MUTUAL ADMITTANCE BETWEEN SLOTS ON A CYLINDER,

S. W. Lee,
S. Safavi-Naini
R. Mittra

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Washington, D.C. 20361
In the design of conformal slot array on the surface of a conducting cylinder, the calculation of the mutual admittance $Y_{12}$ is a crucial step, which has been studied extensively in recent years. In this paper, we summarize, in a handbook format, all of the final formulas of $Y_{12}$ and present some typical numerical data.
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1. INTRODUCTION

In the design of a conformal slot array on the surface of a conducting cylinder, the calculation of the mutual admittance $Y_{12}$ is a crucial step, which has been studied extensively in recent years. In this paper, we summarize, in a handbook format, all of the final formulas of $Y_{12}$, and present some typical numerical data.

2. STATEMENT OF PROBLEM

Referring to Figure 1, two identical slots, circumferential or axial, are located on the surface of an infinitely long cylinder. The geometrical parameters are

$$R = \text{radius of the cylinder}$$

$$\begin{align*}
(a,b) &= \text{dimensions of the slot along } (\phi,z) \text{ directions (a is the arc length along the cylinder)} \\
(z_0,R\phi_0) &= \text{center-to-center distances between slots}
\end{align*}$$

$$s_0 = \sqrt{z_0^2 + (R\phi_0)^2}$$

$$\theta_0 = \tan^{-1}(z_0/R\phi_0)$$

The problem is to determine the mutual admittance between these two slots when $kR$ is large.
Figure 1. Two identical slots on the surface of a cylinder.
First let us define mutual admittance. Throughout this work we always assume that

(i) the slots are thin, and
(ii) their length is roughly a half-wavelength.

Then the aperture field in each slot can be adequately approximated by a simple cosine distribution, which is the so-called "one-mode" approximation. For example, if slot 1 is a circumferential (lower slot in Figure 1a), its aperture field under the "one-mode" approximation is given by

\[ \vec{E} = V_1 \hat{e}_1, \quad \vec{H} = I_1 \hat{h}_1 \]  

where

\[ \hat{e}_1 = \hat{z} \sqrt{\frac{2}{ab}} \cos \frac{n \pi}{a} y, \quad \hat{h}_1 = \hat{x} \times \hat{e}_1 \]  

\[ y = R \phi . \]

\((V_1, I_1)\) are respectively the modal (voltage, current) of slot 1. The mutual admittance \(Y_{12}\) is defined by

\[ Y_{12} = Y_{21} = \frac{I_{21}}{V_1} \]

where \(I_{21}\) is the induced current in slot 2 when slot 1 is excited by a voltage \(V_1\) and slot 2 is short-circuited. An alternative expression for \(Y_{12}\) is

\[ Y_{12} = \frac{1}{V_1 V_2} \iint_{A_2} \vec{E}_2 \times \vec{H}_1 \cdot ds_2 \]

where

- \(A_2\) = aperture of slot 2
- \(\vec{H}_1\) = magnetic field when slot 1 is excited with voltage \(V_1\), and slot 2 is covered by a perfect conductor
- \(\vec{E}_2\) = electric field when slot 2 is excited with voltage \(V_2\), and slot 1 is covered by a perfect conductor.
Because $\hat{H}_1 = I_{21} \hat{h}_2$ and $\vec{E}_2 = V_2 \vec{e}_2$, it is a simple matter to verify that 
(2.8) and (2.9) are equivalent [1].

There is an alternative definition of mutual admittance. Instead of (2.7), a modal voltage $\bar{V}_1$ (with a bar) may be defined through the expression for the aperture field of slot 1 as follows:

$$\vec{E} = \hat{z} \frac{1}{b} \bar{V}_1 \cos \frac{\pi}{a} y$$

or equivalently

$$\bar{V}_1 = \int_{-b/2}^{b/2} (\hat{z} \cdot \vec{E})_{y=0} \, dz.$$  

(2.10a)

(2.10b)

Then a different mutual admittance $\bar{Y}_{12}$ is defined by (2.9) after replacing $(V_1, V_2)$ by $(\bar{V}_1, \bar{V}_2)$. It can be easily shown that

$$\bar{Y}_{12} = \frac{a}{2b} Y_{12}.$$  

(2.11)

Two remarks are in order: (i) In the limiting case that $b \to 0$, $Y_{12}$ goes to zero as $b$, whereas $\bar{Y}_{12}$ approaches a constant independent of $b$. (ii) For the special case $a = \lambda/2$ and $R \to \infty$, it is $\bar{Y}_{12}$ that is identical to the mutual impedance $Z_{12}$ between two corresponding dipoles calculated by the classical Carter's method [2], [3], [4]. (iii) When the slots are excited by waveguides (transmission lines), one often uses $Y_{12}$ ($\bar{Y}_{12}$). From here on, we will concentrate on $Y_{12}$ instead of $\bar{Y}_{12}$.

The mutual admittance defined in (2.8) and (2.9) includes the self admittance $Y_{11}$ as a special case which occurs when two slots coincide. (All the formulas of $Y_{12}$ given in this paper, except for the one in Section 4, can be used for calculating $Y_{11}$ by setting $\varphi_0 \to 0$ and $z_0 \to 0$.)
3. EXACT HUGHES (GSP) MODAL SOLUTION

Once the one-mode approximation in (2.7) is accepted, \( Y_{12} \) can be determined exactly in terms of cylindrical modal functions, as has been done by Stewart, Golden, and Pridmore-Brown [5], [6]. The final result reads:

**Circumferential slots**

\[
Y_{12} = \int_{-\infty}^{\infty} dk_z \sum_{m=-\infty}^{\infty} \psi(m, k_z) G(m, k_z) e^{-j(m\phi_0 + k_z z_0)}
\]

where

\[
\psi(m, k_z) = \frac{ab}{8R} \frac{\sin^2(k b/2)}{(k z/2)^2} \cdot \frac{\sin(m\phi_a + \pi/2) - \sin(m\phi_a - \pi/2)}{\left[\frac{m\phi_a + \pi/2}{m\phi_a - \pi/2}\right]^2}
\]

\[
\phi_a = \left(\frac{a}{2R}\right)
\]

\[
G(m, k_z) = \frac{j k}{k t} \left[ \frac{H_m^{(2)'}(k_z R)}{H_m^{(2)}(k_z R)} + \left(\frac{k z}{k t}\right)^2 \frac{H_m^{(2)}(k_z R)}{H_m^{(2)'}(k_z R)} \right]
\]

and

\[
k_t = \begin{cases} 
\sqrt{k_z^2 - k^2}, & \text{if } k \geq k_z \\
-j \sqrt{k_z^2 - k^2}, & \text{if } k < k_z
\end{cases}
\]

**Axial slots**

\[
Y_{12} = \int_{-\infty}^{\infty} dk_z \sum_{m=-\infty}^{\infty} \phi(m, k_z) F(m, k_z) e^{-j(m\phi_0 + k_z z_0)}
\]

where

\[
\phi(m, k_z) = \frac{ab}{8R} \left(\frac{\sin(m\phi_a)}{(m\phi_a)^2} \cdot \frac{\cos(k b/2)}{(k z/2)^2 - (\pi/2)^2}\right)^2
\]

\[
F(m, k_z) = \frac{j k}{k t} \left(\frac{H_m^{(2)'}(k_z R)}{H_m^{(2)}(k_z R)} \right)
\]
This solution is suitable for numerical calculation if (i) \( z_0 < b \) for circumferential slots, and \( z_0 < a \) for axial slots, (ii) \( kR \) is less than 20, and (iii) the medium is slightly lossy so that \( k \) has a small (negative) imaginary part. Based on this solution, extensive numerical results have been reported by Hughes Aircraft Company at Culver City [7], [8], [9].

4. EXACT UI MODAL SOLUTION

Under the one-mode approximation, another exact modal solution is given in [10]. This solution is derived from the Hughes (SGP) solution in Section 3 by a deformation of integration contour and an application of the Duncan transform [11]. The final result reads

Circumferential slots

\[
Y_{12} = G + jB
\]  

\[
G = \int_0^\infty \sum_{m=0}^{\infty} \frac{\cos m\phi \Omega}{\epsilon_m} \cos k z_0 \psi(m,k_z) R(m,k_z) \, dk_z
\]  

\[
B = \int_0^\infty \sum_{m=0}^{\infty} \frac{\cos m\phi \Omega}{\epsilon_m} \left\{ \int_0^k \left[ R(m,k_z) \psi(m,k_z) \sin k z_0 \, dk_z 
\right.ight.
\]

\[+ \int_0^\infty R(m,j\eta) \psi(m,j\eta) e^{-j\eta z_0} \, d\eta \right\}.
\]  

where

\[
R(m,k_z) = \frac{2}{\pi k R} \cdot \frac{k}{k_c} \cdot \left[ \frac{1}{N_m^2(k_z R)} + \frac{(mk_z R)^2}{k \cdot k_c} \cdot \frac{1}{N_m^2(k_z R)} \right]
\]  

\[
N_m^2(\chi) = J_m^2(\chi) + Y_m^2(\chi)
\]  

\[
N_m^2(\chi) = J_m^2(\chi) + Y_m^2(\chi)
\]  

\[
\epsilon_m = \begin{cases} 2, & m = 0 \\ 1, & m \neq 0 \end{cases}
\]
\( \psi(m, k_z) \) is defined in (3.2) and \( k_t \) in (3.4) (4.6)

**Axial slots**

\[
Y_{12} = \frac{\sqrt{2}}{\pi k R} \sum_{m=0}^{\infty} \frac{\cos m \phi_0}{e_m} \left( \int_0^k \phi(m, k_z) e^{-jk \zeta_0} \frac{dk_z}{N_m(k R)} + \int_0^z \theta(m, j \eta) e^{-\eta \zeta_0} \frac{d\eta}{N_m^2(R^2 + k^2)} \right)
\]

(4.7)

where \( \phi(m, k_z) \) is defined in (3.5)

This solution is valid only if \( z_0 > b \) for circumferential slots and \( z_0 > a \) for axial slots. It is suitable for numerical calculation if \( k R \) is less than 20.

5. **ASYMPTOTIC SOLUTION**

The two modal solutions given in Sections 3 and 4 are based on fields in the Fourier transform domain. An alternative calculation of \( Y_{12} \) involves the field in the spatial domain, namely,

**Circumferential slots**

\[
Y_{12} = \frac{-2}{ab} \int_{A_1} dy_1 dz_1 \int_{A_2} dy_2 dz_2 \left[ \cos \frac{\pi}{a} y_1 \right] \left[ \cos \frac{\pi}{b} (y_2 - R \phi_0) \right] g_\phi(s, \theta)
\]

(5.1)

**Axial slots**

\[
Y_{12} = \frac{-2}{ab} \int_{A_1} dy_1 dz_1 \int_{A_2} dy_2 dz_2 \left[ \cos \frac{\pi}{b} z_1 \right] \left[ \cos \frac{\pi}{b} (z_2 - z_0) \right] g_z(s, \theta)
\]

(5.2)

where \( (y_n, z_n) \) = a typical point in the aperture of slot \( n \) (\( n = 1 \) or 2).

\( A_n = \) aperture of slot \( n \)

(5.3)

\[
s = \sqrt{(y_2 - y_1)^2 + (z_2 - z_1)^2}
\]

(5.4)

\[
\theta = \tan^{-1}[(z_2 - z_1)/(y_2 - y_1)]
\]

(5.5)
Several versions of the Green's functions $g_\phi$ and $g_z$ have been approximately determined under the condition that $kR >> 1$. They are listed as follows:

**OSU Asymptotic solution** [12] [13]

\[
\begin{align*}
g_\phi & \sim G(s) \left[ v(\xi) \sin^2 \theta + \left( \frac{i}{k_s} \right) u(\xi) \cos^2 \theta \right] \quad (5.7) \\
g_z & \sim G(s) \left[ v(\xi) \cos^2 \theta + \left( \frac{i}{k_s} \right) u(\xi) \sin^2 \theta \right] \quad (5.8)
\end{align*}
\]

**PINT Asymptotic solution** [9] [14]

\[
\begin{align*}
g_\phi & \sim G(s) \left[ v(\xi) \sin^2 \theta + \frac{i}{k_s} (1 - 3 \sin^2 \theta) \right] \\
& \quad + \frac{i}{k_s} \sec^2 \theta [u(\xi) - v_1(\xi) \sin^2 \theta] \quad (5.9) \\
g_z & \sim G(s) \left[ \cos^2 \theta + \frac{i}{k_s} (2 - 3 \cos^2 \theta) \right] \quad (5.10)
\end{align*}
\]

**UI Asymptotic solution** [15]

\[
\begin{align*}
g_\phi & \sim G(s) \left[ v(\xi) \sin^2 \theta + \frac{i}{k_s} \cos^2 \theta \right] + \left( \frac{i}{k_s} \right) u(\xi) \left[ \cos^2 \theta (1 - \frac{2i}{k_s}) + \frac{1}{k_s} \sin^2 \theta \right] \\
& \quad + j(\sqrt{2} kR/\cos^2 \theta)^{-2/3} \left[ v'(\xi) \sin^2 \theta + (\tan^4 \theta + \frac{1}{k_s}) u'(\xi) \cos^2 \theta \right] \quad (5.11) \\
g_z & = G(s) \left[ v(\xi) \cos^2 \theta - \frac{i}{k_s} \cos^2 \theta \right] + \left( \frac{i}{k_s} \right) u(\xi) \left[ \sin^2 \theta (1 - \frac{2i}{k_s}) + \frac{1}{k_s} \cos^2 \theta \right] \\
& \quad + j(\sqrt{2} kR/\cos^2 \theta)^{-2/3} \left[ v'(\xi) \cos^2 \theta + (1 + \frac{i}{k_s}) u'(\xi) \sin^2 \theta \right] \quad (5.12)
\end{align*}
\]

where

\[
G(s) = \frac{k^2 Y_0 e^{-jks}}{2\pi} \quad , \quad Y_0 = (120\pi)^{-1} \quad (5.13)
\]

\[
\xi = (k \cos^4 \theta/2R^2)^{1/3} \quad (5.14)
\]

The Fock functions, $u$, $v$, etc., can be calculated from the following two representations:

