COMPUTER PROGRAM OPERATION MANUAL ON "SHABERTH" A COMPUTER PROGRAM FOR THE ANALYSIS OF THE STEADY STATE AND TRANSIENT THERMAL PERFORMANCE OF SHAFT-BEARING SYSTEMS

RESEARCH LABORATORY
SKF INDUSTRIES, INC.
ENGINEERING & RESEARCH CENTER
KING OF PRUSSIA, PA.

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This manual describes the work performed by SKF Industries, Inc. at its Technology Center in King of Prussia, Pa. for the United States Government, including the United States Air Force Systems Command, Air Force Aero Propulsion Laboratory, Wright-Patterson Air Force Base, Ohio and for the Naval Air Propulsion Test Center, Trenton, N.J. Work was performed over a seven month period starting in February 1976 under U.S. Air Force Contract No. F33615-76-C-2061 and Navy MPR No. N62376-76-MP-00005. Mr. John Schrand administered the project for the Air Force and Mr. Raymond Valori administered the project for the Navy.

The project was conducted at SKF under the direction of Messrs. P. S. Given and T. E. Tallian. The SKF report designation is No. AL76P030.

The manual contains the results of analytical modeling and computer program development.

This report has been reviewed by the Information Office, (ASD/OIP) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication.

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Project Engineer

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Howard F. Jones
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Al Lockwood, Supervisor
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AIR FORCE/56780/10 August 1977 – 210
This report is a self contained manual describing an advanced state-of-the-art analytical computer program (SHABERTH) for the study of steady state and transient thermal performance of rolling element bearing and shaft systems. This program embodies mathematical models describing the role of the lubricant in bearing behavior. Descriptions of these models are set forth along with a detailed description of the program organization method of solution and convergence criteria. Input data preparation forms and...
19. Key Words
and Housing Fits and Bearing Clearance Change

20. Abstract
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</tr>
<tr>
<td>A</td>
<td>a constant in Walther's equation</td>
<td>(-)</td>
</tr>
<tr>
<td>A</td>
<td>surface of contact between media</td>
<td>(mm^2)</td>
</tr>
<tr>
<td>A_c</td>
<td>cage-land surface area</td>
<td>(mm^2) or (in.(^2))</td>
</tr>
<tr>
<td>A_e</td>
<td>area of outer cylindrical surface</td>
<td>(mm^2)</td>
</tr>
<tr>
<td>A_i</td>
<td>area of inner cylindrical surface</td>
<td>(mm^2)</td>
</tr>
<tr>
<td>A_v</td>
<td>ball frontal area</td>
<td>(mm^2)</td>
</tr>
<tr>
<td>B</td>
<td>auxiliary variable</td>
<td>(-)</td>
</tr>
<tr>
<td>B</td>
<td>(B = \Delta x/a), a constant in Walther's equation</td>
<td>(-)</td>
</tr>
<tr>
<td>C</td>
<td>a constant tabulation by Fresco</td>
<td>(mm^2/N or in.(^2)/lb)</td>
</tr>
<tr>
<td>C_o</td>
<td>a non dimensional fluid-geometry parameter</td>
<td>(-)</td>
</tr>
<tr>
<td>C_p</td>
<td>specific heat at constant pressure</td>
<td>(W/kg-DegC)</td>
</tr>
<tr>
<td>C_r</td>
<td>cage pocket clearance</td>
<td>(mm)</td>
</tr>
<tr>
<td>C_v</td>
<td>drag coefficient</td>
<td>(-)</td>
</tr>
<tr>
<td>D</td>
<td>ball or roller diameter</td>
<td>(mm)</td>
</tr>
<tr>
<td>D</td>
<td>a constant tabulated by Fresco</td>
<td>(mm^2/N or in.(^2)/lb)</td>
</tr>
<tr>
<td>E</td>
<td>a constant tabulated by Fresco</td>
<td>(mm^2/N or in.(^2)/lb)</td>
</tr>
<tr>
<td>E_1, E_2</td>
<td>Young's modulus for the contacting bodies</td>
<td>(N/mm(^2) or psi)</td>
</tr>
<tr>
<td>F_A</td>
<td>axial force</td>
<td>(N or lb)</td>
</tr>
<tr>
<td>F_{n1}, F_{n2}</td>
<td>normal components of resultant force of the inlet pressure distribution</td>
<td>(N or lb)</td>
</tr>
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*Where multiple units are indicated, the first units given are those associated with the computer program input and output.
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<th>Definition</th>
<th>Units*</th>
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<tbody>
<tr>
<td>( F_R )</td>
<td>sliding force acting on the ball</td>
<td>(N or lb)</td>
</tr>
<tr>
<td>( F_{R1}, F_{R2} )</td>
<td>pumping forces acting on the ball</td>
<td>(N or lb)</td>
</tr>
<tr>
<td>( F_{R3}, F_{S3} )</td>
<td>tangential forces due to inlet rolling and shearing between ball and cage</td>
<td>(N or lb)</td>
</tr>
<tr>
<td>( F_S )</td>
<td>shearing force acting on the ball</td>
<td>(N or lb)</td>
</tr>
<tr>
<td>( F_{S1}, F_{S2} )</td>
<td>inlet friction forces</td>
<td>(N or lb)</td>
</tr>
<tr>
<td>( F_{x}, F_{y}, F_{z} )</td>
<td>force components in the ( x,y,z ) coordinate system</td>
<td>(N or lb)</td>
</tr>
<tr>
<td>( F_W )</td>
<td>windage force or drag force</td>
<td>(N or lb)</td>
</tr>
<tr>
<td>( F )</td>
<td>the vector of inertia and drag forces</td>
<td>(N or lb)</td>
</tr>
<tr>
<td>( \rightarrow F_m )</td>
<td>the vector sum of the hydrodynamic forces acting on the ball at the ( m )-th contact</td>
<td>(N or lb)</td>
</tr>
<tr>
<td>( F_b )</td>
<td>a vector of bearing loads and moments</td>
<td>(N or lb &amp; mm-N or in.-lb)</td>
</tr>
<tr>
<td>( F_{si} )</td>
<td>a vector of shaft loads and moments</td>
<td>(N or lb &amp; mm-N or in.-lb)</td>
</tr>
<tr>
<td>( G )</td>
<td>lubricant coefficient of thermal expansion</td>
<td>(1/DegC or 1/DegF)</td>
</tr>
<tr>
<td>( H )</td>
<td>non dimensional film thickness parameter</td>
<td>(-)</td>
</tr>
<tr>
<td>( J )</td>
<td>moment of inertia of the ball</td>
<td>kg-mm(^2)</td>
</tr>
<tr>
<td>( K_f )</td>
<td>conductivity of the film</td>
<td>(lb/degF-sec)</td>
</tr>
<tr>
<td>( K_{ij} )</td>
<td>the proportion of the heat flow from node ( i ) going to node ( j ).</td>
<td>(-)</td>
</tr>
<tr>
<td>( K_9, K_{10} )</td>
<td>constants in expression for heat transfer coefficient</td>
<td>(-)</td>
</tr>
<tr>
<td>( L )</td>
<td>characteristic length</td>
<td>(mm or in.)</td>
</tr>
<tr>
<td>( L_{10\infty} )</td>
<td>full film fatigue life</td>
<td>(hrs)</td>
</tr>
<tr>
<td>( M_c )</td>
<td>moment due to fluid friction between the cage and the ring land</td>
<td>(mm-N or in.-lb)</td>
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*Where multiple units are indicated, the first units given are those associated with the computer program input and output.
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<td>$M_{x,y,z}$</td>
<td>ball moment components in the $x,y,z$ coordinate system</td>
<td>(mm-N or in.-lb)</td>
</tr>
<tr>
<td>$M$</td>
<td>ball moment vector</td>
<td>(mm-N or in.-lb)</td>
</tr>
<tr>
<td>$N_u$</td>
<td>Nusselt's number</td>
<td>(-)</td>
</tr>
<tr>
<td>$P_r$</td>
<td>Prandtl's number</td>
<td>(-)</td>
</tr>
<tr>
<td>$P_d$</td>
<td>diametral clearance</td>
<td>(mm or in.)</td>
</tr>
<tr>
<td>PE</td>
<td>bearing end play</td>
<td>(mm or in.)</td>
</tr>
<tr>
<td>$P_1, P_2$</td>
<td>forces acting normal to the ball surface within the outer and inner raceway contact ellipse</td>
<td>(N or lb)</td>
</tr>
<tr>
<td>$P_3$</td>
<td>ball-cage normal force</td>
<td>(N or lb)</td>
</tr>
<tr>
<td>$Q$</td>
<td>load</td>
<td>(N or lb)</td>
</tr>
<tr>
<td>$Q_a$</td>
<td>average asperity borne load</td>
<td>(N or lb)</td>
</tr>
<tr>
<td>$Q_r$</td>
<td>the radial component of the minimum rolling element-race normal force</td>
<td>(N or lb)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>non dimensional load parameter</td>
<td>(-)</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>the vector normal load per unit length (N/mm or lb/in.) of the contact ellipse</td>
<td>(-)</td>
</tr>
<tr>
<td>$R$</td>
<td>radius of outer ring groove centers</td>
<td>(mm)</td>
</tr>
<tr>
<td>$R_e$</td>
<td>Reynolds's number</td>
<td>(-)</td>
</tr>
<tr>
<td>$R_x, R_y$</td>
<td>effective radii of curvature parallel and transverse to the rolling direction respectively</td>
<td>(mm or in.)</td>
</tr>
<tr>
<td>$S$</td>
<td>coordinate along the contact in the direction perpendicular to rolling friction</td>
<td>(mm or in.)</td>
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<td>Sd</td>
<td>diametral play</td>
<td>(mm or in.)</td>
</tr>
<tr>
<td>T</td>
<td>time</td>
<td>(sec)</td>
</tr>
<tr>
<td>T₀</td>
<td>long time duration</td>
<td>(sec)</td>
</tr>
<tr>
<td>Tₛ</td>
<td>starting time</td>
<td>(sec)</td>
</tr>
<tr>
<td>T₁,T₂</td>
<td>traction forces</td>
<td>(N or lb)</td>
</tr>
<tr>
<td>T</td>
<td>traction force vector at a general location within the contact</td>
<td>(N or lb)</td>
</tr>
<tr>
<td>U</td>
<td>characteristic speed</td>
<td>(m/sec or in./sec)</td>
</tr>
<tr>
<td>V</td>
<td>fluid entrainment velocity at the contact center</td>
<td>(m/sec or in./sec)</td>
</tr>
<tr>
<td>V</td>
<td>volume of the nodal element</td>
<td>(m³ or in.³)</td>
</tr>
<tr>
<td>V</td>
<td>voltage</td>
<td>(volt)</td>
</tr>
<tr>
<td>Vᵢ</td>
<td>volume flow rate through node i</td>
<td>(m³/sec)</td>
</tr>
<tr>
<td>Vₒ</td>
<td>voltage over long time duration</td>
<td>(volt)</td>
</tr>
<tr>
<td>Vₓ</td>
<td>rolling velocity in x direction</td>
<td>(m/sec or in./sec)</td>
</tr>
<tr>
<td>Vᵧ</td>
<td>rolling velocity in y direction</td>
<td>(m/sec or in./sec)</td>
</tr>
<tr>
<td>X,Y,Z</td>
<td>inertial coordinate system</td>
<td>(-)</td>
</tr>
<tr>
<td>DCL</td>
<td>diametral clearance</td>
<td>(mm or in.)</td>
</tr>
<tr>
<td>EPSFIT</td>
<td>user specified convergence criterion</td>
<td>(-)</td>
</tr>
<tr>
<td>EP₁, EP₂</td>
<td>a user supplied convergence criterion</td>
<td>(-)</td>
</tr>
<tr>
<td>EQ</td>
<td>temperature equilibrium convergence criteria for Eq. (3-41)</td>
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</tr>
<tr>
<td>NEQ</td>
<td>number of equations in bearing solution</td>
<td>(-)</td>
</tr>
<tr>
<td>XCAV</td>
<td>volume fraction of lubricant in bearing</td>
<td>(-)</td>
</tr>
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<th>Definition</th>
<th>Units*</th>
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<tr>
<td>a</td>
<td>a constant coefficient in Nusselt's number</td>
<td>(-)</td>
</tr>
<tr>
<td>a</td>
<td>contact ellipse semi-major axis</td>
<td>(mm or in.)</td>
</tr>
<tr>
<td>a</td>
<td>free convection temperature-exponent</td>
<td>(-)</td>
</tr>
<tr>
<td>b</td>
<td>an exponent in Nusselt's number</td>
<td>(-)</td>
</tr>
<tr>
<td>b</td>
<td>half the contact width</td>
<td>(mm or in.)</td>
</tr>
<tr>
<td>c</td>
<td>an exponent in Nusselt's number</td>
<td>(-)</td>
</tr>
<tr>
<td>c</td>
<td>coefficient of specific heat</td>
<td>(W/kg-Deg C)</td>
</tr>
<tr>
<td>d</td>
<td>exponent in free convection heat transfer equations</td>
<td>(-)</td>
</tr>
<tr>
<td>d_κ</td>
<td>cage-land diameter</td>
<td>(mm)</td>
</tr>
<tr>
<td>d_m</td>
<td>bearing pitch diameter</td>
<td>(mm or in.)</td>
</tr>
<tr>
<td>f_m</td>
<td>the vector of friction force per unit length of the contact ellipse</td>
<td>(N/mm)</td>
</tr>
<tr>
<td>g</td>
<td>gravitational constant</td>
<td>(m/sec(^2)/or (\text{in./sec}^2))</td>
</tr>
<tr>
<td>h</td>
<td>elastohydrodynamic film thickness</td>
<td>(mm or (\mu)-in.)</td>
</tr>
<tr>
<td>h_c</td>
<td>critical value of film thickness</td>
<td>(mm or (\mu)-in.)</td>
</tr>
<tr>
<td>h_f</td>
<td>the film thickness under fully flooded conditions</td>
<td>(mm or (\mu)-in.)</td>
</tr>
<tr>
<td>h_s</td>
<td>starved plateau thickness</td>
<td>(mm or (\mu)-in.)</td>
</tr>
<tr>
<td>h_A.C.</td>
<td>film thickness calculated by Archard-Cowking formula</td>
<td>(mm or (\mu)-in.)</td>
</tr>
<tr>
<td>h_D.H.</td>
<td>film thickness calculated by Dowson-Higginson formula</td>
<td>(mm or (\mu)-in.)</td>
</tr>
<tr>
<td>i, j</td>
<td>indices of heat flow nodes</td>
<td>(-)</td>
</tr>
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*Where multiple units are indicated, the first units given are this associated with the computer program input and output.*
### NOMENCLATURE (CONT'D)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Units*</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l$</td>
<td>separation distance between temperature nodes</td>
<td>(mm)</td>
</tr>
<tr>
<td>$l_{rc}$</td>
<td>contact length, or in the case of an elliptical contact area, 0.8 times the contact length</td>
<td>(mm)</td>
</tr>
<tr>
<td>$n$</td>
<td>number of rolling elements, total number of heat flow nodes</td>
<td>(-)</td>
</tr>
<tr>
<td>$P_0$</td>
<td>maximum contact pressure</td>
<td>$(N/mm^2$ or psi)</td>
</tr>
<tr>
<td>$q$</td>
<td>heat generation rate, net heat transfer</td>
<td>(W)</td>
</tr>
<tr>
<td>$q_c$</td>
<td>heat generated by fluid shearing between the cage and land</td>
<td>(W)</td>
</tr>
<tr>
<td>$q_f$</td>
<td>fluid drag heat</td>
<td>(W)</td>
</tr>
<tr>
<td>$q_I$</td>
<td>heat generated by shearing force in the ball-raceway and ball-cage inlet region</td>
<td>(W)</td>
</tr>
<tr>
<td>$q_i$</td>
<td>heat energy in the $i$-th nodal element</td>
<td>(W)</td>
</tr>
<tr>
<td>$q_T$</td>
<td>heat generated by traction in the contact zone</td>
<td>(W)</td>
</tr>
<tr>
<td>$q_{ti}$</td>
<td>heat carried by mass flow from node $i$</td>
<td>(W)</td>
</tr>
<tr>
<td>$q_{gi}$</td>
<td>heat generated at node $i$</td>
<td>(W)</td>
</tr>
<tr>
<td>$q_{oi}$</td>
<td>heat flow from all neighboring nodes to node $i$</td>
<td>(W)</td>
</tr>
<tr>
<td>$q_{Ri,j}$</td>
<td>the heat energy transferred by radiation between nodes $i$ and $j$</td>
<td>(W)</td>
</tr>
<tr>
<td>$q_{ci,j}$</td>
<td>heat flow transferred by conduction from node $i$ to node $j$</td>
<td>(W)</td>
</tr>
<tr>
<td>$q_{ui,j}$</td>
<td>the heat flow between nodes $i$ and $j$</td>
<td>(W)</td>
</tr>
<tr>
<td>$q_{vi,j}$</td>
<td>heat flow transferred by free convection from node $i$ to node $j$</td>
<td>(W)</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Units*</th>
</tr>
</thead>
<tbody>
<tr>
<td>(q_{wi,j})</td>
<td>heat flow by forced convection from node (i) to node (j)</td>
<td>(W)</td>
</tr>
<tr>
<td>(r)</td>
<td>groove radius</td>
<td>(mm or in.)</td>
</tr>
<tr>
<td>(\mathbf{r}_m)</td>
<td>a vector from the rolling element center to the point of contact</td>
<td>(mm or in.)</td>
</tr>
<tr>
<td>(r^*)</td>
<td>meniscus distance from center of contact along direction of rolling</td>
<td>(mm or (\mu)-in.)</td>
</tr>
<tr>
<td>(t)</td>
<td>temperature</td>
<td>(Deg C or Deg K)</td>
</tr>
<tr>
<td>(u)</td>
<td>sliding velocity at the contact center</td>
<td>(m/sec or in./sec)</td>
</tr>
<tr>
<td>(u)</td>
<td>sliding velocity vector</td>
<td>(m/sec or in./sec)</td>
</tr>
<tr>
<td>(u_1, u_2)</td>
<td>surface velocity of bodies 1 and 2 relative to the contact</td>
<td>(m/sec or in./sec)</td>
</tr>
<tr>
<td>(u_s)</td>
<td>sliding speed</td>
<td>(m/sec or in./sec)</td>
</tr>
<tr>
<td>(u_s^*)</td>
<td>sliding speed at which traction coefficient is a maximum</td>
<td>(m/sec or in./sec)</td>
</tr>
<tr>
<td>(x, y, z)</td>
<td>a local coordinate system established at each ball location</td>
<td>(-)</td>
</tr>
<tr>
<td>(x)</td>
<td>sliding velocity scaled by (u_s^*)</td>
<td>(m/sec or in./sec)</td>
</tr>
<tr>
<td>(x_1)</td>
<td>ball axial position relative to the outer race</td>
<td>(mm or in.)</td>
</tr>
<tr>
<td>(x_m)</td>
<td>maximum variation of (x)</td>
<td>(mm)</td>
</tr>
<tr>
<td>(y_1)</td>
<td>ball radial position relative to the outer race</td>
<td>(mm or in.)</td>
</tr>
<tr>
<td>(y_m)</td>
<td>maximum variation of (y)</td>
<td>(mm)</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Units*</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_c$</td>
<td>ball center-cage pocket offset</td>
<td>(mm or in.)</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>diametral clearance between cage and land</td>
<td>(mm or in.)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>shaft displacement at a bearing location</td>
<td>(mm or in.)</td>
</tr>
<tr>
<td>$\Delta_{DCL}$</td>
<td>change in bearing diametral clearance</td>
<td>(mm or in.)</td>
</tr>
<tr>
<td>$\Delta_T$</td>
<td>a small increment of time</td>
<td>(sec)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>angular distance between rolling elements</td>
<td>(deg)</td>
</tr>
<tr>
<td>$\hat{\delta}_b$</td>
<td>bearing deflection vector</td>
<td>(mm or rad)</td>
</tr>
<tr>
<td>$\Delta_L$</td>
<td>lubricant replenishment layer thickness</td>
<td>(mm)</td>
</tr>
<tr>
<td>$\Phi(C)$</td>
<td>cumulative distribution function of standard normal distribution</td>
<td>(-)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>resistance of heat flow</td>
<td>(degC/W)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>angular velocity</td>
<td>(rad/sec)</td>
</tr>
<tr>
<td>$\omega_c$</td>
<td>cage angular velocity</td>
<td>(rad/sec)</td>
</tr>
<tr>
<td>$\alpha_{res}$</td>
<td>resultant resistance to heat flow</td>
<td>(degC/W)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>contact angle</td>
<td>(deg)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>scaling factor in modified Newton-Raphson technique</td>
<td>(-)</td>
</tr>
<tr>
<td>$\alpha_p$</td>
<td>pressure-viscosity index</td>
<td>(in.$^2$/lb)</td>
</tr>
<tr>
<td>$\alpha_i$</td>
<td>inner race contact angle</td>
<td>(deg)</td>
</tr>
<tr>
<td>$\alpha_o$</td>
<td>outer race contact angle</td>
<td>(deg)</td>
</tr>
<tr>
<td>$\alpha_a$</td>
<td>auxiliary contact angle</td>
<td>(deg)</td>
</tr>
<tr>
<td>$\alpha_v$</td>
<td>film coefficient of heat transfer by free convection</td>
<td>(W/m$^2$-degC)</td>
</tr>
</tbody>
</table>

Where multiple units are indicated, the first units given are those associated with the computer program input and output.
### NOMENCLATURE (CONTD)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Units*</th>
</tr>
</thead>
<tbody>
<tr>
<td>α&lt;sub&gt;w&lt;/sub&gt;</td>
<td>film coefficient of heat transfer by forced convection</td>
<td>(W/m&lt;sup&gt;2&lt;/sup&gt;-degC)</td>
</tr>
<tr>
<td>β</td>
<td>temperature-viscosity coefficient</td>
<td>(l/degC)</td>
</tr>
<tr>
<td>ρ&lt;sub&gt;b&lt;/sub&gt;</td>
<td>ball speed vector pitch angle</td>
<td>(deg)</td>
</tr>
<tr>
<td>δ</td>
<td>the first variation</td>
<td>(-)</td>
</tr>
<tr>
<td>δ&lt;sub&gt;e&lt;/sub&gt;</td>
<td>elastic deformation</td>
<td>(mm)</td>
</tr>
<tr>
<td>δ&lt;sub&gt;x&lt;/sub&gt;, δ&lt;sub&gt;y&lt;/sub&gt;, δ&lt;sub&gt;z&lt;/sub&gt;</td>
<td>the linear deflection components of δ&lt;sub&gt;b&lt;/sub&gt;</td>
<td>(mm)</td>
</tr>
<tr>
<td>ε</td>
<td>surface emissivity</td>
<td>(-)</td>
</tr>
<tr>
<td>ε&lt;sub&gt;a&lt;/sub&gt;</td>
<td>a small arbitrary constant</td>
<td>(-)</td>
</tr>
<tr>
<td>η</td>
<td>dynamic viscosity</td>
<td>(centipoise or lb&lt;sub&gt;f&lt;/sub&gt; sec/in.&lt;sup&gt;2&lt;/sup&gt;)</td>
</tr>
<tr>
<td>θ&lt;sub&gt;y&lt;/sub&gt;, θ&lt;sub&gt;z&lt;/sub&gt;</td>
<td>the angular deflection components of δ&lt;sub&gt;b&lt;/sub&gt;</td>
<td>(rad)</td>
</tr>
<tr>
<td>λ</td>
<td>thermal conductivity</td>
<td>(W/M-degC)</td>
</tr>
<tr>
<td>λ&lt;sub&gt;a&lt;/sub&gt;</td>
<td>a viscoelastic constant (oil parameter)</td>
<td>(-)</td>
</tr>
<tr>
<td>μ</td>
<td>traction coefficient</td>
<td>(-)</td>
</tr>
<tr>
<td>μ&lt;sub&gt;a&lt;/sub&gt;</td>
<td>coulomb friction coefficient</td>
<td>(-)</td>
</tr>
<tr>
<td>μ&lt;sub&gt;r&lt;/sub&gt;</td>
<td>μ scaled by μ&lt;sub&gt;a&lt;/sub&gt;</td>
<td>(-)</td>
</tr>
<tr>
<td>μ&lt;sub&gt;EHD&lt;/sub&gt;</td>
<td>fluid traction coefficient</td>
<td>(-)</td>
</tr>
<tr>
<td>μ&lt;sup&gt;*&lt;/sup&gt;</td>
<td>maximum EHD traction coefficient</td>
<td>(-)</td>
</tr>
<tr>
<td>ν</td>
<td>kinematic viscosity</td>
<td>(centistokes)</td>
</tr>
<tr>
<td>ν&lt;sub&gt;1&lt;/sub&gt;, ν&lt;sub&gt;2&lt;/sub&gt;</td>
<td>Poisson's ratio for contacting bodies</td>
<td>(-)</td>
</tr>
</tbody>
</table>

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<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Units*</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>density</td>
<td>(kg/m$^3$)</td>
</tr>
<tr>
<td>$\rho_o$</td>
<td>density of the oil</td>
<td>(kg/m$^3$)</td>
</tr>
<tr>
<td>$\rho_l$</td>
<td>dimensionless meniscus distance</td>
<td>(-)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stefan-Boltzmann radiation constant</td>
<td>W/m$^2$-degK$^*$</td>
</tr>
<tr>
<td>$R_{\Theta}$</td>
<td>RMS value of the distribution of asperity slope angles</td>
<td>(deg)</td>
</tr>
<tr>
<td>$R_{\phi}$</td>
<td>RMS value of surface roughness</td>
<td>(micrometers)</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>density function of standard normal distribution</td>
<td>(-)</td>
</tr>
<tr>
<td>$\phi_s$</td>
<td>starvation reduction factor</td>
<td>(-)</td>
</tr>
<tr>
<td>$\phi_t$</td>
<td>the film thickness reduction factor, due to heating</td>
<td>(-)</td>
</tr>
<tr>
<td>$\psi$</td>
<td>thermal diffusivity</td>
<td>(mm$^2$/sec)</td>
</tr>
<tr>
<td>$\omega_c$</td>
<td>cage orbital velocity</td>
<td>(rad/sec)</td>
</tr>
<tr>
<td>$\omega_o$</td>
<td>ball orbital velocity</td>
<td>(rad/sec)</td>
</tr>
<tr>
<td>$\omega_x$</td>
<td>ball angular velocity component about the x axis</td>
<td>(rad/sec)</td>
</tr>
<tr>
<td>$\omega_y$</td>
<td>ball angular velocity component about the y axis</td>
<td>(rad/sec)</td>
</tr>
<tr>
<td>$\omega_z$</td>
<td>ball angular velocity component about the z axis</td>
<td>(rad/sec)</td>
</tr>
<tr>
<td>$\omega_o$</td>
<td>first derivative of $\omega_o$ with respect to time</td>
<td>(rad/sec$^2$)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>angular velocity of ball in x, y, z coordinate system</td>
<td>(rad/sec)</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>Symbol</th>
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</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>refers to point where traction curve becomes nonlinear</td>
</tr>
<tr>
<td>C</td>
<td>refers to cage or conduction</td>
</tr>
<tr>
<td>N</td>
<td>refers to current iteration</td>
</tr>
<tr>
<td>R</td>
<td>refers to rolling or radiation</td>
</tr>
<tr>
<td>a, asp</td>
<td>denotes asperity effect</td>
</tr>
<tr>
<td>f</td>
<td>refers to fluid or flooded</td>
</tr>
<tr>
<td>i</td>
<td>denotes the i-th ball, i-th node, inner ring</td>
</tr>
<tr>
<td>j</td>
<td>denotes j-th node</td>
</tr>
<tr>
<td>k</td>
<td>index denoting a specific time interval</td>
</tr>
<tr>
<td>m</td>
<td>an index-denoting bearing component</td>
</tr>
<tr>
<td>o</td>
<td>denotes outer ring</td>
</tr>
<tr>
<td>s</td>
<td>refers to sliding, starvation effect, or shaft</td>
</tr>
<tr>
<td>t</td>
<td>refers to thermal effect</td>
</tr>
<tr>
<td>v</td>
<td>refers to free convection</td>
</tr>
<tr>
<td>w</td>
<td>refers to forced convection</td>
</tr>
<tr>
<td>x, y, z</td>
<td>denotes components of vector quantities with respect to x, y, and z coordinates</td>
</tr>
<tr>
<td>1, 2</td>
<td>refers to bodies 1 and 2</td>
</tr>
</tbody>
</table>
I. INTRODUCTION

The computer program described herein, SHABERTH, "A Computer Program for the Steady State and Transient Analyses of Shaft Bearing Systems," is the third generation of SKF Computer Program AE72Y003. Program AE72Y003 was developed by Kellstrom (1) under U. S. Army Contract DAAD05-73-C-0011, sponsored by the Ballistics Research Laboratory at Aberdeen Proving Grounds. The original as well as the succeeding generations of the program consists of the following major subprograms.

The master program consists of the following major subprograms.

1) Bearing Analysis. These subprograms are largely based upon the methods of Harris, (2,3).

2) Three Dimensional Shaft Deflection Analysis developed by Norlander and Friedrichson. (See Appendix I 3).

3) Bearing Dimensional Change Analysis based on the methods of Timoshenko, (4), and adapted to the shaft-bearing-housing system by Crecelius, (5). (See Appendix I 2).

4) Generalized Steady State and Transient Temperature Mapping and Heat Dissipation Analyses based on the methods of Harris: (6), Fernlund, (7) and Andreason, (8).

Although the primary function of all three generations of the program is to predict general bearing performance characteristics, and the bulk of the coding reflects this emphasis, the steady state and transient heat dissipation and temperature mapping subprogram may be used on a stand alone basis to model the thermal behavior of any system which can be represented by discrete temperature nodes.

The differences between the successive generations of the program reflect the development and installation of improved bearing lubrication and friction models, improved analysis of the bearing cage and improvements in the program structure which increased the program versatility and solution procedures.

The first generation of the program used the Newtonian lubricated friction models developed by Harris (2, 3).
The second generation of the program which carries the designation AT74Y001 was created by Crecelius, Liu and Chiu under Air Force Contract No. F33615-72-C-1467 and Navy MIPR No. M52376-3-000007 and is documented by McCool, et al (9).* In that effort, with Program AE72Y003 as the basis, the ball bearing subprogram was modified to include new models as follows:

1) An EHD film thickness model that accounts for i) thermal heating in the contact inlet using a regression fit to results obtained by Cheng (10) and ii) lubricant film starvation using theoretical results derived by Chiu (11).

2) A new semi-empirical model for fluid traction in an EHD contact (9), is combined with an asperity load sharing model developed by Tallian (12) to yield a model for traction in concentrated contacts that reflects the state of lubrication as it varies from dry, through partial EHD to the full EHD regime.

3) A model for the hydrodynamic rolling and shear forces in the inlet zone of lubricated contacts accounting for the degree of lubricant film starvation, (9).

4) Normal and friction forces between a ball and a cage pocket are modelled in a way that accounts for the transition between the hydrodynamic and elasto-hydrodynamic regimes of lubrication (9).

5) A model for the effect on fatigue life of the ratio of the EHD plateau film thickness to the composite surface roughness, (9).

Additionally, models for temperature viscosity and pressure viscosity variation as functions of temperature given by Walther (13) and Fresco (14) respectively, were adopted.

Program AT74Y001 is capable of analyzing only a single axially loaded ball bearing. The program cannot be used to analyze a multi-bearing system. All other capabilities are present however.

*Due to the similarities between major segments of SHABERTH AT75Y004 and AT74Y001, many sections of (9) have been included in this text, without modification.
The basis for the present program, SHABERTH, was AT74Y001. The latent capability for the analysis of up to five ball and roller bearings subjected to general, (5 degrees of freedom) loading, has been utilized. The models added to AT74Y001 are used in the calculation of both the ball and roller bearing friction forces and frictional heat generation rates. The present program also includes a new model for the hydrodynamic rolling and slip forces in the inlet zone of lubricated line contacts, based on the work of Chiu, Ref. (15). Additionally, a cage model developed under NASA Contract No. NAS3-19739 Ref (15) has been added which allows the cage to move with up to three degrees of freedom versus the one degree of freedom permitted in Program AT74Y001. This cage model may be used in the analysis of both ball and roller bearings.

Under Air Force Contract No. F33615-76-C-2061 and Navy NAPTC MIPR No. N62376-76-MP-00005, the capabilities of SHABERTH were expanded to solve the combined set of multi-rolling element and cage quasidynamic equilibrium equations.

This expansion required changes in the concentrated contactasperity friction model as well as changes in the cage-rolling element and cage-ring interaction calculations. Additionally, the mathematical definition of the range of permitted variable values was made substantially more accurate.

SHABERTH is intended to be as general as possible with the following limits on system size.

Number of bearings supporting the shaft - five (5) maximum
Number of rolling elements per bearing - thirty (30) maximum
Number of temperature nodes used to describe the system - one hundred (100) maximum

The program structure is modular and has been designed to permit substitution of new mathematical models and refinements to the existing models as the needs and opportunities develop.

The third generation program, SHABERTH, exists as two versions, SHABERTH/SKF, SKF Program No. AT75Y004 and SHABERTH/NASA, SKF Program No. AT76Y001. The differences between the two versions reside in the calculation of the elastohydrodynamic (EHD) film thickness and traction forces which develop in the rolling element-raceway and rolling element-cage concentrated contacts. The calculation of these factors as performed in the SKF version is detailed herein. The details of the calculations performed by the NASA version are presented in Ref. (15).
2. PROBLEM FORMULATION AND SOLUTION

The purpose of the program is to provide a tool with which the shaft-bearing system performance characteristics can be determined as functions of system temperatures. These system temperatures may be a function of steady state operation or a function of time variant conditions brought on by a change in the system steady state condition. Such a change would be the termination of lubricant supply to the bearings and other lubricated mechanical elements.

The program is structured with four nested, calculation schemes as follows:

1. **Thermal**, steady state or transient temperature calculations which predict system temperatures at a given operating state.

2. **Bearing dimensional equilibrium** which uses the bearing temperatures predicted by the temperature mapping subprograms and the rolling element raceway load distribution, predicted by the bearing subprograms, to calculate bearing diametral clearance at a given operating state.

3. **Shaft-bearing system load equilibrium** which calculates bearing inner ring positions relative to the respective outer rings such that the external loading applied to the shaft is equilibrated by the rolling element loads which develop at each bearing inner ring at a given state.

4. **Bearing rolling element and cage load equilibrium** which calculates the rolling element and cage equilibrium positions and rotational speeds based upon the relative inner-outer ring positions, inertia effects and friction conditions, which if lubricated, are temperature dependent.

The above program structure allows complete mathematical simulation of the real physical system. The program has been coded to allow various levels of program execution which prove useful and economical in bearing design studies.

These levels of execution are explained fully in Sections 3, 4, and 5.
The structure of the program and the nesting of the solution loops noted above can be seen clearly in the Program Flow Chart which is discussed in Appendix II 1.

The sections below present the systems of field equations which are solved in each of the nested calculation schemes. A more detailed discussion is contained in (1, 9 and 15).

2.1 Temperature Calculations

Subsequent to each calculation of bearing generated heat rates, either the steady state or transient temperature mapping solution scheme may be executed. This set of sequential calculations is terminated as follows:

1. For the steady state case, when each system temperature is within EPA °Centigrade of its previously predicted value, EPA is specified by the user. If it is zero or left blank, a default value of 10 °Centigrade is used. This criteria implies that the steady state equilibrium conditions has been reached.

2. The transient calculation terminates when the user specified time up is reached or when one of the system temperatures exceeds 600°C.

2.1.1 Steady State Temperature Map

The mechanical structure to be analyzed is thought of as divided into a number of elements or nodes, each represented by a temperature. The net heat flow to node i from the surrounding nodes j, plus the heat generated at node i, must numerically equal zero. This is true for each node i, i going from 1 to n, n being the number of unknown temperatures.

