<table>
<thead>
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<th>REPORT DOCUMENTATION PAGE</th>
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</thead>
<tbody>
<tr>
<td>1. REPORT NUMBER:</td>
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<tr>
<td>4. TITLE: GREENLAND'S RAPID POSTGLACIAL EMERGENCE: A RESULT OF ICE-WATER GRAVITATIONAL ATTRACTION; COMMENT AND REPLY.</td>
</tr>
<tr>
<td>5. TYPE OF RUPT &amp; PERIOD COVERED:</td>
</tr>
<tr>
<td>6. PERFORMING ORG. REPORT NUMBER:</td>
</tr>
<tr>
<td>8. AUTHOR(S): CHRISTOPHER CAPSCOTT (N.H.O.I.)</td>
</tr>
<tr>
<td>9. PERFORMING ORGANIZATION NAME AND ADDRESS: Massachusetts Institute of Technology Department of Earth &amp; Planetary Sciences Cambridge, Massachusetts 02139</td>
</tr>
<tr>
<td>10. PROGRAM ELEMENT, PROJECT, TASK AREA &amp; WORK UNIT NUMBERS:</td>
</tr>
<tr>
<td>11. CONTROLLING OFFICE NAME AND ADDRESS:</td>
</tr>
<tr>
<td>12. REPORT DATE: AUG 1976</td>
</tr>
<tr>
<td>13. NUMBER OF PAGES: 3</td>
</tr>
<tr>
<td>14. MONITORING AGENCY NAME &amp; ADDRESS (IF DIFFERENT FROM CONTROLLING OFFICE): Office of Naval Research</td>
</tr>
<tr>
<td>15. SECURITY CLASS. (OF THIS REPORT): Unclassified</td>
</tr>
<tr>
<td>16. DISTRIBUTION STATEMENT (OF THIS REPORT): Approved for public release; distribution unlimited.</td>
</tr>
<tr>
<td>17. DISTRIBUTION STATEMENT (OF THE ABSTRACT ENTERED IN BLOCK 20, IF DIFFERENT FROM REPORT):</td>
</tr>
<tr>
<td>18. SUPPLEMENTARY NOTES:</td>
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<tr>
<td>19. KEY WORDS (CONTINUE ON REVERSE SIDE IF NECESSARY AND IDENTIFY BY BLOCK NUMBER):</td>
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<tr>
<td>20. ABSTRACT (CONTINUE ON REVERSE SIDE IF NECESSARY AND IDENTIFY BY BLOCK NUMBER): N/A</td>
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Greenland's rapid postglacial emergence:
A result of ice-water gravitational attraction:
Comment and reply

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Clark (1976) presents a simple method by which to calculate the change in sea level caused by the gravitational attraction of an ice cap. In the same paper he also presents some results of a more complex numerical solution of the problem, and the two calculations are in disagreement. In fact, his more powerful solution is correct. His simple calculation contains an error. It may be useful to point out the reason for this, lest doubt be cast on his numerical method and the many interesting consequences of his thesis.

When a point mass $M$ is created on the surface of a perfectly rigid earth, Clark concludes that the change in sea level $\theta$ degrees away from the mass is

$$S(\theta) = \frac{GM}{2ag} \csc\frac{\theta}{2}$$

where $G$ is the gravitational constant, $a$ is the radius of the earth, and $g$ is the acceleration of gravity at the earth's surface. This might better be written as

$$S(\theta) = \frac{ax}{2} \csc\frac{\theta}{2}$$

where $x$ is the ratio of $M$ to the mass of the earth. This equation is obviously incorrect, as $s$ is always positive, leading to a rise in sea level everywhere. This cannot happen if the
mass of the ocean is conserved, as Clark demands. The error
enters Clark's derivation, described in his Appendix 1, in the
equation \( \phi^*(s) = \phi_0 \); that is, that the gravitational potential
on the sea surface equipotential after the creation of the
mass is equal to that on the sea surface equipotential before
the creations of the mass. If the mass of the ocean is to be
held constant this will not be true. The two sea surfaces will,
of course, be equipotential surfaces, but their potential values
(relative to a zero value at infinite radius) will in general
differ.

Clark's equation does give the correct shape for an equipotential surface near the sea level surface, and one can use
it to reach the correct solution by making two assumptions:

1) In the region of sea level, the gravitational gradi-
ent is of equal magnitude everywhere.

2) In terms of the variable \( \theta \), oceans and continents
are uniformly distributed over the earth.

The first of these assumptions has already been made by neglect-
ing the partial derivatives of \( \phi^* \) in Clark's equation A3 and by
equating \( \frac{\delta \phi_0}{\delta r} \) with \(-g\). One can then say

\[
S(\theta) = \frac{\alpha x}{2} \csc(\theta) + C
\]
where \( C \) is a constant whose value can be determined by requiring no net change in the volume of the ocean,

\[
\int_0^\pi S(\theta) \sin \theta \, d\theta = 0.
\]

This leads to

\[
S(\theta) = \frac{a x}{2} \left\{ \csc \left( \frac{\theta}{2} \right)^2 - 2 \right\}
\]

Thus, gravitational forces cause sea level to rise over the area within 60° of the mass and to fall over the rest of the sphere.

One can get an idea of the significance of the second assumption by noting that over a wide range of ocean distributions (from the near hemisphere all ocean, and the far hemisphere all continent to the opposite case), \( C \) varies from \(-2.83 \, \alpha x\) to \(-1.17 \, \frac{\alpha x}{2}\).

Clark uses his method to calculate the gravitation component of sea level change near Greenland due to the melting of Greenland ice cap since 9000 years B.P. He finds a 27 m component. If one uses the more correct equation, one finds a 25 m component. The difference between the two results is small because the mass of ice involved is small (\( \alpha x = 1.07 \, \text{m} \)). A much larger mass of ice, however, can lead to serious discrepancies.

Using a more complex numerical method, Clark considers the melting of the Laurentide and Fennoscandian ice caps. This nu-
merical approach includes the effects of the lateral extent of 
the ice caps and the self-gravitation of the oceans. If the 
melting raises the average sea level by \( 85 \text{ m} \left( \frac{ax}{2} = 16.4\text{m} \right) \), 
Clark's more simple method predicts a sea level rise of 69 m 
in the South Pacific and 52 m about 60° from the ice cap cen-
ter. This disagrees with the results of his numerical analy-
sis, as shown in his Figure 2. The method derived here, how-
ever, predicts a rise of 101 m in the South Pacific and 85 m 
about 60° from the ice cap center, in complete agreement with 
Clark's numerical solution. This indicates that Clark's numer-
ical approach is correct, in spite of the error in his simple 
calculation. If Clark has not included the land-sea distribu-
tion in his solution, however, and he does not state that he 
has, he should do so before continuing with his stated plan of 
refining his model to look at small fluctuations in ice sheets. 
While the correction will have little effect on the size of 
small fluctuations in local sea level, it will noticeably ef-
fect large changes. Including the correction will be generally 
useful and will add to the value of the result.

REFERENCES CITED

Clark, J. A., 1976, Greenland's rapid postglacial emergence: 
A result of ice-water gravitational attraction: Geology, 
v. 4, no. 5, p. 310-312.