NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

WIND TUNNEL WALL CORRECTIONS FOR ARBITRARY PLANFORMS AND WIND TUNNEL CROSS-SECTIONS

by

Chester Arthur Heard
June 1977

Thesis Advisor: L.V. Schmidt

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A computer program was developed to obtain the wind tunnel wall corrections for wing angle of attack, induced drag, and pitching moment in incompressible flow. The vortex lattice method is used for computation of these corrections.
(20. ABSTRACT Continued)

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Wind Tunnel Wall Corrections for Arbitrary Planforms and Wind Tunnel Cross-Sections

by

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Lieutenant, United States Navy
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June 1977

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ABSTRACT

A computer program was developed to obtain the wind tunnel wall corrections for wing angle of attack, induced drag, and pitching moment in incompressible flow. The vortex lattice method is used for computation of these correction factors. The program can be applied to wind tunnels of arbitrary cross-sectional shape, and wings of any desired planform, subject to the constraint of straight leading and trailing edges.
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<tr>
<td>[AERO]</td>
<td>Wing aerodynamic influence coefficient matrix at the 0.75 chord position</td>
</tr>
<tr>
<td>AR</td>
<td>Wing aspect ratio = $b^2/s$</td>
</tr>
<tr>
<td>$b$</td>
<td>Wing Span</td>
</tr>
<tr>
<td>C</td>
<td>Wind tunnel cross-sectional area</td>
</tr>
<tr>
<td>$c$</td>
<td>Local wing chord, streamwise direction</td>
</tr>
<tr>
<td>$c_{ave}$</td>
<td>Average wing chord, $(s/b)$</td>
</tr>
<tr>
<td>$c_{\ell}$</td>
<td>Wing span loading, $(\ell/q)$</td>
</tr>
<tr>
<td>$C_L$</td>
<td>Wing lift coefficient = Lift/$qS$, dimensionless</td>
</tr>
<tr>
<td>$C_{L_g}/C_{L_{\alpha}Fa}$</td>
<td>Ground effect ratio</td>
</tr>
<tr>
<td>$C_m$</td>
<td>Pitching moment coefficient = Moment/$qSc\bar{c}$</td>
</tr>
<tr>
<td>$C_R$</td>
<td>Root chord</td>
</tr>
<tr>
<td>$C_T$</td>
<td>Tip chord</td>
</tr>
<tr>
<td>$C_{D_i}$</td>
<td>Wing induced drag coefficient</td>
</tr>
<tr>
<td>$C_{dC}$</td>
<td>Spanwise induced drag distribution $(d/q)$</td>
</tr>
<tr>
<td>$d$</td>
<td>Wing induced drag distribution, drag/unit span</td>
</tr>
<tr>
<td>[DAERO]</td>
<td>Wing aerodynamic influence coefficient matrix at the wing quarter-chord</td>
</tr>
<tr>
<td>[GAMAL]</td>
<td>Wall vortex loop strength</td>
</tr>
<tr>
<td>$h$</td>
<td>Wing height above the ground</td>
</tr>
<tr>
<td>$H$</td>
<td>Wind tunnel height</td>
</tr>
<tr>
<td>$\ell$</td>
<td>Wing running span load, lift/unit span</td>
</tr>
<tr>
<td>NS</td>
<td>Number of horseshoe vortices per semispan</td>
</tr>
<tr>
<td>q</td>
<td>Dynamic pressure</td>
</tr>
</tbody>
</table>
Wall aerodynamic influence coefficient matrix

Influence coefficient matrix for wall-on-tail

Influence coefficient matrix for wall on wing

Wing area

Wing tape ratio, $C_T/C_R$

Freestream velocity

Induced downwash velocity

Wind tunnel width

Induced downwash angle at 0.75 chord position

Influence coefficient matrix for wing-on-wall

Influence coefficient matrix for wing-on-tail

Axial coordinate

Vertical coordinate

Spanwise coordinate

Angle of attack

Wind tunnel wall interference factor

Vortex strength

Induced downwash angle at the tail

Wing spanwise stations ($\frac{2Z}{b}$)

Tail efficiency factor

Subscripts:

Free air

ground

Wall vortex loops

Wind tunnel
TL tail
w wing
( )α θ( )/∂α

Matrix Notation:
[ ] Rectangular matrix
{ } Column matrix
T T Diagonal matrix
[ ]⁻¹ Inverse of a square matrix
ACKNOWLEDGMENT

My sincere gratitude goes to Professor L. V. Schmidt for his invaluable assistance throughout the course of this work and to my wife Anne for her patience and understanding.
I. INTRODUCTION

In the process of wind-tunnel testing the experimental procedure itself constrains and alters the flow past the wing under examination. This modified flow induces errors which must be corrected if accurate results are to be obtained. The major interference effects in subsonic wind tunnels result from the constraining influence of wind tunnel walls, disturbances from model supports and measuring equipment, and turbulence and non-uniformity in the airflow itself.

Russell [Ref. 10] treats the problems resulting from model support interference and introduces a method for reduction of this effect. The differential equations governing flow within a wind tunnel are the same as those in free air. However, the presence of the wind tunnel walls changes the outer boundary conditions. The result is a series of perturbations of the velocity-potential gradient normal to the model surface. The effect is pervasive in that almost all of the measured aerodynamic quantities must be corrected.

Generally, Prandtl is acknowledged as having established the foundations for research on wind-tunnel wall interference. His lifting-line theory and concept of trailing vortices opened the doors to much experimental investigation in this area. The resulting studies considered two-dimensional
and three-dimensional effects, open and closed wind tunnels, and various geometrically shaped test sections.

References 1 and 8 offer a comprehensive treatment of wind-tunnel wall interferences. Much tabular and graphical data are presented for different tunnel shapes and wing spans. Pankhurst and Holder included much data on octagonal wind tunnels, while Pope concentrated more on a uniformly loaded wing.

With the advent of sweptback, slender wings designed for high-speed flight, the lifting line model was no longer suitable. In these situations, the wing lifting system may be represented by distributed horseshoe vortex elements [Refs. 6 and 7]. In Ref. 9, data, graphs, and formulas are assembled from various sources and systematically analyzed for various types of wings in wind tunnels. Although the mathematical formulations are tedious, the resulting graphs and tables summarized very well the data obtained from each theoretical treatment.

Even with all this information available, it is still an arduous task to approach tabulated data with a specific wind tunnel shape and geometric configuration, and hope for a solution to the wall interference problem. It is the purpose of this paper to develop a computerized, closed-form solution to this problem. The design goal was to develop a digital computer code that would be applicable to wind tunnels of arbitrary cross-section and wings with
varying planforms. Such versatility would enhance the accuracy of test data and provide a valuable, accurate tool in wind-tunnel testing.
II. METHODS USED

A. GENERAL

Wind tunnels provide an invaluable tool in the design and performance prediction of wings and wings-and-bodies. However, the wind-tunnel testing environment introduces its own set of problems. The interference action from the walls of the wind tunnel alters the development of induced velocities by the wing lifting system. As a result, downwash formation is suppressed and the wing aerodynamic characteristics are those of a wing having greater aspect ratio. The accurate determination of wind tunnel wall interference is, therefore, an important consideration in wing experiments.

The classical wall correction factor, $\delta$, for the case of a single horseshoe vortex representing the complete wing is defined by:

$$\Delta \alpha = \delta \frac{S}{C} \alpha_L$$

(1)

where

$$\Delta \alpha = \tan^{-1} \frac{W}{V}$$

Using small angle approximations, the tangent is approximately equal to the angle. The wall-correction factor may be expressed in terms of wing circulation, vortex span and average induced downwash velocity by:
\[ \delta = \frac{W}{2B} \frac{c}{r_{w}} \]  

Calculation of \( \delta \) is dependent on the determination of the downwash velocity induced by the wind tunnel walls. Classical methods involve the utilization of an image system of vortices which cancel the effect of the wing lifting system at the wall. The boundary condition of no flow through the wall is, therefore, preserved. This system can provide an accurate, closed-form solution to the ground effect problem. However, when four walls are present, each wall must be represented by an image system which affects other image systems, as well as the model itself. The result is a series of infinite systems. Truncation of this system introduces inaccuracies of unknown magnitude and significance.

This method of images has been applied to tunnels of rectangular cross-section with success. However, it is inadequate when arbitrary tunnel cross-sections are encountered. Add to this a wing with sweep and taper, and the problem becomes significantly altered.

Therefore, a new method was sought to solve this problem specifically for the NPS 3.5 X 5 ft. octagonal wind tunnel. The technique chosen was presented by Joppa in Refs. 4 and 5. This approach lends itself very well to computer solution and can be applied to wind tunnels of any cross-sectional shape.
In this study, an extension of Joppa's vortex lattice method was made to allow the treatment of wings with varying shape, sweep, and aspect ratio. Such treatment requires a modification of the single horseshoe vortex representation of the wing lifting system. The method of Weissinger as presented in Ref. 6 provides an accurate means of representing variable planforms.

B. VORTEX LATTICE METHOD

Since the method of images was considered to be limited in application, it was necessary to find another, more versatile method of representing the wind-tunnel walls. The problem is essentially one of finding a vorticity distribution which will accurately model the wind-tunnel walls. This vortex system must satisfy the boundary condition of no flow through the walls.

In Ref. 4, Joppa presents a method which accurately models the wind-tunnel walls. This was accomplished by representing the tunnel walls with a lattice network of rectangular vortex loops. This vortex system lies in the plane of the tunnel walls and satisfies Helmholtz's theorem that a vortex filament can neither begin nor end in the flow. Fig. 1 shows a representation of this system as applied to the NPS 3.5 by 5 ft. wind tunnel.

Each vortex segment in a given rectangle has vortex strength \( \Gamma_i \), and has a corresponding control point located
Figure 1. Wind Tunnel Geometry
in the center of each rectangle where the no-flow boundary condition is satisfied. The last vortex lattice rectangles located far downstream are represented by horseshoe vortices trailing to infinity. The reasoning is that far downstream, where the influence upon the wall vortex lattice array of the bound vortex elements on the wing is small relative to the trailing vortex elements, one finds that the vortex strengths of the lattice terms do not change in the streamwise direction. Consequently, by a straightforward cancellation argument, one can visualize replacing the vortex lattice rectangles by horseshoe vortices trailing to infinity.

The mathematical presentation of the theory is based on the Biot-Savart law describing the velocity induced at a point by a vortex segment. Joppa in Refs. 4 and 5 follows the mathematical formulation beginning with the Biot-Savart law. This development is summarized in Appendix B.

