TUNNEL WALL INTERFERENCE FOR BODIES OF REVOLUTION IN NON-STEADY MOTION

William R. Hall

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This research was performed for the Applied Research Laboratory at The Pennsylvania State University under contract with the Naval Ordnance Systems Command.
This thesis considers the problem of determining the effects of tunnel wall interference on the measured values of the stability derivatives for axisymmetric bodies. The tunnel wall interference corrections developed apply equally well to both subsonic wind tunnel or water tunnel testing in tunnels with circular test sections.

A linearized slender body theory is developed which can be used to predict corrections for:

1. the static lift and pitching moment derivatives, $C_{L\alpha}$ and $C_{M\alpha}$
2. the rotary damping derivatives, $C_{Lq\alpha}$ and $C_{Mq\alpha}$
3. the "Add Mass" derivatives, $C_{Lq\alpha}$ and $C_{Mq\alpha}$
4. the "Accession-to-Inertia" derivatives $C_{Lq\alpha}$ and $C_{Mq\alpha}$

Limited evaluation of the developed theory is presented in the form of static pitching moment tests of three ellipsoids-of-revolution. In addition, comparison is made with two alternative theories developed by Peirce (1) and Goodman (2).
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LIST OF SYMBOLS

\( d_M \) model diameter as function of \( x \)
\( d_T \) tunnel test section diameter, possibly a function of \( x \)
\( C_{L_a} \) static lift coefficient curve slope, per degree
\( C_{M_a} \) static pitching moment coefficient curve slope, per degree
\( C_{L_q} \) lift rotary damping stability derivative
\( C_{M_q} \) pitching moment rotary damping stability derivative
\( C_{L_d} \) "Add-Mass" acceleration stability derivative, lift
\( C_{M_d} \) "Add-Mass" acceleration stability derivative, moment
\( C_{L_d} \) "Accession-to-Inertia" acceleration stability derivative, lift
\( C_{M_d} \) "Accession-to-Inertia" acceleration stability derivative, moment
\( U \) local velocity parallel to the body \( x \)-axis, at the body surface
\( U \) free-stream velocity parallel to body \( x \)-axis
\( V \) resultant velocity
\( k \) reduced frequency, \( \frac{\omega}{U} \)
\( \ell \) overall body length
\( f \) fineness ratio, \( \ell/d \)
\( d \) maximum model diameter
\( Re \) Reynolds number, \( \frac{U \ell}{\nu} \)
\( S \) \( \frac{\pi d^2}{4} \), body cross-sectional area, function of \( x \)
LIST OF SYMBOLS

r  radial coordinate, Figures 3 and 4
x  axial coordinate, Figure 3
t  time
b  body center of rotation, Figure 3
q  angular velocity, \( \dot{\theta}_1 \)
h  body plunging velocity, \( q = 0 \)
Z_o  total cross flow velocity in the \( r\theta \) plane, Equation (9)
R  tunnel test section radius, function of \( x \)
a  local model radius, function of \( x \)
\( \Delta P_a \)  local pressure at the body periphery
L  total body lift
M  total body pitching moment
C_p  static wall pressure coefficient, \( C_p = \frac{P - P_0}{1/2\rho U^2} \)
R_o  tunnel test section radius where \( C_p = 0 \)
C_p'  static wall pressure gradient, \( \frac{dC_p}{dx} \)
C_L  lift coefficient, Appendix A
C_M  pitching moment coefficient, Appendix A
V  body volume
S_M  maximum body cross-sectional area, \( S_M = \frac{\pi d^2}{4} \)
K  calibration constants for torque cell, pressure transducers, etc.
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<tr>
<td>$\nu$</td>
<td>kinematic viscosity</td>
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<tr>
<td>$\phi$</td>
<td>velocity potential</td>
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<tr>
<td>$\Theta$</td>
<td>azimuth angle, Figure 3 and 4</td>
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<td>$\Theta_l$</td>
<td>body pitch angle</td>
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<tr>
<td>$\mu$</td>
<td>doublet strength, function of $x$</td>
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<tr>
<td>$\alpha$</td>
<td>body angle-of-attack</td>
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<tr>
<td>$\rho$</td>
<td>density</td>
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<tr>
<td>$\gamma$</td>
<td>prismatic coefficient, $\frac{4,\nu}{\pi,d^2\lambda}$</td>
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<tr>
<td>$\omega$</td>
<td>circular frequency</td>
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**Subscripts**

- **FS**: free-stream or unrestricted medium cases
- **IT**: in-tunnel or in-tube cases
- $(\_)_v$: subscript $v$ indicates differentiation with respect to $v$, where $v = x, r, \theta, \alpha, \dot{\alpha}, q, \dot{q}$

**Superscripts**

- $(\_)'$: differentiation with respect to $x$
- $(\_\_)$: differentiation with respect to time, $t$
- $\overline{\_}$: overbar indicates non-dimensional quantity, see Appendix A
CHAPTER I

INTRODUCTION

Origin and Statement of the Problem

The motion of a body through a fluid gives rise to forces and moments which are functionally dependent on the body velocities, accelerations and control angles. For large changes in the velocities and accelerations, these body forces and moments are related to the body motion in a non-linear manner. Classically, in order to develop mathematical expressions relating these forces to the body motions, a Taylor series expansion in the body velocities, accelerations and control angles is made. Experience has shown, for the motion of slender bodies, a linear Taylor series expansion in these variables is adequate to describe small motions of the body from an equilibrium condition. Thus, a typical aerodynamic force, \( A \), may be written as

\[
A = A_0 + A_a a + A_\dot{a} \dot{a} + \ldots A_q q
\]

where \( A_a = \left. \frac{\partial A}{\partial a} \right|_0 \).

The subscript zero indicates that the expression is to be evaluated at the equilibrium condition. The proportionality constants, such as \( A_a \), are called the stability derivatives. A valid Taylor series expansion of Equation (1) requires the forces and moments to be
proportional to the instantaneous values of the body velocities and accelerations. Thus, the stability derivatives are constants. Etkin\(^{(3)}\) has shown that the derivatives are actually time dependent but converge to a constant value very rapidly. This implies that the stability derivative concept is valid except during the very beginning of the body motion.

The techniques used for experimentally determining the stability derivatives of a hydrodynamic body are quite valid. The techniques include static wind tunnel tests, rotating-arm tests\(^{(4)}\), free flight trajectory studies\(^{(5),(6)}\), and forced-oscillation tests\(^{(7),(8),(9),(10)}\). Tests in this last category can be conducted in the 48.0-inch diameter water tunnel\(^{(11)}\) operated by the Applied Research Laboratory (ARL) of The Pennsylvania State University using a Planar Motion Mechanism\(^{(12)}\). The Planar Motion Mechanism incorporates in one device of the forced-oscillation type the means for experimentally determining all the stability derivatives of a hydrodynamic body. By forcing the body to follow a known sinusoidal path, the velocity and acceleration-dependent stability derivatives may be related to measured dynamic forces and moments. The development of the techniques required for determining stability derivatives with a Planar Motion Mechanism at the Applied Research Laboratory was accomplished through testing of the small basic finner missile model shown in Figure 1. A comparison of stability derivatives obtained with this model and similar data\(^{(13)}\) from other sources is given by Table 1. In these tests, a small basic finner missile with a model-to-tunnel diameter ratio of 0.104 was used to