After each calculation of bearing generated heat, which results from a solution of the shaft-bearing system portion of the program, a set of system temperatures is determined which satisfy the system of equations:

\[ q_i = q_{oi} + q_{gi} = 0 \text{ for all temperature nodes } i \quad (2.1) \]

where \( q_{oi} \) is the heat flow from all neighboring nodes to node i

\( q_{gi} \) is the heat generated at node i. These values may be input or calculated by the shaft bearing program as bearing frictional heat.
This scheme is solved with a modified Newton-Raphson method which successfully terminates when either of two conditions are met:

\[ \frac{\Delta t_i}{t_i} \leq \text{EP2} \text{ for all nodes } i \]  

(2.2)

where \( \Delta t \) represents the Newton-Raphson correction to the temperature \( t \) at a given iteration such that, \( t_{N+1} = t_N + \Delta t \) and \( N + 1 \) and \( N \), refer to the next and current iteration respectively.

EP2 is a user specified constant. If EP2 is left blank or set to zero (0) a default value of 0.001 is used.

A second convergence criterion dependent upon EP2 is also used. In the system of equations, \( q_{oi} + q_{gi} = 0 \) for all nodes \( i \), absolute convergence would be obtained if the right hand side (EQ) in fact reduced to zero (0). Usually a small residue remains at each node, such that \( (q_{oi} + q_{gi}) = (\text{EQ})i \).

The second convergence criterion is satisfied if

\[ \sum_{i=1}^{n} \left( \frac{(\text{EQ})_i}{n} \right)^2 \leq 100 \times \text{EP2} \]  

(2.3)

where \( n = \text{number of equations in bearing solution} \)

2.1.2 Transient Temperatures

In the transient case, the net heat \( q_i \) transferred to a node \( i \) heats the element. It is thus necessary for heat balance at node \( i \) that the following equations are satisfied.

\[ \rho C V_i \frac{dt_i}{dT_i} = q_i \]  

(2.4)
where

\[ p = \text{density} \]
\[ C_p = \text{specific heat} \]
\[ V = \text{volume of the element} \]
\[ t = \text{temperature} \]
\[ T = \text{time} \]

The temperatures, \( t_{oi} \), at the time of initiation \( T - T_s \) are assumed to be known, that is

\[ t_i(T_s) = t_{oi} \quad i = 1, 2, ..., n \quad (2.5) \]

The problem of calculating the transient temperature distribution in a bearing arrangement thus becomes a problem of solving a system of non-linear differential equations of the first order with certain initial values given. The equations are non-linear since they contain terms of radiation and free convection, which are non-linear with temperature as will be shown later. The simplest and most economical way of solving these equations is to calculate the rate of temperature increase at the time \( T = T_k \) from equation 2.4 and then calculate the temperatures at time \( T_k + \Delta T \) from

\[ t_{k+1} = t_k + \frac{dt_k}{dT} \Delta T = t_k + \frac{q_k}{C_p V} \Delta T \quad (2.6) \]

If the time step \( \Delta T \) used as program input is chosen too large, the temperatures will oscillate, and if it is chosen too small the calculation will be costly. It is therefore desirable to choose the largest possible time step that does not give an oscillating solution. The program optionally calculates such a time step. The step is obtained from the condition, (16)

\[ \frac{dt_{i,k+1}}{dt_{i,k}} \geq 0 \quad i = 1, 2, ..., n \quad (2.7) \]

If this derivative were negative, the implication would be that the local temperature at node \( i \) has a negative effect on its future value. This would be tantamount to asserting that the hotter a region is now, the colder it will be after an equal time interval. An oscillating solution would result.
Differentiating equation (2.6) for node \(i\), one has as condition (2.8),

\[
\frac{dt_{i,k+1}}{dt_{i,k}} = 1 + \frac{\Delta T_i}{C_i \rho_i V_i} \cdot \frac{dq_i}{dt_i} \quad 0, \quad i = 1, 2, \ldots, n
\]  

(2.8)

The derivative \(dq_i/dt_i\) is calculated numerically

\[
\frac{dq_i}{dt_i} = \frac{q_i(t_i + \Delta t_i) - q_i(t_i)}{\Delta t_i}
\]  

(2.9)

For each node, the value of \(\Delta T_i\) giving a value of zero to the right hand side of Eqn. (2.8) is calculated.

A value of \(\Delta T\) rounded off to one significant digit smaller than the smallest of the \(\Delta T_i\) given by Eqn. (2.8) is used.

If the transient thermal scheme is being used interactively with the bearing subprograms, the user must specify a small enough time step between calls to the bearing subprograms in order that the variation in bearing generated heats, with time, accurately reflects the physical situation. At first, a trial and error procedure will be required to effectively use the program in its mode, however, experience will increase the user's effectiveness.

2.1.3 Calculation of Heat Transfer Rate

The transfer of heat within a medium or between two media can occur by conduction, convection, radiation and fluid flow.

All these types of heat transfer occur in a bearing application as the following examples show.

1. Heat is transferred by conduction between inner ring and shaft and between outer ring and housing.

2. Heat is transferred by convection between the surface of the housing and the surrounding air.

3. Heat is transferred by radiation between the shaft and the housing.

4. When the bearing is lubricated and cooled by circulating oil, heat is transferred by fluid flow.

Therefore, in calculating the net flow to a node all the above-mentioned modes of heat transfer will be considered.

2.1.3.1 Generated Heat

There may be a heat source at node \(i\) giving rise to a heat flow to be added to the heat flowing from the neighboring nodes.
In the case that the heat source is a bearing, it may either be considered to produce known amounts of power, in which case constant numbers are entered as input to the program, or the shaft-bearing program may be used to calculate the bearing generated heat as a function of bearing temperatures.

2.1.3.2 Conduction

The heat flow \( q_{ci,j} \) which is transferred by conduction from node \( i \) to node \( j \), is proportional to the difference in temperature \( (t_i - t_j) \) and the cross-sectional area \( A \) and is inversely proportional to the distance \( L \) between the two points, thus

\[
q_{ci,j} = \frac{\lambda A}{L} (t_i - t_j)
\]

(2.10)

where \( \lambda \) = the thermal conductivity of the medium.

2.1.3.3 Free Convection

Between a solid medium such as a metallic body and a liquid or gas, heat transfer is by free or forced convection. Heat transfer by free convection is caused by the setting in motion of the liquid or gas as a result of a change in density arising from a temperature differential in the medium. With free convection between a solid medium and air, the heat energy \( q_{vi,j} \) transferred between nodes \( i \) and \( j \) can be calculated from the equation, (2.11)

\[
q_{vi,j} = \alpha_v A \ |t_i - t_j| ^ d \cdot \text{SIGN} (t_i - t_j)
\]

(2.11)

where \( \alpha_v \) = the film coefficient of heat transfer by free convection
\( A \) = the surface area of contact between the media
\( d \) = is an exponent, usually = 1.25, but any value can be specified as input to the program

\[
\text{SIGN} = \begin{cases} 
1 & \text{if } t_i \geq t_j \\
-1 & \text{if } t_k < t_j
\end{cases}
\]

9
The last factor is included to give the expression $q_{vi,j}$ a correct sign.

The value of $\alpha_y$ can be calculated for various cases, see Jacob and Hawkins, (16).

2.1.3.4 Forced Convection

Heat transfer by forced convection takes place when liquid or gas moves around a solid body, for example, when the liquid is forced to flow by means of a pump or when the solid body is moved through the liquid or gas. The heat flow $q_{wi,j}$ transferred by forced convection can be obtained from the following equation.

$$q_{wi,j} = \alpha_w \Delta(t_i - t_j) \tag{2.12}$$

where $\alpha_w$ is the film coefficient of heat transfer during forced convection. This value is dependent on the actual shape, the surface condition of the body, the difference in speed, as well as the properties of the liquid or gas.

In most cases, it is possible to calculate the coefficient of forced convection from a general relationship of the form,

$$N_u = a R_e^b P_r^c \tag{2.13}$$

where $a$, $b$, and $c$ are constants obtained from handbooks, such as (17). $R_e$ and $P_r$ are dimensionless numbers defined by

- $N_u =$ Nusselt's number = $\alpha_w \Delta L/\lambda$
- $L =$ characteristic length
- $\Delta =$ conductivity of the fluid
- $R_e =$ Reynold's number = $UL \rho/\mu$
- $U =$ characteristic speed
- $\rho =$ density of the fluid
- $\mu =$ dynamic viscosity of the fluid
- $P_r =$ Prandtl's number = $\mu C_p/\lambda$
- $C_p =$ specific heat
The program can use a value of the coefficient of convection, or let it vary with actual temperatures, the variation being determined by how the viscosity varies. Input can be given in one of four ways, for each coefficient.

**Constant viscosity**

1. Values of the parameters of equation (2.13) are given as input and a constant value of $\alpha_w$ is calculated by the program.

**Temperature dependent viscosity**

2. The coefficient $\alpha_w$ for turbulent flow and heating of petroleum oils is given by

$$\alpha_w = k_9 \cdot \left\{ \eta(t) \right\}^{k_{10}}$$  (2.14)

where $k_9$ and $k_{10}$ are given as input together with viscosity at two different temperatures.

3. Values of the parameters of equation (2.13) are given as input. Viscosity is given at two different temperatures.

2.1.3.5 **Radiation**

If two flat parallel, similar surfaces are placed close together and have the same surface area $A$, the heat energy transferred by radiation between nodes $i$ and $j$ representing those bodies will be,

$$q_{Ri,j} = \epsilon \sigma A \left[ (t_i + 273)^4 - (t_j + 273)^4 \right]$$  (2.15)

where $\epsilon$ is the surface emissivity. The value of the coefficient $\epsilon$ is an input variable and varies between 1 for a completely black surface and 0 for an absolutely clean surface. In addition, $\sigma$ is Stefan-Boltzmann's radiation constant which has the value $5.76 \times 10^{-8}$ watts/m$^2$·(°K)$^4$ and $t_i$ and $t_j$ are the temperatures at points $i$ and $j$.

Heat transfer by radiation under other conditions can also be calculated, (16). The following equation, for instance, applies
between two concentric cylindrical surfaces

\[
q_{Ri,j} = \frac{\epsilon \sigma A_i [(t_i + 273)^4 - (t_j + 273)^4]}{1 + (1-\epsilon) (A_i/A_e)}
\]

(2.16)

where \(A_i\) is the area of the inner cylindrical surface

\(A_e\) is the area of the outer cylindrical surface

2.1.3.6 Fluid Flow

Between nodes established in fluids, heat is transferred by transport of the fluid itself and the heat it contains.

\[
q_{oi} = \rho \cdot C_p \cdot V_i \cdot t_i = K_i t_i
\]

(2.17)

where \(V_i\) = the volume flow rate through node \(i\)
The heat input to node i is the sum of the heat generated at node i (if any) and the sum over all other nodes of the heat transferred to node i by conduction, radiation, free and forced convection.

\[ q_{oi} = q_{G,i} + \sum_j (q_{ci,j} + q_{vi,j} + q_{wi,j} + q_{Ri,j}) \]  

(2.18)

The heat flow between the nodes of Fig. 2.2 is then,

\[ q_{ui,j} = q_{fi} + q_{oi}/2 \]  

(2.19)

If the flow is dividing between node i and j, Figure 2-2 then the heat flow is calculated from

\[ q_{ui,j} = K_{ij} (q_{fi} + q_{oi}/2) \]  

(2.20)

where \( K_{ij} \) = the proportion of the flow at i going to the node j, o \( K_{ij} = 1 \). \( K_{ij} \) is specified at input.

Figure 2.2 Divided Fluid Flow from Node i

2.1.3.7 Total Heat Transferred

The net heat flow rate to node i can be expressed as,

\[ q_i = q_{G,i} + \sum_j (q_{ci,j} + q_{ui,j} + q_{vi,j} + q_{wi,j} + q_{Ri,j}) \]  

(2.21)
The summation should include all nodes \( j \), both with unknown temperatures as well as boundary nodes, at which the temperature is known so long as they have a direct heat exchange with node \( i \).

This expression is a non-linear function of temperatures because of the terms \( q_w \) and \( q_R \). Therefore, the equations to be solved for a steady state solution are non-linear. The sub-program SOLVXX for solving non-linear simultaneous equations is used for this purpose.

2.1.4 Conduction Through a Bearing

As described in Section 2.1.3.2, the conduction between two nodes is governed by the thermal conductivity parameter \( \lambda \) of the medium through which conduction takes place. The value of \( \lambda \) is specified at input.

An exception is when one of the nodes represents a bearing ring and the other a set of rolling elements. In this case, the conduction is separately calculated using the principles described below.

2.1.4.1 Thermal Resistance

It is assumed that the rolling speeds of the rolling elements are so high that the bulk temperature of the rolling elements are the same at both the inner and outer races, except in a volume close to the surface. The resistance to heat flow can then be calculated as the sum of the resistance across the surface and the resistance of the material close to the surface.

The resistance \( \mathcal{R} \) is defined implicitly by

\[
\Delta t = \mathcal{R} \cdot q
\]

(2.22)

where

\( \Delta t \) is temperature difference
\( q \) is heat flow

The resistance due to conduction through the EHD film is calculated as

\[
\mathcal{R}_1 = (h/\lambda) \cdot \Delta
\]

(2.28)
where $h$ is taken to be the calculated plateau film thickness.

$A$ is the Hertzian contact area at the specific rolling element-ring contact under consideration.

$\lambda$ is the conductivity of the oil.

The geometry is shown in Figure 2.3(a). Asperity conduction is not considered.

So far, a constant temperature difference between the surfaces has been assumed. But during the time period of contact, the difference will decrease because of the finite thermal diffusivity of the material near the surface, Fig. 2.3(b).

To points at a distance from the surface this phenomenon will have the same effect as an additional resistance $\mathcal{R}_2$ acting in series with $\mathcal{R}_1$.

This resistance was estimated in (18) as,

$$\mathcal{R}_2 = \frac{1}{\lambda \ell_{re,1}} \left( \frac{\psi}{2b_1} \right)^{b}$$

(2.24)

where $\ell_{re} = $ contact length, or in the case of an elliptical contact area, 0.8 times the major axis.

$\lambda = $ heat conductivity

$\psi = $ thermal diffusivity $= \lambda/(\rho \cdot C_p)$

$\rho = $ density

$C_p = $ specific heat

$b = $ half the contact width

$V = $ rolling speed

The resultant resistance is

$$\mathcal{R}_{res} = \mathcal{R}_1 + \mathcal{R}_2$$

(2.25)
FIGURE 2.3
CONTACT GEOMETRY AND TEMPERATURES
There is one such resistance at each rolling element. They all act in parallel. The resultant resistance, \( r_{\text{res}} \), is thus obtained from
\[
\frac{1}{r_{\text{res}}} = \sum_{i=1}^{n} \frac{1}{r_{\text{res},i}}
\]  

(2.26)

2.2 Bearing Dimensional Change Analysis

The program calculates the changes in bearing diametral clearances according to the analysis described originally in (5) and herein in Appendix I 2, and expressed in generalized equation form as,

\[\Delta DCL = f \left\{ (\text{Fits})_m, t_i, \beta_m, (Q_r)_m \right\}, \ m = 1, 2 \text{ for inner and outer rings} \]  

(2.27)

\[i = 1, 2, 3, 4, 5 \text{ for shaft, inner ring, outer ring, housing and rolling element respectively} \]
where: \( \Delta DCL \) is the change in bearing diametral clearance
fits are the cold mounted shaft and housing fits
\( t_i \) are the component temperatures
\( \Delta \omega_m \) refers to the ring rotational speeds
\( Q_r \) refers to the radial component of the minimum
rolling element-race normal force.

A bearing clearance change criterion is satisfied when
the change in bearing diametral clearance remains within a
narrow, user specified range, for two successive iterations
as follows:

\[
\left| (\Delta DCL)_N - (\Delta DCL)_{N-1} \right| \leq \frac{\text{EPSFIT}}{D} \text{ for all bearings} \quad (2.28)
\]

where: \( N \) denotes the most recent iteration and
\( N-1 \) denotes the previous iteration
\( D \) denotes the ball or roller diameter and
\( \text{EPSFIT} \) is a user specified value = .0001D

It should be noted that although ring rotational speeds,
and initial, i.e. cold, shaft and housing fits are considered
in the clearance change analysis, these two factors are fixed
at input and remain constant through the entire solution.
Although component temperatures may change as a consequence of
the thermal solution, temperatures remain constant through a
complete set of clearance change iterations. As a result, only
the change in bearing load distribution affects the change in
bearing clearance within a set of clearance change iterations.

2.3 Bearing Inner Ring Equilibrium

The bearing inner ring equilibrium solution is obtained
by solving the system:

\[
(F_b)_i - (F_s)_i = 0 \text{ for all bearings, } i \quad (2.29)
\]

where: \( \vec{F}_b \) denotes a vector of bearing loads and
moments resulting from rolling element/race forces and moments.
If the bearing solution considers friction, $F_b$ is comprised of the rolling-element race friction forces as well as the normal forces.

If the bearing solution is, at the user's option, frictionless, $F_b$ is comprised only of rolling element/race normal contact forces.

$(F_s)_i$ denotes a similar vector of loads, exerted on the inner ring by the shaft. The calculation of $(F_s)_i$ is presented in Appendix I 3.

The variables in this system of equations are the bearing inner ring deflections $\Delta_b$ and the shaft displacements $\Delta$ at all bearing locations. The bearing loads may be expressed as a function of the inner ring deflections.

\[
\begin{bmatrix}
F_{bxi} \\
F_{byi} \\
F_{bzi} \\
M_{byi} \\
M_{bzi}
\end{bmatrix} \quad \text{Forces}
\]

\[
\begin{bmatrix}
F_{sx} \\
F_{sy} \\
F_{sz} \\
M_{sy} \\
M_{sz}
\end{bmatrix} \quad \text{Moments}
\]

\[
(F_b)_i = (2.30)
\]

\[
(F_s)_i = (2.31)
\]

\[
\begin{bmatrix}
F_{bxi} \\
F_{byi} \\
F_{bzi} \\
M_{byi} \\
M_{bzi}
\end{bmatrix} \quad \text{Forces}
\]

\[
\begin{bmatrix}
F_{sx} \\
F_{sy} \\
F_{sz} \\
M_{sy} \\
M_{sz}
\end{bmatrix} \quad \text{Moments}
\]

The deflection $\Delta_b$ of a bearing is described by two radial deflections $\phi$ and $\phi_z$, two angular deflections $\theta_y$ and $\theta_z$ and one axial deflection $\phi_x$. The axial deflection is assumed to be the same for all bearings on a shaft, i.e.
The solution scheme is ended when

\[
\frac{\delta (\Delta)_{ij}}{(\Delta)_{ij}} \leq \begin{cases} 
\text{EPS1 (frictionless)} \\
\text{EPS2 (friction)} 
\end{cases} 
\]  

(2.37)

\[i = 1, \ldots \text{(Number of bearings)}
\]

\[j = 1, 5 \quad \text{- for the 3 linear and two angular deflections at each bearing}
\]

If for some \(i\) or \(j\), \((\Delta)_{ij} = 0\), Eq. (2.38) is used in place of (2.37)

\[
\frac{\delta (\Delta)_{ij}}{(0.001 \times \text{NBRG})} \leq \begin{cases} 
\text{EPS1 (frictionless)} \\
\text{EPS2 (friction)} 
\end{cases} 
\]  

(2.38)

NBRG denotes the number of bearings in the system.

EPS1 and EPS2 are used depending on whether the bearing solutions are frictionless or include friction, respectively. If the bearing deflections are extremely small, computer-generated numerical inaccuracies may prevent convergence according to the above criteria although a perfectly good solution has been obtained. To overcome this problem, the iteration is terminated if all angular deflections are less than \(2 \times 10^{-6}\) radians and all linear deflections are less than \(5 \times 10^{-8}\) inches. Any one of the above criteria imply that inner ring equilibrium is satisfied.

2.4 Bearing Quasi-Dynamic Solution

The bearing quasi-dynamic solution is obtained through a two-step process:


2. Elastic and Quasi-dynamic Solution*

*Quasi-dynamic equilibrium is used to connote that the true dynamic equilibrium terms containing first derivatives of the ball rotational speed vectors and the second derivatives of rolling element position vectors with respect to time are replaced by numerical expressions which are position rather than time dependent.
2.4.1 Rolling Element Equilibrium Equations

The equations which define rolling element quasi-dynamic force equilibrium take the form

\[ \sum_{m} \left[ \int_{-a_m}^{a_m} (Q_m + f_m)dt + F_m \right] + \mathbf{F} = 0 \]

where: \( a_m \) refers to the outer, inner and cage rolling element contacts (2.39) respectively

where: \( Q_m \) is the vector normal load per unit length of the contact. See Appendix I 4.

\( f_m \) is the vector of friction force per unit length of the contact. See Appendices I 5 and I 6.

\( \mathbf{F} \) is the vector of inertia and drag forces. See App. I 6 and I 7.

t is a coordinate along the contact, perpendicular to the direction of rolling (usually the major axis).

\( a \) is half the contact length. See Ref. (1).

\( F_m \) is the vector sum of the hydrodynamic forces acting on the rolling element at the m-th contact. See Appendix I 6.

Rolling element moment equilibrium is defined by:

\[ \int_{-a_m}^{a_m} \mathbf{r}_m \times (Q_m + f_m)dt + \mathbf{r}_m \times \mathbf{F}_m + \mathbf{M}_T = 0 \] (2.40)

\( \mathbf{r}_m \) and \( t \) are defined above, \( \mathbf{M}_T \) is a vector of inertia moments. For the definition of \( \mathbf{M}_T \), refer to Appendix I 7.

\( \mathbf{r}_m \) is a vector from the rolling element center to the point of contact, see Appendix I 4.

The solution to the equation sets represented (2.39) and 2.40) generate the necessary data to calculate bearing fatigue life. See Appendix I 9.

In the frictionless elastic solution \( \mathbf{F}_m \) and \( f_m = 0 \). Additionally, the only rolling element inertia term considered in the frictionless solution is centrifugal force. As a consequence, only the axial and radial force equilibrium equations are solved for each ball. For each roller, the radial force equilibrium and the tilting moment about the Z axis of Fig. 2.4 is solved. A dummy equation for axial force equilibrium is included in the solution matrix which keeps the roller centered.
FIGURE 2.4

Bearing Inertial (XYZ) and Rolling Element (xyz), Coordinate Systems
with respect to the outer race. The cylindrical roller bearing considered by the program cannot carry axial loading.

The friction solution determines ball quasi-dynamic equilibrium for six degrees of freedom. A roller is permitted four degrees of freedom. The rolling-element variables in this solution are \( x_1, y_1, \omega_x, \omega_y, \omega_z, \) and \( \omega_o. \)

where \( x_1 \) is the rolling element axial position relative to the outer race. For a roller, this is a dummy variable.

\( y_1 \) is the rolling element radial position relative to the outer race.

\( \omega_x, \omega_y, \omega_z \) are the orthogonal rolling element rotational speeds relative to the cage speed, about the \( x, y \) and \( z \) axes and \( \omega_o \) is the rolling element orbital speed.

For the roller, \( \omega_z \) is a dummy variable.

The variables \( x_1 \) and \( y_1 \) are the ball variables in the frictionless solution. The variables in the roller frictionless solution are \( x_1 \), a dummy, \( y_1 \) and \( \theta z = \tan^{-1} \left( \frac{\omega y}{\omega x} \right) \).

2.4.2 Cage Equilibrium Equations

The cage equations and cage-rolling element interactions are not considered when the friction forces are omitted from the rolling element equilibrium equations.

The number of degrees of freedom given to the cage is one, if the cage will tend to rotate concentrically with respect to the ring on which it is riding. This condition is determined as a function of the rolling element orbital speed variation and prevails with most roller bearings and with ball bearings subjected only to axial loading. In both cases, orbital speed variation is often inconsequential. Also, single degree of freedom is allowed when the cage is rolling element riding. The single degree of freedom corresponds to a small angular rotation about the bearing axis, measured with respect to rolling element 1. The angular displacement is converted to a linear dimension by a multiplication by the bearing pitch diameter and is noted in Fig. 2.4 as \( \gamma \). When a single degree of freedom is permitted, the sum of moments acting on the cage about the bearing \( x \) axis is required to be zero. This moment equation considers the cage-rolling element normal and friction forces as well as the torque generated at the cage-ring surface.

If there is significant rolling element orbital speed variation, the cage is permitted to move to an eccentric position with respect to the land on which it is piloted. Two additional degrees of freedom are required to describe the eccentric position. These are the cage center of mass radial displacement, \( e \), and the angular displacement of the center of mass, with respect to the bearing \( Y \) axis, \( \theta_c \), see Figures 2-5 and 2-6. These radial and angular displacement variables are determined when the sum of forces acting on the cage, resolved along the bearing \( Y \) and \( Z \) axes, reduce to zero. The rolling element-cage normal and friction forces as well as the pressure
FIGURE 2-5
Inner Ring-Cage Land Contact Geometry

\[ \theta_c = \theta_{c'} + \eta \]
FIGURE 2-6
Outer Ring-Cage Land Contact Geometry

\[ \theta_c = \Theta_c' \]
buildup between the cage and its piloting surface are considered in the equilibrium equations. The effect of the cage mass is neglected.

Figure 2-7 depicts the cage pocket normal and friction forces acting on a rolling element which are considered in the cage equilibrium solution. These forces are functions of the rolling speeds $\omega_x$ and $\omega_y$ and the contact geometry are calculated in the $x,y,z$ frame. The forces exerted on the cage due to the $i$-th rolling element are, in the $XYZ$ frame of reference:

$$
M_{xi} = -(F_{yi} + F_{y2})iR_m + (P_1 - P_2)iR_m
$$
$$
F_{yi} = -(F_{yi} - F_{y2})i\cos \phi_i - (P_1 - P_2)i\sin \phi_i
$$
$$
F_{zi} = (F_{yi} - F_{y2})i\sin \phi_i + (P_1 - P_2)i\cos \phi_i
$$

when the forces of Eq. (2.41) are summed over all of the rolling elements, and the total added to the cage land contact forces, the cage equilibrium equations for the three degree of freedom model are obtained as:

$$
M_X = 0 = (M_{xi}) + M_{CX}
$$
$$
F_Y = 0 = (F_{yi}) + F_{CY}
$$
$$
F_Z = 0 = (F_{zi}) + F_{CZ}
$$

where $M_{xi}$, $F_{yi}$ and $F_{zi}$ are defined for each rolling element by Eq. (2.41), $M_{CX}$ is the cage land friction torque, $F_{CY}$ and $F_{CZ}$ are the cage land hydrodynamic forces.

Within SHABERTH, Eq. (2.42) which defines cage equilibrium, are solved simultaneously with the set of all ball or roller quasidynamic equilibrium equations.

Details for calculating the rolling element/cage pocket forces and the cage land/ring land forces are presented in Appendix I 9.

The ball bearing friction solution is thus obtained by solving $2Z+1$ or 3 equations where $Z$ is the number of rolling elements. The ball bearing frictionless solution is obtained by solving 1, $(Z/2)$, $(Z/2+1)$ or $Z$ sets of 2 equations, depending upon the number of rolling elements in the bearing and the degree of load symmetry which prevails. The various symmetry conditions are explained below.

The roller bearing friction solution contains $4Z+1$ or 3 equations and the frictionless solution contains $Z/2$, $Z/2+1$ or $Z$ sets of three equations again depending upon the number of rolling elements and whether or not load symmetry exists.
FIGURE 2-7
Cage Pocket Normal and Friction Forces Affecting Equilibrium
2.4.3 Load Symmetry Conditions

The various load symmetry conditions are as follows.

Axial symmetry is utilized if, for a ball bearing the load is axial only, then only one set of two equations is solved for the frictionless case. Six ball and one cage equilibrium equations are solved when friction is included. All balls are assumed to behave identically.

Radial load symmetry is utilized if the non-axial shaft loading is comprised of only radial components parallel to the Y axis and moment components parallel to the Z axis and the position of the first rolling element is on the Y axis, then symmetry exists, only half the rolling elements need be considered if the number of rolling elements is even and one half plus one need be considered if the number is odd. Because of inertia terms, radial load symmetry can only be utilized in the frictionless solution.

If load symmetry is not present, then Z sets of two (ball bearing) or Z sets of three (roller bearing) equations must be solved to obtain the frictionless solution.

2.4.4 Bearing Quasidynamic Solution Criteria

As with the steady state temperature mapping scheme, the Newton-Raphson scheme in subprogram SOLVXX is used to solve the sets of equations for each bearing. The iteration scheme terminates when either:

\[
\left| \frac{\Delta X_{iN}}{X_{iN-1}} \right| \leq \begin{cases} \text{EPS1 frictionless} \\ \text{EPS2 friction} \end{cases} \quad (2.43)
\]

or

\[
\left( \frac{\sum_{i=1}^{n} \frac{E Q_i}{n}}{n} \right) \leq 100 X \begin{cases} \text{EPS1 frictionless} \\ \text{EPS2 friction} \end{cases} \quad (2.44)
\]

Experience has shown that the second criteria is usually responsible for terminating the solution. However, when rolling element loads are extremely large, on the order of 10^5 Newtons, it becomes difficult to reduce the equation residues to less than 10 Newtons. In those instances, the first criteria usually terminates the iteration scheme.
3. PROGRAM INPUT

3.1 Types of Input Data

A complete set of input data comprises data of four distinct categories. Within these categories, cards which convey specific kinds of information are referred to as card types. Depending on the complexity of the problem, the input data set may contain none, one or several cards of a given type. The categories are listed below.

I. Title Cards
   A title card plus a second card which provides the program control information for the shaft-bearing solution.

II. Bearing Data Cards
   A set of up to sixteen (16) card types, each set describing one bearing in the assembly. All bearings must be so described. The card sets must be input sequentially in order of increasing distance from a selected end of the shaft.

III. Thermal Data Cards
   A set of up to nine (9) card types to describe the thermal model of the assembly.

IV. Shaft Data Cards
   A set of three (3) card types to describe the shaft geometry, bearing locations on the shaft and shaft loading.

If the program is being used to predict the performance of a bearing assembly, cards from all four sets must be included in the runstream. If the program is being used to thermally model a mechanical system wherein no bearing heat generation rates are required, and therefore, no bearing calculations need be performed, the cards from sets II and IV are omitted.

The review of required input information which follows is broken into the four sets of data categories given above, with special emphasis on program control data.

The input data instructions are given in Appendix II 2, and are for the most part self explanatory. They are laid out in the format of an eighty column data card. A description of the variables is given in the input instruction forms.
The units used for input data are as follows:

- Linear Dimensions - (mm)
- Angles - (degrees)
- Surface Roughness - (microns)
- Bearing Angular Mounting Errors - (radians)
- Rotational Speeds - (RPM)
- Force - (Newtons) (N)
- Moments - (N-mm)
- Pressure, Elastic Modulus - (N/mm²)
- Density - (gm/cm³)
- Kinematic Viscosity - (cs)
- Temperature - (degrees centigrade) (°C)
- Coefficient of Thermal Expansion - (°C⁻¹)
- Thermal Conductivity - (Watts/m/°C)

3.2 Data Set I - Title Cards

3.2.1 Title Card 1

This card should contain the computer run title and any information which might prove useful for future identification. The full eighty (80) columns are available for this purpose. The title will appear at the top of each page of Program output.

3.2.2 Title Card 2

This card provides the control information for the shaft bearing solution.

Item 1: Shaft Speed in rpm, GOV (1). All bearings have the same shaft, i.e. inner ring speed.

Item 2: Number of Bearings on the shaft (NBRG), a minimum of zero is permitted if no bearing solution is being sought. A maximum of five is permitted.

Item 3: Print Flag (NPRINT), NPRINT equal to zero is normal and will result in no intermediate or debug output. With a value of one, a low level intermediate print is obtained at the end of each shaft bearing iteration. The values of the variables, the inner ring displacements (DEL), and the equation residues are printed.

At the end of each bearing iteration, wherein the rolling element and cage equilibrium equations are solved, an error parameter is printed which has the value:
Error Parameter = $X_N/X_{N-1}$

$X_N$ is the change in the variable $X$ specified at iteration $N$.

$X_{N-1}$ is the value of the variable specified at the previous iteration.

The Error Parameter is calculated for each of the bearing variables, but only the largest one is printed.

Additionally, at the end of each Clearance Change iteration, the clearance change error parameter is printed. This error is defined by Eq. 2.28.

If NPRINT is set at 2, all of the above information is printed. Additionally, the variable values and residue values are printed for each iteration of the rolling element and cage equilibrium solution.

**Item 4:** ITFIT controls the number of iterations allowed to satisfy the bearing clearance change iteration scheme. If ITFIT is set to zero (0), or left blank, the clearance change portion of the program is not executed. If a positive integer is input, the clearance change scheme is utilized with a maximum iteration limit of five (5). If a negative integer is input, the scheme is used with a maximum iteration limit equal to the absolute value of the negative integer.

**Item 5:** ITMAIN limits the number of iterations attempted during the solution of the shaft and bearing inner ring equilibrium problems, i.e., establishing the equilibrium of bearing reactions and applied shaft loads. If ITMAIN is left blank, set to zero, or to a positive integer, then (15) iterations are permitted. If ITMAIN is set to a negative integer, the number of iterations is limited to the absolute value of that integer.
Item 6: GOV(2) or EPSFIT is the convergence criterion for the diametral clearance change portion of the analysis. As mentioned under item 3 above, this error parameter is defined by Eq. 2.28.

The iteration scheme is terminated when the error parameter is less than the input value of EPSFIT. If EPSFIT is left blank or is set to zero (0), the program default value of 0.0001 is used.

Items 7 & 8: Main loop accuracy for frictionless elastic (EPS1) and friction solution (EPS2). These accuracy values control the accuracy of the shaft bearing deflection solution as well as the quasi-dynamic solution of the component dynamics (cf. Section 3). If EPS1 and EPS2 are left blank or set to zero (0), default values of 0.001 and 0.0001 respectively are used.

Item 9: IMT, if set to 1, the Material properties for both bearing rings and the rolling elements are to be input on card types B 11 through B 19. If IMT is zero or blank, the rings and rolling elements are assumed to be steel. Card types B 11 through B 14 are required only if the change in bearing diametral clearance is to be calculated.

Item 10: NPASS controls the level of the bearing solution

0 Elastic Contact Forces are calculated. No lubrication or friction effects are considered.

1 Elastic Contact Forces are calculated. Lubrication and friction effects are considered using raceway control (ball bearing) or epicyclic (roller bearing) assumptions to estimate rolling element and cage speeds.

2 Inner Equilibrium is satisfied considering only the Elastic Contact Forces. Using the inner ring positions thus obtained, rolling element and cage equilibrium are determined considering friction.

3 Complete Solution. The inner ring, rolling element and cage equilibrium is determined considering all elastic and friction forces.

3.3 Data Set II - Bearing Data

Most of the input instructions are self-explanatory. Where certain items are deemed to require more explanation than given in the input data format instructions, they are treated on an individual basis by card type and item number.
Most of the bearing input data is read into a two dimensional array named "BD," which has the dimensions (1830, 5). For each of the five bearings permitted on a shaft, a total of 1830 pieces of data may be stored. Denoting BD(I,J), I represents a specific piece of bearing data, J represents the bearing number. The bearing input data of Data Set II occupies the first 106 locations of the 1830 allocated. On the input data format sheets, the designation BD(I) where I=1...106, denotes the location within the BD array where each piece of input data is stored.

3.3.1 Card Type 1 - Bearing Type and Material Designations

Item 1: Bearing type, columns 1-10 must be specified, left justified, i.e., "B" or "C" in column 1. This format must be followed since the Program recognition of bearing type, (ball bearing or cylindrical roller bearing), is derived from reading the "B" or "C" in the first column of this card.