The general matrix representation of this method centers around the boundary condition of no flow through the wall. As mentioned previously, this condition is satisfied at a control point located at the center of each vortex loop. Therefore, it follows that the velocity induced at each control point by the vortex loops must be exactly cancelled by the velocity created by the wing lifting system. This is presented below in matrix representation

\[
[R_{OL}] \{\Gamma_L\} = [W_{OL}] \{\Gamma_W\} \quad (3)
\]
which allows a solution of vortex lattice strengths in terms of the wing vortex strengths.

\[
\{\Gamma_L\} = [\text{ROL}]^{-1} [\text{WOL}] \{\Gamma_w\}
\]  

(4)

Note that the [ROL] matrix, although possibly quite large, is square and hence may be inverted.

[ROL] represents the wall influence coefficient matrix. It is obtained by calculating the downwash induced at each wall control point by a unit value of each vortex loop. Therefore, \( ROL_{ij} \) is the downwash at wall control point "i" induced by vortex loop "j". [WOL] is the influence coefficient matrix describing the wing effect on each loop control point. \( WOL_{ij} \) is the downwash produced at control point "i" by the \( j^{th} \) wing horseshoe vortex.

By inverting [ROL] and premultiplying with [WOL], the strength of the wall vortex loops can be found in terms of the wing circulation. The inversion of [ROL] can be very time-consuming, since it may be a very large matrix. However, by using symmetry, the order of this matrix can be reduced by a factor of four. This requires the wing to be placed on the horizontal and vertical centerline of the wind tunnel.

C. AIRFOIL REPRESENTATION

In Ref. 5, Joppa stated that the wing can be represented by a lifting system more complicated than a single
elementary horseshoe vortex. In order to model swept wings of arbitrary planform, a more definitive lifting system model was employed. Gray and Schenk, Ref. 6, present a modified Weissinger method designed to find the load distribution of wings with arbitrary planform. The wing is represented by a series of horseshoe vortices. The midspan of each horseshoe vortex bound element is located at the wing quarter-chord position. The condition of flow tangency is satisfied at the wing three-quarter-chord point in the center of each horseshoe vortex. Figure 2 illustrates the configuration chosen.

Pearson [Ref. 7] modified this technique by allowing the bound vortices to be swept, and including chord-wise horseshoe vortices. For this presentation it was felt that unswept bound vortices would accurately represent the lifting system of a swept wing. In addition it was felt that little deterioration of accuracy would result if the chordwise system of bound vortices was combined into a single bound vortex at the quarter-chord. Figure 2 illustrates the configuration chosen.

An important concept in this representation of the wing is that of flow tangency at the three quarter-chord. The downwash angle at the wing can, therefore, be determined by placing a control point at this position in the center of each horseshoe vortex. The rationale behind the three-quarter chord flow tangency condition is a product of
Figure 2. Wing Geometry
two-dimensional flow considerations. Extension of this technique to the three-dimensional wing has been shown to produce experimentally confirmed results. However, it is still unclear why this technique works so well.

The wing aerodynamic influence coefficient matrix was then developed. [AERO] represents the downwash induced at each 0.75 chord wing control point by a unit value of wing horseshoe vortices. The matrix formulation for induced downwash formation in free air is presented below:

\[
\begin{align*}
\{w\} &= [\text{AERO}] \{\Gamma_w\} \\
\end{align*}
\]

Dividing the above expression by freestream velocity, one obtains an expression for 0.75 chord control point induced downwash angles. The flow tangency requirement equates this column matrix to the wing angle of attack column vector.

\[
\begin{align*}
\{\frac{w}{V}\}_{3c/4} &= [\text{AERO}] \{\frac{\Gamma}{V}\} \\
\end{align*}
\]

and

\[
\{\alpha\} = \{\frac{w}{V}\}_{3c/4}
\]

It should be noted here that \(\frac{g}{d}\) is equal to \(\frac{2\Gamma}{V}\), from the Kutta-Joukowski Law. Substitution into equation 6 yields the following results:
Solving equation 7 for \( \frac{\dot{\gamma}}{q} \), one can obtain the wing span load distribution. Since \( \frac{\dot{\gamma}}{q} \) is equivalent to the sectional value of \( C_{L,C} \), integration of the \( \frac{\dot{\gamma}}{q} \) distribution vs. span provides a direct measure of the wing lift coefficient, \( C_L \).

When one assumes a uniform angle of attack value of one radian along the span, the resultant integration of the span load solution will yield the wing lift curve slope, \( C_{L_a} \). It should be borne in mind that these techniques are predicated upon principles from thin airfoil theory and in the limit for an infinite aspect ratio wing will yield a wing lift curve slope of \( 2\pi \text{ (rad}^{-1}) \).

D. DETERMINATION OF THE WALL CORRECTIONS

The classical wall interference factor \( \delta \), which includes a ratio of wing to tunnel cross-section area, is generally found by representing the wing as a single horseshoe vortex. Since the present formulation utilizes ten horseshoe vortices per semi-span, \( \delta \) in a modification can best be found by continuing our matrix algebra.

\[
\alpha_{FA} = \alpha_T + \delta C_L
\]

\[
\alpha_{FA} = \frac{C_L}{C_L \alpha_{FA}} \quad (8)
\]
\[ \alpha_T = \frac{C_L}{C_L \alpha_T} \]

By assuming that the lift coefficient is the same in both tunnel and free air, the following equations result:

\[
\frac{C_L}{C_L \alpha_{FA}} = \frac{C_L}{C_L \alpha_{T}} + \delta C_L \tag{9}
\]

Cancelling the lift coefficient and solving for \( \delta \):

\[
\delta = \frac{1}{C_L \alpha_{FA}} - \frac{1}{C_L \alpha_{T}} \tag{10}
\]

The free air lift slope was found in the previous section. The wing lift curve slope for the wing in the wind tunnel can be found by a simple extension of this same method. The total downwash at the wing control points is equal to that induced by the wing added to that created by the tunnel walls. The effect produced by the tunnel walls is represented by the vortex rectangle-on-wing influence coefficient matrix \([\text{ROW}]\). \( \text{ROW}_{ij} \) is the downwash induced at the \( i \)\(^{th} \) wing control point by the \( j \)\(^{th} \) wall vortex loop. Multiplying this matrix by the vortex loop strength, one obtains the total wall effect.

\[
\{w\} = [\text{AERO}] \{\Gamma_w\} + [\text{ROW}] \{\Gamma_L\} \tag{11}
\]
Substitution of equation 4 for \( \Gamma_L \) forms the following set of equations:

\[
\{w\} = [\text{AERO}] \{\Gamma_w\} + [\text{ROW}] [\text{ROL}]^{-1} [\text{WOL}] \{\Gamma_w\}
\]

\[
\frac{\{w\}}{V} = 0.5 \left([\text{AERO}] + [\text{ROW}][\text{ROL}]^{-1}[\text{WOL}]\right) \left(\frac{\varphi}{q_T}\right)
\]  

(12)

By establishing one radian angle of attack, solving for \( \frac{\varphi}{q_T} \), and integrating the result, the wind-tunnel lift curve slope can be determined. It should be noted that the correction factor obtained from this process is different from the classical \( \delta \), since the correction factor computed above already contains the wing area to tunnel area ratio influence. Therefore, the correction to angle of attack can be obtained by a direct application of this \( \delta \).

\[
\Delta \alpha = \delta C_L
\]

(13)

In most literature, the correction for induced drag is obtained by application of the same \( \delta \) as that used for angle of attack.

\[
\Delta C_{D_i} = \delta C_L^2
\]

(14)
However, in this presentation, the induced drag correction is calculated in a different manner because of added available aerodynamic information. The spanwise induced drag distribution in free air is given by the following formula:

\[
\frac{d}{q}_{FA} = \left[ \frac{2}{q} \right] [DAERO] \left[ \frac{2}{q} \right]_{FA}
\] (15)

[DAERO] is the wing aerodynamic influence coefficient matrix computed with the wing control point now located at the quarter-chord position. The wall effect can similarly be found by including additional matrix terms for the wall effect on the wing quarter-chord position.

\[
\frac{d}{q}_{T} = \left[ \frac{2}{q} \right] \left[ [DAERO] + [ROW][ROL]^{-1}[WOL] \right] \left[ \frac{2}{q} \right]_{T}
\] (16)

Integration of equations 15 and 16 result in the free air and wind tunnel induced drag, respectively. The induced drag correction factor can then be found in the following manner.

\[
C_{D_{iFA}} = K_{FA} C_{L_{FA}}^2
\]

\[
C_{D_{iT}} = K_{T} C_{L_{T}}^2
\]
When testing a model in the tail-on configuration, the presence of the wind tunnel walls serves to modify the downwash angle induced at the tail. This altered flow field can have a significant effect on the pitching moment coefficient. The following increment in pitching moment coefficient is generated by the horizontal tail:

\[
\Delta C_{D_i} = K C_{L_T}^2
\]

At a given value of wing lift coefficient an incremental change in downwash angle at the tail due to the wall effect can be computed:

\[
\alpha_{TL} = \alpha - \epsilon + \delta_i
\]
\[ \Delta \varepsilon = \varepsilon_T - \varepsilon_{FA} \]  

(20)

Therefore, the correction to pitching moment coefficient is given by the following relation:

\[ \Delta C_m = C_{m_i} \Delta \varepsilon \]  

(21)

Since the downwash angle \( \varepsilon \) is equal to \( \frac{W}{V} \) at the tail position, \( \varepsilon \) for free air and for the wind tunnel can be found from the following relations, respectively:

\[ \varepsilon_{FA} = 0.5[WOT] \left( \frac{\theta}{q_{FA}} \right) \]

\[ \varepsilon_T = 0.5[WOT] + [ROT][GAMAL] \left( \frac{\theta}{q_T} \right) \]  

(22)

[WOT] is the wing effect on the tail position and [ROT][GAMAL] represents the wall effect on the tail. Equation 20 can then be used to find \( \Delta \varepsilon \). The pitching moment correction, as evaluated in equation 21, is thus dependent on being given an experimental value for the tail effectiveness parameter.
III. RESULTS AND DISCUSSION

A. COMPUTER PROGRAM

The computer program developed consists of two major subroutines. Subroutine Loop calculates the downwash at any point due to a vortex rectangle. Subroutine HSHOE computes the downwash at any arbitrary point due to a horseshoe vortex. HSHOE is used for both the wing horseshoe and those of the wall system located far downstream in the tunnel walls. The program offers two basic options. One option establishes whether ground effect or wind-tunnel wall corrections are to be calculated. The other option indicates whether the tail-on or tail-off case is to be considered.