For $0 < \xi < 0.7$
\[ v(\xi) = 1 - \frac{v}{4} e^{\frac{i\pi}{4}/\xi^3/2} + \frac{71}{60} \xi^3 + \frac{7v}{512} e^{-\frac{i\pi}{4}/\xi^3/2} - 4.141 \times 10^{-3\xi^6} \] (5.15)

\[ u(\xi) = 1 - \frac{v}{2} e^{\frac{i\pi}{4}/\xi^3/2} + \frac{51}{12} \xi^3 + \frac{5v}{64} e^{-\frac{i\pi}{4}/\xi^3/2} - 3.701 \times 10^{-2\xi^6} \] (5.16)

\[ v_1(\xi) = 1 + \frac{v}{2} e^{-\frac{i\pi}{4}/\xi^3/2} - \frac{71}{12} \xi^3 - \frac{7v}{64} e^{-\frac{i\pi}{4}/\xi^3/2} + 4.555 \times 10^{-2\xi^6} \] (5.17)

\[ v'(\xi) = \frac{3v}{8} e^{-\frac{i\pi}{4}/\xi^3/2} + \frac{21}{20} \xi^2 + \frac{63v}{1024} e^{-\frac{i\pi}{4}/\xi^3/2} - 2.485 \times 10^{-2\xi^6} \] (5.18)

\[ u'(\xi) = \frac{3}{4} v e^{-\frac{i\pi}{4}/\xi^3/2} + \frac{51}{4} \xi^2 + \frac{45v}{128} e^{-\frac{i\pi}{4}/\xi^3/2} - 2.221 \times 10^{-1\xi^5} \] (5.19)

For \( 0.7 \leq \xi \leq \infty \)

\[ v(\xi) = e^{-\frac{i\pi}{4}/\xi^3/2} \xi^{1/2} \sum_{n=1}^{10} \frac{(t')^{-1} e^{-j\xi t' n}}{n^2} \] (5.20)

\[ u(\xi) = e^{\frac{i\pi}{4}/\xi^3/2} \xi^{3/2} \sum_{n=1}^{10} \frac{-j\xi t' n}{e^n} \] (5.21)

\[ v_1(\xi) = e^{\frac{i\pi}{4}/\xi^3/2} \xi^{3/2} \sum_{n=1}^{10} \frac{-j\xi t' n}{n^2} \] (5.22)

\[ v'(\xi) = \frac{1}{2} e^{-\frac{i\pi}{4}/\xi^3/2} \xi^{-1/2} \sum_{n=1}^{10} (1 - j2\xi t' n)(t')^{-1} e^{-j\xi t' n} \] (5.23)

\[ u'(\xi) = e^{\frac{i\pi}{4}/\xi^3/2} \xi^{1/2} \sum_{n=1}^{10} \left(1 - j\frac{2}{3} t' n\right) e^{-j\xi t' n} \] (5.24)

where \( t_n = |t_n| \exp(-j\pi/3) \), \( t'_n = |t'_n| \exp(-j\pi/3) \), and

| \( n \) | \( |t_n| \) | \( |t'_n| \) | \( n \) | \( |t_n| \) | \( |t'_n| \) |
|---|---|---|---|---|---|
| 1 | 2.33811 | 1.01879 | 6 | 9.02265 | 8.48849 |
| 2 | 4.08795 | 3.24820 | 7 | 10.04017 | 9.53545 |
| 3 | 5.52056 | 4.82010 | 8 | 11.00852 | 10.52766 |
| 4 | 6.78671 | 6.16331 | 9 | 11.93602 | 11.47506 |
| 5 | 7.99413 | 7.37218 | 10 | 12.82878 | 12.38479 |
It has been verified through several hundred numerical examples that the UI asymptotic solution given above is in excellent agreement (within a quarter db in magnitude and a few degrees in phase) with the exact model solution for all slot separations \((\zeta_0', z_0')\) provided that \(kR \geq 5\).

In using the asymptotic solutions for calculating the self admittance \(Y_{11}\), care must be exercised in avoiding the singularity in the Green's function which occurs at \(s = 0\). A most convenient way to avoid this apparent difficulty is to (i) use a large number of points for the two surface integrals in (5.1) and (5.2), and (ii) shift slightly the integration nets for these two surface integrals.

6. EXACT PLANAR SOLUTION

In the limit \(kR \to \infty\) the Green's function of the UI solution in (5.11) and (5.12) is reduced to

\[
g_\phi = G(s)[\sin^2 \Theta + \frac{1}{k s} (2 - 3 \sin^2 \Theta)(1 - \frac{1}{k s})] \tag{6.1}
\]

\[
g_z = G(s)[\cos^2 \Theta + \frac{1}{k s} (2 - 3 \cos^2 \Theta)(1 - \frac{1}{k s})] \tag{6.2}
\]

When (6.1) and (6.2) are used in (5.1) and (5.2), we obtain the exact solution (under the "one-mode" approximation of course) for two slots on an infinitely large, conducting plane.

7. APPROXIMATE SOLUTION

Based on the UI asymptotic solution, a simple approximate solution, is reported in [10], i.e.,

Circumferential slots

\[
Y_{12} = \frac{8ab}{\pi^2} [S(b \sin \Theta) C(a \sin \Theta)]^2 g_\phi \tag{7.1}
\]
Axial slots

\[ Y_{12} = -\frac{8\alpha b}{\pi^2} [S(a \cos \theta) C(b \sin \theta)]^2 \tilde{g}_z \]  \hspace{1cm} (7.2)

where

\[ S(x) = \frac{\sin (kx/2)}{(kz/2)} , \quad C(x) = \frac{\cos (kx/2)}{1 - (kx/\pi)^2} . \]  \hspace{1cm} (7.3)

The (simplified) Green's functions \( \tilde{g}_\phi \) and \( \tilde{g}_z \) are given by

\[ \tilde{g}_\phi = G(s) \left[ v(\xi) \left( \sin^2 \theta + \frac{1}{k_s} \cos 2\theta \right) + \frac{1}{k_s} u(\xi) \cos^2 \theta \right. \]
\[ \left. + ju'(\xi)(\sqrt{2} kr \cos \theta)^{-2/3} \sin^4 \theta \right] \]  \hspace{1cm} (7.4)

\[ \tilde{g}_z = G(s) \left[ v(\xi) \left( \cos^2 \theta - \frac{1}{k_s} \cos 2\theta \right) + \frac{1}{k_s} u(\xi) \sin^2 \theta \right]. \]  \hspace{1cm} (7.5)

This solution gives an accurate numerical result (within several percent in magnitude and less than 5° in phase) provided that

\( kr \geq 10 \) and the slot separation is greater than two wavelengths.

8. CONCLUDING REMARKS

Based on extensive numerical data, we conclude that \( Y_{12} \) (including \( Y_{11} \) as a special case) can be best calculated by

(i) Hughes modal solution if \( kr \leq 5 \) and \( z_0 \) is less than the axial dimension of the slot,

(ii) UI modal solution if \( kr \leq 5 \) and \( z_0 \) is greater than the axial dimension of the slot, and

(iii) UI asymptotic solution if \( kr \geq 5 \) for all slot separations.

If several percents of error are acceptable, the approximate solution can be used if \( kr \geq 10 \) and the slot separation is greater than two wavelengths.
REFERENCES


APPENDIX A: NUMERICAL RESULTS

By using the formulas of $Y_{12}$ presented in the text, we have analyzed the following 6 slots:

<table>
<thead>
<tr>
<th>Slot</th>
<th>Type</th>
<th>Dimension</th>
<th>Suggested by</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Circumf.</td>
<td>$0.9'' \times 0.4''$ (f = 9 GHz)</td>
<td>Aerospace Hughes</td>
</tr>
<tr>
<td>B</td>
<td>Circumf.</td>
<td>$0.5\lambda \times 0.01\lambda$ (R in inch)</td>
<td>Hansen</td>
</tr>
<tr>
<td>C</td>
<td>Axial</td>
<td>$0.4'' \times 0.9''$ (f = 9 GHz)</td>
<td>Aerospace Hughes</td>
</tr>
<tr>
<td>D</td>
<td>Circumf.</td>
<td>$0.5\lambda \times 0.01\lambda$ (R in \lambda)</td>
<td>Hansen</td>
</tr>
<tr>
<td>E</td>
<td>Circumf.</td>
<td>$0.5\lambda \times 0.2\lambda$</td>
<td>Hansen</td>
</tr>
<tr>
<td>F</td>
<td>Axial</td>
<td>$0.5\lambda \times 0.2\lambda$</td>
<td></td>
</tr>
</tbody>
</table>

In all tables, $Y_{12}$ is listed in (db, phase in degree) format where $\text{db} = 20 \log_{10} (Y_{12} \text{ in mho})$. In all figures, the normalized phase of $Y_{12}$ is equal to $\text{Arg}(Y_{12} \text{exp} jk_0 \lambda)$. 

14
DATA SET A OF MUTUAL ADMITTANCE

(1) The mutual admittance $Y_{12}$ between two circumferential slots on an infinitely long cylinder is calculated from the

* (Exact) Hughes modal solution
* (Exact) UI modal solution
* UI asymptotic solution
* OSU asymptotic solution
* PINY asymptotic solution.

The parameters are

* Frequency: $f = 9$ GHz, $k = 4.7878$ (inch)$^{-1}$, $\lambda = 1.3123''$
* Cylinder: $R = 1.991''$ unless specified otherwise
* Slot A: Circumferential
  
  $a = 0.9'' = 0.6858\lambda$
  
  $b = 0.4'' = 0.3048\lambda$

\[
|Y_{11}| = 1.70747 \times 10^{-3} \text{ mho} = -55.35 \text{ db}
\]

\[
Y_g = 1.8155 \times 10^{-3} \text{ mho}
\]

* Center-to-center distance between two slots is $(R\phi_0, z_0)$.

(2) $Y_{12}$ is listed in (db value, phase in degree), where

\[
\text{db value} = 20 \log_{10} (|Y_{12}| \text{ in mho})
\]

(3) Data are presented in

TABLE A-1: $\phi_0 = 0$ and various $z_0$

A-2: $z_0 = 2''$ and various $\phi_0$

A-3: $z_0 = 0$ and various $\phi_0$

A-4: $\phi_0 = 0$ and various $z_0$.

Figure A-1: Mutual admittance $Y_{12}$ between two circumferential slots as a function of $\phi_0$.

A-2: Mutual admittance $Y_{12}$ between two circumferential slots as a function of $z_0$.

A-3: $|Y_{12}|$ on a cylinder (UI modal solution) and that on a plane as a function of $z_0$.

A-4: $Y_{12}$ on a cylinder as a function of the radius $R$ of the cylinder.
<table>
<thead>
<tr>
<th>$z_d$</th>
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<th>Asymptotic</th>
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<th>Exact Planar</th>
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<tr>
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<td>UI</td>
<td>UI</td>
<td>OSU</td>
<td>PINY</td>
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### TABLE A-2

$Y_{12}$ OF SLOT A FOR $z_o = 2''$

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<th>Asymptotic</th>
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<td>UI</td>
</tr>
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### TABLE A-3

$Y_{12}$ OF SLOT A FOR $z_o = 0$

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<td>UI</td>
</tr>
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<tr>
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</table>
### TABLE A-4

UI SOLUTIONS OF $y_{12}$ OF SLOT A FOR $\phi_o = 0$

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<th>$z_o$</th>
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<th>Asymptotic</th>
<th>$z_o$</th>
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</table>
Figure A-1. Mutual admittance $Y_{12}$ between two circumferential slots as a function $\phi_0$. 

$R = 1.99''$ 
$Z_0 = 2''$ 
$f = 9$ GHz
Figure A-2. Mutual admittance $Y_{12}$ between two circumferential slots as a function of $z_0$. 

$$R = 1.991''$$

$$\phi_0 = 0^\circ, f = 9 \text{ GHz}$$
Figure A-3: $|Y_{12}|$ on a cylinder (UI modal solution) and that on a plane as a function of $z_0$. 

CIRCUMFERENTIAL SLOTS

$\alpha = 0.9''$, $b = 0.4''$

$\phi_0 = 0^\circ$, $R = 1.991''$

$f = 9 \text{GHz}$
DATA SET B OF MUTUAL ADMITTANCE

(1) The mutual admittance $Y_{12}$ between two circumferential slots on an infinitely long cylinder is calculated from the
* (Exact) UI modal solution
* UI asymptotic solution.

The parameters are
* Frequency: $f = 9$ GHz, $k = 4.787787$ (inch)$^{-1}$, $\lambda = 1.3123''$
* Cylinder: $R = 1.991''$, 3.777'', 6''
* Slot B: Circumferential
  $a = 0.656168'' = 0.50\lambda$
  $b = 0.013123'' = 0.01\lambda$

* Center-to-center distance between two slots is $(r_0, z_0)$.

(2) $Y_{12}$ is listed in (db value, phase in degree), where

  \[
  \text{db value} = 20 \log_{10} (|Y_{12}| \text{ in mho}).
  \]

(3) Data are presented in

TABLE B-1: $\phi_0 = 0$ and various $z_0$
  B-2: $z_0 = 2''$ and various $\phi_0$
  B-3: $z_0 = 8''$ and various $\phi_0$
  B-4: Comparison of Hughes and UI solutions
| $\theta$ | $R = 1.991''$ | $R = 3.777''$ | $R = 6''$ | Exact Planar
<table>
<thead>
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TABLE B-2
UI SOLUTIONS OF $Y_{12}$ OF SLOT B FOR $z_o = 2''$

<table>
<thead>
<tr>
<th>$\phi_0$</th>
<th>$R = 1.991''$</th>
<th>$R = 3.777''$</th>
<th>$R = 6.0''$</th>
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<td>Modal</td>
</tr>
<tr>
<td>$10^\circ$</td>
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<td>$-102.04$</td>
<td>$-104.18$</td>
</tr>
<tr>
<td></td>
<td>$-125^\circ$</td>
<td>$-125^\circ$</td>
<td>$-140^\circ$</td>
</tr>
<tr>
<td>$20^\circ$</td>
<td>$-103.94$</td>
<td>$-104.11$</td>
<td>$-109.18$</td>
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<tr>
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<td>$-149^\circ$</td>
<td>$-148^\circ$</td>
<td>$142^\circ$</td>
</tr>
<tr>
<td>$30^\circ$</td>
<td>$-106.86$</td>
<td>$-107.20$</td>
<td>$-115.53$</td>
</tr>
<tr>
<td></td>
<td>$172^\circ$</td>
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</tr>
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<td>$45^\circ$</td>
<td>$-112.51$</td>
<td>$-112.98$</td>
<td>$-125.07$</td>
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<td>$92^\circ$</td>
<td>$93^\circ$</td>
<td>$169^\circ$</td>
</tr>
<tr>
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<td>$-119.28$</td>
<td>$-134.48$</td>
</tr>
<tr>
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<td>$-9^\circ$</td>
<td>$-81^\circ$</td>
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<td>$90^\circ$</td>
<td>$-131.40$</td>
<td>$-131.83$</td>
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<td>-------</td>
</tr>
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<td>-111.74</td>
<td>-113.45</td>
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<td>34°</td>
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<td>34°</td>
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<td>-114.40</td>
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<td>26°</td>
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<td>-112.83</td>
<td>-113.18</td>
<td>-115.94</td>
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<tr>
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<td>13°</td>
<td>-32°</td>
<td>-32°</td>
</tr>
<tr>
<td>$45^\circ$</td>
<td>-114.41</td>
<td>-115.12</td>
<td>-119.29</td>
</tr>
<tr>
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<td>-16°</td>
<td>-122°</td>
<td>-121°</td>
</tr>
<tr>
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<td>-117.70</td>
<td>-123.69</td>
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<td>-55°</td>
<td>118°</td>
<td>121°</td>
</tr>
<tr>
<td>$90^\circ$</td>
<td>-122.98</td>
<td>-124.10</td>
<td>-134.62</td>
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<tr>
<td>-161°</td>
<td>159°</td>
<td>169°</td>
<td>172°</td>
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TABLE B-4
COMPARISON OF HUGHES AND UI SOLUTIONS