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<th>$C_{L\alpha}$</th>
<th>$C_{M\alpha}$</th>
<th>$C_{Lq}$</th>
<th>$C_{Mq}$</th>
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<td>a. 4.0 in. dia. model</td>
<td>-0.0395</td>
<td>-0.0195</td>
<td>--------</td>
<td>--------</td>
<td>-0.0059</td>
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<tr>
<td>b. 2.0 in. dia. model</td>
<td>-0.1100</td>
<td>-0.0205</td>
<td>--------</td>
<td>--------</td>
<td>-0.0075</td>
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<td>a. Angular Balance</td>
<td>-0.1107</td>
<td>-0.0251</td>
<td>--------</td>
<td>-0.0409</td>
<td>-0.0138</td>
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<td>b. Translat'1 Balance</td>
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<td>--------</td>
<td>-0.0134</td>
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<td>c. Static Tests</td>
<td>-0.0997</td>
<td>-0.0243</td>
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<td>a. Static Tests</td>
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<td>b. Dynamic Tests</td>
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<td>-0.0108</td>
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<td>-0.0149</td>
<td>-0.0357</td>
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<td>-0.0248</td>
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<td>--------</td>
<td>--------</td>
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<tr>
<td>b. Dynamic Tests</td>
<td>-0.123</td>
<td>-0.0265</td>
<td>-0.0125</td>
<td>-0.0436</td>
<td>-0.0142</td>
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avoid tunnel wall interference. However, for typical models with large model-to-tunnel diameter ratios, a question remains as to the extent of the tunnel wall interference on the various stability derivatives. Until such time as the extent of the tunnel wall interference on the various stability derivatives as a function of model-to-tunnel diameter ratio is resolved, the experimental stability derivative data obtained for these models from tests with the planar motion mechanism are suspect. Obviously, the conditions under which a model is tested in a water or wind tunnel are not the same as those in an unrestricted medium. A longitudinal static pressure gradient is usually present in the test section. In addition to the tunnel walls, model supports, local variations in flow velocity and angularity, and tunnel blockage frequently result in extraneous interference forces for which the experimental data must be corrected. In the past, much work has been done to correct experimental data for flow angularity, local velocity variations, tunnel blockage, and support interference and several references (14) indicating common practices are available for the purpose. In the case of the tunnel walls, the walls constrain the flow around the body resulting in unwanted interference velocities not present in the unrestricted medium. The extent of these unwanted velocities is functionally dependent on the model-to-tunnel diameter ratio and the body geometry. Since these unwanted velocities can produce extraneous forces and moments characterized by errors in the measured stability derivatives, some form of correction must be applied to the experimental data.
This thesis explores both theoretically and experimentally the question of tunnel wall interference. The theoretical phase of this work attempts to develop a relatively simple theory for correcting the experimental stability derivatives of a general axisymmetric body for tunnel wall interference effects based on slender body theory. The general body shape implies that the body radius need not be an exact mathematical function, but rather the body radius as a function of axial position may be specified by a numerical tabulation. The tunnel wall interference corrections are presented in the form of multiplicative factors which may readily be applied to experimental test results. These tunnel wall interference corrections are defined as the ratio of in-tunnel to free stream stability derivatives. Corrections are indicated for:

1) the static lift and pitching moment derivatives, $C_{L_a}$ and $C_{M_a}$;

2) the rotary damping derivatives, $C_{L_q}$ and $C_{M_q}$;

3) the "Add Mass" derivatives, $C_{L_A}$ and $C_{M_A}$; and

4) the "Accession-to Inertia" derivatives, $C_{L_q}$ and $C_{M_q}$.

In the experimental phase, a limited experimental verification of the developed theory is presented in the form of wind tunnel tests of several ellipsoids-of-revolution. The ellipsoids-of-revolution were chosen on the basis of earlier works by Peirce\(^1\) and Goodman\(^2\) which provide some basis for comparison. Also, the ellipsoids-of-revolution permit exact formulation of the various stability
derivative integrands which may be compared with numerically specified
integrands of the kind required if the developed theory to be applied
to a general axisymmetric body.

Previous Investigations

Much of the early work concerning tunnel wall interference for
axisymmetric bodies was stimulated by the airship development efforts
prior to 1936. Prime examples of this work are the papers of Munk\textsuperscript{(15)},
Abbott\textsuperscript{(16)}, Freeman\textsuperscript{(17)} and Upson, et al\textsuperscript{(18)}. Peirce indicates that
most of the previous theoretical work on tunnel wall interference was
done prior to 1933, while interest in the experimental determination
of the aerodynamic characteristics of airships declined about 1936.
This decline was spurred by the airship disasters of the mid-thirties
and the rapid progress in airplane design. Even so, the bulk of this
early work\textsuperscript{(19)} on tunnel interference was concerned with blockage
effects and increases in measured body drag due to the static pressure
gradient in the tunnel created by the growth of the tunnel wall
boundary layer. Also, in the majority of the experimental work on air-
ships, the model-to-tunnel diameter ratio was 0.143 or less and most
of the testing was performed in open-jet tunnels. Peirce indicates that
the magnitude of the drag interference correction for an open-jet tunnel
is approximately 75 percent that of a closed-section tunnel. Therefore,
tunnel wall drag interference was small, and completely ignored by
most investigators\textsuperscript{(16)}. 
The advent of the guided missile, supersonic airplane, nuclear submarine, and high-speed torpedo, which are all essentially axisymmetric bodies incorporating very low aspect ratio wings, has resulted in further interest in the hydrodynamic characteristics of such bodies. Since experimental testing of such bodies is desirable, there has also been a resurgence of interest in the tunnel wall interference problem for them. Further impetus for research in these tunnel interference problems may be found in the development of high speed tube transportation systems, where the aerodynamic forces experienced by the body in the tube may be an order of magnitude greater than predicted from unrestricted medium data\(^{(2)}\).

A literature search\(^{(20)}\) prepared by NASA covering the period 1962 to 1972 has indicated that most research concerning aerodynamic characteristics of an axisymmetric body in a tunnel has been concerned with correcting the body drag for the static pressure gradient of the tunnel. Two notable exceptions are papers by Peirce and Goodman.

Peirce conducted a theoretical and experimental investigation of tunnel interference aimed at extending the capabilities of the ARL 48-inch diameter water tunnel. This tunnel was originally designed to study the problems of wake-operating propellers. Lehman states that for mechanical reasons such propellers must be at least 5 inches in diameter. For model submarine testing, propellers are approximately 7.5 inches in diameter and model body diameters are of the order of 16 inches. In testing such models in this tunnel, it can be expected that the tunnel wall interference, both to drag and to the stability
derivatives, may be significant. Indeed, during the initial testing of submarine models at ARL, it was discovered that existing methods of correcting for tunnel interference were inadequate.

For correcting drag, Peirce examined two approaches to the tunnel wall interference problem. One approach forces the flow around the body to conform to the stream pattern in an unrestricted medium by means of a flow correcting tunnel liner; the second considers further correction factors applied to the tunnel data. Due to the large corrections indicated, the latter alternative was quickly discarded.

Subsequent testing of submarine models has verified that the flow correcting liner is quite useful in providing the correct operating environment for the wake-operating submarine propeller. That is, the liner supplies the correct pressure distribution along the body resulting in the proper boundary layer growth on the body. In addition, Peirce examined the effect of tunnel liner manufacturing deviations on body drag measurements and wall interference on the body pitching moment derivative, $C_{M\alpha}$.

Following the slender body approach of Sprueter (21), Peirce predicts a maximum pitching moment derivative correction of

$$
\left( \frac{C_{M\alpha \text{IT}}}{C_{M\alpha \text{FS}}} \right) = \frac{1 + \left( \frac{d_M}{d_T} \right)_{\text{max}}^2}{\left[ 1 - \left( \frac{d_M}{d_T} \right)_{\text{max}}^2 \right]^2}, \tag{2}
$$
based on the maximum model-to-tunnel diameter ratio, \(d_{MT\text{max}}\).

Subsequent experimental verification of Equation (2) shows agreement to within 3 percent of predicted theoretical values. However, this maximum correction factor ignores body geometric details and further refinement may be useful. In any case, the pitching moment data of this reference appears to be the only experimental tunnel wall correction data available. Independent substantiation of these data constitutes part of the experimental phases of this thesis. Also, Peirce's application of slender body theory is reasonable, and by applying the techniques suggested by Ribner, it may be extended to indicate corrections for all the stability derivatives.

