AN INVENTORY PLANNING MODEL FOR NAVY ENLISTED PERSONNEL.

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CNA - Professional Paper 181
May 1977

The ideas expressed in this paper are those of the author. The paper does not necessarily represent the views of either the Center for Naval Analyses or the Department of Defense.

CENTER FOR NAVAL ANALYSES
1401 Wilson Boulevard
Arlington, Virginia
Prepared for presentation at the Joint National Meeting of the Operations Research Society of America and The Institute for Management Science. 9 May 1977, San Francisco, California
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AN INVENTORY PLANNING MODEL
FOR NAVY ENLISTED PERSONNEL

ABSTRACT
The model specifies annual accessions plus minimum allocations to formal and on-the-job training needed to maintain future inventories within specified limits of manpower requirements. Plans are derived simultaneously for many skill categories over several years. Restrictions are imposed on the size of annual inventories, flows between skill categories and smoothness of flows into formal training. Experience levels within skill category are explicitly accounted for by allowing specification of up to 3 length-of-service groups. The methodology is linear programming, which can be extended to stochastic programming to account for uncertainty in projections of future requirements. Plans derived using actual Navy data are presented.

INTRODUCTION
This paper presents an application of linear programming to a manpower and personnel planning problem. The work is being conducted as part of an ongoing study at the Center for Naval Analyses, called the Enlisted Manpower and Personnel Planning System (EMPPS) Study.1 The problem is to determine

1See reference 1.
an efficient allocation of scarce resources (annual Navy accessions) to Navy ratings (skill groups) which meets the demand (annual requirements) for each skill in each of the next seven years. An efficient allocation is one that meets requirements at minimum cost.

The manpower-personnel system is not assumed to be in steady state. Annual requirements fluctuate, not only as to the total number required within each rating, but also with respect to the experience mix within the totals. For each rating, the Navy specifies annual requirements for nine separate experience groups, called paygrades.

Total accessions and authorized endstrength (inventory size at the end of each planning year) are constrained by law to be less than specified levels. In recent years, authorized endstrength levels have been less than total requirements. This situation is expected to persist. Because of this, requirements cannot be fully met. This gives rise to an alternate interpretation of "meeting" requirements in which a manager specifies how close he wants projected inventories to be to projected requirements, for each rating (and even for subsets of experience groups within ratings). This option allows full manning in high priority ratings, and programmed shortages in others, and reveals the impact of programmed undermanning.
The problem is further complicated in that, for the most part, accessions into the Navy do not flow directly into ratings; rather, they enter one of two training pipelines, either formal or on-the-job training. The durations of these pipelines vary by type and rating. Generally, formal training (rating "A" schools) lasts less than one year, sometimes only a few weeks, while on-the-job training usually takes in excess of 2 years.

In our model, inventory flows are modeled after those observed in the recent past. However, the model allows the user to easily modify those flows to reflect planned or proposed shifts in continuation management policies during the planning horizon.

FORMULATION

The mathematical formulation of the linear program is presented in appendix A. This section presents a non-technical description of the formulation.

The solution to the linear program represents a proposed management plan for all ratings over the next seven years. For each rating and each year, the following quantities are specified:

1. the size of allocations of accessions to formal training
2. the size of allocations of accessions to on-the-job training
3. the number of lateral transfers into the rating
4. the number of lateral transfers out of the rating
5. the number of direct inputs to the rating (from sources other than training)

The quantities, along with the starting inventory and projected continuation rates, are then used to derive the inventory expected to result from following the proposed plan. The allocation schedule and resulting inventories are then summarized for management review. This will be illustrated later with an example.

Each type of constraint will now be discussed separately, followed by a discussion of the objective.

Constraints

Constraints fall into one of two categories: those which reflect "real world" relationships beyond the control of the manager (user), and those which reflect policies selected by the manager (user) and can be freely changed as desired.

