LIMIT TEMPERATURE STATE OF PERMAFROST EARTH-FILL DAM AND RESERVOIR FLOOR

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DAMS
RESERVOIRS

FROZEN GROUND
PERMAFROST

Dam-building and reservoir formation have a substantial influence on the natural temperature mode of permafrost soils that comprise the floor of a reservoir. These alterations are most strongly manifested in permafrost type dams within the upper thaw zone, particularly during the first years following the filling of the reservoir. The calculated limit temperature state of the base sets in after many years of temperature changes. During the operation of a freezing system permafrost dam stability is determined on the basis of...
the following factors: (1) boundary and size of the thaw zone located within
the top wedge of the dam; (2) outline, size and temperature field of the perma-
frost zone, including the central part of the profile (permafrost core) and
the entire lower wedge, and also the surrounding part of the upper wedge,
which behaves as an insulation liner; (3) temperature-dependent strength, and
water-resistant properties of the soils that make up the permafrost zone; (4)
elimination of all local manifestations of natural filtration in closed thaw
zones before the filling of the reservoir. This report discusses these factors
in some detail.
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FOREIGN TITLE: (PREDEL'NOYE TEMPERATURNOYE SOSTOYANIYE MERZLOY ZEMLYANOY PLOTINY I LOZHA VODOKHRANILISHCHAA)

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Dam-building and reservoir formation have a substantial influence on the natural temperature mode of permafrost soils that comprise the floor of a reservoir. These alterations are most strongly manifested in permafrost type dams within the upper thaw zone, particularly during the first years following the filling of the reservoir.

The calculated limit temperature state of the base of a dam sets in after many years of temperature changes. During the operation of a freezing system permafrost dam stability is determined on the basis of the following factors:

1) boundary and size of the thaw zone located within the top wedge of the dam;

2) outline, size and temperature field of the permafrost zone, including the central part of the profile (permafrost core) and the entire lower wedge, and also the surrounding part of the upper wedge, which behaves as an insulation liner;

3) temperature-dependent strength and water-resistant properties of the soils that make up the permafrost zone;

4) elimination of all local manifestations of natural filtration in closed thaw zones before the filling of the reservoir.

The resistance of the thaw-filled upper slope to stresses and the stress state of the permafrost soil that receives the dynamic head should be calculated for several characteristic modes of development of the slowly forming temperature mode of the dam, in particular its limiting steady temperature state. This state will determine in most practical cases, assuming conditions 4 to be satisfied, basic stability factors 1 and 2 [several words illegible] development of the thaw zone in the base and contiguous shore line. On the basis of this assumption the most important task of calculation of the temperature mode is analysis of factors 1 and 2 in the final, limiting development of the temperature field.

The above-mentioned factors and the rather high strength characteristics and virtually perfect water impermeability of most kinds of layer-frozen and solid-frozen soil provide a basis for assuming the criterion of approximation
of the stability of a dam to be the so-called "[one word illegible] limit criterion" \( k[?] \) [3].

\( k[?] \) characterizes the relative (in comparison with the profile width at a given level) limiting width of the [one word illegible] zone (Figure 1) as a function of the height of the dam, outline of the slopes, width of the crest of the dam, mean annual temperatures of dry and inundated soil surfaces, thermal conductivity coefficients of the soil in the thaw and permafrost zones (assuming that the soil mass of the body and foundation of the dam are homogeneous in terms of thermal properties).

Thus the most important special problem of calculating the limit (stationary) temperature mode of the dam, in addition to construction of the temperature field of the permafrost zone, is determination of its outlines. Ignoring phase transitions in the temperature spectrum, the zero isotherm may be considered as the sufficiently well defined boundary of the thaw and permafrost zones. In some cases the solution of this special problem, i.e., determination of the zero isotherm, without construction of the complete temperature field in the permafrost part of the profile of a dam, completes analysis of the temperature state as a function of its [one or two words illegible] of the installation. This simplified approach to the problem under examination here is completely justified for temporary, low-pressure installations and in preliminary planning stages, when the necessary initial data are not available for more exact calculation.

The stationary distribution of temperatures in the body and base of a dam, comprising a homogeneous earth fill with some constant thermal conductivity coefficients of soil in the thaw and permafrost states \( (\lambda_t \neq \lambda_p) \), is characterized by the Laplace equation

\[
\frac{\partial}{\partial x} \left( \lambda \frac{\partial \lambda}{\partial x} \right) + \frac{\partial}{\partial y} \left( \lambda \frac{\partial \lambda}{\partial y} \right) + \frac{\partial}{\partial z} \left( \lambda \frac{\partial \lambda}{\partial z} \right) = 0, \tag{1}
\]

where \( \lambda = f(t) \) is the thermal conductivity coefficient of the soil, which varies as a function of the phase state of the moisture in the pores of the soil.

