FAILURE RADIUS: THEORY AND PREDICTION

BY D. J. PASTINE
D. J. EDWARDS
RESEARCH AND TECHNOLOGY DEPARTMENT

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A theory is presented which relates the Chapman-Jouquet pressure of a detonating explosive to its charge radius and also predicts the failure radius. This theory results in a one parameter equation. This equation fits the experimental data for cast TNT reasonably well when the parameter is obtained using one datum point.
FAILURE RADIUS: THEORY AND PREDICTION

This work was carried out under Task SF33354316. The present results and conclusions on failure diameter should be of interest in the area of explosive applications.

J. W. ENIG
By direction
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INTRODUCTION

The failure diameter of an explosive composition is an important characteristic which, to some extent, determines the composition's usefulness from an application's viewpoint. The failure diameter is that dimension through which a steady detonation cannot be propagated. One class of explosives which has evoked a great deal of interest in recent years has been labeled "non-ideal" explosives. This class of explosives is not thought to react completely within the steady state detonation wave. For this discussion non-ideal explosives are compositions containing an ideal explosive (RDX, HMX), aluminum, and an oxidizer (ammonium nitrate, ammonium perchlorate). The failure diameters of non-ideal explosives are usually much greater than those of ideal explosives. The usual method of determining failure diameter is to fire a series of cylindrical test charges of different diameter. This is an expensive and time consuming process, especially if one is developing a new composition. The ultimate goal of this work is to predict the failure diameter of an explosive composition on the basis of the physical properties of its constituents.

To be able to predict failure diameter in this manner, the physical properties related to failure diameter must be determined. Two properties which seem clearly related to detonation failure are the speed of sound, $c$, in the steady detonation wave and the reaction zone length, $l$. Rarefaction waves entering from the cylindrical edges of the steady detonation front can certainly quench or slow reaction if they penetrate deeply enough into the steady wave. They can also cause curvature of the detonation front. The depth of penetration will depend on the sound speed and the time available for propagation into the steady wave regime. The latter time will in turn depend on reaction zone length. It is rather interesting that in adding aluminum to explosives both of these quantities may be expected to increase. The sound speed which ordinarily decreases when small quantities of aluminum are added to explosives will probably increase (due to the same aluminum presence) in the dense regions behind a detonation front. In addition, the slower energy release rate associated with reacting aluminum should extend the length of the steady detonation wave (reaction zone). Such explosives should, therefore, be expected to have larger failure diameters than their non-aluminized counterparts.

DIAMETER EFFECT

The phenomenon known as diameter effect in condensed explosives is no more than the experimental fact that the detonation velocity, $D$, of a cylindrical charge decreases as the diameter of the charge decreases. This relationship between detonation velocity and diameter
has been studied theoretically by Jones, Eyring, Powell, Duffey, and Parlin, Wood and Kirkwood, Sichel, and Dubnov, to mention a few. (Actually, Wood and Kirkwood were concerned with radius of curvature of the detonation wave and not the diameter). The relationships which have been derived by these researchers can be put into the form,

\[
\frac{D}{D_0} = 1 - \frac{A\ell}{R}
\]  

where \(D_0\) is the detonation velocity at infinite diameter, \(\ell\) is the reaction zone length, \(R\) is the radius of the charge, and \(A\) is a constant (which is dependent on the model used to derive it). This equation could be used to predict failure diameter \(D\) if \(\ell\) was a constant for varying \(R\) or if a relationship between \(\ell\) and \(R\) could be obtained. If condition (1) were true, the problem would be trivial. However, \(\ell\) is not constant as will be shown below. Condition (2) is a possibility because \(\ell\) does vary with \(R\).