The real world constraints are:
1. The total number of personnel in the Navy is bounded above in each year. (These are called "endstrength" constraints.)
2. The total allocated to training, across all skills, cannot exceed the maximum flow out of recruit training in each year. (These are referred to as "recruit input" constraints.)
3. The sum of lateral transfers within specified subsets of skills must equal zero. This means that the sum of lateral
ins must equal the sum of lateral outs within each group. Groupings will correspond to Navy practice with regard to control of lateral flows. (These are called "lateral flow" constraints.)

4. Flows into skills via direct inputs and lateral transfers are bounded above in accordance with empirically derived maximums. (These constraints are simple upper bounds.)

5. The allocation to formal training in the first year is predetermined and not subject to change. This is because the long lead time required to reprogram resources in formal training precludes changing next year's allocation.

Policy related constraints include:

1. "Inventory Flow" constraints are generated which force projected rating inventories to be within a specified percentage of requirements in each year. The procedure used to construct these constraints will be illustrated in the next section. These constraints are applied not only to the total in each rating, but to several experience groups within each rating simultaneously, as long as the groups are describable in terms of intervals of LOS cells. (This will be illustrated later in an example where plans are derived which meet requirements for the total rating, that is for LOS cells 1 through 31, and
also for careerists within the rating, which are defined to be those in LOS cells 5 through 31.)

2. "School smoothing" constraints require that allocations to formal training be smooth. That is, the input in a year cannot differ from the input in the preceding year by more than an amount specified by the user.

3. The flow of lateral transfers can be controlled by the user. This means, for example, that no lateral transfers will be allowed into a rating which is grossly in excess, and no lateral transfers will be allowed out of a rating which has a significant shortage. This control is affected by appropriately setting simple upper bounds on lateral transfers.

4. Limits on the flows through OJT are controlled by the user. That is, for each skill, an upper bound is specified beyond which no further flow is either desired or can be expected.

The solution space described by a given set of constraints may be empty. In this case, the user can selectively relax constraints until feasibility is achieved.

Objective Function

The objective function in the linear program represents the cost of meeting requirements as decided upon by the user.
The solution to the linear program is the minimum cost allocation of new inputs which meets requirements as desired. At the current stage of development, meaningful cost coefficients have not yet been derived for the objective function. Instead, a substitute objective function is being used which represents the total allocation to training over the planning years. A solution to this problem gives that allocation which puts the least number of people into training, while meeting requirements as desired. In this version, broken service reenlistments and lateral transfers are free goods, and are constrained to reasonably attainable limits.

INVENTORY PLANNING MODEL

The linear programming routine, along with the routines which preprocess and postprocess data, together comprise what will be called the Inventory Planner. This model will be explained by describing the sequence of steps involved in its use. This will serve to illustrate the interaction between the model and manager (user), and also how the model can be used to investigate and contrast the impact of proposed policy shifts.

Step 1: The user specifies deviations from current continuation management policy he wishes to investigate (such as changes in bonus levels for some ratings, or a change in
enlistment contract length, etc.). If no changes are desired, the current policy set will automatically be simulated.¹

Step 2: For each rating, the manager identifies up to three LOS groups he is concerned about. LOS 1 through 31 includes the total rating, and is generally of interest. LOS 5 through 31 includes those usually referred to as "careerists." (These two LOS groups will be treated in the example to follow.) Other LOS groups which may be of interest include LOS 3 through 7 and LOS 11 through 15, which comprise operators and supervisors in nuclear propulsion ratings.

Step 3: The user specifies how close he wants projected inventories to be to projected requirements for each LOS group within each rating. This is done by specifying an interval (or band, or funnel), about annual requirements totals within which projected inventories must remain. This is illustrated in figure 1. The solid line represents total requirements (LOS 1-31) for one skill over the next seven years. The circle represents the total inventory as of the end of the last fiscal year. The dotted lines represent a possible funnel about the requirements line described by the following two 7-element vectors.