In conjunction with studies of a high speed tube transportation system, Goodman assumes potential flow and formulates the expression for the aerodynamic characteristics of a slender body traveling in a tube. This is accomplished by following the perturbation techniques suggested by Hoffman and Platzer for obtaining the aerodynamic characteristics of a slender body oscillating in supersonic flow. This perturbation technique represents a refinement of classical slender body theory permitting inclusion of transverse body forces other than those produced by the cross-flow velocity. Generation of the tunnel wall interference corrections by Goodman's method requires solution of the Neumann problem for the axisymmetric flow past a body-of-revolution in a tunnel and in an unrestricted medium. Goodman obtains a simple expression for the required axial and radial velocity components on the body surface by assuming a line source of variable strength along the body centerline. In this case, the axial velocity
expression, $U_x/U$, at an axial station, $x$, becomes:

$$\frac{U_x}{U} = \frac{1}{1 - (d_M/d_T)^2}$$

(3)

where $d_M/d_T$ is the local model-to-tunnel diameter ratio at $x$. The radial velocity expression is more complex. Equation (3) is identical to the results obtained from a one-dimensional conservation of mass approach and reduces to the proper slender body assumption, $U_x/U = 1$, for the free stream case. Figure 2 indicates the extent of the error in the axial velocity at the body surface, by comparing Equation (3) to an exact formulation of $U_x/U$ available using Smith for an ellipsoid-of-revolution. Except for regions near the body nose and tail, the use of Equation (3) for axial velocity does not appear unreasonable. Since the local radial velocity component at the body surface is not required except for predicting those transverse forces produced by means other than the cross flow velocities in the classical slender body approach, no attempt has been made to assess the error in local radial velocity between the Goodman and Smith references as was done for the longitudinal velocity in Figure 2. Unfortunately, in attempts to combine the techniques of Goodman and the exact solution of the Neumann problem by Smith, the numerical techniques required quickly become unwieldy. In any case, the tunnel wall interference corrections for ellipsoids-of-revolution predicted by Goodman provide further comparison data.
Figure 2  Errors in the Axial Velocity at the Body Surface Using a One-Dimensional Continuity Equation as Compared to a Numerical Solution of the Neumann Problem Due to Smith for an Ellipsoid
Several other approaches are suggested by the literature. Using potential theory, Newman \(^{(25)}\) calculates the forces and moments on a slender body moving parallel to an infinite straight wall. In this case, assuming the body-wall separation to be small, and small perturbations about a mean body position, leads to the use of slender body theory for the body-wall combination.

Levine \(^{(26)}\) formulates the steady incompressible axisymmetric flow about a body in a tunnel without resorting to the slenderness assumption. However, the body geometry is determined a posteriori, limiting the usefulness of this technique.

**Scope of the Investigation**

In order to limit the scope of this investigation, several restrictions and assumptions will be applied. A slender body is assumed. Following Spieter and Ribner, for simplicity, this assumption restricts the body forces and moments to those predicted by the imposed cross-flow velocities. Only potential flow will be considered. Segel \(^{(27)}\) considers the problem of one circular cylinder oscillating laterally inside a second cylinder; the fluid between being considered viscous. The problem is specialized to the case of large Reynolds numbers and the force on the inner cylinder determined. This force consists of a component in-phase with the motion which is of the add-mass type and an out-of-phase component which is the damping force. That part of Segel's add-mass force per unit length which is independent of Reynolds number could be obtained following the inviscid theory suggested by Spieter and Ribner. However, Segel's damping force per unit length is purely viscous in nature, while that outlined by Spieter and Ribner is created
by the axial variation of cross-sectional area of the body. Goodman indicates that the ratio of Segel's viscous damping force to that predicted by the inviscid theory is of the order:

\[
\frac{(kf^2)}{(Re)}^{1/2}
\]

where \( k \) equals the reduced frequency, \( \frac{\omega k}{U} \); \( Re \) the Reynolds number \( \frac{U^2}{\nu} \); and \( f \) the body fineness ratio. Typical values of these quantities for testing with the planar motion mechanism in the ARL 48-inch diameter water tunnel are: \( k = 1.5 \), \( Re = 2.0 \times 10^7 \) and, \( f = 7.0 \). Therefore, the viscous damping force per unit length is of the order of 0.002 the inviscid damping force per unit length and may safely be ignored.

As a result of these first two assumptions for closed bodies, which have the property \( s(0) = s(h) = 0 \), the tunnel wall interference corrections for \( C_{L_a} \) and \( C_{M_q} \) are lost. However, for bodies which are not closed, corrections for \( C_{L_a} \) and \( C_{M_q} \) may be defined. One approximate method for overcoming this limitation for the closed bodies forwarded by Sprietz is the definition of an equivalent body geometry including the body boundary layer thickness. This technique has been used to successfully predict \( C_{L_a} \) for bodies of revolution. However, the prediction of the boundary layer thickness of an axisymmetric body in an unrestricted medium or in a tunnel is beyond the scope of this investigation.
For the present investigation, the body is considered axisymmetric at all axial stations. No wings or other appendages are considered. Modification of the appropriate cross flow velocity potential to cover a slender wing-body combination in an unrestricted medium has been given by Spriter, but the cross-flow velocity potential for a slender wing-body combination in a tunnel is unknown.

The present theory is further limited to the prediction of the various stability derivatives, and thus the tunnel wall interference corrections, at the mean body position, where the model and tunnel centerlines coincide.
CHAPTER II

THEORETICAL CONSIDERATIONS

Introduction to Slender Body Theory

The approximate theory for the flow past a slender body-of-revolution, elongated in the direction of flight, which is generally known as "Slender Body Theory" was introduced by Munk for the calculation of the aerodynamic forces on airships. Jones (28) extended this theory to the study of low-aspect-ratio pointed wings and Ribner applied it to determine the stability derivatives of low-aspect-ratio triangular wings. Sprueter further extended the theory to the study of slender wing-body combinations. Finally, Adams and Sears (29) have reviewed slender body theory, its limitations and extension extensively. In all these studies, the essential point in the study of bodies or wings by "Slender Body Theory" is the fact that, near the body, the flow is approximately two-dimensional when viewed in planes perpendicular to the direction of motion. This follows directly from the nature of the slender body.

Laplace's Equation in cylindrical coordinates for an incompressible flow is well known and may be written:

\[ \phi_{xx} + \phi_{rr} + \frac{1}{r} \phi_r + \frac{1}{r^2} \phi_{\theta\theta} = 0, \]

(5)

where \( \phi \) the velocity potential is a function of the independent variables \( x, r, \) and \( \theta \). A coordinate system is presented in
Figure 3. When written in the form of Equation (5), the subscripts \( x, r, \theta \) indicate partial differentiation with respect to that variable. The solution of Equation (5), given the boundary condition of no normal velocity at the body surface, is the Neumann problem. For a general axisymmetric body, the accurate solution of the Neumann problem requires the integral equation formulation and numerical integration techniques of Smith. In avoiding the complexity of Smith’s technique "Slender Body Theory" in its classical presentation (1), begins by neglecting the term \( \phi_{xx} \) in Equation (5), leaving an approximate equation in the \( r\theta \) or cross flow plane of Figure 3 of

\[
\phi_{rr} + \frac{1}{r} \phi_r + \frac{1}{r^2} \phi_{\theta\theta} = 0 \quad .
\]

Equation (6), then, is identically the continuity equation in the cross flow variables \( r \) and \( \theta \). Justification for this simplification is the slender elongated form of the body. Intuitively, since the cross-sectional area of the body may vary only slowly in the \( x \)-direction, it is clear that, \( \phi_{xx} \), which is the rate of change of that perturbation velocity \( \phi_x \) in the same direction must also be small, at least near the body surface. Near the nose and tail of the body, the accuracy of this assumption is questionable, since the local longitudinal curvature of the body, and hence \( \phi_{xx} \), may be rather large. These features are illustrated by Figure 2. Indeed, the inaccuracy of this assumption near the body nose and tail is one reason for predicting tunnel wall interference corrections rather than predicting absolute values for the stability derivatives from slender body theory.
Figure 3 The Problem Coordinate System
The argument for proceeding from Equation (5) to Equation (6) can be made more precisely by considering a non-dimensional form of Equation (5). Let

\[
\tilde{x} = x/l, \quad \tilde{r} = r/d, \quad \text{and} \quad \tilde{\phi} = \phi/Uk,
\]

where the overbar indicates a non-dimensional parameter following Appendix A. Upon substituting Equation (7) into Equation (5), one obtains

\[
\left(\frac{d}{\tilde{r}}\right)^2 \frac{\tilde{\phi}_{xx}}{\tilde{r}} + \frac{\tilde{\phi}_{rr}}{\tilde{r}} + \frac{1}{\tilde{r}} \frac{\tilde{\phi}}{\tilde{r}} + \frac{1}{\tilde{r}^2} \frac{\tilde{\phi}_{\theta\theta}}{\tilde{r}} = 0.
\]

It is clear that for sufficiently large values of the fineness ratio, \(l/d\), the leading term of Equation (8) may be neglected when compared to the remaining terms. In this case, Equation (8) becomes a non-dimensional form of Equation (6).

