence of strict similarity, we find it convenient for purposes of data presentation to identify an apparent virtual origin, recognizing that this is not the correct virtual origin for the third, far-field region. The momentum thickness, which is constant in wakes and jets provided the external flow has constant velocity, is used as a non-dimensionalizing length; for our case in which the initial velocity of the helium is higher than that in the moving stream, the appropriate definition of momentum thickness is

\[ \theta = 2 \int_0^\infty \left( \frac{\rho}{\rho_\infty} \right) \left( \frac{u}{U_\infty} \right) \left( \frac{u}{U_\infty} - 1 \right) dy \]

where the overbar denotes the usual time-averaged quantity and the notation is clear from Figure 1.
II. ARRANGEMENT AND TECHNIQUES OF THE EXPERIMENT

In this section we describe the flow conditions and the experimental and data reduction techniques used in the present work.

Description of the flow

A symmetric, airfoil shaped tube with a chord of 5.14 cm and a maximum thickness of 2.18 cm is mounted across the working section of a low-speed wind tunnel having a 76 x 76 cm cross section. Helium is injected at the trailing edge of the tube from a line of uniform holes of diameter 1.59 mm and with a spacing of 0.254 cm between centers. The helium is supplied by a bank of high pressure bottles, heated to within 1.5°C of the airstream temperature prior to being introduced at both ends of the airfoil tube, and distributed within the airfoil by a perforated inner tube of 9.52 mm diameter. A temperature probe at one end of the airfoil provides the sensor for the control of the helium heater.

The velocity, $U_\infty$, and turbulence level in the free stream are $4.6 \pm 0.06$ m/s and less than 0.07% respectively. The initial jet velocity $U_j$ is about 85 m/s as estimated from the momentum thickness.
Probe and Support

Velocity and concentration measurements are made with a two-sensor probe similar to that described in Way and Libby\(^6\) and LaRue and Libby.\(^7\) Briefly, the probe consists of a hot film normal to the flow and a hot-wire in the thermal field of the film.

A standard height gauge, resting on the floor of the tunnel, is used to support the probe to an accuracy of \(2.5 \times 10^{-2}\) mm. Vertical traverses across the jet are made at four streamwise locations corresponding to values of the x-coordinate measured from the trailing edge of the tube of \(x = 43, 87, 131,\) and \(174\) cm.

The frequency response of the probe is estimated to be at least \(500\) Hz on the basis of measurements on a similar probe by LaRue and Libby.\(^7\) The spatial resolution corresponds to a rectangle normal to the flow of \(0.25 \times 0.5\) mm; the longest side of the rectangle is about 3-4 times the estimated Kolmogoroff length.

Probe Calibration

The probe is calibrated in a calibration jet installed temporarily in the wind tunnel at velocities within the expected range of velocities in the turbulent jet, namely from four to twelve m/s and at five helium-air mixtures with helium mass fractions of 0, 0.019, 0.049, 0.087, and 0.203. The calibration techniques are essentially those described in LaRue and Libby\(^7\) and Stanford and Libby\(^8\) for the
determination of u-c pairs from a digital pair of voltages from the film and wire.

Data Collection and Reduction

Both calibration and turbulent data are processed by an ADC having twelve bit resolution and recorded on digital tape at a sample rate of 2086 samples per second per channel. At each probe position 30 seconds of data are recorded; for the unconditioned and zone averages presented here only ten seconds of data are used, but range conditioned, point statistics require all thirty seconds of data in order to obtain adequate statistics even for the mean values and lowest moments. To make possible future data reduction, the signals from the sensors and their derivatives are recorded on a Honeywell 7610 FM instrumentation tape at 15 ips.

Consider the data reduction technique; let \( X \) denote the square of the voltage from the normal film and \( Y \) the square of the voltage from the wire. The mass fraction of helium is obtained from a polynomial of the form

\[
  c = \sum_{n=0}^{2} A_n (Y) X^n
\]

where the \( A_n \) coefficients are taken to be third-order polynomials in the indicated variable and are based on the calibration data. With \( c \) determined for a X-Y pair of voltages, the \( u \)-velocity component is obtained from another polynomial of the form

\[
  X = \sum_{n=0}^{2} B_n (c) u^n
\]

where the \( B_n \) are second order polynomials in the indicated variable.
Measures of Accuracy

As part of the assessment of experimental errors, measurements are made to determine the sensitivity of the probe to variations in temperature. It is found that at low helium concentrations and at velocities up to about five m/s a two-degree change in temperature leads to errors of $3 \times 10^{-3}$ in concentration and of 20 cm/s in velocity. Accordingly, as noted earlier, the temperature of the helium is maintained within 1.5°C of the airstream temperature.

