Values of Diffusion Coefficients Deduced From the Closing Times of Helicopter-Produced Clearings in Fog

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12 January 1977

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VALUES OF DIFFUSION COEFFICIENTS DEDUCED FROM THE CLOSING TIMES OF HELICOPTER-PRODUCED CLEARINGS IN FOG.

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Diffusion coefficients in fog
Closings times of artificially created clearings in fog
Fog, Lewisburg, W. Virginia
Fog experiments

Cloud physics
Temperature structure in artificial clearings in fog
Liquid water content

Values of diffusion coefficients determined from the observed closing times of nine conical-shaped clearings in fog produced by hovering helicopters at Lewisburg, West Virginia, in September 1969 are presented. The values were established following the method of Elliott, assuming that the geometric and diffusive properties of the clearings and surroundings could be approximated by theoretical equations of the type governing the diffusion of heat and water substance in a bounded, circular cylinder of infinite length, with appropriate specification of the condensation conditions.
20. Abstract (Continued)

The diffusion coefficients for the experiments ranged in value from 0.7 \( \times 10^5 \) to 1.9 \( \times 10^5 \) cm\(^2\)/sec. The values were consistent for experiments performed on the same days, but no other correlations with meteorological or geometric parameters of clearing were found. The values are larger than those reported previously for fog situations and, although they are certainly the "effective values" pertaining to helicopter clearing, there is a question whether they are characteristic of the ambient fog surroundings. This matter is discussed.

Summary diagrams are presented to illustrate how a cylindrical or slot-shaped clearing will close-in with time, dependent on the values of the diffusion coefficient and on the initial temperature and humidity differences between clearing and surrounding.
Preface

The authors would like to thank Dr. Bernard A. Silverman* and Mr. Chankey N. Touart, of the Meteorology Division, AFGL, for reviewing the manuscript and offering helpful suggestions.

*Dr. Silverman is now with the Bureau of Reclamation, Denver, Colorado.
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Values of Diffusion Coefficients Deduced From 
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Clearings in Fog

1. INTRODUCTION

Elliott\textsuperscript{1} indicated methods whereby values of the mixing coefficients in stable atmospheric situations might be estimated from the closing times of clearings created in fog layers by artificial means. He considered the case in which a slot of clearing, of sufficient length and depth to be considered infinite, had been created in a fog layer, and presented equations describing the "closing in" characteristics of the slot as a function of the initial slot width and time. He also pointed out that similar equations could be written for a circular hole of clearing which could be treated mathematically as an infinite cylinder.

Clearing data that can be employed to estimate the values of diffusion coefficients in the manner of Elliott's suggestion have been acquired by Plank, Spatola, and Hicks.\textsuperscript{2,3} The clearings were created by helicopter downwash during hover experiments performed 5 miles north of Lewisburg, West Virginia, in September

(Received for publication 11 January 1977)

1969. The geometric and thermodynamic properties and observed closing times of certain of these clearings are summarized herein, and the modifications to Elliott's problem-treatment method required to estimate the diffusion coefficients for the particular Lewisburg situations are discussed. The estimation methods are indicated and the values presented, with qualifying comments regarding their significance.

2. THE CLEARING DATA

Nine helicopter-hover experiments were conducted during the Lewisburg program, and they yielded data that were suitable for estimating the mixing coefficients. The dates and times of these experiments are listed in Table 1 together with information about the geometric sizes and thermodynamic properties of the clearings at the surface and at fog-top levels. The measurements were obtained under steady-state conditions of clearing, as described by Plank, Spatola, and Hicks, or they were derived by methods indicated in the table caption. The ESTs listed are for when the helicopters ceased hovering and when aircraft observations of the closing of the clearings first began. The closing times of the clearings are shown in the last column of the table.