¹Imbedding the effect of policy changes in projected continuation rates is performed by the VACATE model (see reference 2), which is a separate model which works in concert with the linear programming model. After VACATE derives projected continuation rates, the rates are used by the linear programming model to derive the management plan.
Figure 1 Funnel About Requirements Line
Percentages of requirements for:

<table>
<thead>
<tr>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
<th>Year 6</th>
<th>Year 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper bound</td>
<td>1.05</td>
<td>1.05</td>
<td>1.05</td>
<td>1.05</td>
<td>1.05</td>
<td>1.05</td>
</tr>
<tr>
<td>Lower bound</td>
<td>.85</td>
<td>.87</td>
<td>.89</td>
<td>.91</td>
<td>.93</td>
<td>.95</td>
</tr>
</tbody>
</table>

The lower boundary of the funnel is chosen to force projected inventories to more closely align with requirements. In this example, it might appear that the upper boundary of the funnel is superfluous when using an objective of minimizing the cost of staying within the funnel; however, other constraints, such as those on the career part of the rating, might force the total in the rating well above the lower boundary shown in some years, and even significantly above the requirements line. For ratings that are overmanned, it is the upper boundary of the funnel which will tend to drive projected inventories towards projected requirements. When describing a funnel for an LOS group within some rating, the manager is describing his indifference region for that group. In effect, he is saying that he is indifferent as to the precise number projected to be in that group, as long as it is within the limits described by the funnel.

**Step 4:** The user has the option of smoothing school inputs either by percentages or by fixed amounts. If the percentage method is selected, the user provides, for each year, a percentage interval, such as 90 percent, 105 percent, and constraints are generated which require inputs in a year to be greater than 90
percent and less than 105 percent of the preceding years' input. If the "fixed amount" method is selected, 2 numbers must be provided, for each year, which specify annual shrinkage and expansion possible within the year.

**Step 5:** The user may override automatically imposed simple upper bounds or controls on lateral transfers. The model will automatically prevent lateral transfers into ratings which are not undermanned, and also prevent lateral transfers out of ratings which are not overmanned. Simple upper bounds on such things as annual direct inputs, lateral flows, or inputs to on-the-job training are preset to reasonably attainable limits, but can be altered by the user as desired.

**Step 6:** The linear program then derives the minimum cost schedule of allocations to each skill which results in the projected inventories for each LOS group and each rating remaining within the prescribed funnels, if this is possible. If it is not, the program indicates that the manager's desires cannot be accommodated without either

- Changing some continuation management policy, or
- Relaxing the boundaries on one or more of the funnels.

If a feasible (optimal) plan is derived, it is printed out for the manager's review. The plan includes the schedule of allocations to each rating, by type, and the inventories.
of each rating which are projected to result. If no feasible plan exists, the process reverts to step 1, where the manager is required to interact again.

EXAMPLE

This section presents an example which illustrates, in part, the operation of the Inventory Planning Model. Since the focus of this paper is on the linear program, continuation management policy is assumed to remain unchanged over the planning horizon. (Reflecting changes in such policies as changes in continuation rates is achieved by the VACATE part of the Inventory Planning Model, which is discussed in reference 2.)

For the sake of simplicity, only 3 of the Navy's 82 ratings are the number of ratings treated in the example. The strating inventory data for these 3 ratings is actual Navy data. Continuation flow parameters model flows observed during fiscal year 1976. Actual requirements for these ratings have been adjusted here to derive a scenario of interest. The example will demonstrate how both excesses and shortages can be reduced while maintaining smooth flows through training.

For each rating, two LOS groups (experience groups) are planned: LOS 1-31, representing the total rating; and LOS 5-31, the "career" part of the rating.
One of the 3 ratings is designated as having "priority manning status," which means that this rating is to be manned at least at 100 percent of requirements in all LOS groups (where manning is defined as the ratio of inventory to requirements).

Manning in the ratings, as of the end of the last fiscal year, is as shown in table 1. Rating 2 is priority manned and is overmanned in both total (110 percent) and careerists (110 percent). Rating 1 is undermanned in total (90 percent) and careerists (90 percent). Rating 3 is neither overmanned or undermanned. Aggregate manning across all ratings is 96 percent in both total and careerists.

Annual requirements for the example ratings are shown in table 2. The magnitudes of these values have been adjusted.