In any application of slender body theory, the definition of a velocity potential, \(\phi\), satisfying Equation (6) and of the appropriate boundary conditions in the \(r\theta\) plane at any axial station \(x\) are required. From this velocity potential, \(\phi\), the pressures on the body periphery at \(x\), and hence the local incremental body forces are available. Axial variations in \(\phi\) result from a variable body cross-sectional area distribution along the body \(x\) axis. In Figure 3, this axial variation of \(\phi\) is equivalent to the body moving through a stationary \(r\theta\) plane with a velocity \(U\), or conversely moving the \(r\theta\)
plane past the body at a velocity $U$. This interpretation has the interesting property of turning a steady three-dimensional problem into an unsteady two-dimensional one. Determination of the total body lift and pitching moment follows from the integration of the local incremental body lift over the body length. Since the local body lift increment follows from Equation (6), no allowance can be made for lift and pitching moment contributions to the total for flows other than the cross flows without resorting to the perturbation techniques of Goodman.

The use of slender body theory to predict the tunnel wall interference corrections reduces the problem to defining the appropriate potential form satisfying Equation (6) subject to boundary conditions reflecting the total body motions.

Development of the Body Lift and Pitching Moments from Slender Body Theory

In the $r\theta$ plane of Figure 3, the total cross-flow velocity, $Z_o$, is governed by the total body motion assumed. Generally, this cross-flow velocity, $Z_o$, at $x$ is created by a body plunging motion, $h$, perpendicular to the $x$-axis, and a simultaneous rotation $\phi_1$ at an angular rate $\dot{\phi}_1$ or $q$, about a point $b$ on the body $x$-axis, as shown. Thus, at any axial station $x$, a first-order approximation of the total cross-flow velocity, $Z_o$, in the $r\theta$ plane is

$$Z_o(x,t) = h(t) + q(t)(x - b). \tag{9}$$

In Figure 3, $U$ is the constant free stream or unblocked tunnel velocity. A small angle assumption is introduced by restricting the magnitude of $Z_o$ in Equation (9) to values much less than $U$. It is convenient
to decompose the local body motion of Equation (9) into a plunging motion where \( q = \dot{q} = 0 \) and a pitching motion where \( \dot{h} = \ddot{h} = 0 \).

For the plunging motion Equation (9) reduces to

\[
\dot{Z}_0(x,t) = \dot{h}(t), \quad q(t) = 0.
\]  

(10)

Equation (10) may be related to the body angle—of—attack, \( \alpha \), since as a result of the small angle assumption and by definition:

\[
V = U = \text{constant}
\]

and

\[
\dot{Z}_0(x,t) = \dot{h}(t) = U \alpha_0(t)
\]  

(11)

Similarly for the pitching motion Equation (9) reduces to

\[
\dot{Z}_0(x,t) = q(t) (x - b), \quad \dot{h}(t) = 0.
\]  

(12)

Again, the small angle assumption, of \( Z_0(x,t) \ll U \) limits the validity of Equation (12) to small angular displacements about the mean body position. The mean body position being defined as the model and tunnel centerlines coincident.

Another result of the small angle assumption is that in the \( r^9 \) plane of Figure 3, the body cross—sections are approximately cylindrical. Looking downstream with \( x \)-axis and free stream velocity of Figure 3 into the page at the instant the model and tunnel centerlines coincide, the \( r^9 \) plane coordinate system and problem geometry is shown in
Figure 4. It should be noted that the cross sections are only approximately cylindrical, and the use of a velocity potential based on that cylindrical cross-section only applies strictly when the model and tunnel centerlines do coincide. For example, in the unrestricted medium of Figure 4a, rotation about b as defined by Equation (12) results in the body cross sections becoming slightly elliptical in the r0 plane. For the model in the tunnel, Figure 4b, the body rotation further complicates the picture; the body cross-section being eccentric in the tunnel. That is, even if the body cross-section were cylindrical as in Figure 4b, it would still be displaced from the tunnel centerline as a function of x and t. In addition, the body plunging motion as defined by Equation (10) displaces the model and tunnel centerlines as a function of t alone. Peirce considers a similar problem in letting the resultant velocity V of Figure 3 be the free-stream or unblocked tunnel velocity and considering the body to be a constant angle-of-attack. Since the r0 plane is now perpendicular to V, the body cross-sections are slightly elliptical and displaced from the tunnel centerline. By limiting the theory to small angle of attacks, Peirce also neglects the eccentricity of the body cross-sections and displacement from the tunnel centerline.

A simple potential form satisfying Equation (6) and the geometry of Figure 4 is the doublet. Then for the absolute flow about a cylinder moving through the unrestricted medium of Figure 4a, the velocity potential is given by
Figure 4 The Problem Coordinate System in the $r\theta$ Plane Looking Downstream with the x Axis into the Page
\[ \phi_{FS} = \frac{\nu(x)}{r} \cos \theta . \]  (13)

Similarly, for the absolute flow about a cylinder moving inside a second cylinder as shown in Figure 4b, one obtains the velocity potential

\[ \phi_{IT} = \mu(x) \left( \frac{1}{r} + \frac{r}{R^2} \right) \cos \theta , \]  (14)

where \( \mu(x) \) is the doublet strength defined by the appropriate boundary conditions on the body surface and tunnel walls.

In the \( r\theta \) plane of Figure 4, there can be no flow through the cylindrical surface element defined by \( a(x) \). Mathematically, the boundary condition on the body surface, \( a(x) \) is written

\[ \phi_r = \frac{\partial \phi}{\partial r} = z_o(x,t) \cos \theta \text{ at } r = a(x), \]  (15)

where \( \phi_r \) equals the radial velocity component and \( z_o(x,t) \) is defined by either Equation (10) or Equation (12). In addition, for the cylinder moving in the unrestricted medium, the fluid at infinity must be at rest, or

\[ \phi_r = 0 \text{ as } r \to \infty . \]  (16)

For the model in the tunnel, there can be no flow through the tunnel walls defined by \( R(x) \). That is,

\[ \phi_r = 0 \text{ at } r = R(x), \]  (17)
for the body in the tunnel. Performing the indicated operations with Equations (13) and (14) and substituting the appropriate boundary conditions from Equations (15), (16), or (17), one gets for the required velocity potential forms:

\[
\Phi_{FS} = -\frac{Z_0 a^2(x)}{r} \cos \theta ,
\]

(18)

for the body in free stream, and

\[
\Phi_{IT} = -\frac{Z_0 a^2(x)}{1-(a/R)^2} \left\{ \frac{1}{r} + \frac{r^2}{R^2} \right\} \cos \theta ,
\]

(19)

for the model in the tunnel, respectively.

Once the velocity potentials are known, the incremental body lift in the \( r\theta \) plane, \( \frac{dL}{dx} \), can be obtained by integrating the pressure around the body periphery \( r = a(x) \). Then,

\[
\frac{dL}{dx} = \int_{0}^{2\pi} \Delta P_a \ a \cos \theta \ d\theta ,
\]

(20)

where \( \Delta P_a \) is the local pressure on the body surface as a function of azimuth angle, \( \theta \). Appendix B develops a general Bernoulli Equation relating pressure and fluid velocities in a coordinate system moving with the body permitting use of the velocity potentials of Equations (18) and (19). From this general Bernoulli Equation, Equation (B-7), the incremental pressure at a point in the \( r\theta \) plane is
Performing the indicated operations of Equations (B-7) and evaluating at the body periphery, \( r = a \), the local pressure on the body periphery as a function of azimuth angle, \( \theta \), becomes

\[
\frac{\Delta P}{\rho} = -\frac{\partial \phi}{\partial t} - \frac{1}{2} \left( \phi_r^2 + \frac{1}{r} \phi_\theta^2 \right) + \frac{1}{2} Z_0 \left( \phi_r \cos \theta - \frac{1}{r} \phi_\theta \sin \theta \right).
\]  

(B-7)

for the model in the unrestricted medium and

\[
\Delta P_a = -\rho \frac{\partial \phi_{FS}}{\partial t} + \frac{1}{2} Z_0^2 \left( 1 - 4 \sin^2 \theta \right), \text{ at } r = a \quad (21)
\]

for the model in the tunnel.