The clearings had truncated, conical shapes and the air temperature within the clearings was warmer than that of the surrounding fog due to the adiabatic warming of the helicopter downwash air and the heat of the engine exhaust that was added to the air. The water vapor content of the clearings, although less than saturated, was likewise greater, in all but one case, than the water mass content (vapor plus liquid) of the surroundings. This was the result of the particular fog situations at Lewisburg in which the clear air above the fog, which was transported downward into the fog by the helicopter rotors during the clearing process, had a larger water mass content than that of the fog air itself.

3. MODIFICATIONS TO THE PROBLEM TREATMENT OF ELLIOTT

The equations of Elliott cannot be applied directly to estimate the diffusion coefficients for the Lewisburg clearings because (a) they pertain to a slot, or swath, of clearing, whereas the Lewisburg clearings had conical shape, and (b) they describe the diffusion of water substance in isothermal situations, whereas the Table 1 data reveal that the Lewisburg clearings were distinctly non-isothermal in character. These differences require treatment modifications as indicated in the following sections.
Table 1. Dates, Times, and Dimensional-Thermodynamic Data Pertaining to Nine Helicopter-Produced Clearings of the Lewisburg Experiments. The parameters are listed for the surface and fog-top levels. The pressures (p) aloft and within the clearings were derived from the measured pressures of the surface surroundings by use of the hyposometric equation and the assumptions of (a) hydrostatic equilibrium within the environment and clearings, and (b) constant pressure at the helicopter flight altitude, within wake and environment, at the time of the termination of the helicopter clearing efforts. The temperature (T) and specific humidity (q) values within the clearing at the fog-top level were derived assuming constant \( \frac{\partial T}{\partial z} \) and \( \frac{\partial q}{\partial z} \) within the helicopter wake corresponding to the differences in these parameters measured within the clearing at the surface level and within the wake of the helicopter at the flight altitude at the assumed instant of clearing effort termination. The closing times of the clearings and the fog top appearance were observed, visually and/or photographically, from a C-119 aircraft that was circling the scene of the operations. (M = mixing ratio)

<table>
<thead>
<tr>
<th>Experiment No.</th>
<th>Date</th>
<th>Helicopter Departure Time</th>
<th>Fog Depth</th>
<th>Mean Horizontal Dimensions of Clearing Surface Level</th>
<th>Mean Horizontal Dimensions of Clearing Top Level</th>
<th>Surface Level Parameters Environmental</th>
<th>Within Clearing Hor. Sect. Mean</th>
<th>Fog-Top Level Parameters Environmental</th>
<th>Within Clearing Hor. Sect. Mean</th>
<th>Fog-Top Appearance</th>
<th>Observed Closing Times of Clearings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12 Sept 1969</td>
<td>0720</td>
<td>68</td>
<td>470 340</td>
<td>942.10 4.3 5.5 0.20</td>
<td>942.04 5.5 6.1</td>
<td>934.32 6.8 6.7</td>
<td>934.30 8.4 6.5</td>
<td>S</td>
<td>5-10</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>13 Sept 1969</td>
<td>0732</td>
<td>68</td>
<td>230 150</td>
<td>941.80 5.3 5.9 0.11</td>
<td>941.72 6.8 6.6</td>
<td>934.04 7.4 7.0</td>
<td>934.01 9.4 7.0</td>
<td>MC</td>
<td>3-4</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>13 Sept 1969</td>
<td>0732</td>
<td>68</td>
<td>220 150</td>
<td>941.80 5.5 6.0 0.11</td>
<td>941.69 8.1 7.0</td>
<td>934.05 7.4 7.0</td>
<td>934.02 10.7 7.2</td>
<td>MC</td>
<td>3-4</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>13 Sept 1969</td>
<td>0824</td>
<td>76</td>
<td>190 160</td>
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<td>933.16 7.9 7.2</td>
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<tr>
<td>5</td>
<td>14 Sept 1969</td>
<td>0900</td>
<td>160</td>
<td>155 85</td>
<td>941.10 10.3 8.4 0.21</td>
<td>943.99 10.6 8.4</td>
<td>926.23 11.8 9.5</td>
<td>926.21 14.5 9.0</td>
<td>C</td>
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<td></td>
</tr>
<tr>
<td>6</td>
<td>16 Sept 1969</td>
<td>0729</td>
<td>61</td>
<td>300 210</td>
<td>939.70 9.3 7.8 0.17</td>
<td>939.63 10.5 8.3</td>
<td>932.83 11.2 9.0</td>
<td>932.82 13.3 9.1</td>
<td>S</td>
<td>5-6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>16 Sept 1969</td>
<td>0743</td>
<td>68</td>
<td>120 240</td>
<td>939.70 9.3 7.8 0.17</td>
<td>939.62 10.8 8.5</td>
<td>932.08 11.5 9.2</td>
<td>932.06 14.2 9.4</td>
<td>S</td>
<td>5-6</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>27 Sept 1969</td>
<td>0731</td>
<td>145</td>
<td>185 85</td>
<td>937.30 6.6 6.5 0.14</td>
<td>937.16 8.5 7.1</td>
<td>921.06 10.6 8.8</td>
<td>921.03 12.5 8.7</td>
<td>S</td>
<td>1-3</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>27 Sept 1969</td>
<td>0802</td>
<td>145</td>
<td>180 85</td>
<td>937.30 7.0 6.7 0.14</td>
<td>937.12 9.4 7.7</td>
<td>921.07 10.6 8.8</td>
<td>921.03 13.2 8.8</td>
<td>S</td>
<td>1-3</td>
<td></td>
</tr>
</tbody>
</table>