The value of \( \lambda \) in equation (1) that does not depend on temperature \( (\lambda_t = \lambda_p) \) may be written in less complex form:

\[
\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0. \tag{2}
\]

Expression (2) characterizes the distribution of relative temperatures in the examined mass of homogeneous soil on the assumption of a constant thermal conductivity coefficient. In the relative temperature scale \( U \) the temperature of the surface of the soil in contact with air, the lowest of all the temperatures on the contours of the examined region [1-5], is taken as zero. The temperature of the underwater surface of the soil in the
upper reach, the lowest of all the temperatures on the contours of the examined region, is taken as unity. Consequently the temperature difference $t_1$ and $t_2$ in relative scale $U$ is assumed equal to unity (the total relative temperature drop $\Delta U = 1$).

The numerical relation between the actual temperature in the scale of degrees $t$ and the relative temperature $U$ corresponding to it is established by the following formulas [2]:

a) for negative temperatures (permafrost zone)

$$u_i = \frac{\lambda_m (t_i - t_m)}{\lambda_m |t_m| + \lambda_r f_i} ;$$  \hspace{1cm} (3)

b) for positive temperatures (thaw zone)

$$u_i = \frac{\lambda_r |t_m| + \lambda_r f_i}{\lambda_m |t_m| + \lambda_r f_i} .$$  \hspace{1cm} (4)

The temperatures on the contours of the region may be characterized not only by the values $t_1$ and $t_2$, but also by any values in the range $t_1 < t_k < t_2$.

In particular, to solve the problem of the stationary temperature distribution in a dam it is necessary to consider the temperatures of internal sources, for example drains, such as the contours of cooling systems, freezers, galleries, ventilated large pore layers, and the contours of air-permeable rock-filled prisms. In this case the temperature on the
contour of internal cooling systems $t_{cool}$ is the lowest of all contour temperatures, whereas conservation of the air cooling system during summer maintains a lower mean annual (actually mean winter) temperature in it than the natural temperature $t_{nat}$ on the "dry" external contour of the dam. Therefore the relative temperature $U[?] = 0$ is given on the contour of the cooling system, and the relative temperature $U[?]$ on the "dry" external contour is determined by formula (3).

In many cases there may be a difference between $t[?]$ and the temperature $t[?]$ on the contour of the reservoir or deep thaw in the lower reach, determining the external temperature conditions at the base of the lower wedge of the dam. The relative temperature $U[?]$ on these contours is determined by formula (4) if $t[?] < t[?]$. The case when $t[?] > t_1$ may occur very rarely and need not be examined.

The electrothermal analogies method (ETA) is based on the known analogy between the stationary temperature distribution $t$, $U$ in a heat-conducting medium of homogeneous soil (equations 1 and 2) and the distribution of electrical potentials $\phi$ in a geometrically similar current-conducting medium -- a continuous ETA model. The Laplace equation for this model is

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0.$$  (5)

On the basis of the electrothermal analogy (see Table 2) one may easily and accurately construct the stationary temperature field in the body of a dam with any outlines and temperatures on the external and internal contours. For this purpose one may use stationary network integrators (for example EI-12, MSM-1), ZGDA electrolytic [possibly modeling] systems and electrical integrators with models made of electrically conducting paper (for example EGDA 9/60).

The scale of an electrical model is arbitrary and is based on the required calculation accuracy and size of the model stand. The technique of making models and assigning boundary conditions during modeling with the EGDA 9/60 instrument is basically the same as that used to solve problems of plane and plan filtration. Some additional explanations should be given concerning the choice of dimensions of the modeled region, assignment of the lower boundary condition, which considers the natural temperature of the natural permafrost mass and influx of heat from the depths of the earth, and assignment of the boundary conditions in the lower reach and on the internal contours.

The size of the modeled region should be such that it is possible to correctly consider the influence of the lower boundary condition and minimize distortions of the modeled temperature field caused by the influence of the lateral contours of the model.
On the basis of experience in the electrical modeling of certain real objects [6-12], it may be assumed that the lateral contours of a model will have no significant effect on the temperature field of a dam for a model with length

\[ t_m > 5B, \quad (6) \]

where \( B \) is the width of the dam (Figure 1).

The height of the model is determined in consideration of the lower boundary condition -- the temperature of the natural permafrost mass at the depths where the natural temperature field of the base of the dam is virtually not affected by external temperature conditions -- \( t_{\text{nat}}, t_\langle \rangle, t_\rangle \).