### TABLE 1

**MANNING IN THE STARTING INVENTORY**

<table>
<thead>
<tr>
<th></th>
<th>Rating 1</th>
<th>Rating 2</th>
<th>Rating 3</th>
<th>Aggregate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total in rating</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Required(^a)</td>
<td>25,920</td>
<td>11,065</td>
<td>5,430</td>
<td>42,405</td>
</tr>
<tr>
<td>Inventory</td>
<td>23,342</td>
<td>12,146</td>
<td>5,431</td>
<td>40,919</td>
</tr>
<tr>
<td>Manning</td>
<td>90%</td>
<td>110%</td>
<td>100%</td>
<td>96%</td>
</tr>
<tr>
<td>Career part</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Required(^a)</td>
<td>12,330</td>
<td>5,410</td>
<td>2,631</td>
<td>20,371</td>
</tr>
<tr>
<td>Inventory</td>
<td>11,090</td>
<td>5,939</td>
<td>2,616</td>
<td>19,645</td>
</tr>
<tr>
<td>Manning</td>
<td>90%</td>
<td>110%</td>
<td>100%</td>
<td>96%</td>
</tr>
</tbody>
</table>

\(^a\)Requirements adjusted to produce manning levels for illustration.
### TABLE 2

**ANNUAL REQUIREMENTS**

<table>
<thead>
<tr>
<th>Planning year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Career requirements</strong>&lt;sup&gt;a&lt;/sup&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rating 1</td>
<td>12,322</td>
<td>12,362</td>
<td>12,357</td>
<td>12,415</td>
<td>12,415</td>
<td>12,415</td>
<td>12,415</td>
</tr>
<tr>
<td>Rating 2</td>
<td>5,407</td>
<td>5,449</td>
<td>5,448</td>
<td>5,467</td>
<td>5,467</td>
<td>5,467</td>
<td>5,467</td>
</tr>
<tr>
<td>Rating 3</td>
<td>2,631</td>
<td>2,646</td>
<td>2,645</td>
<td>2,652</td>
<td>2,652</td>
<td>2,652</td>
<td>2,652</td>
</tr>
<tr>
<td><strong>Total rating requirements</strong>&lt;sup&gt;a&lt;/sup&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rating 2</td>
<td>11,060</td>
<td>11,116</td>
<td>11,093</td>
<td>11,115</td>
<td>11,115</td>
<td>11,115</td>
<td>11,115</td>
</tr>
<tr>
<td>Rating 3</td>
<td>5,391</td>
<td>5,432</td>
<td>5,419</td>
<td>5,437</td>
<td>5,437</td>
<td>5,437</td>
<td>5,437</td>
</tr>
<tr>
<td><strong>Aggregate requirements</strong></td>
<td>42,366</td>
<td>42,497</td>
<td>42,403</td>
<td>42,546</td>
<td>42,546</td>
<td>42,546</td>
<td>42,546</td>
</tr>
</tbody>
</table>

<sup>a</sup>Adjusted as in table 1.

<sup>b</sup>Sum of total rating requirements.
slightly as mentioned earlier, however, the annual fluctuations reflect those of actual Navy projections for these ratings. The year-to-year change is slight for all ratings, and the last 3 years are projected here to be the same as the fourth year. The bottom row displays aggregate annual requirements across all 3 ratings.

To illustrate how plans are developed when authorized endstrengths are less than aggregate requirements, authorized endstrength levels were arbitrarily selected to be 42,000 in each year. This will force the Inventory Planner to derive a plan in which priority-manned rating 2 is manned at least at 100 percent in all years, and non-priority ratings 1 and 3 have to absorb all ensuing shortages.

Total Navy accessions for these 3 ratings were limited to 10,650 in the first 2 planning years, 10,750 for the next 2 years, and 10,850 for the last 3 years. Attrition out of the Navy before completion of recruit training was arbitrarily chosen to be 14 percent. This means that 86 percent of annual accessions will be available for assignment to training in one of the 3 ratings.