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<tr>
<th>$\phi$</th>
<th>$\theta$</th>
<th>Hughes</th>
<th>UI</th>
<th>Hughes</th>
<th>UI</th>
<th>Hughes</th>
<th>UI</th>
<th>Hughes</th>
<th>UI</th>
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<td>Modal</td>
<td>Asymp</td>
<td>Modal</td>
<td>Asymp</td>
<td>Modal</td>
<td>Asymp</td>
</tr>
<tr>
<td>0°</td>
<td>0.5&quot;</td>
<td>-92.3 db</td>
<td>-79°</td>
<td>-92</td>
<td>-92.03</td>
<td>-78°</td>
<td>-92.83</td>
<td>-77°</td>
<td>-92.48</td>
</tr>
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<td>1&quot;</td>
<td>-96.5</td>
<td>153°</td>
<td>-96.31</td>
<td>152°</td>
<td>-96.28</td>
<td>153°</td>
<td>-97.18</td>
<td>157°</td>
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<td>33°</td>
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<td>-111.56</td>
<td>37°</td>
<td>-113.65</td>
<td>40°</td>
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<td>-6°</td>
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<td>-116.35</td>
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<td>-119.27</td>
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<td>-112.51</td>
<td>92°</td>
<td>-112.98</td>
<td>93°</td>
<td>-125.43</td>
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<td>-125.40</td>
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<td>-125.40</td>
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</tbody>
</table>

R = 1.991" R = 3.777" R = 6"
DATA SET C OF MUTUAL ADMITTANCE

(1) The mutual admittance $Y_{12}$ between two axial slots on an infinitely long cylinder is calculated from the

* (Exact) UI modal solution
* UI asymptotic solution

The parameters are

* Frequency: $f = 9$ GHz, $k = 4.7877$ (inch)$^{-1}$, $\lambda = 1.3123''$
* Cylinder: $R = 1.991''$, and other values
* Slot C: Axial
  \[ a = 0.4'' = 0.3048\lambda \]
  \[ b = 0.9'' = 0.6858\lambda \]
* Center-to-center distance between two slots is $(R\phi_0, z_0)$.

(2) $Y_{12}$ is listed in (db value, phase in degree), where db value = $20 \log_{10}$ ($|Y_{12}|$ in mho)

(3) Data are presented in

| TABLE C-1: $\phi_0 = 0, R = 1.991''$, and various $z_0$. |
|--------|--------|--------|
| C-2:   | $z_0 = 1.5''$, $R = 1.991''$, and various $\phi_0$. |
| C-3:   | $\phi_0 = 0$, $z_0 = 8''$, and various $R$. |

**Figure C-1:** $|Y_{12}|$ on a cylinder (UI modal solution) and that on a plane as a function of $z_0$. 

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### TABLE C-1

$Y_{12}$ OF SLOT C FOR $\phi_0 = 0^\circ$

<table>
<thead>
<tr>
<th>$z_0$</th>
<th>Modal</th>
<th>Asymp</th>
<th>$z_0$</th>
<th>Modal</th>
<th>Asymp</th>
</tr>
</thead>
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<tr>
<td>1&quot;</td>
<td>-77.38$_{db}$</td>
<td>-77.28</td>
<td>12&quot;</td>
<td>-123.86</td>
<td>-123.55</td>
</tr>
<tr>
<td></td>
<td>-590</td>
<td>-59°</td>
<td></td>
<td>134°</td>
<td>130°</td>
</tr>
<tr>
<td>2&quot;</td>
<td>-92.00</td>
<td>-91.86</td>
<td>14&quot;</td>
<td>-127.50</td>
<td>-126.23</td>
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<tr>
<td></td>
<td>8°</td>
<td>6°</td>
<td></td>
<td>-51°</td>
<td>-59°</td>
</tr>
<tr>
<td>3&quot;</td>
<td>-99.48</td>
<td>-99.25</td>
<td>16&quot;</td>
<td>-128.96</td>
<td>-128.55</td>
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<tr>
<td></td>
<td>89°</td>
<td>86°</td>
<td></td>
<td>115°</td>
<td>112°</td>
</tr>
<tr>
<td>4&quot;</td>
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<td>-104.36</td>
<td>18&quot;</td>
<td>-131.64</td>
<td>-130.60</td>
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<tr>
<td></td>
<td>172°</td>
<td>170°</td>
<td></td>
<td>-68°</td>
<td>-76°</td>
</tr>
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<td>-108.28</td>
<td>20&quot;</td>
<td>-133.39</td>
<td>-132.43</td>
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<tr>
<td></td>
<td>-103°</td>
<td>-106°</td>
<td></td>
<td>102°</td>
<td>95°</td>
</tr>
<tr>
<td>6&quot;</td>
<td>-111.94</td>
<td>-111.48</td>
<td>24&quot;</td>
<td>-136.07</td>
<td>-135.59</td>
</tr>
<tr>
<td></td>
<td>-17°</td>
<td>-21°</td>
<td></td>
<td>81°</td>
<td>77°</td>
</tr>
<tr>
<td>7&quot;</td>
<td>-114.61</td>
<td>-114.17</td>
<td>28&quot;</td>
<td>-138.79</td>
<td>-138.27</td>
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<tr>
<td></td>
<td>68°</td>
<td>64°</td>
<td></td>
<td>72°</td>
<td>60°</td>
</tr>
<tr>
<td>8&quot;</td>
<td>-116.93</td>
<td>-116.5</td>
<td>32&quot;</td>
<td>-141.24</td>
<td>-140.59</td>
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<tr>
<td></td>
<td>151°</td>
<td>149°</td>
<td></td>
<td>59°</td>
<td>42°</td>
</tr>
<tr>
<td>9&quot;</td>
<td>-119.28</td>
<td>-118.55</td>
<td>36&quot;</td>
<td>-143.68</td>
<td>-142.63</td>
</tr>
<tr>
<td></td>
<td>-122°</td>
<td>-126°</td>
<td></td>
<td>39°</td>
<td>25°</td>
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</tbody>
</table>
TABLE C-2

Y₁₂ OF SLOT C FOR z₀ = 1.5"

<table>
<thead>
<tr>
<th>θ₀</th>
<th>Modal</th>
<th>Asymptotic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-86.58 db</td>
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<tr>
<td></td>
<td>151°</td>
<td>149°</td>
</tr>
<tr>
<td>0°</td>
<td>-86.41</td>
<td>-85.15</td>
</tr>
<tr>
<td></td>
<td>-26°</td>
<td>-38°</td>
</tr>
<tr>
<td>30°</td>
<td>-87.43</td>
<td>-85.77</td>
</tr>
<tr>
<td></td>
<td>84°</td>
<td>72°</td>
</tr>
<tr>
<td>60°</td>
<td>-93.02</td>
<td>-91.04</td>
</tr>
<tr>
<td></td>
<td>169°</td>
<td>156°</td>
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</tbody>
</table>
TABLE C-3

Y_{12} OF SLOT C FOR \phi = 0 and z_o = 8''

<table>
<thead>
<tr>
<th>R (in)</th>
<th>Modal</th>
<th>Asymptotic</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.995''</td>
<td>-118.07 db</td>
<td>-116.55</td>
</tr>
<tr>
<td></td>
<td>150°</td>
<td>148°</td>
</tr>
<tr>
<td>1.991''</td>
<td>-116.93</td>
<td>-116.50</td>
</tr>
<tr>
<td></td>
<td>151°</td>
<td>149°</td>
</tr>
<tr>
<td>3.982''</td>
<td>-116.91</td>
<td>-116.47</td>
</tr>
<tr>
<td></td>
<td>150°</td>
<td>149°</td>
</tr>
<tr>
<td>5.973''</td>
<td>-116.90</td>
<td>-116.46</td>
</tr>
<tr>
<td></td>
<td>154°</td>
<td>149°</td>
</tr>
<tr>
<td>7.964''</td>
<td>-116.89</td>
<td>-116.45</td>
</tr>
<tr>
<td></td>
<td>154°</td>
<td>149°</td>
</tr>
<tr>
<td>11.946''</td>
<td>-116.84</td>
<td>-116.45</td>
</tr>
<tr>
<td></td>
<td>153°</td>
<td>149°</td>
</tr>
<tr>
<td>15.928''</td>
<td>-116.82</td>
<td>-116.45</td>
</tr>
<tr>
<td></td>
<td>153°</td>
<td>149°</td>
</tr>
<tr>
<td>19.910''</td>
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<td>-116.44</td>
</tr>
<tr>
<td></td>
<td></td>
<td>149°</td>
</tr>
</tbody>
</table>
Figure C-1: $|Y_{12}|$ on a cylinder (UI modal solution) and that on a plane as a function of $z_0$. 

**AXIAL SLOTS**

$a = 0.4''$  
$b = 0.9''$  
$\phi_0 = 0^\circ$  
$R = 1.991''$  
$f = 9 \, \text{GHz}$
DATA SET D OF MUTUAL ADMITTANCE

(1) The mutual admittance $Y_{12}$ between two circumferential slots on an infinitely long cylinder from the

* (Exact) UI modal solution
* UI asymptotic solution

The parameters are

* Cylinder: $R = 1\lambda$, $2\lambda$, $4\lambda$, $10\lambda$, $\infty$ (planar)
* Slot D: Circumferential
  
  a = 0.5\lambda

b = 0.01\lambda

* Center-to-center distance between two slots is $(R_0, z_0)$

(2) $Y_{12}$ is listed in (db value, phase in degree), where

$$\text{db value} = 20 \log_{10} (|Y_{12}| \text{ in mho})$$

(3) Data are presented in

<table>
<thead>
<tr>
<th>TABLE</th>
<th>$\phi_0$</th>
<th>R</th>
<th>z_0</th>
</tr>
</thead>
<tbody>
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<td>various</td>
</tr>
<tr>
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<td>0</td>
<td>various R</td>
<td>z_0</td>
</tr>
<tr>
<td>D-3</td>
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<td>z_0</td>
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<td>various R</td>
<td>$\phi_0$</td>
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<td>0</td>
<td>1\lambda</td>
<td>various R</td>
</tr>
<tr>
<td>D-6</td>
<td>0</td>
<td>5\lambda</td>
<td>various R</td>
</tr>
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</table>
**TABLE D-1**

UI SOLUTIONS OF $y_{12}$ OF SLOT D FOR $\phi_o = 0$ and $R = 2\lambda$

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>Modal</th>
<th>Asymptotic</th>
</tr>
</thead>
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<tr>
<td>1$\lambda$</td>
<td>$-98.60 \text{ db}$ 71°</td>
<td>$-98.56$ 71°</td>
</tr>
<tr>
<td>2$\lambda$</td>
<td>$-103.87 \text{ db}$ 74°</td>
<td>$-103.84$ 75°</td>
</tr>
<tr>
<td>3$\lambda$</td>
<td>$-106.98 \text{ db}$ 75°</td>
<td>$-106.96$ 75°</td>
</tr>
<tr>
<td>4$\lambda$</td>
<td>$-109.17 \text{ db}$ 74°</td>
<td>$-109.16$ 75°</td>
</tr>
<tr>
<td>5$\lambda$</td>
<td>$-110.84 \text{ db}$ 73°</td>
<td>$-110.85$ 75°</td>
</tr>
<tr>
<td>6$\lambda$</td>
<td>$-112.19 \text{ db}$ 73°</td>
<td>$-112.21$ 74°</td>
</tr>
<tr>
<td>7$\lambda$</td>
<td>$-113.32 \text{ db}$ 72°</td>
<td>$-113.35$ 74°</td>
</tr>
<tr>
<td>8$\lambda$</td>
<td>$-114.28 \text{ db}$ 72°</td>
<td>$-114.33$ 73°</td>
</tr>
<tr>
<td>9$\lambda$</td>
<td>$-115.12 \text{ db}$ 71°</td>
<td>$-115.18$ 73°</td>
</tr>
<tr>
<td>10$\lambda$</td>
<td></td>
<td>$-115.94$ 72°</td>
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</table>
### TABLE D-2

UI ASYMPTOTIC SOLUTIONS OF $Y_{12}$ OF SLOT D FOR $\phi_o = 0$

<table>
<thead>
<tr>
<th>$z_o$</th>
<th>$R = 1\lambda$</th>
<th>$R = 2\lambda$</th>
<th>$R = 4\lambda$</th>
<th>$R = 10\lambda$</th>
<th>Planar ($R = \infty$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>81.51 90°</td>
<td>81.51 90°</td>
<td>81.51 90°</td>
<td></td>
</tr>
<tr>
<td>1 λ</td>
<td>-97.49 67°</td>
<td>-98.56 71°</td>
<td>-99.45 74°</td>
<td>-99.51 76°</td>
<td>-99.76 77°</td>
</tr>
<tr>
<td>2 λ</td>
<td>-102.39 69°</td>
<td>-103.84 75°</td>
<td>-104.63 79°</td>
<td>-105.13 81°</td>
<td>-105.47 83°</td>
</tr>
<tr>
<td>3 λ</td>
<td>-105.26 69°</td>
<td>-106.96 75°</td>
<td>-107.92 80°</td>
<td>-108.52 83°</td>
<td>-108.93 86°</td>
</tr>
<tr>
<td>4 λ</td>
<td>-107.25 68°</td>
<td>-109.16 75°</td>
<td>-110.25 80°</td>
<td>-110.94 84°</td>
<td>-111.40 87°</td>
</tr>
<tr>
<td>5 λ</td>
<td>-108.76 67°</td>
<td>-110.85 75°</td>
<td>-112.05 80°</td>
<td>-112.81 84°</td>
<td>-113.33 87°</td>
</tr>
<tr>
<td>6 λ</td>
<td>-109.97 67°</td>
<td>-112.21 75°</td>
<td>-113.51 80°</td>
<td>-114.34 84°</td>
<td>-114.91 88°</td>
</tr>
<tr>
<td>7 λ</td>
<td>-110.98 66°</td>
<td>-113.35 74°</td>
<td>-114.74 80°</td>
<td>-115.63 84°</td>
<td>-116.25 88°</td>
</tr>
<tr>
<td>8 λ</td>
<td>-111.85 65°</td>
<td>-114.33 73°</td>
<td>-115.80 79°</td>
<td>-116.75 84°</td>
<td>-117.40 88°</td>
</tr>
<tr>
<td>9 λ</td>
<td>-112.60 65°</td>
<td>-115.18 73°</td>
<td>-116.72 79°</td>
<td>-117.73 84°</td>
<td>-118.43 89°</td>
</tr>
<tr>
<td>10 λ</td>
<td>-113.27 64°</td>
<td>-115.94 72°</td>
<td>-117.55 79°</td>
<td>-118.61 84°</td>
<td>-119.34 89°</td>
</tr>
<tr>
<td>$z_o$</td>
<td>$R = 1\lambda$</td>
<td>$R = 2\lambda$</td>
<td>$R = 4\lambda$</td>
<td>$R = 10\lambda$</td>
<td>Planar ($R = \infty$)</td>
</tr>
<tr>
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<td>----------------</td>
<td>----------------</td>
<td>----------------</td>
<td>----------------</td>
<td>----------------</td>
</tr>
<tr>
<td>0.5\lambda</td>
<td>-93.01 db</td>
<td>-93.83</td>
<td>-94.27</td>
<td>-94.55</td>
<td>-94.74</td>
</tr>
<tr>
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<td>-116°</td>
<td>-115°</td>
<td>-114°</td>
<td>-113°</td>
</tr>
<tr>
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### TABLE D-4