**MODIFICATION OF D VERSUS 1/R EQUATION**

The reaction zone length of an explosive is related to the sound speed in the reaction zone and the reaction time, \(\tau\), by

\[
\ell = \int_0^\tau c(t) \, dt. \tag{2}
\]

At the present time \(c(t)\) is an unknown function which depends on the reaction kinetics of the explosive. In a steady detonation, \(c(t)\) is thought to be a monotonically decreasing function of time, thus Equation (2) can always be rewritten in the form

\[
\ell = \bar{c} \, \tau \tag{3}
\]

where \(\bar{c} = \frac{1}{\tau} \int_0^\tau c(t) \, dt\), and should satisfy \(c_f > \bar{c} > c_{CJ}\), \(c_f\) is the sound speed at the detonation front, and \(c_{CJ}\) is the sound speed at the end of the reaction zone. Substituting Eq. (3) into Eq. (1) yields

\[
\frac{D}{D_0} = 1 - \frac{A\bar{c}\tau}{R}. \tag{4}
\]

To proceed further requires a set of data on an explosive which
gives D and T as a function of R for various R's including the
failure radius, Rf. Dremin, Saviov, Trofimov, and Shvedov\(^6\) have
published such data for cast TNT at densities of 1.62 and 1.60 g/cc.
Table 1 lists this data (Table 2 of reference 6). The difference
in density (and failure radius) is due to different methods of
casting. "...The trotyl [TNT] castings of the first type were
obtained by cooling the melt with the continuous stirring. They had
a homogeneous fine-crystalline structure throughout the entire cross
section of the charge. The density of such castings, determined by
analytic suspension in water, amounted to 1.62 g/cm\(^3\). Castings of
the second type were manufactured similarly, but the melt was cooled
without stirring. As a result castings were obtained with large
crystals, extending from the periphery to the center, and their
density was 1.6 g/cm\(^3\) on the average..."\(^6\) P\(_{\text{CJ}}\) and u\(_{\text{CJ}}\) in Table 1
are the pressure and particle velocity at the end of the reaction
zone. The detonation parameters in Table 1 were obtained by Dremin
as follows. D was determined by streak camera measurements. u\(_{\text{CJ}}\) and
\(\tau\) were obtained from the break point in the u(t) profile obtained
from electromagnetic velocity gages located on the axis of the charge.
P\(_{\text{CJ}}\) was obtained from P\(_{\text{CJ}}\) = \(\rho_0\) u\(_{\text{CJ}}\) D; \(\lambda\) was obtained from \(\lambda = \tau \left( D - 1.25 \right)\) u\(_{\text{CJ}}\).* Since there is only a 1% difference in density, one value of
D\(_0\) (7.04 mm/\mu s) will be used for these two densities.

It is obvious from Table 1 that \(\lambda\) is not a constant for varying
R. The next question is: Is A of Equation (1) a constant? Solving
Equation (1) for A one obtains

\[
A = \left(1 - \frac{D}{D_0}\right) \frac{R}{\lambda}.
\]  

For 1.62 g/cc TNT, A varies from 0.071 at R = 30 mm to 0.461 at
R = 8 mm.

Up to this point, only the variation of \(\frac{D}{D_0}\) with \(\lambda\) and R has
been considered. The data of Table 1 can also be used to look
at the variation of P\(_{\text{CJ}}\) with \(\lambda\) and R. An equation similar to
Equation (1) or Equation (4) involving P\(_{\text{CJ}}\) can be obtained using
the approach of Dubnov.\(^5\) The Infinite Diameter C-J pressure,
P\(_{\text{CJ}}\), of an explosive is defined\(^5\) by

\[
P_{\text{CJ}} = G(\rho_0) Q (\gamma - 1)
\]

*Russian researches have noted that the extrapolated particle velocity
at the detonation front using the EMV gage is usually 1.5-1.6 u\(_{\text{CJ}}\).
The particle velocity-time profiles obtained are essentially linear.
Thus they use the average value of the particle velocity in the reaction
zone (1.25 u\(_{\text{CJ}}\)) to compute \(\lambda\). This approach, however, ignores any
possibility of an induction zone behind the detonation front.