Item 2 & 3: Columns 11-30 and 31-50, "Steel designations," inner and outer rings, respectively. The alphabetic-literal description of the steel types such as "M-50" or "AISI 52100" is input.

Items 4 & 5: Columns 51-60 and 61-70, the numbers input for items 4 (inner ring) and 5 (outer ring) are used to account for improved materials and multiply the raceway fatigue lives as determined by Lundberg-Palmgren methods. Typical life factor values for modern steels are in the neighborhood of 2.0 to 3.0. If the ASME Publication Life Adjustment Factors for Ball and Roller Bearings, is referenced by the user, the Material Factor D and the Material Process Factor E should be used multiplicatively as inputs for items 4 and 5. Additionally, if the user is accustomed to using a lubricant life multiplier he must also multiply the material factor by the lubricant life multiplier. The program considers EHD film thickness and RMS surface roughness but generates a life multiplier having a maximum value of 1 and a minimum of 0.479, i.e. Lube-Life Factor Programmed only serves to reduce predicted Fatigue Life.

Item 6: Columns 71-78, "Orientation angle of the first rolling element." (\(\phi_1\)) (degrees). Refer to Fig. (2.4). The quasi-dynamic rolling element bearing problem has an infinite number of solutions which fall within a narrow envelope having a periodic shape. The solution obtained is a function of the rolling element positions relative to the bearing system coordinate axes. \(\phi_1 = 0\), places a rolling element on the Y axis and is the choice customarily made. \(\phi_1\) can be desig-
nated as any value \(0 \leq \frac{360}{Z} \) where \(Z\) is the number of rolling elements. For each different value assigned to \(\phi_1\), a different, although similar, bearing solution will be obtained. To take advantage of bearing symmetry and the computer time savings which result, \(\phi_1\) must be specified as zero or left blank.

**Item 7:** Column 80, a signal, termed the crown drop flag, which specifies for a cylindrical roller bearing, whether the roller-race crown drops will be calculated, or read directly. If item 7 is blank or zero, the crown drops are calculated based on the roller-race crown radius, and effective flat length input information. If the crown drop flag is other than zero or blank, the non-uniform separation of the roller and raceway must be specified at the center of each slice into which the roller-raceway effective contact length is divided. The slice widths are identical. The number of slices is input as item 7 card type B4. The non-uniform roller-raceway separation is input on card types B5 and B6.

### 3.3.2 Card Type B2 - Bearing Geometry and Outer Ring Speed

Item 1, 2 and 5 need no explanation, however, items 3 and 4 require substantial explanation as they apply to the various types of ball bearings.

#### 3.3.2.1 Ball Bearing Geometry

Through the proper specification of the diametral clearance and contact angle, the Program can properly handle deep groove, split inner, and angular contact ball bearings.

The deep groove ball bearing requires the specification of zero contact angle and either the operating diametral clearance \(P_d\) or the off-the-shelf diametral clearance, if the dimensional change analysis is utilized.

The angular contact bearing is fully described through specification of the contact angle which obtains under a gauge, axial load. However, this method of input does not accurately define the system if there is more than one angular contact supporting the shaft and at least one of those bearings has its grooves offset in the direction opposite to the other bearings and if the shaft is capable of axial and/or radial play. In other words, if what are known as angular contact ball bearings are mounted such that some diametral shaft play is permitted, an auxilliary angle as well as the diametral play must be specified at input. The angle input is not the
manufacturer's designated contact angle, \( \alpha \), but an auxiliary angle, \( \alpha_o \), the calculation for which shall be demonstrated.

Refer to Figure 3.1. The manufacturer's contact angle is calculated as follows:

\[
\alpha = \cos^{-1} \left[ \frac{2A - P_d}{2A} \right] \quad (3.1)
\]
\[
A = r_o + r_i - D \quad (3.2)
\]

where: \( r_o \) and \( r_i \) are the outer and inner raceway groove radii respectively.

\( D \) is the ball diameter.

Under a gauge axial load \( \alpha \) is obtained at both inner and outer raceways for each ball. Under this condition, the outer and inner raceways are axially offset an amount \( s_{\alpha} \).

\[
s_{\alpha} = A \sin \alpha \quad (3.3)
\]

When angular contact ball bearings are mounted with some diametral play, the grooves are offset an amount \( s_{\alpha_0} \) such that \( s_{\alpha_0} < S_{\alpha} \). The diametral play which obtains at this condition is \( S_d \). This diametral play is usually known by the engineer or designer and is usually required to allow some forgiveness when thermal gradients are encountered. Assuming that the user has the values for \( \alpha \), \( r_o \), \( r_i \), \( D \) and \( S_{\alpha} \) then:

\[
\alpha_o = \tan^{-1} \left( \frac{S_{\alpha_0}}{A - \frac{P_d}{2}} \right) \quad (3.4)
\]

where: \( P_d \) and \( A \) may be calculated from Eqs. (3.1) and (3.2).

The manufacturer might be able to provide the value of \( S_{\alpha_0} \) at the \( S_d \) value of interest. If not, the following equation may be solved for \( \alpha_o \).
\[
\frac{S_d}{2} \cos^2 \alpha_o - A \cos \alpha_o + A - \frac{P_d}{2} = 0
\]  
(3.5)

Note: \( \alpha_o \) should be less than \( \alpha \) and \( \alpha_o \) should be less than \( \alpha \).

Equation (3.6) is derived by developing two expressions for the radial separation \( (A_r) \) of the outer and inner raceway curvature centers.

\[
A_r = A - \frac{P_d}{2}
\]

\[
A_r = (A - \frac{S_d \cos \alpha_o}{2}) \cos \alpha_o
\]  
(3.6)

\[
A - \frac{P_d}{2} = A \cos \alpha_o - \frac{S_d \cos^2 \alpha_o}{2}
\]

In order that the Program properly handle split inner ring ball bearings an auxiliary angle and diametral play must be input. Referring to Figure 3.2, the auxiliary angle \( \alpha_o \) and diametral play \( S_d \) must be determined and input. Typically, the values of \( D, r_o, \alpha_s \) and \( S_d' \) are known. \( P_d \) and \( S_d \) may be calculated as follows:

\[
P_d = S_d' + (2r_i - D) (1 - \cos \alpha_s)
\]  
(3.7)

\[
S_d = \left[ \frac{P_d}{2} - 2A (1 - \cos \alpha_o) \right] / \cos^2 \alpha_o
\]  
(3.8)

The unloaded half of the inner ring must be removed from consideration and the ball moved such that its center lies on the line connecting the origins of \( r_i \) and \( r_o \) and positioned such that the auxiliary clearance \( S_d/4 \) exists at both the inner and outer raceways. The auxiliary angle is

\[
\alpha_o = \tan^{-1} \left[ \frac{(r_i - D/2) \sin \alpha_s}{r_o - D/2 - S_d'/2 + (r_i - D/2) \cos \alpha_s} \right]
\]  
(3.9)
FIGURE 3-2
Split Inner Ring Ball Bearing Geometry
The angle associated with each ball bearing must be specified with the correct sign. A positive contact angle allows the bearing to accept a positively directed axial load transmitted by the shaft.

3.3.3 Card Type B3 - Rolling Element Geometry

These data are self explanatory. Although space has been set aside for the specification of roller end radius and roller included angle, this has been done for future considerations and are not used by the program. The items may be left blank.

3.3.4 Card Type B4 - Raceway and Roller Raceway Contact Geometry

3.3.4.1 Ball Bearing

Items one and two refer to the outer and inner raceway curvatures respectively where curvature is defined as the cross groove radius divided by the ball diameter. Typical values range from 0.515 to 0.57.

3.3.4.2 Roller Bearing Contact Geometry Data

All items are used to define the roller-race contact geometry, see Fig. 3.3 "Flat length" and "Crown Radius" are used to calculate roller-race separation along the roller profile if this information is not specifically input. See Item 7 of the Bearing Data title card and Bearing Data Cards B5 and B6.

Items 1 and 4 "Effective Contact Length" refer to the longest possible length which can obtain at a roller-race contact. Typically, this is the roller total length less the corner radii. If, however, the raceway undercuts are exceptionally large so that the tract width is smaller than the effective roller length then the tract width should be input.

Item 7 refers to the number of slices into which the roller raceway contact may be divided. A maximum value of twenty is permitted a default value of two is used if Item 7 is blank or zero. Each slice is the same width.

3.3.5 Roller-Raceway Non-Uniform Profile Definition

3.3.5.1 Card Type B5 - Inner Roller Raceway Contact

These cards are used to input the inner and outer race roller-race separation along the roller profile. With the high points of the roller and race in contact, i.e. with all clearance between roller and raceway removed. These cards must be omitted if item 7 of the Bearing Data Title card is zero or blank.
FIGURE 3.3

Roller-Race Lamination Showing Relative Approach ($\delta_n$) and Crown Drop ($\delta_c$)
3.3.6 Card Type B6 - Outer Roller Raceway Contact

Same as Card Type B5.

3.3.7 Card Type B7 - Raceway-Rolling Element Surface Data

Items 1 through 6 define the statistical surface micro-geometry parameters of the rollers and raceways. Items 1 through 3 require the input of center line average CLA surface roughness. Within the program CLA values are converted to RMS by multiplying by 0.9.

Items 4 through 6 are RMS values of the slopes measured in degrees of the surface asperities as measured in a traverse across the groove for rings, longitudinally for rollers and in any arbitrary direction for balls. Typical values for raceway and rolling element surfaces are 1 to 2 degrees. This card is omitted if item 10 of title card 2 is zero or blank.

3.3.8 Card Type B8 - Cage Data

This card is omitted if item 10 of the title card 2 is zero or blank. These data are self explanatory. Note that the cage weight is an input item. The weight, however, is not used in any calculation. It is included only for future consideration of cage stability predictions.

3.3.9 Card Type B9 - Shaft and Housing Fit Dimensions

These cards are to be included only if the change in bearing diametral clearance with operating conditions is to be calculated, i.e. if item 4 ITFIT on the Bearing Title Card 2 is non-zero. On Card Type B9, tight interference fits bear a positive sign and loose fits, a negative sign.

Items 3 and 6 on Card No. 9 are termed the shaft and housing effective widths, respectively. The value specified for these effective widths may be as great as twice the ring width.

Use of an effective width is an attempt to account for the greater radial rigidity of a shaft than the ring that is pressed on to it, owing to the fact that the shaft deflects over a distance that extends beyond the ring width. In the program, the calculated internal pressure on the ring due to
its interference fit with the shaft, is distributed over the shaft effective width and this (lower) pressure is used in computing the shaft deflection. Using double the actual width as the effective width is customary.

### 3.3.10 Card Type B10 - Shaft Housing Fit Dimensions

These items are self explanatory, and are used to describe equivalent ring sections, see Fig. I 2.1.

### 3.3.11 Card Type B11

This card defines the elastic modulus for the shaft, inner ring, rolling element, outer ring, and housing, respectively. This card is to be included only if the change in bearing diametral clearance with operating conditions is to be calculated, i.e., if item 4 ITFIT on the Bearing Title Card is non-zero.

### 3.3.12 Card Type B12

This card defines the Poisson's ratio for the shaft, inner ring, rolling element, outer ring, and housing, respectively. This card is to be included only if the change in bearing diametral clearance with operating conditions is to be calculated, i.e., if item 4 ITFIT on the Bearing Title Card is non-zero.

### 3.3.13 Card Type B13

This card defines the density for the shaft, inner ring, rolling element, outer ring, and housing, respectively. This card is to be included only if the change in bearing diametral clearance with operating conditions is to be calculated, i.e., if item 4 ITFIT on the Bearing Title Card is non-zero.

### 3.3.14 Card Type B14

This card defines the coefficient of thermal expansion for the shaft, inner ring, rolling element, outer ring, and housing, respectively. This card is to be included only if the change in bearing diametral clearance with operating conditions is to be calculated, i.e., if item 4 ITFIT on the Bearing Title Card is non-zero.

### 3.3.15 Card Type B15 - Lubrication and Friction Data

This card is omitted if item 10 of title card 2 is zero or blank.
Items 1 and 2

These items are the amounts by which the combined thickness of the lubricant film on the rolling track and rolling element is increased during the time interval between the passage of successive rolling elements, from whatever replenishment mechanisms are operative. Items 1 applies to the outer and Item 2 to the inner race-rolling contacts, respectively. If Item 1 is zero or blank, the mode of friction is assumed to be dry.

At the present time, the magnitude of the inner and outer raceway replenishment layers has not been correlated to lubricant flow rate, lubricant application methods and bearing size and speed factors. At this point then, the user is forced to establish proper values for the replenishment layer thickness. As a rough guide, the following suggestions are made.

1. To avoid starvation, the replenishment layer thicknesses should be one or two times the EHD film thickness which develops in the rolling element raceway contacts.

2. Because of centrifugal force effects, intuition suggests that the outer raceway replenishment layer should be several times thicker than that prescribed at the inner raceway.

Item 3, XCAV, describes the percentage of the bearing cavity, estimated by the user to be occupied by the lubricant. It is recommended to be approximately one percent. As with the replenishment layer thicknesses, the amount of free lubricant should be able to be correlated with lubricant flow rate, lubricant application methods and bearing size and speed factors. At this time, such correlations do not exist. XCAV values of approximately one percent are recommended at this point.

Item 4 is the coefficient of coulomb friction applicable for the contact of asperities. If Items 1 and 2 are zero, then Item 4 serves as the coulomb friction coefficient which prevails in all contacts.

3.3.16 Card Type B16

This card is omitted if Item 10 title card 2 is zero or blank or if Item 1 card 15 is zero or blank which implies dry friction.

This card specifies the lubricant type. If Item 1, NCODE is 1, 2, 3, or 4, the Program uses preprogrammed lubricant properties as presented in Table 1, and no further information is required.
<table>
<thead>
<tr>
<th>LUBRICANT NO.</th>
<th>LUBRICANT TYPE</th>
<th>KINEMATIC VISCOSITY 37.78°C (100°F)</th>
<th>(cs) 98.89°C (210°F)</th>
<th>DENSITY 015.56°C (60°F) gm/cm³</th>
<th>THERMAL CONDUCTIVITY W/m°C</th>
<th>THERMAL COEFF. OF EXPAN. 1/°C* 10^-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mineral Oil</td>
<td>64.0</td>
<td>8.0</td>
<td>0.88</td>
<td>0.116</td>
<td>6.336</td>
</tr>
<tr>
<td>2</td>
<td>MIL-L-7808G</td>
<td>17.8</td>
<td>3.2</td>
<td>0.95</td>
<td>0.152</td>
<td>7.092</td>
</tr>
<tr>
<td>3</td>
<td>C-Ether</td>
<td>25.4</td>
<td>4.13</td>
<td>1.20</td>
<td>0.119</td>
<td>7.470</td>
</tr>
<tr>
<td>4</td>
<td>MIL-L-23699</td>
<td>28.0</td>
<td>5.1</td>
<td>1.01</td>
<td>0.152</td>
<td>7.452</td>
</tr>
</tbody>
</table>
3.4 Data Set III - Thermal Model Data

Appendix I 1 has been included to aid the user in data preparation and calculation of heat transfer coefficients required at input.

3.4.1 Card Type 1

Card type 1 is a control card. If no temperature map is to be calculated, this card is to be included as a blank card followed by a Type 2 card for each bearing on the shaft. Card Type 1 contains control input for both steady state and transient thermal analyses. It is not intended, however, that both analyses be executed with the same run.

Item 1: The highest node number (M). The temperature nodes must be numbered consecutively from one (1) to the highest node number. The highest node number must not exceed one hundred (100).

Item 2: Node Number of the Highest Unknown Temperature Node (N). This number should equal the total number of unknown node temperatures. It is required that all nodes with unknown temperatures be assigned the lowest node numbers. The nodes which have known temperatures are assigned the highest numbers.

Item 3: Common Initial Temperature (TEMP)°C: The temperature solution iteration scheme requires a starting point, i.e., guesses of the equilibrium temperatures. Card Type 3 allows the user to input guesses of individual node temperatures. When a node is not given a specific initial temperature, the temperature specified as Item 3 of Card Type 1 is assigned.

Item 4: Punch Flag (IPUNCH): If the Punch Flag is not zero (0) or blank, the system equilibrium temperature along with the respective node numbers will be punched according to the format of Card T3. This option is useful if, for instance, the user makes a steady state run with lubrication, and then wishes to use the resultant temperature as the initiation point for a transient dry friction run in order to assess the consequence of lubricant flow termination.

<table>
<thead>
<tr>
<th>NCODE</th>
<th>Lubricant</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A specific mineral oil</td>
</tr>
<tr>
<td>2</td>
<td>A MIL-L-7808G</td>
</tr>
<tr>
<td>3</td>
<td>C-Ether</td>
</tr>
<tr>
<td>4</td>
<td>A MIL-L-23699</td>
</tr>
</tbody>
</table>

NCODE may also be specified as negative (-1 to -4), in which case, the traction characteristics of the respective lubricant NCODE noted above are used but the actual properties specified by Items 2 through 7 override the hard coded data. This option is most useful in specifying various mineral oils i.e. NCODE = -1.
Item 5: "Output Flag" (IUB). If the "Output Flag" is not zero, the bearing program output and a temperature map will be printed after each call to the shaft bearing solution scheme. This printout will allow the user to observe the flow of the solution and to note the interactive effects of system temperatures and bearing heat generation rates. Since the temperature solution is not mathematically coupled to the bearing solution, the possibility exists that the solution may diverge or oscillate. In such a case, study of the intermediate output produced by the "Output Flag" option may provide the user with better initial temperature guesses that will effect a steady state solution.

Item 6: "Maximum Number of Calls to the Shaft Bearing Program" (IT1). IT1 is the limit on the number of Thermal-Shaft-Bearing iterations, i.e., the external temperature equilibrium calculation. The user must input a non-zero integer such as 5 or 10 in order for the Program to iterate to an equilibrium condition. If IT1 is left blank or set to zero (0) or 1, shaft bearing performance will be based on the initially guessed temperatures of the system. The temperatures printed out will be based on the bearing generated heats. It is unlikely that an acceptable equilibrium condition will be achieved. However, the temperatures which result may provide better initial guesses, for a subsequent run, than those specified by the user.

IT1 also serves as a limit on the transient temperature solution scheme, by limiting the number of times the shaft-bearing solution scheme is called. Each call to the shaft-bearing scheme will input a new set of bearing heats to the transient temperature scheme until a steady state condition is approached or until the transient solution time up limit is reached.

Item 7: "Absolute Accuracy of Temperatures for the External Thermal Solution" (EPl). In the steady state thermal solution scheme, each calculation of system temperatures occurs after a call to the shaft-bearing scheme which produces bearing generated heats. After the system temperatures have been calculated for each iteration, using the internal temperature solution scheme, each node temperature is checked against the nodal temperature at the previous iteration.

If \( |t(N)_i - t(N-1)_i| < EPl \) for all nodes \( i \) then equilibrium has been achieved and the iteration process stops.
Item 8: "Iteration Limit for the Internal Thermal Solution" (IT2). After each call to the shaft-bearing program, the internal temperature calculation scheme is used to determine the steady state equilibrium temperatures based on the calculated set of bearing heat generation rates. If the program is used to calculate the temperature distribution of a non-bearing system, it is the internal temperature scheme which is employed. If IT2 is left blank or set to zero, the number of internal iterations is limited to twenty (20).

Item 9: "Accuracy for Internal Thermal Solution" (EP2). The use of EP2 is explained in Section 2.1.1. If EP2 is left blank or set to zero (0), a default value of 0.001 is used.

Item 10: "Starting Time" (START) is a time $T_s$ at which the transient solution begins; usually set to zero (0).

Item 11: "Stopping Time" (STOP) is the time in seconds at which the transient solution terminates, $T_f$. The transient solution will generate a history of the system performance which will encompass a total elapsed time of

$$(T_f - T_s) \text{ seconds}$$

Item 12: "Calculation Time Step" (STEPIN). The transient internal solution scheme solves the system of equations

$$t_{k+1} = t_k + \frac{q_k}{\rho c_p V} \Delta T$$

$$\Delta T = \text{STEPIN}$$

The user may specify STEPIN. If left blank or set to zero (0), the Program calculates an appropriate value for STEPIN using the procedure described in Section 2.1.2.

Item 13: "Time Interval Between Printed Temperature Maps" (TTIME) seconds. The user must specify the length of time which will elapse between each printing of the temperature map. The interval will always be at least as large as the "calculation timestep" (STEPIN).

Item 14: "Time Interval Between Calls of the Shaft Bearing Portion of the Program" (BTIME). BTIME will always have a value larger than or equal to (STEPIN) even if the user inadvertently inputs a shorter interval. Computational time savings result if BTIME is greater than STEPIN, however, accuracy might be lost.
3.4.2 Card Type 2

Card Type T2 is required, one card for each bearing if no thermal analysis is being performed. The temperature data is used within the shaft-bearing analysis portion of the program to fix temperature dependent properties of the lubricant in which case the inner race, outer race and lubricant bulk cavity temperatures are used. The assembly component temperatures at each bearing location are used in the analysis which calculates the change in bearing diametral clearance from "off the shelf" to operating conditions.

Item 9: "Flange" temperature is not currently used in the analysis. It simply provides for future consideration of tapered roller bearings.

3.4.3 Card Type 3

In the steady state analysis this card is used to input initial guesses of individual nodal temperatures for unknown nodes as well as the constant temperatures for known nodes, such as ambient air and/or an oil sump.

In the transient analysis, Card Type T3 is used to input the nodal temperatures of all nodes at (START) = T_s, i.e., at the initiation of the transient solution.

3.4.4 Card Type 4

With this card, node numbers are assigned to the components of each bearing, one card per bearing. With this information, the proper system temperatures are carried into each respective bearing analysis. The inner race and inner ring node numbers may or may not be the same at the user's discretion. Similarly, the outer race and outer ring node numbers may or may not be the same.

3.4.5 Card Type 5

The shaft bearing system analysis accounts for frictional heat generated at four locations in the bearing, i.e., at the inner race, the outer race, between the cage rail and ring land, and in the bulk lubricant due to drag. The heat generated at the hydrodynamic cage-rolling element contact is added to the bulk lubricant. Heat generated at the flange is not presently considered. This card allows the heat generated to be distributed equally to two nodes. For instance, the heat generated at the inner race-rolling element contact should be
distributed half to the rolling element and half to the inner race. The heat developed between the cage and inner ring land may be distributed half to the inner ring and half to the cage if a cage node has been defined, otherwise, half to the bulk lubricant.

3.4.6 Card Type 6

This card specifies the node numbers and the heat generation rate for those nodes where heat is generated at a constant rate such as at rubbing seals or gear contacts.

3.4.7 Card Type 7

This card type is used to input the numerical values of the various heat transfer coefficients which appear in the equations for heat transfer by conductivity, free convection, forced convection, radiation and fluid flow. Up to ten coefficients of each type may be used. Separate values of each type of coefficient are assigned an index number via card T7 and in describing heat flow paths (Card Type T8 below) it is necessary only to list the index number by which heat transfers between node pairs.

Incides 1-10 are reserved for the conduction coefficient $\lambda$, 11-20 for the free convection parameters, 21-30 for forced convection, 31-40 for emissivity and 41-50 for fluid flow (product of specific heat, density and volume flow rate).

As an example, for heat transfer by conduction with coefficient $\lambda$ of 53.7 watts/MOC one could prepare a card type T7 with the digit 1 punched in column 10 and the value 53.7 punched in the field corresponding to card columns 11-20. If a conduction coefficient of 46.7 were applicable for certain other nodes in the system, one could punch an additional card assigning index No. 2 to the value $\lambda = 46.7$ by punching a "2" in card column 10 and 46.7 anywhere within card columns 11-20.

Rather than inputting constant forced convection coefficients, optionally, these coefficients can be calculated by the program in one of three ways. If the calculation option is exercised a pair of cards is used in place of a single card containing a fixed value of $\alpha$. The contents of the pair of cards depends upon which of the three optional methods are used.

Option 1) is independent of temperature but is calculated as a function of the Nusselt number which in turn is a function of the Reynolds number $Re$, the Prandtl number $Pr$ as follows, (cf. [17])
\[ \alpha = \frac{\ln \lambda_{\text{oil}}}{L N_u} \quad (3.11) \]
\[ N_u = a R_e^b p^c \quad (3.12) \]

where \( \lambda_{\text{oil}} \) is the lubricant conductivity, \( L \) is a characteristic length (with a unit of meters) and \( K, a, b \) and \( c \) are constants.

Option 2) \( \alpha \) is a function only of fluid dynamic viscosity and viscosity is temperature dependent.

\[ \alpha = c \eta^d \quad (3.13) \]

where \( c \) and \( d \) are constants.

Option 3) \( \alpha \) is a function of the Nusselt, Reynolds and Prandtl numbers, and viscosity is temperature dependent.

3.4.8 Card Type 8

This card defines the heat flow paths between pairs of nodes. Every node must be connected to at least one other node, i.e., two or more independent node systems may not be solved with a single Program execution.

The calculation of heat transfer areas is based on lengths, \( L_1 \) and \( L_2 \) input using Card Type T8. Additionally, the type of surface for which the area is being calculated is indicated by the sign assigned to the heat transfer coefficient index. If the surface is cylindrical or circular, the index should be positive, if the surface is rectangular the index should be input as a negative integer.

In the case of radiation between concentric axially symmetric bodies, \( L_3 \) is the radius of the larger body. For radiation between two parallel flat surfaces or for conduction between nodes, \( L_3 \) is the distance between them.

Fluid flow heat transfer accounts for the energy which the fluid transports across a node boundary. Along a fluid node at which convection is taking place, the temperature varies. The nodal temperature which is output is the average of the fluid temperature at the output and input boundaries. If the emerging temperature of the fluid is of interest, it is necessary to have a fluid node at the fluid outlet. At this auxiliary node, only fluid flow heat transfer occurs and the fluid temperature would be constant throughout the node. Thus, the true fluid outlet temperature will be obtained.
Conduction of heat through a bearing is controlled by index 51. The actual heat transfer coefficient which contains a conductivity, area and a path length term is calculated in the bearing portion of the program. The term is based upon an average outer race and inner race rolling element contact.

3.4.9 Card Type 9

This card inputs data required to calculate the heat capacity of each node in the system. This card type is required only for a transient analysis.

3.5 Data Set IV - Shaft Input Data

The Shaft-bearing analysis requires all loading to be applied to the shaft. The loads applied to each bearing are a product of the shaft-bearing solution. There is no need for the user to solve the statically determinant or indeterminant system for bearing loads. Even if a single bearing is being analyzed, with the applied load acting through the center of the bearing, data for a dummy shaft must be supplied.

In the analysis, the housing is assumed to be rigid. Provision has been allowed to input data for housing radial and angular spring characteristics. However, this has been done for future consideration of an elastic housing and is therefore currently unavailable.

The shaft input data consists of three card types:

1. Shaft Geometry and Elastic Modulas Data
2. Bearing Position and Mounting Error Data
3. Shaft Load Data

3.5.1 Card Type 1

This card type is used to describe shaft geometry at up to twenty locations along the shaft. The user must place his shaft in a cartesian coordinate system with the end of the shaft at the origin and with the shaft lying along the X-axis.

The shaft may have stepwise and linear diameter variations. The stepwise variations require a single card which specifies different diameters immediately to the left and right of the relevant X shaft coordinate. The shaft analysis assumes a linear diameter variation if on two successive cards, i.e. two successive X coordinates, the diameters to the right of the location differ from the diameters to the left of the location
of the following card. Complex shaft geometries may be approximated with a set of linear diameter variations spaced at close intervals.

If an Elastic Modulus is not specified at the designated input location, the modulus of steel is assumed, 204083 N/mm².

3.5.2 **Card Type 2**

This card type locates the bearing inner ring on the shaft in the X-Y and X-Z planes. For a ball bearing, the X coordinate specified locates the inner ring center of curvature. For cylindrical roller bearings, the X coordinate locates the center of the inner race roller path.

In addition to specifying bearing location, the Type 2 card is also used to specify housing radial and angular mounting errors. As mentioned previously, space has been reserved for inputting housing radial and angular spring characteristics, however, these characteristics are not used in the system analysis.

Two sets of Type 2 cards may be required. The first set is always required and defines housing alignment errors in the shaft X-Y plane. The second set defines the housing alignment errors in the shaft X-Z plane and is required only if non-zero errors exist for the particular bearing in question.

The first set of Type 2 cards must contain a card for each bearing.

3.5.3 **Card Type 3**

Type 3 cards are used to specify shaft loading at a given X coordinate. Loading may be applied in the x-y and x-z planes, thus requiring two distinct sets of Type 3 cards. Applied loads may have the form of concentrated radial forces, concentrated moments, linearly distributed radial forces and concentrated axial loads which may be eccentrically applied. If an axial load is eccentrically applied, the moment which results must not be separately calculated and input as a concentrated moment.

Variations in distributed radial loads are handled at input just as shaft linear diameter variations are handled.

Note that each set of Type 3 cards must be followed by a blank card.
Also, note that in order for symmetry conditions (see section 2.4.2) to be considered the second type 3 card must be void of any loading data.
4. COMPUTER PROGRAM OUTPUT

4.1 Introduction

The Program Output is intended to provide the engineer or
designer with a complete picture of the shaft-bearing system
performance.

In addition to the calculated output data, the input data
is listed, thus producing a complete record of the computer
run.

A sample set of program output is included for reference
as Appendix II 3 and represents an NPASS=2 solution for a two
bearing system comprised of a 209 size cylindrical roller
bearing, a single 110 mm bore angular contact ball bearing
operating at 10,000 rpm under a thrust load of 2,000 lbs.
(8,896 Newtons) with MIL-L-23699 lubricant and a 6220 size
split inner ring angular contact ball bearing operating at
15000 RPM under shaft loads of 8896 Newtons axial, 2248 Newtons
radial and a moment load of 4000 Newton millimeters. The
bearings are lubricated with an MIL-L-7808G lubricant.

The first seven pages of output essentially consists of
a summary of the input data categorized into bearing, cage,
steel, lubricant, fit temperature and shaft geometry and
loading data.

For four specific lubricants, see Table 1, the relevant
lubricant data has been coded into the Program. In this case,
the lubricant input information consists only of a single number
which designates the particular lubricant but the relevant
information for the lubricant is printed in the input data list.

Except as just noted, the actual results of calculations
are printed under the headings "Bearing Output" and "Rolling
Element Output."

Key output items are discussed briefly below.
4.2 Bearing Output

4.2.1.1 Linear and Angular Deflections

These deflections refer to the bearing inner ring relative to the outer ring and are defined in the inertial coordinate system of Figure 2.4. The bearing deflections are not necessarily equal to the shaft displacements since the bearing outer ring radial or angular mounting errors may be specified as non-zero input.

4.2.1.2 Reaction Forces and Moments

These values reflect bearing reactions to shaft applied loading and outer ring mounting errors.

When the bearing inner ring has achieved an equilibrium position, the summation of all bearing reaction loads should numerically equal the shaft applied loading. When the level of solution indicated by "NPASS" = 2 is employed, as discussed in Section 5, differences between shaft applied and bearing reaction loads will exist but will typically be less than 10%.

This difference is a consequence of friction forces contributing to the reaction loads whereas the inner ring equilibrium position has been determined considering elastic contact forces only.

4.2.2 Fatigue Life Data

The $L_{10}$ fatigue life of the outer and inner raceways as well as the bearings are presented. The bearing life represents the statistical combination of the two raceway lives. These lives reflect the combined effects of the lubricant film thickness and material life factors. The lubricant film thickness life factor is described in detail in Section 3.

4.2.2.1 $h/\sigma$

The ratio $h/\sigma$, also referred to as $h/\sigma$, is printed for the most heavily loaded rolling element. The variable $h$, represents the EHD plateau film thickness with thermal and starvation effects considered. The variable $\sigma$, represents the composite root mean square surface roughness of the rolling element and the relevant raceway.
4.2.2.2 Life Multipliers

4.2.2.2.1 Lubrication - This life multiplier is a function of \( \frac{h}{\nu} \) at each concentrated contact and is in the form of a derating factor. Its value ranges from 0.479 for \( \frac{h}{\nu} = 0 \) to 1.0 at \( \frac{h}{\nu} \geq 4 \). Since the lubricant life multiplier is decremental the normal multiple of 3 used for thick film lubrication must be multiplied by the material life factor normally used and this product should be specified at input. This subject is covered in more detail in Section 3.3.1.

4.2.2.2.2 Material - This output simply reflects the input value. Again, it is covered in Section 3.3.1.

4.2.3 Temperatures Relevant to Bearing Performance

These temperatures fully describe the temperature conditions which affect the performance of a given bearing. If one of the temperature mapping options is used, the temperatures printed reflect the results of the particular option. If, neither temperature option was used, the list is simply a repeat of the input data. Note that there are separate temperatures for outer and inner raceway and ring temperatures. The raceway temperature is used to determine lubricant properties. The ring temperatures are used in the bearing dimension change analysis. The raceway and ring temperatures may be the same value.

4.2.4 Frictional Heat Generation Rate and Bearing Friction Torque

4.2.4.1 Frictional Heat Generation Rate

The various sources of frictional heat generated within the bearings are listed. The values printed for "OUTER RACE, OUTER RINGS, INNER RACE, INNER RINGS, R.E. DRAG AND R.E.-CAGE" represent the sum of the generated heats for all rolling elements. Additionally, the heats printed for the outer and inner raceways plus the rolling element-cage, reflect the friction developed outside the concentrated contacts, i.e., the HD friction as well as the EHD friction developed within the concentrated contacts. The raceway data also include any heat generated as a consequence of asperity contacts. "R.E. DRAG" should be interpreted as the heat resulting from lubricant churning as the rolling elements plow through the air-oil mixture. Items 2 and 4 relevant to rolling element-flange contacts are present for future program expansion.
4.2.4.2 **Torque**

The torque value is calculated as a function of the total generated neat and the sum of the inner and outer ring rotational speeds. The intent is to present a realistic value of the torque required to drive the bearing. Under conditions of inner ring rotation, the torque value reflects the torque required to drive the inner ring. The inner ring torque includes that fraction of torque required to impart an angular velocity to the lubricant in the bearing. A considerable portion of the lubricant will come to rest within the housing and not at the outer ring. Thus, the measured outer ring torque may not equal the torque at the inner ring.

4.2.5 **EHD Film and Heat Transfer Data**

4.2.5.1 **EHD Film Thickness**

These values refer to the calculated EHD plateau film thickness at both contacts of the most heavily loaded rolling element and include the effects of the thermal and starvation reduction factors.

4.2.5.2 **Starvation Reduction Factor**

These factors give for the inner and outer ring contacts, the reduction in EHD film thickness ascribable to lubricant film starvation according to the methods of Chiu, (11).

These factors pertain to the EHD film thickness for both the inner and outer race contacts of the most heavily loaded rolling elements, but are applied to the respective inner and outer race film thickness for each rolling element in the bearing.

4.2.5.3 **Thermal Reduction Factor**

These factors are calculated according to the methods of Cheng, (10) and pertain to the EHD film thickness for both the inner and outer race contacts of the most heavily loaded rolling elements, but are applied to the respective inner and outer race film thickness for each rolling element in the bearing.

4.2.5.4 **Meniscus Distance**

These factors are calculated according to the methods of Chiu, (11) and pertain to the EHD film thickness for both the inner and outer race contacts of the most heavily loaded
rolling elements, but are applied to the respective inner and outer race film thickness for each rolling element in the bearing.

4.2.5.5. Raceway-Rolling Element Conductivity

These data reflect the amount of heat transfer between rolling element and raceway for each degree centigrade difference between the two components. These data reflect the average of all outer and inner contacts, respectively.

4.2.6 Fit and Dimensional Change Data

4.2.6.1 Fit Pressures

These data refer to the pressures built up as a consequence of interference fits between shaft and inner ring and housing and outer ring. Pressures are presented both for the standard cold-static condition (16°C) and at operating conditions.