Each computer run offers the inclusion of many wings of different span, aspect ratio, taper ratio, and sweep. The wing parameters are read as input data after the inversion of the large wall-on-wall coefficient matrix. Therefore, with a given tunnel configuration, a variety of different wings can be included with a minimum of computer time consumed.

Integration of the wing span loads in accomplished by a subroutine obtained from the NPS source library. It should be noted that in the integration scheme the assumption is made that the wing loading at $\eta = 0.0$ is the same as that for $\eta = .05b$. Any inaccuracy resulting from this assumption
appears to be negligible. Appendix C describes the necessary input to the program.

B. GROUND EFFECT

Originally, ground effect calculations were conducted merely to check the accuracy of the vortex lattice method. As previously stated, the method of images provides a closed-form solution to the ground effect problem. Therefore, this provided an excellent opportunity for a convergence check. As an object of academic interest an option for ground effect computation was retained in the final computer program.

The results of the convergence check showed that the vortex lattice method could very accurately compute the velocity induced at a point by the presence of a wall. By applying the theory previously developed, the lift curve slopes for free air and ground effect were computed. Results were then compared with those presented in Etkin [Ref. 3]. The free air lift curve slope was found to correlate extremely well for both straight and sweptback wings.

Furthermore, in Ref. 3, Etkin defines a ground effect ratio as the ground effect lift curve slope divided by that in free air. The results are tabulated in Table 1, and Figure 3 compares the results obtained from the computer program with those presented in Etkin. As can be seen, agreement is very favorable.
Figure 3. Ground Effect Ratio Plotted as a Function of Wing Height Above the Ground
<table>
<thead>
<tr>
<th>( \frac{2h}{b} )</th>
<th>0.0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>1.0248</td>
<td>1.0266</td>
<td>1.0259</td>
<td>1.0244</td>
<td>1.022</td>
</tr>
<tr>
<td>0.667</td>
<td>1.0488</td>
<td>1.0541</td>
<td>1.0521</td>
<td>1.0481</td>
<td>1.0426</td>
</tr>
<tr>
<td>0.50</td>
<td>1.072</td>
<td>1.080</td>
<td>1.079</td>
<td>1.073</td>
<td>1.065</td>
</tr>
<tr>
<td>0.333</td>
<td>1.126</td>
<td>1.145</td>
<td>1.139</td>
<td>1.125</td>
<td>1.01</td>
</tr>
</tbody>
</table>

Note: Values apply to a wing with
AR = 6.0
TR = 0.3

Table 1
Ground Effect Factor Variation with Sweep Angle
Several computer runs were conducted at different wing heights above the ground. For each run, sweep was varied from 0 to 40 degrees. Results are tabulated in Figure 4. It can be noted that the ground effect ratio initially increases with sweepback but returns to the unswept value at about 30 degrees.

As a note of caution, care should be exercised when the wing gets very close to the ground. It is necessary to greatly decrease the vortex loop size for accurate results. This procedure seems to provide a much more finely defined ground representation. Otherwise, calculations will result in inordinately high values.

C. WIND TUNNEL APPLICATION

A convergence check of the wind tunnel wall correction factor was obtained by applying the program to a 3 X 5 ft rectangular tunnel. The correction factor obtained was converted to the classical $\delta$ by including the factors $\frac{S}{C}$ and 57.3 (radians to degrees). The result of 0.1216 was compared to that presented by Joppa in Ref. 5. The wing geometric configuration presented in Ref. 5 consisted of a single horseshoe vortex. The value of 0.127 from Ref. 5 correlates very well with the results of this study.

Tables 2 and 3 present the data obtained from computations with the NPS 3.5 X 5.0 ft octagonal wind tunnel. Computer runs were conducted using wings of varying span.
Figure 4. Wing Span Load Distribution
<table>
<thead>
<tr>
<th>SPAN RATIO</th>
<th>.4</th>
<th>.5</th>
<th>.6</th>
<th>.7</th>
<th>.8</th>
<th>.9</th>
<th>.10</th>
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<td>4.2545</td>
<td>4.2545</td>
<td>4.2545</td>
<td>4.2545</td>
<td>4.2545</td>
<td>4.2545</td>
<td>4.2545</td>
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<tr>
<td>$C_{L_{u_{T}}}$</td>
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<td>4.4135</td>
<td>4.4907</td>
<td>4.594</td>
<td>4.7489</td>
<td>5.6563</td>
<td>6.2838</td>
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<tr>
<td>$C_{D_{FA}}$</td>
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<td>.9342</td>
<td>.9342</td>
<td>.9342</td>
<td>.9342</td>
<td>.9342</td>
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<td>$C_{D_{T}}$</td>
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<td>.8583</td>
<td>.8240</td>
<td>.7793</td>
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<td>.7086</td>
<td>.9955</td>
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<td>.00755</td>
<td>.0107</td>
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<td>.0517</td>
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Table 2

Wing Parameter Variation With Span
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<tr>
<th>c/4 Chord Sweep</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
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<td>$C_{L_{\alpha_{FA}}}$</td>
<td>4.2545</td>
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<td>$C_{L_{\alpha_{T}}}$</td>
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<td>4.6347</td>
<td>4.5966</td>
<td>4.5312</td>
<td>4.4355</td>
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<td>$C_{D_{FA}}$</td>
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<td>.9948</td>
<td>.9924</td>
<td>.9806</td>
<td>.9585</td>
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<td>.7480</td>
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<td>$C_{D_{T}}$</td>
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<td>.8190</td>
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<td>.01061</td>
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</table>

Table 3
Wing Parameter Variation with Sweep Angle
and sweep angle. Figures 4 and 5 illustrate the span loads calculated for a typical wing. It is interesting to note that as sweep angle increased, negative induced drag appears on the outboard wing stations. Therefore, total induced drag is seen to steadily decrease with increasing sweep angle.

As sweep angle is increased, the free air lift curve slope decreases. Correspondingly, the corrections to angle of attack and induced drag are reduced. Figure 6 graphically displays the data for a straight wing. In the past, the correction factors for the NPS 3.5 X 5.0 ft octagonal wind tunnel were obtained by assuming an elliptic cross-section. The correction factors presented in Ref. 1 for elliptic cross-section are 0.610 for angle of attack and 0.0106 for induced drag. In comparison, the vortex lattice program computes values of 0.7086 and 0.01075, respectively. It is interesting to note that the induced drag correction is almost identical, whereas the correction to angle of attack is slightly higher for the octagonal wind tunnel. Figure 7 illustrates the angle of attack correction factor with varying tunnel span ratios. The correction factor can be seen to steadily increase with the span ratio.

It is an interesting observation that the value of induced drag for a wing span of 5.0 ft was almost zero. This compares very well with airfoil theory in that a wing
Figure 5. Induced Drag Distribution
Figure 6. Angle of Attack Correction Factor Plotted as a Function of Span Ratio
Figure 7. Angle of Attack Correction Factor Plotted As A Function Of Sweep Angle
which spans the wind tunnel represents an approximation to the two-dimensional wing. Induced drag for a two-dimensional wing should equal zero. In addition, the wind-tunnel lift curve slope is within 0.0006 of the theoretical two-dimensional value of $2\pi \text{ rad}^{-1}$.

A sample computer run was made for the moment correction factor. The tail quarter-chord was placed two feet behind the wing bound vortex, on the tunnel centerline. The correction factor obtained for a straight wing of aspect ratio 6.0 was -0.2297. For a wing with sweep angle of 30 degrees and taper of 0.3, a value of -0.1887 was calculated. The above correction factors must be multiplied by the stabilizer effectiveness parameter, using consistent angular measures, in order to find the pitching moment correction.
IV. CONCLUSIONS AND RECOMMENDATIONS

It is felt that the vortex lattice computer program as presented in this paper offers a very reliable and flexible tool for the determination of wind-tunnel wall corrections. It is capable of treating wings of any span, taper ratio, aspect ratio, and quarter-chord sweep, subject to the constraint of straight leading and trailing edges. The geometric shape of the wind tunnel cross-section can similarly be varied at the discretion of the program user. Resultant accuracy appears to be excellent. However, caution should be exercised in determining the vortex loop size. As the wing is positioned closer to the walls, a smaller, more definitive vortex loop representation must be used.

A logical extension of this program would be the inclusion of wings with an antisymmetric load distribution. Furthermore, slight modifications could incorporate the ability to consider the wall effects on other stability derivatives. In its present form, this program offers the wind tunnel user a versatile tool for determination of wind tunnel wall corrections.
APPENDIX A

The general assumptions used in the compilation of this program are listed below.

1. The two-dimensional lift-curve slope is $2\pi$ rad$^{-1}$.
2. The wing load distribution is symmetrical.
3. Aeroelastic effects were not considered.
4. The wing is located on the horizontal and vertical centerline of the wind tunnel.
5. The leading and trailing edges of the wing are straight.
APPENDIX B
DEVELOPMENT OF INDUCED VELOCITY EQUATIONS

The equations required to solve for induced velocities consist of two main types. One equation solves for the velocity induced by an arbitrarily oriented vortex segment, whereas the other basic equation applies to horseshoe vortex elements. These equations have been incorporated in subroutines LOOP and HSHOE, respectively. The development of these equations presented below follows closely that found in Ref. 4 and 5.

According to the Biot-Savart law, the velocity induced at a point \( p \) is found from:

\[
\mathbf{w} = \frac{\mu}{4\pi h} (\cos B_1 + \cos B_2) \mathbf{v} \quad (1B)
\]

For vortex segments oriented in the \( Y-Z \) plane,

\[
\cos B_1 + \cos B_2 = \frac{R_1 + R_2}{2R_1 R_2 S} \left( S^2 - (R_1 - R_2)^2 \right) \quad (2B)
\]

It is now necessary to consider \( \mathbf{v} \), the unit vector which establishes direction. From Fig. 8, it can be seen that

\[
\mathbf{v} = \frac{R_1 \times S}{|R_1 \times S|} \quad (3B)
\]
Figure 8. Velocity Induced By A Vortex Segment
Noting from Fig. 8 that

$$hs = |\mathbf{R}_1 \times \mathbf{S}|$$

(4B)

then

$$hs = [(R_{1y}S_z - R_{1z}S_y)^2 + (R_{1z}S_x - R_{1x}S_z)^2 + (R_{1x}S_y - R_{1y}S_x)^2]^{1/2}$$

Calculating the induced velocity in the direction of outward normal of point P per unit Γ,

$$\mathbf{w} = \frac{\mathbf{v}}{Γ} \cdot \mathbf{n}$$

(5B)

where the unit normal vector is defined by:

$$\mathbf{n} = \cos θp \mathbf{j} + \sin θp \mathbf{k} + \cos θp \mathbf{k}$$

Therefore, one finally obtains:

$$\frac{\mathbf{w}}{Γ} = \frac{R_1 + R_2}{8πR_1R_2(hs)^2} \left[ s^2 - (R_1 - R_2)^2 \right]$$

$$\cdot \left[ (\sin θp)(R_{1y}S_x - R_{1x}S_y) + (\cos θp)(R_{1x}S_y - R_{1y}S_x) \right]$$

(6B)
A conditional statement must be included in the program to set the downwash equal to zero if $h$s is equal to zero. This will preclude having a zero term in the denominator.

