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A REEXAMINATION OF THE ADAPTIVE EXPECTATIONS HYPOTHESIS WHEN APPLIED TO A COBWEB MODEL

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**20. ABSTRACT (Continue on reverse side if necessary and identify by block number)**

In this note we point out a certain type of inconsistency which appears when the familiar adaptive expectations hypothesis is applied to the supply equation of a basic cobweb model with a simple error structure. We show that there is a difference between the minimum mean square error forecast function of the price (when the price is viewed as a stationary time series process) and the implicit assumption of the adaptive expectations hypothesis. We show that the inconsistency disappears when a certain...

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1. **Introduction**

The compatibility between dynamic simultaneous equation econometric models and autoprojective methods was pointed out in an earlier paper [Nussbaum and Singpurwalla (1977)]. In that paper we show that two versions of the familiar cobweb model are compatible with an autoregressive process of the first order. We also point out other advantages of viewing these models in terms of their associated time series (stochastic) process.

In this note we discuss Nerlove's (1958) modification to the cobweb model, and demonstrate, by looking at the modified model as a time series process, that a certain inconsistency arises. This strengthens our thesis that further insight into the structure of economic models can be obtained by looking at the associated time series processes.

2. **The Cobweb Model**

We discussed the cobweb model in our earlier paper. Here we shall reproduce the more important aspects so that we may facilitate an analysis of Nerlove's modification.
Briefly, the cobweb model is used as our introductory model to explain supply-demand dynamics. It portrays the agricultural sector of a competitive economy. It states that farmers generate the quantity of a perishable product to be supplied next period as a function of this period's price. Thus,

\[ Q^S_t = \alpha + \beta P_{t-1} , \]  

where

\[ Q^S_t = \text{quantity supplied in period } t \]
\[ P_t = \text{market price in period } t. \]

The demanders of the commodity, on the other hand, determine the quantity desired at time \( t \) as a function of the current (time \( t \)) price as follows:

\[ Q^D_t = \gamma + \delta P_t , \]  

when

\[ Q^D_t = \text{quantity demanded in period } t. \]

By economic considerations, the signs of \( \alpha, \beta, \) and \( \gamma \) are assumed positive. The sign of \( \delta \) is assumed negative. To the above behavioral relationships, a clearing equation identity is added as follows:

\[ Q^S_t = Q^D_t. \]  

A minor variant of the basic cobweb model was also considered in the earlier paper. This variant suggested that quantity decisions be regarded as a function of the change in price between successive periods. This resulted in a modified supply equation as follows:

\[ Q^S_t = \alpha + \beta (P_{t-1} - P_{t-2}) , \]  

and a modified demand equation

\[ Q^D_t = \gamma + \delta (P_t - P_{t-1}) . \]  

The clearing equation (2.3) is still presumed to hold.
We showed that the only difference between the original cobweb model and this variant is that all results for the cobweb model held with the replacement of $P_t - P_{t-1}$ for $P_t$.

To incorporate a probabilistic structure to the model, random disturbances $u_t$ and $v_t$ were then imposed on Equations (2.1) and (2.2), respectively. Under certain assumptions on $u_t$ and $v_t$, we were able to demonstrate that the price series was an autoregressive process of order 1. Specifically, the assumptions on $u_t$ and $v_t$ were:

1. $u_t$ and $v_t$ are both normally distributed with mean zero and variances $\sigma_u^2$ and $\sigma_v^2$, respectively;
2. $E[u_t u_{t-j}] = E[v_t v_{t-j}] = 0$ for all $t$ and $j > 0$, thus the individual disturbances are independent;
3. $E[u_t v_{t-j}] = 0$ for all $t$ and $j$, thus the $u_t$ and the $v_t$ are mutually independent.

The form of the resultant autoregressive process (for large $t$) was:

$$
\Delta p_t = \frac{\alpha - \gamma}{\delta} + \frac{\beta}{\delta} p_{t-1} + \frac{u_t - v_t}{\delta}.
$$

If the minor variant is considered, then the form of the process becomes

$$
\Delta p_t = \frac{\alpha - \gamma}{\delta} + \frac{\beta}{\delta} \Delta p_{t-1} + \frac{u_t - v_t}{\delta}.
$$

where $\Delta p_t = p_t - p_{t-1}$.