In Equations (21) and (22), the bracketed term of the right-hand side is the steady state pressure distribution at the body periphery shown in Figure 5a. Upon integration of these bracketed terms as indicated by Equation (20), the resultant pressure disappears, since no pressure forces can occur in potential flow. Indeed, this is a shortcoming of the inviscid slender body theory. Allen and Perkins\(^{(30)}\) attempt to ameliorate this difficulty by considering viscous flow in the \( r\theta \) plane. The approximate method consists of the addition of a cross-flow drag term to the potential flow incremental body lift. This is based on the assumption that each circular element along the body will experience an additional cross force equal to the drag force of a section operating at a velocity \( Z_0 \). In practice, this approximate
a) STEADY STATE PRESSURE DISTRIBUTION OVER THE SURFACE OF A CYLINDER

\[ \frac{\Delta P}{1/2 \rho v^2} \]

\[ a(x) \]

b) ACCELERATION PRESSURE DISTRIBUTION OVER THE SURFACE OF A CYLINDER

\[ \Delta P = -\rho \frac{\partial \phi}{\partial t} \]

\[ a(x) \]

Figure 5 The Pressure Distribution Over the Surface of a Cylinder in Two-Dimensional Potential Flow
method does give good agreement between measured and calculated lifts. However, when considering potential flows, it is necessary to consider only the acceleration pressures of the form

\[ \Delta P_a = -\rho \frac{\partial \phi}{\partial t} \]  \hspace{1cm} (23)

shown in Figure 5b. Thus \( \Delta P_a \), and hence, the incremental body lift \( \frac{dL}{dx} \) can only be generated by changes in body and/or tunnel cross-sections and accelerations in the \( r\theta \) plane. By chain rule differentiation the local acceleration in the \( r\theta \) plane is

\[ \frac{\partial \phi}{\partial t} = U_x \left( a' \frac{\partial \phi}{\partial a} + R' \frac{\partial \phi}{\partial R} \right) + Z_o \frac{\partial \phi}{\partial Z_o}, \]  \hspace{1cm} (24)

where \( U_x \) is the local axial velocity, \( a' \) the local body slope and \( R' \) the local tunnel wall slope, are all functions of \( x \), and where

\[ Z_o = \frac{\partial Z}{\partial t}. \]  

For a model in a tunnel, the application of Equation (24) to Equation (19), the evaluation at the body periphery, and the integration according to Equation (20) yields the incremental body lift.

\[ \frac{dL}{dx} = \rho U_x Z_o \frac{1 + (a/R)^2}{[1 - (a/R)^2]^2} S' - 4 a \frac{(a/R)^3}{[1 - (a/R)^2]^2} R' \]

\[ + \rho Z_o \left[ \frac{1 + (a/R)^2}{1 - (a/R)^2} \right] S, \]  \hspace{1cm} (25)
where \( S = \pi a^2 \) is the local body cross-section in the \( r \theta \) plane, and \( S' \) the local change of that cross section. Both of these quantities are functions of \( x \).

The unrestricted medium incremental body length can be obtained from the in-tunnel expression, Equation (25) by letting the tunnel radius approach infinity. Then,

\[
\frac{dL}{dx}_{FS} = \lim_{R \to \infty} \left( \frac{dL}{dx} \right)_{IT} = \rho U_x \int S' + \rho \int S.
\]

Alternatively, \( \frac{dL}{dx}_{FS} \) may be obtained by direct application of Equation (24) to Equation (18), evaluation at the body periphery, and integration according to Equation (20) as was done for the model in the tunnel.

Inclusion of a term proportional to \( R' \), the local tunnel radius slope, in Equation (25) for completeness permits estimation of the interference on a body in an axially varying tunnel section such as a nozzle or diffuser. In practice, since most closed-jet tunnels have constant radius test sections, this term is zero for an inviscid fluid solution. However, the term does indicate one approximate method for correcting the lift and pitching moment for the static pressure gradient for the test section.

The static pressure gradient along the tunnel test section is typically of the form presented in Figure 6. Using Bernoulli's Equation and the one-dimensional continuity equation for the unblocked
Figure 6  Wall Pressures in the Working Section of the ARL 48.0-inch-Diameter Water Tunnel
tunnel, this negative static pressure gradient can be considered as an effective decrease in the cross-sectional area due to the growth of the boundary layer on the test section walls. Then, an equivalent body radius \( R \) may be defined:

\[
\frac{R}{R_o} = \frac{1}{(1 - C_p')^{1/2}},
\]

(27)

where \( R_o \) is the tunnel radius at the point the reference static pressure was measured, usually the nozzle throat. \( R' \) follows directly from Equation (27)

\[
\frac{d}{dx} \left[ \frac{R}{R_o} \right] = \frac{C_p'}{(1 - C_p')^{5/4}},
\]

(28)

where \( C_p' \) is the local static pressure gradient along the tunnel working section. For the ARL 48-inch diameter water tunnel, \( C_p' \) equals \(-0.003\) per foot. Using this \( C_p' \) as typical, and a 7:1 fineness ratio ellipsoid of model-to-tunnel diameter ratio 0.25 for the calculations, the contribution of the \( R' \) term in Equation (25) to the total is negligible. Thus, even for a real tunnel Equation (25) may be further simplified to

\[
\left( \frac{dL}{dx} \right)_T = \rho U^2 \frac{Z_0}{1 - (a/R)^2} \left[ \frac{1 + (a/R)^2}{1 - (a/R)^2} \right] \left( S' + \rho Z_0 \left[ \frac{1 + (a/R)^2}{1 - (a/R)^2} \right] S \right),
\]

\( (29) \)

without a loss in accuracy.
From the free-body diagram, Figure 7, the total body lift, \( L \), may be obtained by integrating the appropriate incremental body lifts over the body length \( \ell \). That is,

\[
L = \int_{x=0}^{x=\ell} \frac{dL}{dx} \, dx ,
\]

(30)

where \( \frac{dL}{dx} \) is specified by either Equation (26) for the model in the unrestricted medium, or by Equation (29) for the model in the tunnel. Also, the pitching moment, \( M \), about the rotation center, \( b \), may be written

\[
M = - \int_{x=0}^{x=\ell} (x - b) \frac{dL}{dx} \, dx .
\]

(31)

The Stability Derivatives

For a body undergoing a plunging motion, the substitution of Equation (26) along with Equation (11) in Equations (30) and (31) yields the total lifts and pitching moments for the body in an unrestricted medium. Then, by non-dimensionalizing following Appendix A, but dropping the overbars for notational simplicity, and expanding the resultant lift and pitching moment coefficients in a linear Taylor series, the stability derivatives for plunging motion in an unrestricted medium are

\[
C_{L_{\alpha}} = \frac{2}{S} \frac{\alpha^2}{M} \int_{x=0}^{x=1} \frac{U}{U} \left( \frac{U}{U} \right)_{PS} \, dS ,
\]

(32)
Figure 7 The Coordinate System Perpendicular to the rθ Plane When the Model and Tunnel Centerlines Coincide
\[ C_{M_{\alpha}}_{FS} = -2 \frac{k^2}{S_M} \int_{x=1}^{x=0} \left[ \frac{U_x}{U} \right]_{FS} (x-b) \, dS, \quad (33) \]

\[ C_{L_{\alpha}}_{FS} = 2 \frac{k^2}{S_M} \int_{x=1}^{x=0} dV, \quad (34) \]

and

\[ C_{M_{\alpha}}_{FS} = -2 \frac{k^2}{S_M} \int_{x=1}^{x=0} (x-b) \, dV, \quad (35) \]

where \( dV \) is the volume increment, \( Sdx \).

Note that in contrast to slender wing theory which is applied only to wings of increasing span, Equations (32) and (33) may be applied to bodies of revolution in regions of either increasing or decreasing radius, since the Kutta condition does not apply to bodies-of-revolution. Thus, in general, the lift and pitching moment for a body-of-revolution is different from that of a wing of equal planform.