The trailing elements of a vortex loop can be treated with more simplicity. For these segments,

$$\cos B_1 = \frac{(R_1^2 - h^2)^{1/2}}{R_1}$$

$$\cos B_2 = \frac{(R_2^2 - h^2)^{1/2}}{R_2}$$

(7B)

However, it can be seen from Fig. 8 that

$$R_1^2 = (x_p - x_A)^2 + (y_A - y_p)^2 + (z_A - z_p)^2$$

$$h^2 = (y_A - y_p)^2 + (z_A - z_p)^2$$

(8B)

Combining equations 7 and 8,

$$\cos B_1 = \frac{x_p - x_A}{R_1}$$

$$\cos B_2 = \frac{x_B - x_p}{R_2}$$

(9B)

As a result, the equation for the trailing elements of a vortex loop is simply
\[
\frac{w}{\Gamma} = \frac{\cos B_1 + \cos B_2}{4\pi h^2} \left[ (\sin \theta p)(z_A - z_p) + (\cos \theta p)(y_p - y_A) \right]
\]

(10B)

For the horseshoe vortex, the equation for the bound segment is identical to equation 6. Similarly, the trailing elements can be treated as in equation 11 with a slight modification. In this case, the cosine of one enclosed angle is equal to 1, since the trailing element extends downstream to infinity. The induced velocity equation becomes

\[
\frac{w}{\Gamma} = \frac{1 + \cos B_1}{4\pi h^2} \left[ (z_A - z_p) + (y_p - y_A) \right]
\]

(11B)
## APPENDIX C

### COMPUTER PROGRAM INPUT

<table>
<thead>
<tr>
<th>Card Number</th>
<th>Column Number</th>
<th>Description</th>
<th>Format</th>
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<tr>
<td>1</td>
<td>1-72</td>
<td>Program title</td>
<td>18A4</td>
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<tr>
<td>2</td>
<td>1-5</td>
<td>MM=Number of Spanwise Stations</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>6-10</td>
<td>NN=Number of Downstream Stations</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>11-20</td>
<td>Deltax=Loop length</td>
<td>FL0.0</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>NOPT=1 for Ground Effect</td>
<td>I1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>NTAIL=1 for tail-on</td>
<td>I1</td>
</tr>
<tr>
<td>4</td>
<td>1-72</td>
<td>Tunnel y-coordinates (one quadrant only)</td>
<td>8FL0.0</td>
</tr>
<tr>
<td>5</td>
<td>1-72</td>
<td>Tunnel z-coordinates (one quadrant only)</td>
<td>8FL0.0</td>
</tr>
<tr>
<td>6</td>
<td>1-10</td>
<td>Span</td>
<td>FL0.0</td>
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<tr>
<td></td>
<td>11-20</td>
<td>Aspect Ratio</td>
<td>FL0.0</td>
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<tr>
<td></td>
<td>21-30</td>
<td>Taper Ratio</td>
<td>FL0.0</td>
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<tr>
<td></td>
<td>31-40</td>
<td>c/4 Sweep (degrees)</td>
<td>FL0.0</td>
</tr>
<tr>
<td>7</td>
<td>1-10</td>
<td>x-Coordinate of tail quarter-chord</td>
<td>FL0.0</td>
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<tr>
<td></td>
<td>11-20</td>
<td>z-Coordinate of tail quarter-chord</td>
<td>FL0.0</td>
</tr>
</tbody>
</table>

Input cards 5 and 6 can be repeated for different planforms and tail locations.
C******************************************************************************
C THIS PROGRAM IS DESIGNED TO CALCULATE THE INTERFERENCE PARAMETERS
C FOR ARBITRARY PLANE FLAPS IN THE PRESENCE OF THE GROUND OR A WIND-
C TUNNEL OF ANY CROSS-SECTIONAL SHAPE. CORRECTION FACTORS ARE COMPUTED
C FOR ANGLE OF ATTACK, INDUCED DRAG, AND PITCHING MOMENT (FAR TAIL-
C CONFIGURATIONS).
C******************************************************************************

C INPLICIT REAL*8 (A-H, O-Z)
C COMMON/C1/(DELTA,P,P,P,P,PM,PH,P4)
C COMMON/C2/(SPAN,AR,TR,SC4,B2,NS)
C COMMON/C3/(XH(10),XH(10),ZH(10),ZH(10),ZC(10),ETA(12)
C COMMON/C4/XP(50),XL(50),YL(50),Z(50),XLI(50),YLI(50)
C COMMON/C5/YP(50),XL(50),YL(50),THETA(50),XL(10),YL(10)
C DIMENSION ROL(50,50),ROL(50,100),ROH(10,50)
C DIMENSION GAMAL(50,10),W(10,10),CLC(10),CDC(10)
C COMMON TITHE(20),AERO(10,10),COORD(20)
C COMMON ROT(1,50),WCT(1,10),WT(1,10),CLCFA(10),CLCT(1)

1001 FCRMAT(18A4)
1002 FCRMAT(4F10.0)
1003 FCRMAT(2I5,F10.0)
1004 FCRMAT(8F10.0)
1005 FCRMAT(2I)
1006 FCRMAT(2I)
2001 FCRMAT(1H1,18A4//)
2002 FCRMAT(*,T12,5HM = ,12,1X,41HVCRTEX RECTANGLES IN TUNNEL CIRCUM
2ERENCE/T12,5HMN = ,12,1X,28HVORTEX RECTANGLES DOWNSTREAM/
2TB,9HDDELTA = ,F3,1X,375FEET (REACTANGLE LENGTH IN X-DIRECTION) //)
2003 FCRMAT(*,T5,FREE-AIR SPAN LOADS//)
2004 FCRMAT(*,T7,3HTA,T20,3HL/0,3T4,3HD/Q)
2005 FCRMAT(*,T5,F7.4,T18,F9.6,132,F9.6)
2006 FCRMAT//IX,T5,32THTHE FREE-AIR LIFT CURVE SLOPE = ,F6.4,1X,
2007 FCRMAT(*,T5,22HIND-TUNNEL SPAN LOADS//)
2008 FCRMAT(*,T7,35THTHE WIND-TUNNEL LIFT CURVE SLOPE = ,F6.4,
21X,10FRE RADIUS/T7,35THTHE WIND-TUNNEL INDUCED DRAG = ,F6.4//)
2009 FCRMAT(*,T7,29THWIND-TUNNEL WALL CORRECTIONS/T7,12CEL-ALPHA = ,
21X,8HCL (0E/1,7T7,12DEL-CD)
210C FCRMAT(*,T7,24HGRND EFFECT SPAN LOADS//)
2111 FCRMAT(*,T7,37THE GROUND-EFFECT CURVE SLOPE = ,F6.4,
21X,10FRE RADIUS/T7,37THE GROUND-EFFECT INDUCED DRAG = ,F6.4//)
2112 FCRMAT//IX,T5,35THTHE WING FEIGHT ABOVE THE GROUND = ,F5.2,
21X,4FEET//)
2113 FCRMAT(*,T7,26THTHE GROUND EFFECT RATIO = ,F6.4//)
2114 FCRMAT(*,T10,7HSPAN = ,F3.1,1X,4FEET/T2,12ASPECT RATIO = ,
2F,1T3,14HTAPER RATIO = ,F3.1/T5,12HC/4 SWEET = ,F3.1,1X,
27° DEGREES/T12, SHNS = 12, 1X, 24HCRSHOES PER SEMI-SPAN//
2015 FCRMAT1 * TT, 12odel-cm = 1, F7.5, 1X, 10PER RADIUS, 1X,
240 TIMES STABILIZER EFFECTIVENESS PARAMETER
216 FCRMAT1/IX TT 20 THE TAIL POSITION = F6.3, 1X,
217 FEET(X-DIRECTION)/TT, 20 THE TAIL POSITION = F6.3, 1X,
217 FEET(Z-DIRECTION)!!

CC
REACTS(5,10C1) TITLE(1), I = 1, 18
REAL TUNNEL PARAMETERS.
ACFT=0 : CALCULATIONS ARE FOR WIND TUNNEL,
NOPT=1 : CALCULATIONS ARE FOR GROUND EFFECT.
NTAIL=1 : CALCULATIONS ARE MADE FOR TAIL-ON CONFIGURATION.
READ(5,10C5) NOPT,NTAIL
READ(5,10C3) MN, NN, DELTA
M4=MW/4
IF (NOPT.EQ.1) M4=MW/2
KAM4=KAM*4
NEW=NNM4-M4+1
K41=M4+1

C REAL IN WALL COORDINATES
READ(5,10C4) Y(11), I = 1, M41
READ(5,10C4) X(11), I = 1, M41
F1=3.141592700
P4=4.0000*3.1415926400
PB=8.0000*PB

C CALL GECMT
C CALCULATION OF WALL EFFECT ON WALL CONTROL PTS.
C ESTABLISH COORD FOR THIS CONTROL PT.
CC 160 I = 1, NN4
CCRD(I) = XLP(I)
CCRD(1) = YLP(I)
CCRD(3) = ZLP(I)
CCRD(14) = IFETAP(I)*57.2957795DC