6. Dremin, Saviov, Trofimov, and Shvedov, Detonation Waves in
Condensed Media, (English Translation).
Table 1

Detonation Parameters of Cast TNT versus Charge Radius*

<table>
<thead>
<tr>
<th>R (mm)</th>
<th>D (mm/μsec)</th>
<th>( u_{CJ} ) (mm/μsec)</th>
<th>( P_{CJ} ) (Kbar)</th>
<th>( \tau ) (μsec)</th>
<th>( l ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>6.98</td>
<td>1.62</td>
<td>183.2</td>
<td>0.26</td>
<td>1.27</td>
</tr>
<tr>
<td>20</td>
<td>6.95</td>
<td>1.60</td>
<td>180.1</td>
<td>0.28</td>
<td>1.36</td>
</tr>
<tr>
<td>14</td>
<td>6.89</td>
<td>1.52</td>
<td>169.5</td>
<td>0.30</td>
<td>1.47</td>
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<tr>
<td>12.5</td>
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<td>150.7</td>
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<td>1.57</td>
</tr>
<tr>
<td>11</td>
<td>6.75</td>
<td>1.31</td>
<td>143.2</td>
<td>0.30</td>
<td>1.51</td>
</tr>
<tr>
<td>10</td>
<td>6.64</td>
<td>1.24</td>
<td>133.3</td>
<td>0.34</td>
<td>1.71</td>
</tr>
<tr>
<td>9</td>
<td>6.52</td>
<td>1.20</td>
<td>126.7</td>
<td>0.33</td>
<td>1.64</td>
</tr>
<tr>
<td>8**</td>
<td>6.36</td>
<td>1.10</td>
<td>113.3</td>
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<td>1.77</td>
</tr>
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<td>1.43</td>
</tr>
<tr>
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<td>159.6</td>
<td>0.315</td>
<td>1.51</td>
</tr>
<tr>
<td>17.5</td>
<td>6.70</td>
<td>1.43</td>
<td>152.3</td>
<td>0.31</td>
<td>1.50</td>
</tr>
<tr>
<td>15</td>
<td>6.37</td>
<td>1.39</td>
<td>140.8</td>
<td>0.34</td>
<td>1.55</td>
</tr>
<tr>
<td>13.75**</td>
<td>6.20</td>
<td>1.20</td>
<td>119.0</td>
<td>0.35</td>
<td>1.67</td>
</tr>
</tbody>
</table>

* From Table 2 of Reference 6
** Failure radius of the charge
where \( G(\rho_0) \) is a function of the initial density of the explosive, \( Q \) is the total heat of the explosion, and \( \gamma \) is the index of polytropy of the detonation products.* Taking into account the losses due to the lateral spreading of the products away from the reaction zone, Equation (6) is rewritten as

\[
P_{\text{CJ}} = G(\rho_0) Q (\gamma - 1) \eta \quad (7)
\]

where \( P_{\text{CJ}} \) is the actual pressure and \( \eta \) is a factor characterising that part of \( Q \) that goes into the shock wave. Thus

\[
\frac{P_{\text{CJ}}}{P_{\text{CJO}}} = \eta \quad (8)
\]

Taking the square root of Equation (8) one obtains

\[
\left( \frac{P_{\text{CJ}}}{P_{\text{CJO}}} \right)^{1/2} = \sqrt{\eta} \quad (9)
\]

Equation (9) is similar to Equation (4) of reference (5). From reference (5), \( \eta \) is the ratio of unaffected volume to the total volume in the reaction zone, and

\[
\left( \frac{P_{\text{CJ}}}{P_{\text{CJO}}} \right)^{1/2} \approx 1 - \frac{l}{2R} \quad (10)
\]

Since the coefficient is dependent on the approach, Equation (10) will be generalized to

\[
\left( \frac{P_{\text{CJ}}}{P_{\text{CJO}}} \right)^{1/2} = 1 - \frac{A' l}{R} \quad (11)
\]

\( P_{\text{CJO}} \) (in k bar) is obtained from

\[
P_{\text{CJO}} = 10 \rho_0 \omega_{\text{CJO}} D_0 \quad (12)
\]

where \( D_0 = 7.04 \text{ mm/\mu sec} \) for both densities.** \( \omega_{\text{CJO}} \) for the TNT in Table 1 is obtained by using \( \gamma = 3 \) in

\[
\omega_{\text{CJO}} = \frac{D}{\gamma + 1} \quad (13)
\]

*The assumption is made here that detonation products are describable by an isentropic equation of the form \( P V^\gamma = \text{constant} \).