4.2.6.2 Speed Giving Zero Fit Pressure (Between the Shaft and Inner Ring)

This is a calculated value based upon operating conditions and provides a measure of the adequacy of the initial shaft fit.

4.2.6.3 Clearances

"Original" refers to cold unmounted clearance which is specified at input if the diametral clearance change analysis is executed. "Change" refers to the change in diametral clearance at operating conditions relative to the cold unmounted condition. A minus sign indicates a decrease in clearance. "Operating" refers to the clearance at operating conditions. For all types of ball bearings, the decrease in clearance can be combined with the initial diametral clearance, and the free operating contact angle at operating conditions may be calculated. Note that the change in clearance should be compared against the diametral play of the split inner ring ball bearing in order to determine if the possibility for three point contact exists. The Program does not account for three point contacts even though the change in clearance might suggest that three point contact is obtained.
4.2.7 Lubricant Temperatures and Physical Properties

The lubricant properties, particularly the dynamic viscosity and to a lesser degree, the pressure viscosity coefficient, are heavily temperature dependent. These factors enter the EHD film thickness calculation and the HD and EHD friction models. The lubricant is assumed to be at the same temperature as the relevant raceway. As noted elsewhere, these temperatures may be either input directly or calculated by the Program.

The physical properties printed are self-explanatory. The units are enumerated.

4.2.8 Cage Data

4.2.8.1 Cage-Land Interface

The cage data indicates the performance parameters at the interface between the cage rail and the ring land on which the cage is guided. The torque, heat rate and separating force require no explanation. The eccentricity ratio defines the degree to which the cage approaches the ring on which it is guided at the point of nearest approach. The radial displacement of the cage relative to the bearing axis is divided by one half the cage-land diametral clearance. An eccentricity ratio of one indicates cage-land contact. A ratio of zero indicates that the cage rotation is concentric with the bearing axis.

Only the cage-land and rolling element pocket forces are considered in determining the cage eccentricity. The cage weight and centrifugal force which result from the eccentricity, although available, are not considered in the analysis. The omission of these considerations helps reduce convergence problems.

4.2.8.2 Cage Speed Data

Cage speed data presents the comparison between the cage speed calculated based upon the quasidynamic equilibrium considerations and the speeds calculated with raceway control theory for ball bearings and the epicyclic speeds of the roller bearing components.

4.3 Rolling Element Output

4.3.1 Rolling Element Kinematics
4.3.1.1 Rolling Element Speeds

All of the rolling element speeds tend to vary from position to position when the bearing is subjected to combined loading.

The total rolling element speed is with reference to the cage and represents the vector sum of the three orthogonal components.

4.3.1.2 Speed Vector Angles

The rolling element speed vector angles, \(\arctan \left( \frac{\omega_y}{\omega_x} \right)\) and \(\arctan \left( \frac{\omega_z}{\omega_x} \right)\) are presented in order to show a clearer picture of the predicted ball kinematics. The ball speed vector tends to become parallel with the bearing X axis with increasing shaft speed and decreasing contact friction.

4.3.2 Rolling Element Raceway Loading

4.3.2.1 Normal Forces

The normal forces acting on each rolling element are printed. The rolling element race normal forces are self-explanatory. The cage force is calculated only when the friction solution is employed and is always directed along the rolling element Z axis. If the rolling element orbital speed is positive, a positive cage force indicates the cage is pushing the rolling element, tending to accelerate it. Cage force is a function of rolling element position within the cage pocket. Its magnitude is derived using hydrodynamic lubrication assumptions, when the distance between the rolling element and cage web is large, and EHD assumptions when the separation is of the order of the EHD film thickness or when rolling element-cage web interference exists.

4.3.2.2 Hertz Stress

The stress printed represents the maximum normal stress at the center of each ball race contact or at the most heavily loaded slice of the roller raceway contact.

4.3.2.3 Load Ratio \(Q_{asp}/Q_{tot}\)

If the EHD film thickness is small compared to the RMS composite rolling element-race surface roughness, the rolling element-race normal load will be shared by the EHD film and asperity contacts. The load ratio reflects the portion of the total load carried by the asperities.
4.3.2.4 Contact Angles

A ball bearing, subject to axial loading, misalignment or mounted such that the inner ring is always displaced axially relative to the outer rings, (i.e., a duplex set of angular contact ball bearings) will have non-zero contact angles. At low ball orbital speeds, the inner and outer race angles will be substantially the same. At high speeds, ball centrifugal force will cause the outer race contact angle to be less than the inner race angle.

4.4 Thermal Data

As in the case for bearing output, all of the input data is printed. The calculated output data is presented in the form of a temperature map in which a node number and the respective node temperature appear. The appearance of the steady state and transient temperature maps are identical. The transient temperature map also includes the time (T) at which the temperature calculations were made.

4.5 Shaft Data

These data simply reflect the input information.

4.6 SHABERTH Error Messages

4.6.1 Introduction

For various reasons, SHABERTH execution may terminate before the desired results have been achieved. This section is intended to give insight to the user as to the nature of the problem which caused termination.

In some instances, error messages are printed and execution proceeds. These messages indicate that in one of the internal iterative loops, a particular solution failed to converge to the desired accuracy. These messages should be taken as caution signals to the user to check the results carefully. In particular, compare the calculated bearing reaction forces against the applied shaft loading. If these results check to within 10 percent with an NPASS = 2 solution and to within 1 percent with an NPASS = 3 solution, the solutions should be sufficiently accurate.

Additional means of evaluating solution accuracy are presented in section 4.6.10.
4.6.2 From ALLT - Message: "STEADY STATE SOLUTION WITH (EP1) DEGREES ACCURACY WAS NOT OBTAINED AFTER (IT1) ITERATIONS." Explanation: This message pertains to the thermal equilibrium solution in which bearing generated heat and system temperatures must be consistent.

4.6.3 From AXLBOJ - Message: "ERROR MESSAGE, KX = (IER) SINGULAR SET OF SHAFT EQUATIONS." Explanation: This message indicates an error in the input data which describes the shaft.

4.6.4 From DAMPCO - Message: "THE NUMBER OF EQUATIONS CALCULATED BY SUMMING THE NUMBER OF EQUATIONS IN EACH SUBSET IS (NTOT). THIS DOES NOT EQUAL THE TOTAL NUMBER OF EQUATIONS SPECIFIED (N) AN ERROR EXISTS AND EXECUTION TERMINATES." Explanation: If the nonlinear equations are comprised of M independent subsets, then N must equal the summation of NSIZE(K) = 1,M.

\[
NTOT = \sum_{K=1}^{M} NSIZE(K) \quad (4.1)
\]

In SHABERTH M is always 1 and NSIZE(1) is N. This message should thus never be written.

4.6.5 From EHDSKF - Message: "AN IMPROPER LUBRICANT TYPE CODE HAS BEEN PASSED TO EHD SKF. EXECUTION TERMINATES." Explanation: Coming into EHD SKF N must have an integer value 1, 2, 3 or 4, a test has shown that 1\neq 4.

4.6.6 From INTFIT - Message: "SINGULAR MATRIX ON TIGHT SHAFT FIT." Message: "SINGULAR MATRIX ON LOOSE SHAFT FIT." Explanation: These messages indicate bad data entering INTFIT.

4.6.7 From SHABE - Message: "AFTER (ITFIT) ITERATIONS, ERRMAX = (ERRMAX) WHILE THE REQUIRED FIT ACCURACY WAS (ERFIT)." Explanation: The bearing diametral clearance change analysis did not converge in ITFIT iterations. Either increase the number of iterations or set the number to -2 for a good approximate solution.

4.6.8 From SONRI - Message: "SINGULAR SET OF EQUATIONS, NPASS = (NPASS)." Explanation: This message pertains to shaft equilibrium solution and has never been known to occur.

4.6.3.1 From SONRI - Message: "THE RELATIVE ACCURACY EPS HAS NOT BEEN OBTAINED AFTER IT ITERATIONS IN ROUTINE SONRI." Explanation: This message indicates that shaft bearing inner ring equilibrium has not been achieved within the specified number of iterations. This may occur under light loading.
4.6.9 From STARFC - Message: "***IN STARFC/ROOTI*** ROOT OF F(X) DOES NOT EXIST BELOW HO." Explanation: The iterative solution for the meniscus distance for a rolling element raceway contact has not converged. This occurs only when the specified replenishment layers are extremely thin.

4.6.10 From SOLVXX

The majority of the error messages printed by SHABERTH will be printed from SOLVXX, indicating that SOLVXX has been unable to fully solve a particular set of nonlinear equations. Within SHABERTH failure has never occurred during the solution of a set of steady state thermal equations. Failure does occur, however, in the solution of the bearing quasidynamic set of rolling element and cage equations. The major portion of this section should be read with this in mind.

4.6.10.1 Message: "ONLY (NDER) EQUATIONS WERE FOUND TO VARY WITH X(J), ND(J) WERE EXPECTED." THE DIFFERENCES EQ (X+DX) - EQ(X) + DIFF(I)" IF DIFF(I) IS LESS THAN SF8*EQ(I), THEN EQ(I) IS NOT CONSIDERED TO VARY WITH X SF8 = (SF8)." "FOR THE FOLLOWING EQUATIONS THE DIFFERENCES ARE BIG ENOUGH" CC(N*J-N+I), I = 1, COUNT. Explanation: The matrix of partial derivatives calculated within SHABERTH must have at least N, nonzero diagonal elements. It is possible to specify a minimum number of nonzero elements greater than one, for each variable with the use of the array ND. If that minimum number is not obtained, the above set of error messages is printed. In SHABERTH, only the diagonal elements are required.

4.6.10.2 Message: "**** ERROR MESSAGE FROM THE EQUATION SOLVING ROUTINE AT ITERATION LOOP (LOOP) ****." One of the four following situations has occurred. Situations 1. and 2. have never been known to occur; 3. and 4., however, have, and are explained.


This message (No. 3) indicates that the solution accuracy is not as good as desired. To achieve the required accuracy, the problem might be rerun with the iteration limit increased. This is accomplished at solution level NPASS = 2, through changing the "20" in CALL BEAR statement in "SHABE" from 20 to a larger number. At solution level NPASS = 3, the "30" in the CALL BEARC statement in SONRI must be increased to a larger number.

Prior to making these Program changes, however, the magnitudes of the equation residues should be examined in the same manner as suggested above. The solution may be sufficiently good, as discussed in 4 below, so that further computations are unnecessary.
4. "THIS IS THE BEST WE CAN DO. IT MAY BE USABLE."
IER = 4.

This message indicates that computation has stopped before the desired results have been obtained. Fifty attempts have been made to increment the variable values without finding a set of increments which would reduce the equation residues.

After failing at these numerous attempts to improve the solution, it is concluded that the best solution has been achieved and that further changes to the variables will serve only to increase the equation residues. (In this discussion equation values and residue values are synonymous.) It is believed that this situation arises when large changes in variable values introduce small changes in equation values, i.e. when the force versus variable function has a very shallow slope.

When the differences in equation values are of the same order of magnitude as the numerical accuracy of the particular computer being used, these kinds of convergence problems can be expected.

It is possible that when this "BEST WE CAN DO" message is printed, that even though the solution is not as accurate as desired, it may be sufficiently accurate to be usable. The accuracy can be assessed by comparing the magnitude of the equation residue to the magnitude of the individual terms which comprise the equation. Since most convergence problems arise in the quasi-dynamic equilibrium solution of the rolling element and cage equations, the method of assessing the accuracy of this set of equations shall be addressed.

As noted earlier, the set of equations used to define the quasidynamic problem is comprised of \((6Z + Mcage)\) equations where \(Z\) is the number of rolling elements and \(Mcage\) is the number of degrees of freedom (1 or 3) assigned to the cage. To assess the accuracy of the solution, the magnitude of the equation residue should be compared to the magnitude of the components which make up the equation. The residue values are printed under the heading "CORRESPONDING EQ-VALUES." The residues are printed in a six per line format such that the residues pertaining to a given element are all on one line. These six values represent the following six equilibrium equations:

1) \(\Sigma F_x = 0\)  
2) \(\Sigma F_y = 0\)  
3) \(\Sigma F_z = 0\)  
4) \(\Sigma M_x = 0\)  
5) \(\Sigma M_y = 0\)  
6) \(\Sigma M_z = 0\)

Equations 1 and 2 are dominated by the normal raceway contact forces and the rolling element centrifugal force. Compared to the magnitude of these forces the residues of \(\Sigma F_x\) and \(\Sigma F_y\) are usually very small and thus acceptable. The remaining four equilibrium equations, however, have as their major terms, various components of the friction forces which act upon the element. Large values of these residue values are a manifestation of an unstable
operating condition, where the instability is defined qualita-
tively in terms of large change in component and raceway relative
speeds producing small changes in internal bearing forces. The
shallow slopes of these "Force vs. Relative Speed Functions"
promote dynamic and numerical instabilities.

The \( \Sigma F_z \) equation should be examined for each rolling
element. Typically this may be done by inspection with close
comparison and calculations made for only one or two elements.

The three cage equilibrium equations are:

1) \( \Sigma M_x = 0 \)  
2) \( \Sigma F_y = 0 \)  
3) \( \Sigma F_z = 0 \)

If only one cage equation is considered, it is equation 1); if
more than one is considered, all three are used.

The components of these equations are the rolling element
cage normal and friction forces as well as the cage-ring normal
and friction forces. Magnitudes of the components of these
forces and moments are printed as part of the output. Thus,
comparison of components against residues is straight forward
after converting to a consistent set of units. Residues are in
English while the output is in metric units.

It should be noted that in those solutions in which the
cage has only one degree of freedom, that the cage interaction
with the rolling elements has only minor impact upon the rolling
element dynamics. Therefore, a relatively large residue for
the cage equation is not terribly significant. The rolling
element \( \Sigma F_z \) equations should be the basis for the judgement as
to whether a solution is good enough.

Although the message "THIS IS THE BEST WE CAN DO. IT MAY BE
USABLE" may be written during a steady state temperature calcu-
lation scheme, numerical instabilities in those schemes are rather
uncommon. None have been experienced with this program after
three years of operation.

The following data are printed subsequent to printing messages
1 through 4 above.

RELATIVE ACCURACY (ERREL) ITERATION.
LIMIT (ITEND), NUMBER OF UNKNOWNS (N) ABSOLUTE ACCURACIES EXA(J).
DAMPING FACTORS 1-5 OTHER STEP FACTORS 6-10 (SF), MAXIMUM STEP
FACTORS (SMX(J), J=1,N).
CORRECTIONS OF THE X-ES FROM SMQ D(J), J=1,N.
NUMBER OF DERIVATIVES EXPECTED FOR EACH X ND(J), J=1,N
X=VALUE X X(J), J=1,N.
CORRESPONDING EQ-VALUES EQ(J), J=1,N.
5. GUIDES TO PROGRAM USE

The Computer Program is a tool. As with any tool, the results obtained are at least partially dependent upon the skill of the user. Certainly, the economics of the Program usage are highly dependent upon the user's technical need and discriminate use of Program options.

Some general guides for efficient use of the Program are listed below:

1. **Attempt to use the lowest level of solution possible.** For instance, if the prime object of a given run is to obtain bearing fatigue lives, execute only the elastic solution \( (\text{NPASS} = 0) \). If an estimate of bearing frictional heat is required, execute the low level friction evaluation \( (\text{NPASS} = 1) \). Execute the friction solution \( (\text{NPASS} = 2) \) only if rolling element and cage kinematics are of interest. Execute the highest level of solution \( (\text{NPASS} = 3) \) if kinematics are of interest and the bearing reaction loads deviate substantially from the shaft applied loading, i.e., a deviation greater than ten percent.

2. **Attempt to input bearing operating diametral clearance rather than calculate it.** Or, execute the diametral clearance change analysis once for a group of similar runs and use the output from the first run as input to the subsequent runs omitting the clearance change analysis.

3. **Attempt to input accurate operating temperatures rather than calculate them.**

4. **The more non-linear the problem, the more computer time required to solve it.** In the bearing friction solution, large coefficients of friction seem to increase the degree of non-linearity. In the thermal solutions, if possible, eliminate non-linearities by omitting radiation terms and by using constant rather than temperature dependent free and forced convection coefficients.

5. **In the transient thermal solution, space the calls to the shaft-bearing solution \( (\text{BTIME}) \) to as large an interval as prudently possible.** Be careful, however, too long an interval will produce large errors in heat rate predictions.
6. In the steady state thermal analysis, attempt to estimate nodal temperatures on a node-by-node basis. Nodes which are heat sources should have higher temperatures than the surrounding nodes.

The above suggestions are intended to encourage the use of the Program on a cost-effective basis. The intent is not to discourage the use of important program capabilities, but to emphasize how the program should be most effectively used.

It is suggested that the user take a simple, axially loaded ball bearing problem and execute the program through the full range of options beginning with a frictionless solution proceeding to the three levels of friction solution with a low (0.01) and high (0.1) friction coefficient. The diametral clearance change analysis and the thermal solutions should also be executed on an experimental basis. This exercise will provide the user with some insight into economics of the Program usage on his computer as well as the results obtained from various levels of solution of the same problem.

It is also suggested that a constant user of the program should study the hierarchical Program flow chart, Appendix II, along with the Program listing to gain an appreciation of the program complexity and the flow of the problem solution. The Program is comprised of many small functional subroutines. Knowledge of these small elements may allow the user to more easily piece together the philosophy of the total problem solution.

SHABERTH is intended to be used for the analysis of a multi-bearing system. It may, however, be used to analyze single bearings, mounted on dummy shafts under certain conditions of limited applied loading. These loading conditions are outlined and explained below as they apply separately for ball and cylindrical roller bearings.

**Ball Bearings**

Of major value is the capability of SHABERTH to treat, in a simple, economic manner, ball bearings subjected to axial loading only. Use of the program in this manner is recommended.

SHABERTH is not recommended for the solution of a single ball bearing subjected to radial load only. The Program attempts to satisfy axial and angular equilibrium as well as radial. A ball bearing is elastically very soft in those directions which causes mathematical instabilities during the solution scheme. This makes the Program uneconomical for this particular situation.
Whereas the use of SHABERTH to solve single radially loaded ball bearing problems is not recommended, because of economics, the solution of a single, radially and axially loaded problem is impossible. The impossibility arises because a moment reaction will develop when a ball bearing is subjected to both radial and axial loading. In order for the user to solve this problem he must specify at input the bearing reaction moment. The user must know the answer to a portion of the problem before he can begin to solve it.

Cylindrical Roller Bearing

A single, cylindrical roller bearing may be subjected to radial loading or combinations of radial and moment loading. When SHABERTH is used in this manner, it is important that bearing misalignments be specified indirectly through specification of a non-zero applied moment. If a radial load and an initial outer ring misalignment are specified along with a zero applied amount, SHABERTH will attempt but will be unable to solve the problem since it will be impossible to equilibrate the non-zero reaction moment, resulting from the offset, against the zero applied moment.

The cylindrical roller bearing cannot accept applied axial loading and thus when a single cylindrical roller bearing is being examined, the applied axial load must be specified to be zero.

When a single cylindrical roller bearing is being examined all loading should be referred to the X-Y plane in order to take advantage of the symmetry of load distribution among rolling elements.
LIST OF REFERENCES


APPENDIX I 1

HEAT TRANSFER INFORMATION
APPENDIX I 1

HEAT TRANSFER INFORMATION

I 1.1 BACKGROUND

The temperature portion of SHABERTH is designed to produce temperature maps for an axisymmetric mechanical system of any geometrical shape. The mechanical system is first approximated by an equivalent system comprising a number of elements of simple geometries. Each element is then represented by a node point having either a known or an unknown temperature. The environment surrounding the system is also represented by one or more nodes. With the node points properly selected, the heat balance equations can be set up accordingly for the nodes of unknown temperature. These equations become non-linear when there is convection and/or radiation between two or more of the node points considered. The problem is, therefore, reduced to solving a set of linear and/or non-linear equations for the same number of unknown nodal temperatures. It is obvious that the success of the approach depends largely on the physical subdivision of the system. If the subdivision is too fine, there will be a large number of equations to be solved; on the other hand, if the subdivision is too crude, the results may not be reliable.

In a system consisting of rolling bearings, for the sake of simplicity, the elements considered are usually axially symmetrical, e.g., each of the bearing rings can be taken as an element of uniform temperature. For an element which is not axially symmetrical, its temperature is also assumed to be uniform and its presence is assumed not to distort the uniformity in temperature of a neighboring element which is axially symmetrical. That is, the non-symmetrical element is represented by an equivalent axially symmetrical element with approximately the same surface area and material volume. This kind of approximation may seem to be somewhat unrealistic, but with properly devised equivalent systems, it can be used to solve complicated problems with results satisfying some of the important engineering requirements.

The computer program can solve the heat-balance equations for either the steady state or the transient state conditions and produce temperature maps for the mechanical system when the input data are properly prepared.

I 1.2 BASIC EQUATIONS

I 1.2.1 Heat Conduction

The rate of heat flow $q_{ci,j}(W)$ that is conducted from node $i$ to node $j$ may be expressed by
\[ q_{ci,j} = \frac{\lambda_{ij} \Delta t_{ij}}{L_{ij}} \quad (t_i - t_j) \quad I 1.1 \]

t_i and t_j are the temperatures at i and j, respectively, \( \lambda_{ij} \) the area normal to the heat flow, (m²) \( L_{ij} \) the distance (m) and \( \lambda_{ij} \) the thermal conductivity between i and j, (W/m°C).

Assuming that the structure between point i and j is composed of different materials, an equivalent heat conductivity may be calculated as follows:

**Fig. 1.1 Parallel Conduction**
\[
\lambda_{ij} = \frac{\lambda_1 A_1 + \lambda_2 A_2}{A_{ij}}
\]

\( A_{ij} = A_1 + A_2 \)

**Fig. 1.2 Series Conduction**
\[
\lambda_{ij} = \frac{\ell_{ij}}{\ell_1/\lambda_1 + \ell_2/\lambda_2}
\]

\( \ell_{ij} = \ell_1 + \ell_2 \)

The calculation of the areas will be discussed in Section I 1.2.5

**I 1.2.2 Convection**

The rate of heat flow that is transferred between a solid structure and air by free convection may be expressed by

\[ I 1-3 \]
The rate of heat flow that is transferred between a solid structure and a fluid by forced convection may be expressed by

\[ q_{ni,j} = \alpha_{i,j} A_{i,j} (t_i - t_j) \quad (I.1.3) \]

in which \( \alpha_{ij} \) is the heat transfer coefficient.

Now, with \( \alpha = \alpha_{ij} \), introduce the Nusselt number

\[ N_u = \frac{\alpha L}{\lambda} \quad (I.1.4) \]

the Reynolds number

\[ R_e = \frac{UL}{\nu} \quad (I.1.5) \]

and the Prandtl number

\[ P_r = \frac{\rho \nu c_p}{\lambda} \quad (I.1.6) \]

where

\( \lambda \) is a characteristic length which is equal to the diameter in the case of a cylindrical surface and is equal to the plate length in case of a flat surface.
\( U \) is a characteristic velocity which is equal to the difference between the fluid velocity at some distance from the surface and the surface velocity (m/sec)

\( \lambda \) is the fluid thermal conductivity (W/M°C)

\( \nu \) is the fluid kinematic viscosity (m²/sec)

\( \rho \) is the fluid density (kg/m³)

\( c_p \) is the fluid specific heat (J/kg°C)

For given values of \( Re \) and \( Pr \), the Nusselt number \( Nu \) and thus, the heat transfer coefficient may be estimated from one of the following expressions:

Laminar flow along a flat plate: \( Re < 2300 \)

\[
Nu = 0.323 \sqrt{Re} \cdot \sqrt[P]{Pr}
\]

Laminar flow of a liquid in a pipe:

\[
Nu = 1.36 \sqrt{Re} \cdot Pr \left( \frac{L}{D} \right)
\]

where \( D \) is the pipe diameter and \( L \) the pipe length

Turbulent flow of a liquid in a pipe:

\[
Nu = 0.027 \cdot Re^{0.8} \cdot \sqrt[P]{Pr}
\]

Gas flow inside and outside a tube:

\[
Nu = 0.3 Re^{0.57}
\]

Liquid flow outside a tube:

\[
Nu = 0.6 Re^{0.5} \cdot Pr^{0.31}
\]

Forced free convection from the outer surface of a rotating shaft

\[
Nu = 0.11 \left[ 0.5 Re^2 \cdot Pr \right]^{0.35}
\]

where the Reynolds number \( Re \) is developed by the shaft rotation.

\[
Re = \frac{\omega \pi D^2}{\nu}
\]
in which \( \omega \) is the angular velocity (rad/sec) and 
\( D \) is the roll diameter (m).

The average coefficient of forced convection to the lubricating oil within a rolling contact bearing may be approximated by,

\[
\alpha = 0.0986 \left( \frac{N}{v} \left[ 1 + \frac{D \cos (\theta)}{d_m} \right] \right) ^{1/4} \lambda (Pr)^{1/3} \quad (I \ 1.14)
\]

using + for outer ring rotation
- for inner ring rotation

in which
- \( N \) is the bearing operating speed (rpm)
- \( D \) is the diameter of the rolling elements (mm)
- \( d_m \) is the bearing pitch diameter (mm)
- \( \alpha \) is the bearing contact angle (degrees)

I 1.2.3 Fluid Flow

The rate of heat flow that is transferred from fluid node \( i \) to fluid node \( j \) by fluid flow is

\[
q_{fi,j} = \rho V_{ij} C_p (t_i - t_j) \quad (I \ 1.15)
\]

\( V_{ij} \) is the volume rate of flow from \( i \) to \( j \). It must be observed that the continuity of mass requires the following equation to be satisfied

\[
\sum V_{ij} = 0 \quad (I \ 1.16)
\]

provided the fluid density is constant. The summation should be extended over all nodes \( i \) within the fluid which have heat exchange with node \( j \) by fluid flow.

I 1.2.4 Heat Radiation

The rate of heat flow that is radiated to node \( j \) from node \( i \) is expressed by

\[
q_{Ri,j} = \sigma_{i,j} \left\{ (T_i+273)^4 - (T_j+273)^4 \right\} \quad (I \ 1.17)
\]

where

\[ T_j = t_j + 273.16 \]
\[ T_i = t_i + 273.16 \]
and the value of the coefficient $\delta_{i,j}$ depends on the geometry and the emissivity or the absorptivity of the bodies involved.

For radiation between large, parallel and adjacent surfaces of equal area, $A_{i,j}$ and emissivity, $\varepsilon_{i,j}$, $\delta_{i,j}$ is obtained from the equation

$$\delta_{i,j} = \varepsilon_{i,j} \sigma A_{i,j}$$

where $\sigma$, the Stefan-Boltzmann constant, is

$$\sigma = 5.76 \times 10^{-8} \text{ W/m}^2/(\text{degK})^4$$

For radiation between concentric spheres and coaxial cylinders of equal emissivity, $\varepsilon_{i,j}$, $\delta_{i,j}$ is given by the equation

$$\delta_{i,j} = \frac{\varepsilon_{i,j} \sigma A_{i,j}}{1 + (1 - \varepsilon_{i,j}) \frac{A_{i,j}}{A^*_{i,j}}}$$

where $\sigma$ is as above $A_{i,j}$ is the area of the enclosed body and $A^*_{i,j}$ is the area of the surrounding body, i.e. $A_{i,j}$ $A^*_{i,j}$.

Expressions for $\delta_{i,j}$ that are valid for more complicated geometries or for different emissivities may be found in the heat transfer literature.

I 1.2.5 Calculation of Areas

In the case of heat transfer in the axial direction $A_{i,j}$ is given by the equation (I 1.3)

$$A_{i,j} = 2\pi r m \cdot \Delta r$$

Referring to the input instructions, Section 5, but recalling $L$ must be input in mm not m.

$$L_1 = r_m = \frac{r_1 + r_2}{2}$$

$$L_2 = \Delta r = r_2 - r_1$$
In the case of heat transfer in the radial direction, \( A_{i,j} \) is obtained from the expression

\[
A_{i,j} = 2\pi r_m \cdot H; \quad L_1 = r_m; \quad L_2 = H
\]

and similarly for the radiation term above

\[
A_{i,j}^* = 2\pi r_{m*} H
\]

\[
L_3 = r_{m*}
\]

\[
L_2 = 2H
\]

in which \( H \) is the length of the cylindrical surface; where heat is conducted between \( i \) and \( j \), \( r_m \) is given by the same equation as above (Fig. I 1.3(d)); where heat is convected between \( i \) and \( j \), \( r_m \) is the radius of the cylindrical surface (Fig. I 1.3(c)); where heat is radiated between \( i \) and \( j \), \( r_m \) is the radius of the enclosed cylindrical surface and \( r_{m*} \) the radius of the surrounding cylindrical surface (Fig. I 1.3(d)).

![Figure I 1.3(a)](image1)

![Figure I 1.3(b)](image2)

![Figure I 1.3(c)](image3)

![Figure I 1.3(d)](image4)
I 1.3 Transient Analysis

For the transient analysis, all of the data pertaining to the node-to-node heat transfer coefficients must be provided by the input. Additionally, the volume and the specific heat at each node is required. For metal nodes, this input is straightforward. However, when fluid flow is being considered, there is no easy way to approximate the fluid nodal volume in a free space such as the bearing cavity. However, through use of the Program, the user's ability to make appropriate estimates will improve.
APPENDIX I 2

BEARING DIAMETRAL CLEARANCE CHANGE ANALYSIS, FROM COLD UNMOUNTED TO MOUNTED OPERATING CONDITIONS
APPENDIX I 2

BEARING DIAMETRAL CLEARANCE CHANGE ANALYSIS, FROM COLD UNMOUNTED TO MOUNTED OPERATING CONDITIONS*

I 2.1 INTRODUCTION

As a bearing is taken from the shelf, mounted in a housing and on a shaft, turned up to speed and subjected to operating loads and temperature, the diametral clearance of the bearing will change. To accurately analyze the performance of a bearing, its operating clearance must be known. An analysis has been developed to account for the following effects:

1. Temperature changes and gradients.
2. Initial and operating shaft and housing fits.
3. Rotation induced, ring radial growth.
4. Uniform radial components of the rolling element-raceway normal loads.

The basis for the major portion of this analysis is taken from Timoshenko, 4. The bearing rings are treated as thick walled circular cylinders of constant wall thickness subjected to the action of uniformly distributed internal and external pressure.

The external pressure arises in the case of the outer ring from a press fit into the bearing housing. The internal pressure on the outer ring arises from the discrete rolling element loads which are regarded as uniform internal pressure acting on the outer ring. Similarly for the inner ring the press fit on the shaft provides a uniform internal pressure and the rolling element loads are regarded as a uniform external pressure. Fig. I 2.1 show the idealized sections used in the analysis.

*This Appendix is based upon the original work [5]
2.2 BEARING SHAFT AND HOUSING EQUIVALENT SECTIONS

Timoshenko's analysis requires that a ring be fully defined by specification of only internal and external radii. If two concentric rings are pressed together the analysis assumes that the rings are of equal width. Perhaps the outside surface of the actual inner ring section cannot be defined by a single radius. However, an equivalent radius may be found such that the cross sectional areas of the real and equivalent rings are equal.

A second significant dimension associated with both the outer and inner rings is the radius of the rolling element path at each section. Ultimately, it is the change in these path radii, in addition to a change in the rolling element diameter, which permit a change in the bearing diametral clearance.

The bearing diametral clearance change problem requires the solution to similar problems for both the inner ring-shaft (shaft) and the outer ring-housing (housing) sections. To help eliminate repetition and cumbersome subscripting, only the shaft section will be covered thoroughly. Differences in analysis which occur at the housing section will be noted.

2.3 INTERNAL AND EXTERNAL EQUIVALENT PRESSURES

Both the shaft and housing sections are represented by two rings. The rings may or may not be in intimate contact. Intimate contact occurs when due to any of several factors, the outside radius of the inside ring is greater than the inside radius of the outer ring. A pressure is developed at that interface, which tends to expand the outside ring and collapse the inside ring. Although the pressure acting on the actual interfering surfaces is identical, the use of equivalent geometric sections requires the
use of equivalent pressure, such that the calculated pressure acting to collapse the inside ring will not be identical to the pressure acting to expand the outside ring.

The equivalent pressure concept allows the effect of unequal component widths to be considered.

The effect of unequal inner ring and shaft width is taken into account by the factor $X$ which applies to the pressure where, referring to Fig. 1

$$0.5 \leq (X_\text{s} = \frac{W_\text{s}}{W_\text{e}}) \leq 2. \quad (I \ 2.1)$$

$$0.5 \leq (X_\text{h} = \frac{W_\text{h}}{W_\text{e}}) \leq 2. \quad (I \ 2.2)$$

Where $W$ denotes the element width and the subscripts $I$, $O$, $S$ and $H$ refer to inner, outer, shaft and housing respectively.

The equivalent pressures acting on the shaft and housing are thus

$$P_{\text{s_0}} = X_\text{s} P_{\text{i_i}} \quad (I \ 2.3)$$

$$P_{\text{h_i}} = X_\text{h} P_{\text{o_0}} \quad (I \ 2.4)$$

$P_{\text{s_0}}$ denotes the pressure on the shaft O.D. as related to the pressure on the inner ring $P_{\text{i_i}}$, $P_{\text{h_i}}$ and $P_{\text{o_0}}$ are similarly defined. The pressures ($P$) in Eq. (I 2.3 and I 2.4) are non-zero only if an interference fit exists at the section in question. An interference fit may exist under cold mounted conditions but due to thermal gradients and high speed rotation, clearance may develop at operating conditions.
To account for the elastic effect of the uniform radial components of the rolling element loads on the ring dimensions, an equivalent inner ring external pressure and outer ring internal pressure are calculated based on the minimum radial component of the rolling element loads, such that for the inner ring:

$$P_{I\sigma} = \frac{\varepsilon (Q_{i} \cos \alpha_{i})_{\min}}{2\pi C W_{I}}$$  \hspace{1cm} (I 2.5)

where \( Z \) is the total number of rolling elements

\( (Q_{i} \cos \alpha_{i})_{\min} \) is the smallest rolling element raceway radial load component at the inner raceway

\( W_{I} \) is the inner ring total width, or one half the total width if the inner ring is split.

An expression corresponding to Eq. (I 2.5) can be developed for the internal pressure acting to expand the outer ring. In fact \( P_{I\sigma} \) will exist only when the applied load is predominantly axial or when the bearing is preloaded, but the internal pressure acting on the outer ring, \( P_{O\sigma} \), will always exist to some extent, as a result of rolling element centrifugal force.

I 2.4 TEMPERATURE EFFECTS

Temperature effects at operating conditions are considered in two ways. A bulk effect based on the radii to the rolling paths (\( R \)) is calculated from Eq. (I 2.6)

$$\delta = R \lambda \xi$$  \hspace{1cm} (I 2.6)

where

\( \delta \) - is the change in rolling path radius
\( \lambda \) - is the component coefficient of thermal expansion,
\( \xi \) - is the difference between the component effective temperature and 68°F.
A similar expression may be written for the change in roller diameter.

The second effect of temperatures is the change in interference fit from cold to operating conditions such that the interference fit as a result of the initial fit and the temperature difference between the two components can be calculated. Cold (293°K, (68°F)), shaft and housing fits \( F_{\text{cold}} \) are input to the analysis. A positive value reflects an interference fit. The analysis neglects asperity crushing. The fit \( F_{\text{hot}} \) at operating temperature is given by

\[
(F_{\text{hot}})_s = (F_{\text{cold}})_s + 2b (\lambda_3 t_{(3)} - \lambda_1 t_{(1)})
\]  

(I 2.7)

\( F_{\text{hot}} \) is the fit considering operating temperatures.