The Inventory Planner uses manning levels in the starting inventory to establish lateral transfer flow constraints for each rating. Due to manning as shown in table 1, these flows were constrained as follows:

- no lateral transfers are to be planned into overmanned rating 2;
o no lateral transfers are to be planned out of under-
manned rating 1;
o no lateral transfers are to be planned into or out of
properly manned rating 3.

The user can override or modify these constraints, as
desired. Because of the simplified nature of the example, the
only lateral flow allowed is from rating 2 to rating 1.

In the example, flows into formal training in all ratings
are constrained to fluctuate no more than 10 percent from year
to year, and inputs in the first planning year are fixed and
not subject to change by the manager or the planning model.

A good plan is one which efficiently reduces both over-
manning in rating 1 and undermanning in rating 2, while main-
taining rating 3 at its proper manning level, and does all this
within endstrength and recruit input limitations.

Before the Inventory Planner is run, the manager must
specify how he wants requirements "met" for each rating. This
is illustrated for rating 1 in figure 2. The solid lines graph
the annual requirements from table 2. The starting inventories
for each LOS group are shown on the left-hand axis with darkened
circles. The dotted lines show the boundaries of the funnel
chosen for each LOS group. Since rating 1 is undermanned at
the start, the lower boundaries are chosen to force projected
inventories in both LOS groups nearer to requirements. The
Figure 2  Graph of Funnel Boundaries for Rating 1
left-hand edge of the lower boundary is selected to be below the starting inventory level to allow derivation of a feasible plan. The right-hand portion of this lower boundary indicates the minimum manning level the manager will accept in this rating. The rate at which the boundary approaches the requirements line, starting at the left and moving to the right, indicates the manager's desired rate of improvement.

These boundaries for rating 1 result from selection of the vectors displayed in table 3. The decimal fractions shown specify how close the inventory projections must be to requirements, given that the derived input plan is followed. Fractions are shown here for all ratings, and are those used in the example. Note that the lower boundary for rating 2 is selected to be the same as requirements because of priority manning status.

The rating plans derived by the Inventory Planning Model are presented in figure 3 through 5, and a 3-rating summary is presented in figure 6.

Figure 3 contains the input plan and projections for rating 1. Planned inputs, by type, are listed at the bottom of the figure below the year for which they are planned. Projections for both total and careerists approach the requirements line as desired. This improvement, particularly for the career LOS group, is enhanced by the indicated lateral flows. This reduction is accommodated without large fluctuations in the school input plan.
### TABLE 3

**FUNNEL BOUNDARY SPECIFICATION VECTORS**

<table>
<thead>
<tr>
<th>Planning years</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rating 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Upper limit</td>
<td>1.05</td>
<td>1.05</td>
<td>1.05</td>
<td>1.05</td>
<td>1.05</td>
<td>1.05</td>
<td>1.05</td>
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<tr>
<td>Lower limit</td>
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<td>.90</td>
<td>.91</td>
<td>.92</td>
<td>.93</td>
<td>.94</td>
<td>.95</td>
</tr>
<tr>
<td>Career</td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>Upper limit</td>
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<td>1.05</td>
<td>1.05</td>
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<tr>
<td>Lower limit</td>
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<td>.91</td>
<td>.92</td>
<td>.93</td>
<td>.94</td>
<td>.95</td>
<td>.95</td>
</tr>
<tr>
<td><strong>Rating 2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
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<td>1.08</td>
<td>1.07</td>
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<tr>
<td>Career</td>
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<tr>
<td>Upper limit</td>
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<tr>
<td>Lower limit</td>
<td>1.0</td>
<td>1.0</td>
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<tr>
<td><strong>Rating 3</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
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<td>Total</td>
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<tr>
<td>Career</td>
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<tr>
<td>Upper limit</td>
<td>1.05</td>
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<td>1.05</td>
<td>1.05</td>
<td>1.05</td>
<td>1.05</td>
</tr>
<tr>
<td>Lower limit</td>
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<td>.95</td>
<td>.95</td>
<td>.95</td>
<td>.95</td>
<td>.95</td>
<td>.95</td>
</tr>
</tbody>
</table>
FIG. 3: PLAN AND PROJECTION FOR RATING 1
No on-the-job training is planned, which is not surprising, because in general, those provided with formal training become more productive, faster, and stay in the Navy longer. These characteristics are captured by the model and, under the objective used, result in a clear preference for formal training.