**This value was obtained by fitting \( D \) vs \( 1/R \).

with the result that $u_{CJO} = 1.76 \text{ mm/us}$. Thus $P_{CJO} = 200.7 \text{ Kbar}$ for $\rho_0 = 1.62$ and $P_{CJO} = 198.2 \text{ Kbar}$ for $\rho_0 = 1.60 \text{ g/cc}$. Table 2 lists $A$, $D/D_0$, $A'$, and $(P_{CJ}/P_{CJO})^{1/2}$. For $1.62 \text{ g/cc TNT}$, $A$ varies by a factor of $2.39$ while $A'$ varies by a factor of $1.46$; for $1.60 \text{ g/cc}$, $A$ varies by $1.73$ while $A'$ varies by $1.50$.

Changing from $D$ to $P_{CJ}$ does not seem to matter as far as the terms $A$ and $A'$ are concerned. However, close examination of the data in Table 2 shows that $P_{CJ}$ is more sensitive to $R$ than $D$ is. For $\rho_0 = 1.62 \text{ g/cc TNT}$ $P_{CJ}$ at failure diameter is reduced to $56\%$ of its infinite diameter value while $D$ is reduced to only $90\%$; for $\rho_0 = 1.60 \text{ g/cc TNT}$ $P_{CJ}$ is reduced to $60\%$ while $D$ is reduced to $88\%$.

In Equation (4), $\bar{c}$ can be related to $P_{CJ}$ as follows. The quantity $\bar{c}$ lies between $c_f$ and $c_{CJ}$ and, it is reasonable to assume that $\bar{c}$ can be written as a linear function of $c_f$ and $c_{CJ}$.

The quantity $c_{CJ}$ is given by

$$c_{CJ} = D - u_{CJ} = D - \frac{D}{\gamma+1}.$$ (14)

Since it follows from Equations (12) and (13) that $D$ is proportional to $\sqrt{P_{CJ}}$, then the result $c_{CJ} \propto \sqrt{P_{CJ}}$ also follows from Equation (14). The quantity $c_f$ can be closely approximated by terms linear in $D$ and $u_f$, i.e. $c_f = D + bu_f$ where $b$ is a constant approximately equal to two. Since $D \propto \sqrt{P_{CJ}}$, and, since it has been observed that $u_f \approx 1.5 u_{CJ}$, it follows the $c_f$ is also approximately proportional to $\sqrt{P_{CJ}}$. Accordingly, $\bar{c}$ is written in the form

$$\bar{c} = A'' \frac{\sqrt{P_{CJ}}}{P_{CJO}}.$$ (15)

where Equation (15) has been normalized to $P_{CJO}$.

The next question is: Can $\tau$ be related to $P_{CJ}$. Walker and Wasley have proposed a critical energy fluence, $E_c$, for shock initiation of explosives

$$E_c = \frac{P^2 \tau}{\rho_0 U_s}.$$ (16)

where $P$ is the pressure, $\tau$ is the duration of constant pressure shock pulse, and $U_s$ is the shock velocity in the explosive at pressure $P$. Does an equation similar to Equation (12) hold for the steady detonation regime? Three possible equations were investigated:

$$E = \frac{P_{CJ}^2 \tau}{\rho_0 U_s}.$$ (17)

$$E = \frac{P_f^2 \tau}{\rho_0 U_s}.$$ (18)