Repeating the above procedure using Equation (29), provides one with stability derivatives from the plunging motion for the model in the tunnel. Thus,

\[ C_{L_{\alpha}}_{IT} = 2 \frac{k^2}{S_M} \left[ \frac{1}{1 - \left( \frac{a}{R} \right)^2} \right]_{IT} \left[ \frac{U_x}{U} \right]_{IT} \, dS, \quad (36) \]
\[
C_{M_{IT}} = -2 \frac{k^2}{S_M} \int_{x=0}^{x=1} \left[ \frac{1}{1 - (a/R)^2} \right] \left[ \frac{U_x}{U} \right]_{IT} (x-b) dS, \tag{37}
\]

\[
C_{L_{IT}} = 2 \frac{k^2}{S_M} \int_{x=0}^{x=1} \left[ \frac{1}{1 - (a/R)^2} \right] dV, \tag{38}
\]

and

\[
C_{M_{IT}} = -2 \frac{k^2}{S_M} \int_{x=0}^{x=1} \left[ \frac{1}{1 - (a/R)^2} \right] (x-b) dV. \tag{39}
\]

The free stream stability derivatives may also be obtained by letting the tunnel radius, \( R \), approach infinity in Equations (26) to (39).

For pure pitching motion, the substitution of Equation (26) along with Equation (12) in Equations (30) and (31) provides the appropriate lifts and pitching moments. Then, following the non-dimensionalization and expansion procedures outlined above, the stability derivatives in the unrestricted medium from the pitching motion become

\[
C_{L_{FS}} = 2 \frac{k^2}{S_M} \int_{x=0}^{x=1} \left[ \frac{U_x}{U} \right]_{FS} (x-b) dS, \tag{40}
\]
\[
C_{Mq_{FS}} = -2 \frac{k^2}{sM} \int_{x=0}^{x=1} \frac{(x-b) \, dS}{(x-b)^2} , \quad (41)
\]
\[
C_{Lq_{FS}} = 2 \frac{k^2}{sM} \int_{x=0}^{x=1} (x-b) \, dV \quad , \quad (42)
\]
and
\[
C_{Mq_{FS}} = -2 \frac{k^2}{sM} \int_{x=0}^{x=1} (x-b) \, dV . \quad (43)
\]

In the tunnel, the stability derivatives from Equation (29) for the pitching motion of the body are

\[
C_{Lq} = 2 \frac{k^2}{sM} \int_{x=0}^{x=1} \frac{1 + (a/R)^2}{[1 - (a/R)^2]^2} \left[ \frac{U_x}{U} \right]_{IT} (x-b) \, dS , \quad (44)
\]
\[
C_{Mq} = -2 \frac{k^2}{sM} \int_{x=0}^{x=1} \frac{1 + (a/R)^2}{[1 - (a/R)^2]^2} \left[ \frac{U_x}{U} \right]_{IT} (x-b)^2 \, dS , \quad (45)
\]
\[
C_{Lq} = 2 \frac{k^2}{sM} \int_{x=0}^{x=1} \frac{1 + (a/R)^2}{1 - (a/R)^2} (x-b) \, dV , \quad (46)
\]
and
\[
C_{Mq} = -2 \frac{k^2}{sM} \int_{x=0}^{x=1} \frac{1 + (a/R)^2}{1 - (a/R)^2} (x-b)^2 \, dV . \quad (47)
\]
However, because of the inaccuracies inherent in the slender body assumptions, it is reasonable to predict tunnel interference corrections rather than to attempt to predict absolute values of the stability derivatives from Equations (32) to (47). It is expected that experimentally-determined derivatives would prove to be more accurate.

For example, $C_{L_a}$ is known to be seriously in error when compared with experimentally determined values of $C_{L_a}$ for closed bodies-of-revolution. To ameliorate this difficulty, numerous techniques have been suggested including:

1. The inclusion of the cross-flow drag coefficient in the definition of the incremental body lift, by Allen and Perkins,
2. The inclusion of the body boundary thickness in the definition of the body geometry by Sprieter, and
3. The resorting to strictly empirical prediction techniques by Abkowitz (31).

These techniques have met with varying degrees of success.

Similarly, for $C_{M_a}$, the development following Sprieter differs from the more exact formulation of Munk by a factor $(K_2 - K_1)$ where $K_2$ and $K_1$ are the so-called transverse and longitudinal "add mass" coefficients, respectively. In addition, Munk's formulation for $C_{M_a}$ is known to over-estimate $C_{M_a}$ by about 13 percent due to real fluid effects.
CHAPTER III

THE TUNNEL WALL INTERFERENCE CORRECTIONS

By definition, the tunnel wall interference correction factors are the ratio of the stability derivatives for the model in the tunnel to those for the model in an unrestricted medium. Then, for the plunging motion Equation (11), these corrections become

\[
\frac{C_L^{\alpha_{IT}}}{C_L^{\alpha_{FS}}} = \frac{\text{Equation (36)}}{\text{Equation (32)}},
\]

\[
\frac{C_M^{\alpha_{IT}}}{C_M^{\alpha_{FS}}} = \frac{\text{Equation (37)}}{\text{Equation (33)}},
\]

\[
\frac{C_L^{\delta_{IT}}}{C_L^{\delta_{FS}}} = \frac{\text{Equation (38)}}{\text{Equation (34)}},
\]

and

\[
\frac{C_M^{\delta_{IT}}}{C_M^{\delta_{FS}}} = \frac{\text{Equation (39)}}{\text{Equation (35)}},
\]
Similarly for the pitching motion, Equation (12), the following corrections apply:

\[
\frac{C_L q_{IT}}{C_L q_{FS}} = \frac{\text{Equation (44)}}{\text{Equation (40)}},
\]

\[
\frac{C_M q_{IT}}{C_M q_{FS}} = \frac{\text{Equation (45)}}{\text{Equation (41)}},
\]

\[
\frac{C_L \dot{q}_{IT}}{C_L \dot{q}_{FS}} = \frac{\text{Equation (46)}}{\text{Equation (42)}},
\]

and

\[
\frac{C_M \dot{q}_{IT}}{C_M \dot{q}_{FS}} = \frac{\text{Equation (47)}}{\text{Equation (43)}}.
\]
CHAPTER IV

APPLICATION OF THE DEVELOPED THEORY

The definition of the general axisymmetric body implies that the body radius as a function of the axial station \( x \) along the body cannot easily be defined by a simple mathematical function, but rather must be specified numerically. Generally, models for which stability derivatives are desired will fall into this category. In addition, it is possible to define three separate axial velocity distributions at the body surface, \( U_x / U \), for use in the \( C_{L_a} \) and \( C_{M_a} \) equations. First, one may assume \( U_x / U = 1 \). This assumption corresponds to that of classical slender body theory where the longitudinal velocity everywhere on the body surface is equal to the free stream velocity. This implies small body thicknesses and curvatures compared to overall body length. Alternatively, one may choose to use Equation (3)

\[
\frac{U_x}{U} = \frac{1}{1 - \left(\frac{d_x}{T}\right)^2},
\]

for the model in the tunnel. Figure 2 shows that this is a somewhat better assumption for the axial velocity distribution for a body in a tunnel than \( U_x / U = 1 \). Finally, one may consider \( U_x / U \) defined from the numerical solution of the Neumann problem by Smith.
The effect of these three different axial velocity functions on the $C_M$ integrand for an ellipsoid may be seen in Figure 8. While an exact evaluation of the integrals may be available for simple body shapes and the slender body theory axial velocity assumption, $U_x/U_1 = 1$, the integrations in Equations (32) to (47) involving more complex body shapes and axial velocity assumptions would be tedious and time-consuming, at best. The modern high-speed digital computer offers a more rewarding approach encompassing all the possible axial velocity assumptions and body shapes, by means of numerical integration.

The body shape and the axial velocity $U_x/U_1$; and hence, the integrands of Equations (32) to (47) may be specified as a function of $x$ using the mathematical equivalent of the draftman's spline curve. This mathematical spline curve accurately defines the curve and its first and second derivatives, from a set of points. The only constraint on the use of the mathematical spline curve being the points to be fitted must be single-valued in the independent variable.

Also, numerous integration techniques, e.g., Runge-Kutta, Simpson's Rule, Gauss-Legendre, etc., are available. The technique chosen for the calculations of this thesis, primarily for its speed and accuracy, was the Gauss-Legendre Quadrature.

Then, the procedure used for the evaluation of the integrals, Equations (32) to (47) regardless of the body shape or axial velocity assumption used was as follows:
Figure 8 Pitching Moment Coefficient Integrands as a Function of Body Axial Coordinate x
(1) The body shape and axial velocity as a function of $x$ were specified numerically and fitted using the mathematical spline curve techniques.