Figure 4 shows the plan and projection for rating 2. Manning was forced downward more towards requirements, as desired. The lateral flows out of this rating were helpful in this regard. Note, however, that maintaining career manning above 100 percent in later years forced increased allocations to formal training and also some allocations to on-the-job training in years 1 and 2.

Figure 5 shows the plan and projection for rating 3. No lateral flows were planned. Inputs to formal training are very smooth, except for year 2 when the large recruit inputs needed in rating 2 and the upper limit on total recruit inputs force the maximum allowed reduction in school input in rating 3, 10 percent, between year 1 and year 2.

Figure 6 shows how the projected endstrengths (total inventories) compare to arbitrarily selected endstrength authorization levels, and also how planned recruit accession totals compare to arbitrarily selected limits on accessions. Projected endstrengths quickly approach the authorized level. In each year except the last, the maximum number of accessions
FIG. 4: PLAN AND PROJECTION FOR RATING 2
FIG. 5: PLAN AND PROJECTION FOR RATING 3
Figure 6  Annual Accessions and Projected Endstrength
are used. In the last year, a lesser amount is needed because endstrength is not allowed to sharply increase, as was the case in preceding years.

It is reemphasized that the usefulness of this planning model does not end with the development of one plan. The marginal cost of a proposed shift in continuation management policy, such as bonus level, or non-continuation, can be derived through comparison of plans developed with and without the proposed shift. It is in this fashion that the manager can compare such things as reductions in accession levels possible through application of a reenlistment bonus, or perhaps the increase in accession levels implied by reduction in direct procurement programs (e.g., inputs from sources other than training).

EXTENSION TO ACCOUNT FOR UNCERTAINTY IN REQUIREMENTS

The problem structure allows direct extension from linear to probabilistic programming. The most worrisome uncertainty lies in projections of future requirements. Probability distributions based on duration of projection can be estimated using available historical data. In the linear program, these projections appear on the right-hand side of the inventory flow constraints. Through application of chance constrained programming, the manager's aversion to the risk of not meeting future requirements can be explicitly entered into the formulation. This is a topic for future work.
REFERENCES


APPENDIX A

LINEAR PROGRAMMING FORMULATION OF THE INVENTORY PLANNING PROBLEM

Let

\[ x_{i,j}^k = \text{allocation of type } k, \text{ in year } i, \text{ to rating } j \]

where

\[ i = 1, \ldots, 7 \text{ (years in the planning horizon)} \]
\[ j = 1, \ldots, n \text{ (ratings)} \]

and

\[ k = 1 \text{ if "A" school} \]
\[ 2 \text{ if OJT} \]
\[ 3 \text{ if lateral transfer into rating } j \]
\[ 4 \text{ if lateral transfer out of rating } j \]
\[ 5 \text{ if direct input, such as broken service reenlistee} \]

Then \( x_{ij} \) is a vector of allocations in year \( i \) to rating \( j \), and

\[ \bar{x}_j = (\bar{x}_{1j}, \bar{x}_{2j}, \ldots, \bar{x}_{7j}) \]

is a vector of allocations to rating \( j \) over all years in the planning horizon.