Known values of velocity and concentration for the calibration data provide a means of evaluating the error in the polynomial inversion schemes. Calibration data corresponding to $4.0 \text{ m/s} \leq u \leq 8.0 \text{ m/s}$ and $0.0 \leq c \leq 0.05$ result in maximum mean and RMS errors of $\pm 0.07 \text{ m/s}$ and $\pm 0.05 \text{ m/s}$ respectively for velocity and $\pm 6(10)^{-4}$ and $\pm 8(10)^{-4}$ respectively for concentration.

Determination of the Intermittency Function

The helium concentration is used to discriminate between turbulent and irrotational fluid. Although the wind tunnel is operated in an open fashion, a slight increase in the background helium concentration is detected by the probe during successive measurements at each downstream measuring station. Figures 2a-c show the probability density function for concentration at various downstream and transverse locations. The peak of the spike on the left of each
distribution is taken as the level of the helium in the external flow due to contamination. The thickness of the spike is associated with a variety of effects: experimental inaccuracies, spurious fluctuations in helium concentration in the external flow, and contributions from the interfaces between turbulent and irrotational fluid. Our explanation for the increase in apparent helium concentration in the external flow is consistent with the observed increase in the peak value of the pdf's at a given downstream measuring station as the probe is moved sequentially through the jet.

In Figures 3a-b we indicate the influence on the intermittency and crossing frequencies of various increments above the peak value. We note that at probe positions corresponding to values of \( \gamma < 0.5 \) a low threshold value increases the apparent intermittency and crossing frequency while in regions of \( \gamma > 0.50 \) a low threshold value increases the intermittency and decreases the crossing frequency. On the basis of these variations we have selected a threshold 0.001 greater than the peak value as indicated by the shaded symbols. Note that these values are in the range of \( c_T \) in which the intermittency and crossing frequency are relatively insensitive to \( c_T \).
III. RESULTS

In this section we compare our results for the development of the jet with earlier experiments and then present our results in terms of unconditioned and conditioned statistics. First the location of the fluid-dynamic centerline and the location of the apparent virtual origin are discussed.

Fluid-dynamic Centerline

The $y$-coordinate used in our data presentation relates to the normal distance from a fluid mechanical centerline which can deviate from the geometric centerline due to slight buoyancy effects; these are expected to arise in an intermediate downstream location where the maximum velocity and maximum helium concentration are diminishing. In fact we find the corrections to be small. Nevertheless the location of the fluid mechanical centerline and the values of the maximum velocity, $u_0(x)$, and of the maximum helium concentration, $c_0(x)$, are determined at each streamwise location by fitting in a least square sense a parabola through at least five sets of data in the neighborhood of the centerline. We find offsets of the fluid mechanical centerline from 5.5 mm above to 1.8 mm below the geometric centerline.
Apparent Virtual Origin

The apparent virtual origin for the flow is calculated by assuming a similarity profile of the form
\[
\frac{\bar{u} - U_{\infty}}{\bar{U}_o - U_{\infty}} = \exp \left( -\frac{\sigma y^2}{(x - x_o)} \right).
\]
At each downstream location the values of
\[
\frac{y^2}{\left( \frac{\bar{u} - U_{\infty}}{\bar{U}_o - U_{\infty}} \right)}, \text{ where } y, \bar{u}, \text{ and } U_{\infty}
\]
are measured quantities and where \( \bar{U}_o \) is computed as part of the determination of the fluid-dynamic centerline, are averaged and plotted as a function of \( x \). Likewise a similarity profile for \( c \) is assumed and the procedure repeated. A least square approximation of the resulting distribution shown in Figure 4 leads to a virtual origin for velocity at \( x_o = 17.8 \text{ cm} \) and for concentration at \( x_o = 16.0 \text{ cm} \). These apparent virtual origins are used in the data presentation.

Momentum Thickness

The momentum thickness, used as a non-dimensionalizing length, is determined by graphically integrating smoothed profiles of \( \frac{\rho}{(\bar{U}_\infty)} \left( \frac{\overline{\bar{u}/U_{\infty}}}{(\bar{u}/U_{\infty}) - 1} \right) \) across the jet at each downstream location. All of the stations yield values of \( \theta \) within 16% of an average value of 7.1 cm, which is used in the data presentation.