Detailed analyses of the stationary temperature mode of a number of permafrost dams, conducted by the author and engineer V. T. Shugayeva (dams with permafrost [one word illegible] on the Sytykan River and Irellykh River in Yakut ASSR, frozen earth-fill dam of the Anadyrskaya [possibly thermoelectric power plant] on the river Kazachka, earth-fill dam of a reservoir in Chitinskaya Oblast, etc.), disclosed that for a dam height up to [illegible] with a wide variety of combinations of permafrost soil conditions and temperatures, the thermal influence of the reservoir on the temperature field of the natural mass under the dam extends to a depth of 25-35 m. Therefore the natural temperature of permafrost at a depth of 35-40 m may be used as the lower boundary condition, determining the height of the model and given relative temperature \( U_\langle \rangle \) on its [possibly lower] contour (see Table 2). This recommendation requires refinement during analysis of the temperature field in the body and base of a dam with a height greater than 20 m.

The thermal influence of the reservoir on the soils of its floor within the confines of the dam is manifested to a considerable depth and is estimated by the outline of the limiting thaw line [1, 2, 15], in accordance with which the height of the model \( h_m \) is refined for calculation of the temperature field at some distance from the dam.

The temperature field in the base of the dam near the floor of the reservoir depends on the influx of heat from the depths of the earth, characterized by the geothermal gradient or geothermal stage in the vicinity of the dam construction site. By considering the heat of the earth it is possible to draw a more complete picture of the relation between the permafrost zone of the dam and the thickness of the permafrost soils of the base.

Following N. A. Bogoslovskiy's recommendations [1], we use for this purpose the superposition method and examine the equations of two special problems.

The first problem is to construct an ETA model (Figure 2a) of the stationary relative temperature field without consideration of the lower
boundary condition and influx of heat from the depths of the earth. In this case the depth of the mass of the base in the model should not exceed the height of the dam by less than a factor of 10.

By modeling the second special problem it is possible to consider the earth's heat and to use the model (Figure 2a) to construct the relative temperature field for a "dry" dam (with an unfilled reservoir). In this problem the relative temperature field $U_2 = 0$, corresponding to the real temperature field, is given on the contour of the dam and the upper and lower reaches.

The relative temperature $U_2 = 1$ is applied on the lower horizontal contour of the model. The position of this contour (the height of the model for the second problem) is determined as follows.

For a known thickness $M$ of the permafrost and known temperatures on the outer surface ($t_s$) and on the floor ($0^\circ$), the geothermal stage in the thickness of the permafrost zone is given by the formula

$$ S = \frac{M}{t[?]} \text{ m/deg.} \quad (5) $$

Assuming the value $S$ to be constant within the limits of the height of the model, we determine the depth $h_k$, at which the temperature of the thaw mass is equal to the temperature $t[?]$:

$$ \text{illegible, not reproducible} \quad (6) $$

At that depth, which is the lower contour of the model for the second problem, applies the contour line with the relative potential $\phi_2$, corresponding to the relative temperature $U_1 = 1$.

The summary relative isotherms $U_1$ plotted in Figure 2b as the result of the superposing of two temperature [possibly fields] of the first and second special problems, pass through the points of intersection of the isotherms $U_1$ and $U_2$, for which the equality $U = U_1 + U_2$ is satisfied.

The desired temperature field is constructed in Figure 2c, where the relative temperature values are converted to real temperatures $t_1$ by formulas (3) and (4).

In the most general case the ETA method can be used to reproduce on the model a complex set of topographic conditions of construction, the geological structure and temperature mode of the permafrost base, construction of the dam and external temperature factors, and also the thermophysical properties of the soils and materials.
Based on the results of our analyses we recommend that the boundary conditions be assigned for certain important practical cases. These plans and brief explanations of them are given in Table 1 and Figure 3.

![Figure 3. Analyses of the limiting temperature state of certain dams and tailings dumps surrounding dams, conducted under actual operating conditions\(^1\), lead to the following important practical conclusion, which applies to most practical permafrost soil construction plans.

During the construction of a homogeneous, monolithic, completely frozen dam body during the time of construction, and also during [one word illegible] filtering thaw soils beneath a dam whose lower reach slope has no [one word illegible] sediments, the thaw basin forms within the underwater part of the upper reach slope. The zero isotherm, the conventional boundary between the thaw and permafrost zones of the profile, passes within the upper wedge, bounded by a vertical line, drawn from the water line on the upper reach slope (points 1 and A' in Figure 1).

This conclusion substantially facilitates preliminary evaluation of the depth of freezing of the thaw-permafrost profile of the dam and its strength. Assuming that the construction technology ensures the complete freezing of the dam and thaw zones of the base by the time the reservoir is filled, it is possible, without resorting to complex mathematical calculations and modeling, to assume vertical line \(y\) (Figure 1) to be the

\(\text{\textsuperscript{1}During the period of 1967-1971 25 electrical models of dams were analyzed (Noril'sk, Yakut ASSR), during which the structure of the profiles, outlines of underwater and dry surfaces, temperature and thermophysical factors were carefully reproduced.}\)
boundary between the thaw and permafrost zones. This provides some margin in the calculation, since even with the most unfavorable combination of boundary conditions, the zero isotherm falls to the left of vertical line y and approaches it only in the uppermost part.