CC ESTABLISH COORD FOR LOOPS
CC 140 J = 1, NN4
WASF=0.000
WASF=0.000
HL3=0.000
HL4=0.000
IF (J.GE.NNEW) GO TO 135
CCRD(5) = XLI(J)
CCRD(6) = XLI(J)
CCRD(7) = YLI(J)
C CALCULATE EFFECT IN QUADRANT 1
CALL LCCP (COORD, WASH1)
C ESTABLISH COORD FOR QUADRANT 2
CCERC(7) = -YL2(J)
CCERC(8) = -YL1(J)
COORD(6) = -ZL2(J)
COORD(10) = -ZL1(J)
C CALCULATE EFFECT IN QUADRANT 2
CALL LCCP (COORD, WASH2)
IF (NOPT.EQ.1) GO TO 134
C ESTABLISH COORD FOR QUADRANT 3
CCERC(7) = -YL2(J)
CCERC(8) = -YL1(J)
COORD(5) = -ZL2(J)
COORD(10) = -ZL1(J)
C CALCULATE EFFECT IN QUADRANT 3
CALL LCCP (COORD, WASH3)
C ESTABLISH COORD FOR QUADRANT 4
CCERC(7) = -YL1(J)
CCERC(8) = -YL2(J)
COORD(5) = ZL1(J)
COORD(10) = ZL2(J)
C CALCULATE EFFECT IN QUADRANT 4
CALL LCCP (COORD, WASH4)
134 CONTINUE
PCL(1, J) = WASH1+WASH2+WASH3+WASH4
CC TO 139
135 CONTINUE
C CALCULATE EFFECT FROM END FORESHOES
C ESTABLISH COORD FOR QUADRANT 1
CCERC(5) = XL1(J)
CCERC(6) = YL1(J)
COORD(7) = YL2(J)
COORD(8) = ZL1(J)
COORD(5) = ZL2(J)
C CALCULATE EFFECT IN QUADRANT 1
CALL +SHOE (COORD, HL1)
C ESTABLISH COORD FOR QUADRANT 2
CCERC(6) = YL2(J)
CCERC(7) = YL1(J)
COORD(8) = -ZL2(J)
COORD(5) = -ZL1(J)
C CALCULATE EFFECT FROM QUADRANT 2
CALL +SHOE (COORD, HL2)
IF (NOPT.EQ.1) GO TO 138
C ESTABLISH COORD FOR QUADRANT 3
CCORD(6)=-YL2(J)
CCORD(7)=-YL1(J)
CCORD(8)=-ZL2(J)
CCORD(9)=-ZL1(J)
C CALCULATE EFFECT FROM QUADRANT 3
CALL PSHELH (CCORD,HL3)
C ESTABLISH COORD FOR QUADRANT 4
CCORD(6)=-YL1(J)
CCORD(7)=-YL2(J)
CCORD(8)=ZL1(J)
CCORD(9)=ZL2(J)
C CALCULATE EFFECT FROM QUADRANT 4
CALL PSHELH (CCORD,HL4)
138 CONTINUE
FCL(I,J)=FL1+HL2+L3+HL4
139 CONTINUE
140 CONTINUE
16C CONTINUE

C INVERT THE WALL INFLUENCE COEFFICIENT MATRIX.
CALL INVR (ROL,ANM4,50)

C 16S CONTINUE
C READ IN NEW WING PARAMETERS FOR THIS COMPUTATION.
READ(5,1002) SPAN,AR,TR,SC4
C IF THERE IS NO MORE WING PARAMETER INPUT, GO TO END
C CF PDCRAW.
IF (SPAN.EQ.0.300) GO TO 500
C ENTER THE NUMBER OF WING HORSESHOE VORTICES.
NS = NC. CF HORSESHOE VORTICES (ONE SIDE ONLY)
NS=10
WRITE(2,2001) (TITLE(I),I=1,18)
C WRITE TUNNEL PARAMETERS.
WRITE(5,2002) MM,NN,DELTAX
C WRITE WING PARAMETERS.
WRITE(6,2014) SPAN,AR,TR,SC4,NS
CALL GECMW
C
C CALCULATE EFFECT OF WING ON WALL CONTROL PTS.
C ESTABLISH COORD FOR WALL CONTROL PTS.
CC 1901 =1,NM4
CCRCD(1)=XLP(I)
CCRCD(2)=YLP(I)
CCRCD(3)=ZLP(I)
CCRCD(4)=TCP(1)*57.295779500
C ESTABLISH COORD FOR LEFT WING
CC 170 J=1,NS
CCRCD(5)=XF(J)
CCRCD(6)=0.000
CCRCD(7)=0.000
CCRCD(8)=ZF(J)
CCRCD(9)=ZF-J(J)
C CALCULATE EFFECT FROM LEFT WING
CALL FSDE(COORD,WASHL)
C
C ESTABLISH COORD FOR RT. WING
CCRCD(6)=-ZH2(J)
CCRCD(7)=-ZH1(J)
C CALCULATE EFFECT FROM RT. WING
CALL HSHOE(COORD,WASHR)
WCL(I,J)=WASHL+WASHR
17C CONTINUE
15C CONTINUE
C SOLVE THE MATRIX EQU: (RGL)*(GAMAL)=-(WOL)*(GAMAW)
CC 1951 =1,NM4
CC 1941 =1,NS
W=0.000
CC 193K =1,NM4
W=W+ROL(I,K)*WOL(K,J)
152 CONTINUE
GAMAL(I,J)=W
154 CONTINUE
155 CONTINUE
C
C CALCULATE THE EFFECT OF WALL ON WING CONTROL PTS.
C ESTABLISH COORD FOR WING CONTROL PTS.
CC 220I =1,NS
CCRCD(1)=XWP(I)
CCRCD(2)=0.000
CCRCD(3)=ZWP(I)
CCRCD(4)=-50.000
C ESTABLISH COORD FOR LCOP
CC 210J =1,NM4
WASH3=0.000
WASH4=0.000
HL3=J,JO0
CALCULATE EFFECT FROM QUADRANT 1
CALCULATE EFFECT FROM QUADRANT 2
CALCULATE EFFECT FROM QUADRANT 3
CALCULATE EFFECT FROM QUADRANT 4
CALCULATE EFFECT FROM QUADRANT 1
CALCULATE EFFECT FROM QUADRANT 2
CCED(6) = YL2(J)
CCED(7) = YL1(J)
CCED(8) = -ZL2(J)
CCED(9) = -ZL1(J)

C CALCULATE EFFECT FROM QUADRANT 2
CALL HSHOW (COORD, HL2)
IF (NOPT = EC = 1) GO TO 206

C ESTABLISH COORD FOR QUADRANT 3
CCED(6) = -YL1(J)
CCED(7) = -YL2(J)
CCED(8) = ZL1(J)
CCED(9) = ZL2(J)

C CALCULATE EFFECT FROM QUADRANT 3
CALL HSHOW (COORD, HL3)

C ESTABLISH COORD FOR QUADRANT 4
CCED(6) = -YL1(J)
CCED(7) = -YL2(J)
CCED(8) = ZL1(J)
CCED(9) = ZL2(J)

C CALCULATE EFFECT FROM QUADRANT 4
CALL HSHOW (COORD, HL4)

205 CONTINUE
CCED(1,J) = HL1 + HL2 + HL3 + HL4

206 CONTINUE

210 CONTINUE

220 CONTINUE

C CALCULATE THE DOWNWASH PRODUCED BY THE WING FOR SHOES
C ON THE WING CONTROL PTS. (AERO).

CC 40 I = 1, NS

C DEFINE CONTROL PT. COORDINATES FOR THIS ITERATION
CCED(1) = XWFP(I)
CCED(2) = C = 0.000
CCED(3) = ZWFP(I)
CCED(4) = -SC = 0.000

DC 30 J = 1, NS

C ESTABLISH COORD FOR LEFT SIDE OF WING
CCED(5) = XH(J)
CCED(6) = C = 0.000
CCED(7) = 0.000
CCED(8) = ZH1(J)
CCED(9) = ZH2(J)

C CALCULATE DOWNWASH FROM LEFT SIDE OF WING
CALL HSHOW (COORD, WSHL)