Table 2
Values of A and A' for Cast TNT

<table>
<thead>
<tr>
<th>R (mm)</th>
<th>D/Do</th>
<th>A (mm)</th>
<th>((P_{CJ}/P_{CJO})^{1/2})</th>
<th>A' (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.991</td>
<td>0.213</td>
<td>0.955</td>
<td>1.063</td>
</tr>
<tr>
<td>20</td>
<td>0.987</td>
<td>0.191</td>
<td>0.947</td>
<td>0.779</td>
</tr>
<tr>
<td>14</td>
<td>0.979</td>
<td>0.200</td>
<td>0.919</td>
<td>0.771</td>
</tr>
<tr>
<td>12.5</td>
<td>0.972</td>
<td>0.223</td>
<td>0.866</td>
<td>1.067</td>
</tr>
<tr>
<td>11</td>
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<td>0.299</td>
<td>0.845</td>
<td>1.129</td>
</tr>
<tr>
<td>10</td>
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</tr>
<tr>
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<td>0.926</td>
<td>0.406</td>
<td>0.795</td>
<td>1.125</td>
</tr>
<tr>
<td>8</td>
<td>0.903</td>
<td>0.438</td>
<td>0.751</td>
<td>1.125</td>
</tr>
</tbody>
</table>

\(\phi_0 = 1.62 \text{ g/cc}\)

<table>
<thead>
<tr>
<th>R (mm)</th>
<th>D/Do</th>
<th>A (mm)</th>
<th>((P_{CJ}/P_{CJO})^{1/2})</th>
<th>A' (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.973</td>
<td>0.566</td>
<td>0.941</td>
<td>1.238</td>
</tr>
<tr>
<td>20</td>
<td>0.957</td>
<td>0.569</td>
<td>0.897</td>
<td>1.364</td>
</tr>
<tr>
<td>17.5</td>
<td>0.951</td>
<td>0.571</td>
<td>0.877</td>
<td>1.435</td>
</tr>
<tr>
<td>15</td>
<td>0.905</td>
<td>0.919</td>
<td>0.842</td>
<td>1.529</td>
</tr>
<tr>
<td>13.75</td>
<td>0.881</td>
<td>0.980</td>
<td>0.775</td>
<td>1.853</td>
</tr>
</tbody>
</table>

\(\phi_0 = 1.60 \text{ g/cc}\)
where \( P_f \) is the pressure at the detonation front. \( P_f \) is obtained by using \( P_f = 10 \rho_0 U_f D \). \( U_f \) is the particle velocity at the detonation front and is obtained from the unreacted Hugoniot. For TNT at \( \rho_0 = 1.614 \text{ g/cc} \), the unreacted Hugoniot \(^6\) is \( U = 2.39 + 2.05 u \) where \( U \) is the shock velocity in mm/µsec. The third equation is

\[
E = \frac{\int_{0}^{\tau} P^2 dt}{\rho_0 D}.
\]

The integral in Equation (19) is evaluated by assuming that \( P \) varies linearly with \( t \) from \( P_f \) to \( P_{CJ} \). This assumption was used because the measured \( u(t) \) profiles were essentially linear. \(^6\) \( E \) is the energy flux along the axis of the charge.

The results of Equation (17), (18), and (19) are given in Table 3. Except for data close to the failure diameter the individual results for Equations (18) and (19) are within \(-10\%\) of the average for each equation. This is exceptional considering the possible error in the measurements. The results for Equation (17) are not quite as good. The individual results are within \(-30\%\) of the average. This is not bad considering that there is more error in measured values of \( P_{CJ} \) than \( D \) (and hence \( P_f \)). Again, if an equation of state existed for the reaction products and intermediate products, then \( P_{CJ} \) could be calculated from and perhaps be more accurate than the measured values. Thus, the indications are that \( \tau \) can reasonably be related to \( P_{CJ} \) as follows:

\[
\tau = A'^{''}/(P_{CJ}/P_{CJO})^2
\]

where Equation (20) has been normalized to \( P_{CJO} \).