I 2.5 RING RADIAL DISPLACEMENT CONSIDERING SURFACE PRESSURES

At this point we consider the problem of evaluating thick walled cylinder, (ring) radial displacements as a function of the ring radial dimensions, physical properties, and uniform internal and external pressures. Timoshenko develops Eq. (I 2.8) for determining the ring/radial displacement \( \delta \), as a function of a general radius \( r \) to any point within the ring when considerations of ring rotation are omitted.

\[
\frac{d^2\delta}{dr^2} + \frac{1}{r} \frac{d\delta}{dr} - \frac{\delta}{r^2} = 0
\]  

(I 2.8)

The general solution of this equation is

\[
\delta = \kappa_1 r + \frac{\kappa_2}{r}
\]  

(I 2.9)

The expression for normal stress in the ring is given by:

\[
\sigma_r = \frac{E}{1-\nu} \left[ \kappa_1 (1+\nu) - \kappa_2 \left( \frac{1-\nu}{r^2} \right) \right]
\]  

(I 2.10)
The constants of integration $K_1$ and $K_2$ can be determined from Eqs. (I 2.11 and I 2.12) which make use of the boundary conditions at the inner and outer surfaces of the ring. Note that the dimensions of inner ring section are used in Eq. (I 2.11 and I 2.14). These equations, however, are general expressions and are valid for all rings.

\[ (\sigma_r)_{r+c} = -P_{lo} = \frac{E_x}{1-\nu^2_x} \left[ K_1 (1+\nu_x) - K_2 \left( \frac{1-\nu_x}{C^2} \right) \right] \]  \hspace{1cm} (I 2.11)

\[ (\sigma_r)_{r-b} = -P_{l-} = \frac{E_x}{1-\nu^2_x} \left[ K_1 (1+\nu_x) - K_2 \left( \frac{1-\nu_x}{C^2} \right) \right] \]  \hspace{1cm} (I 2.12)

Where $E$ and $\nu$ are the modulus of elasticity and Poisson's ratio respectively. The negative sign associated with $P_{lo}$ and $P_{l-}$ reflect the sign convention, wherein a positive normal stress indicates tension. Solving Eqs. (I 7.11-12) for $K_1$ and $K_2$ and substituting the results into Eqs. (I 2.9 and I 2.10) gives the general expressions for normal stress and radial deflection as functions of the internal and external pressures acting on the ring:

\[ \sigma_{r-} = \frac{b^2 P_{l-} - C^2 P_{lo}}{C^2 - b^2} - \frac{c^2 b^2 (P_{lo} - P_{l-})}{r^2 (C^2 - b^2)} \]  \hspace{1cm} (I 2.13)

\[ \delta_{I-} = \frac{(1-\nu)}{E_x} \left[ \frac{b^2 P_{l-} - C^2 P_{lo}}{C^2 - b^2} + \frac{c^2 b^2 (P_{lo} - P_{l-})}{r E_x} \right] \]  \hspace{1cm} (I 2.14)

Using the appropriate dimensions and pressures, Eq. (I 2.14) can express the inward displacement of the shaft and the outward displacement of the inner ring when assembled with an interference fit $F_{HOT}$.

\[ \delta_{Ib} - \delta_{sIb} = F_{HOT} \]  \hspace{1cm} (I 2.15)

\[ \delta_{Ib} = \frac{(1-\nu)}{E_x} \left[ \frac{b^2 P_{l-} - C^2 P_{lo}}{C^2 - b^2} + \frac{c^2 b^2 (P_{lo} - P_{l-})}{r E_x} \right] \]  \hspace{1cm} (I 2.16)

\[ \delta_{sb} = -\frac{b P_{lo}}{E_x} \left[ \frac{a^2 + b^2}{b^2 - a^2} - \nu \right] \]  \hspace{1cm} (I 2.17)
and $P_{ho}$ are assumed zero. Using Eq. (I 2.5) to obtain $P_{ho}$, the following expression may be developed for $P_{xi}$ which accounts for all variables in the interference fit problem except for the effect of high speed rotation.

$$P_{xi} = \frac{F_{hot} + 2\left(1 - \frac{\nu_z}{E_x}\right) \left(\frac{b c^3 P_{ho}}{c^2 - b^2}\right)}{\left(1 - \frac{\nu_z}{E_x}\right) b \left(\frac{b^2 + c^2}{c^2 - b^2}\right) + b X \left(\frac{a^2 + b^2}{b^2 - c^2} + \nu_z\right)}$$  \hspace{1cm} (I 2.18)

Note, $P_{xi}$ may not be negative, i.e. the effect of fitting one ring on another cannot result in placing the common surfaces in tension.

Having determined $P_{xi}$ and $P_{ho}$, Eq. (I 2.14) may be used to determine $\delta_{xi}(F, t, q)$ by replacing the general radius ($r$) with the radius to the roller path, $R$. $\delta_{xi}(F, t, q)$ is defined as the change in rolling path radius resulting from:

- $F$: The initial fit
- $t$: The change in fit resulting from a temperature gradient
- $q$: The effect of the rolling element load.

I 2.6 RING ROTATION

We must now examine the effect on the rolling path which results from high speed rotation. If the rotational speed is less than 100 rpm, the rolling path radius is assumed to be unaffected.

Timoskenko presents Eq. (I 2.19) to define ring displacement $\delta$ in terms of a general radius ($r$), the weight density of the ring material, ($\rho$), and the ring angular velocity ($\Omega$).

$$\frac{d^2 \delta}{dr^2} + \frac{1}{r} \frac{d \delta}{dr} - \frac{\delta}{r^2} + (1 - \nu_z) \frac{\rho \Omega^2 r^2}{4 \mu} = 0$$  \hspace{1cm} (I 2.19)
Using the notation;

\[ N = (1-\nu^2) \frac{E \sigma^2}{3} \]  

(1.2.20)

the general solution to Eq. (1.2.19) may be written:

\[ \delta = -N \frac{r^3}{6} + K_1 r + K_2 \]  

(1.2.21)

The general expression for the normal stress is given by:

\[ N = \frac{E}{(1-\nu^2)} \left[ \frac{\nu^2}{6} N r^3 + (1-\nu) K_1 r - (1-\nu) K_2 \frac{1}{r^3} \right] \]  

(1.2.22)

Using Eqs. (1.2.21 and 1.2.22) and the principle of superposition, the effects of ring rotation can be considered. In this case, superposition allows a set of integration constants \( K_1 \) and \( K_2 \) to be calculated for a ring based on a change in pressure \( P^* \) at the internal and external surfaces. Eq. (1.2.22) may be written for the four specific surfaces of the shaft section as follows;

**Inner Ring External Surface**

\[ -P_{s0} = \frac{E_t}{1-\nu_t} \left[ \frac{\nu^2}{6} N x r^3 + (1+\nu_t) K_{1x} r - (1-\nu_t) K_{2x} \frac{1}{r^3} \right] = 0 \]  

(1.2.23)

**Inner Ring Internal Surface**

\[ -P_{s0} = \frac{E_t}{1-\nu_t} \left[ \frac{\nu^2}{6} N x r^3 + (1+\nu_t) K_{1x} r - (1-\nu_t) K_{2x} \frac{1}{r^3} \right] = 0 \]  

(1.2.24)

**Shaft External Surface**

\[ -P_{s0} = \frac{E_s}{1-\nu_s} \left[ \frac{\nu^2}{6} N s r^3 + (1+\nu_s) K_{1s} r - (1-\nu_s) K_{2s} \frac{1}{r^3} \right] = 0 \]  

(1.2.25)

**Shaft Internal Surface**

\[ -P_{s0} = \frac{E_s}{1-\nu_s} \left[ \frac{\nu^2}{6} N s r^3 + (1+\nu_s) K_{1s} r - (1-\nu_s) K_{2s} \frac{1}{r^3} \right] = 0 \]  

(1.2.26)

From Eq. (1.2.3)

\[ P_{s0} = P_{s0} \]  

(1.2.27)

An additional useful relationship derived from Eq. (1.2.21) defines the difference \( \delta^* \) between the inner ring and shaft.
displacement at their common surface, as a result of their rotational speed, in terms of the four integration constants of Eq. (I 2.23 - I 2.26).

\[ \delta^* = K_{11} b + \frac{K_{12}}{b} \rightleftharpoons N_2 b^3 - \frac{K_{13} b}{b} - \frac{K_{14}}{b} + N_4 b^3 \]  

(1 2.28)

Under press fit conditions \( \delta^* = 0 \).

We will now examine the application of Eq. (I 2.23 - I 2.26) in determining the ring behavior as a function of rotational speed.

The following conditions might be encountered:
1. A tight fit remains tight
2. A tight fit loosens
3. A loose fit remains loose
4. A loose fit tightens

For all four conditions, \( P_{\text{fit}} \) and \( P_{\text{loose}} \), i.e. it is assumed that no change in pressure occurs at the internal surface of the shaft or the external surface of the inner ring resulting from ring rotation. Also \( \Omega_x \) and \( \Omega_y \) are identical. All four of the integration constants \( K_{11}, K_{12}, K_{13}, \) and \( K_{14} \) are unknowns. Also, either the change in pressure \( P_{\text{fit}}^* \) or the rotational speed \( \Omega \) embodied in \( N_x \) and \( N_y \) are additional unknowns.

Now the formulations of Eq. (I 2.23 - I 2.28) are presented which are required to solve each of the four conditions.

I 2.6.1 SITUATION 1 AND 2, INITIALLY TIGHT FIT

In addition to the integration constants, \( P_{\text{fit}}^* \) is the unknown, where \( P_{\text{fit}}^* \) is the change in inner ring internal pressure resulting from the rotational speed. If \( P_{\text{fit}}^* \) is less than \( P_{\text{fit}} \) from Eq. (I 2.18), situation 1 is realized. The operating fit pressure is the difference between \( P_{\text{fit}} \) and \( P_{\text{fit}}^* \) and the fit remains tight. The resulting radial displacement at the inner ring rolling path is given by:

\[ \delta^*_r = -N_4 b^3 + K_{11} R + \frac{K_{12}}{b} \]  

(1 2-11)
and the total change in the rolling path radius at the inner ring is

\[ \delta_{\text{TOT}} = \delta_{(L, q)} + \delta_T + \delta_L \]  

(I 2.30)

\( \delta_T \) is the change in rolling path radius which occurs as a result of a temperature change from 68°F.

If \( P_{T^*} \) is greater than \( P_{L^*} \) the solution to the problem requires several additional steps. Eq. (I 2.23), must be re-solved to determine the rotational speed, \( \Omega_{TL} \) at which the change in fit pressure is just equal to the initial fit pressure and \( \delta^* \cdot 0 \).

This rotational speed is termed the tight fit speed limit and is subscripted with the letter (TL). Using the integration constants thus determined and Eq. (I 2.21), \( \delta_{TL} \) may be determined.

Note, the subscripts in parenthesis should be interpreted as follows: The first subscript refers to the state of the initial fit (T-tight) or (L-loose), the second subscript refers to the rotational speeds (T-Total) or (L-Limit). For instance, \( U_{TL} \) refers to the change in inner ring rolling path radius due to the rotational speed at which the initially tight shaft fit is lost.

After determining \( U_{TL} \), Eqs. (I 2.23 - I 2.26) are resolved twice for the four integration constants, with \( P_{L^*} \) set to zero. First, for the full rotational speeds, yielding \( U_{TL} \), and then for the tight fit speed limit, yielding \( U_{LL} \). This is accomplished by changing the values of \( N_{L^*} \) and \( N_s \) through a change in the value of \( \Omega \) in Eq. (I 2.20). Utilizing super-position, the change in rolling path radius resulting from rotation is given in Eq. (I 2.31).

\[ \delta_N = \delta_{TL} + (\delta_{LT} - \delta_{LL}) \]  

(I 2.31)

Eq. (I 2.31) is presented graphically in Fig. I 2.2. Then is calculated from Eq. (I 2.30).
FIGURE I 2.2
Ring Radial Expansion vs. Rotational Speed Squared
I 2.6.2 SITUATIONS 3 AND 4 INITIALLY LOOSE FIT

The integration constants $K_{11}$, $K_{12}$, $K_{13}$, and $K_{15}$ are unknowns. $P_{11}$, $P_{12}$, $P_{13}$, and $P_{15}$ are all set to zero, and only Eqs. (I 2.23 - I 2.26) are solved. Using the integration constants thus obtained, Eq. (I 2.28) is solved for $S^*$. If $S^* < |F_{not}|$, where $F_{not}$ is negative, the loose fit remains loose at operating conditions. The constants $K_{11}$ and $K_{13}$ are used to calculate $U_{tot}$. Using Eq. (I 2.30), $U_{tot}$ is calculated. If $S^* > |F_{not}|$, the shaft has expanded further under the effects of rotation than the inner ring, and the initially loose fit becomes tight. This requires additional solutions to Eqs. (I 2.23 - I 2.26).

$S^*$ is set equal to $F_{not}$ and the equations are solved for the integration constants plus the speed at which the initially loose fit becomes tight $\omega_{ll}$. Using the integration constants $K_{12}$ and $K_{12}$ along with Eq. (I 2.32), $S_{ll}$ is determined. Eq. (I 2.23 - I 2.28) are again resolved twice at $\omega_{ll}$ and $\omega_{lt}$ for $P_{11}$ and the integration constants, after setting $S^*$ to zero. $S_{tl}$ and $S_{tt}$ can be obtained by from eq. (I 2.29) using the two sets of integration constants. The operating fit pressure is then given by:

$$P_{11} = P_{11}^* - P_{11}^*_{tl}$$

(I 2.32)

$$S_{ot} = S_{lt} + (S_{tt} - S_{tl})$$

(I 2.33)

and Eq. (I 2.30) is used to determine $S_{tot}$. Where $S_{tot}$ represents the total change in the radius to the rolling path of the section as the bearing is taken off the shelf at 68°F, mounted on the shaft, subjected to load and rotated up to operating speed.

I 2-14
**I 2.7 BEARING DIAMETRAL CLEARANCE**

The preceding sections dealt primarily with the shaft-inner ring section. However, the equations presented are valid for all four of the assembly sections. For both sections, a value is determined for $\delta_{\text{TOT}}$, cold fit pressure, operating fit pressure and the speed at which an initially tight fit loosens.

As noted earlier, Eq. (I 2.6) can be used to determine this change in rolling element diameter such that:

$$2\delta_{et} = D(\Delta t)$$  \hspace{1cm} (I 2.34)

The change in bearing diametral clearance is given by

$$\Delta P_d = 2 \left[ \delta_{\text{HTOT}} - \delta_{\text{STOT}} - 2\delta_{et} \right]$$  \hspace{1cm} (I 2.35)

The bearing operating diametral clearance equals the initial clearance plus the change in clearance $\Delta P_d$
APPENDIX I 3

ELASTIC SHAFT ANALYSIS
APPENDIX I 3

ELASTIC SHAFT ANALYSIS *

I 3.1 COORDINATE SYSTEM, LOAD AND GEOMETRY CONSIDERATIONS

The shaft subprogram calculates the deflection characteristics of a general shaft in two planes, one plane at a time, by solving the differential equation for the deflection curve of the shaft,

\[ \frac{d^2\gamma}{dx^2} = -\frac{M(x)}{EI(x)} \]  

(I 3.1)

where

- \( M \) = moment
- \( E \) = modulus of elasticity
- \( I \) = moment of inertia

The coordinates \( X \) and \( Y \) are shown in Fig. I 3.1

![Figure I 3.1 Shaft Coordinate System and Shaft Loading](image)

Figure I 3.1 Shaft Coordinate System and Shaft Loading

The shaft load may consist of concentrated loads, moments, or linearly varying distributed loads, Fig. I 3.1. The shaft may be hollow, and the inner and outer diameters may vary stepwise or linearly, Fig. I 3.2

![Figure I 3.2 Shaft Schematic Showing Stepwise and Linear Diameter Variation](image)

*This Appendix is based upon the original work [1]
The shaft may be supported by bearings at up to five locations. The bearings may take force and/or moments and they may be initially displaced \((\delta y_0, \Theta_{a_0})\).

![Figure I 3. Schematic-Shaft Supports](image)

The bearing subprograms calculate the reaction forces and moments on the shaft from the bearings as functions of bearing deflections. The bearing deflections can be looked upon as shaft support displacements, \((\delta y, \Theta)\).

I 3.2 SHAFT BEARING EQUATIONS

The shaft reaction at any location \(i\) is calculated as the shaft reaction, \(F_{oi}\), at \(i\) when all additional displacements of the shaft supports are zero, plus the additional reactions at \(i\) caused by all additional displacements, or bearing deflections:

\[
F_i = F_{oi} + \sum_{j=1}^{N_i} \frac{\partial F_i}{\partial \Theta_j} \Theta_{j_i} + \sum_{j=1}^{N_i} \frac{\partial F_i}{\partial \Delta y_j} \delta y_j \quad (I 3.2)
\]

The reactions \(F_o\) are calculated using the shaft program with the given initial displacements and spring constants. The constant derivatives \(\partial F/\partial \Theta\) and \(\partial F/\partial \Delta y\) are calculated by introducing one initial displacement at a time. If the bearing mounts are rigid, i.e. spring constants are zero, then the reaction at location \(i\) depends only on the deflections at \(i-1\), \(i\), and \(i+1\). The other derivatives are zero. The present program uses the shaft program this way, and has not yet been extended to include all derivatives. Thus the influence of housing elasticity is not taken into account.

I 3-3
I 3.3 METHOD OF SOLUTION

The object is to calculate the shaft deflections and slopes at the bearing locations along the shaft. The shaft may be arbitrarily loaded and have arbitrarily varying cross-section. The supports may take force or moment load, they can be initially displaced. These deflections and slopes are found by going through the following steps:

Step 1. Take away all supports (support and bearing are synonymous) except two. In the following, these two supports are called the initial supports. In the steps to follow, slopes, deflections and initial displacements refer to a coordinate system with the X-axis along the center-line of the unloaded shaft when it is supported by the initial supports. The relation between this coordinate system and the original system is given by the coordinates \( \xi_0 \) and \( \psi_0 \) of the left end of the shaft as defined in the original coordinate system.

The problem is now statically determinate and the reactions of the initial supports are calculated.

Step 2. Define a new coordinate system. Fix the left end of the shaft in that system so that deflection and slope are zero at that end. Let the load acting on the shaft still be the external load plus the reactions from the initial supports as calculated in Step 1.

Since these reactions were obtained from equilibrium equations of the shaft, there is no reaction at the fixed end. Therefore, all shaft loads are known and the moment \( M \) of the shaft can be calculated at any section \( X \)
from an equation of equilibrium of the shaft to the left of the section. The slopes and deflections are calculated from the Moment-Area theorems by numerical integration of the following equations.

\[ \theta(x) = \int_0^x \frac{M(x)}{EI(x)} \, dx \]  
\[ \delta(x) = \int_0^x \frac{M(x)(x-x)}{EI(x)} \, dx \]  

**Step 3.** Put the shaft back on the initial supports and correct the slopes and deflections accordingly. The resulting calculated slopes and deflections are denoted by \( \Delta_c \).

**Step 4.** The reaction forces and moments, here called \( F_i \) at all supports except the initial supports are now introduced. Their magnitude is determined from the condition that the deflection at each support location goes back to zero + initial displacement, \( \Delta_0 \).

\[ F_i \beta_{ii} + \cdots + F_i \beta_{ij} + \cdots + F_{Ni} \beta_{Ni} = -\Delta_i - \Delta_i \]  

where

- \( \beta \) are influence coefficients
- \( \beta_{ij} \) is the deflection or slope at location i from a unit force or moment at location j.

The influence coefficients are obtained by introducing a unit force or moment at one support at a time and then calculating the deflections and slopes starting from Step 2.

There is one equation for each of the n bearings except the two initial supports. The linear system of equations
can be written
\[ A \cdot F = B \]  \hspace{1cm} (I 3.6)

where \( A \) is a matrix \( N_b \times N_b \)
\[ A_{ij} = \beta_{ij} \]

\( F_b \) = the vector of unknown reactions
\[ B_i = -\Delta c_{i1} - \Delta o_i \]  \hspace{1cm} (I 3.7)

The reactions are obtained by solving this system of equations.

**Step 5.** Calculate the reactions at the initial supports again, this time with both the external loads and the calculated reactions acting on the shaft.

**Step 6.** The deflections and slopes are calculated again as in Step 2, this time with external loads and all reactions acting on the shaft.

**Step 7.** The deflections are expressed in the original coordinate system by addition of \( \delta v_o \) to the lateral deflection and \( \Theta z_o \) to the slopes.
CONCENTRATED CONTACT CALCULATIONS *

I 4.1 INTRODUCTION

This section deals with the calculation of the forces which develop in rolling element-raceway contacts in ball and cylindrical roller bearings.

I 4.2 COORDINATE SYSTEMS

Consider a plane through the bearing axis and the center of a rolling element Fig. 2.4. The position of the center of the rolling element relative to the outer ring is described as a coordinate system fixed to the rolling element center, having its x-axis parallel to the bearing X-axis and y-axis radially outward from the bearing axis.

I 4.3 BALL RACEWAY FORCES

In a ball bearing, the outer and inner ring groove curvature centers EC and IC, are defined by the vectors $\mathbf{g}_1$ and $\mathbf{g}_2$, respectively, Fig. 4.1 and 4.2.

The normal force $Q$ is obtained from the Hertz theory for bodies in contact under load.

$$Q = K_1 \delta_{el}^{3/2} \quad (I~4.1)$$

where $K_1$ = a function of the initial geometry and of the material properties.

$\delta_{el}$ = the elastic deflection

*This Appendix is based upon the original work {1}
The elastic deflection at one ring is obtained from the distance between the ball center and the groove curvature center, that is \(|\mathcal{G}_1| - \mathcal{G}_2|\). When the elastic deflection is zero, this distance is \(r_g - 0.5D\) where \(r_g\) is the groove curvature radius. When there is elastic deflection, this distance is greater by the amount \(\delta_{e\theta}\) of the elastic deflection. Thus,

\[
\delta_{e\theta,m} = |\mathcal{G}_m| - (r_g - 0.5D), \ m = 1, 2
\]

The dimensions of the contact ellipse are obtained from the same theory,

\[
a = K_2 Q^{1/3} \tag{I 4.3}
\]

\[
b = K_3 Q^{1/3} \tag{I 4.4}
\]

where \(a\) = the major half-axis

\(b\) = the minor half-axis

\(K_2\) and \(K_3\) = functions of the initial geometry and of the material properties

\[
\mathcal{G}_1 = \begin{pmatrix} -x_1 \\ -y_1 \\ 0 \end{pmatrix} \tag{I 4.5}
\]

\[
\mathcal{G}_2 = \mathcal{G}_1 + \mathcal{G}_0 + \mathcal{G}_\Delta
\]
FIGURE 14.1
Ball Bearing Geometry
FIGURE I 4.2
Ball Coordinate System Showing Ball Center Position Vectors
where \( \mathbf{g}_o = \) a vector from ED to the original position of IC\(_0\).

\( \mathbf{g}_o = \) a vector between IC\(_0\) and IC

\[
\mathbf{g}_o = \begin{pmatrix}
g_o \sin \alpha_0 \\
g_o \cos \alpha_0 \\
0
\end{pmatrix}
\]

(I 4.6)

where

\[
\mathbf{g}_o = \mathbf{g}_1 + \mathbf{g}_2 - \mathbf{D} - \mathbf{\delta}_o.
\]

\( \mathbf{\delta}_o \) = the total clearance at one ball in the direction of \( \mathbf{g}_1 \)

\[ G \] is the displacement of the inner ring groove curvature center at a ball from original position to loaded position in the bearing coordinate system. \( \mathbf{g}_1' \) is the vector \( G \) in the rolling element coordinate system.

\[
\mathbf{g}_1' \cdot \mathbf{\Pi} \cdot \mathbf{G}
\]

(I 4.7)

The vector \( \mathbf{g}_1' \) is the projection of \( \mathbf{g}_1' \) on the x-y plane, i.e. \( \mathbf{g}_1' \) with the z-component equal to zero.
\( T \) is the rotational transformation matrix which transforms forces and moments described in the bearing coordinate system to their equivalents in the rolling element coordinate system.

\[
T = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi & \cos \phi
\end{bmatrix}
\]  

(I 4.8)

where \( \phi \) is the rolling element azimuth angle described in Fig. 2.4.

The vector \( G \) is obtained from the bearing linear deflection vector \( D \) and angular deflection vector \( \Theta \)

\[
G = D + \Theta \times R_{lc}
\]

(I 4.9)

where

\[
D = \begin{pmatrix}
\delta_x \\
\delta_y \\
\delta_z
\end{pmatrix}, \quad \Theta = \begin{pmatrix}
0 \\
\Theta_y \\
\Theta_z
\end{pmatrix}, \quad R_{lc} = \begin{pmatrix}
0 \\
R_{lc} \cos \phi \\
R_{lc} \sin \phi
\end{pmatrix}
\]

I 4.4 ROLLER RACEWAY FORCES

In a roller bearing the position \( x_1 = 0 \) and \( y_1 = 0 \) for the roller center is defined as the position of the roller center when it is centrally located in relation to the raceway in the \( x \)-direction, and when the roller under no load and no misalignment touches the outer raceway, Fig. I 4.3.
FIGURE 4.3
Roller Bearing Geometry and Roller Coordinate System
To obtain the normal forces on the roller, it is thought of as divided into $n_k$ slices. The force on each individual slice is calculated separately, as though the entire length of the roller was deflected to the amount which obtains at the actual slice. Shear forces between slices are neglected. The relation between deflection and force is given by the following formula,

$$Q = K_4 \cdot \frac{Q}{l_{re}} \cdot l_{re}^{10/3}$$  \(1.4.10\)

where

- $Q$ = roller load
- $l_{re}$ = roller length
- $\delta_{el}$ = elastic deflection

The load per unit length of the contact is $q$.

$$q = \frac{Q}{l_{re}}$$  \(1.4.11\)

The slice load, $Q_k$, is obtained from:

$$Q_k = q \cdot \delta_{el} = \frac{Q}{l_{re}} \cdot \delta_{el} = \frac{Q}{l_{re}} \cdot \frac{K_4 l_{re}}{l_{re}} \cdot \delta_{el} \cdot \frac{\Delta l_{re}}{\delta_{el}} = \frac{K_4 l_{re}}{n_k} \cdot \delta_{el} \cdot \frac{\Delta l_{re}}{\delta_{el}}$$  \(1.4.12\)

At the outer race, the deflection $\delta_{el}$ at slice $k$ is the difference between the normal approach $\delta_n$ and the crown drop $\delta_{ck}$. See Fig. 3.3.

$$\delta_{el,k} = \delta_{nk} - \delta_{ck}$$  \(1.4.13\)
where $\delta_{c1k}$ = sum of roller and race crown drops

$$\delta_{nk} = (-g_i + \Theta \times \vec{r}_{\kappa}) \cdot \hat{y}$$

(I 4.14)

$\hat{y}$ = a unit vector in the y-direction

$$g_i = \begin{pmatrix} -x_i \\ -y_i \\ 0 \end{pmatrix} \quad \Theta = \begin{pmatrix} 0 \\ \Theta_y \\ \Theta_z \end{pmatrix}$$

The vector $\vec{r}_{\kappa}$ from the rolling element center to the center of slice $k$ in the roller raceway contact is defined as follows:

$$\vec{r}_{\kappa} = \begin{pmatrix} x_{\kappa} \\ D/2 \\ 0 \end{pmatrix}$$

(I 4.15)

$$x_{\kappa} = -\ell_{re}/2 + (k - 0.5) \cdot \ell_{re}/n_{\kappa}$$

At the inner race, the deflection at slice $k$ is:

$$\delta_{e12k} = -\delta_{nk} + (D + \Theta \times \vec{R}_{ck}) \cdot \hat{y} - \delta_0 - \delta_{c2k}$$

(I 4.16)

where the vector from the inner ring center to the contact at the inner race, $\vec{R}_{ck}$ is

$$\vec{R}_{ck} = \begin{pmatrix} x_{\kappa} \\ R_{ic} \cos \phi \\ R_{ic} \sin \phi \end{pmatrix}$$
Since the roller rotates only about its geometric axis, the angular alignment of the roller may be described in terms of the orthogonal components of the roller rotational speed \( \omega \) as follows:

\[
\begin{align*}
\theta_y &= \tan^{-1} \left( -\frac{\omega_z}{\omega_x} \right) - \frac{\omega_z}{\omega_x} \\
\theta_z &= \tan^{-1} \left( \frac{\omega_y}{\omega_x} \right) - \frac{\omega_y}{\omega_x}
\end{align*}
\]  

(I 4.17)  

(I 4.18)

4.5 SUMMATION OF CONTACT TRACTION FORCES

Of all the forces which develop within various concentrated contacts, only the elastic Hertz force in a ball raceway contact may be treated as a point force. The elastic forces which develop in a roller raceway contact and both ball and roller traction forces cannot be treated as point forces since they are not necessarily symmetric. Within the extremely small areas which develop in concentrated contacts, the variation in surface pressure is extreme. Despite the small area, significant variation in rolling element-raceway, relative surface velocities may occur within the contact. The acute pressure variations plus the changes in surface velocities necessitate that concentrated contact areas be subdivided into smaller elemental areas. Over the latter the assumptions of a circular pressure distribution and constant relative sliding velocity closely approach the physical situation.

For the roller raceway contact, the same contact sub-areas used to calculate the elastic force distribution are subsequently used to evaluate the traction force distribution. For the ball raceway contact, the sub-areas consist of slices perpendicular to the contact major axis.
14.5.1 VECTOR TO THE CONTACT CENTER FROM THE ROLLING ELEMENT CENTER

For roller bearings the vector $\mathbf{r}$ has been given previously.

In a ball bearing the vector $\mathbf{r}$ is required to calculate the ball raceway friction forces and is given by its components $n$ and $t$. See Fig. 14.4.

$$\mathbf{r} = n \hat{\mathbf{n}} + t \hat{\mathbf{t}}$$

$$\hat{\mathbf{n}} = \begin{pmatrix} \pm \sin \alpha \\ \pm \cos \alpha \\ 0 \end{pmatrix}$$

$$\hat{\mathbf{t}} = \begin{pmatrix} \mp \cos \alpha \\ \pm \sin \alpha \\ 0 \end{pmatrix}$$

The upper and lower signs apply for outer and inner ring contacts respectively.

The radius of the deformed surface, Fig. 14.5, is given by Eq. 14.20.

$$r_{def} = \frac{D}{2} \cdot \frac{2r_e}{r_2 + D/2}$$

$$n = (r_{def}^2 - t^2)^{1/2} - n_0$$

$$n_0 = (r_{def}^2 - a^2)^{1/2} \left( (D/2)^2 - a^2 \right)^{1/2}$$

The coordinates $t$ and $n$ are used with the other components of the ball rotational speed to calculate the ball surface velocity components.

14.5.2 SUMMING THE TRACTION FORCE OVER A CONCENTRATED CONTACT AREA

In computing the traction force acting in a contact, the high pressure contact region is divided into a number of slices parallel to the minor axis. In a roller raceway contact, these slices are the same as those used to calculate the normal force. In a ball raceway contact the total contact length as determined by the Hertz analysis is divided into approximately 21 slices. The tractive force is computed for each slice and then summed to give the total.
FIGURE I 4.4
Ball Coordinate System Showing Ball-Race Contact Position Vectors
FIGURE I 4-5

Ball Race Deformed Contact and Deformed Surface Radius
Figure 4.6a shows a Hertzian contact area with the semi-elliptical distribution of pressure due to elastic deformation (effects of the lubricant on the pressure distribution are neglected) and a local coordinate system established at the contact center ($t = \omega$).

By considering a sufficient number of slices the variation of pressure in the x-direction over a slice width may be neglected, i.e. each slice is regarded as the contact zone due to a cylindrical disk (without edge effects).

Sliding velocities at a typical race slice are shown in Fig. 4.6b. A sliding velocity in the x-direction results if the ball rotational vector has a component $\omega_z$. The sliding velocity $u_{sx}$ is always equal for all slices.

Because of groove curvature the sliding velocity component $u_{sz}$ will vary from slice to slice across the contact ellipse.

Figure 4.6c shows the traction components $T_x$ and $T_z$ on the slice. $T$ indicates the resultant for the given slice. The forces $T_x$ and $T_z$ are computed for each slice and summed to give the components of the total traction force acting at the contact.

Additionally, during the integrations, friction moments of the tractions about the rolling element center are obtained, using the radius vector coordinates $n$ and $t$. 
FIGURE I 4.6
Calculation of Traction Force Components

a. CONTACT ELLIPSE AND PRESSURE DISTRIBUTION

b. SLIDING VELOCITIES ON TYPICAL SLICE

c. TRACTION COMPONENTS AND THEIR RESULTANTS AT A TYPICAL SLICE
APPENDIX 15

LUBRICANT PROPERTY AND EHD FILM THICKNESS MODELS
I 5.1 LUBRICANT PROPERTY MODELS

Many of the calculations performed by computer program SHABERTH require that the viscosity $\eta$ and the pressure-viscosity coefficient $\alpha$ be known at a given temperature.

Accordingly, the program employs subroutines which, when given lubricant kinematic viscosity $\nu$ at 100°F (37.78°C) and 210°F (98.89°C), density $\rho$ at 60°F (15.56°C) and the thermal coefficient of expansion, determine the lubricant density, viscosity, and the pressure-viscosity coefficient at any temperature required.

The kinematic viscosity $\nu$ (cs) at atmospheric pressure is calculated at a given temperature $t$ (°F) from Walther's relation \cite{13}

$$\log_{10} \log_{10} (\nu + 0.6) = A - B \log_{10} (t + 460) \quad (I 5-1)$$

where $A$ and $B$ are constants determined by substituting the known values of $\nu$ at $t = 100°F$ and $t = 210°F$ into Eq. (I 5-1) and solving the two equations which result for $A$ and $B$.

Having calculated $\nu$ at a specific $t$, $\eta$ is computed as

$$\eta = \nu \rho \quad (I 5-2)$$

* This Appendix is based upon the original work \cite{9}.
### Table I 5-1

**LUBRICANT PROPERTIES OF FOUR OILS USED IN PROGRAM SHABERTH**

<table>
<thead>
<tr>
<th>Oil No.</th>
<th>Oil Type</th>
<th>Kinematic Viscosity, 100°F (cs)</th>
<th>Kinematic Viscosity, 210°F (cs)</th>
<th>Walther Equation Constants A</th>
<th>Walther Equation Constants B</th>
<th>Density @ 60°F (gm/cm³)</th>
<th>Thermal Conductivity Kf (Btu/Hr/Ft/°R)</th>
<th>Thermal Coeff. Viscosity Temp. Coeff. (°R⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mineral Oil</td>
<td>64</td>
<td></td>
<td>8.0</td>
<td>10.349</td>
<td>3.673</td>
<td>0.8800</td>
<td>0.0671</td>
</tr>
<tr>
<td>2</td>
<td>MIL-L-7808G</td>
<td>12.8</td>
<td></td>
<td>3.2</td>
<td>10.215</td>
<td>3.698</td>
<td>0.9526</td>
<td>0.0879</td>
</tr>
<tr>
<td>3</td>
<td>C-Ether</td>
<td>25.4</td>
<td></td>
<td>4.13</td>
<td>11.452</td>
<td>4.113</td>
<td>1.201</td>
<td>0.0690</td>
</tr>
<tr>
<td>4</td>
<td>MIL-L-23699</td>
<td>28.0</td>
<td></td>
<td>5.1</td>
<td>10.207</td>
<td>3.655</td>
<td>1.010</td>
<td>0.0879</td>
</tr>
</tbody>
</table>
where \( \rho \), the lubricant density at temperature \( t \) is given by,

\[
\rho(t) = \rho(60^\circ F) - G(t - 60^\circ F)
\]

(I 5-3)

where \( G \) is the lubricant coefficient of thermal expansion.