An important property of the autoregressive process of order 1 is its forecast function. The price series $p_t$ has a minimum mean square error forecast [cf. Box and Jenkins (1970)] of

$$
\hat{p}_{t+k} = p^* + (p_t - p^*)(\frac{\beta}{\delta})^k,
$$

- 3 -
where $\hat{p}_{t+k}$ is the forecast at time $t$ for time $t+k$, and $p^*$ is the equilibrium value of price. For the cobweb model, the value of $p^*$ is

$$p^* = \frac{\alpha - \gamma}{\delta - \beta}.$$

In particular, we will find it important to know the one-step-ahead forecast. Thus, if we are at time $t-1$ and wish to forecast the price at time $t$, our best forecast would be

$$\hat{p}_t = p^* + (p_{t-1} - p^*) \frac{\beta}{\delta} = p^*(1 - \frac{\beta}{\delta}) + \frac{\beta}{\delta} p_{t-1} = \frac{\alpha - \gamma}{\alpha - \beta} \frac{\delta - \beta}{\delta} p_{t-1} + \frac{\beta}{\delta} p_{t-1};$$

thus,

$$\hat{p}_t = \frac{\alpha - \gamma}{\delta} + \frac{\beta}{\delta} p_{t-1}.$$

3. **The Adaptive Expectation Modification**

Nerlove (1958) proposed a modification to the cobweb model in an attempt to enlarge the scope of the model. He suggested that the supply equation (2.1) should not be a function of last period's price, but rather a function of this period's expected (forecasted) price. This expected price is a subjective forecast which is a function of the previous period's price and the error made in forecasting the last period's price.

This scheme is known as the "adaptive expectations hypothesis," since price estimates are adapted in proportion to previous forecasting errors. The forecasted price in this period, $\hat{p}_t$, is the forecasted price in the previous period, $\hat{p}_{t-1}$, adjusted by a proportion of the error made in predicting last period's price, $p_{t-1} - \hat{p}_{t-1}$.

Thus, according to Nerlove

$$\hat{p}_t = \hat{p}_{t-1} + (1 - \lambda)(p_{t-1} - \hat{p}_{t-1}),$$

where $\lambda$ is a constant such that $0 < \lambda < 1$. Equivalently,
\[ \hat{p}_t = \lambda \hat{p}_{t-1} + (1-\lambda)p_{t-1}. \] 

(3.1)

An important outcome of Nerlove's modification is that the parameter space for which the price is stationary is enlarged. Note [Nussbaum and Singpurwalla (1977)] that in the original cobweb model the price series is stationary only when \( |\frac{\beta}{\delta}| < 1 \). For Nerlove's modification, it can be shown [Nerlove (1958), Wallis (1972)] that when \( \beta > 0 \), \( \delta < 0 \), and when

\[ 0 < -\frac{\delta + \beta}{\delta - \beta} < \lambda < 1, \]

the resultant price series is stationary even if \( |\frac{\beta}{\delta}| > 1 \). In this note, we shall argue that even though the price series is stationary for the enlarged parameter space, a certain kind of inconsistency creeps up, suggesting a reconsideration of the enlarged space.

It is easy to show [cf. Nerlove (1958)] that the adaptive expectation hypothesis leads us to the following equation for the price series:

\[ p_t = \frac{(1-\lambda)(\alpha - \gamma)}{\delta} + \left[ \frac{\beta}{\delta} (1-\lambda) + \lambda \right] p_{t-1} + \frac{\eta_t}{\delta}, \] 

(3.2)

where \( \eta_t = (u_t - v_t) - \lambda(u_{t-1} - v_{t-1}) \).

Since \( \eta_t \) involves the terms \( u_t, u_{t-1}, v_t, \) and \( v_{t-1}, \) it is easy to verify that \( E(\eta_t \eta_{t-1}) \neq 0 \), but that \( E(\eta_t \eta_{t-k}) = 0 \), for \( k \geq 2 \). Furthermore, if we let \( w_t = (u_t - v_t)/\delta \), then Equation (3.2) can be written as

\[ p_t - \left[ \frac{\beta}{\delta} (1-\lambda) + \lambda \right] p_{t-1} = \frac{(1-\lambda)(\alpha - \gamma)}{\delta} + w_t - \lambda w_{t-1}, \]

where the \( w_t \)'s are independent and identically distributed. If we identify \( \frac{\beta}{\delta} (1-\lambda) + \lambda \) with \( \phi_1 \), \( \lambda \) with \( \theta_1 \), and \( \frac{(1-\lambda)(\alpha - \gamma)}{\delta} \) with a constant \( \mu \), then the above equation for \( p_t \) is of the form

\[ (1 - \phi_1 B)p_t = \mu + (1 - \theta_1 B)w_t, \]
where $B$ is the backward shift operator.

Clearly then, Equation (3.2) represents, in the terminology of Box and Jenkins, an ARIMA ($1,0,1$) process. This process is stationary if

$$|\frac{\beta}{\delta} (1-\lambda) + \lambda| < 1,$$

and thus for certain values of $\lambda$ we can have

$$|\frac{\beta}{\delta}| > 1.$$ Note that when $\beta = \delta$, the $P_t$ process is not stationary for any value of $\lambda$; however, the differenced process $\Delta P_t = P_t - P_{t-1}$ is stationary.