(2) From this spline-fitted data, the integrands required in Equations (32) to (47) were defined as a function of $x$ and similarly fitted using the mathematical spline curve techniques.

(3) Evaluation of the integrals was accomplished using the Gauss-Legendre Quadrature.

The accuracy of this procedure was checked by computing the volume of an ellipsoid and comparing it with known exact values for the ellipsoid volume. This comparison produced errors of less than 0.03 percent.

In addition, since tunnel interference corrections were to be defined for ellipsoids for comparison with experimental results later, and for which the required integrands could be defined exactly as a function of $x$, a comparison was made with the integrands obtained following the above procedure. There were no observable differences, and the technique may be safely applied to any axisymmetric body-of-revolution.

The majority of the theoretical work was specialized to the ellipsoids-of-revolution for two reasons. First, the integrands of Equations (32) to (41) could be formulated exactly from the equation describing the ellipsoid.

$$ S = 4s_{MN}x(1-x), \quad 0 \leq x \leq 1,$$  \hspace{1cm} (50)
where \( x \), the axial coordinate, is measured from the nose. In this case, a step-by-step evaluation of the numerical techniques outlined is possible. Secondly, a significant, though limited, body of data \((1),(2)\) both theoretical and experimental are available for comparison with the present simple theory. Such data are not available for any other body shape.

The listed assumption, presented in the scope of the investigation, and the choice of the ellipsoidal body shape imply several things. Since the ellipsoid is a closed body, \( C_L \) is lost. Similarly, in the choice of the body midpoint as the rotation center, fore and aft body symmetry produces the loss of corrections for \( C_M \), \( C_L \), and \( C_M^a \). In the case of a body with no fore and aft symmetry, tunnel interference corrections for these derivatives could be defined. Of course, the selection of a different rotation center for the ellipsoids would permit the definition of tunnel interference corrections for \( C_M^q \), \( C_M^a \) and \( C_L^q \); but care must be exercised in interpreting the resultant corrections. Also, from comparison of Equation (37) and (40), \( C_L^q = -C_M^a \); thus, for these stability derivatives the tunnel interference corrections are identical.

The theoretical tunnel wall interference corrections developed for the ellipsoids-of-revolution from the present theory are presented in Figures 9 and 10 as a function of the maximum model to tunnel diameter ratio. For the corrections involving the axial velocity at the body surface, \( C_M^a \) and \( C_L^q \), Figure 9 shows the effect of each of the three possible axial velocity assumptions on the correction.
Figure 9 Tunnel Wall Interference Corrections for the Static Pitching Moment Coefficient Per Degree as a Function of the Model-to-Tunnel Diameter Ratio
Figure 10  Tunnel Wall Interference Corrections for the Acceleration Stability Derivative as a Function of the Model-to-Tunnel Diameter Ratio
Also, Figure 9 includes the corrections from Goodman for an ellipsoid and the maximum tunnel interference correction for $C_{N}$ of Peirce, Equation (2).

Similarly, Figure 10 presents the theoretical corrections for the acceleration stability derivatives, $C_{L_{\alpha}}$ and $C_{M_{\alpha}}$. Note that in Figure 10, the corrections from Goodman and the present theory are identical. Also, Figure 10 includes a maximum tunnel wall interference correction for $C_{L_{\alpha}}$ or $C_{M_{q}}$. Following Peirce and assuming the maximum correction factor occurs at the maximum local model/tunnel diameter ratio, the maximum $C_{L_{\alpha}}$ or $C_{M_{q}}$ interference correction is defined as

$$1 + \left(\frac{d_{m}}{d_{T}}\right)^{2}_{\text{max}}$$

$$1 - \left(\frac{d_{m}}{d_{T}}\right)^{2}_{\text{max}}$$

Equation (51)

This correction, corresponds to an infinite cylinder oscillating inside a second cylinder.

These maximum corrections, Equation (2) for $C_{L_{\alpha}}$, $C_{M_{\alpha}}$, $C_{L_{q}}$, and $C_{M_{q}}$, and Equation (51) for $C_{L_{\alpha}}$, $C_{M_{\alpha}}$, $C_{L_{q}}$, and $C_{M_{q}}$, may be considered as applicable to a body with a prismatic coefficient, $a$, of 1.0. The ellipsoid which was used to evaluate the various theories have a prismatic coefficient of 0.667. This prismatic coefficient, 0.667, is close to the minimum for typical hydrodynamic bodies-of-revolution. It may be that the present theory and that of Goodman represent minimum tunnel wall interference corrections.
However, some care must be exercised in making this statement, since the prismatic coefficient does not specify the distribution of cross-sectional area, $S$, along the body $x$-axis. Indeed, the ellipsoidal distribution, Equation (50), is but one of many possible distributions with a prismatic coefficient of 0.667. Goodman indicates that correction calculations made for a circular cylinder with ogival ends produces only minor deviations from the tunnel interference correction for the ellipsoids, but no further specification of this body is given. In any case, for the purposes of this thesis, no attempt has been made to assess the effect of cross-sectional area distribution at a constant prismatic coefficient on the tunnel interference corrections.
CHAPTER V

EXPERIMENTAL STUDIES

The Experimental Apparatus

In order to provide a comprehensive evaluation of the developed slender body theory, at least three geometrically similar models of varying model-to-tunnel diameter ratios would have to be fabricated for testing with the ARL Planar Motion Mechanism in the 48.0-inch-diameter water tunnel. Since model fabrication for tests with the Planar Motion Mechanism and the testing itself are quite expensive and time consuming, it was decided that at least a partial confirmation could be obtained from wind tunnel tests. Although no evaluation of the dynamic stability derivatives, $C_{L_{q}}$, $C_{M_{q}}$, $C_{L_{q}}$, and $C_{M_{q}}$, could be obtained, tests yielding model pitching moments as a function of angle of attack for three ellipsoids-of-revolution would provide experimental tunnel interference corrections for comparison with the various theories of Figure 9.

All the experimental work was performed in the subsonic wind tunnel operated by the Department of Aerospace Engineering, The Pennsylvania State University.

This tunnel is a closed-return type tunnel which may be operated in either a closed-throat or an open-jet test section configuration. In the closed-throat mode, the rectangular test section measures 4 ft. high, 5 ft. wide and 6 ft. long. Maximum closed-throat test section
velocities of the order of 180 ft./sec. may be obtained, but safe practical speeds are somewhat lower. Associated with this tunnel is a permanent six-component pyramidal balance system manufactured by Aerolab., Incorporated, of Gaithersburg, Maryland. Unfortunately, for the present experiments, the very low moments of less than 2 ft. lbs. maximum generated by the ellipsoids precluded the use of this balance except for the largest of the ellipsoids. Since it was desirable to make all moment measurements with a common measurement system, the permanent wind tunnel balance was not used except as a device for setting the body angles of attack.

The experimental phase of this thesis was divided into two major sections. First, the ellipsoid pitching moment coefficients as a function of angle-of-attack were obtained in an essentially unrestricted medium; i.e., the model's maximum cross-section was sufficiently small when compared to that of the tunnel to justify neglecting any residual tunnel interference. Secondly, similar pitching moment data were obtained for the ellipsoids in a 12.0-inch diameter tube inserted in the tunnel as shown in Figure 11, where the tunnel interference was predicted to be significant. From these tests, the ratio of the pitching moment coefficient slope for the ellipsoids in the tube and unrestricted medium was the experimental tunnel wall interference correction for comparison with the theories of Figure 9.

To model a tunnel test section, the 12.0-inch diameter tube 5.0 ft. long was rolled from 0.0625 in. thick aluminum sheet. Stiffening rings were provided for fastening the tube in the tunnel test section as shown in Figure 11. In addition, a crude inlet fairing was installed
in an attempt to prevent any local flow separation occurring as a result of tube misalignment in the tunnel. To insure that the tube was a fair representation of a tunnel test section, the velocity profile near the tube inlet, Figure 12, and the wall static pressure gradient along the tube, Figure 13, were obtained. From comparison of these figures with similar data for typical tunnels, it was clear that the tube was representative of typical test sections.

As models, three ellipsoids-of-revolution of fineness ratio 7, Figure 14, and with maximum diameters of 2.0, 4.0 and 6.0 inches, resulting in model to tunnel diameter ratios of 0.1667, 0.3333 and 0.500, respectively, were fabricated from white pine. Boundary layer trip wires were installed on all the models at approximately 0.05.