UI ASYMPTOTIC SOLUTIONS OF $Y_{12}$ OF SLOT D FOR $z_0 = 0$

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**TABLE D-5**

UI ASYMPTOTIC SOLUTIONS OF $\psi_{12}$ OF SLOT D FOR $z_{0} = 1\lambda$

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<tr>
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<td>-105.62</td>
<td>-116.60</td>
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<tr>
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<td>48$^\circ$</td>
<td>4$^\circ$</td>
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<td>-115$^\circ$</td>
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<tr>
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<td>-111.94</td>
<td>-126.33</td>
<td>-148.10</td>
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<td>-18$^\circ$</td>
<td>-17$^\circ$</td>
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<tr>
<td>$45^\circ$</td>
<td>-107.99</td>
<td>-121.35</td>
<td>-138.27</td>
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<td>134$^\circ$</td>
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<td>111$^\circ$</td>
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<td>-129.91</td>
<td>-148.52</td>
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<tr>
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**TABLE D-6**

UI ASYMPTOTIC SOLUTIONS OF $\bar{V}_{12}$ OF SLOT D FOR $z_o = 5\lambda$

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<td>-111.98</td>
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<td>-130.02</td>
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<td>9°</td>
<td>14°</td>
<td>159°</td>
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</table>
DATA SET E OF MUTUAL ADMITTANCE

(1) The mutual admittance $Y_{12}$ between two circumferential slots on an infinitely long cylinder is calculated from the

* UI asymptotic solution

The parameters are

*Cylinder: $R = 1\lambda, 2\lambda, 4\lambda, 10\lambda$

*Slot E: Circumferential

\[ a = 0.5\lambda \]

\[ b = 0.2\lambda \]

*Center-to-center distance between two slots is $(R_o, z_0)$

(2) $Y_{12}$ is listed in (db value, phase in degree), where

\[ \text{db value} = 20 \log_{10} \left( |Y_{12}| \text{ in mho} \right) \]

(3) Data are presented in

**TABLE E-1:** $z_0 = 0$, various $\phi_0$ and $R$

E-2: $z_0 = 0.5\lambda$, various $\phi_0$ and $R$

E-3: $z_0 = 1\lambda$, various $\phi_0$ and $R$

E-4: $z_0 = 2\lambda$, various $\phi_0$ and $R$

E-5: $z_0 = 4\lambda$, various $\phi_0$ and $R$

E-6: $z_0 = 8\lambda$, various $\phi_0$ and $R$

E-7: Comparison of UI asymptotic and UI modal solutions

E-8: Comparison of UI asymptotic and UI modal solutions

**Figure E-1:** Mutual admittance $Y_{12}$ between two circumferential slots as a function of $\phi_0$.

E-2: Mutual admittance $Y_{12}$ between two circumferential slots as a function of $\phi_0$.

E-3: Mutual admittance $Y_{12}$ between two circumferential slots as a function of $\phi_0$.

E-4: Mutual admittance $Y_{12}$ between two circumferential slots as a function of $z_0$.

E-5: Mutual admittance $Y_{12}$ between two circumferential slots as a function of $z_0$. 

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**TABLE E-1**

UI ASYMPOTIC SOLUTIONS OF $Y_{12}$ OF SLOT E FOR $z_0 = 0$

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<th>$R = 2\lambda$</th>
<th>$R = 4\lambda$</th>
<th>$R = 10\lambda$</th>
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<tbody>
<tr>
<td>30°</td>
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<td>-105.83</td>
<td>-124.96</td>
</tr>
<tr>
<td></td>
<td>$25^0$</td>
<td>$153^0$</td>
<td>$121^0$</td>
<td>$52^0$</td>
</tr>
<tr>
<td>45°</td>
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<td>-102.35</td>
<td>-117.50</td>
<td>-138.57</td>
</tr>
<tr>
<td></td>
<td>$-110^0$</td>
<td>$-54^0$</td>
<td>$8^0$</td>
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<tr>
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<td>-127.44</td>
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<tr>
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<td>$440^0$</td>
<td>$101^0$</td>
<td>$49^0$</td>
<td>$77^0$</td>
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TABLE E-2
UI ASYMPTOTIC SOLUTIONS OF $y_{12}$ FOR $z_o = 0.5\lambda$

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<tbody>
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<td>$-68.46$ -114°</td>
<td>$-68.89$ -112°</td>
<td>$-69.16$ -111°</td>
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</tr>
<tr>
<td>10°</td>
<td>$-69.00$ -122°</td>
<td>$-72.97$ -132°</td>
<td>$-81.72$ 170°</td>
<td>$-98.38$ -146°</td>
</tr>
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<td>20°</td>
<td>$-72.67$ -137°</td>
<td>$-82.21$ 164°</td>
<td>$-95.39$ -39°</td>
<td>$-113.59$ -49°</td>
</tr>
<tr>
<td>30°</td>
<td>$-77.77$ -165°</td>
<td>$-90.67$ 64°</td>
<td>$-105.02$ 75°</td>
<td>$-124.52$ 32°</td>
</tr>
<tr>
<td>45°</td>
<td>$-85.89$ 130°</td>
<td>$-100.98$ -116°</td>
<td>$-116.60$ 50°</td>
<td>$-138.17$ 150°</td>
</tr>
<tr>
<td>60°</td>
<td>$-93.37$ 47°</td>
<td>$-109.75$ 51°</td>
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<td>$-149.69$ -90°</td>
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TABLE E-3
UI ASYMPTOTIC SOLUTIONS OF $Y_{12}$ FOR $Z_o = 1\lambda$

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<td>-74.28 78°</td>
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<td>-80.15 9°</td>
<td>-94.52 105°</td>
</tr>
<tr>
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<td>-91.02 161°</td>
<td>-110.75 116°</td>
</tr>
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<td>-100.56 -20°</td>
<td>-122.14 -18°</td>
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<tr>
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<td>-95.69 132°</td>
<td>-112.38 -22°</td>
<td>-136.13 111°</td>
</tr>
<tr>
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<td>-104.13 -42°</td>
<td>-122.57 -41°</td>
<td>-148.13 -124°</td>
</tr>
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<td>$R = 4\lambda$</td>
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<td>----------------</td>
<td>----------------</td>
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UI ASYMPTOTIC SOLUTIONS OF $Y_{12}$ FOR $z_o = 4\lambda$

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<td>-87.48 5°</td>
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TABLE E-6
UI ASYMMETRIC SOLUTIONS OF $y_{12}$ FOR $z_o = 8\lambda$

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TABLE E-7
COMPARISON OF UI ASYMPTOTIC AND UI MODAL SOLUTIONS

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<td>-79.91</td>
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<th>$R = 2\lambda$</th>
<th>Planar (Exact)</th>
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<td>Modal</td>
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<td>-114°</td>
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<td>73°</td>
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<td>-77.46</td>
<td>-77.24</td>
<td>-78.98</td>
<td>-78.67</td>
</tr>
<tr>
<td></td>
<td>68°</td>
<td>70°</td>
<td>75°</td>
<td>76°</td>
</tr>
<tr>
<td>4\lambda</td>
<td>-82.22</td>
<td>-82.10</td>
<td>-84.3</td>
<td>-84.01</td>
</tr>
<tr>
<td></td>
<td>66°</td>
<td>68°</td>
<td>75°</td>
<td>75°</td>
</tr>
<tr>
<td>8\lambda</td>
<td>-86.65</td>
<td>-86.7</td>
<td>-89.41</td>
<td>-89.18</td>
</tr>
<tr>
<td></td>
<td>62°</td>
<td>66°</td>
<td>72°</td>
<td>73°</td>
</tr>
</tbody>
</table>
Figure E-1. Mutual admittance $Y_{12}$ between two circumferential slots as a function of $\phi_0$. 
Figure E-2. Mutual admittance $Y_{12}$ between two circumferential slots as a function of $\phi_0$. 

Figure E-4. Mutual admittance $Y_{12}$ between two circumferential slots as a function of $z_0$. 

\[ |Y_{12}| \]
Figure 4-5. Mutual admittance $Y_{12}$ between two circumferential slots as a function of $z_0/\lambda$. 

- UI ASYM.
- APPROX.
- UI EXACT MODAL
DATA SET F OF MUTUAL ADMITTANCE

(1) The mutual admittance $Y_{12}$ between two axial slots on an infinitely long cylinder is calculated from the
*UI asymptotic solution

The parameters are
*Cylinder: $R = 1\lambda, 2\lambda, 4\lambda, 10\lambda$
*Slot F: Axial
  $a = 0.2\lambda$
  $b = 0.5\lambda$
*Center-to-center distance between two slots is $(R\phi, z_o)$

(2) $Y_{12}$ is listed in (db value, phase in degree), where
  $\text{db value} = 20 \log_{10} \left( |Y_{12}| \text{ in mho} \right)$.

(3) Data are presented in

<table>
<thead>
<tr>
<th>TABLE F-1</th>
<th>$z_o = 0$, various $\phi_o$ and R</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-2:</td>
<td>$z_o = 0.5\lambda$, various $\phi_o$ and R</td>
</tr>
<tr>
<td>F-3:</td>
<td>$z_o = 1\lambda$, various $\phi_o$ and R</td>
</tr>
<tr>
<td>F-4:</td>
<td>$z_o = 2\lambda$, various $\phi_o$ and R</td>
</tr>
<tr>
<td>F-5:</td>
<td>$z_o = 4\lambda$, various $\phi_o$ and R</td>
</tr>
<tr>
<td>F-6:</td>
<td>$z_o = 8\lambda$, various $\phi_o$ and R</td>
</tr>
<tr>
<td>F-7:</td>
<td>Comparison of UI asymptotic and UI modal solutions</td>
</tr>
<tr>
<td>F-8:</td>
<td>Comparison of UI asymptotic and UI modal solutions</td>
</tr>
<tr>
<td>F-9:</td>
<td>Comparison of asymptotic solutions</td>
</tr>
</tbody>
</table>

**Figure F-1**: Mutual admittance $Y_{12}$ between two axial slots as a function of $\phi_o$.

F-2: Mutual admittance $Y_{12}$ between two axial slots as a function of $\phi_o$.
F-3: Mutual admittance $Y_{12}$ between two axial slots as a function of $\phi_o$.


<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$R = 1\lambda$</th>
<th>$R = 2\lambda$</th>
<th>$R = 4\lambda$</th>
<th>$R = 10\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10°</td>
<td>-63.59 -12°</td>
<td>-67.11 -66°</td>
<td>-72.13 178°</td>
<td>-80.11 167°</td>
</tr>
<tr>
<td>20°</td>
<td>-67.13 -69°</td>
<td>-72.57 173°</td>
<td>-78.93 -75°</td>
<td>-88.04 -110°</td>
</tr>
<tr>
<td>30°</td>
<td>-70.46 -131°</td>
<td>-76.90 -49°</td>
<td>-83.98 -26°</td>
<td>-94.11 -32°</td>
</tr>
<tr>
<td>45°</td>
<td>-74.93 -130°</td>
<td>-82.36 -154°</td>
<td>-90.41 -6°</td>
<td>-102.17 -82°</td>
</tr>
<tr>
<td>60°</td>
<td>-78.97 -28°</td>
<td>-87.25 -6°</td>
<td>-96.20 -39°</td>
<td>-109.73 -164°</td>
</tr>
</tbody>
</table>
TABLE F-2
UI ASYMPTOTIC SOLUTIONS OF $Y_{12}$ FOR $z_o = 0.5\lambda$

<table>
<thead>
<tr>
<th>$\phi_o$</th>
<th>$R = 1\lambda$</th>
<th>$R = 2\lambda$</th>
<th>$R = 4\lambda$</th>
<th>$R = 10\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>-70.14 db 25°</td>
<td>-70.11 25°</td>
<td>-70.10 26°</td>
<td>-70.09 26°</td>
</tr>
<tr>
<td>10°</td>
<td>-74.24 -20°</td>
<td>-76.61 -94°</td>
<td>-77.20 139°</td>
<td>-81.28 144°</td>
</tr>
<tr>
<td>20°</td>
<td>-76.84 -101°</td>
<td>-77.58 133°</td>
<td>-80.64 -102°</td>
<td>-88.34 -123°</td>
</tr>
<tr>
<td>30°</td>
<td>-77.48 -112°</td>
<td>-79.63 10°</td>
<td>-84.79 6°</td>
<td>-94.24 -41°</td>
</tr>
<tr>
<td>45°</td>
<td>-79.13 90°</td>
<td>-83.71 -179°</td>
<td>-90.78 -19°</td>
<td>-102.22 77°</td>
</tr>
<tr>
<td>60°</td>
<td>-81.68 -6°</td>
<td>-88.04 -14°</td>
<td>-96.49 -50°</td>
<td>-109.76 -168°</td>
</tr>
</tbody>
</table>
TABLE F-3

UI ASYMPTOTIC SOLUTIONS OF $Y_{12}$ FOR $z_0 = 1\lambda$

<table>
<thead>
<tr>
<th>$\phi_o$</th>
<th>$R = 1\lambda$</th>
<th>$R = 2\lambda$</th>
<th>$R = 4\lambda$</th>
<th>$R = 10\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^\circ$</td>
<td>-86.65 db 173°</td>
<td>-86.63 172°</td>
<td>-86.61 172°</td>
<td>-86.60 172°</td>
</tr>
<tr>
<td>$10^\circ$</td>
<td>-87.35 172°</td>
<td>-87.92 134°</td>
<td>-86.18 33°</td>
<td>-84.30 78°</td>
</tr>
<tr>
<td>$20^\circ$</td>
<td>-88.37 128°</td>
<td>-86.51 26°</td>
<td>-84.78 -180°</td>
<td>-89.22 -160°</td>
</tr>
<tr>
<td>$30^\circ$</td>
<td>-88.07 70°</td>
<td>-85.58 -81°</td>
<td>-86.95 -51°</td>
<td>-94.63 -66°</td>
</tr>
<tr>
<td>$45^\circ$</td>
<td>-87.12 -15°</td>
<td>-87.09 109°</td>
<td>-91.80 -60°</td>
<td>-102.39 60°</td>
</tr>
<tr>
<td>$60^\circ$</td>
<td>-87.51 -99°</td>
<td>-90.13 -72°</td>
<td>-97.06 -81°</td>
<td>-109.84 179°</td>
</tr>
</tbody>
</table>