C ESTABLISH COORD FOR RIGHT SIDE OF WING
CCED(10) = -ZL2(J)
CCORD(9) = -2*1(I)
C CALCULATE DOWNWASH FROM RIGHT SIDE OF WING
  CALL HSPOE (COORD,WSHR)
  AERO(I,J) = (WSHL+WSHR)*0.5D)
30  CONTINUE
40  CONTINUE
C
C ESTABLISH COORD FOR INDUCED DRAG CALCULATION
CC 60 I=1,NS
C CEFORE C/4 CONTROL POINTS FOR THIS ITERATION
CCORD(1) = XH(I)
CCORD(2) = C.COO
CCORD(3) = 2*PI(I)
CCORD(4) = -50.0D0
C ESTABLISH COORD FOR LEFT WING
CC 60 J=1,NS
CCORD(5) = XH(J)
CCORD(6) = 0.0D0
CCORD(7) = C.COO
CCORD(8) = ZH(1J)
CCORD(9) = ZH(2J)
C CALCULATE C/4 DOWNWASH FOR LEFT WING
  CALL HSPOE (COORD,WSHL)
C ESTABLISH COORD FOR RIGHT WING
CCORD(8) = 2*Z(2J)
CCORD(9) = -2*Z(1J)
C CALCULATE C/4 DOWNWASH FOR RIGHT WING
  CALL HSPOE (COORD,WSHR)
  AERO(I,J) = (WSHL+WSHR)*0.5D0
50  CONTINUE
60  CONTINUE
C
C FIND THE TOTAL DOWNWASH AT WING CONTROL PTS.
C MULTIPY [ROW]*[GAMAL] TO GET WALL EFFECT ON WING CCNTROL PTS.
CC 250 J = 1,NS
C SET 1.0 RADIUS ANGLE OF ATTACK ACROSS SEMI-SPAN.
  CLC(I) = 1.0D0
CC 240 J = 1,NS
   WW = 0.0D0
CC 230 K = 1,NNM4
   WW = WW+ROW(I,K)*GAMAL(K,J)
23C  CONTINUE
C SET (W) EQUAL TO TOTAL WING DOWNWASH (AERO-WALL EFFECT)
  WW(I,J) = AERO(I,J) - (WW)*0.5D0)
24C  CONTINUE
25C CONTINUE
C ELL ANG SLVB TO SOLVE : (L/Q)*(AERC)=(ALPHA)
   CALL ELL (AERO,NS,10)
   CALL SLVB (AERO,NS,10,CLC)
   WRITE(6,2003)
   WRITE(6,2004)
C SOLVE THE MATRIX EQN : (D/Q)=(L/Q)*CAERO*(L/Q)
   CC 60 I=1,NS
   CI=C.000
   CC 60 J=1,NS
   CI = D1*DAERO(I,K)*CLC(K)
50 CONTINUE
C CERTAIN CDC
   CCC(I) = CLC(I)*D1
   ETA2=2*WP(I)/B2
C WRITE WING SPAN LOADS IN FREE AIR
   WRITE(6,2005) ETA2,CLC(I),CDC(I)
80 CONTINUE
   CLC(1)=CLC(1)
   CLC2(12)=0.000
   CCC(1)=CCC(1)
   CCC2(12)=0.000
   CC 10 I=2,11
   CCC2(I)=CCC(I-1)
   CLC2(I)=CLC(I-1)
   CLCFA(I-1)=CLC(I-1)
1C1 CONTINUE
C INTEGRATE SPAN LOADS TO OBTAIN LIFT-CURVE SLOPE
C AND INDUCED DRAG IN FREE AIR.
   CALL LINTG (ETA,CLC2,CLC2,12)
   CALL CINTG (ETA,CDC2,CDC2,12)
C AVE=SPAN/AR
   CLFA=CLC2(NS+2)/CAVE
   COFA=CLC2(NS+2)/CAVE
   WRITE(6,2006) CLFA,COFA
C
C CALCULATE WAIL EFFECT ON WING C/4 CONTROL PTS FOR
C INDUCED DRAG CALCULATIONS.
C ESTABLISH CDCRD FOR WING C/4 PTS.
   CC 2801 = 1,NS
   CCCRD(1)=XH(I)
   CCCRD(2)=0.000
   CCCRD(3)=2*WP(I)
CCCRC(4) = -90.000
CC 270 J = 1, NM4
WASF3 = 0.000
WASF4 = 0.000
HL3 = 0.000
HL4 = 0.000
IF (J.GE.NNEM) GO TO 265
C CALCULATIONS FOR QUADRANT 1
CCCRCD(5) = XL1(J)
CCCRC(6) = XL3(J)
CCCRC(7) = YL1(J)
CCCRC(8) = YL2(J)
CCCRC(9) = ZL1(J)
CCCRC(10) = ZL2(J)
C CALCULATE EFFECT FROM QUADRANT 1
CALL LCOP (COORD, WASH1)
C ESTABLISH COORD FOR QUADRANT 2
CCCRC(7) = YL2(J)
CCCRC(8) = YL1(J)
CCCRC(9) = ZL2(J)
CCCRC(10) = ZL1(J)
C CALCULATE EFFECT FROM QUADRANT 2
CALL LCOP (COORD, WASH2)
IF (NOT.EQ.1) GO TO 264
C ESTABLISH COORD FOR QUADRANT 3
CCCRC(7) = -YL2(J)
CCCRC(8) = -YL1(J)
CCCRC(9) = -ZL2(J)
CCCRC(10) = -ZL1(J)
C CALCULATE EFFECT FROM QUADRANT 3
CALL LCOP (COORD, WASH3)
C ESTABLISH COORD FOR QUADRANT 4
CCCRC(7) = -YL1(J)
CCCRC(8) = -YL2(J)
CCCRC(9) = ZL1(J)
CCCRC(10) = ZL2(J)
C CALCULATE EFFECT FROM QUADRANT 4
CALL LCOP (COORD, WASH4)
264 CONTINUE
C ACCRUMULATE CONTRIBUTIONS FROM EACH QUADRANT
FCAW(J,J) = WASH1+WASH2+WASH3+WASH4
GO TO 269
265 CONTINUE
C SET CORD FOR END HORSSES
CCCRC(5) = XL1(J)
CCCRC(6) = YL1(J)
CCCRC(7) = YL2(J)
CCCRC(8) = ZL1(J)
CCRC(9)=ZL2(J)
C CALCULATE EFFECT FOR QUADRANT 1
CALL HSHCE (COORD,HL1)
C ESTABLISH COORD FOR QUADRANT 2
CCRC(6)=YL2(J)
CCRC(7)=YL1(J)
CCRD(8)=-ZL2(J)
CCRD(5)=-ZL1(J)
C CALCULATE EFFECT FROM QUADRANT 2
CALL HSHCE (COORD,HL2)
IF (NOPT.EQ.1) GO TO 268
C ESTABLISH COORD FOR QUADRANT 3
CCRC(6)=-YL2(J)
CCRD(7)=-YL1(J)
CCRD(8)=-ZL2(J)
CCRD(9)=-ZL1(J)
C CALCULATE EFFECT FROM QUADRANT 3
CALL HSHCE (COORD,HL3)
C ESTABLISH COORD FOR QUADRANT 4
CCRC(6)=-YL1(J)
CCRD(7)=-YL2(J)
CCRC(8)=ZL1(J)
CCRC(5)=ZL2(J)
C CALCULATE EFFECT FROM QUADRANT 4
CALL HSHCE (COORD,HL4)
268 CCCTINUE
RCW(I,J)=HL1+HL2+HL3+HL4
269 CCCTINUE
270 CCCTINUE
280 CCCTINUE
C (RCW)*GAMAL TO GET WALL EFFECT CN WING
C C/4 CONTROL PTS.
C 320I =1,NS
C 310J =1,NS
C WW =0.000
C 300K =1,AM4
C WW=WW+RCW(I,K)*GAMAL(K,J)
330 CCCTINUE
C SET (WD) EQUAL TO TOTAL DOWNWASH AT C/4 PTS : (DAERC-WALL EFFECT)
C WW(I,J)=(EAERO(I,J)-(WW*.500))
31C CCCTINUE
CL(C)=1.000
32C CCCTINUE
C EQU AND SLVB TO SOLVE THE MATRIX EQUATION :
C (ALPHA)=(AERC-W)*L/Q).
CALL ELU (W,NS,10)
CALL SLVB (W,NS,10,CLC)
IF(NOPT.EQ.1) GO TO 321
WRITE(6,2007)
CC TO 322
321 CONTINUE
WRITE(6,2010)
CC 322 CONTINUE
WRITE(6,2004)
C
SOLVE THE MATRIX EQN. : (D/Q)= (L/Q)*(WD)*(L/Q)
CC 330 I=1,NS
CC 329 K=1,NS
C 325 CONTINUE
C KEEP CDC
CDC(I)=CLC(I)*F1
ETA2=2*W(I)/B2
C WRITE WING SPAN LOADS IN PRESENCE OF WALL.
WRITE(6,2005) ETA2,CLC(I),CDC(I)
33C CONTINUE
CLC2(I)=CLC(I)
CCL2(I)=CC2(I)
CLC2(12)=0.00D0
CLC2(12)=0.00D0
CC 340 I=2:11
CLC2(I)=CLC(I-1)
CCL2(I)=CC2(I-1)
CLC(I-1)=CLC(I-1)
34C CONTINUE
C INTEGRATE SPAN LOADS TO OBTAIN LIFT-CURVE SLOPE
C AND INDUCED DRAG IN WALL EFFECT.
CALL CCTFG (ETA,CLC2,CLC2,12)
CALL CLTFG (ETA,CC2,CC2,12)
CLT=CLC2(NS+2)/CAVE
CCT=CC2(NS+2)/CAVE
IF(NOPT.EQ.1) GO TO 342
WRITE(6,2008) CLT,COT
DKT=CCT/(CLT**2)
DKFA=CDFA/(CLFA**2)
DELCD=DKFA-DKT
C CONVERT LIFT-CURVE SLOPE TO DEGREES.
CLFA=CLFA*.01745329D0
CLT=CLT*.01745329D0
DELA=(1.0D0/CLFA-1.0D0/CLT)
WRITE(6,2005) DELA,DELCD
CC TO 341
342 CONTINUE
C WRITE WING HEIGHT ABOVE THE GROUND.
WRITE(6,2012) Y(1)
WRITE(6,2011) CLT,CDT
C CALCULATE THE GROUND-EFFECT RATIO.
GER= CLT/CLFA
WRITE(6,2013) GER
341 CONTINUE
C CC TO THE END OF PROGRAM FOR TAIL-OFF CONFIGURATIONS.
IF (NTAIL.NE.1) GO TO 400
C
C READ IN X AND Z COORDINATES OF TAIL QUARTER-CORD.
READ(5,1006) XT,ZT
WRITE(6,2016) XT,ZT
C CALCULATE COWNASH PRODUCED BY THE WING ON THE TAIL.
C ESTABLISH COORD FOR TAIL POSITION.
   CCCRD(1)=XT
   CCCRC(2)=0.000
   CCCRD(3)=ZT
   CCCRC(4)=-5.000
C ESTABLISH COORD FOR LEFT WING.
   DO 350 J=1,NS
      CCCRD(5)=XF(J)
      CCCRC(6)=0.000
      CCCRD(7)=0.000
      CCCRC(8)=ZF(J)
      CCCRC(5)=ZF2(J)
C CALCULATE EFFECT FROM LEFT WING
   CALL HSHOE (COORD,WASHL)
C ESTABLISH COORD FOR RT. WING.
   CCCRC(8)=-ZH2(J)
   CCCRD(5)=-ZH1(J)
C CALCULATE EFFECT FROM RT. WING.
   CALL HSHOE (COORD,WASHR)
   WCT(J)= (WASHL+WASHR)*0.5CO
350 CONTINUE
C
C CALCULATE EFFECT OF WALL ON TAIL.
C ESTABLISH COORD FOR LEOP
C ESTABLISH COORD FOR QUADRANT 1
C 370 J=1,NM4
   WAS3=0.000
   WAS4=0.000
HL3=0.000
HL4=0.000
IF(J.GE.NNEW) GO TO 360
CCORD(5)=XL1(J)
CCORD(6)=XL3(J)
CCORD(7)=YL1(J)
CCORD(8)=YL2(J)
CCORD(9)=ZL1(J)
CCORD(10)=ZL2(J)
C CALCULATE EFFECT FROM QUADRANT 1
CALL LCOP (COORD,WASH1)
C ESTABLISH COORD FOR QUADRANT 2
CCORD(7)=YL2(J)
CCORD(8)=YL1(J)
CCORD(9)=ZL2(J)
CCORD(10)=ZL1(J)
C CALCULATE EFFECT FROM QUADRANT 2
CALL LCOP (COORD,WASH2)
C SKIP QUADRANT 3 AND 4 FOR GROUND EFFECT CALCULATIONS.
IF (NOPT.EQ.1) GO TO 359
C ESTABLISH COORD FOR QUADRANT 3
CCORD(7)=YL2(J)
CCORD(8)=YL1(J)
CCORD(9)=ZL2(J)
CCORD(10)=ZL1(J)
C CALCULATE EFFECT FROM QUADRANT 3
CALL LCOP (COORD,WASH3)
C ESTABLISH COORD FOR QUADRANT 4
CCORD(7)=YL1(J)
CCORD(8)=YL2(J)
CCORD(9)=ZL1(J)
CCORD(10)=ZL2(J)
CALL LCOP (COORD,WASH4)
355 CONTINUE
FOT(1,J)=WASH1+WASH2+WASH3+WASH4
36C CONTINUE
C CALCULATE EFFECT FROM END HORSES+CES.
C ESTABLISH COORD FOR QUADRANT 1
CCORD(5)=XL1(J)
CCORD(6)=XL3(J)
CCORD(7)=YL1(J)
CCORD(8)=YL2(J)
CCORD(9)=ZL1(J)
CCORD(10)=ZL2(J)
C CALCULATE EFFECT FROM QUADRANT 1
CALL HSHE (COORD,HL1)
C ESTABLISH COORD FOR QUADRANT 2
CCORD(6)=YL2(J)
CCORD(7)=YL1(J)
CCRC(8)=-ZL2(J)
CCRD(5)=-ZL1(J)
C CALCULATE EFFECT FROM QUADRANT 2
  CALL FSHE (COORD,HL2)
  IF (NOPT.EQ.1) GO TO 369
C ESTABLISH COORD FOR QUADRANT 3
  COORD(6)=-YL2(J)
  CCCR(7)=-YL1(J)
  CCCRC(8)=-ZL2(J)
  CCCRD(5)=-ZL1(J)
C CALCULATE EFFECT FROM QUADRANT 3
  CALL FSHE (COORD,HL3)
C ESTABLISH COORD FOR QUADRANT 4
  COORD(6)=-YL1(J)
  CCCR(7)=-YL2(J)
  CCCRC(8)=ZL1(J)
  CCCRD(5)=ZL2(J)
C CALCULATE EFFECT FROM QUADRANT 4
  CALL FSHE (COORD,HL4)
365 CONTINUE
  RCT(1,J)=HL1+HL2+HL3+HL4
370 CONTINUE
C CALCULATE TAIL DOWNWASH ANGLE IN FREE AIR.
  EFA=0.000
  CC 380 I=1:NS
  EFA=EFA+ROT(1,I)*CLCFA(I)
38C CONTINUE
C CALCULATE TAIL DOWNWASH ANGLE IN THE PRESENCE OF THE WALL.
C MULTIPLY (ROT)*(GAMAL) TO GET WALL EFFECT ON TAIL POSITION.
  CC 390 I=1:NS
  WHT=0.000
  CC 385 J=1:NM4
  WHT=WHT+ROT(1,J)*GAMAL(J,I)
39S CONTINUE
C SET (WT) EQUAL TO TOTAL DOWNWASH AT THE TAIL.
  WTI(1,I)=(ROT(1,I)-(WHT*0.500))
39C CONTINUE
C CALCULATE TAIL DOWNWASH ANGLE IN THE PRESENCE OF THE WALL.
  EWT=0.000
  CC 395 I=1:NS
  EWT=EWT+WHT(1,I)*CLCT(I)
39S CONTINUE
  CELE=EWT-EFA
  WRITE(6,2015) CELE
4CC CONTINUE
C RETURN TO MAIN BODY OF PROGRAM FOR CALCULATIONS
C WITH A NEW WING CONFIGURATION.
CC TO 165
5CC CONTINUE
END
C***********************************************************************
C GECI T... TUNNEL GEOMETRY
C SUBROUTINE GEOM ESTABLISHES VORTEX RECTANGLE AND CONTROL
C POINT COORDINATES REQUIRED TO CALCULATE LOOP DOWNWASH.
C***********************************************************************
C CELTAX=VORTEX RECTANGLE LENGTH (X-DIR)
SIDE=VORTEX RECTANGLE WIDTH
NN=NUMBER OF VORTEX RECTANGLES IN TUNNEL LENGTH
INCLUDING THE END HORSESHOE
M4=1/4 OF THE VORTEX RECTANGLES COMPRISING
WING TUNNEL CIRCUMFERENCE.
M4=1/2 OF THE VORTEX RECTANGLES COMPRISING
THE GROUND REPRESENTATION.
C***********************************************************************
C SUBROUTINE GEOM
C IMPLICIT REAL*8(A-H,O-Z)
COMMON/C1/ DELTAX, P8, P4, PI, MM, NN, M4
COMMON/C4/ XL1(50), XLL(50), YL1(50), YL2(50), ZL1(50), ZL2(50)
COMMON/C5/ YLP(50), XLP(50), ZLP(50), THETAPI(50), X(10), Y(10), Z(10)
DIMENSION SIDE(50)
XN2=DFLOAT(NN/2)
X(I)=-(XN2*DELTAX)
CC 101 =2, NN
X(I)=X(I-1)+DELTAX
CC CONTINUE
K=0
CC 30 I = 1, NN
CC 20 J=1, M4
K=K+1
XL1(K)=X(I)
XL3(K)=X11(K)+DELTAX
XLP(K)=XLP(K)+XL3(K))/2.000
YL1(K)=Y(J)
YL2(K)=Y(J+1)
ZL1(K)=Z(J)
ZL2(K)=Z(J+1)
YLP(K)=(YLP(K)+YLP(K))/2.000
ZLP(K)=(ZLP(K)+ZLP(K))/2.000
SIDE(K)=DSQRT((YL2(K)-YL1(K))**2+(ZL2(K)-ZL1(K))**2)
THETAPI(K)=DARCOS((YL2(K)-YL1(K))/SIDE(K))
CC CONTINUE
3C CONTINUE
RETURN
**COMMENT: Slab Wing Geometry**