After inserting slack and surplus variables, the constraints are of the form
\[
\begin{bmatrix}
\bar{A}_1 & \bar{A}_2 & \ldots & \bar{A}_r \\
\bar{B}_1 & & & \\
& \bar{B}_2 & & \\
& & \ldots & \\
& & & \bar{B}_r
\end{bmatrix}
\begin{bmatrix}
\bar{x}_1 \\
\bar{x}_2 \\
\bar{x}_3 \\
\vdots \\
\bar{x}_r
\end{bmatrix} =
\begin{bmatrix}
\bar{b}_0 \\
\bar{b}_1 \\
\bar{b}_2 \\
\vdots \\
\bar{b}_r
\end{bmatrix}
\]

with dimensions: \( (m_j \text{ and } n_j \text{ are rows and columns in the } j^{th} \text{ matrix.}) \)

\[
\bar{A}_j \quad 23 \times n_j, \quad 21 \leq n_j \leq 35
\]

\[
\bar{B}_j \quad m_j \times n_j, \quad 14 \leq m_j \leq 42
\]

\[
\bar{x}_j \quad 1 \times n_j, \quad 21 \leq n_j \leq 35
\]

\[
\bar{b}_0 \quad 23 \times 1
\]

\[
\bar{B}_j \quad m_j \times 1, \quad 14 \leq m_j \leq 42
\]

Constraints

\[
(\bar{A}_1 \quad \bar{A}_2 \ldots \quad \bar{A}_r) \begin{bmatrix}
\bar{x}_1 \\
\bar{x}_2 \\
\vdots \\
\bar{x}_r
\end{bmatrix} = \bar{b}_0
\]

are overall coordinating constraints, including

- Endstrength constraints
- Recruit input constraints
- Lateral flow constraints
are rating specific constraints, including

- One set of inventory flow constraints for each LOS interval specified
- School smoothing constraints.

Vector $\bar{L}$ contains lower bounds which are preset to zero, and vector $\bar{U}$ contains upper bounds which are preset to reasonably attainable values under current policy. (For those variables which are not to be bounded above, sufficiently large values are inserted.)

This system of equations has the "staircase" structure which lends itself to accelerated solution by decomposition methods (see reference 3).

The development of constraints is as follows.

Inventory Flow Constraints

Let

$$a^k_{i,j,l,m} = \text{fraction of allocation of type } k, \text{ in year } l, \text{ to rating } j \text{ that is or becomes designated in rating } j \text{ and survives till the end of year } l \text{ and is in LOS interval } m \text{ at that time}$$

where

- $i$, $j$, and $k$ are as above for $x^k_{i,j}$, and
- $l = 1, \ldots, 7$
- $m = 1, 2, 3$
Let

\[ b_{j,\ell,m} = \text{requirements in year } \ell \text{ for LOS interval } m \text{ in rating } j \]

\[ (\ell j, \ell, m, u_{j, \ell, m}) = \text{interval of percentages which defines} \]
\[ \text{what constitutes "satisfaction" of requirements for rating } j \]

\[ I_{j, \ell, m} = \text{total in base inventory who are projected} \]
\[ \text{to survive until the end of year } \ell \text{ and} \]
\[ \text{be in interval } m \text{ in rating } j \]

Then, for each year \( \ell = 1, \ldots, 7 \), and each rating \( j = 1, \ldots, r \)
and each interval \( m = 1, 2, 3 \) there is a constraint

\[ a_{i,j,\ell,m} x_{i,j} \leq u_{j,\ell,m} b_{j,\ell,m} - I_{j,\ell,m} \]

which forces new additions during the year, plus aged additions
from previous years, to satisfy requirements.

**School Smoothing Constraints**

Let

\[ f_j = \text{predetermined input to "A" school in rating } j \text{ in} \]
\[ \text{year } 1 \]

Then, constraints

\[ x_{1,j} = f_j \quad j = 1, \ldots, r \]

set the first year inputs to predetermined levels.
Let

\[ s_{i,j} = \text{percentage change in "A" school input allowed for rating } j \text{ in year } i \]

Then, constraints

\[
(1 - s_{i,j}) x_{i,j}^1 \leq x_{i+1,j}^1 \leq (1 + s_{i,j}) x_{i,j}^1
\]

\[ i = 1, \ldots, 6 \]

force annual inputs to change no more than \( s_{i,j} \) percent from one year to the next.