The pressure-viscosity index \( \alpha \) is defined implicitly by the relation

\[
y(p) = y(p_0) \cdot e^{\alpha p}
\]

(I 5-4)

where \( y(p) \) denotes the viscosity at pressure \( p \) at an arbitrary temperature.

The value of \( \alpha \) itself varies with pressure. The appropriate value of \( \alpha \) to use in the film thickness prediction equations wherein it appears, is the value applicable at the inlet, i.e., at atmospheric pressure.

The value of \( \alpha \) at a given temperature and at atmospheric pressure is calculated by the relation developed by Fresco,

\[
\alpha = (2.303) \cdot 10^{-4} [C + D \log_{10} y + E(\log_{10} y)^2](t + 460)(\text{in.}^2/\text{lb})
\]

(I 5-5)

wherein \( y \) is evaluated at temperature \( t \) (°F) and \( C, D, \) and \( E \) are constants tabulated by Fresco as a function of \( S = 0.2B \) where \( B \) is the coefficient in Walther's equation.
Another lubricant property that appears in the model for film thickness reduction due to inlet heating is the temperature viscosity coefficient $\theta$, that appears in Reynolds' exponential temperature - viscosity relationship.

$\theta$ is computed from the viscosity values at $t = 100^\circ F$ and $210^\circ F$ as follows,

$$\theta = 0.00909 \ln \frac{\eta(t = 100^\circ F)}{\eta(t = 210^\circ F)}$$  \hspace{1cm} (I 5-6)

Relevant lubricant properties for the oils whose properties have been preprogrammed in SHABERTH are listed in Table I 5-1. These property values have been supplied by the manufacturers.

I 5.2 LUBRICANT FILM THICKNESS

The elastohydrodynamic (EHD) film thickness, $h$, at each contact is computed as the product of the film thickness predicted by the Archard-Cowking 20 (point contact) or by the Dowson-Higginson 21 (line contact) formulas and two reduction factors $\phi_t$ and $\phi_s$. The factors $\phi_t$ and $\phi_s$ account respectively for the reduction in film thickness due to heating in the contact inlet and the decrease in film due to lubricant starvation, i.e., due to the finiteness of the distance between the contact zone and the inlet oil meniscus. In equation form,

$$h = \phi_t \cdot \phi_s \cdot \{ h_{A.C.} \} \quad \text{D.H.}$$  \hspace{1cm} (I 5-7)
The Archard-Cowking and Dowson Higginson film thickness formulas take the following forms respectively.

\[ h_{A.C.} = 2.04 \left[ 1 + \frac{2R}{3R_y} \right]^{-0.93} (\alpha \eta V)^{0.740} R^{0.407} (Q/E')^{-0.074} \]  \hspace{1cm} (I 5-8)

\[ h_{D.H.} = 1.6.R_x^0.6 \left( \alpha E \right)^{0.4} \left( \frac{\eta V}{2E'R_x} \right)^{0.4} \left( \frac{q}{E'R_x} \right)^{0.13} \]  \hspace{1cm} (I 5-9)

- \( R_x, R_y \) - effective radii of curvature parallel and transverse to the rolling direction respectively
- \( \alpha \) - pressure viscosity coefficient
- \( V \) - lubricant entrainment velocity
- \( R = \left[ R_x^{-1} + R_y^{-1} \right]^{-1} \)
- \( q \) - maximum load per unit length
- \( Q = \) load
- \( E' = 2 \left[ \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right]^{-1} \)
- \( \eta \) - absolute viscosity
- \( E_1, E_2 \) - Young's modulus for the contacting bodies
- \( \nu_1, \nu_2 \) - Piosson's ratio for the contacting bodies
5.2.1 Inlet Heating Factor $\phi_t$

A Grubin type inlet film thickness analysis considering full thermal effects was developed for line contact by Cheng [10]. Results presented in [10] covering wide ranges of loads, speeds, and lubricant parameters were used to develop regression formulas for the thermal reduction factor $\phi_t$. Based on 28 sets of data, each containing 15 data points, the regression formulas obtained take the following form:

$$\phi_t = e^{x_o}$$  \hspace{2cm} (I 5-10)

where

(1) $x_o = -0.3011 - 0.00432 \ln \left(\frac{p_o}{E'}\right) - 0.03469 \ln (1 + S)$

$$-0.16423 \ln Q_m - 0.01728 (\ln Q_m)^2 + 0.00389 \ln \alpha'$$

$$- 0.06316 \ln \beta'$$

for $0 < Q_m < 0.1$, $\beta' > 11.5$ and $0 < Q_m < 0.4$, $\beta' < 11.5$

(2) $x_o = -1.119304 - 0.16192 \ln \left(\frac{p_o}{E'}\right) - 0.0895 \ln (1 + S)$

$$- 0.29 \ln Q_m - 0.04572 (\ln Q_m)^2 + 0.13615 \ln \alpha'$$

$$- 0.31615 \ln \beta'$$

for $0.1 < Q_m < 1$, $\beta' > 11.5$ and $0.4 < Q_m < 1$, $\beta' < 11.5$

(3) and $x_o = -3.66426 - 0.48511 \ln \left(\frac{p_o}{E'}\right) + 0.00568 \alpha'$

$$- 0.05491 \beta' - 0.1678 \ln (1 + S) - 0.19573 \ln \alpha_m$$

$$- 0.09392 \ln (\ln Q_m) + 0.20908 - \ln \alpha'$$

for $Q_m > 1$

where, $\alpha' = \frac{\pi'}{2} \alpha p_o$  \hspace{2cm} I 5-7
\[ \phi' \approx \phi_t \]
\[ S = (u_2-u_1)/2V \]
\[ Q_m = 2 V^2/K_f \tau_0 \]

**\( \alpha \)** - pressure viscosity coefficient, in \( 2/lb. \)

**\( b \)** - half of Hertzian width in the rolling direction, in

**\( p_0 \)** - maximum Hertz pressure, \( lb/in^2 \)

**\( \theta \)** - temperature viscosity coefficient, \( oR^{-1} \) computed via Eq. (4-6)

**\( t \)** - ambient temperature, \( oR \)

**\( \eta \)** - ambient viscosity, \( (lb.sec/in^2) \)

**\( u_1, u_2 \)** - surface velocity of bodies No. 1 and 2, (relative to the contact) \( in/sec \)

**\( K_f \)** - conductivity of the film \( (lb/°F.sec) \)

**\( V \)** - \( (u_1+u_2)/2 \), lubricant entrainment velocity \( (in/sec) \)

It is noted that for small values of \( Q_m \), \( \phi_t \) computed from Eq. (I 5-10) may be larger than unity. \( \phi_t = 1.0 \) is used whenever the value computed using Eq. (I 5-10) is larger than 1.0.

In evaluating \( \phi_t \) for the elliptical point contacts in a ball bearing, \( p_0 \) is taken to be the maximum of the Hertzian pressure ellipse.

The point contact is thus treated as if it were a line contact having a maximum contact pressure \( p_0 \) along its entire length of contact. This is a conservative approximation inasmuch as it will tend to underestimate \( \phi_t \) and hence underestimate film thickness.
The magnitude of the error resulting from this approximation is small as \( p_0 \) is not a highly influential variable in the expression for \( \phi_t \).

### 5.2.2 Starvation Reduction Factor \( \phi_s \)

A hydrodynamic analysis of an elliptical point contact having two equivalent principal radii of curvature \( R_x \) and \( R_y \) (parallel and transverse to the rolling direction, respectively) is used for calculating \( \phi_s \). Finite thickness of the half films \( (h_{1,1} \text{ and } h_{1,2}) \) upstream from the inlet are set. The flow rate in the center plane at the meniscus line is set equal to the incoming flow rate at the contact centerline. The complete analysis is given in \( \{11\} \) and \( \{23\} \).

Figure I 5-1 shows the geometry considered. The starved plateau film thickness is \( h_s \) and the meniscus distance from the contact center along the direction of rolling is \( r^* \). The ambient film layers move toward the contact zone with velocities \( u_1 \) and \( u_2 \).

The analysis shows \( h_s \) and \( r^* \) to be related to \( h_1 \), the sum of the upstream ambient film layer thicknesses, \( (h_1 = h_{1,1} + h_{1,2}) \) through the following two equations:

\[
5.5V y \propto \frac{R_x^{1/2}}{h_s^{3/2}(3+2k)} - \frac{12V y \propto r^*}{(3+2k)(h_s^+ r^*/2R_x)^2} = 1.0 \quad (I 5-11)
\]

\[
\text{and } h_1 = \frac{2(2+k)}{3+2k} \frac{h_s^+ kr^2}{(3+2k)R_x} = \quad (I 5-12)
\]
FIGURE I 5-1
Film Geometry

\[ h_1 = h_{1,1} + h_{1,2} \]
where $k = R_x/R_y$, $\eta$ is the viscosity of oil at the contact inlet, $\alpha$ the pressure-viscosity coefficient and $V = (u_1 + u_2)$ is the entrainment velocity in the x-direction.

The simultaneous solution of Eqs. (I 5-11) and (I 5-12) for given values of $R_x$, $R_y$, $V$, $\eta$, $\alpha$ and $h_1$ yields the associated values $h_s$ and $r^*$. Subroutine STARFC performs this calculation.

As the meniscus distance $r^*$ increases, the film thickness $h_s$ increases, asymptotically approaching a value $h_f$ as $r^* \rightarrow \infty$. Therefore, $h_f$ is the film thickness under fully flooded conditions.

On letting $r^* \rightarrow \infty$ and $h_s \rightarrow h_f$ in Eq. (I 5-11), the second term on the left hand side vanishes and one may solve for $h_f$ as

$$h_f = \left[ \frac{5.5V\eta \alpha (R_x)^{1/2}}{(3+2k)} \right]^{2/3} \quad (I 5-12)$$

Note that for line contacts, the contact curvature ratio $k$ is set equal to a small value.

$$K = R_x/R_y = 0.01.$$  

This $k$ value results in a contact length to width ratio of 18 : 1.

The lubricant meniscus distance in the rolling element-cage contact is assumed to be proportional to the rolling element radius.

$$r^* = 0.25 \cdot r$$
Subroutine STARFC evaluates $h_f$ from Eq. (I 5-13) and then calculates the ratio $\phi_s = h_s/h_f$. As given by Eq. (I 5-13), it does not indicate a dependence upon load. This is characteristic of film thickness formulas derived from a Grubin type assumption applied to rigid bodies. It is considered preferable to use the Archard-Cowking and Dowson Higginson formulas for the unstarved cases rather than $h_f$ since these formulas better describe the dependence of film thickness upon the influential physical variables. The only role played by $h_f$ is to scale $h_s$ to yield the ratio $\phi_s$, which is applied as a multiplicative factor on the Archard-Cowking and Dowson Higginson predictions of unstarved film thickness.

5.3 Film Replenishment

As noted above, it is necessary to know the combined oil layer thickness $h_1$ to calculate $\phi_s$ and $r^*$. As a rolling element passes a point on the inner or outer raceway of a bearing, a very thin lubricant film remains on each of the components and is of the same order of magnitude as half the plateau film thickness in the EHD contact. Replenishment of the lubricant layer on the raceway is required in order to assure sufficient lubricant in the inlet region of the succeeding contact, so that the EHD film thickness will be the same as in the preceding contact. If replenishment fails to occur, each successive rolling element pass would have a thinner EHD film and steady state operation with EHD lubrication would not be possible.
Many mechanisms serve to replenish the lubricant in the track of a high speed bearing. Of prime concern is replenishment at the inner race in high speed ball bearings since centrifugal force tends to direct free fluid away from that surface. Seven replenishment mechanisms have been identified:

1. Centrifugal flinging of the lubricant from the ball.
2. Centrifugal travel of oil along the surface.
3. Random splashing of lubricant in the bearing cavity.
4. Direct deposition from a jet.
5. Back flow along the surface into the track, from its edges resulting from lubricant surface tension.
6. Carrying into the contact, of lubricant adhering to the ball.
7. Back flow into the gap behind the contact exit due to vacuum in the cavitated area.

A model exists for item (5) above. In view of the other, possibly more influential sources of replenishment enumerated above, this model has not been adopted in this program. Instead, it is assumed that an externally supplied replenishment, amount \( \Delta \), adds to the plateau film thickness to yield \( h_1 \).

As a simplification, since \( \Delta \) is usually much larger than \( h \), the approximation \( h \sim h_s \) is used so that,

\[
 h_1 = h_s + \Delta 
\]  

(I 5-13)

Subroutine STARFC uses Eq. (I 5-13) in solving Eqs. (I 5-10) and (I 5-11), with a user specified \( \Delta \) value.

Criteria for estimation of \( \Delta \) are presented in Sect. 3.3.15.
APPENDIX I 6

TRACTION AND INLET FRICTION CALCULATIONS
APPENDIX I 6

TRACTION AND INLET FRICTION CALCULATIONS

I 6.1 INTRODUCTION

The traction model developed for use in program SHABERTH is applicable to the partial EHD regime in which the lubricant film separating the contacting surfaces may be small enough to permit some degree of asperity contact. The model computes the traction coefficient as a function of the ratio $(h/c_f)$, film thickness $h$ to composite surface roughness $c_f$. For small values of $h/c_f$ ($h/c_f < 0.04$) the model represents dry friction. For large values of $h/c_f$ ($h/c_f > 3$) the model becomes a non-Newtonian semi-empirical fluid film model in which the traction coefficient, $\mu_{EHD}$, depends upon sliding rate, as well as the load, rolling speed and lubricant properties at operating temperature. For intermediate values of $h/c_f$ the model is a combination dry and fluid film friction. Essential features of the model are described below.

A single, two dimensional, functional relationship is used to model aspects of both wet and dry friction. The function is plotted in Fig. I 6-1 and is valid over the range.

\[ 0 \leq x^* < \infty \]

and has the following characteristics.

\[ y^* = \frac{Y_b}{X_b} x^* \quad 0 \leq x^* \leq Y_b \quad (I 6-1) \]

\[ y^* = Y_b (1 + X_b - X^*) / (Y_b + X_b - Y_b - X_b) \quad (I 6-2) \]

where:

\[ X_b = Y_b - X^* \]

\[ Y_b = 1 - Y_b \]

\[ 0 < Y_b < 1, Y_b \] defines the value of $y^*$ below which $y^*$ is a linear function of $x^*$.

\[ 0 < X_b < \infty, X_b \] defines the value of $X^*$ below which $y^*$ is a linear function of $X^*$.

I 6-2
Note that $y^*$ increases monotonically with $x^*$ and that $\frac{dy^*}{dx^*}$ is continuous at $x^* = x_B$.

Both wet and dry friction coefficients will be defined in terms of Eqs. (I 6.1 and I 6.2).

I 6.2 ASPERITY TRACTION MODEL

If $Q$ is the total load applied to a concentrated contact (or to a suitable sub-element of a contact), a portion of this load, designated $Q_a$, will be carried by elastically deformed asperities and the remaining, $Q - Q_a$, will be carried by the EHD film. The traction force $T_s$ then is,

$$T_s = Q_a + T_{EHD}(Q - Q_a)$$  \hspace{1cm} (I 6.3)

Under the assumption that the rough surface consists of two-dimensional ridges of random height and slope angle, the average asperity borne load $Q_a$ is the following function of the ratio $h/\varphi$.

$$Q_a = \frac{F_1'}{4\pi A} \varphi \Phi^I(h/\varphi, \alpha)$$  \hspace{1cm} (I 6.4)

where

- $F_1'$ = $2 \left[ \frac{1 - \frac{E_2}{E_1} - \frac{1 - \frac{E_2}{E_+}}{2} }{E_2} \right]$
- $F_1', E_2$ = Young's moduli of the contacting bodies
- $A$ = contact area
- $\varphi$ = RMS value of the distribution of asperity slope angles (radians)
- $\Phi^I(h/\varphi, \alpha)$ = function defined in (121 of the film parameter $h/\varphi$ and a statistical micro-geometry parameter $\Phi$ defined in Nayak (26).

It is shown in (27) that $\alpha = 2$ is a reasonable value to use for rolling bearing surfaces. The following polynomial fit to the function $I(h/\varphi, \alpha = 2.0)$ is used in computation:

$$I(h/\varphi, 2) = 2.31 e^{-1.84 h/\varphi} + 0.1175(h/\varphi - 0.4)^2(2-h/\varphi)$$  \hspace{1cm} (I 6.5)

$0.4 \leq h/\varphi \leq 2$

$$I(h/\varphi, 2) = 17 e^{-2.84 h/\varphi} + 1.44 \times 10^{-4} (h/\varphi - 2)^{1.1}(4-h/\varphi)^{7.8}$$

$2 < h/\varphi$

When $h/\varphi < 0.4$ the asperities are assumed to carry the entire contact load.
The asperity friction coefficient ($\mu_a$) is calculated as follows:

$$\mu_a = M^* \gamma^*$$  \hspace{1cm} (I 6.6)

$M^*$ is supplied by the user, 0.1 to 0.2 is the recommended range of values.

where $\gamma^*$ is calculated from Eq. (I 6.1 or I 6.2) in which:

$$\gamma_B = 0.66$$
$$X_B = 0.005$$
$$x^* = \left\{ \begin{array}{l}
2(U_2 - U_1) \\
U_2 + U_1
\end{array} \right. \hspace{1cm} \text{calculated, local slide to roll ratio}

U_2$ and $U_1$ are defined in Fig. I 6.4

This relationship is believed to better approximate the asperity friction phenomena than the coulomb friction model.

This relationship causes the friction coefficient to be a function of the small tangential displacement of the surface of one body with respect to the other. The slide to roll ratio provides a measure of the displacement.

In the numerical solutions being employed, it is important that the functional relationships produce a unique set of forces for a given set of rolling element rotational speeds. The use of the above asperity friction model helps guarantee this uniqueness.

The value $M^* = 0.1$ was recommended in (277) as being consistent with values deduced in traction measurements in the partial EHD regime.

I 6.3 FLUID TRACTION COEFFICIENT $\mu_{EHD}$

The general behavior of the fluid traction coefficient $\mu_{EHD}$ as a function of sliding rate is illustrated by the curves in Fig. (I 6.2). (Throughout this section the subscript on $\mu_{EHD}$ will be omitted.)
In the curve in Fig. (I 6.2a) the traction coefficient increases linearly, at low sliding speeds, reaches a maximum $\mu = \mu^*$ at speed $u_s = u_s^*$, and decreases thereafter.

In the curve in Fig. (I 6.2b) the traction coefficient increases linearly at low sliding speeds and then approaches an asymptotic value $\mu^*$. Both types of traction curves have been experimentally observed. Both signify a departure from isothermal Newtonian fluid behavior since for this situation the traction coefficient increases linearly with sliding speed.

For the curve of Fig. (I 6.2b), $\mu^*$ denotes the asymptotic traction coefficient. Define $u_s^* = 3 \times u_{sc}$, where $u_{sc}$ is the sliding speed at which the line $\mu^*$ intersects the extended linear portion of the curve $\mu$ vs. $u_s$.

It has been found that $\mu^*$ for either type of curve increases with the contact pressure and decreases with rolling velocity and lubricant viscosity (and hence ambient temperature); $\mu_s^*$ on the other hand decreases with pressure and increases with rolling velocity and viscosity. This joint variation has been found to result in a scale change in the two axes but not in a substantial change in the character of the traction curve.

This means that traction curves obtained under widely different conditions of pressure, rolling velocity and temperature when plotted on a grid with coordinates $\mu^*_r = \mu^*/\mu^*$ and $x = u_s/u_s^*$, yield substantially the same curve \cite{23,27}.

A characteristic of the curves of the type of Fig. (I 6.2a) is that $\mu^*_r$ decreases indefinitely with large $x$. Curves of this type can cause convergence difficulties in a bearing computer program because there are two sliding speeds associated with each value of the traction coefficient. The solution may cycle.
FIGURE I 6.2
Typical Traction Curves

a) A CURVE EXHIBITING A PEAK

b) A CURVE EXHIBITING ASYMPTOTIC BEHAVIOR
between two values in seeking an equilibrium condition.

An approach to avoiding this difficulty is to use the previously mentioned monotonically increasing curve Eq. (I 6.1 and I 6.2) that is a good match to the actual curve over the increasing portion of that curve. If the program then converges at a sliding speed that is within the range where the curves match, the solution is valid.

In the present context, let \( y^* \) and \( x^* \) from Eq. (I 6.1 and I 6.2) correspond to the relative traction coefficient (\( \mu_R \)) and the sliding speed \( U_s \), respectively.

If we now define \( \mu^* \) as the asymptotic EHD traction coefficient the actual coefficient \( \mu_{EHD} \) is:

\[
\mu_{EHD} = \mu^* + \mu_R \tag{I 6.8}
\]

For a given oil, the maximum traction coefficient \( \mu^* \) has been found \cite{27} to vary with pressure, viscosity, rolling velocity, and film thickness in the following manner:

\[
\mu^* \sim f(p_o)^{0.61} p_o^{(-1.14)} y^{(0.59)} V^{(0.46 - 0.61x)} h^{(-0.48)} \tag{I 6.9}
\]

where

- \( p_o \) = the maximum contact pressure (lb/in\(^2\))
- \( h \) = plateau EHD film thickness (in)
- \( f(p_o) \) = a function governing the dependence of viscosity on pressure \( p_o \) (oil parameter)
- \( x \) = a visco-elastic constant (oil parameter)

Similarly, \( U_s^* \) exhibits the following dependence on \( p_o, y, V \) and \( h \):

\[
U_s^* \sim p_o^{(-0.14)} f(p_o)^{(-0.40)} y^{(-1.1)} V^{(0.40x - 0.09)} h^{(-0.55)} \tag{I 6.10}
\]

The quantities \( \mu^*, \mu_s^* \) and \( h \) can be measured experimentally for a set of given values of \( p_o, y, \) and \( V \). For the oils thusfar examined, the function \( f(p_o) \) has been found to follow a law of the form:

\[
f(p_o) \sim (p_o/p_i)^{A_1} \quad p_o < p_i \tag{I 6.8}
\]

\[
\sim (p_o/p_i)^{A_2} \quad p_o > p_i \tag{I 6.11}
\]
where $A_1$, $A_2$, and $p_1$ are lubricant dependent constants.

Values of $A_1$, $A_2$, and $p_1$ for four oils are listed in Table (I 6.1). The procedure for determining these values for other oils is given in [23, 24].

Making use of Eq. (I 6.11), Eqs. (I 6.9) and (I 6.10) can be expressed in the following form on introducing $C_1$ and $C_2$ as proportionality constants.

$$
\mu^* = C_1 p_o^{-1.14} \left( \frac{p_o}{p_1} \right)^{0.61} A_i \eta (0.59) \sqrt{(0.48 - 0.61 \lambda) - 0.45} \frac{h}{\eta}
$$  \hspace{1cm} (I 6.12)

$$
u_s^* = C_2 p_o^{-0.14} \left( \frac{p_o}{p_1} \right)^{0.4} A_i \eta (-1.1) \sqrt{(0.40 \lambda - 0.09) h^{0.55}}
$$  \hspace{1cm} (I 6.13)

The values of $C_1$ and $C_2$ are evaluated by substituting measured $\mu^*$, $\nu_s^*$, and $h$ values for a specific test condition. Then knowing values of $C_1$, $C_2$, $p_1$, $A_i$ and $h$ (starved), it is possible to predict values of $\mu^*$ and $\nu_s^*$ as functions of the operating parameters $p_o$, $V$, and $\eta$ by Eqs. (I 6.12) and (I 6.13). Values for $C_1$ and $C_2$ thus calculated for four oils are tabulated in Table (I 6.1). The units in Eqs. (I 6.12) and (I 6.13) are $V$ (in/sec), $\eta$ (cp), $p_o$ (ksi) and $h$ (microinches). Also shown in Table (I 6.1) are the values of $(\mu T)_B$ and $(\nu_s T)_B$ used for the four oils. These data have been preprogrammed into SHABERTH.

In summary, the calculation of $m_{\text{EHD}}$ at a given sliding velocity $u_s$ and for a given pressure $p_o$, film thickness $h$, rolling velocity $V$ and temperature $t$, proceeds as follows:

1. Calculate viscosity $\eta$ at temperature $t$.
2. Using appropriate constants from Table (I 6.1), calculate $\mu_s^*$ and $\nu_s^*$ for given $p_o$, $h$, $V$ and $\eta(t)$.
3. Calculate $x = u_s / \nu_s^*$
### TABLE I 6.1

**TABULATION OF CONSTANT FOR FOUR OILS**

<table>
<thead>
<tr>
<th>Oil No.</th>
<th>Oil Type</th>
<th>λ</th>
<th>A₁</th>
<th>A₂</th>
<th>P₁ × 10⁵</th>
<th>C₁</th>
<th>C₂</th>
<th>(M₁)B</th>
<th>(V₂)B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mineral (1) Oil</td>
<td>0.3</td>
<td>3.42</td>
<td>2.14</td>
<td>1.5</td>
<td>1.8</td>
<td>115.9</td>
<td>0.65</td>
<td>0.10</td>
</tr>
<tr>
<td>2</td>
<td>MIL-L-7808 (2)</td>
<td>0.94</td>
<td>4.08</td>
<td>1.48</td>
<td>1.7</td>
<td>17.6</td>
<td>39.0</td>
<td>0.68</td>
<td>0.25</td>
</tr>
<tr>
<td>3</td>
<td>C-Ether</td>
<td>0.70</td>
<td>3.29</td>
<td>1.50</td>
<td>1.7</td>
<td>26.2</td>
<td>6.6</td>
<td>0.65</td>
<td>0.15</td>
</tr>
<tr>
<td>4</td>
<td>MIL-L-23699 (2)</td>
<td>0.93</td>
<td>6.88</td>
<td>3.44</td>
<td>2.2</td>
<td>10.4</td>
<td>47.3</td>
<td>0.68</td>
<td>0.25</td>
</tr>
</tbody>
</table>

**NOTES:**

(1) obtained from Traction Data reported by Johnson and Cameron (28)

(2) S K F tests

(3) obtained from Traction Data reported by Smith et al (29)
4. Use Eq. (6.1 and 6.2) with values of \((\mu_r)_B\) and 
\((us)_B\) from Table I 6.1 to calculate \(\mu_r\) associated with 
the value of \(x\) calculated in Step 3.

5. Compute 
\[ \mu_{EHD} = \mu_r \cdot \mu^* \]

I 6.4 INLET REGION HYDRODYNAMIC FRICTION FORCES

I 6.4.1 ELLIPSOIDAL CONTACT

The contacts between ball and race and between ball and 
cage pockets are "point" contacts. Under lubricated conditions,
the surfaces are separated by a fluid film and there is a 
pressure build-up around the contact caused by the sweeping-in 
motion of the surfaces. This pressure build-up contributes to 
the friction in rolling. Tangential surface forces are required to 
pump the oil into the high pressure zone.

The pumping forces are due to rolling, \((F_R)\), and sliding \((F_S)\).
Expressions as a function of a finite meniscus distance \(r^*\) 
have been found for these forces. The complete analytical development is contained in \(\{11\}\) and \(\{27\}\).

Since these forces arise in the contact inlet region, 
elastic deformation is not considered to have a significant effect 
and the analysis invokes a rigid body assumption.

Figure (I 6-3) shows the relevant geometry. Two rigid bodies 
are shown in nominal point contact separated by an oil film and 
undergoing relative rolling and sliding. A local cartesian co-
ordinate system is established with the \(x-y\) plane parallel to 
the tangent plane of the two bodies and with the origin coincident 
with the surface of body 2. The coordinate system remains fixed 
in the contact as the surfaces of the two bodies move. The prin-
cipal radii of curvature of the two bodies are \((R_x)_i\) and \((R_y)_i\), 
\(i = 1,2\). The equivalent radii are
\[ \varepsilon_{x*} = \left( \frac{1}{\varepsilon_{x1}} + \frac{1}{\varepsilon_{x2}} \right)^{-1}, \varepsilon_{y*} = \left( \frac{1}{\varepsilon_{y1}} + \frac{1}{\varepsilon_{y2}} \right)^{-1} \] 
(I 6.14)
FIGURE 1 6.3
Notation for Rolling Sliding Point Contact
FIGURE I 6.4

Friction Forces on Sliding and/or Rolling Disks

FORCES ON SOLID BODIES

FORCES ON FLUID

I 6-13
The normal separation of points on the two bodies near the origin is given by

\[ h = h_0 + \frac{x^2}{2R_x} + \frac{y^2}{2R_y} \]  

(I 6.15)

where \( h_0 \) is the minimum film thickness at the origin. This separation function is applicable if the width of the Hertzian flat region in the contact is negligible compared to the relevant \( x \) and \( y \) dimensions in the inlet and outlet.

The surfaces are assumed to be moving with velocities \( u_i \) and \( v_i \) \((i = 1, 2)\) relative to the origin in the \( x \) and \( y \) directions. The rolling velocities in the \( x \) and \( y \) directions are defined respectively as \( V_x = (u_1 + u_2)/2 \) and \( V_y = (v_1 + v_2)/2 \) and sliding velocities in the respective directions are \( u_{sx} = u_1 - u_2 \) and \( u_{sy} = v_1 - v_2 \).

\( \Omega \) is the portion of the normal load that is being supported by hydrodynamic forces in the inlet. For elastic ball race contact \( \Omega \) is considered negligible when compared to the load supported over the Hertzian contact zone.

The forces \( F_R \), \( F_S \) and \( F_N \) are displayed in Fig. (I 6.4). For clarity the special case of contact between two disks is illustrated. In this case, \( R_y \) is infinite and the forces are directed along the \( x \) axis. In the general point contact case the forces have both \( x \) and \( y \) components.

The force \( F_R \) acts in the same direction on both contacting bodies and opposite to the direction of motion. \( F_S \) acts in opposite directions on the two bodies in such a way as to tend to increase the speed of the slower body and to decrease the speed of the faster body.

The forces \( F_1 \) and \( F_2 \) are the resultants of the hydrodynamic pressure distribution in the inlet and act through the centers of the two bodies. The component of these forces in the \( y \) direction
represents the (small) portion of the total load supported hydrodynamically. The components \( F_{n1} \) and \( F_{n2} \) acting in the \( x \) direction contribute to the force balance in the rolling direction.

The magnitudes of \( F_R, F_S \) and \( F_n \) depend upon the meniscus location \( r^* \). The calculation of \( r^* \) for the rolling element race and rolling element cage contacts is discussed in Appendix I 5.

Expressions for the \( x \) and \( y \) components of the forces \( F_R, F_S \) and \( F_n \) are given below in terms of the dimensionless quantities \( F_T \) and \( F_s \).

### Pumping Forces

#### Rolling Component

\[
F_{Ry} = \frac{1}{2} C_o \overline{F_R} \cos \gamma \]

(I 6.16)

\[
F_{Rx} = \frac{1}{2} C_o \overline{F_R} (\sin \gamma) (R_y/R_x)^{1/2} \]

(I 6.17)

#### Sliding component

\[
F_{sy} = \overline{F_s} \sqrt{u_{sx}} (R_x R_y)^{1/2} \]

(I 6.18)

\[
F_{sx} = \overline{F_s} \sqrt{u_{sy}} (R_x R_y)^{1/2} \]

(I 6.19)

### Normal Forces (on ball)

\[
F_{ny} = C_o \overline{F_R} \frac{R_x}{r} \cdot \cos \gamma \]

(I 6.20)

\[
F_{nx} = C_o \overline{F_R} \frac{R_y}{r} \cdot (\sin \gamma) (R_y/R_x)^{1/2} \]

(I 6.21)

(For a ring, \( r \) is replaced by the raceway radius) where

\[
C_o = \sqrt{V_x (R_x R_y)} \frac{1}{4} \left[ \frac{(3+2k)^{-2}}{(3+2k^{-1})^{2k-1}} \right] \frac{1}{2} \]

(I 6.22)

\[
\gamma = \tan^{-1} \left[ \frac{3 + 2k}{k^4 (3+2k^{-1})} \cdot \frac{V_x}{V_y} \right] \]

(I 6.23)

\[
k = R_y/R_x \]

\( \eta \) = absolute ambient viscosity
The quantities $\bar{F}_R$ and $\bar{F}_S$ are dimensionless and depend upon two further dimensionless parameters $\bar{A}$ and $\bar{P}_1$. $\bar{P}_1$ is a dimensionless meniscus distance defined as

$$\bar{P}_1 = \left[ \frac{r^*}{(2h_oR_x)^{\frac{1}{2}}} \right] \left[ (\cos^2 \gamma + (1/k)\sin^2 \gamma)^{\frac{1}{2}} \right] \quad (I.6.24)$$

$h_o$ for ball-race contact is taken as the plateau EHD film thickness.

For ball-cage contact $h_o$ is as calculated in Appendix I 8.

$r^*$ is the distance of the oil meniscus from the contact center along the rolling direction.

$\bar{A}$ is the product of the pressure viscosity coefficient of the lubricant and the maximum fluid pressure $q_{\text{max}}$ that prevails if the lubricant is isoviscous.

$\bar{A}$ assumes values between 0 and 1 with $\bar{A} = 0$ indicative of purely hydrodynamic and $\bar{A} = 1$ of purely elasto-hydrodynamic conditions. $\bar{A} = 1$ is taken for ball-race and $\bar{A} = 0$ for ball-cage pocket calculations.

Plots of $\bar{F}_S$ and $\bar{F}_R$ as a function of $\bar{P}_1$ for various values of $\bar{A}$ are given as Figs. (I 6.5) and (I 6.6) taken from [23].

For the EHD ball race contacts the following expressions have been fit to the $\bar{F}_R$ vs. $\bar{P}_1$ curve in Fig. (I 6.6) for $\bar{A} = 1$.

$$\bar{F}_R = 23.59 \text{ in } \bar{P}_1 - 10.1; \quad \bar{P}_1 < 5 \quad (I.6.25)$$

$$\bar{F}_R = 36.57 \text{ in } \bar{P}_1 - 22.85; \quad \bar{P}_1 > 5$$

$\bar{F}_S$ is not considered for a ball-race contact because the amount of sliding is so small.

For the predominantly hydrodynamic contacts, ($\bar{A} = 0$) which arise between the ball and the cage web, both pumping and sliding friction are considered. The following equations were fit to the $\bar{A} = 0$ curves in Fig. (I 6.5) and (I 6.6).

$I 6^*16$
FIGURE I 6.5

Variation of $F_s$ with $\rho_1$
FIGURE 6.6
Variation of $F_R$ With the Dimensionless Meniscus Distance $\rho_1$

$\tilde{\alpha} = 1.0$
$\tilde{\alpha} = 0.5$
$\tilde{\alpha} = 0$

$\tilde{\alpha} = \alpha q_{max}$

$I 6-18$
\[ F_R = 8.53 \times 10^{-14} \left[ \ln(1000 \, \rho_1) \right]^{15.570} ; \quad \rho_1 < 5.4 \quad (I \, 6.26) \]
\[ F_R = 36.576 \ln \rho_1 - 29.32 ; \quad \rho_1 \geq 5.4 \]

\[ F_S = 1.71 \times 10^{-6} \left[ \ln(1000 \, \rho_1) \right]^{11.01} ; \quad \rho_1 < 2 \quad (I \, 6.27) \]
\[ F_S = 10.115 \ln \rho_1^{.965} + 1.5 ; \quad \rho_1 \geq 2 \]

In applying the above results to ball-cage web and ball-race contacts, it is necessary to interpret the geometrical parameters of the general configuration of Fig. (I \, 6.7) in terms of the appropriate bearing dimensions.

Figure I \, 6.7 shows the relevant geometry for the two contact types as well as the direction of the various force components.

In this figure \( r \) denotes the ball radius, \( r' \) the cage pocket radius, \( r_g \) the outer ring groove radius, \( R \) the radius to the center of the contact ellipse and \( \alpha \) the outer ring contact angle.