Notes: The parameters underlined are derived parameters.
The fog-top appearance symbols are: S = Stable, C = Convective, MC = MIdly Convective.
4. DIFFUSION EQUATIONS OF WATER SUBSTANCE AND HEAT FOR AN INFINITE CYLINDER

It is assumed, for first estimation purposes, that the conical-shaped clearings of the Lewisburg experiments can be approximated by the theoretical diffusion equation for an erect, right cylinder of sufficient length, that is, fog depth, to be considered infinite. (The details of how the cones were approximated by cylinders is described in Section 7.) The governing equation that describes the diffusion of water substance into or out of the interior of the cylinder and from or to the boundary (in the temporarily presumed absence of any second interacting diffusion process) is, in cylindrical coordinates, 4

\[
\frac{\partial C(r, t)}{\partial t} = K_C \left[ \frac{\partial^2 C(r, t)}{\partial r^2} + \frac{1}{r} \frac{\partial C(r, t)}{\partial r} \right],
\]

where \( r \) is radial distance outward from the centerline of the cylinder, \( t \) is time measured from some initial time \( t = 0 \), \( K_C \) is the eddy diffusion coefficient, and

\[ C = \rho q + M \tag{2} \]

is the "water substance parameter," where \( \rho \) is the air density, \( q \) is the specific humidity of the water vapor in the air, and \( M \) is the liquid water content of the fog, when present. Equation 1 is analogous to that employed by Elliot, 1 and its application to the fog problem is based on the assumptions that the coordinate system moves with the cloud, that horizontal wind shear effects can be ignored, that the eddy diffusion of water vapor and liquid water in droplet form are both governed by the same constant coefficient \( K_C \), and that evaporation and condensation occur instantaneously.

If differences of temperature (\( T \)) exist initially between the interior of the cylinder and its boundary, and if we assume temporarily that no second diffusion process is operating that would modify the heat flow by interaction, then the temporal and spatial changes of temperature within the cylinder will be governed by the equation

\[
\frac{\partial T(r, t)}{\partial t} = K_H \left[ \frac{\partial^2 T(r, t)}{\partial r^2} + \frac{1}{r} \frac{\partial T(r, t)}{\partial r} \right],
\]

which is of type similar to Eq. (1), where $K_H$ is the eddy coefficient of thermometric conductivity.