Downstream Development

In Figures 5a-b we compare our results on gross jet behavior with earlier measurements and with the theory of wakes.
and jets based on far-field approximations. The standard
presentation of gross behavior involves the variation with \( \frac{x - x_{ou}}{\theta} \)
of the centerline velocity in the form \( \left( \frac{U_z}{u_o - U_z} \right)^2 \) and the
variation of the half-width, i.e., the value of the \( y \)-coordinate where
the velocity excess is one-half its centerline value, in the form
\( \left( \frac{\delta_x}{\theta} \right)^2 \). In Figure 5a-b we compare our results with those of
Weinstein et al. \(^2\) and of Bradbury and Riley. \(^3\) The three sets of
data are in reasonable agreement within the context of the sensitivity
of jet data to a variety of flow effects. We also show on these
figures the theory for the far field of wakes and jets (cf., e.g.,
Schlichting \(^9\) and Libby \(^10\)) based on the experimentally determined
apparent origin. The considerable discrepancy is noted. Thus as
indicated earlier we have another example of apparent but faulty
similarity. We note that in order to bring our experimental data
into accord with the far-field theory we would require an un-
reasonable \( x_{ou} \), namely 53 cm. Finally, we note that in order
for us to obtain farther downstream data without compromising
excessively the two-dimensionality of the jet would require a
reduced momentum thickness, i.e., a reduced initial velocity at
the jet exit.

In Figures 6a-b we compare the gross behavior of the velocity
and concentration. The mean concentration of helium on the
centerline decays more rapidly and the half-width based on a
mean helium concentration equal to one half its centerline value is
found to grow more rapidly than the corresponding quantities
associated with the velocity. These are expected results, now
understood to be associated with the relatively high level of a scalar
in the turbulent fluid far from the centerline. The difference in the
behavior of the two quantities, velocity and scalar concentration,
is undoubtedly due to the influence of the pressure on the former
and to the absence of a corresponding distributing force on the
latter.

Intermittency Factor and Crossing Frequency

The distributions of the intermittency factor and of the crossing
frequency are shown in Figures 7 and 8. The peak in the crossing
frequency near $y/\left((x-x_0)\bar{\theta}\right)^2 = 0.32$ corresponds to the $y/\left((x-x_0)\bar{\theta}\right)^2$
location where the intermittency factor equals 0.50, as generally
expected. Note that we have non-dimensionalized the crossing
frequency so as to form a Strouhal number by introducing the
characteristic length and $U_x$; the resulting correlation of the data
from various streamwise stations is considered satisfactory.

Unconditioned and Zone Statistics

Unconditioned and turbulent zone mean, root mean square,
skewness and kurtosis distributions for the velocity and concentration
are shown in Figures 9a–d and 10a–d, where symbols representing quantities in the turbulent zones are flagged. The corresponding data for the velocity in the non-turbulent zones are shown in Figures 11a–d. Here we adopt the notation of Kovaszany et al. \(^{11}\) for a turbulent zone average, and \((\sim)\) for a non-turbulent, i.e., irrotational, zone average.

We discuss some of the interesting results shown in this large number of figures. Consider first the distributions of mean velocity: Figure 9a indicates that the turbulent fluid moves faster than the external airstream. In fact except for one data point at \(\left(y \left(\frac{x-x_0}{\tau} \right) \theta \approx \right) = 0.43\), which may be in slight error, the velocity within the turbulent fluid at the edge of the jet moves faster than \(U_x\) with an increment of about five percent of the maximum velocity difference. Close to the jet centerline the velocity in the irrotational zones is higher than \(U_x\), suggesting that the turbulent fluid carries along the air between the large turbulent structures. Near the outer edges of the jet, the velocity in the irrotational fluid appears to be somewhat less than \(U_x\), consistent with motion of the external flow over the turbulent structures.

The view which emerges from these velocity distributions is in accord with that given by Kovaszany et al. \(^{11}\) for the turbulent boundary layer and by Fabris \(^{12}\) for the turbulent wake of a cylinder.
in both of these previous cases the turbulent fluid moves slower than the external flow so that behavior is reversed but is qualitatively consistent with our results in which the turbulent fluid moves faster than the external stream.

