COMPARATIVE STUDY OF ELECTRON-PHONON INTERACTION
IN THREE VANADIUM SAMPLES OF VARYING PURITY*

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Comparative Study of Electron-Phonon Interaction in Three Vanadium Samples of Varying Purity

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Recently a theory for the transport coefficients of pure type II superconductors (\(T > T_c\)) near the upper critical field \(H_{c2}\) was developed by A. Houghton and K. Maki (HM). The electron-phonon interaction is expected to exhibit two types of behavior: very near \(H_{c2}\), the ultrasonic attenuation should exhibit gapless behavior \((\Delta n/\Delta n_{c2})/\Delta n_{c2}-H\); further away from \(H_{c2}\), the absorption should go as \(\Delta n/\Delta n_{c2}-H\). Although this theory has been compared to niobium where qualitative agreement was found, there has been no definitive study performed on vanadium. The present work relates the results of ultrasonic attenuation measurements as a function of temperature and applied magnetic field in three vanadium samples with resistivity ratios of 8, 245, and about 400, hereafter referred to as samples 1, 11, and 111 respectively.

HM have developed a theory for the low frequency response functions for clean type II superconductors when the magnetization \(M\) is small. Their expression for the damping factor provides simple analytical forms for both the thermal conductivity and the ultrasonic attenuation in terms of a single dimensionless parameter \(\alpha = 2T^2/\Delta^2/kT_c\) for the limit \(T = 0\ K\). In terms of this parameter, the normalized attenua-

<table>
<thead>
<tr>
<th>Sample Length</th>
<th>Diameter</th>
<th>(T_c) (a)</th>
<th>(T_c) (b)</th>
<th>(\Delta n/\Delta n_{c2}) (H_{c2})</th>
<th>(\Delta n/\Delta n_{c2})</th>
<th>(\Delta n/\Delta n_{c2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.9 cm</td>
<td>0.6 cm</td>
<td>4.81</td>
<td>4.69</td>
<td>3.5±1</td>
<td>1.0</td>
</tr>
<tr>
<td>11</td>
<td>3.8 cm</td>
<td>0.6 cm</td>
<td>5.26</td>
<td>5.26</td>
<td>3.5±2</td>
<td>2.1</td>
</tr>
<tr>
<td>111</td>
<td>0.6 cm</td>
<td>0.5 cm</td>
<td>5.34</td>
<td>5.27</td>
<td>3.3±2</td>
<td>4.0</td>
</tr>
</tbody>
</table>

(a) from extrapolation of \(H_{c2}\) data to \(H_{c2} = 0\)
(b) from attenuation vs temperature data
tion for transverse waves with phonon wave vector \( \mathbf{q} \) parallel to the applied field \( H \) is
\[
\frac{\alpha_n - \alpha_0}{\alpha_n} = \Delta \alpha / \alpha = 15u (1-u^2) J_1 + \pi / 16 \cdot u^2 / 3 - \pi u^2 / 4 + u^2 \]
where
\[
J_1 = \pi / 2 \cdot (2u^2 / u^2 - 1) \tan^{-1} [ (u-1) / (u+1) ]^{\frac{1}{2}},
\]
\( \Delta \) is the spatially averaged order parameter, \( k_c \) is the reciprocal lattice vector of the lattice, \( v_F \) is the Fermi velocity, and \( l \) is the electron mean free path. Relating these parameters to experimental observables, we find
\[
\mu = \frac{e}{\pi h^2 v_F^2 N(0)} \left[ \frac{\phi_0}{2 \pi k_c^2} \right]^2 \left[ H_{c2} - \frac{dH_{c2}}{dt} \right]^{\frac{1}{2}} \frac{H_{c2} - H}{1.16 [2k_c^2 - 1] + n}
\]
is the flux quantum, \( N(0) \) is the density of states at the Fermi surface, \( n \) is the demagnetization factor, and \( K_2 \) is the Maki-Tsuzuki parameter. This expression implies \( \mu = f(t) \cdot \left[ H_{c2} - H \right] \) where \( f(t) \) is an experimentally determined function of the reduced temperature \( t = T / T_c \).