Let

\[ \bar{s}_{i,j} = \text{absolute value of change in "A" school input allowed for rating } j \text{ in year } i \]

Then, constraints

\[
x_{i,j}^1 - \bar{s}_{i,j} \leq x_{i+1,j}^1 \leq x_{i,j}^1 + \bar{s}_{i,j}
\]

\[ i = 1, \ldots, 6 \]

force annual inputs to change by no more than \( s_{i,j} \) from one year to the next. The user can select one or the other of these sets of constraints.

**Lateral Flow Constraints**

Let

\[ R_t, (t = 1, \ldots, g) \]

be \( g \) subsets of ratings in which lateral transfers are authorized. Then, for each \( t = 1, \ldots, g \), in each year \( i = 1, \ldots, 7 \), constraints

\[
\sum_{j \in R_t} (x_{i,j}^3 - x_{i+1,j}^4) = 0
\]

require that net lateral transfers within each of the \( g \) subsets equals zero in each year.
Recruit Input Constraints

Let

\[ rt_i = \text{maximum flow out of recruit training expected to be allowed in year } i, \]

Then constraints

\[
\sum_{k=1,2}^{k} x_{i,j} \leq rt_i \quad \text{for } i = 1, \ldots, 7
\]

force total inputs to "A" school and OJT to be within the limit \( rt_i \) in each year \( i \).

Endstrength Constraints

Since only one interval \((1,31)\) is considered, subscript \( m \) is omitted.

Let

\[ i = 0 \text{ correspond to the base year. (This is necessary in order to account for undesignated personnel in training pipelines in the base inventory.)} \]

\[ x_{0,j}^k = \text{total in training pipeline } k \text{ for rating } j \text{ in the base inventory} \]

\[ ES_k = \text{authorized strength less recruit training strength in year } k \]

\[ p_{i,j,k}^k = \text{fraction of those who enter training pipeline } k \text{ in year } i \text{ in rating } j \text{ who will be in the Navy at the end of year } k \]

Then, for each year \( k = 1,2,\ldots,7 \), constraints

\[
\sum_{j=1,\ldots,r}^{j} p_{i,j,k}^k x_{i,j}^k + \sum_{k=1,2}^{k} a_{i,j,k}^k x_{i,j}^k = \ldots, k
\]

A-6
\[ \sum_{j=1,\ldots,r}^{I} l_{j,k} \leq ES_{k} \]

require that the total number of people in the Navy at the end of each year be within authorized levels.

The Objective Function

Let

\[ c_{i,j}^{k} = \text{the discounted cost to recruit, train, and maintain an input of type } k, \text{ in year } i, \]

until he becomes designated in rating \( j \),

then, one objective to be used during testing is to select that set of new additions \( x_{i,j}^{k} \), \( i = 1,\ldots,7 \), \( j = 1,\ldots,r \), and \( k = 1,\ldots,5 \) which minimizes

\[ \sum_{i=1,\ldots,7}^{\gamma} \sum_{j=1,\ldots,r}^{\gamma} \sum_{k=1,\ldots,5}^{\gamma} c_{i,j}^{k} x_{i,j}^{k}, \]

the cost of all new additions over all planning years.

Another objective is to select that set of inputs to training ("A" and OJT), which minimizes

\[ \sum_{i=1,\ldots,7}^{\gamma} \sum_{j=1,\ldots,r}^{\gamma} \sum_{k=1,2}^{\gamma} x_{i,j}^{k}, \]

the total input to training over the planning years. This objective function was used in the example.
DISCUSSION

The dimensions of the full constraint matrix depend on 2 factors: first, the number of ratings, which can be between 1 and 82, and second, the number of LOS groups within each rating being planned. For a one rating, one LOS group problem, the constraint matrix has only 37 rows and 35 columns, excluding slack variables and upper bound constraints. This matrix has a density of less than 7 percent. For 82 ratings and 3 LOS groups, the full matrix has about 3,500 rows and 3,000 columns, and is approximately one percent dense.

A matrix generator has been written to derive the full constraint matrix as a function of the number of ratings and LOS groups, and also as a function of the upper bounds placed on each variable. For those variables given an upper bound of zero, the corresponding columns are deleted from the problem matrix. This reduces the number of columns by 10 to 20 percent.
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