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TABLE F-4
UI ASYMPTOTIC SOLUTIONS OF \( y_{12} \) FOR \( \rho = 2\lambda \)

<table>
<thead>
<tr>
<th>( \phi )</th>
<th>( R = 1\lambda )</th>
<th>( R = 2\lambda )</th>
<th>( R = 4\lambda )</th>
<th>( R = 10\lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>-99.37 db 177°</td>
<td>-99.34 176°</td>
<td>-99.33 176°</td>
<td>-99.33 176°</td>
</tr>
<tr>
<td>10°</td>
<td>-99.72 176°</td>
<td>-100.00 157°</td>
<td>-98.96 93°</td>
<td>-92.20 144°</td>
</tr>
<tr>
<td>30°</td>
<td>-100.83 115°</td>
<td>-97.05 4°</td>
<td>-93.13 109°</td>
<td>-96.09 163°</td>
</tr>
<tr>
<td>45°</td>
<td>-99.82 49°</td>
<td>-95.40 126°</td>
<td>-95.21 150°</td>
<td>-103.04 7°</td>
</tr>
<tr>
<td>60°</td>
<td>-98.70 15°</td>
<td>-96.10 89°</td>
<td>-99.12 161°</td>
<td>-110.19 129°</td>
</tr>
<tr>
<td>( \phi_0 )</td>
<td>( R = 1\lambda )</td>
<td>( R = 2\lambda )</td>
<td>( R = 4\lambda )</td>
<td>( R = 10\lambda )</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>0°</td>
<td>-111.56 db</td>
<td>-111.54 -178°</td>
<td>-111.53 -178°</td>
<td>-111.52 -178°</td>
</tr>
<tr>
<td>10°</td>
<td>-111.78 177°</td>
<td>-111.97 168°</td>
<td>-111.81 132°</td>
<td>-105.41 -27°</td>
</tr>
<tr>
<td>20°</td>
<td>-112.38 164°</td>
<td>-112.36 126°</td>
<td>-108.23 21°</td>
<td>-100.03 -35°</td>
</tr>
<tr>
<td>30°</td>
<td>-113.06 143°</td>
<td>-111.16 67°</td>
<td>-104.80 -104°</td>
<td>-100.65 -154°</td>
</tr>
<tr>
<td>45°</td>
<td>-113.38 97°</td>
<td>-108.48 24°</td>
<td>-103.49 28°</td>
<td>-105.31 100°</td>
</tr>
<tr>
<td>60°</td>
<td>-112.64 47°</td>
<td>-107.29 -123°</td>
<td>-104.94 114°</td>
<td>-111.46 -67°</td>
</tr>
<tr>
<td>$\phi_0$</td>
<td>$R = 1\lambda$</td>
<td>$R = 2\lambda$</td>
<td>$R = 4\lambda$</td>
<td>$R = 10\lambda$</td>
</tr>
<tr>
<td>---------</td>
<td>----------------</td>
<td>----------------</td>
<td>----------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>$0^0$</td>
<td>-123.63 db -179°</td>
<td>-123.61 -179°</td>
<td>-123.61 -179°</td>
<td>-123.60 -179°</td>
</tr>
<tr>
<td>$10^0$</td>
<td>-123.78 -178°</td>
<td>-123.92 173°</td>
<td>-124.05 155°</td>
<td>-120.63 54°</td>
</tr>
<tr>
<td>$20^0$</td>
<td>-124.23 171°</td>
<td>-124.56 150°</td>
<td>-122.90 83°</td>
<td>-113.13 178°</td>
</tr>
<tr>
<td>$30^0$</td>
<td>-124.87 159°</td>
<td>-124.73 113°</td>
<td>-119.69 -2°</td>
<td>-110.52 137°</td>
</tr>
<tr>
<td>$45^0$</td>
<td>-125.87 131°</td>
<td>-123.26 46°</td>
<td>-116.53 145°</td>
<td>-111.57 36°</td>
</tr>
<tr>
<td>$60^0$</td>
<td>-126.32 94°</td>
<td>-121.57 -21°</td>
<td>-115.88 38°</td>
<td>-115.46 50°</td>
</tr>
</tbody>
</table>
TABLE F-7  
COMPARISON OF UI ASYMPTOTIC AND UI MODAL SOLUTIONS

<table>
<thead>
<tr>
<th>( z_0 )</th>
<th>( \phi_0 )</th>
<th>( R = 1\lambda )</th>
<th>( R = 2\lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Modal</td>
<td>Asym.</td>
<td>Modal</td>
</tr>
<tr>
<td>( 0^\circ )</td>
<td>-87.06 db</td>
<td>-86.65</td>
<td>-86.83</td>
</tr>
<tr>
<td></td>
<td>-171°</td>
<td>-173°</td>
<td>-172°</td>
</tr>
<tr>
<td>( 10^\circ )</td>
<td>-87.69</td>
<td>-87.32</td>
<td>-88.23</td>
</tr>
<tr>
<td></td>
<td>176°</td>
<td>172°</td>
<td>139°</td>
</tr>
<tr>
<td>( 20^\circ )</td>
<td>-88.91</td>
<td>-88.37</td>
<td>-87.64</td>
</tr>
<tr>
<td></td>
<td>139°</td>
<td>128°</td>
<td>35°</td>
</tr>
<tr>
<td>( 30^\circ )</td>
<td>-89.40</td>
<td>-88.97</td>
<td>-87.01</td>
</tr>
<tr>
<td></td>
<td>85°</td>
<td>70°</td>
<td>-72°</td>
</tr>
<tr>
<td>( 45^\circ )</td>
<td>-89.39</td>
<td>-87.32</td>
<td>-88.67</td>
</tr>
<tr>
<td></td>
<td>2°</td>
<td>-15°</td>
<td>119°</td>
</tr>
<tr>
<td>( 60^\circ )</td>
<td>-89.84</td>
<td>-87.72</td>
<td>-91.86</td>
</tr>
<tr>
<td></td>
<td>-83°</td>
<td>-99°</td>
<td>-61°</td>
</tr>
</tbody>
</table>
## TABLE F-8
COMPARISON OF UI ASYMPTOTIC AND UI MODAL SOLUTIONS

<table>
<thead>
<tr>
<th>$\phi_0$</th>
<th>$z_o/\lambda$</th>
<th>$R = 1\lambda$</th>
<th>$R = 2\lambda$</th>
<th>Planar (Exact)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Modal</td>
<td>Asym.</td>
<td>Modal</td>
<td>Asym.</td>
</tr>
<tr>
<td>0.5$\lambda$</td>
<td>--</td>
<td>-70.14 db 29$^\circ$</td>
<td>--</td>
<td>-70.11 29$^\circ$</td>
</tr>
<tr>
<td>1$\lambda$</td>
<td>-87.06 -171$^\circ$</td>
<td>-86.65 -173$^\circ$</td>
<td>-86.83 -172$^\circ$</td>
<td>-86.63 -172$^\circ$</td>
</tr>
<tr>
<td>2$\lambda$</td>
<td>-99.97 -174$^\circ$</td>
<td>-99.37 -177$^\circ$</td>
<td>-99.61 -176$^\circ$</td>
<td>-99.34 -176$^\circ$</td>
</tr>
<tr>
<td>4$\lambda$</td>
<td>-112.43 -175$^\circ$</td>
<td>-111.56 -178$^\circ$</td>
<td>-111.93 -177$^\circ$</td>
<td>-111.54 -178$^\circ$</td>
</tr>
<tr>
<td>8$\lambda$</td>
<td>-124.33 -174$^\circ$</td>
<td>-123.63 -179$^\circ$</td>
<td>-124.12 -177$^\circ$</td>
<td>-123.61 -179$^\circ$</td>
</tr>
</tbody>
</table>

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## TABLE F-9
### COMPARISON OF ASYMPTOTIC SOLUTIONS

<table>
<thead>
<tr>
<th>( \phi_0 )</th>
<th>( R = 2\lambda )</th>
<th></th>
<th></th>
<th>( R = 10\lambda )</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( z_0 )</td>
<td>UI Asym.</td>
<td>PINY</td>
<td>OSU</td>
<td>UI Asym.</td>
<td>PINY</td>
<td>OSU</td>
</tr>
<tr>
<td>10°</td>
<td>-100.00</td>
<td>-99.93</td>
<td>-105.37</td>
<td>-92.2</td>
<td>-92.51</td>
<td>-92.53</td>
</tr>
<tr>
<td>30°</td>
<td>-97.05</td>
<td>-97.85</td>
<td>-98.23</td>
<td>-96.09</td>
<td>-96.30</td>
<td>-96.30</td>
</tr>
<tr>
<td>45°</td>
<td>-95.40</td>
<td>-96.16</td>
<td>-96.09</td>
<td>-103.04</td>
<td>-103.25</td>
<td>-103.25</td>
</tr>
<tr>
<td>60°</td>
<td>-96.10</td>
<td>-96.68</td>
<td>-96.6</td>
<td>-110.19</td>
<td>-110.41</td>
<td>-110.41</td>
</tr>
<tr>
<td>89°</td>
<td>97°</td>
<td>96°</td>
<td>129°</td>
<td>131°</td>
<td>131°</td>
<td>131°</td>
</tr>
</tbody>
</table>
Figure F.1: Mutual admittance $Y_{12}$ between two axial slots as a function of $\phi_0$. 

Normalized phase $\phi_0$ vs. $\lambda_1^{1/2}$.
Figure P-2: Mutual admittance $Y_{12}$ between two axial slots as a function of $\phi_0$. Normalize phase.
Figure F-3: Mutual admittance $y_{12}$ between two axial slots as a function of $z_0/\lambda$. 
APPENDIX B: COMPUTER PROGRAM LISTING

This appendix contains the program listing of all solutions, except the exact Hughes modal solution, discussed in the text.
ASYMPTOTIC SOLUTIONS OF \( y_{12} \)

This program is used to solve for mutual admittance of dots either on a cylinder or on a plane.
There are two types of slots: 1) Circumferential; 2) Axial.
This program involves a lot of integrations which are basically solved
by summation method.

---

**Input Parameters of this Program**

---

**IPLAN** - control the program in dealing with 2 different cases:

1) In planar case, set \( IPLAN = 1 \)
2) In cylindrical case, set \( IPLAN = 2 \)

Following parameters are common to both \( IPLAN = 1 \) and \( IPLAN = 2 \):

- **CUM** - assign a logic value 'true' if we are interested in the
  circumferential case; otherwise assign 'false' to it.
- **AXIAL** - assign a logic value 'true' if we are interested in the axial case;
  otherwise assign 'false' to it.
- **A** - the longer length of a slot (measured in wavelength).
- **B** - the shorter length of a slot (measured in wavelength).
- **IPA** - number of subdivisions over the longer length.
- **IN** - number of subdivisions over the shorter length.
- **OSU** - OSU = 2 if we use OSU asymptotic. If not, OSU = 0.
- **PINY** - PINY = 3 if we use PINY asymptotic. If not, PINY = 0.
- **Z** - array of 20 elements at most. Each element stands for the
  separation between 2 points along Z axis (measured in mho).
- **NDE** - number of elements in Z. Don't be greater than 20.

Following parameters are only good for cylindrical case.
Just forget them if we are dealing with the planar case:

- **RADIUS** - array of different radii of a cylinder (measured in wavelength).
- **NP** - number of elements in radius. Don't be greater than 6.
- **PHI** - array of 6 elements at most. Each element represents the angular
  separation between two slots (measured in degree).
- **NPHE** - number of elements in PHI. Don't be greater than 6.

Following parameters are for the planar case:

- **YP** - array of 20 elements at most. Each of them is the distance
  between 2 slots along Y-axis (measured in wavelength).
- **NDE** - number of elements in YP. Don't be greater than 20.

---

Before each run, check the following parameters and make appropriate correction

Set a value to the normalization factor

**Note:**

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ASYMPTOTIC SOLUTIONS OF Y12

In order to solve for initial amplitudes of dots either on
or cylinders, a non-linear equation is obtained for each set of
parameters which are basically solved

By identifying the following parameters (if necessary):

- I PLAN=1
- I PLAN=2

- The program in dealing with 2 different cases:
  1) IN CYLINDRICAL CASE, SET I PLAN=1
  2) IN PLANAR CASE, SET I PLAN=2

- The parameters are common to both I PLAN=1 and I PLAN=2

- Assumed values of parameters: HAT=0, WAVELENGTH=10, I1=0.5

- The height between the dots (measured in wavelength)

- The following parameters are only valid for CYLINDRICAL CASE.

- The following parameters are only valid for PLANAR CASE.

- The following parameters are valid for both cases.