Consider next the distributions of the mean helium concentration as shown in Figure 10a. Of particular interest is the relatively high value for the average within the turbulent fluid near the edges of the jet. In fact the concentration within the turbulent fluid appears to approach a constant value of about 40% of the centerline mean at the outer edge; this is in agreement with the results of Fabris and LaRue and Libby for the temperature in the wake of a heated cylinder and is in contrast with the behavior of the zone averaged mean velocity.

Figure 9b and 11b indicate the data on the intensity of the velocity fluctuations. On the centerline of the jet we find

\[ \frac{(u')^2}{(u_o - U_s)} = 0.27. \]

There is considerable variation among the published values for this relative intensity on the centerline of wakes and jets. Bradbury and Riley indicate that this value increases as the far-field is approached and becomes in reasonable agreement with the corresponding value given by Townsend for the wake of a cylinder, namely 0.25. However, Fabris gives 0.20 for this same quantity in the wake of a cylinder. Thus, while our value is somewhat high, it is not inconsistent with previous measurements.
The distributions of relative intensity of the velocity fluctuations across the jet have the expected behavior; the peak intensity occurs off-axis but in the region of the jet with almost fully turbulent flow. The intensity within the turbulent fluid at the outer edge of the jet as a percent of centerline value is high, roughly 65%. Also the intensity of the velocity fluctuations within the irrotational zones in the middle of the jet is also high but decays to low levels near the outer edge.

These results, which are in accord with those of Kovaszny et al. and Fabris for the boundary layer and cylinder wake respectively, indicate that the observed, well-known decay of un-conditioned intensities as the outer edge of a shear layer is approached is due largely to the decrease in the percentage of time the flow is turbulent and that the irrotational fluid is far from quiescent.

The corresponding results for the intensity of the fluctuations of helium concentration are shown in Figure 10b. On the centerline we find a relative intensity of 0.23; there are several sets of data for the relative intensity of temperature fluctuations on the centerline of the wake of a cylinder; again there is a range of experimental values. LaRue and Libby give 0.28, Freymuth and Uberoi 0.20, and from Fabris we deduce 0.25. Thus our value is well within the range of previous results.

The zone averaged intensity of the concentration fluctuations actually increases across the jet and appears to approach a value
30% higher than at the centerline. This is in agreement with the results for temperature fluctuations as given by LaRue and Libby but is less than the increase indicated by Fabris whose results in this regard may suffer from insufficient samples for statistical reliability.

These results on the intensity of the fluctuations of the helium concentration indicate that the decay of the unconditioned intensity as the outer edge of the jet is approached is due solely to the decreased percent of time the flow is turbulent and that the fluctuations within the turbulent fluid remains high throughout the jet.

The data on the skewness and kurtosis complete the picture of the statistical behavior of the velocity and concentration but are of less interest. Comparison of the distributions of the two quantities suggests that they are dominated by the variation of the intensities. Also the distributions of skewness and kurtosis indicate that nowhere in the jet is a Gaussian distribution a reasonable approximation.

In concluding this discussion of the unconditioned and zone statistics we remark that some of the results are comparable to those in the far-field of the wake of a cylinder and thus correspond to apparent similarity, despite the earlier indication in terms of gross jet behavior that our measurements do not correspond to the far-field.
Range conditioned point statistics

A picture of the structure of the turbulent zones of various durations can be established by employing a conditioning technique which we term range conditioned point statistics. This technique was developed independently by Antonia. To understand the technique consider a point within the jet where the flow is intermittent to a significant extent, e.g., where $\gamma = 0.5$. The time intervals for turbulent fluid to pass the point in question have a complete distribution from short times corresponding either to small turbulent structures or to grazes off the centerplane of larger structures and to long times corresponding to passages through the center of large structures. We can select a subset of these passages that are within a small, specified tolerance of a nominal duration, can divide the passage time into equal time intervals from the upstream and downstream crossings, and can carry out statistical analysis of the velocity and helium concentration at these times. The result is termed range conditioned point statistics and provides a statistical picture of turbulent structures of a given size provided the time durations are converted to spatial dimensions by application of Taylor's hypothesis. The zone averages discussed earlier correspond to averaging over all points within a turbulent structure of a given size and again over all sizes. It can be appreciated that because of the heavy
conditioning involved in this technique extended time records are required to achieve statistical reliability. For the present results there are at least 50 samples used for each data point.