An equation involving only \( P_{CJ} \) and \( R \) can now be obtained. Substituting Equation (3) into Equation (11) results in

\[
\left( \frac{P_{CJ}}{P_{CJO}} \right)^{1/2} = 1 - \frac{A' \bar{c} \tau}{R}.
\]

Substituting Equation (15) for \( \bar{c} \) and Equation (20) for \( \tau \) there results

\(^6\) Dremin, Saviov, Trofimov, and Shvedov, Detonation Waves in Condensed Media, (English Translation).

### Table 3

$p^2$ versus $\tau$ for Cast TNT

<table>
<thead>
<tr>
<th>$R$ (mm)</th>
<th>Equation (17)</th>
<th>Equation (18)</th>
<th>Equation (19)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Kbar$^2$-cc-\text{\mu}sec$^2$/mm-gm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_0 = 1.62$ g/cc</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>$7.7 \times 10^2$</td>
<td>$1.43 \times 10^3$</td>
<td>$1.065 \times 10^3$</td>
</tr>
<tr>
<td>20</td>
<td>$8.0$</td>
<td>$1.55$</td>
<td>$1.123$</td>
</tr>
<tr>
<td>14</td>
<td>$7.7$</td>
<td>$1.61$</td>
<td>$1.130$</td>
</tr>
<tr>
<td>12.5</td>
<td>$6.4$</td>
<td>$1.61$</td>
<td>$1.058$</td>
</tr>
<tr>
<td>11</td>
<td>$5.6$</td>
<td>$1.49$</td>
<td>$0.959$</td>
</tr>
<tr>
<td>10</td>
<td>$5.6$</td>
<td>$1.57$</td>
<td>$0.997$</td>
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<td>9</td>
<td>$5.0$</td>
<td>$1.42$</td>
<td>$0.895$</td>
</tr>
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<td>8</td>
<td>$4.5$</td>
<td>$1.40$</td>
<td>$0.854$</td>
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<tr>
<td>$\rho_0 = 1.60$ g/cc</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>$8.4 \times 10^2$</td>
<td>$1.55 \times 10^3$</td>
<td>$1.139 \times 10^3$</td>
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<tr>
<td>13.75</td>
<td>$5.0$</td>
<td>$1.19$</td>
<td>$0.800$</td>
</tr>
</tbody>
</table>
\[
\left( \frac{P_{CJ}}{P_{CJO}} \right)^{1/2} = 1 - \frac{A'A'''}{R\left( \frac{P_{CJ}}{P_{CJO}} \right)^{3/2}} 
\]

or

\[
\left( \frac{P_{CJ}}{P_{CJO}} \right)^{1/2} = 1 - \frac{\beta}{R(P_{CJ}/P_{CJO})^{3/2}} 
\]

where \( \beta = A'A'''. \)

**FAILURE DIAMETER PREDICTION**

The main question is: Does Equation (23) predict a point of failure? And if so, how does it compare with experiment? The first question is the same as asking: Does Equation (23) have an infinite derivative in the \( (P_{CJ}/P_{CJO})^{1/2} \) versus \( 1/R \) plane? To determine this the derivative \( d(P_{CJ}/P_{CJO})^{1/2}/d(1/R) \) must be obtained. For simplicity let \( (P_{CJ}/P_{CJO})^{1/2} = x \) and \( 1/R = q \). Then Equation (23) becomes

\[
x = 1 - \frac{\beta q}{x^3} \tag{24}
\]

or

\[
x^4 - x^3 = - \beta q \tag{25}
\]

Differentiating Equation (25) yields

\[
4x^3 dx - 3x^2 dx = - \beta dq \tag{26}
\]

or

\[
\frac{dx}{dq} = \frac{- \beta}{4x^3 - 3x^2} \tag{27}
\]

\[
\frac{dx}{dq} = \frac{- \beta/4}{x^2(x-3/4)} \tag{28}
\]

At failure

\[
\frac{dx}{dq} = - \infty + x_f = 0.75 .
\]
Thus Equation (23) has a vertical node in the $p^{1/2}$ versus $1/R$ plane when

$$\left(\frac{P_{CJ}}{P_{CJO}}\right)^{1/2} = 0.75$$

or

$$\left(\frac{P_{CJ}}{P_{CJO}}\right)_{\text{failure}} = 0.56.$$  

Thus Equation (23) predicts that there is value of $R$, designated $R_f$, for which no solutions exist when $R < R_f$. Therefore, $R_f$ is the predicted failure radius of an explosive.