- Values of 2 should be inverted when Y=0 (measured in wavelength)

- Values of 2 should be greater than 25

- Values of a should be inverted when Y=0 (measured in wavelength)

- Values of a should be greater than 25

- Values of 2 should be inverted when Y=0 (measured in wavelength)

- Values of a should be greater than 25

- Values of b should be inverted when Y=0 (measured in wavelength)

- Values of b should be greater than 25

- Values of c should be inverted when Y=0 (measured in wavelength)

- Values of c should be greater than 25

- Values of d should be inverted when Y=0 (measured in wavelength)

- Values of d should be greater than 25

- Values of e should be inverted when Y=0 (measured in wavelength)

- Values of e should be greater than 25

- Values of f should be inverted when Y=0 (measured in wavelength)

- Values of f should be greater than 25

- Values of g should be inverted when Y=0 (measured in wavelength)

- Values of g should be greater than 25

- Values of h should be inverted when Y=0 (measured in wavelength)

- Values of h should be greater than 25

- Values of i should be inverted when Y=0 (measured in wavelength)

- Values of i should be greater than 25

- Values of j should be inverted when Y=0 (measured in wavelength)

- Values of j should be greater than 25

- Values of k should be inverted when Y=0 (measured in wavelength)

- Values of k should be greater than 25

- Values of l should be inverted when Y=0 (measured in wavelength)

- Values of l should be greater than 25

- Values of m should be inverted when Y=0 (measured in wavelength)

- Values of m should be greater than 25

- Values of n should be inverted when Y=0 (measured in wavelength)

- Values of n should be greater than 25

- Values of o should be inverted when Y=0 (measured in wavelength)

- Values of o should be greater than 25

- Values of p should be inverted when Y=0 (measured in wavelength)

- Values of p should be greater than 25

- Values of q should be inverted when Y=0 (measured in wavelength)

- Values of q should be greater than 25

- Values of r should be inverted when Y=0 (measured in wavelength)

- Values of r should be greater than 25

- Values of s should be inverted when Y=0 (measured in wavelength)

- Values of s should be greater than 25

- Values of t should be inverted when Y=0 (measured in wavelength)

- Values of t should be greater than 25

- Values of u should be inverted when Y=0 (measured in wavelength)

- Values of u should be greater than 25

- Values of v should be inverted when Y=0 (measured in wavelength)

- Values of v should be greater than 25

- Values of w should be inverted when Y=0 (measured in wavelength)

- Values of w should be greater than 25

- Values of x should be inverted when Y=0 (measured in wavelength)

- Values of x should be greater than 25

- Values of y should be inverted when Y=0 (measured in wavelength)

- Values of y should be greater than 25

- Values of z should be inverted when Y=0 (measured in wavelength)

- Values of z should be greater than 25
C AS SIGN THE AP PROPRIATE LOGIC VALUES TO AX IAL AN D CUM
CUM = FALSE,
AX IAL = TRUE.
C SET A = THE LENGTH OF LONGER SIDE OF SLOT AND B = THE LENGTH OF SHORTER ONE
A = 0.5
B = 0.2
C CHOOSE A PROPER INTEGRATION GRID
IPA = 14
IP8 = 2
C CHOOSE WHICH ASYMPTOTIC METHOD TO BE USED
UI = 0
OSU = 2
PI = 3
C AS SIGN THE NUMBER OF DIFFERENT VALUES OF SEPARATION BETWEEN 2 SLOTS
C ALONG X -DIRECTION.
C THEN CONSTRUCT A CORRESPONDING ARRAY Z OF KXZ ELEMENTS
NDZ = 1
N(1) = 0.5
C SET IPLAN = 1 FOR PLANAR CASE AND IPLAN=2 FOR CYLINDRICAL CASE.
C IF IPLAN=1, SKIP THIS SECTION TO THE NEXT ONE
C IF IPLAN=2, SET THE NUMBER OF DIFFERENT RADIUS OF CYLINDER TO NDR
C AND THEN CONSTRUCT A CORRESPONDING ARRAY RADIUS OF NDR ELEMENTS
C SET NUMBER OF DIFFERENT VALUES OF ANGULAR SEPARATION BETWEEN 2 SLOTS
C THEN CONSTRUCT A ARRAY OF PHI OF NDKH ELEMENTS (MAX. NO. OP ELEMENTS IS 6)
NDR = 4
RADIUS(1) = 1,
RADIUS(2) = 2,
RADIUS(3) = 4
RADIUS(4) = 10,
NPHI = 2
PHI(1) = 0,
PHI(2) = 10,
PHI(3) = 40,
PHI(4) = 60
C IF IPLAN = 2 YOU CAN PUT THE WHOLE DECK INTO THE READER NOW
C IF IPLAN=1, SET THE NUMBER OF DIFFERENT VALUES OF DISTANCE BETWEEN 2 SLOTS
C THE, CONSTRUCT A CORRESPONDING ARRAY YP OF NDY ELEMENTS
NDY = 1
YP(1) = 0.0001
C AFTER MAKING THE CORRECTIONS, YOU CAN PUT THE DECK INTO THE READER
CUM = 1
AXIAL = 2
NP(X - NOT.CUM) (CUM = 2
NP(X - NOT.AXIAL) (AXIAL = 1
IL = 1
TEST : PINY - OSU - UT
EP. TEST.EQ. 1 OR TEST.EQ. -2) IL = 2
IU = MAXD(PINY, OSU, UI)
PHI = PHITAN(1, DO)
GO TO (1, 2), IPLAN
1 WRITE (6, 47) 
GO TO 3
2 WRITE (6, 466) 
3 WRITE (6, 455) 
XK = 2, EP
DO 5 (I = 1, ICUM, IAXIAL
GO TO (4, 5), IU
4 A = A + XK
GO TO 6
5 SAVE A 
A = B
B = SAVE A
GO TO 1
J AS SIGN THE APPROPRIATE LOGIC VALUES TO AXIAL AND CIRCULAR

J 2 FALS = FALSE

J 3 3 = THE LENGTH OF LONGER SIDE OF SLOT AND R= THE LENGTH OF SHORTER ONE

J 4 L = A PROPER INTERPRETATION GRID

J 5 1 = 1

J 6 F IN A WHICH ASYMPOTIC METHOD TO BE USED

J M = 2

J N = 3

J 7 AS K A TABLE OF DIFFERENT VALUES OF SEPARATION BETWEEN 2 SLOTS

J 8 O R D = 0 liberty

J 9 P = 1 THE CORRESPONDING ARRAY Z OF 182 ELEMENTS

J 0 Q R = 1 FOR PLANAR CASE AND UPLAN=2 FOR CYLINDRICAL CASE.

J 1 F = 1 SKIP THIS SECTION TO THE NEXT ONE

J 2 A = 1 THE NUMBER OF DIFFERENT VALUES OF RADIUS TO USE

J 3 A = 1 THE NUMBER OF CORRESPONDING ARRAY FORMS OF SPRING ELEMENTS

J 4 A = 1 THE NUMBER OF DIFFERENT VALUES OF ANGULAR SEPARATION BETWEEN 2 SLOTS

J 5 A = 1 THE CORRESPONDING ARRAY YP OF 16Y ELEMENTS (MC, NO. OF ELEMENTS IN 6)

J 6 A = 1 THE NUMBER OF ELEMENTS

J 7 A = 1 IF YES, YOU CAN PUT THE WHOLE DECK INTO THE READER NOW.

J 8 A = 1 IF NO, THEN THE NUMBER OF DIFFERENT VALUES OF DISTANCE BETWEEN 2 SLOTS

J 9 A = 1 THE CORRESPONDING ARRAY YP OF 16Y ELEMENTS

J 0 A = 1 IF YES, YOU CAN PUT THE WHOLE DECK INTO THE READER NOW.

J 10 A = 1

J 11 A = 1

J 12 A = 1

J 13 A = 1

J 14 A = 1

J 15 A = 1

J 16 A = 1

J 17 A = 1
AO = ANX
BO = BKN
TEMP = IPB
IPB = IPA
IPA = TEMP
WIDTH1 = AO/IPA
WIDTH2 = BO/IPA
C1 = COEXP (DCXPLX (0, 0, -PI/3,))
C2 = COEXP (DCXPLX (0, 0, PI/4,))
F2 = DSQRT (PI)
Z1 = -AO/2, -WIDTH1/2
Z2 = -BO/2, -WIDTH2/2
GO TO (7, 3), -PLAN
DO 40 IOP = IL, IU
DO 30 TRAD = 1, HDR
RHO = XK*RADIUS (TRAD)
WRITE (6, 144) XK
IF (IOP = 2, 2) GO TO 9
WRITE (6, 333)
GO TO 11
WRITE (6, 222) A, AO, B, BO
IF (IPLAN, EQ. 1) GO TO 12
WRITE (6, 99) RADIUS (TRAD), RHO
GO TO (91, 92, 93), IOP
WRITE (6, 1233)
GO TO 12
WRITE (6, 1214)
GO TO 12
WRITE (6, 1235)
DO 60 JZ = 1, NDZ
Z0 = Z (JZ) * KK
IF (IPLAN, EQ. 1) NO PHI = NDY
DO 70 IY = 1, NDPHI
IF (IPLAN, EQ. 1) GO TO 13
Y0 = RHO*PI*PHI (IY) / 180.
IF (PHI (IY), EQ. 0.0) Y0 = 0.001
Y = Y0/KK
GO TO 14
Y = YP (IY)
Y = Y*KK
WRITE (6, 77) PHI (IY), Y, Z (JZ), Z0
YZ = Y0 - AO/2, -WIDTH1/2
ZZ = Z0 - BO/2, -WIDTH2/2
ZSUM = 0.
DO 60, K = 1, IP3
TY1 = Y1 + WIDTH1*K
DO 90, L = 1, IP3
TZ1 = Z1 + WIDTH2*L
DO 100 H = 1, IPA
TY2 = Y2 + WIDTH1*M
DO 110 H = 1, IPA
TZZ = ZZ + WIDTH2*N
R = DSQRT ((TY1 - TY1)**2 + (TZZ - TZ1)**2)
THEHA = DATAN2 ((TZZ - TZ1) / (TY2 - TY1))
IF (IPLAN, EQ. 1) GO TO 60
CALL CYLIND (IY, ZSUM)
GO TO 110
600 CALL PLANAR (IY, ZSUM)
110 CONTINUE
100 CONTINUE
90 CONTINUE
90 CONTINUE
YZ = ZSUM * (WIDTH1*WIDTH2)**2*(-2)/(AO*BO)
MAC = CDABS (YZ)
PHASE = DATAN2 (IMAG (YZ), DREAL (YZ)) * 180. / PI
ZEXPON = CDEXP (DCXPLX (0, 0, DSQRT (Y0**2 + Z0**2)))
ZPROD = ZY1*ZEXPON
PHN = DATAN2 (DCMAG (ZPROD), DREAL (ZPROD)) * 180. / PI
SUBROUTINE FOCK(X)
IMPLICIT COMPLEX*16 (C,Z)
IMPLICIT REAL*8 (A-D,P-Y)
REAL (10), TNP(10)
COMPLEX*16 CDMPUX
COMMON/CVF,CUP,CVPF,CUPF,CUP
COMMON P1,T1,T2,T1,T2,T1,T2,T1,T2,THETA
COMMON DATA,TN,TNPT,TH0,E1,E2,F2
F1=DSQRT(X)
F3=X/2.
CVF=3.
CUP=0.
CVPF=0.
CUPF=0.
DO 20 N=1,10
TH=TNP(N)
C3=CDMPUX(DCMLK(0,DO,-X))
C4=CDMPUX(DCMLK(0,DO,-X))
CVPF=CVF+C3*ZTNPI
CUPF=CUP+C4
20 CONTINUE
RETURN
ENTRY FOCK(X)
F3=DSQRT(X)
F2=F3*2.
71
THIS SUBROUTINE IS USED TO GET THE 'PLANAR' SOLUTION

SUBROUTINE PLANAR (IJ, ZSH)
IMPLICIT COMPLEX*16 (C,H,Z)
IMPLICIT REAL*8 (A,B,P,R,Y)
REAL*8 ZJ
COMMON PL, T1, T2, T3, T4, R, Theta
COMMON/OPT\*4, A, B, X, Y
GO TO (10, 20), IJ
10 XSH = 1.1, X1, X2, X3
XPL = X*(1/3)
X2 = X*(1/3)
X3 = X*(1/3)
HA = CD\*\*2 + DC\*\*2 + DC\*\*2
FACTOR = DC\*\*2 + DC\*\*2 + DC\*\*2
RETURN
20 Z1 = DC\*\*2 + DC\*\*2 + DC\*\*2
Z2 = DC\*\*2 + DC\*\*2 + DC\*\*2
HA = Z1 + Z2 + Z3
FACTOR = DC\*\*2 + DC\*\*2 + DC\*\*2
RETURN
END

THIS SUBROUTINE IS USED TO GET THE 'CYLINDRICAL' SOLUTION

SUBROUTINE CYLIND (IJ, ZSNH)
IMPLICIT COMPLEX*16 (C,H,Z)
IMPLICIT REAL*8 (A,B,P,R,Y)
REAL*8 Z20, KA
CALL TN\*10, THE\*10
COMMON T1, T2, T3, T4, R, Theta
COMMON/OPT\*4, A, B, X, Y
COMMON/OPT\*4, T2, T3, T4, R, Theta
COMMON/OPT\*4, A, B, X, Y
COMMON/OPT\*4, A, B, X, Y
COMMON/OPT\*4, A, B, X, Y
COMMON/OPT\*4, A, B, X, Y
COMMON/OPT\*4, A, B, X, Y
COMMON/OPT\*4, A, B, X, Y
COMMON/OPT\*4, A, B, X, Y
COMMON/OPT\*4, A, B, X, Y
HA = (0.1, 1./10.) \* CD\*\*2 + DC\*\*2 + DC\*\*2
FACTOR = DC\*\*2 + DC\*\*2 + DC\*\*2
RETURN
END
- HZ = A * DCO5 (THETHA) ** 2 + TH * DSIN (THETHA) ** 2
- HUH = H * DSIN (THETHA) ** 2 + TH * DCO5 (THETHA) ** 2
GO TO 500

22 HZ = ZGR * CVF
HZ = H * DCO5 (THETHA) ** 2 + TH * DSIN (THETHA) ** 2
HPHI = HP * DSIH (THETHA) ** 2 + TH * DCO5 (THETHA) ** 2
GO TO 500

23 HZ = (5, 1, 1) * (1 - 3, * DSIN (THETHA) ** 2) / 8
HZ = ZGR * CVF * (DCOS (THETHA) ** 2 + TH * 1, 1 * (2, -3, * DCO5 (THETHA) ** 2) / 8
HPHI = ZGR * (CVF * (DSIN (THETHA) ** 2 + TH * 1, 1) / 2)

500 ZGREEN = HPME
IF (JH, EQ, 2) ZGREEN = HZ
FACTOR = DCS (PL * FY1/A) * DCO5 (PL * (FY2 - YO) / A)
IF (JH, EQ, 2) FACTOR = DCO5 (PL * TW2/A) * DCO5 (PL * (TZY - ZC) / A)
RETURN
END
PROGRAM FOR COMPUTATION OF THE MUTUAL ADMITTANCE BETWEEN TWO
IDENTICAL CIRCUMFERENTIAL SLOTS ON A CYLINDER (UI MODAL SOLUTION)
REAL KO,KZ,KT,K2,KAKIBG
COMPLEX*15 Z1,Y12,ESINXP,YN12
REAL*8 PI(400), FM(400), FN(400), DIMAG, DREAL, DATAN2

INPUT PARAMETERS:
KO= WAVE NUMBER IN FREE SPACE IN TERMS OF 1/INCH
RHO=RADIUS OF CYLINDER <INCH>
A*= SLOT DIMENSION, A>B <INCH>
PHI0=ANGULAR SEPARATION OF THE SLOTS (CENTER TO CENTER) <RADIANS>
Z0= SEPARATION OF THE SLOTS IN Z-DIRECTION <INCH>
Y11=NORMALIZATION FACTOR
NMAX= MAXIMUM NO. OF TERMS WHICH HAS BEEN USED IN CALCULATION OF
INFINITE SERIES
NCYCLE=NO. OF SUBSECTIONS BETWEEN ANY TWO SUCCESSIVE ZEROS OF INTEGRAND
IN TRAPEZOIDAL RULE FOR NUMERICAL INTEGRATION
PI=3.14153265
Y0=1./(120.*PI)
KO=2.*PI
ESINXP=3.810*KZ/(2.*PI*2.54)
A=0.5
B=0.2
AKA=KO*A
BKZ=KO*B
NMAX=300
NCYCLE=40
Y11=1.
RHO=2.
RK=KO*RHC