Table 1

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperatures</td>
<td>Contour of application</td>
<td>Value</td>
<td>Bottom of upper reach</td>
<td>&quot;Dry&quot; surface</td>
<td>Internal cooling contour</td>
<td>Bottom of lower reach</td>
<td>Thaw in lower reach</td>
<td>Lower boundary condition</td>
</tr>
</tbody>
</table>

Key: 1, Temperatures; 2, Contour of application; 3, Value; 4, Bottom of upper reach U[?]; 5, "Dry" surface U[?]; 6, Internal cooling contour U[?]; 7, Bottom of lower reach U[?]; 8, Thaw in lower reach U[?]; 9, Lower boundary condition U[?].

Comment to Table 1

To simplify the calculation a row of columns with diameter d was replaced by a vertical slit, in which the direction, velocity and pressure of the air, and also the assigned vertical temperature and air pressure drop in the annular space were left unchanged. The validity of this assumption was checked by a comparison with the data on the full-scale [illegible] dam [6]. The length of the slit through the dam was set equal to the length of the permafrost curtain.

The [one word illegible] that model different values of U₁ are separated by an open space of 5-10 mm.

The height of the model in Figure 2 is h[?]. Modeling was done in two runs and U[?] was taken into consideration one time (see the text).

In Figure 5, where the core of the longitudinal gallery is cooled U[?] is considered along the external perimeter.
Another important practical conclusion was drawn from a comparison of the zero isotherms, plotted in an ETA model of a [possibly half-dam] (split dam in a flat valley), for a wedge (calculation diagram of the profile of a dam as calculated by I. S. Moyseyev), and for a trapezoid (real profile of dam). The isotherms were plotted for the following conditions:

- The layout of the compared isotherms agrees satisfactorily with the above conclusion. The isotherms for the wedge and trapezoid virtually coincide, which is reason to assume that it is completely permissible to replace the real profile of a dam by a wedge in an analytical calculation such as I. S. Moyseyev's. The position of the zero isotherm on the half-plane is characterized by a higher value of critical limiting depth of freezing. Hence it may be concluded that it is advisable to plan the flattest possible layered profile of a naturally cooled permafrost dam (the influence of slope steepness on the value $K_m$ is examined in greater detail in another article by the author).

Table 2. Electrothermal Analogies for Modeling Stationary Temperature Mode of Dam

<table>
<thead>
<tr>
<th>Analyzed and assigned thermal processes and parameters (full-scale)</th>
<th>Equivalent thermoelectric processes and parameters (model)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual temperature $t$, deg.</td>
<td>Absolute electrical potential $\phi$</td>
</tr>
<tr>
<td>Relative temperature $U$, in fractions of unity</td>
<td>Relative electrical potential in fractions of unity; on instrument 9/60 -- in percent</td>
</tr>
<tr>
<td>Relative temperature difference $\Delta U$</td>
<td>Relative potential difference $\Delta \phi$</td>
</tr>
</tbody>
</table>

[Continued on next page]
<table>
<thead>
<tr>
<th>Stationary fields of real and relative temperatures</th>
<th>Stationary electrical field in solid current-conducting medium (on EGDA 9/60 instrument -- in model on electrically conducting paper)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative temperature gradient</td>
<td>Relative potential gradient</td>
</tr>
<tr>
<td>Thermal conductivity gradient of [one word illegible] soil</td>
<td>Specific conductance or resistivity of electrically conducting material of model (arbitrary for modeling homogeneous soil mass)</td>
</tr>
<tr>
<td>[One word illegible] value -- ratio of thermal conductivity coefficients of two adjacent zones of different soils</td>
<td>Choice of two different kinds of electrically conducting paper in accordance with equality [formula illegible] (for electrolytic and network models their resistivities are also chosen)</td>
</tr>
<tr>
<td>[One word illegible] of equal relative temperatures (relative [possibly isotherms])</td>
<td>Isolines of equal relative potentials (relative equipotentials)</td>
</tr>
<tr>
<td>External and internal contours of [one word illegible] soil mass with temperatures ( t[?] ), ( t[?] ), ( t[?] ) assigned on them</td>
<td>Geometrically similar contours of electrically conducting model with given relative potentials ( \phi ) on contours modeling boundary conditions of problem (on EGDA 9/60 instrument -- brass bars and open-ended contours of paper model)</td>
</tr>
<tr>
<td>[One word illegible] ( t = 0^\circ ) -- boundary between [one word illegible] and permafrost zones -- determined by equations (3) and (4) after construction of relative temperature field</td>
<td></td>
</tr>
</tbody>
</table>
BIBLIOGRAPHY