Figure 1 illustrates the linear relationship between \( \mu \) and \( H_{c2} - H \) at a few temperatures for sample 11, while figure 2 illustrates the resulting fit of the theory to the experimental points at the same temperatures. Note that \( H_{c2} \) is also easily obtained at each temperature and its resulting temperature dependence for sample 11 is shown in figure 3.

We have intentionally eliminated the data points for extremely small \( \Delta H \) values in samples 11 and 311 because of a one
dimensional fluctuation effect\(^4\) which occurs here in pure samples, causing a rounding in the attenuation. The effect was minimal in sample 1 and has not been included in the HM theory.

The temperature dependence of \(\xi\) was determined by using Pippard's\(^5\) result for attenuation in normal metals. His theory shows that the attenuation in the normal state \(\alpha_n\) is proportional to \(\omega^2t\) for \(q<1 \) and to \(\omega\) for \(q>1\) (\(t=\nu_F; \omega=\nu_S\)). Having applied a magnetic field sufficient to turn the sample normal, and having varied the temperature from 1.5\(^\circ\) to \(T_c\), we found the attenuation constant in all three samples. Since doubling the applied field also had no effect we assumed that the electron mean free path in the normal state was independent of temperature and magnetic field in the region of interest. Finnemore\(^6\) has experimentally found that for "not too pure" samples, \(K_2(t)/K_2(1)\) obeys the same temperature dependence for a wide range of sample purities. Using this fact and the data of Sekula and Kernohan\(^7\) and the relation \(K_2(1) = K = K_0 + 7.53 \times 10^3 \rho_{07}\) we were able to determine \(K_2(t)\) for all three samples.

Fig. 3 - \(H_{c2}\) as function of \(t\) for sample 11.

Fig. 4 - \(f(t)\) vs \(t\) for each sample. Same \(t\) scale for each curve. Sample 1, 45MHz; 11, 15MHz; 111, 255MHz; all transverse waves \(\vec{q}\parallel \vec{H}\).
The final result is an independent determination of \( f(t) = \mu / \Delta H \) which can be compared to the discrete values obtained above. The agreement can be seen in figure 4. In these plots, the mean free path is used as a fitting parameter to the experimental point at the lowest temperature. This value \( (t_1) \) can be found for each sample in table 1.

In sample 111 we were able to obtain frequencies high enough to manifest the transition region between \( qL < 1 \) and \( qL > 1 \) for normal metals as described by Pippard (see fig. 5). This allowed an independent determination of \( t \) and by comparison of residual resistance data, similar values were obtained for samples 1 and 11 (labeled \( t_2 \) in table 1). As is evident from figure 4 and table 1 there is reasonable agreement between our experimental results and the HN theory.

References:

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**Key Words:**
- Electron-phonon interaction, gapless behavior, magnetization, vanadium, ultrasonic attenuation, damping factor, transport coefficients, niobium, thermal conductivity, pure type II superconductors, magnetic field, transverse waves, upper critical field, low frequency response function.

**Abstract:**
- The ultrasonic attenuation coefficient has been measured in the mixed state of three samples of single crystal vanadium of varying purity in order to determine the effect of impurities on the electron phonon interaction in the mixed state of vanadium. The resistivity ratio of the three samples was 8, 245, and 400. Within experimental error the superconducting energy gap of the three samples was 3.5 kT~c~. The electron mean free path determined ultrasonically was 0.4, 2.1 and 4.0 x 10^4 cm. respectively. A recent theory is presented.
19. Key Words - continued

phonon wave vector
order parameter
reciprocal lattice vector
Fermi velocity
electron mean free path
flux quantum
density of states
demagnetization factor
one dimensional fluctuation
normal metal
residual resistance

20. Abstract - continued

which indicates that electron phonon interaction is a strong function of the electron mean free path in the mixed state. The relative attenuation decreases more rapidly for the purer samples as the magnetic field is decreased from the upper critical field and therefore the electron phonon interaction term also decreases more rapidly. There appears to be good quantitative agreement between the experimental results and the theoretical predictions.