WRITE (6,30)
30 FORMAT (40X,'MUTUAL ADMITTANCE OF SLOTS ON A CYLINDER/)
WRITE (6,50)
50 FORMAT (52X,'CIRCUMFERENTIAL/)
WRITE (6,51) PREC,NO
51 FORMAT (25X,'FREQUENCY',10X,'F=',E12.5,'<H2>',10X,'K=',E12.5,
      ,'<1/<INCH>/')
WRITE (6,52)
52 FORMAT (10X,'SLOT DIMENSIONS')
WRITE (6,53) AK,AKB,BKB
53 FORMAT (10X,'A=',E10.4,'<INCH>',4X,'KA=',E10.4,15X,'B=',E10.4,
      ,'<INCH>',4X,'KB=',E10.4/)
WRITE (6,54) RHO,3K
54 FORMAT (35X,'CYLINDER',10X,'R=',E12.5,'<INCH>',4X,'KR=',E12.5/)
WRITE (6,55)
55 FORMAT (35X,'METHOD OF SOLUTION Modal*/)
WRITE (6,56) Y11
56 FORMAT (50X,'NORMALIZATION [Y11]=',E10.4///)
WRITE (5,57)
57 FORMAT (55X,'DATA OUTPUT/)
PH10=PI/6.
CO=2.
PHIN=HALF ANGULAR WIDTH OF THE SLOT
PHIN=ARCSIN(A/(2.*RHO))
COMPUTATION OF INFINITE SERIES

C

1. MAX1 = MAX+1
2. J = 0, MAX1
3. T = MAX, 1
4. P = (M, EQ. 1), DPA = 2,
5. PHI1 = PHI

IF (ABS(PHI2*(1-E2/2.) + LE.1.E-7) PHIB1 = PHI1 = 1.001

1. F1 (1, N) = COS (1 + PHI2) * (-PI * COS (1 + PHI1) / (((1 + PHI1) ** 2 - (PI/2.) ** 2

IF (1)) ** 2 * (1/2PI)

C INTEGRATION OF KZ*H1(KZ) * EXP (-J*KZ*ZC) BETWEEN 0 AND KO

C DELTA = NEAREST HOURGLASS OF THE SINGULAR POINT KZ=KO TO WHICH THE INTEGRAL

C AND BEEN CALCULATED ANALYTICALLY

C DELTA = 0, J = 1.01*KO

C DELTA1 = NEAREST HOURGLASS OF THE SINGULAR POINT KZ=KO WHERE THE INTEGRAND

C VARIES RAPIDLY AND 'NDelta' SAMPLES HAVE BEEN USED.

C DELTA1 = 0, J = 1.01*KO

C JKZZ = (DELTA1 - DELTA) / NDDELTA

C NSECT1 = NO. OF SUBSECTIONS BETWEEN 0 AND KO-DELTA1

C NSECT1 = (IPIAX (B + ZC + RHO) * KO/PI) ** 2 * NCYCLE

C JKZ1 = (KO - DELTA1) / NSECT1

C NSECT1 = NSECT1 * DELTA + 2

C 1 = FIRST INTEGRAL (BETWEEN 0 AND KO)

C 1 = (3.9, 1)

C IF J = 0, NSECT1

C PI = 1) * NSECT1 + 1) GC TO 120

C = KO-DELTA1 + (1 - NSECT1 - 2) * JKZZ

C JKZ = JKZZ

C TO 140

C K = (L-1) * JKZ1

C F(KZ, EQ, J) A = C, C = 0.0001*KO

C JKZ = JKZ1

C 140 CI = 1.

C F (L, EQ, J, 1K) (L, EQ, NSECT1 + 1), OR (L, EQ, NSECT1 + 2), OR (L, EQ, NSECT)

C IF (CI = 0.5

C J = J + 1

C ELSE (2

C (KZ*KO - KZ*KZ)

C SIN(KZ/E/2) / (KZ/E/2)) ** 2 * CI ** 1 * BKZ * CEXP ((0.1, -1.1) * KZ * ZO)

C MAX1 = MAX

C 2 = 20.0 * KZ

C COMPUTATION OF FM(N) = 1/(JNK(X) ** 2 + YN(X) ** 2) AND FN(N) = 1/(JDN(X) ** 2

C JNK(X) ** 2) FOR X = E0*KX AND N = 0 TO MAX1, WHERE MAX1 IS A NUMBER AFTER

C WHICH THE CONTRIBUTIONS OF FM(N) AND FN(N) TO THE INFINITE SUM

C SCHEME NEGLIGIBLE. MAX1 IS A FUNCTION OF THE ARGUMENT X AND IS ALWAYS

C LESS THAN 1.0 EQUAL TO MAX1. MAX1, FM(N) AND FN(N) ARE CALCULATED

C BY SUBROUTING FM(N), MAX1, FM, FN).

C CALL FMN (ROK, MAX1, FM, FN)

C KZ = KZ (X/AT*KO*RHO) ** 2, 2 = 200, KZ = MAX1

C 1 = 1.

C 2 = (L, KZ ** 2) * (F (1, J) + 1.1, ** 2 * KZ**2 * RO * FN(N))

C 2 = 1, 1 = 1 + 1, SJ = 1 * JEXP (* PI (J)

C 1 = (2. + KO/PI*RHO) * (1 - F1(1) * CEXP ((0.1, -1.1) * KO * ZO) * (PI * PI) / (2. * KO))
\[
\begin{align*}
\text{(5)(2)} & =\frac{(243.032)}{(50.625)} \times 2/2, \quad \text{or}\quad (\text{5)(2)} = 2.667928728
\end{align*}
\]
PROGRAM TO COMPUTE THE MUTUAL ADMITTANCE BETWEEN TWO IDENTICAL
AXIAL SLOTS ON A CYLINDER ( UI NORMAL SOLUTION)

REAL X0, X1, Y, Z, ZKETO
COMPLEX*16 Y1, Y12, Y112
ALAN=3, F1(400), F2(400), IMAG, DEEL, DATAN2

PARAMETERS:
AO=Wave number in free space in terms of 1/inch
N=0. OF SUBSECTIONS BETWEEN ANY TWO SUCCESSIVE ZEROS OF INTEGRAND
M=TRAPEZOIDAL RULE FOR NUMERICAL INTEGRATION
MB=SLIT DIMENSION >A <INCH>
R=RADIUS OF CYLINDER <INCH>
MT=ANGULAR SEPARATION OF THE SLOTS (CENTER TO CENTER) <Radian>
ND=SEPARATION OF THE SLOTS IN Z-DIRECTION <INCH>
N11=NORMALIZATION FACTOR
NMAX=MAXIMUM NO. OF TERMS WHICH HAS BEEN USED IN CALCULATION OF
INFINITE SERIES

N=0
N=3.141525
ND=1.2(12.5*2)
R=2.*PI
ST=3.21*KO/(2.*PI*2.54)
(inertia=10)
A=0.5
x=0.2
KX=TA*A
K=TA*X*3
IO=2.
A=AO/3.

**

FORMAT (6, 5)
FORMAT (8, 5)
FORMAT (25, 'FREQUENCY',10X,'F=',E16.5,'<H2>',10X,'K=',E12.5,
'<INCH>',10X,'X=',E12.5)

1 FORMAT (6, 5)
2 FORMAT (15, 'SLOT DIMENSIONS')
3 FORMAT (6, 5) X, KA, X, RB
4 FORMAT (10, 'X=',E10.4,'<INCH>',4X,'KA=',E10.4,15X,'B=',E10.4,
'<INCH>',4X,'X=',E10.4)
5 FORMAT (6, 5) X, KA, X, RB
6 FORMAT (10, 'CYLINDER',10X,'F=',E12.5,'<INCH>',4X,'RR=',E12.5)

11 
11 FORMAT (6, 5)
12 FORMAT (15, 'METHOD OF SOLUTION *MODEA*)&
13 FORMAT (6, 5) N11
14 FORMAT (6, 5)
57 FORMAT (55, 'DATA OUTPUT')
ALAN=1174.
PHIA=HALF ANGULAR WIDTH OF THE SLOT
PHIA=2.*ASIN(A/(2.*RHO))

COMPUTATION OF INFINITE SERIES
M1AX12=M1AX+1
DO 10 C=M1,MMAX12
1=MM1
F(D,N)=IC IC 99
F(1)=COS(M1*PHIA)*SIN(M1*PHIA/2.)/(M1*PHIA/2.*)**2
GO TO 10
C
F(D)=.5
10 CONTINUE

INTEGRATION OF FSI(KZ)*F1(N,KZ)*EXP(-J*KZ*Z) BETWEEN 0 AND KO
INTEGRATION OF THE SINGULAR PART OF THE INTEGRAL HAS BEEN CALLED ANALYTICALLY
DELTA=K.E-7*KO
LSCT=IC IC OF SAMPLES IN THE INTERVAL (C., K0-DELTA).
LSCT=IC IC ((2.+C0+C0)*K0/PI+2.)*NCYCLE
KOZ=(K0-DELTA)/NSCT
LSCT=NSCT1+1
CL=(C.,K0)
LS=FIRST INTEGRAL (BETWEEN 0 AND KO)
DO 20 Z=1,NSCT
AK=(1-1.)*KS
F(KZ-E2.)*KS=0.00001*KO
JNC1=KZ
F((L.EG.1.)*KS,(L.EC. NCOUNT)) C1N=0.5
KZ=SQRT(AK*KZ-KZ*KZ)
F(ABS(KZ-E2.)*KS,LE.1.E-8)*KZ=1.00001*KZ
F*EXP=(COS((KZ-E2.)/(KZ-E2.)*2.-(E2.)*2.)*2.*C1N*KZ
*IC*EXP((C..-1.)*KZ**2)

F(JAX1=M1AX
J0K=AK*K1
COMPUTATION OF F1(K)=1./(JN(X)**2+YN(X)**2) AND FN(N)=1./(JN(X)**2+
YN(X)**2) FOR X=K0T AND N=0 TO M1AX1; WHERE M1AX1 IS A NUMBER AFTER
WHICH THE CONTRIBUTION OF FN(N) AND FN(N) TO THE INFINITE SUM
IS NEGLECTIBLE. M1AX1 IS A FUNCTION OF THE ARGUMENT X AND IS ALWAYS
LESS THAN OR EQUAL TO M1AX. FN(N) AND FN(N) ARE CALCULATED
BY SUBROUTINE FEPH(X,M1AX1,FN,FN).
CALL FPN(K0T,M1AX1,FN,FN)
DO 20 Z=1,MMAX1
4=MM1

E00 L=MM1+FN(D)*F1(D)*F1(D)
COMPUTATION OF 12 (BETWEEN ZERO AND ETAMAX; WHERE ETAMAX IS A NUMBER
AFTER WHICH THE INTEGRAND BECOMES VERY SMALL)
ZD=0.
ETAMAX=14.*(C..-E)
THE INTEGRATION IS CARRIED OUT BY TRAPEZOIDAL RULE. AT FIRST THE WHOLE
ANOE OF INTEGRATION (0.,ETAMAX) IS DIVIDED INTO TWO SUBINTERVALLS :
(0.,ETAT1) AND (ETAT1,ETAMAX), WHERE ETAT1=ETAMAX/2. THEN THE NUMERICAL
COMPUTATION OF THE INTEGRAL IS PERFORMED IN THESE SUBINTERVALS WITH THE
D N OF SAMPLES IN THE FIRST SUBINTERVAL TWO TIMES THAT IN THE SECOND ONE.
ETAT1=ETAMAX/4.0
20 CONTINUE
SUBROUTINE JESY

PURPOSE

COMPUTE THE Y BESSEL FUNCTION FOR A GIVEN ARGUMENT AND ORDER

USAGE

CALL JESY(X,N,BY,IER)

DESCRIPTION OF PARAMETERS

X - THE ARGUMENT OF THE Y BESSEL FUNCTION DESIRED

N - THE ORDER OF THE Y BESSEL FUNCTION DESIRED

BY - THE RESULTANT Y BESSEL FUNCTION

IER - RESULTANT ERROR CODE WHERE

IER=0 NO ERROR

IER=1 N IS NEGATIVE

IER=2 X IS NEGATIVE OR ZERO

IER=3 X HAS EXCEEDED MAGNITUDE CF 10**70

REMARKS

VERY SMALL VALUES OF X MAY CAUSE THE RANGE OF THE LIBRARY
FUNCTION ALG TO BE EXCEEDED

X MUST BE GREATER THAN ZERO

N MUST BE GREATER THAN OR EQUAL TO ZERO
SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
NONE

METHOD
RECURRENCE RELATION AND POLYNOMIAL APPROXIMATION TECHNIQUE
AS DESCRIBED BY A.J.M. HITCHCOCK, 'POLYNOMIAL APPROXIMATIONS
TO BESSEL FUNCTIONS OF ORDER ZERO AND ONE AND TO RELATED
FUNCTIONS', M.T.A.C., V.11, 1957, PP.86-88, AND G.N. WATSON,
'A TREATISE ON THE THEORY OF BESSEL FUNCTIONS', CAMBRIDGE
UNIVERSITY PRESS, 1958, P. 62

SUBROUTINE EESY(X,N,BY,IER)
CHECK FOR ERRORS IN N AND X
IF(N) 180,10,10
IER=0
IF(X) 190,190,20
SEARCH IF X LESS THAN OR EQUAL 4
IF(X-4.0) 40,40,30

COMPUTE Y0 AND Y1 FOR X GREATER THAN 4
30 T1=4.0/X
Z1=Z1*T1
P0=(((---0.00037043*Z2+.0000173565)*Z2-.0000487613)*T2
+0.00017343)*Z2+.01753062)*T2+.3989423
P1=(((0.00032312*Z2-.0000142078)*Z2+.0003424668)*T2
+0.003869744)*Z2+.003564324)*T2-.01246694
P2=(((0.000003214*Z2-.00000708)*Z2+.0000040920)*T2+.000580759)*T2
+0.0022223233)*T2+.0002921526)*T2+.3989423
J1=(((---0.00035694*Z2+.00001622)*Z2-.0000396708)*T2
+.0001069741)*T2-.0006390400)*T2+.03740084
a=2.0/SQRT(X)
J=AT1
C=X+.7853982
Z0=A*C*SIN(C)+B*C*COS(C)
Y1=-A*P1+COS(C)+B*Q1*SIN(C)
30 TO 90

COMPUTE Y0 AND Y1 FOR X LESS THAN OR EQUAL TO 4
40 Z=X/2.
Z2=Z*X**Z
Z=ALOG(XX)*.5772157
SUM=0.
ZSUM=T
I=T
20 TO 70 L=1,15
IF (L-1) 50, 60, 50
50 SUM = SUM + 1./FLOAT(L-1)
60 FL = L
70 IS = I-SUM
80 TERM = (TERM * (-2)/FL**2) * (1. - 1./ (FL*TS))
90 XO = Y0 + TERM
100 TERM = XX * (I-.5)
110 SUM = 0.
120 I = I + 1.
130 GO TO L-2, 15
140 SUM = SUM + 1./FLOAT(L-1)
150 FL = FL - 1.
160 IS = I-SUM
170 TERM = (TERM * (-2)/(FL*FL)) * ((TS-.5/FL) / (TS+.5/FL1))
180 I = I + 1.
190 PI2 = 6366198
200 YF = FL2*Y0
210 Y1 = PI2/Y + PI2*Y1

CHECK IF Y0 OR Y1 IS DESIRED
220 IF (N-1) 10, 110, 130
230 RETURN EITHER Y0 OR Y1 AS REQUIRED
240 IF (N) 110, 120, 110
250 Y0 = Y1
260 GO TO 170
270 GO TO 170

PERFORM RECURSIVE OPERATIONS TO FIND YN(X)
280 YA = Y0
290 YB = Y1
300 X0 = 1
310 XI = FLOAT(Z*K)/X
320 XC = YB - YA
330 IF (ABS(XC) - 1.0E70) 145, 145, 141
340 KBR = 3
350 RETURN
360 X0 = K + 1
370 IF (K-N) 150, 160, 150
380 XA = YB
390 XB = YC
400 GO TO 140
410 XA = YC
420 RETURN
430 KBR = 1
440 RETURN
450 KBR = 2
460 RETURN
END
SUBROUTINE BSLJZ(X, FJ, NMAX, A, ND, IERR, FJAPRX, RR)

THIS IS ONE OF THREE ROUTINES, "BSLJZ", "BSLIZ", AND "BSCJZ", BASED ON ALGORITHM 236 FROM "COMMUNICATIONS OF THE A.C.M.", AUGUST 1964. THIS ONE EVALUATES THE BESSEL FUNCTIONS OF THE FIRST KIND FOR REAL ORDERS AND NON-NEGATIVE REAL ARGUMENTS.