The solutions of Eqs. (1) and (3) for any given set of initial conditions, identified by the subscript "0," and boundary conditions, identified by the subscript "1," are, respectively,

$$
\frac{C_0 - C}{C_0 - C_1} = 1 - \frac{2}{a} \sum_{n=1}^{\infty} \frac{J_0 (r \alpha_n)}{\alpha_n J_1 (a \alpha_n)} \times \exp ( - K_C \alpha_n^2 t ) ,
$$

which satisfies the initial conditions $C = C_0$ at $a > r > 0$, at $t = 0$, and boundary conditions $C = C_1$ at $r = a$, for $t > 0$, and

$$
\frac{T_0 - T}{T_0 - T_1} = 1 - \frac{2}{a} \sum_{n=1}^{\infty} \frac{J_0 (r \alpha_n)}{\alpha_n J_1 (a \alpha_n)} \times \exp ( - K_H \alpha_n^2 t ) ,
$$

which satisfies analogous initial and boundary conditions on $T$. In these equations, $J_0$ is the Bessel function of zero order of the first kind, $J_1$ is the Bessel function of the first order, which fulfills the boundary conditions, and $\alpha_1, \alpha_2, \ldots$ are the positive roots of $J_0 (\alpha a) = 0$.

5. NON-ISOTHERMAL, SIMULTANEOUS DIFFUSION WITHOUT COUPLING

Henry\(^5\) and Frank-Kamenetskii\(^6\) have demonstrated that if two diffusing substances are involved in a given problem (having its particular geometric, boundary, and initial conditions), the problem may be regarded as one in which two time-waves of diffusion occur for each of the substances. There is a "primary wave," which moves independent of the existence of the other substance and is governed by equations of the type (1) and (3) herein, and there is a "coupled wave," which moves dependent on the mutual interaction influences of the two substances.

If we apply the Henry\(^5\) findings to the present problem, in which we have a cylinder of clearing, and if we assume (a) that the boundary conditions within the


environment are invariant with time, (b) that the coupling interaction between the
diffusion of heat and the diffusion of water substance is negligible, and (c) that the
two eddy coefficients of diffusion are equal, constant with time, and independent of
the magnitude changes of either the temperature or water vapor content of the air,
that is,
\[ K_C = K_H = K_e, \]

then we may conclude that the initial, boundary, and variable parameters will, at
all times during the diffusion processes, be related as
\[ \frac{C_0 - C}{C_0 - C_1} = \frac{T_0 - T}{T_0 - T_1}. \]

This relationship implies, in essence, that the fractional time changes of tem-
perature and water-substance within the cylinder, at any point, will proceed at the
same rates, proportional to their initial differences.

Figure 1. Solutions of Equation (4) or (5) for Two Ranges of the Ratio \( (C_0 - C)/(C_0 - C_1) \) or \( (T_0 - T)/(T_0 - T_1) \). The isolines give the values of the non-dimen-
sional quantity \( K_{at}/a^2 \). Presented with the permission of the Clarendon Press,

*The coupling between the water substance and heat in the fog problem involves the
latent heat released by the conversion of water vapor into liquid water during the
mixing processes. The amount of this heat release is small and can be ignored,
to a first approximation. For example, the condensation of fog liquid water in
amounts of the order of 0.10 gm/m², in the temperature range 5°C to 20°C, will
only cause temperature changes of about 0.1°C.
Plots of Eqs (4) and (5) that incorporate assumptions (6) and (7) are shown in Figure 1. The isolines specify the values of the non-dimensional parameter $K_a^2/a^2$ which correspond to particular values of the ratio quantities $(C_0 - C)/(C_0 - C_1)$ or $(T_0 - T)/(T_0 - T_1)$ and to particular radial distances $r/a$ from the centerline axis of the cylinder. The diagram is a conversion of Figure 24 of Carslaw and Jaeger and is analogous to the plot discussed by Elliott. 1