We present data corresponding to two points within the jet at a streamwise station corresponding to $((x - x_{ou})/\theta) = 16.1$; Figures 12 and 13 present the results for the velocity and helium concentration at a lateral position corresponding to $y((x - x_{ou})\theta)^{-\frac{1}{2}} = 0.41$ where $\gamma = 0.86$, i.e., where the flow is turbulent most of the time. Figures 14 and 15 do likewise for a lateral position corresponding to $y((x - x_{ou})\theta)^{-\frac{1}{2}} = 0.57$ where $\gamma = 0.34$. Several nominal durations at each station with a tolerance of 10% are considered.

The durations are presented in terms of multiples of digital time steps; the equal time intervals are measured relative to either the upstream or downstream crossing so that the tolerance accepted for the nominal duration is accommodated in the central region where the various statistical quantities are found to change gradually. The data points located relative to the upstream edge are open symbols, those from the downstream edge are half-shaded.

It should be recalled in the present context that discrimination between turbulent and irrotational fluid is based on a level of helium as discussed earlier and that this discrimination is applied to the velocity as well. Thus we implicitly assume a single interface for momentum and concentration, an assumption which will be seen later
to be consistent with the experimental results. Finally, we note that
in order to facilitate interpretation of the results the distributions of
various statistical quantities have been presented with the upstream
edges aligned.

In Figures 12a and 14a we see the distributions of velocity through
turbulent zones of four durations; the dashed lines on these figures is
the velocity in the external stream. From these data several obser-
vations may be made; the velocity at the two interfaces is continuous
and increases smoothly across the interface with a steeper gradient
at the downstream edges. In the central portion of the zones, i.e.,
remote from the interfaces, the velocity increases from the upstream
to the downstream edges, implying that the stretching of the zones in
the streamwise direction is associated with the movement of the down-
stream edges away from the upstream edges. This also suggests, as
we shall confirm later in connection with the results relative to the
concentration, that entrainment of the slower moving air occurs
primarily on the upstream edges of the zones. Finally, comparison
of the distributions at the two lateral positions indicates that the
shorter turbulent zones further from the jet centerline have velocities
close to $U_\infty$ so that the positive increment of the turbulent zone
averages over $U_\infty$ is due to the large turbulent structures.

The distributions of root-mean-square velocity intensities
shown in Figures 12b and 14b indicate smooth gradients at the two
interfaces and relatively flat regions in the central portions of the zones. The gradients of intensity are greater at the downstream edges, again consistent with greater entrainment of low velocity air at the upstream edges with consequent higher intensities there.

We now turn to the distributions of helium concentration: Figures 13a and 15a display the distributions of mean helium concentration through turbulent zones of various durations. Three reasonably distinct regions in each zone can be identified; there are two interfacial regions involving relatively steep gradients of concentration. The structure of the upstream interfaces are essentially the same independent of duration but those associated with the longer zones involve greater increases in concentration before fairing into the third, central region. The extent of this third region depends on the duration of the zone, but independent of duration, displays a gradual increase in helium concentration from the upstream to the downstream interfacial regions.

For zones of longest duration it appears that the mean concentration approaches a constant. We shall see subsequently that the skewness of the concentration fluctuations for these zones of long duration suggests a well-stirred mixture without the obvious presence of newly entrained air. Finally, comparison of these two figures indicates little difference in the distributions of mean concentration with the two values of $Y$ considered.
We conclude from these data that entrainment of air from the external flow occurs to a greater extent on the upstream edges of the turbulent zones. This supports the observations made earlier on the basis of the range conditioned point statistics of the velocity. Of special interest is to compare these results with the temperature measurements of LaRue and Libby\textsuperscript{14} which imply preferential entrainment on the downstream edges of the turbulent zones. The difference is attributable to the difference in the velocity of the turbulent zones relative to the external flow; in the case of the wake the turbulent fluid moves slower than the external stream and entrainment on the leeward, i.e., downstream, surfaces of the turbulent zones dominates. In the two-dimensional jet considered here the turbulent fluid moves faster than the external stream and again entrainment on the leeward edges dominates. We note that there is no theoretical explanation for these results which would appear to be of fundamental interest.

We note that these observations concerning greater entrainment at the upstream interfacial regions do not establish whether the entrainment mechanism is associated with engulfment or molecular processes. We would deduce on the basis of measurements of the statistical geometry of the interface in the turbulent heated wake by Paizis and Schwartz\textsuperscript{17} and LaRue and Libby\textsuperscript{18} that in our jet flow the number of overhangs suggestive of engulfment is greater on the leeward,
i.e., upstream, edges of the turbulent zones, but whether this coincidence accounts for the observed preferential entrainment is uncertain.