**SUBROUTINE GECM:** Computes pertinent wing geometry which is needed to establish the aerodynamic influence coefficient matrix.

- **SPAN:** Wing span (feet)
- **AR:** Aspect ratio
- **TR:** Taper ratio
- **SC4:** Quarter chord sweep angle (deg)
- **NS:** No. of equal-width semi-span stations

**SLAB ROUTINE GECM**

```plaintext
IMPLICIT REAL*8 (A-H,O-Z)
COMMON/C1/ DELTAX,P8,P4,PI,MM,NN,M4
COMMON/C2/ SFAN,AR,TR,SC4,B2,NS
COMMON/C3/ XH(10),XWP(10),ZHI(10),ZHP(10),C(10),ETA(12)
B2=SPAN/2.0DC
AN2=DFLCAT(NS)
SWF=J17453300*SC4
TC=DIAN(SWF)
BS=B2/AN2
CR=(2.0DC*SPAN)/(1.0DC+TR)*AR

C ASSUMPTION OF WING ON TUNNEL CNTR LINE
YF1=0.0DC
YF2=0.0DC
ETA(12)=1.0DC
ETA(I1)=0.0DC
CC 10 I=1,NS
AI=DFLCAT(I)
ZWP(I)=(AI-.500)*BS
ETA(I1)=ZWP(I)/B2
ZI(I1)=AI*BS
ZI(I1)=(AI-1.000)*BS
C(I)=CR*(1.000+(TR-1.000)*ZWP(I)/B2)
X(I)=ZWP(I)+TC4
XWP(I)=XH(I)+.500*C(I)
10 CONTINUE
RETURN
END
```

**COMMENT: Slab Wash (CCORD,CCMASH)**

Calculates downwash velocity due to horseshoe vortex of unit strength.

- **CCORD:** is a 9 DIM COLUMN VECTOR

```plaintext
I=1: XP=X-CCORD OF CENTREL POINT
I=2: YP=Y-CCORD OF CENTREL POINT
I=3: ZP=Z-CCORD OF CENTREL POINT
```
I=4: TETAF=OUTER NORMAL DIRECTION AT P IN Y-Z PLANE, DEGREES
I=5: XF=X-COORD OF HORSESHOE BOUND ELEMENT
I=6: YHF=Y-COORD OF HORSESHOE CORNER (1)
I=7: YH2=Y-COORD OF HORSESHOE CORNER (2)
I=8: ZH1=Z-COORD OF HORSESHOE CORNER (1)
I=9: ZH2=Z-COORD OF HORSESHOE CORNER (2)
WASH = VELOCITY INDUCED AT (P) IN DIRECTION OF OUTER NORMAL
C (0 ALONG +Z, 90 ALONG +Y)
C******************************************************************************
IMPLICIT REAL*8 (A-H,O-Z)
COMMON/C1/ DELTAX,PA,P4,PI,MM,NN,M4
DIMENSION COORD(2C)
XF=COORD(1)
YF=COORD(2)
ZF=COORD(3)
THETA=COORD(4)*.01745329D0
XT=COORD(5)
YT=COORD(6)
ZT=COORD(7)
TP=COORD(8)
TPZ=COORD(9)
C CALCULATE GEOM TERMS
STF=DSIN(THETA)
CTP=DCOS(THETA)
R1=DSQRT((XF-XP)**2+(YH1-YP)**2+(ZH1-ZP)**2)
CS1=(XP-XH1)/R1
FT1=DSQRT((YH1-YP)**2+(ZH1-ZP)**2)
IF (HT1.LE.0.0001D0) GO TO 15
C CALCULATE DELTA-VEL DUE TO LEFT ELEMENT
DTH1=(1.000D0+CS1)/HT1
C RESOLVE DELTA-VEL COMPONENTS IN THE TAP DIRECTION
WNT1=DTH1*((ZP-ZH1)*(STF)+(YH1-YP)*(CTP))/HT1
GE TO 18
15 CONTINUE
WNT1=0.000
18 CONTINUE
R2=DSQRT((XF-XP)**2+(YH2-YP)**2+(ZH2-ZP)**2)
CS2=(XP-XH2)/R2
FT2=DSQRT((YH2-YP)**2+(ZH2-ZP)**2)
IF (HT2.LE.0.0001D0) GO TO 16
C CALCULATE DELTA-VEL DUE TO RIGHT ELEMENT
DTH2=(1.000D0+CS2)/HT2
C RESOLVE DELTA-VEL COMPONENTS IN THE TAP DIRECTION
WNT2 = DTH2*{(ZH2-ZP)*(STF)+(YP-YP2)*(CTP)}/HT2
GE TO 15
16 CONTINUE
WNT2=C.000
19 CONTINUE
RXS2=(X*XP)**(YH2-YH1)
RXS2=(X*XP)**(YH2-YH1)
RYSZ=(Y1-YP)**(ZH2-ZH1)
RZSY=(Z1-ZE)**(YH2-YH1)
S=DOT{((YH2-YH1)**2+(ZH2-ZH1)**2)
S=DOT{(RYS2-RZSY)**2+(RZS2)**2}
IF (HS.EQ.0.00001) GO TO 30
C FINE DELTA-VEL DUE TO BCUND ELEMENT
CBW=((R1+R2)**2-(R1-R2)**2)/(PE*HS**2*R1**2)
C RESOLVE DELTA-VEL COMPONENTS IN THETAP DIRECTION
WAB = CBW*((-STP*RXS2)+(CSP*RXY2))
GO TO 40
30 CONTINUE
WAB=0.000
40 CONTINUE
WAB=WN+WNT1+WNT2
RETURN
END
C***************************************************************
C SUBROUTINE INVR(A, N, ISIZE)
C SUBROUTINE INVR COMPUTES THE INVERSE OF THE INPUT MATRIX.
C (A) IS DIMENSIONED ISIZE BY ISIZE AND CONTAINS N ROWS
C AND A COLUMNS OF DATA.
C SUBROUTINE INVR WAS OBTAINED FROM THE FOLLOWING REFERENCE SOURCE:
C JCPFA R.C. "WIND TUNNEL INTERFERENCE FACTORS FOR HIGH- 
C***************************************************************
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION A(50,50), PIVC(100)
DIMENSION IPIVC(100), INDEX(100,2)
EQUIVALENCE (AMAX,T,SWAP)
EQUIVALENCE (IREM,JREM),(ICOLUMN,JCOLUMN)
100 CC 101 J=1,N
101 IFIVC(J)=0
102 CC 5501=1A
AMAX=0.000
104 CC 105 J=1,N
106 IF (IPIVC(J)-1) 107,107
107 CC 120 K=1,N
108 IF (IPIVC(K)-1) 109,120,740
109 IF (CABS(AMAX)-DABS(A(J,K))) 111,120,120
111 IREM=J
112 ICOLUMN=K
113 AMAX=A(J,K)
120 CONTINUE
105 CONTINUE
110  IPIVOT(I.COLUMN)=IPIVOT(I.COLUMN)+1
130  IF (IROW-I.COLUMN) > 140, 260, 140
140  CONTINUE
150  CC 200 L=1
160  SWAP=A(IROW,L)
170  A(IROW,L)=A(I.COLUMN,L)
180  A(I.COLUMN,L)=SWAP
190  INDEX(I,1)=IROW
200  INDEX(I,2)=ICOLUMN
210  IPIVOT(I)=A(I.COLUMN,ICOLUMN)
220  A(I.COLUMN,ICOLUMN)=1.0
230  CC 350 L=1
240  A(I.COLUMN,L)=A(I.COLUMN,L)/PIVOT(I)
250  CC 550 L=1
260  IF (LI-I.COLUMN) > 400, 550, 400
270  T=A(LI,ICOLUMN)
280  A(LI,ICOLUMN)=0.0
290  CC 450 L=1
300  A(LI,L)=A(LI,L)-A(ICOLUMN,L)*T
310  CC 710 L=1
320  IF (INDEX(L,1)-INDEX(L,2)) > 630, 710, 630
330  JCOLUMN=INDEX(L,2)
340  CC 705 K=L
350  SWAP=A(K,J.COLUMN)
360  A(K,J.COLUMN)=SWAP
370  CONTINUE
380  CC 710 CONTINUE
390  RETURN