I \, 6.4.2 INLET REGION HYDRODYNAMIC FRICTION FORCES LINE CONTACTS

A model has been developed for calculating hydrodynamic pumping and sliding friction forces in the inlet regions of roller bearing race contacts wherein film variable viscosity and starvation effects are included. The contacting bodies are assumed to be rigid. The model was developed by Dr. Y. P. Chiu of S K F Industries.

Consider two rigid cylinders of radius \( R_{y1} \) and \( R_{y2} \) moving at surface velocity \( \mathbf{u}_1 \) and \( \mathbf{u}_2 \) as shown in Figure I \, 6-4. Floberg in [30] has obtained expressions for the fluid pressure distribution \( P \), normal rolling friction forces \( F_{n1} \) and \( F_{n2} \) and tangential friction forces \( F_R \) as functions of a dimensionless meniscus distance \( x_{01} = r^*/(R-h_{0f}^W) \) Isoviscosity is assumed. \( h_0 \) is the minimum film thickness and \( R = (1/R_{y1} + 1/R_{y2})^{-1} \).
FIGURE I 6-7  Configuration of Contacts
(a) Ball-Cage Contact
(b) Ball-Race Contact (Outer Ring)

\[ R_y = r, \quad R_x = \frac{1}{r - 1/r'} \]

**FIGURE A**

**FIGURE B**

BEARING AXIS

\[ R_x = \frac{1}{r - 1/r_g}, \quad R_y = \frac{1}{r - \cos \theta / R} \]
Specifically, the maximum fluid pressure $P_{\text{max}}$ is given by

$$P_{\text{max}} = P_0^* \left(2y \cdot uR^2\right) h_0^{-3/2} \quad (I 6.29)$$

where $P_0^*$ is a function of $X_{01}$ as shown in Table I 6.2. The normal friction forces acting through the centers of the two cylinders of length $l$ are

$$\begin{pmatrix} F_{n1} \\ F_{n2} \end{pmatrix} = \begin{pmatrix} \frac{R_1}{R} \\ \frac{R_2}{R} \end{pmatrix} l \bar{P}_{x0} \left(2y \cdot u R^{1/2}\right) \left[h^{1/2} \left(R_1 + R_2\right)\right]^{-1} \quad (I 6.30)$$

The pumping forces are

$$F_R = 0.5 \left(F_{n1} + F_{n2}\right) \quad (I 6.31)$$

where $u = (u_2 + u_1)/2$.

The dimensionless coefficients $\bar{P}_{x0}$, $A$, $B$ and $C$, given in Table I 6.2 below, are functions of $X_{01}$ and were calculated by Floberg.

<table>
<thead>
<tr>
<th>$X_{01}$</th>
<th>$\bar{P}_{x0}$</th>
<th>$A$</th>
<th>$B$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>- 0.954</td>
<td>0.11</td>
<td>2.02</td>
<td>0.176</td>
<td>3.22</td>
</tr>
<tr>
<td>- 1.380</td>
<td>0.25</td>
<td>2.27</td>
<td>0.322</td>
<td>2.92</td>
</tr>
<tr>
<td>- 2.245</td>
<td>0.81</td>
<td>2.72</td>
<td>0.877</td>
<td>2.94</td>
</tr>
<tr>
<td>- 4.600</td>
<td>4.6</td>
<td>3.5</td>
<td>4.485</td>
<td>3.41</td>
</tr>
</tbody>
</table>

For the case that the fluid viscosity increases exponentially with pressure, i.e. $\eta = \eta_0 e^{-\alpha P}$, it is desirable to use the Archard-Snidle approximation $\{31\}$ that the minimum film thickness $h_0$ in an EHL contact is determined from Eq. (I 6-25) by setting $P_{\text{max}}$ equal to the reciprocal of the pressure viscosity coefficient ($\alpha^{-1}$).

Using $\alpha = 1$, one then has,

$$P_0^* = h^{3/2} / \left(2y \cdot u \alpha R^{1/2}\right) \quad (I 6.32)$$

and

$$\begin{pmatrix} F_{n1} \\ F_{n2} \end{pmatrix} = \begin{pmatrix} \frac{R_1}{R} \\ \frac{R_2}{R} \end{pmatrix} C \left(\frac{2y \cdot u \alpha}{R}\right)^{1/2} R/\alpha (R_1 + R_2) \quad (I 6.33)$$

$$F_R = \left(\frac{2y \cdot u \alpha}{R}\right)^{1/2} \left[ R/2\alpha \left[ C + \frac{2(1-k)(1-k)}{1+k}B \right] \right] \quad (I 6.34)$$

where

$$C = (\bar{P}_{x0} + \bar{P}_{x0}) / (P_0^*)^{1/3} \quad (I 6.35)$$
The values of C and B, as functions of $X_01$, are tabulated in the last two columns of the above table.

For the case where one has pure rolling such that $k \approx 1.0$, Eq. (I 6.30) yields,

$$F_e = C \left( \frac{2X \mu_0}{R} \right)^{2/3} Re/(2\alpha)$$  \hfill (I 6.36)

The forces $F_n$ exerted on the roller race contacts are,

$$\begin{align*}
F_{n(\text{outer ring})} &= \left( \frac{R_0}{R_i} \right) C \left( \frac{2X \mu_0}{R} \right)^{3/2} \frac{R}{\alpha E_m} \\
F_{n(\text{inner ring})} &= \left( \frac{R_0}{R_i} \right) C \left( \frac{2X \mu_0}{R} \right)^{3/2} \frac{R}{\alpha E_m}
\end{align*}$$  \hfill (I 6.37)

where $R_m = R_0 - r = R_i + r = \text{roller pitch radius}$.

Chiu \cite{15} has determined in his analysis that contact load has a negligible effect on the above pumping forces, and has obtained good agreement with experiment for his rigid body assumption.

**Sliding friction** has been determined to be

$$\begin{align*}
(F_s(\text{outer ring})) &= B \left[ \frac{u(\text{outer})}{u(\text{inner})} \right]^2 \frac{R}{2X} \left[ \frac{2X \mu_0(\text{outer})}{2X \mu_0(\text{inner})} \right]^{2/3} \\
(F_s(\text{inner ring})) &= B \left[ \frac{u(\text{outer})}{u(\text{inner})} \right]^2 \frac{R}{2X} \left[ \frac{2X \mu_0(\text{outer})}{2X \mu_0(\text{inner})} \right]^{2/3}
\end{align*}$$  \hfill (I 6.38)

where $u = \text{entrainment velocity} = (u_2 + u_1)/2$ and $V = \text{sliding velocity} = u_2 - u_1$

**I 6.4.3 HEAT GENERATION RATES**

In the ball-race and ball-cage inlet regions, the heat generated due to the sliding force $F_s$ and sliding force $F_R$ is calculated as,

$$\sigma_I = 2F_R V + F_s u$$  \hfill (I 6.28)

where $V = \text{fluid entrainment velocity at the contact center}$

$u = \text{sliding velocity at the contact center}$

**I 6.5 BALL DRAG FORCE IN BULK LUBRICANT**

In the following form of "churning friction force" is cited, to account for all friction losses on the ball other than EHD sliding traction in the ball/race contacts:

---
\[ F_w = \frac{\rho A_v C_v (d_m \omega_o)^2}{8 g} \]  

where \( F_w \) is the drag force

- \( A_v \): the ball frontal area
- \( C_v \): a drag coefficient given in [35] as a function of the Reynolds number
- \( d_m \): the bearing pitch diameter
- \( \omega_o \): the ball orbital angular velocity
- \( g \): the gravitational constant
- \( \rho \): the density of the air-oil mixture in the bearing cavity

\[ \rho = XCAV \cdot \rho_o \]  

\( XCAV \): the fractional amount of lubricant assumed to be in the bearing cavity

\( \rho_o \): the density of the oil

In the present model, three hydrodynamic force components at each point contact on a ball have been defined.

These components tend to retard ball motion as would \( F_w \). Since two race contacts and a cage contact exist for each ball, 15 force components have been made explicit. After accounting for all contact friction forces, there is left a residual loss due to "windage" or "drag" acting on a ball as it moves through the air-oil mixture in the bearing cavity. Eq. (I 6.37) has been used to model this windage force. Although the effect of the drag force is less significant than calculated in [3], it remains important.

\( XCAV \) values of one percent or less are recommended. In actuality, \( XCAV \) is a function of lubricant supply rate, method of supply, speed and bearing and bearing cavity geometry.
ROLLING ELEMENT INERTIA FORCES AND MOMENTS
APPENDIX I 7
ROLLING ELEMENT INERTIA FORCES AND MOMENTS

A rolling element, Fig 2-4, traveling between azimuth locations, is forced to undergo changes in its rotational velocity components $\omega_x$, $\omega_y$, and $\omega_z$, as well as in its orbital velocity $\omega_o$. The forces which must act on an element to produce time variations in its rotational and orbital velocities may be deduced from Newton's Laws of Motion as follows:

$$\mathbf{F} = \begin{cases} F_x \\ F_y \\ F_z \end{cases} = m \begin{bmatrix} \ddot{x} \\ \ddot{y} - \omega_o^2 (R+y) \\ 2 \omega_o \dot{y} + \omega_o (R+y) \end{bmatrix}$$ (I 7.1)

where $F_x$, $F_y$, and $F_z$ are the components of the forces in the rotating coordinate system attached to the element, $m$ is the ball mass, $x$ and $y$ are the element center displacements shown in Fig. 2-4, and $R$ is the radius of outer ring groove centers.

A rough estimation assuming stable operation yields that the term $x$ is smaller than $\omega_o^2 (R+y)$ by a factor of the order of $x_m/R$, where $x_m$ is the maximum variation of $x$. Similarly, the terms $y$ and $2 \omega_o \dot{y}$ are smaller than $\omega_o^2 R$ by a factor in the order of $y_m/R$ where $y_m$ is the maximum variation of $y$. Note that both $x_m/R$ and $y_m/R$ are very small in magnitude. The second derivatives with respect to time of $x$ and $y$ are thus neglected as is the Coriolis term $2 \omega_o \dot{y}$. The term $\omega_o \dot{y}$ is expressible as follows:
\[ \dot{\omega}_o = \frac{d\omega_o}{dT} \cdot \frac{d\omega_o}{d\phi} \cdot \frac{d\phi}{dT} = \omega_o \cdot \frac{d\omega_o}{d\phi} \quad (I\ 7.2) \]

The term \( \frac{d\omega_o}{d\phi} \) is approximated for ball \( i \) as follows:

\[ \left[ \frac{d\omega_o}{d\phi} \right] = \frac{(\omega_o)_i + 1 - (\omega_o)_i - 1}{2\Delta\phi} \quad (I\ 7.3) \]

where \( \Delta\phi \) is the angular distance between rolling elements.

The moments necessary to cause the element velocity to change are as follows:

\[ \dot{M} = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = [J] \begin{bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{bmatrix} \quad (I\ 7.4) \]

\[ [J] = \begin{bmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{bmatrix} \quad (I\ 7.5) \]

for a ball:

\[ \therefore I_y = J_z = mb^2/10 \quad (I\ 7.6) \]
for a roller:

\[ J_x = \frac{mD^2}{8} \]

\[ J_y = J_z = \frac{m}{12} \left( \frac{3}{4} D^2 + lre^2 \right) \]  

(I 7.7)

m is the element mass

D is the element diameter

lre is the element length

The time variation of the rotational velocity components \( \omega_x, \omega_y \) and \( \omega_z \) are approximated in the same manner as \( \omega_o \), e.g.,

\[
\omega_x = \left[ \frac{(\omega_x)_{i+1} - (\omega_x)_{i=1}}{2\Delta \phi} \right] \omega_o
\]

(I 7.8)

Using D'Alembert's principle, forces \( \vec{F} \) and moments \( \vec{M} \) calculated as described above are imposed on the element along with the other forces and moments due to friction and elastic contact. The combined system of forces is then regarded as being in static equilibrium.
Because the time rates of change of $\omega_o, \omega_x, \omega_y$, and $\omega_z$ are included by approximation as described above, the analytical treatment is considered to be quasi-dynamic as distinct from analyses wherein these terms are neglected and only the centrifugal force $m\omega_o^2(R+y)$ and gyroratory moments $J\omega_o\omega_z$ and $-J\omega_o\omega_y$ are considered. The description "quasi-static" has been applied to solutions of this type.
ROLLING ELEMENT BEARING CAGE MODEL
I 8.1 INTRODUCTION

The cage is driven by normal and friction forces which act at the interfaces between balls or rollers and cage pockets, and at the cage rail(s) and ring land(s). These forces are calculated as functions of the separation and speeds of the interfacing members. In this analysis it is assumed that:

1. normal forces exerted by the rolling element on the cage pocket act in the plane of cage rotation which is coincident with the cage axial midplane.
2. friction forces exerted by the rolling element on the cage pocket act orthogonal to a corresponding normal force and at the normal force point of application.
3. the only friction force components considered in the cage equilibrium equations are those which lie in the plane of cage rotation. It is assumed that each rolling element is axially centered within its pocket.
4. cage rail normal forces act at the cage midplane and pass through the axis of the cage. These forces are coplanar with the rolling element normal forces.
5. cage-land friction forces act in the cage midplane such that any resulting torque tends to drive or retard the cage rotation.

*This Appendix is based upon the original work [15]
The analysis is used to determine the normal and traction forces at each rolling element and ring land on the basis of hydrodynamic, elastohydrodynamic and Hertzian concentrated contact theory.

8.2 GEOMETRY

Figure 2.4 shows a coordinate system (XYZ) with the origin on the outer ring axis in the plane of the outer raceway centers. A local coordinate system (x,y,z) is established at the center of each rolling element. The azimuth angle $\phi$ defined in the (X,Y,Z) coordinate frame locates the x axis penetration through the Y-Z plane. The x axis is parallel to X. The y direction is radially outward and the z direction is tangent to the direction of rolling.

A local coordinate $z_c$ is also defined for each cage pocket, wherein the origin is located on the cage pitch circle. We wish to determine the position of each rolling element center with respect to the cage pocket center along $z_c$, in terms of the rolling element orbital speeds $\omega_o$, the cage rotational speed $\omega_c$ and the cage rotational and translational displacement components.

8.3 CAGE MOTIONS

The equilibrium solution considers that the cage operates in one of three modes:

1. The cage is outer ring land riding such that radial and small circumferential motions of the cage with respect to the rolling elements are resisted by hydrodynamic fluid film forces that develop between the cage rail land outside diametral surface and the bearing outer ring outside diametral surface. Three degrees of freedom
apply to the cage motion. These are the circumferential
to the rolling elements (\( \xi \))
and two components of radial displacement (\( \Delta Y \) and \( \Delta Z \)
in rectangular coordinates, or \( e \) and \( \theta_c \) in polar coord-
andates.) When the bearing is subjected to axial load
only, or when the rolling element speed variation is in-
consequential, the radial degrees of freedom are neglected.

(2) The cage is inner ring land riding when motions are re-
sisted by hydrodynamic forces which develop at the cage
inside surface and the bearing inner ring outside surface.
Three or one degrees of freedom also apply.

(3) The cage is ball or roller riding in which case there
are no net radial fluid film forces between the bearing
rings and the cage, and consequently, no radial motion
of the cage relative to the bearing axis of rotation.
Angular motion of the cage relative to the rolling
elements is the only applicable degree of freedom.

The circumferential displacement of rolling element No. 1 at
azimuth location \( \phi = \phi_1 \), relative to its cage pocket center is
designated \((-\xi)\).

I 8.4 ROLLING ELEMENT MOTIONS

Returning to Fig. 2-4, the velocity with which the moving
coordinate system rotates about the X axis is designated \( \omega_o \)
and is also assumed to be a function of azimuth angle, i.e. \( \omega_o = \omega_o(\phi) \).
The rolling element is assumed to rotate relative to each of the axes in the moving system of coordinates. The angular velocities about each of the axes $x$, $y$, $z$ are denoted $\omega_x$, $\omega_y$, and $\omega_z$ respectively and are shown as the orthogonal components of the rotational velocity vector $\mathbf{\omega}$ in Fig. 2-4.

The value of the ball center-cage pocket center offset $z_c$ applicable at other ball positions is deduced relative to ball position No. 1, which remains fixed at its azimuth position. The cage is assigned a rotation $\xi$, so that the offset of cage pocket no. 1 relative to ball no. 1 is

$$\xi = \frac{d_m}{2} \Delta \Theta_x \quad (I \, 8.1)$$

In so doing, it is assumed that a rolling element orbital velocity remains constant as it traverses the distance corresponding to one half of the pitch spacing on either side of the nominal azimuth position. As a rolling element enters the azimuth location of the next adjacent rolling element the orbital speed undergoes a step change. This is illustrated in Fig. I 8.1 for ball Nos. 1 and 2. The top half of Fig. I 8.1 is a plot of the assumed variation of orbital velocity with respect to ball position.

The cage orbital velocity is denoted by $\omega_c$ and is assumed uniform and equal to the average of the ball orbital velocities, i.e.

$$\omega_c = \frac{1}{n} \sum_{i=1}^{n} (\omega_o)_i \quad (I \, 8.2)$$

where $n$ is the number of rolling elements.
Cage and Rolling Element Speeds and Displacements
The distance between ball positions is the quotient of the circumference $\pi d_m$ of the locus of rolling element centers (neglecting small excursions) and the number of rolling elements. The time $\Delta T$ for the cage to traverse this distance is then

$$\Delta T = \frac{\pi d_m}{n} \cdot \left( \frac{d_m}{2}, \omega_c \right) = \frac{2\pi}{n\omega_c} \quad (I\ 8.3)$$

In this time period the center of rolling element No. 1 moves a circumferential distance of

$$\frac{d_m}{2} (\omega_0)_1 \cdot \frac{\Delta T}{2} + \frac{d_m}{2} (\omega_0)_2 \cdot \frac{\Delta T}{2} = \frac{d_m \Delta T}{4} \left[ (\omega_0)_1, (\omega_0)_2 \right] \quad (I\ 8.4)$$

The circumferential distance between the rolling element and cage pocket center at position No. 2 is obtained as the difference between the rolling element travel and cage travel in time $\Delta T$, less the initial offset of the cage pocket center at rolling element position 1 with respect to the center of rolling element 1, $(\xi)$, see Fig. I 8.1, plus the components of the radial eccentricity ($\Delta Z$ and $\Delta Y$) of the cage axis with respect to the bearing axis. Then

$$(Z_c)_2 = \frac{d_m \Delta T}{4} \left[ (\omega_0)_1 + (\omega_0)_2 - \frac{\pi d_m}{n} - (\xi) - 2Y_{r} \sin \varphi_2 + \Delta Z \cdot \cos \varphi_2 \right] \quad (I\ 8.5)$$

Letting $\Delta T = \frac{2\pi}{n\omega_c}$ then Eq. (I 8.5) becomes for the general $i$-rolling element:

\[ I\ 8-7 \]
FIGURE I 8-2
Cage Pocket Geometry
CALCULATION OF CAGE POCKET NORMAL FORCES

A means for calculating the cage driving forces due to the balls was developed in §23 I. The analysis has been extended to include roller-cage pocket, line contacts, by approximating the line contact to be an elliptical contact that has a large curvature ratio (e.g.: \(a^*/b^* = 18\)).

The analysis is applied to determine the normal forces acting at two diametrically opposite points on a rolling element, i.e., the points of nearest and furthest approach of the ball or roller relative to the cage. The net normal force acting on the rolling element is the resultant of these two forces. The discussion below considers ball-cage, but it applies to roller contact.

The typical ball geometry is shown in Fig. I 8.2. \(z_c\) denotes the offset between the ball and cage pocket centers in the direction of rolling. \(\omega_x\) and \(\omega_y\) denote the components of the ball rotational velocity vector that result in relative surface speeds of the ball and cage pocket.

The closest approach \(h_0\) is the minimum film thickness when the cage is lubricated.

\[
(z_c)_i = \frac{\pi d_w}{n} \sum_{j=2}^{l} \left[ \frac{(\omega_j) + (\omega_j)}{2\omega_c} - 1 \right] - \left[ \xi + AY \sin \phi_i - A2 \cos \phi_i \right]
\]

(I 8.6)
Given the ball cage pocket eccentricity the associated cage pocket load $P$ can be calculated. When $z_c$ is small the load is small and borne hydrodynamically by the lubricant film, which then has minimum thickness $h_o = r' - r - z_c$. In this regime, the load for a given value of the ball and the cage pocket clearance is that supported by a hydrodynamic contact of minimum thickness $h_o$.

Elastic deformation is negligible in this regime. As $z_c$ increases, $h_o$ decreases until it reaches a critical value $h_c$, below which a further increase in $z_c$ results in elastic deformation but no further decrease in film thickness. In this regime

$$h = h_c \quad \text{(I 8.7)}$$

and the elastic deformation is calculated from,

$$\delta_e = z_c + h_c - C_r \quad \text{(I 8.8)}$$

where $C_r$ is the cage pocket radial clearance ($r' - r$).

The load $P$ in this case is assumed to be the sum of the load $P_c$ hydrodynamically related to the film thickness $h_c$, and an additional load $P_e$ associated with the elastic deformation through the Hertzian and flexural equations of contact elasticity.

An analysis was performed as described in \cite{23} of the relationship between normal load $P$ and minimum film thickness $h_o$ in a lubricated point contact between two rigid bodies, each having two
principal radii of curvature, assuming that the lubricant viscosity increases exponentially with pressure. The analysis yielded a relationship between the nondimensional load parameter $\bar{Q}$ and the nondimensional film thickness parameter $\bar{H}$, as shown by the solid curve in Figure I 8.3. These nondimensional variables are defined as:

$$\bar{Q} = P \cdot \left( \frac{\alpha R_y}{C_0^2} \right)^{1/3} \left( \frac{R_x R_y}{R_x R_y} \right)^{-1/2} = P \cdot \frac{D}{1} \quad \text{(I 8.9)}$$

$$\bar{H} = h_0 P_x \left( \frac{C_0 R_x \alpha}{\alpha} \right)^{-2/3} = \frac{h_0}{C_r} \quad \text{(I 8.10)}$$

where:

$$C_0 = \frac{h_0}{V_y} \left( R_x R_y \right)^{1/2} \quad \text{(I 8.11)}$$

$$k_1 = \left[ (3+2k)^{-2} + (3+2k+1)^{-2} \right]^{1/2} \quad \text{(I 8.12)}$$

$$k = \frac{R_y}{R_x} \quad \text{(I 8.13)}$$

$$D \equiv \left( \frac{\alpha R_y}{C_0^2} \right)^{1/3} \left( \frac{R_x R_y}{R_x R_y} \right)^{-1/2} \quad \text{(I 8.14)}$$

$$B \equiv C_r P_x \left( \frac{C_0 R_x \alpha}{\alpha} \right)^{-2/3} \quad \text{(I 8.15)}$$

$$R_x = \frac{1}{r - \frac{1}{r}} - \frac{1}{r} \quad \text{(I 8.16)}$$

$$R_y = r \quad \text{(I 8.17)}$$

$$C_r = r' - r \quad \text{pocket radial clearance} \quad \text{(I 8.11)}$$
FIGURE I 8-3
Load Capacity Vs. Film Thickness for Hydrodynamic and Elastohydrodynamic Operating Regimes

- ELASTIC OR EHD REGION
- INTERMEDIATE REGION
- RIGID ISOVISCOUS SOLUTION
- RIGID ISOVISCOUS REGION

DIMENSIONLESS LOAD PARAMETER, $\pi$
DIMENSIONLESS FILM THICKNESS, $H_c$
\[ V_x = \frac{1}{2} \omega y \cdot r \]

\[ \eta_o = \text{bulk viscosity} \]

\[ V_y = -\frac{1}{2} \omega x \cdot r \]

It has been found that the relationship between \( \overline{Q} \) and \( \overline{H} \) for an unstarved point contact can be approximated by the following formula:

\[
\overline{Q} = 53.3 \ (\overline{H})^{-1/2} + 163 \ (\overline{H})^{-3} \quad (I\ 8.17)
\]

provided that:

\[ \overline{Q} < \overline{Q}_c = 37.6 \]

I 8.5.1 ELASTOHYDRODYNAMIC (EHD) CONTACT

For \( \overline{Q} \geq 37.6 \), the film thickness is independent of load and the nondimensional parameter \( \overline{H} \) remains constant at \( \overline{H}_c \). Operation in this case is in the EHD region.

\[ \overline{H} = \overline{H}_c = 3.122 \quad (I\ 8.18) \]

for EHD Contact

\[ \overline{Q} \geq \overline{Q}_c = 37.6 \quad (I\ 8.19) \]

Equations (I 8.18) and (I 8.19) result in the following for the EHD region of operation,

\[ P_c = \overline{Q}_c/D = 37.6/D \quad (I\ 8.20) \]

\[ h_o = h_c = 3.122 \ C_r/B \quad (I\ 8.21) \]

I 8-13
The elastic deformation $\delta_e$ is given by,

$$\delta_e = z_c + h_c - C_r$$

(I 8.22)

$P_e(\delta_e)$ was originally calculated according to Hertz Theory. This model has been changed, however, in an attempt to reduce the nonlinearity of rolling element-cage load displacement relationships. This was done through the assumption that the cage will respond to large rolling element loads through flexing as well as through the local contact displacements.

It was assumed that 95 percent of $\delta_e$ would be accommodated by cage flexing and the remainder would be accommodated by the Hertz deformation. The Hertz calculations are made based on the assumptions of a 9:1 major to minor contact axis ratios for a ball-cage contact and an 18.2:1 ratio for the roller-cage contact. The cage flexure deflection is calculated from

$$P_e(\text{flex}) = (0.95 \delta_e) R \times 13500$$

(I 8.23)

The spring constant 13500 $R$ where $R$ is the cage rail radius, was derived using circular ring theory, considering the cage material to be steel. The total ball-cage contact load $P$ is thus

$$P = P_c + P_{HZ} (0.05 \delta_e) + P_{(\text{flex})} (0.95 \delta_e)$$

(I 8.24)

### I 8.5.2 HYDRODYNAMIC (HD) CONTACT

If the contact film thickness $h_0$ is greater than the critical value $h_c$, the contact is assumed to be hydrodynamic:

$$H \geq H_c = 3.122 \text{ for HD contact}$$

The minimum film thickness for this case is given in terms of the ball-cage clearance and eccentricity $z_c$ as:

I 8-14
\[ h_o = C_r - z_c \]  
(I 8.25)

The calculation procedure is,

(a) Calculate \( h_o \) as above

(b) Evaluate \( H = h_o R_x (C_o R_x)^{-2/3} \)

(c) For \( H \) from (b) find \( \bar{Q} = 53.3 (H)^{-1/2} + 163 (H)^{-3} \)

(d) Calculate cage-ball load \( P \) as \( \bar{Q}/D \)

The procedure for determining the ball-cage normal load is performed for the points of nearest and further approach (\( h_o \) min and \( h_o \) max). The net normal load acting on the ball is given by:

\[ \sum P = P (h_o \text{ minimum}) - P (h_o \text{ maximum}) \]  
(I 8.26)

I 8.5.3 DRY CONTACT

Normal cage-pocket rolling element contact forces for conditions of dry contact are calculated for the \( h_o \) (minimum) contact only. A continuous force displacement function is assumed for this calculation. A soft spring and low force values occur for \( \bar{e} < 0.99 \), where \( \bar{e} = 2 z_c / (r' - r) \); such that,

\[ P = 10 . r . e \text{ for } e \leq 0.99 \]  
(I 8.27)

A hard spring function is assumed for \( \bar{e} > 0.99 \), such that

\[ \bar{e}_1 = 0.99 + 0.01 (\bar{e} - 0.99) / \bar{e} \]

\[ \bar{e}_p = \bar{e}_1 + 1.01 \]

\[ P = 0.0506r, \left\{ \bar{e}_1 / [1 - \bar{e}(1/\bar{e}_p)] \right\} (\bar{e}_p / 2)^{3} \bar{e}_1 \]  
(I 8.28)
I 8.6 CAGE POCKET/ROLLING ELEMENT FRICTION FORCES

Friction forces which arise in the rolling element/cage pocket contacts are calculated according to Appendix 16 for wet friction. Dry friction forces are calculated with a Coulomb model.

I 8.7 CALCULATION OF CAGE LAND NORMAL FORCES AND FRICTION MOMENT

The lubricant forces which develop between a cage rail and its supporting ring surface are obtained using the hydrodynamic solution for self-acting short-journal bearings. According to the resultant of the pressure distribution on the cage has orthogonal force components, one of which lies along the cage line of centers (the line which passes through the cage center and its point of closest approach to the ring). Both components pass through the cage center.
Figure 2.5 depicts the geometric and operating parameters for the inner land riding situation, and Figure 2.6 the outer land riding situation. $u_i$, $u_o$, $u_c$ are the surface speeds of the inner ring, outer ring, and cage land, respectively. The cage undergoes a displacement in the bearing XYZ frame of a magnitude $e$ and direction $\Theta_c$. An xyz frame is attached to the cage, such that the y axis passes through the point of minimum film thickness. The y axis is rotated $\Theta_c$ from the ring Y axis. The short bearing solution for an isoviscous, Newtonian fluid gives the magnitude of the normal first terms in Eq. (18.29) through Eq. (18.31). The second terms account for cage elastic flexure.

\[ W_y = \frac{y_o UL^3}{C^2} \cdot \frac{e^2}{(1 - e^2)^2} + 13500 R \beta e |e| = \pm F_y \] (18.29)

\[ W_z = \frac{y_o UL^3}{C^2} \cdot \frac{e}{4(1 - e^2)^{3/2}} - 475 R \beta e |e| \frac{V}{|V|} = \pm F_z \] (18.30)

and of the friction torque as,

\[ M_c = \frac{y_o VR^2L}{C} \cdot \frac{2\pi}{(1 - e^2)^{1/2}} + 475 R^2 \beta e |e| \frac{V}{|V|} \] (18.31)

where $\beta e 1 = e - 0.999 C$

- $C$ = radial clearance, (in.)
- $y_o$ = viscosity, (lb-sec/in.²)
- $L$ = cage ring width, (in.)
- $R$ = cage ring radius, (in.)
- $U$ = entrainment velocity, (in/sec)
- $V$ = $(u_i + u_c)$ for inner land riding cage
- $V$ = $(u_o + u_c)$ for outer land riding cage
- $V$ = $(u_i - u_c)$ sliding velocity
- $e$ = eccentricity ratio, $e/C$

I 8-17
$W_y = \text{cage land normal force component along line of centers}, \ (lb)$

$W_z = \text{cage land normal force component normal to line centers}, \ (lb)$

$M_c = \text{cage land friction torque}, \ (\text{in-lb})$

In using Eqs.(I 8.29) and (I 8.30), the upper sign applies to an inner ring riding cage and the lower to an outer.

Subroutine CGWET makes the calculations.

For dry contact a load displacement relationship is assumed which has the following form.

\[
W_y = XK \cdot \varepsilon = \pm F_y \tag{I 8.32}
\]

\[
XK = \frac{L^3}{C^2} \text{ for } \varepsilon \leq 0.9 \tag{I 8.33}
\]

\[
XK = 0.2111 \cdot \frac{L^3}{C^2} \cdot \frac{\varepsilon}{(1 - \varepsilon^2)} \quad \varepsilon > 0.9
\]

\[
W_z = \frac{f}{\varepsilon} W_y \cdot \frac{U}{U_T} = +F_z \tag{I 8.34}
\]

\[
M_c = W_z \cdot R \tag{I 8.35}
\]

Subroutine CGDRY makes the calculations.

In order to insert the values for $W_y$, $W_z$ and $M_c$ into the cage equilibrium equations, the following transformations are made for inner and outer rings:

\[
\begin{pmatrix}
M_cX \\
F_{cY} \\
F_{cZ}
\end{pmatrix} = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \theta_c & -\sin \theta_c \\
0 & \sin \theta_c & \cos \theta_c
\end{bmatrix} \begin{pmatrix}
M_c \\
F_y \\
F_z
\end{pmatrix} \tag{I 8.36}
\]

where $\tan \theta_c = (\Delta Z / \Delta Y)$ for an inner ring riding cage and $\tan \theta_c = \Delta Z / \Delta Y$ for an outer ring riding cage.
I 8.7  FRICTION HEAT GENERATION RATES

The heat generated by fluid shearing between the cage and land is calculated as the product of the cage friction moment and rotational speed, i.e.,

\[ q_c = M_c \cdot |\omega - \omega_c| \]  \hspace{1cm} (I 8.37)

where \( M_c \) is calculated according to Equation I8.31 and \( |\omega - \omega_c| \) is the absolute value of the difference between the cage speed and the speed of the ring that guides the cage.
APPENDIX I 9

ING FATIGUE LIFE CALCULATIONS
APPENDIX I 9

BEARING FATIGUE LIFE CALCULATIONS

I 9.1 INTRODUCTION

Within SHABERTH, ball and roller bearing raceway fatigue life is calculated with the methods of Lundberg Palmgren \cite{33} and \cite{34}. The life thus calculated is modified by multiplicative factors which account for material and lubrication effects.

I 9.2 BALL BEARING RACEWAY LIFE

Bearing raceway $L_{10}$ fatigue life in millions of revolutions as determined by Lundberg-Palmgren, \cite{33} is expressed by

$$L_{10m} = \left( \frac{Q_{cm}}{Q_{cem}} \right)^3$$ \hspace{1cm} (I 9.1)

$Q_{cm}$ is the raceway dynamic capacity, the load for which the bearing raceway will have 90 percent assurance of surviving 1 million revolutions. From ref. \cite{33},

$$Q_{cm} = \frac{7140}{2} \left( \frac{f_m}{2m-1} \right)^{0.41} \left[ \frac{(1+\gamma_m)^{1.39}}{2 \left(1+\gamma_m\right)^{0.33}} \left( \frac{\gamma_m}{\cos \alpha_m} \right)^{0.3} \right] B^{1.8} \hspace{1cm} (I 9.2)$$

where:

- $f = \text{groove curvature, raceway radius/ball diameter}$
  $(r_m/D)$
- $\gamma = D \cos \alpha_m/d_m$
- $D = \text{Ball diameter}$
- $\alpha = \text{Raceway contact angle}$
- $d_m = \text{Bearing pitch diameter}$
- $Z = \text{Number of rolling elements}$
- $m = \text{is a subscript, it is 1 for the outer raceway and 2 for the inner raceway.}$
The upper sign is used for the outer race, the lower for the inner race.

Q_{em} is the raceway equivalent load,

\[ Q_{em} = \left[ \frac{1}{Z} \left( \sum_{j=1}^{Z} Q_{mj} \right) \right]^{1/\epsilon} \quad m = 1, 2 \]  

(1 9.3)

where Q_{mj} is the individual ball contact load, and \( \epsilon = 3 \) or \( \epsilon = 3.3 \) depending respectively upon whether the applied load rotates or is stationary with respect to the raceway in question.

I 9.3 ROLLER BEARING RACEWAY LIFE

To account for non symmetrical load distributions across a line contact, the roller and raceways are thought of as being comprised of a number of sliced discs. Raceway L_{10} fatigue life, in millions of revolutions at a given slice as determined by Lundberg-Palmgren, \( \{34\} \) is expressed by

\[ L_{10mk} = \left( \frac{Q_{cmk}}{Q_{emk}} \right)^2 \]  

(I 9.4)

Q_{cmk} is the dynamic capacity of a raceway slice, defined as the load for which the slice will have a 90 percent assurance of surviving 1 million revolutions. n refers to raceway, k refers to slice, n is the index of the last slice, from \( \{34\} \)

\[ Q_{cmk} = \frac{49500 \lambda}{(1 \pm \beta)} \left( \frac{10^3}{L_{re}} \right)^{1.034} A \Delta l_{re} \left( \frac{\pi}{2} \right)^{0.122} \]  

(I 9.5)

The upper sign is used for the outer race, the lower sign refers to the inner race.