The distributions of the intensities of the concentration fluctuations are shown in Figures 13b and 15b. They support our earlier conclusions concerning entrainment and show smooth increases in intensities in the two interfacial regions and gradual increases from the upstream to the downstream edges.

Although the statistical reliability may be marginal, we show in Figure 13c the distributions of the skewness of the concentration fluctuations for the position closest to the jet centerline. Of interest is the large skewness in the two interfacial regions and the small skewness in the central portions of each turbulent zone. The implications from these data are that the central regions involve nearly symmetrically distributed values of the concentration whereas the freshly entrained fluid involves highly skewed distributions. These results are in agreement with those for temperature given by LaRue and Libby.14

An examination of the interface regions given by these data exposes several points of interest. Consider the downstream interfaces as given, e.g., by the intensity of the velocity fluctuations in Figure 12b and by the mean helium concentration in Figure 13a. We find that the thickness of the two interface regions is essentially the same.
despite the low value for the Schmidt number for dilute helium in helium-air mixtures, namely 0.24. It might be expected that this low value would lead to a significantly thicker interface for helium than for velocity. We thus appear to have evidence in support of the commonly employed assumption, frequently implicit, that the velocity and scalar interface between the turbulent and irrotational fluid are coincident.

We can use the range conditioned point statistics of the mean concentration to estimate the thicknesses of the interfacial regions. We select the downstream edges for examination since they are somewhat cleaner. We estimate from application of Taylor's hypothesis that the downstream interface is typically 9 mm thick. This can be compared with the Kolmogoroff and Batchelor lengths as follows:

From the spectrum of the velocity in the fully turbulent region of the jet we estimate the Kolmogoroff length $l_k$ to be 0.14 mm. The Batchelor length is $l_k (Sc)^{-\frac{1}{2}}$ or 0.29 mm. Thus a typical interface is found to be 55 times the Batchelor length. This is considerably thicker than the estimate of $8l_k$ obtained by LaRue and Libby\textsuperscript{14} for the thickness of the temperature interface in the heated wake.

\*We intend to repeat the present experiment with the helium heated; in this case it will be of interest to compare the concentration and temperature interfaces and to establish whether the differences in Schmidt and Prandtl numbers will result in distinguishable interface thicknesses.
The reason for this increase in relative thickness is unknown; it may be due to the reduction in the Schmidt number relative to the Prandtl number but it might be expected that this should be accounted for by the comparison of the interface thickness with the Batchelor scale rather than the Kolmogoroff scale.

IV. CONCLUSIONS

We find the downstream development of the turbulent helium jet to be in reasonable agreement with earlier experiments of plane jets in moving streams. Although strict similarity does not apply in this region of jet flow, turbulent zone statistics of concentration for the mean and relative intensity distributions resemble turbulent zone statistics of temperature in the wake of a heated cylinder.

The conditioned velocity statistics show that the non-turbulent fluid well within the jet is moving faster than the free stream velocity.

Distributions through turbulent zones of various durations obtained by ranged conditioned point statistics show that entrainment at the upstream interfacial regions is dominant, that the concentration and momentum interfaces cannot be distinguished, and finally that the concentration interface is considerably thicker than that inferred.
from measurements of temperature in the wake of a heated cylinder. The findings relative to entrainment are consistent with previous results in that entrainment at the leeward interface of the turbulent zone dominates.
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\[ y_0 \left[ \begin{array}{c} 2(x-x_0c) \end{array} \right]^{1/2} \]

\( \frac{(x-x_{0u})}{\theta} \)

\( 3.86 \)

\( 10.0 \)

\( 16.1 \)

\( 22.3 \)
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Measurements of the streamwise velocity component and the concentration of helium are made in a two-dimensional helium jet discharging into a moving airstream. The transverse distribution of the unconditioned and conditioned statistics of the velocity and helium concentration at various downstream positions are presented. In addition range conditioned point statistics provide information on the structure of turbulent zones of various durations.

The gross properties of the jet agree with previous data. Entrainment is found to occur on the leeward edges of the turbulent zones; because the
turbulent fluid is moving faster than the external stream these edges are on the upstream end of the zones. The interfaces as given by the velocity and concentration cannot be distinguished and are found to be relatively thicker than previously measured temperature interfaces.