6. SATURATION ASSUMPTIONS, CONCERNING THE THRESHOLD OF CONDENSATION AT ANY GIVEN RADIUS AND TIME WITHIN THE CYLINDER

It is assumed that the fog—no fog boundary within the cylinder will move radially inward with time, in accordance with the observations of the Lewisburg experiments, and that the air at any particular point on this boundary, at the radius $r_5$ and time $t_5$, when condensation first occurs, will be saturated but devoid of liquid water, such that

$$M = 0 \quad (8)$$

and, from Eq. (2),

$$C = C_s = \rho_s q_s \quad (9)$$

If the pressure in the cylinder at any horizontal level is considered to be the same as that of the environment, the air density at saturation, $\rho_s$, is given by the equation of state as

$$\rho_s = \frac{P_1}{R(1 + 0.6078 q_s)T_s} \quad (10)$$

where $R$ is the gas constant for dry air and $T_s$ is the absolute temperature, in °K, at which saturation occurs. The saturation specific humidity and the saturation temperature are related by the equation of Tetens. 7

\[
q_s = \frac{17.2694(T_s - 273.16)}{3.7989 \exp \frac{T_s}{-35.86}} \quad \text{and} \quad p_1 = 2.3089 \exp \frac{17.2694(T_s - 273.16)}{T_s - 35.86}
\]

where \( p_1 \) is specified in millibars and the constants are those of Murray. If the initial and boundary conditions are specified for any given fog situation for which the cylinder approximation is presumed to apply, Eqs (7), (9), (10), and (11) may be solved simultaneously to determine the particular values of \( T_s, q_s, \) and \( c_s \) that pertain to the "threshold of condensation" condition within the cylinder. The \( c_s \) and \( T_s \) values may then be introduced in the ratio quantities on the left of diffusion Eqs (4) and (5). From these diffusion equations, thus evaluated, the radius-time characteristics of the diffusion within the cylinder can be ascertained, as is illustrated in Figure 1.

7. DETERMINATION OF THE DIFFUSION COEFFICIENTS

It was assumed that the conical-shaped clearings of the Lewisburg experiments could be approximated by cylinders having a radius \( a \) equal to the average of the surface and fog-top level radii that were observed. These \( a \) values are listed in Table 2, together with the mean values of the initial and environmental parameters \( T_0, q_0, p_1, T_1, q_1, \) and \( M_1 \). These means, except for \( M_1 \), are the vertical averages of the measured surface and fog-top-level values of Table 1. Fog liquid water was measured only at the surface level, hence the \( M_1 \) values are those of the surface.

The closing times of the clearings, \( t_c \), are also listed in Table 2; it should be noted that these correspond to the second of the observed times of Table 1. (The first of the times specified in Table 1 is the approximate time required for the initial whisps of fog to diffuse inward to the center of the clearings from the surroundings; the second is the time required for the vertical visibility at the center of the former cleared zones to become approximately the same as that of the surroundings. It is assumed herein that the latter times are the most appropriate for estimating the diffusion coefficients for the layer-mean conditions of clearing under the cylinder approximation.)

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Table 2. Cylinder Radii, Initial and Boundary Conditions, Clearing Closing Times, and Derived Mixing Coefficient Values for Nine Clearing Experiments. The dates and times of the experiments are given in Table 1. The initial and boundary values listed are simple averages of the surface and fog-top-level values of Table 1, except for the fog liquid water content, which was assumed equal to that measured at the surface level. See text for description of saturation conditions.