THE PARAMETERS ARE DESCRIBED AS FOLLOWS, WITH "(I)" "(O)" AND "(I/O)" INDICATING, RESPECTIVELY, THAT A PARAMETER IS TO BE SET ON ENTRY, WILL BE SET BY THE ROUTINE, OR BOTH:

*** ALL PARAMETERS EXCEPT "ND", "IERR", "NMAX" ARE ***

(I) Y = THE (NON-NEGATIVE) ARGUMENT TO THE BESSEL FUNCTIONS.

(I) FJ = AN ARRAY IN WHICH THE VALUES OF THE BESSEL FUNCTIONS ARE STORED. AS FOLLOWS: LET J(X;B) DENOTE THE VALUE OF THE BESSEL FUNCTION OF ORDER B WITH ARGUMENT X. THEN FOR I = 1 TO ABS(NMAX) + 1,

J(I) = J(I;A) + (I-1)*SIGN(NMAX). 

(I) NMAX = REFER TO "FJ".

(I) A = REFER TO "FJ". NORMALLY, 0 <= A < 1 BUT THE ALGORITHM WORKS WITH SOME LOSS OF ACCURACY, FOR A >= 1. SEE THE PROGRAM NOTES BELOW.

(I) ND = THIS GIVES THE NUMBER OF SIGNIFICANT FIGURES OF ACCURACY DESIRED IN THE FUNCTION VALUES.

(O) IERR = THIS IS AN ERROR FLAG WHICH IS SET TO 0 IF THE INPUT PARAMETERS ARE OKAY, AND TO SOME POSITIVE VALUE IF ONE OF THE PARAMETERS IS INVALID. REFER TO THE ERROR EXITS AT THE END OF THE CODE FOR A DETAILED LIST OF THE VALUES OF IERR.

(O) FJAPRX = A SCRATCH ARRAY USED BY THE ROUTINE. IT MUST HAVE AT LEAST AB(NMAX) + 1 ENTRIES.

(O) RR = ANOTHER SCRATCH ARRAY. IT TOO MUST HAVE AT LEAST AB(NMAX) + 1 ENTRIES.

OTHER ROUTINES CALLED: ( * INDICATES A LOCAL ROUTINE )

* NSO1Z = INVERSE FUNCTION OF X*LOG(X).

* UNDEPZ = ROUTINE TO CONTROL UNDERFLOW INTERRUPTS.

* DLOG = DOUBLE PRECISION LOGARITHM.

* DABS = ABSOLUTE VALUE.

* MOD = REMAINDER.

* DMAX1 = MAXIMUM OF 2 REALS.

NOTES:

THE METHOD OF COMPUTATION IS A VARIANT OF THE BACKWARD RECURRENCE ALGORITHM OF J.C.P. MILLER (REFERENCE 1). THE PURPORTED ACCURACY IS OBTAINED BY A JUDICIOUS SELECTION OF THE INITIAL VALUE "NO" OF THE RECURSION INDEX (REPRESENTED IN THE CODE BY THE VARIABLE "MNO") TOGETHER WITH AT LEAST ONE REPETITION OF THE RECURSION WITH "MNO" REPLACED BY "MNO+6, NEAR A ZERO OF ONE OF THE BESSEL FUNCTIONS. THE ACCURACY OF THAT PARTICULAR BESSEL FUNCTION MAY DECREASE TO LESS THAN "NO" SIGNIFICANT DIGITS. THE ALGORITHM IS MOST EFFICIENT WHEN X IS SMALL OR MODERATELY LARGE.

THE ABOVE PARAGRAPH IS TAKEN FROM GAUTSCHI'S PRESENTATION OF ALGORITHM 236 IN C.A.C.M. THE SELECTION OF THE INITIAL "NO" IS DONE WITH THE HELP OF THE FUNCTION NSO1Z, ALSO BY GAUTSCHI (AND CALLED "M") BY HIM. IN THIS CODE, THE FOLLLOWING SPECIAL CASES HAVE BEEN ADDED:

A. Y = 0 WHEN NMAX > 0 OR A = 0
B. A = 0 AND NMAX < 0
C. A >= 1: THE ALGORITHM WORKS IN THIS CASE, BUT THE INITIAL CHOICE OF "NO" IS NO LONGER OPTIMAL AND SOME ACCURACY IS LOST. SIMPLE TESTS INDICATE THAT ONLY A FEW DECIMAL PLACES ARE SACRIFICED AT WORST. A LIMIT OF LOGARITHM PLACED ON A TO AVOID OVERFLOW IN THE GAMMA FUNCTION TO AVOID COMPLICATIONS, NMAX IS REQUIRED TO BE NON-NEGATIVE IF A > 1.

83
REFERENCES:
1. GAUTSCHI, W., "RECURSIVE COMPUTATION OF SPECIAL FUNCTIONS", UNIVERSITY OF MICHIGAN ENGINEERING SUMMER CONFERENCES, NUMERICAL ANALYSIS, 1963.

**SUBROUTINE BSLIX**

SUBROUTINE BSLIX(X, PJ, NMAX, A, ND, IERR, PJAPRX, RF)
IMPLICIT REAL*8 (A-H,O-Z)
DOUBLE PRECISION NMAX
DIMENSION PJ(1), PJAPRX(1), RF(1)
LOGICAL NEVEN, AFLAG
DATA ONE/1.0D0/, TWO/2.0D0/, THREE/3.0D0/, C1/1.73205080756888/,
      SMALL/1.0D-15/, C2/1.35651551391306/,
      C3/2.0D0/, ABIG/6.5D0/, T0D5/2.5D0/,
      ALPH/7.0D0/, T16P/1.0D0/, T24P/1.0D0/, T32P/1.0D0/,
      T40P/1.0D0/, T48P/1.0D0/, T56P/1.0D0/,
C* INITIALIZE THE ERROR PARAMETER, TURN UNDERFLOW OFF, AND CHECK THE PARAMETERS FOR VALIDITY AND FOR THESE SPECIAL CASES:
A. X=0 WITH NMAX > 0 OR A=0
B. A=0 AND NMAX < 0
THE CODE DELIBERATELY AVOIDS TESTING MORE THAN ONE THING IN EACH LOGICAL "IF" BELOW BECAUSE OF I.B.M. FORTRAN INEFFECTIVENESS IN THIS REGARD.
C. IF A>1, NMAX MUST NOT BE NEGATIVE.
**********
CJ 1
IERR = 0
CALL UMDFPZ("OFF", SAVE)
IP(A) = LT. ZERO) GOTO 999
IP(A) = GT. ABIG) GOTO 998
IP(A) = LT. ZERO) GOTO 997
IP(A) = GT. C1) GOTO 10
IP(A) = GT. SMALL) GOTO 996
IP(A) = LT. ZERO) GOTO 994
IP(NMAX) = GT. ZERO) GOTO 40
IP(NMAX) = LT. ZERO) GOTO 20
**********
CJ 2
IF NMAX < 0, NMAX IS SET HERE SO THAT ONLY J(Y;A) IS CALCULATED. THE LOOPS FOLLOWING STATEMENTS 200 THEN CALCULATES THE REMAINING FUNCTIONS BY SIMPLE RECURSION.
IF A=0, NMAX IS SET SO THAT J(Y;A+N) = 0...NMAX ARE CALCULATED; THE CODE AFTER 800 THEN REVISES THE SIGN OF EVERY OTHER ONE.

WE FIRST HANDLE THE CASE X=0.
**********
CJ 20
NTEMP = FABS(NMAX) + 1
DO 10 I = 1, NTEMP
CJ 20
PJ(I) = ZERO
CJ 10
999
CJ 2
998
CJ 1
997
CJ 2
10
996
CJ 2
994
CJ 2
40
CJ 2
20
CJ 2
995
**********
**5C**

```
(1) = ZERO) FJ(1) = ONE
G = 1

**6C**

```
```
PLAM = (A-70. ZERO) .AND. (NMAX .LT. 0)
NMAX = NMAX
IP = (NMAX .LT. 0) NMAX = 1
NBSP = NMAX(NMAX+1)/2
IP(IP .LT. NMAX) GOTO 60
NMAX = - NMAX
NBSP = NMAX* + 1
DO 40 I = 1, J + 1
```

**8C**

```
PIJAPPR(Y) = ZERO
SYM = (Y/500)**(1/2)/GAMMA(ONE+1)
DI = C1*ND + C4
R = ZERO
IP = (NMAX .LT. 2) P = NMAX* NBS01Z(HALF*DI/NMAX)
S = C2 * Y * NBS01Z(C1*DI/X)
```

**10C**

```
THE RECIPROCAL Y/N IS DELIBERATELY CALCULATED AS A FLOATING
POINT NUMBER REP. THAN AN INTEGER. AND ALL COMPARISONS WITH IT
ARE DONE AS FLOATING POINT COMPARISONS.
```

```
YNJ = ONE + DMAX1(K,3)
XLIMIT = YNJ/2
TADA = A + Y
KY = Y - Y
PL = ONE
```

**20C**

```
THE OUTER ITERATION LOOP STARTS HERE.
THE FOLLOWING LOOP IS DONE ENTIRELY IN FLOATING POINT FOR
EFFICIENCY.
```

```
KN = YN + ONE
PL = PL * ((YV + 1)/(YV + ONE))
IP (YV .LT. XLIMIT) GOTO 260
OLDPL = PI
OLDYV = YV
```

```
C
```

```
KN = Z*YN
YN = N
NEWYN = *TRUE
R = Z*YN
C = Z*YN
TEMP1 = Z/Y
```

**30C**

```
IN THE FOLLOWING LOOP, THE SUCCESSIVE VALUES OF "R" ARE PARTIAL
FRACTIONS OF A CONTINUED FRACTION.
```

```
DENOM = TEMP1 * (A + YN) - R
IF (DENOM .LT. SMALL) DENOM = DENOM + SMALL
P = ONE/DENOM
```

85
PLMBDA = ZERO
PL (* NOT. NEVEN) GOTO 400
PL = PL * (XN + TWA)/(XN + TWA)
PLMBDA = PL * (XN + A)

400 IF (N * LP * NMAX) RR(N) = R
N = N - 1
XN = XN - ONE
NEVEN = * NOT. NEVEN
IF (N * GE. 1) GOTO 300

C******
C THE LATEST APPROXIMATIONS ARE CHECKED FOR IMPROVEMENT:
C******
500 IF (NMAX .EQ. 0) GOTO 600
500 PJ(N+1) = RR(N) * PJ(N)

C******
C THE FOLLOWING CODE IS A REDUCTION OF THE LOOP
DO 900 N = 2, NMAX + 1
900 PJ(N+1) = PJ(N) * (A - N) * PJ(N) - PJ(N-1)

800 CONTINUE
IF (NMAX .GE. 0) GOTO 1000

C******
IF NMAX < 0, WE HAVE FINISHED OBTAINING J(X;A), AND NOW
ITERATE TO FIND ALL THE DESIRED FUNCTIONS.

C FIRST WE CHECK FOR THE SPECIAL CASE A=0.
C******
850 IF (* NOT. AFLAG) GOTO 850
NMAX = NMAX + 1
850 PJ(2) = TWO * A * PJ(1)/X - PJ(2)
820 RJ(N) = - PJ(N)
GOTO 1000

C******
C THE FOLLOWING CODE IS A REDUCTION OF THE LOOP
DO 900 N = 2, NMAX + 1
900 PJ(N+1) = PJ(N) * (A - N) * PJ(N) - PJ(N-1)

800 CONTINUE
IF (NMAX .GE. 0) GOTO 1000

C******
C THE FOLLOWING CODE IS A REDUCTION OF THE LOOP
DO 900 N = 2, NMAX
900 PJ(N+1) = (2/X) * (A - N) * PJ(N) - PJ(N-1)

NMAX = NMAX + 1

86
FJNM2 = PJ(1)
FJNM1 = PJ(2)
TEMP1 = TWO/X
OVER = ZERO
XNM1 = TWO

C DO 880 N = 3, NMAXT
FJN = TEMP1 * (A - XNM1) * PJNM1 - PJNM2
FJNM2 = PJHM1
PJ(N) = PJN
XNM1 = XNM1 + ONE
OVER = OVER + ONE
FJNM1 = PJNM1/C5
PJNM2 = PJNM2/C5
CONTINUE

880 C IF(NMAXT .LE. 3) GOTO 1000
OVER = ZERO
SCALE = ONE
GOTO 1000
C DO 900 N = 4, NMAXT
FJ = TEMP1 * (A - XNM1) * PJNM1 - PJNM2
OVER = OVER + ONE
FJNM1 = PJNM1/C5
PJNM2 = PJNM2/C5
CONTINUE
GOTO 1000
C

C**** ERROR EXITS FOLLOW. MEANINGS OF THE EXIT VALUES OF "IERR" ARE:
0 :: NO ERROR
1 :: A < 0
2 :: A > ABIG
3 :: NMAX < 0 AND 0 < A < SMALL
4 :: A = 0 AND NMAX < 0, AND A > 0
5 :: NMAX < 0 AND A >= 1

C****
994 IERR = IERR + 1
995 IERR = IERR + 1
996 IERR = IERR + 1
997 IERR = IERR + 1
998 IERR = IERR + 1
1000 CALL UNDERZ('S',SAVE)
RETURN
END

87
End 9-77