The answer to the question on comparison with experimental results can be obtained for cast TNT, at least. First of all, Equation (23) predicts that the radius will be reached when $P_{CJ}/P_{CJO} = 0.56$. As was mentioned earlier, failure occurs at $P_{CJ}/P_{CJO} = 0.56$ and 0.60 for 1.62 g/cc and 1.60 g/cc cast TNT, respectively. Thus as far as the ratio of $P_{CJ}/P_{CJO}$ in concerned, Equation (23) works well for cast TNT.

A calculated curve in the $(P_{CJ}/P_{CJO})^{1/2}$ versus $1/R$ plane can be obtained as follows. A value for $\beta$ is obtained by using one $(P_{CJ}/P_{CJO})^{1/2}$, $R$ datum point and substituting it into Equation (23). Using this $\beta$, the curve is generated by varying either $(P_{CJ}/P_{CJO})^{1/2}$ or $R$ and solving for the other parameter. Figure 1 presents the calculated and experimental results for 1.62 g/cc TNT. The solid circles are from Table 2 and were calculated using Dremin's data. The solid curve was calculated using Equation (23) and the datum point at $R = 30$ mm with $\beta = 1.176$ mm. The predicted $R_f$ associated with this curve is 11.5 mm while the measured failure radius is 8 mm. The predicted $R_f$ is 44% too large. Note, however, that the solid curve in Figure 1 passes through only the datum point used to generate it. All the other data points lie to the right of the solid curve. The dashed curve in Figure 1 was generated using the datum point at $R = 20$ mm ($\beta = 0.900$ mm). This curve appears to fit the data better and $R_f$ is 8.5 mm or 6.3% larger than the measured value. The difference in the calculated curves is due to the error in the CJ pressure, either the measured $P_{CJ}$, or the calculated $P_{CJO}$, or both. Dremin unfortunately does not give a value for the error in pressure. But, for example, an error of 5% (which would be excellent) easily reconciles the difference in the calculated curves.

Figure 2 presents the calculated and experimental results for 1.60 g/cc TNT, the solid circles are from Table 2. The solid curve was generated using Equation (23) and the datum point at $R = 30\text{ mm}$ with $\delta = 1.475\text{ mm}$. The predicted $R_f$ is 13.98 mm compared to the measured value of 13.75 or 1.7% too large. This curve fits the data so well that no other points were used for comparison.

In Table 2, the variation of $A'$ with $(P_{CJ}/P_{CJO})^{1/2}$ was presented. $A'$ varied by a factor of 1.46 for 1.62 g/cc and 1.73 for 1.60 g/cc TNT. In Table 4, the calculated values of $\delta$ for each datum point are presented. For 1.62 g/cc TNT, $\delta$ varies by a factor of 1.39; for 1.60 g/cc TNT, $\delta$ varies by 1.08. For 1.62 g/cc TNT, the variation of $\delta$ and $A'$ are essentially the same. For 1.60 g/cc TNT, $\delta$ is essentially constant while $A'$ varies by 73%. Note also that the actual values of $\delta$ and $A'$ are very close. However, until the physical meaning of $\delta$ is explained, no physical significance is attached to this.

PHYSICAL INTERPRETATION OF THE RESULTS

A theory is presented which relates the Chapman-Jouquet pressure of a detonating explosive to its charge radius and also predicts the failure radius. The question remains: What is the physical meaning of this theory? First, this theory was developed by comparing the ratio of the total energy available to the actual energy which is unaffected by lateral rarefaction. This resulted in Equation (11). The same equation results if the ratio of the energy flux through the unaffected C-J surface to the energy flux through the detonation front is used as a starting point. The important point about either approach are: (1) the unaffected energy is the governing parameter, and (2) the Chapman-Jouquet pressure is the measurable parameter most sensitive to changes in energy.