4. The Inconsistency of the Adaptive Expectations Hypothesis

Following the well-known results on the minimum mean square error forecasts for the ARIMA ($1,0,1$) process, a forecast of the price at time $t$ made from origin $(t-1)$ is

$$\hat{p}_t = \left[\frac{\beta}{\delta} (1-\lambda) + \lambda\right] p_{t-1} - \lambda w_{t-1}.$$ The residuals $w_t$ are the one-step-ahead forecast errors; that is,

$$w_{t-1} = p_{t-1} - \hat{p}_{t-1}.$$ Substituting the above in the expression for $P_t$, we obtain

$$\hat{p}_t = \left[\frac{\beta}{\delta} (1-\lambda)\right] p_{t-1} + \lambda \hat{p}_{t-1}$$

as the minimum mean square error forecast for the price series.

A comparison of Equations (3.1) and (3.3) demonstrates the point of this note. Equation (3.1) represents a forecast for the price as suggested by Nerlove under the adaptive expectations hypothesis. When the adaptive expectations hypothesis is incorporated into the cobweb model, and the resultant price series process is considered, then the minimum mean square error forecast is given by Equation (3.3). Since for $\beta \neq \delta$ the expressions given by Equations (3.1) and (3.3) are different, we claim that a certain type of inconsistency exists in the system. Furthermore,
if in an effort to correct the situation the expression given by Equation (3.3) were incorporated into the model instead of the adaptive expectations hypothesis, the minimum mean square error forecast would be

$$\hat{P}_t = \left[ (\frac{\delta}{\beta})^2 (1-\lambda) \right] P_{t-1} + \lambda \hat{P}_{t-1};$$

for $\beta \neq \delta$, the inconsistency therefore persists. Moreover, it is easy to verify that any adaptive expectations hypothesis of the form

$$\hat{P}_t = A\hat{P}_{t-1} + BP_{t-1},$$

where $A$ and $B$ are constants, would lead us to a minimum mean square error forecast of the type

$$\hat{P}_t = A\hat{P}_{t-1} + \frac{\beta}{\delta} BP_{t-1}.$$

Therefore, for this general form the inconsistency remains.

For $\beta = \delta$, Equations (3.1) and (3.3) are identical, and the inconsistency disappears. However, under this condition, the price series $P_t$ becomes non-stationary. Fortunately, the differenced series $VP_t = P_t - P_{t-1}$ is a stationary moving average process for which the minimum mean square error forecast is precisely that given by Equation (3.1).

In summary, we claim that a modification to the cobweb model based on the adaptive expectations hypothesis (or variants of it) leads us to an inconsistency with respect to the minimum mean square error forecast when $\beta = \delta$. Thus, when an error structure is imposed on the cobweb model, the adaptive expectations hypothesis is consistent with the minimum mean square error forecast only when $\beta = \delta$. This reflects a considerable reduction of the original parameter space.

5. Some Concluding Remarks

We note that Equation (3.1) also represents the one-step-ahead forecast function for an ARIMA (0,1,1) process. Such a process is commonly called an "exponentially weighted moving average process," and can be written as
\[ p_\tau - p_{\tau-1} = a_\tau - \lambda a_{\tau-1}, \]

where \( a_\tau \) is a random shock, i.e., a normally distributed random variable with zero mean and constant variance. For this process, exponential smoothing is an optimum method for generating minimum mean square error forecasts.

It is quite possible that in empirical work the differences between the forecasts from an exponentially weighted moving average process and a mixed autoregressive moving average process are not that significant. In fact, Box and Jenkins (1970) discuss the fact that an autoregressive process of order 1 with parameter \( \beta \) close to one would behave very much like an exponentially weighted moving process with parameter \( \lambda \) close to zero. Also, Anderson (1975) discusses difficulties in the identification between an ARIMA (1,0,0) process and an ARIMA (0,1,1) process. Thus, for purposes of forecasting the price, the particular model selected becomes less and less important.

While it may not be crucial in empirical work, it is the purpose of this paper to point out that when \( \beta \neq \delta \), a logical inconsistency arises if the adaptive expectations hypothesis is applied to the cobweb model, and if the minimum mean square error forecast criteria is considered.

We should also point out that the adaptive expectations hypothesis is not the only modification ever suggested to improve the cobweb model. Muth (1961) has suggested the principle of rational expectations. Muth suggests that information is a scarce commodity, and all information is considered before forecasts are made. The adaptive expectations hypothesis therefore becomes a subset of the rational expectations hypothesis. The ramifications of this principle are not examined in this note. However, the results of this note motivate us to look for hypotheses of the type considered by Nerlove which would lead us to enlarged parameter spaces, and which do not violate any requirements of consistency.
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