<table>
<thead>
<tr>
<th>Experiment No.</th>
<th>Assumed Cylinder Radius</th>
<th>Initial Conditions</th>
<th>Corresponding r.h.</th>
<th>Boundary-Environmental Conditions</th>
<th>Closure Time</th>
<th>Saturation Ratio</th>
<th>Diffusion Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a m</td>
<td>T₀ (°C)</td>
<td>q₀ (gm/kg)</td>
<td>P₁ (mb)</td>
<td>T₁ (°C)</td>
<td>q₁ (gm/kg)</td>
<td>M₁ (gm/m³)</td>
</tr>
<tr>
<td>1</td>
<td>202</td>
<td>6.7</td>
<td>6.3</td>
<td>96</td>
<td>938.2</td>
<td>5.6</td>
<td>6.1</td>
</tr>
<tr>
<td>2</td>
<td>95</td>
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<td>94</td>
<td>937.9</td>
<td>6.4</td>
<td>6.5</td>
</tr>
<tr>
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<td>92</td>
<td>9.4</td>
<td>7.1</td>
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<td>6.5</td>
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<td>929.2</td>
<td>8.8</td>
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</tr>
</tbody>
</table>
With these assumptions about the correspondence of the observational data and theory, the $T_s$, $q_s$, and $C_s$ values were computed for each of the nine experiments, following the method indicated previously. Absolute accuracy of the measured parameters was assumed in the computations, and it might be mentioned that a high degree of computational exactitude was required. The resultant values are given in Table 2.

Knowledge of these values permitted determination of the ratios $(C_0 - C_s)/(C_0 - C_1)$ or $(T_0 - T_s)/(T_0 - T_1)$, and this, in turn, from Eqs (4) and (5), as shown plotted in Figure 1, gave the $(K_e t/a^2)_{r=0}$ values that pertained to the centerline of the cylinder, where $r = r/a = 0$. Finally, the diffusion coefficients,

$$K_e = (K_e t/a^2)_{r=0} \times a^2/t_c,$$

were computed from the thus-established values of $(K_e t/a^2)_{r=0}$ and from the observed $a$ and $t_c$ values. The $K_e$ values are listed in Table 2.

8. DISCUSSION

It is seen that the $K_e$ values for the Lewisburg experiments ranged from $0.7 \times 10^5$ to $1.9 \times 10^5$ cm$^2$/sec. The values were reasonably consistent for experiments performed on the same days, but there was little correlation with the visual-appearance-state of the fog top, or with any of the other meteorological or geometric parameters identified in Tables 1 and 2.

These $K_e$ values are larger, by about a factor of two, than the profile values reported by Vorontsov and Selitskaia\textsuperscript{9} for 250-meter-thick fog in the Dickson Island\textsuperscript{a} region during landward wind conditions, and are likewise larger than the values for sea fog reported by Vorontsov.\textsuperscript{10} The values correspond approximately with the typical values for stratus clouds ($0.44 \times 10^5$ cm$^2$/sec) and altostratus clouds ($1.1 \times 10^5$ cm$^2$/sec), as reported by Smith, Chien, and MacCready.\textsuperscript{11}

\textsuperscript{a}A small island off the NW coast of Siberia at the mouth of the Yenisei River.


There is little question but that the tabulated values herein are the "effective coefficients" that pertained to the closing-in of the helicopter clearings. But there is a question as to whether the values are characteristic of the ambient fog surroundings.

There are several obvious reasons why the computed values might be too large, relative to the true ambient values. But there is also a counter reason for arguing that they might be proper or too small. The actual truth is unknown, of course, and we can only surmise the possibilities.

The tabulated values could readily be overestimates of the true ambient values because, at the initial times and during the closing-in process, residual, helicopter-induced turbulence could have been present in the clearings that was not taken into account in the theoretical treatment. Moreover, since the clearings were initially warmer than the surroundings, solenoidal-type circulations could have existed across the clearing boundaries, which is another form of turbulence not considered. Wind shear effects could also have operated to close the clearings more rapidly than in the homogeneous wind field situation assumed in the theoretical treatment, again leading to an overestimate of the coefficient values. On the other hand, though, the equations employed herein, which presume the cylinder boundary to be fixed at the radius a, with invariant environmental conditions prevailing at and beyond this boundary, are not really good approximations of the true situation. In the real atmosphere, as in the Lewisburg experiments, it would seem that the eddy diffusion of water substance and heat across the clearing walls, which existed initially at t = 0, would proceed in both radial directions with time, both toward center, toward decreasing r, and also outward, toward r > a. If this were true, it would imply that the \( K_e \) values of the table might be underestimates of the actual, by some unknown factor related to this boundary specification problem.