The second crucial step was the assumption that $P_{CJ}^{2}t = \text{constant}$ holds in the reaction zone over the range of pressure considered. An alternate way of stating this is that the reaction along a streamline, which is not affected by lateral rarefaction, will proceed until all the energy is released. The time required for this reaction process is, in turn, directly dependent on the pressure.

SUMMARY AND FUTURE WORK

A one parameter equation (Equation (23)) has been developed which relates $P_{CJ0}$, $P_{CJ}$, and $R$. This equation fits the experimental data for cast TNT reasonably well when the parameter $\delta$ is obtained using one datum point and $P_{CJ0}$. This equation also predicts the failure diameter of cast TNT reasonably well when the error in the pressure is taken into account. Unfortunately, no complete set of data containing $P_{CJ}$, $R$, and the failure diameter has been found for
Table 4

Values of $\beta$ for Cast TNT

<table>
<thead>
<tr>
<th>$R$ (mm)</th>
<th>$\left( \frac{P_{CJ}}{P_{CJo}} \right)^{1/2}$</th>
<th>$\beta$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.955</td>
<td>1.176</td>
</tr>
<tr>
<td>20</td>
<td>0.947</td>
<td>0.900</td>
</tr>
<tr>
<td>14</td>
<td>0.919</td>
<td>0.880</td>
</tr>
<tr>
<td>12.5</td>
<td>0.866</td>
<td>1.088</td>
</tr>
<tr>
<td>11</td>
<td>0.845</td>
<td>1.029</td>
</tr>
<tr>
<td>10</td>
<td>0.815</td>
<td>1.001</td>
</tr>
<tr>
<td>9</td>
<td>0.795</td>
<td>0.927</td>
</tr>
<tr>
<td>8</td>
<td>0.751</td>
<td>0.844</td>
</tr>
</tbody>
</table>

Average = 0.981

$\rho_o = 1.62$ g/cc

<table>
<thead>
<tr>
<th>$R$ (mm)</th>
<th>$\left( \frac{P_{CJ}}{P_{CJo}} \right)^{1/2}$</th>
<th>$\beta$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.941</td>
<td>1.475</td>
</tr>
<tr>
<td>20</td>
<td>0.897</td>
<td>1.487</td>
</tr>
<tr>
<td>17.5</td>
<td>0.887</td>
<td>1.380</td>
</tr>
<tr>
<td>15</td>
<td>0.842</td>
<td>1.415</td>
</tr>
<tr>
<td>13.75</td>
<td>0.775</td>
<td>1.440</td>
</tr>
</tbody>
</table>

Average = 1.439

$\rho_o = 1.60$ g/cc
a non-ideal explosive. Future experimental work will obtain such a set of data to check Equation (23) for a non-ideal explosive.

The derivation of Equation (23) relies heavily on the assumption that $P_{\text{dy}} \gamma$ is a constant, Equation (20). The data for cast TNT indicates that this assumption is reasonable. Future theoretical work will investigate this relationship.
References

FIG. 1 COMPARISON OF PREDICTED AND EXPERIMENTAL $P^{1/2}$ VS. $1/R$ RESULTS FOR CAST TNT, $P_0 = 1.62$ g/cc

TABLE 2

<table>
<thead>
<tr>
<th>Calculation</th>
<th>$R_f$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Calculated using data at</td>
<td>11.5 mm</td>
<td>1.176 mm</td>
</tr>
<tr>
<td>$R = 30$ mm,</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta = 1.176$ mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Calculated using data at</td>
<td>8.54 mm</td>
<td>0.900 mm</td>
</tr>
<tr>
<td>$R = 20$ mm,</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta = 0.900$ mm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
FIG. 2 COMPARISON OF PREDICTED AND EXPERIMENTAL $P^{1/3}$ VS. 1/R RESULTS FOR CAST TNT, $P_o = 1.60$ g/cc
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