C*******************************************************************************
C SUBROUTINE ELU TRI-DIAGONALIZES THE INPUT MATRIX (A).
C*******************************************************************************
SUBROUTINE ELU(A,N,ND)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(ND,ND)
N=1-N-1
CC 100 K=1,AM1
KC1=K+1
CC 100 I=KC1,N
G=A(I,K)/A(K,K)
A(I,K)=G
CC 100 J=KC1,N
A(I,J)=A(I,J)+G*A(K,J)
RETURN
C******************************************************************************
C SLERCLTINE SLVB SOLVES THE TRI-DIAGONALIZED MATRIX (A) OBTAINED
C FROM EQU BY BACK SUBSTITUTION.
C******************************************************************************
SLEROUTINE SLVB(A,N,ND,B)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(ND,ND),B(ND)
N+1=N-1
NP1=N+1
CC 100 K=1,N
K=KP1
DC 100 I=KP1,N
100 B(I)=B(I)+A(I,K)*B(K)
E(K)=B(N)/A(N,N)
CC 300 K=2,N
I=NP1-K
J=I+1
CC 200 J=J1,N
200 B(I)=B(I)-A(I,J)*B(J)
300 B(I)=B(I)/A(I,I)
RETURN
END
C******************************************************************************
C SLERCLTINE LLCP (COORD, WASH)
C
C SLERLTINE LLCP (COORD, WASH)
C
C******************************************************************************
C SLERCLTINE LLCP (COORD, WASH)
C******************************************************************************
IMPLICIT REAL*8 (A-H,O-Z)
COMMON/G1/ DELTAX,P0,P4,P1,MM,NN,N4
DIMENSION CCORD(20)
1 XP=CCORD(1)
YP=COORC(2)
ZP=COORC(3)
THETAP=COORC(4)*0.01745329D0
XH1=COORC(5)
XH3=COORC(6)
YH1=COORC(7)
YH2=COORC(8)
ZH1=COORC(9)
ZH2=COORC(10)

C FIND VALUES USED BY ALL FCUR PARTS OF VORTEX LOC P CALCS.
1C
STP=DSIN(THETAP)

CTP=CCOS(THETAP)
R1=DSQRT((XH1-XP)**2+(YH1-YP)**2+(ZH1-ZP)**2)
R2=DSQRT((XH1-XP)**2+(YH2-YP)**2+(ZH2-ZP)**2)
R3=DSQRT((XH3-XP)**2+(YH1-YP)**2+(ZH2-ZP)**2)
R4=DSQRT((XH3-XP)**2+(YH2-YP)**2+(ZH1-ZP)**2)

C FIND INDUCED VELOCITY DUE TO ELEMENT (1-2)
2C
RXSZ=(XH1-XP)*(ZH2-ZH1)
RXSY=(YH1-YP)*(YH2-YH1)
RYSZ=(YH1-YP)*(ZH2-ZH1)
RZSY=(ZH1-ZP)*(YH2-YH1)
S=DSQRT((YH2-YP)**2+(ZH2-ZP)**2)
HS=DSQRT((RYSZ-RZSY)**2+(RXSZ)**2+(RXSY)**2)

IF(HS.LE.0.00001C0) GO TO 25
LBW1=(R1+R2)*(S**2-(R1-R2)**2)/(P8*HS**2*R1*R2)

KBW1=CBW1*(-STP*RXSZ)+(CTP*RXSY)
CC TO 30

C 25
WH1=0.00D0

C FIND INDUCED VELOCITY DUE TO ELEMENT (2-3)
3C
HT2=DSQRT((YH2-YP)**2+(ZH2-ZP)**2)
IF(HT2.LE.0.00001C0) GO TO 35

CSB2=(XP-XF1)/R2
CSB3=(XH3-XP)/R3
CTH2=(CSB2+CSB3)/(P4*HT2)

KBW2=CTH2*([ZH2-ZP]*STP+(YP-YH2)*CTP)/HT2
CC TO 40

C 35
WH2=0.00D0

C FIND INDUCED VELOCITY DUE TO ELEMENT (3-4)
4C
RXSZ=(XH3-XP)*(ZH1-ZH2)
RXSY=(XH3-XP)*(YH1-YH2)
RYSZ=(YH2-YP)*(ZH1-ZH2)
RZSY=(ZH2-ZP)*(YH1-YH2)
S=DSQRT((YH2-YP)**2+(ZH2-ZP)**2)

HS=DSQRT((RYSZ-RZSY)**2+(RXSZ)**2+(RXSY)**2)

IF(HS.LE.0.00001C0) GO TO 45

KBW3=CBW3*(-STP*RXSZ)+(CTP*RXSY)
CC TO 50
$V_{ind}$ = 0.000

**VARIABLES:**
- $V_{ind}$, induced velocity
- $H_4$, height of element
- $R_4$, radius of element
- $ZP$, zeta parameter
- $Z1$, zeta parameter
- $Cp$, cross-sectional area
- $Cp$, cross-sectional area
- $Cp$, cross-sectional area
- $Cp$, cross-sectional area

**FORTRAN Statements:**

```fortran
45 $V_{ind}$ = 0.000

C FINE INDUCED VELOCITY DUE TO ELEMENT (4-1)
5C $H_4 = \sqrt{(Y1-YP)^2 + (Z1-ZP)^2}$

IF($H_4$.LE.C0000100) GO TO 55

CSB1 = (XP-ZP)/R1
CSB4 = (XH3-XP)/R4
CSB4 = (CSB1+CSB4)/(P4*$H_4$)

WNT4 = CTW4((ZP-Z1)*STP + (Y1-YP)*Cp)/$H_4$

GO TO 60

55 WNT4 = 0.000

C SUM UP INDUCED VELOCITY INCREMENTS.
6C WASH = WNB1 + WNT2 + WNB3 + WNT4
RETURN
END

C********************************************************************
C SLERCUTINE DCTFG
C************************************************************************

C COMPUTES THE VECTOR OF INTEGRAL VALUES FOR A GIVEN GENERAL TABLE OF ARGUMENT AND FUNCTION VALUES.

Y: INPUT VECTOR OF FUNCTION VALUES.
X: INPUT VECTOR OF ARGUMENT VALUES.
Z: RESULTING VECTOR OF INTEGRAL VALUES.

BEGINNING WITH $Z(1)=0$, EVALUATION OF VECTOR Z IS DONE BY MEANS OF TRAPEZOIDAL RULE (SECOND ORDER FORMULA).

SUBCUTINE DCTFG WAS OBTAINED FROM THE NPS SOLVE.

LIBRARY.
C********************************************************************

SLERCUTINE CNTFG(X,Y,Z,NDIM)

DIMENSION X(NDIM),Y(NDIM),Z(NDIM)

CCABLE PRECISION X,Y,Z,SUM1,SUM2

SLM2 = 0.000

IF(NDIM-1)4,3,1

10 DC 2 I=2,NDIM

SLM1 = SUM2

SLM2 = SUM2 + .500*(X(1)-X(I-1))*(Y(I)+Y(I-1))

20 Z(I-1) = SUM1

30 Z(NDIM) = SUM2

40 RETURN
END
```

**Note:** The code snippet provided is a FORTRAN program that calculates induced velocities due to an element. The program includes functions for computing integral values and handling certain mathematical operations. The comments and variable names are indicative of a vector-based calculation approach, likely used in a fluid dynamics or structural engineering context. The specific details and context of the application are not provided in the snippet.
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