In any event, it may be stated that the clearings of the Lewisburg experiments did close-in very quickly, which is prima facie evidence of large turbulent diffusion. But whether the method of Elliott\(^1\) provides the proper coefficient values for the diffusion remains uncertain. Comparison experiments are needed in which the coefficients deduced from hole-closing are related directly to those of conventional measurement.

9. SUMMARY ILLUSTRATIONS

The salient features of the diffusion situations discussed in this paper are illustrated in Figures 2 and 3. The upper diagram of Figure 3 shows how the closure times of a cylindrical clearing having the mean properties stated in the caption will vary as a function of the initial relative humidity of the clearing, for
various values of $C_0 - C_1$ and $T_0 - T_1$. The lower diagram shows the same for a slot, or swath, of clearing, which was the particular geometric shape discussed by Elliott. The $K_e$ value assumed in the plotting was $10^5$ cm$^2$/sec. For $K_e$ values other than this, it might be mentioned that the closing times vary inversely with $K_e$.

The diagrams reveal that the closing times are sensitively dependent on the initial relative humidity of the clearing, particularly in the humidity range from 90 to 100 percent. They also demonstrate that a circular clearing — for a circle diameter equal to the slot width — will close about twice as rapidly as a slot of clearing.
Figure 2. Diagrams illustrating the time changes of the T and C profiles within the cylinder for the three possible gradient situations of $T_0 - T_1$ and $C_0 - C_1$. The T profiles (solid) and the C profiles (dashed) are shown for each situation for the times $t = 0$, $T = t_c/2$, $t = t_c$, and $t = \infty$, where $t_c$ is the closing time of the clearing (see text) when the fog, indicated by the shading, first reaches the centerline axis of the cylinder from the surroundings. A clearing of 100 meters initial radius is assumed in all cases, with a diffusion coefficient of $10^5$ cm$^2$/sec, and boundary conditions of $T_1 = 10^\circ$C, $C_1 = 9.7$ gm/m$^3$, and $M_1 = 0.2$ gm/m$^3$. The $T$ and $C$ scales, shown at the left, are related through the equation of Tenen [1] which is Eq. (11) of the text. The vertical spacing between the $T$ and $C$ profiles, when $C > T$, provides a measure of the fog liquid water content, $M$. The particular closing times for the three situations sketched are 7 minutes for the "warm-dry" situation, 3 minutes for the "warm-moist" situation, and 5 minutes for the "cold-dry" situation.
Figure 3. Closing Times for a Cylindrical Clearing (upper diagram) and for a Slot, or Swath, of Clearing (lower diagram). The diagrams show how the closing times vary as a function of the initial relative humidity for particular values of $C_0 - C_1$ and $T_0 - T_1$. The diagrams are drawn for $K_e = 10^5 \text{ cm}^2/\text{sec}$, $T_1 = 10^0\text{C}$, and $C_1 = 9.6 \text{ gm/m}^3$, with $M_1 = 0.2 \text{ gm/m}^3$. The initial diameter of the cylinder was assumed to be 200 meters, which also corresponds to the assumed initial width of the slot. The $S_e$ values indicated are values of $(C_0 - C_8)/(C_0 - C_1) = (T_0 - T_8)/(T_0 - T_1)$. The A, B, and C points shown in the upper diagram correspond to the "warm dry," "warm moist," and "cold dry" situations illustrated in Figure 2.
References