Q_{mek} is the equivalent load for the slice.

\[ Q_{mek} = \frac{1}{Z} \left( \sum_{j=1}^{Z} Q_{mkj} \right)^{1/\epsilon} \]  

(I 9.6)
$O_{mkj}$ is the individual roller contact load on the $k$-th slice and $\varepsilon = 4.0$ or $\varepsilon = 4.5$ depending respectively upon whether the applied load rotates or is stationary with respect to the raceway in question.

The $L_{10}$ life of a raceway is given by

$$L_{10} = a_2 a_3 a_5 \left\{ \sum_{k=1}^{n} L_{10m_k}^{-e} \right\}^{-1/e} \quad (I \; 9.7)$$

where $e$ is the Weibull slope exponent, here taken to be $9/8$ for roller bearings and $10/9$ for ball bearings.

$a_2$ is a life improvement factor to account for improved materials.

$a_3$ is a life improvement factor to account for full film lubrication which $a_3^*$ is less than 1 when full film lubrication is not obtained. See I 9.5.

I 9.4 BEARING LIFE

The $L_{10}$ life of the bearing considering both raceways is:

$$L_{10} = \left\{ \sum_{m=1}^{2} (L_{10m})^{-e} \right\}^{-1/e} \quad (I \; 9.8)$$

I 9.5 BEARING LIFE REDUCTION DUE TO ASPERITY INTERACTION

In [25] and [27] the form of a reduction factor accounting for the effect of surface asperity interaction was deduced and its parameters were set to best fit a large body of rolling contact life test data.

As employed in Program SHABERTH, the reduced tenth percentile life $L_{10}$ is calculated as follows,

$$a_3^* = \left[ 1 + \frac{\psi^2(h/t)}{\psi(1.5)} \right]^{-1/e} \quad (I \; 9.9)$$
where

\[ \psi(h/\sigma) = \frac{\phi'(h/\sigma)}{1 - \Phi(h/\sigma)} \]  

\( \phi'(\cdot) \) = density function of standard normal distribution

\( \Phi(\cdot) \) = cumulative distribution function of standard normal distribution

\( h/\sigma \) = ratio of plateau film thickness to surface roughness for most heavily loaded ball

\( (L_{10})_\infty \) = the full film life

\[ L_{10} = a_3 \ast (L_{10})_\infty \]  

The term \( (L_{10})_\infty \) is calculated using the principles of Lundberg-Palmgren and multiplying by the user supplied product of two factors which represent by Industry practice the life improvement due to the type of material from which the bearings is fabricated and the life improvement due to full EHD film conditions. \( (L_{10})_\infty \) is then down-rated to actual film conditions by Eq. (I 9.11).
The hierarchical flow chart presents the program structure, listing the program elements in the order in which they would be called to solve the shaft-bearing dynamic, as well as steady state and transient temperature distribution problems. The various solution loops are indicated, as well as notes which indicate the functions of various subroutine groupings.

Each line in the flow chart represents a program element, subroutine, function or the main program ALWAYS. The call of one subroutine by another is denoted by indenting the called subroutine relative to the routine doing the calling. As an example, subroutine SKF calls subroutines FLAGS, TYPE, PROPS1, LUPRPROP, LUBCON, DATOT, CNVRT, CONS and SPRING. Subroutine CONS calls CONST, CONST calls BCON and CRCON and BCON calls ABDEL.

The first mention of a subroutine within the flow chart includes the entire list of subordinate program elements. At subsequent calls to that subroutine the list of subordinate elements is omitted. As an example the first call to subroutine AXLBOJ is followed by the subordinate elements JHVIKT, SNITHT, NUMLOS, DUBSIM, HEIE, MEIL and SIMQ. After the call of AXLBOJ from INDEL, the subordinate elements are not listed but are, nevertheless, employed. The list of subordinate program elements are omitted in repeated calls of subordinate GUESS, BEAR, SOLV13 and DELIV3 as well as AXLBOJ.

As noted earlier, rolling equilibrium is calculated, first without, then if required, with friction forces included. Whether or not friction is considered is highlighted with the words Frictionless or Friction beside subroutine BEAREQ.

If the Program is too large to fit in its entirety on the user's computer, segments of the program may be "overlaid". For this purpose the Program is subdivided into ten (10) modules which can be sequentially "overlaid". The contents of the ten modules are listed below.

The Program segments SKF, TEMPIN, SHAFT and GUESS all perform initiation functions and with the exception of GUESS, are called only once per program execution.

The real problem solving portion of the program is embodied in segment ALLT. Within this segment the shaft bearing solution is obtained through the call to SHAEB, then the steady state or transient temperature distributions are obtained.
This scheme is repeated until the end objective, steady state thermal equilibrium or time up for the transient scheme, is realized.

The nonlinear equation solver SOLV13 is central to the program and deserves special discussion as related to the flow chart. The first call to SOLV13 is from BEAR. Only for this first call are all of the SOLV13 subordinate subroutines listed as noted earlier. These include INSOLV, EQS, PARDER, SIMQ, EOCHER, ERWRIT and ERCHEK. In the subsequent call to SOLV13 in which the steady state temperatures are being calculated, the above listed subroutines are again called but these calls with the exception of EQS are not listed on the flow chart.

EQS is the name given by SOLV13 to a subroutine which sets up the system of equations to be solved. EQS is brought into SOLV13 through the argument list. When the bearing equations are being solved, subroutine BRGGEQ is brought into SOLV13 and within SOLV13 is referenced by the name EQS. When the heat transfer equations are being solved as a consequence of the call of SOLV13 from ALLT, NET is brought into SOLV13 and is referenced as EQS.
Read and set bearing and bearing solution control data

Set lubricant properties and calculate constants for temperature dependency calculations

Write bearing input and hardcoded, preset data

Calculate bearing related constants

Read and write thermal and thermal solution control data and calculate heat transfer coefficients

Read and write shaft geometry, loading and bearing position data

Calculate shaft deflection constants

Make initial guesses of bearing reaction loads and displacements

Calculate shaft influence coefficients

Guess values of bearing variables

Begin the solution of the steady state or transient thermal and temperature dependent shaft-bearing analyses

Calculate bearing diametral clearance

Establish iteration scheme to satisfy inner ring equilibrium

Calculate rolling element raceway normal forces

Sum the rolling element forces and moments acting on the inner ring

Calculate bearing fatigue life

Add bearing inner ring forces and moments to the shaft equilibrium equations and predict new shaft displacements

Calculate rolling element raceway normal loads with the new shaft displacements and guess bearing component speeds

Calculate temperature dependent lubricant properties

Begin the calculation of bearing friction with the guessed component speeds

Calculate ball-race film thickness plus the hydrodynamic and concentrated contact friction forces
Solution Loops
1. Steady State and Transient Thermal
2. Change in Clearance
3. Shaft-Inner Ring Equilibrium
4. Rolling Element and Cage Equilibrium
5. Temperature Equilibrium
APPENDIX - II 2

SKF COMPUTER PROGRAM "SHAPERTH/SKF"

INPUT FORMAT FORMS
HOW TO READ AND USE THE FOLLOWING FORMS

Program Names of input Items, for example: W1

<table>
<thead>
<tr>
<th>Item</th>
<th>DIA</th>
<th>BD(5)</th>
<th>1Z</th>
<th>A(6)</th>
<th>R(2,3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item 1</td>
<td></td>
<td></td>
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<tr>
<td>Item 2</td>
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<td>Item 3</td>
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<tr>
<td>Item 4</td>
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</tbody>
</table>

All values are to be given in the units of the SI-system, the International system. Exceptions are lengths which are expressed in millimeters instead of meters, and variables which are given in Newtons instead of Newtons. Volumes in m³.

Unless noted, all numerical input data is in F format, F10.0 or F5.0.
### TITLE CARD

Card Type TT1

<table>
<thead>
<tr>
<th>Title</th>
<th>2''</th>
<th>30</th>
<th>31</th>
<th>32</th>
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</tbody>
</table>

**Title to be printed on each page.**
**TITLE CARD**

Card 2, One Card

<table>
<thead>
<tr>
<th>E5N(5)</th>
<th>E5N(4)</th>
<th>E5N(3)</th>
<th>E5N(2)</th>
<th>E5N(1)</th>
</tr>
</thead>
<tbody>
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</tbody>
</table>

**IPASS, IFLAG(5), Bearing Solution Level F**

0 Elastic Contact Forces are calculated, no lubrication or friction effects are considered
1 Elastic Contact Forces are calculated. Lubrication and friction effects are considered using raceway contact (ball bearing) or spinctic (roller bearing) assumptions to estimate rolling element and cage speeds
2 Inner Equilibrium is satisfied considering only the Elastic Contact Forces. Using the inner ring positions thus obtained, rolling element and cage equilibrium are determined considering friction
3 Complete Solution. The inner ring, rolling element and cage equilibrium is determined considering all elastic and friction forces
BEARING DATA. One set per bearing cards B1-B16 for bearing 1, followed by cards B1-B16 for bearing 2, etc.

Card Type B1, one card

<table>
<thead>
<tr>
<th>BD(1)</th>
<th>BD(43)</th>
<th>BD(44)</th>
<th>BD(2)</th>
<th>NC</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bearing Type</th>
<th>Steel Designations</th>
<th>Outer Ring</th>
<th>Outer Shell</th>
<th>Inner Ring</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ball or Cylindrical</td>
<td></td>
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<td></td>
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<tr>
<td>Start Begin in Column 1</td>
<td></td>
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</tr>
</tbody>
</table>

NC Normally zero or blank, if set to 1, nonsmooth roller-raceway profile geometry is input on card types B5 and B6.
**BEARING DATA**

Card Type B2, one card

<table>
<thead>
<tr>
<th>Pitch Diameter (mm)</th>
<th>No. of Rolling Elements</th>
<th>Diametral Clearance (mm)</th>
<th>Contact Angle (Deg.)</th>
<th>Outer Ring Speed (RPM)</th>
</tr>
</thead>
<tbody>
<tr>
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</tbody>
</table>
BEARING DATA

Card Type B3, one card

<table>
<thead>
<tr>
<th>BD(4)</th>
<th>BD(10)</th>
<th>BD(11)</th>
<th>BD(12)</th>
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</thead>
<tbody>
<tr>
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</tbody>
</table>

Ball Bearing

<table>
<thead>
<tr>
<th>Ball Diameter (mm)</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
</tbody>
</table>

Roller Bearing

<table>
<thead>
<tr>
<th>Roller Diameter (mm)</th>
<th>Roller Length End to End (mm)</th>
<th>Roller End Radius (mm)</th>
<th>Roller Included Angle (deg.)</th>
</tr>
</thead>
<tbody>
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</tbody>
</table>

II 2-7
**BEARING DATA**

Card Type B4, one card

<table>
<thead>
<tr>
<th>Ball Bearing</th>
<th>Outer Raceway Curvature $r_{4/r}$</th>
<th>Inner Raceway Curvature $r_{1/r}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BD(21)</td>
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<td>BD(22)</td>
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<td>BD(23)</td>
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<td>BD(26)</td>
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<tr>
<td>BD(27)</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Roller Bearing</th>
<th>Outer Raceway</th>
<th>Inner Raceway</th>
<th>No. of Slices (20) max</th>
<th>Blank it is Set to 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Effective Contact Length (mm)</td>
<td>Flat Length (mm)</td>
<td>Crown Radius (mm)</td>
<td>Effective Contact Length</td>
</tr>
<tr>
<td></td>
<td>BD(21)</td>
<td>BD(22)</td>
<td>BD(23)</td>
<td>BD(24)</td>
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</tbody>
</table>
## BEARING DATA

Omit Cards B5 and B6 if crown drop flag is 0

Card Type B5, as many as needed, maximum of 3, for the inner race.

<table>
<thead>
<tr>
<th>BLAN</th>
<th>1.5</th>
<th>2.5</th>
<th>3.5</th>
<th>4.5</th>
<th>5.5</th>
<th>6.5</th>
<th>7.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crown Drop at Laminas 1 (mm)</td>
<td>Crown Drop at Laminas 2</td>
<td>... and so on, use more than one card if No. of laminas is greater than 7</td>
<td></td>
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</tr>
</tbody>
</table>

Card Type B6, as many as needed, maximum of 3, for the outer race.

<table>
<thead>
<tr>
<th>BLAN</th>
<th>1.4</th>
<th>2.4</th>
<th>3.4</th>
<th>4.4</th>
<th>5.4</th>
<th>6.4</th>
<th>7.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crown Drop at Laminas 1 (mm)</td>
<td>Crown Drop at Laminas 2</td>
<td>... and so on</td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**BEARING DATA**

Card Type B7, one card, omit if NPASS, Title Card 2 is zero or blank

<table>
<thead>
<tr>
<th>CLA Roughness Surface (microns)</th>
<th>Outer</th>
<th>Inner</th>
<th>Rolling Element</th>
<th>Outer</th>
<th>Asperity Slope (Degrees)</th>
<th>Inner</th>
<th>Rolling Element</th>
</tr>
</thead>
</table>
**BEARING DATA**

Card Type B8, one card, Omit if NPASS, Title Card 2, is zero or blank

<table>
<thead>
<tr>
<th>Cage Type</th>
<th>Rail-Leg Diameter (mm)</th>
<th>Single Rail Width (mm)</th>
<th>Rail-Leg Diametral Clearance (mm)</th>
<th>Rolling Element Cage Pocket Diametral Clearance (mm)</th>
<th>Cage Weight &amp; No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1. for outer-ring load riding</td>
<td></td>
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<tr>
<td>-1. for inner-ring load riding</td>
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<tr>
<td>0. for rolling element riding</td>
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</tbody>
</table>
BEARING DATA

Omit Card Types B11 through B14 if operating clearances are not to be calculated.

**Card Type B11, one card**

<table>
<thead>
<tr>
<th>Shaft</th>
<th>Inner Ring</th>
<th>Rolling Element</th>
<th>Outer Ring</th>
<th>Housing</th>
</tr>
</thead>
</table>

**Modulus of Elasticity N/mm²**

<table>
<thead>
<tr>
<th>Shaft</th>
<th>Inner Ring</th>
<th>Rolling Element</th>
<th>Outer Ring</th>
<th>Housing</th>
</tr>
</thead>
</table>

**Poisson's Ratio**

<table>
<thead>
<tr>
<th>Shaft</th>
<th>Inner Ring</th>
<th>Rolling Element</th>
<th>Outer Ring</th>
<th>Housing</th>
</tr>
</thead>
</table>

**Density gm/cm³**

<table>
<thead>
<tr>
<th>Shaft</th>
<th>Inner Ring</th>
<th>Rolling Element</th>
<th>Outer Ring</th>
<th>Housing</th>
</tr>
</thead>
</table>

**Coefficient Thermal Expansion 1/°C**

<table>
<thead>
<tr>
<th>Shaft</th>
<th>Inner Ring</th>
<th>Rolling Element</th>
<th>Outer Ring</th>
<th>Housing</th>
</tr>
</thead>
</table>
BEARING DATA

Card Type B15, one card, Omit if NPASS, Title Card 2, is zero or blank.

<table>
<thead>
<tr>
<th>BD(102)</th>
<th>BD(103)</th>
<th>BD(104)</th>
<th>BD(106)</th>
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</thead>
<tbody>
<tr>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Lubricant Replenishment Layer Thickness (mm)</th>
<th>Percent Lubricant in Bearing Cavity</th>
<th>Bearing-Rolling Element Asperity Friction Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer Raceway</td>
<td>Inner Raceway</td>
<td></td>
</tr>
</tbody>
</table>
**BEARING DATA**
Card Type B16, one card, Omit if NPASS, Title Card 2, is zero or blank
Lubricant Properties

<table>
<thead>
<tr>
<th>Lubricant Designation</th>
<th>VISC Dynamic Viscosity (cst) @ 100°F</th>
<th>VISC2 Dynamic Viscosity (cst) @ 210°F</th>
<th>SMOO Density (g/ml) @ 15.5°C</th>
<th>Coefficient of Thermal Expansion 1/°C</th>
<th>Thermal Conductivity (W/m°C)</th>
</tr>
</thead>
<tbody>
<tr>
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</table>

*If NO CODE is zero read this data*
THERMAL DATA
Card Type T1

Once controlling card. If no temp. calculation is desired, then leave the card blank.

<table>
<thead>
<tr>
<th>D</th>
<th>B</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
<th>L</th>
<th>M</th>
<th>N</th>
<th>O</th>
<th>P</th>
<th>Q</th>
<th>R</th>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>SCHEDULE</th>
<th>INITIAL CONDITIONS</th>
<th>PHASE</th>
<th>OUTPUT</th>
<th>TERMINAL</th>
<th>STARTING</th>
<th>STOPPING</th>
<th>CALCULATING</th>
<th>TIME</th>
</tr>
</thead>
<tbody>
<tr>
<td>REGULAR</td>
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<tr>
<td>TRANSIENT</td>
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II-2-16
THERMAL DATA - BEARING TEMPERATURES

T2, One card/brg. Use only if no temperature calc. is desired, and then give no more thermal data.

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</tbody>
</table>

Shaft: Inner Ring, Inner Race, Rolling Element, Outer Race, Outer Ring, Housing, Bulk of Lubricant, Flange
**THERMAL DATA - INDIVIDUAL INITIAL TEMPERATURES**

T3. As many cards as needed, followed by a blank card

<table>
<thead>
<tr>
<th>Node Number</th>
<th>Initial Etc.</th>
<th>Temperature</th>
<th>Node Number</th>
<th>Initial Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
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<tr>
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<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
THERMAL DATA - BEARING NODE NUMBERS

T4, One card/brg.

<table>
<thead>
<tr>
<th>Shaft</th>
<th>Inner Ring</th>
<th>Inner Race</th>
<th>Rolling Element</th>
<th>Outer Race</th>
<th>Outer Ring</th>
<th>Housing</th>
<th>Bulk of Lubricant in the Bearing Cavity</th>
<th>Flange, if any</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

II 2-19
THERMAL DATA - NODES WITH A CONSTANT POWER SOURCE

T6, as many cards as needed, followed by a blank card

<table>
<thead>
<tr>
<th>Node Number</th>
<th>Power</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
THERMAL DATA - HEAT FLOW PATHS

T8, as many cards as needed, max. 500, followed by a blank card

Index


Ex. 9-2-8-3
THERMAL DATA - NODE HEAT CAPACITIES, ONLY FOR TRANSIENT CALCULATIONS

T9, One card/node

<table>
<thead>
<tr>
<th>Node Number</th>
<th>Volume at Nodes $L_1$, $L_2$, $L_3$</th>
<th>Density (KG/m$^3$)</th>
<th>Specific Heat W5/KG°C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$L_1$ mm</td>
<td>$L_2$ mm</td>
<td>$L_3$ mm</td>
</tr>
</tbody>
</table>
### SHAFT DATA - SHAFT GEOMETRY

Omit all shaft data cards if there are no bearings. S1, as many cards as needed.

<table>
<thead>
<tr>
<th>Type of Shaft card.</th>
<th>Blank</th>
<th>X-coordinate of shaft section (mm)</th>
<th>Shaft Inner diameter immediately before the section. (mm)</th>
<th>Shaft Inner diameter immediately after the section. (mm)</th>
<th>Shaft Outer diameter immediately before the section. (mm)</th>
<th>Shaft Outer diameter immediately after the section. (mm)</th>
<th>Shaft Modulus of elasticity. If left blank, steel is assumed. 2.041x10^5 N/mm²</th>
</tr>
</thead>
</table>
SHAFT DATA - BEARING POSITION AND SUPPORT, X-Y PLANE

S2. As many cards as the number of bearings. Arranged in order of increasing X-coordinate.

<table>
<thead>
<tr>
<th>Type of Shaft Gear</th>
<th>Plane</th>
<th>Radial Position Error at the Center Plane of the Gearbox (mm)</th>
<th>Radial Offset Angle of the Gearbox at the Center Plane (deg)</th>
<th>Angular Position Error of the Shaft Gear in the Gearbox (deg)</th>
<th>Angular Error in Special Position of the Shafts in the Gearbox (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
SHAFT DATA - SHAFT LOADING, X-Y PLANE FORCES, Z-MOMENTS

S3, as many cards as needed, followed by a blank card.

<table>
<thead>
<tr>
<th>Type of Shaft card</th>
<th>Blank</th>
<th>X-Coordinate of shaft section (mm)</th>
<th>Force</th>
<th>Concentrated force Fy. N</th>
<th>Concentrated Moment Mz. N-mm</th>
<th>Distributed load. Load intensity immediately before the section, N/mm</th>
<th>Distributed load. Load intensity immediately after the section, N/mm</th>
<th>Concentrated axial load. N</th>
<th>Y-Coordinate for the line of action of the axial load (mm)</th>
</tr>
</thead>
</table>
SHAFT DATA - BEARING POSITION AND SUPPORT, X-Z PLANE

S2, as many cards as needed. Card needed only if any data is different from X-Y plane.

<table>
<thead>
<tr>
<th>Type of shaft card</th>
<th>Bearing number</th>
<th>Blank</th>
<th>Radial Mounting error of the outer ring in the Z-direction (mm)</th>
<th>Radial spring characteristics of the housing in the Z-direction mm/N.</th>
<th>Angular mounting error of the outer ring in the Y-direction (radians)</th>
<th>Angular spring characteristics of the housing in the Y-direction radians/Nm.</th>
</tr>
</thead>
<tbody>
<tr>
<td>This card type is 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**SHAFT DATA - SHAFT LOADING, X-Z PLANE FORCES, Y-MOMENTS.**

S3, as many cards as needed, followed by a blank card

<table>
<thead>
<tr>
<th>No</th>
<th>Type of shaft card.</th>
<th>Blank</th>
<th>X-coordinate of shaft section. (mm)</th>
<th>Concentrated force Fx. N</th>
<th>Concentrated moment My. N-mm</th>
<th>Distributed load. Load intensity immediately before the section. N/mm</th>
<th>Distributed load. Load intensity immediately after the section. N/mm</th>
<th>Concentrated axial load. N</th>
<th>Z-coordinate for the line of action of the axial load (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>3</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

11-2-29
THIS DATA SET CONTAINS 2 BEARINGS

BEARING NO. (1) — CYLINDRICAL ROLLER BEARING.

BEARING NO. (2) — BALL BEARING

THE MAXIMUM NUMBER OFFIT ITERATIONS ALLOWED IS 2 AND THE RELATIVE ACCURACY REQUIRED IS .00010
<table>
<thead>
<tr>
<th>BEARING NUMBER</th>
<th>NUMBER OF ROLLING ELEMENTS</th>
<th>AZIMUTH ANGLE ORIENTATION</th>
<th>PITCH DIAMETER</th>
<th>DIAMETRAL CLEARANCE</th>
<th>CONTACT ANGLE</th>
<th>INNER RING SPEED</th>
<th>OUTER RING SPEED</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14</td>
<td>-0.000</td>
<td>0.6500</td>
<td>0.060</td>
<td>-0.000</td>
<td>15000</td>
<td>-0</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
<td>-0.000</td>
<td>140.660</td>
<td>0.030</td>
<td>25.000</td>
<td>15000</td>
<td>-0</td>
</tr>
</tbody>
</table>

**CAGE DATA**

<table>
<thead>
<tr>
<th>BEARING NUMBER</th>
<th>CAGE TYPE</th>
<th>CAGE POCKET CLEARANCE</th>
<th>RAIL-LAND WIDTH</th>
<th>RAIL-LAND DIAMETER</th>
<th>RAIL-LAND CLEARANCE</th>
<th>WEIGHT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>INNER RING LAND RIDING</td>
<td>0.29000</td>
<td>2.6800</td>
<td>53.8400</td>
<td>0.240</td>
<td>0.050000</td>
</tr>
<tr>
<td>2</td>
<td>INNER RING LAND RIDING</td>
<td>0.82600</td>
<td>4.0000</td>
<td>128.4700</td>
<td>1.270</td>
<td>0.330000</td>
</tr>
</tbody>
</table>

**STEEL DATA**

<table>
<thead>
<tr>
<th>BRG.NO.</th>
<th>INNER RING TYPE</th>
<th>LIFE FACTOR</th>
<th>OUTER RING TYPE</th>
<th>LIFE FACTOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>AISI 52100</td>
<td>3.000</td>
<td>AISI 52100</td>
<td>3.000</td>
</tr>
<tr>
<td>2</td>
<td>M50 CVM</td>
<td>5.000</td>
<td>M50 CVM</td>
<td>5.000</td>
</tr>
</tbody>
</table>
**ROLLING ELEMENT DATA**

<table>
<thead>
<tr>
<th>BEARING NUMBER (1)</th>
<th>TYPE</th>
<th>CYLINDRICAL ROLLER BEARING</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROLLER DIAMETER</td>
<td>ROLLER LENGTH</td>
<td>ROLLER END SPHERE RADIUS</td>
</tr>
<tr>
<td>10.0000 10.0000 200.0000 0.000 0.0000 0.0000 0.0000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>EFF. LENGTH OUTER RACEWAY CROWN RAD.</th>
<th>EFF. LENGTH INNER RACEWAY CROWN RAD.</th>
<th>NO. OF AXIAL LAMINAE</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.0000 3.0000 1000.000 6.0000 3.0000 1000.000</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>BEARING NUMBER (2)</th>
<th>TYPE</th>
<th>BALL BEARING</th>
</tr>
</thead>
<tbody>
<tr>
<td>BALL DIAMETER</td>
<td>OUTER RACEWAY CURVATURE</td>
<td>INNER RACEWAY CURVATURE</td>
</tr>
<tr>
<td>19.0500</td>
<td>0.520</td>
<td>0.540</td>
</tr>
</tbody>
</table>
S U R F A C E  D A T A

<table>
<thead>
<tr>
<th>BEARING NUMBER</th>
<th>OUTER</th>
<th>CLA ROUGHNESS</th>
<th>ROLL ELM</th>
<th>OUTER</th>
<th>RMS ASPERITY</th>
<th>SLOPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.15</td>
<td>.15</td>
<td>.10</td>
<td>2.000</td>
<td>2.000</td>
<td>2.000</td>
</tr>
<tr>
<td>2</td>
<td>.08</td>
<td>.08</td>
<td>.04</td>
<td>2.000</td>
<td>2.000</td>
<td>2.000</td>
</tr>
</tbody>
</table>

L U B R I C A N T  D A T A

<table>
<thead>
<tr>
<th>BEARING NUMBER</th>
<th>DESIGNATION</th>
<th>KINEMATIC VISCOSITY</th>
<th>DENSITY AT</th>
<th>THERMAL EXP/M</th>
<th>THERMAL CONDUCTIVITY</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(37.78 C)</td>
<td>(15.56 C)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>MIL-L-7808G</td>
<td>12.76</td>
<td>3.20</td>
<td>.9526</td>
<td>7.80E-04</td>
</tr>
<tr>
<td>2</td>
<td>MIL-L-7808G</td>
<td>12.76</td>
<td>3.20</td>
<td>.9526</td>
<td>7.80E-04</td>
</tr>
</tbody>
</table>

L U B R I C A T I O N  A N D  F R I C T I O N  D A T A

<table>
<thead>
<tr>
<th>BEARING NUMBER</th>
<th>PERCENT LUBE IN CAVITY</th>
<th>FILM REPLENISHMENT LAYER THICKNESS</th>
<th>ASPERITY FRICTION COEFFICIENT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ROLL ELM + RACEWAY</td>
<td>OUTER</td>
<td>INNER</td>
</tr>
<tr>
<td>1</td>
<td>.08</td>
<td>.9000E-03</td>
<td>.3000E-03</td>
</tr>
<tr>
<td>2</td>
<td>1.00</td>
<td>.6000E-03</td>
<td>.2000E-03</td>
</tr>
</tbody>
</table>
### Fit Data and Material Properties

<table>
<thead>
<tr>
<th>BEARING NUMBER</th>
<th>COLD FITS</th>
<th>SHAFT</th>
<th>HOUSING</th>
<th>EFFECTIVE WIDTHS</th>
<th>SHAFT</th>
<th>INNER RING</th>
<th>OUTER RING</th>
<th>HOUSING</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>0.0380</td>
<td>0.0250</td>
<td>24.000</td>
<td>19.000</td>
<td>19.000</td>
<td>22.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>0.0500</td>
<td>0.0380</td>
<td>22.000</td>
<td>17.000</td>
<td>17.000</td>
<td>20.000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Effective Diameters

<table>
<thead>
<tr>
<th>BEARING NUMBER</th>
<th>SHAFT I.D.</th>
<th>BEARING BORE</th>
<th>INNER RING</th>
<th>OUTER RING</th>
<th>BEARING I.D.</th>
<th>HOUSING O.D.</th>
<th>O.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>45.000</td>
<td>54.000</td>
<td>73.000</td>
<td>65.000</td>
<td>120.000</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>20.000</td>
<td>100.000</td>
<td>127.000</td>
<td>195.000</td>
<td>160.000</td>
<td>265.000</td>
<td></td>
</tr>
</tbody>
</table>

### Bearing Number (1)

<table>
<thead>
<tr>
<th>MODULUS OF ELASTICITY</th>
<th>204083.0</th>
<th>204083.0</th>
<th>204083.0</th>
<th>204083.0</th>
<th>204083.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>POISSONS RATIO</td>
<td>0.3000</td>
<td>0.3000</td>
<td>0.3000</td>
<td>0.3000</td>
<td>0.3000</td>
</tr>
<tr>
<td>WEIGHT DENSITY</td>
<td>7.806</td>
<td>7.806</td>
<td>7.806</td>
<td>7.806</td>
<td>7.806</td>
</tr>
<tr>
<td>COEFF. OF THERMAL EXP.</td>
<td>0.0001224</td>
<td>0.0001224</td>
<td>0.0001224</td>
<td>0.0001224</td>
<td>0.0001224</td>
</tr>
</tbody>
</table>

### Bearing Number (2)

<table>
<thead>
<tr>
<th>MODULUS OF ELASTICITY</th>
<th>204083.0</th>
<th>204083.0</th>
<th>204083.0</th>
<th>204083.0</th>
<th>204083.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>POISSONS RATIO</td>
<td>0.3000</td>
<td>0.3000</td>
<td>0.3000</td>
<td>0.3000</td>
<td>0.3000</td>
</tr>
<tr>
<td>WEIGHT DENSITY</td>
<td>7.806</td>
<td>7.806</td>
<td>7.806</td>
<td>7.806</td>
<td>7.806</td>
</tr>
<tr>
<td>COEFF. OF THERMAL EXP.</td>
<td>0.0001224</td>
<td>0.0001224</td>
<td>0.0001224</td>
<td>0.0001224</td>
<td>0.0001224</td>
</tr>
</tbody>
</table>

Unless otherwise stated, International units are used.

### Given Temperatures

<table>
<thead>
<tr>
<th>BRG</th>
<th>SHAFT</th>
<th>I. RING</th>
<th>I. RACE</th>
<th>ROLL EL.</th>
<th>O. RACE</th>
<th>O. RING</th>
<th>HOUSING</th>
<th>BULK</th>
<th>FLANGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>66.00</td>
<td>71.00</td>
<td>74.00</td>
<td>77.00</td>
<td>74.00</td>
<td>71.00</td>
<td>66.00</td>
<td>57.00</td>
<td>-0.00</td>
</tr>
<tr>
<td>2</td>
<td>177.00</td>
<td>182.00</td>
<td>185.00</td>
<td>189.00</td>
<td>188.00</td>
<td>185.00</td>
<td>177.00</td>
<td>141.00</td>
<td>-0.00</td>
</tr>
</tbody>
</table>
20 GEOMETRIC SECTIONS 1 LOAD SECTION(S), 2 BEARINGS. MODULUS OF ELASTICITY = 2.041E+05

<table>
<thead>
<tr>
<th>POSITION</th>
<th>INNER DIAM., LEFT</th>
<th>OUTER DIAM., LEFT</th>
<th>POINT FORCE</th>
<th>POINT MOMENT</th>
<th>LOAD INTENSITY LEFT</th>
<th>RIGHT</th>
<th>POS. ERR DEFL/FOR</th>
<th>ANG. ERR DEFL/MOM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>29.0</td>
<td>1</td>
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<td>0.0</td>
<td>0.0</td>
<td>29.0</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>63.5</td>
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<td>0.0</td>
<td>45.0</td>
<td>3 -0.000 -0.</td>
<td>-0.000 -0.</td>
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<td></td>
</tr>
<tr>
<td>4</td>
<td>69.0</td>
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<td>0.0</td>
<td>52.0</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>5</td>
<td>71.0</td>
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<td>0.0</td>
<td>44.0</td>
<td>5</td>
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<td></td>
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<tr>
<td>6</td>
<td>74.0</td>
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<td>0.0</td>
<td>44.0</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>73.0</td>
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<td>0.0</td>
<td>42.0</td>
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</tr>
<tr>
<td>8</td>
<td>322.0</td>
<td>0.0</td>
<td>0.0</td>
<td>42.0</td>
<td>8</td>
<td></td>
<td></td>
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<tr>
<td>9</td>
<td>324.0</td>
<td>0.0</td>
<td>0.0</td>
<td>44.0</td>
<td>9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>326.0</td>
<td>0.0</td>
<td>0.0</td>
<td>56.0</td>
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<tr>
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<td>340.0</td>
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<td>56.0</td>
<td>11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>355.0</td>
<td>0.0</td>
<td>0.0</td>
<td>85.0</td>
<td>12</td>
<td>2224.1 -4000.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>366.0</td>
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<td>0.0</td>
<td>85.0</td>
<td>13</td>
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**SNABERTH/SKF**  **TECHNOLOGY DIVISION**  **SKF INDUSTRIES INC.**  **SNABERTH/SKF**

WPABF LUBRICATION BRANCH  * R.E.B. TEST MACHINE ASSEMBLY 1  * SOLUTION LEVEL 2

SHAFT GEOMETRY, BEARING LOCATIONS, AND SHAFT LOAD:  **PLANE X - Z.**

28 GEOMETRIC SECTIONS  1 LOAD SECTION(S).  2 BEARINGS.  MODULUS OF ELASTICITY = 2.841E+05

**THRUST LOAD = 8.896E+03**

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### BEARING SYSTEM OUTPUT METRIC UNITS

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<th>DZ</th>
<th>GY</th>
<th>GZ</th>
<th>FX</th>
<th>FY</th>
<th>FZ</th>
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### FATIGUE LIFE (HOURS)

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<th>BEARING</th>
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### TEMPERATURES RELEVANT TO BEARING PERFORMANCE (DEGREES CENTIGRADE)

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<th>I. RACE</th>
<th>I. FLNG.</th>
<th>ROLL. EL.</th>
<th>O. FLNG.</th>
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### BEARING SYSTEM OUTPUT METRIC UNITS

#### FRICIONAL HEAT GENERATION RATE (Watts) AND FRICTION TORQUE (N-MM)

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<th>I. RACE</th>
<th>I. FLNGS</th>
<th>R.E. DRAG</th>
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#### END FILM THICKNESS, FILM REDUCTION FACTORS AND HEAT CONDUCTIVITY DATA FOR THE OUTER AND INNER RACEWAYS RESPECTIVELY

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<th>Brg.</th>
<th>FILM (MICRONS)</th>
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<th>THERMAL FACTOR</th>
<th>MENISCUS_DIST. (MM)</th>
<th>CONDUCTIVITY (W/DEG.C)</th>
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#### FIT PRESSURES (N/MM²)

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**CAGE DATA** METRIC UNITS

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<th>CALCULATED SPEED (RAD/SEC)</th>
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<th>V W Z</th>
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### ROLLING ELEMENT OUTPUT FOR BEARING NUMBER 2 METRIC UNITS

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### Rolling Element Output for Bearing Number 2 Metric Units

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*Notes:*
- **Angle (Deg.):** The angle in degrees for the bearing number.
- **Cage:** The normal forces in Newtons for the cage.
- **Outer & Inner:** The stress (N/mm²) for the outer and inner races.
- **Load Ratio:** The ratio of the load on the outer and inner races.
- **Contact Angles (Deg.):** The contact angles measured in degrees.