The boundary layer trip wires were considered necessary for two reasons. First, in order to insure the same flow conditions over the models, it was desirable to conduct at least some of the tests at a Reynolds number common to all the models. Since the largest practical common geometric Reynolds number as defined by practical tunnel speeds and the 2.0 in. diameter model was approximately $10^6$, it was possible that consideration of an effective Reynolds number could result in models being tested in the critical Reynolds number range for transition from laminar to turbulent flow over the body. No tunnel turbulence factor measurements were available for defining an effective Reynolds number of the tunnel used. However, subsequent pitching moment tests of the 6.0-inch diameter ellipsoid with and without the trip indicated no measurable differences
Figure 12  Tube Velocity Profile at 3.0 in. Aft of the Tube Inlet
Figure 13 Static Wall Pressures in the Tube as a Function of Distance from the Tube Inlet.
in pitching moment coefficients. Secondly, since the ultimate goal for these tunnel corrections is the correction of water tunnel data, where Reynolds numbers of the order of 10 times the maximum $2.3 \times 10^6$ of the 6.0 in. diameter ellipsoid tested here are common, it was desirable to insure that the flow over the body was indeed turbulent. Also, even if at a later date, it was shown that no effective Reynolds number corrections were required, $2.3 \times 10^6$ is approximately the critical Reynolds number for the ellipsoids.

The ellipsoids were supported at their center-of-gravity by a single 0.375-inch diameter rod manufactured of 416 stainless steel, Figure 15. At the model end of this support rod, machining produced a hollow thin-walled section with a 0.012 inch wall thickness approximately 1.0 inch long. To this hollow section, or torque tube, was epoxied a threaded fitting for attachment to the ellipsoids. The design of this torque tube and model mounting arrangements in the 4.0 inch and 6.0 inch diameter ellipsoids was such that the center of the torque tube and the ellipsoid centerlines were coincident. This was done to minimize any unwanted lift and drag interactions in the moment measuring system. Unfortunately, in the 2.0-inch diameter ellipsoid, space limitations prevented maintaining this arrangement. However, the displacement of the torque tube center, where the electro-mechanical strain gages were placed from the 2.0-inch diameter ellipsoid centerline was held to 0.50 inches. The other end of the support rod was threaded, permitting mounting through an adapter to an existing ARL streamlined strut. Threading the adapter end permitted height and angle adjustments of the
model. This support rod and strut were bolted to the wind tunnel balance which was used to change the body angle of attack. The total model support moment measuring system is shown in Figure 16.

The moment measuring system or torque cell consisted of electromagnetic strain gages placed at 45 degrees to tube axis and centered on the 1.0 inch torque tube to measure strains produced by torques applied to the tube. A basic four-arm 120 ohm strain gage bridge was used to minimize temperature effects.

Calibrations and Velocity Measurements

A schematic of the instrumentation used for the wind tunnel tests is shown in Figure 17. The torque cell was designed to operate at a supply voltage of 5.00 volts and variable gains of either 100 or 1000. To eliminate any unwanted effects due to the strain gage power supply and amplifier, the units to be used in the actual wind tunnel testing were assembled, and the torque cell, power supply and amplifier calibrated as a system.

To calibrate the torque cell, a fixture, mounting on the torque tube in the same manner as the ellipsoids in Figure 16, with accurate moment arms of -6.0, 0.0 and 6.0 inches and applying the loads through the torque tube center, was built. By clamping the strut adaptor plate, model support rod, and calibration fixture to a vertical wall, known moments could be produced by hanging weights at the various moment arm distances. Although it was realized that loading in this manner did not produce a pure moment, but also resulted in a bending force being applied to the torque cell, loading at the zero length moment arm over
Figure 16 The Schematic of the Total Model Support System
Figure 17: The Instrumentation Schematic for the Pitching Moment Tests.
the calibration range produced no measurable torque interaction. Rotating the torque cell 90.0 degrees, and repeating this test indicated there was no torque interaction due to bending forces for all angular orientations of the tube. The torque cell calibration was then obtained by placing a total of 5.0 lbs. at the 6.0 inch moment arm distance in 0.50 and 1.00 lb. increments. The resulting calibration was 0.06026 in. lbs./MV at an amplifier gain of 100. Repeating the calibration for an amplifier gain of 1000 resulted only in shifting the calibration constant decimal point.

Since it was realized that the torque tube would twist, thus changing the ellipsoid angle of attack, dial displacement indicators were placed at the -6.0 inch and zero moment arm distances and the twist measured at 0.0275 deg/in. lb. This constant was used to modify the ellipsoid angle-of-attack set with the wind tunnel balance in the data reduction. Also, the calibration was repeated without the dial indicators, since it was felt they might modify the calibration. However, no measurable change in the calibration was seen due to the presence of the dial indicators in the calibration set-up.

Since all the non-dimensional coefficients derived from tunnel testing use velocity as a basis for calculation, one of the more important measurements was the test section velocity. Fortunately, the average velocity through the test section is one of the easiest of measurements. However, the velocity in the tube, especially when a model is installed, varies significantly from the tunnel test section velocity as can be seen from Figure 18. The proper velocity for the reduction of the in-tube pitching moment data to coefficient form is the
Figure 18 Dynamic Head Losses at the Tube Inlet as a Function of Model-to-Tunnel Diameter Ratio
velocity in the tube inlet. To obtain the tube velocity, a pitot-static probe was placed on the tube centerline approximately 3.0 inches aft of the inlet. A second pitot-static probe was placed at the tunnel test section throat to obtain the tunnel velocity for reduction of the unrestricted medium pitching moment data. Figure 18 shows the ratio of the readings from these two pitot-static probes. Periodic comparison of the total pressures from these two pitot-static probes insured correct velocity measurements.

All the pressure measurements, i.e., the in-tube and unrestricted medium velocity, and the wall static pressures, were made using a Validyne, Model 5370, pressure transducer. The calibration constant for the particular transducer used was 1.516 inch of water/volt at a span setting of 7.0. The span setting is a combination of transducer supply voltage and amplifier gain.

Test and Data Reduction Procedures for the Pitching Moment Measurements

Because of the ease with which yaw angles could be set using the permanent wind tunnel balance the pitching moment measurements were made in a horizontal plane. After crudely centering the ellipsoid in the tube with the aid of a centering guide, the torque cell was zeroed at a fixed tunnel velocity with the model at a zero angle of attack. The centering guide consists of a plexiglass disk of diameter equal to the diameter of the tube with cross hairs inscribed at the center. It was realized that due to errors in the initial positioning of the ellipsoids, a slight angle of attack and hence a small pitching moment would exist. As only the slope of the pitching moment curve was of interest, the change in pitching moment due to angle of attack
was measured. After zeroing the torque cell at a test velocity, the ellipsoid was rotated through a total angle of attack of 4.0 degrees in 0.5 degree increments and measurements taken at each step. A similar procedure was followed for negative angles up to -1.0 degrees. Where feasible, the procedure was repeated for several Reynolds numbers. A similar test procedure was followed for testing in the unrestricted medium.

In both cases, the data reduction followed the nondimensionalization practices outlined in Appendix A. The pitching moment coefficient $C_M$ is

$$C_M = \frac{M}{1/2 \rho U^2 S_m}$$  \hspace{1cm} (52)

Also, allowances were made in the data reduction for the increased angles-of-attack due to twisting of the torque tube under the applied pitching moments.

Results of the Pitching Moment Experiments

Figures 19, 20, 21 are typical plots of the measured pitching moment coefficients as a function of angle of attack for the various ellipsoids, both in the unrestricted medium and in the tube. Figure 22 is a composite plot of all the experimental data.

A series of tests, primarily with the 6.0 inch diameter ellipsoid, Figure 21, were conducted to assess the effects of Reynolds number on the pitching moment coefficient. Over the modest Reynolds number
Figure 19 Static Pitching Moment Coefficient as a Function of the Body Angle of Attack: 2.0-in.-Diameter Ellipsoid
Figure 20 Static Pitching Moment Coefficient as a Function of the Body Angle of Attack: 4.0-in.-Diameter Ellipsoid
Figure 21 Static Pitching Moment Coefficient as a Function of the Body Angle of Attack: 6.0-in.-Diameter Ellipsoid
Figure 22 Pitching Moment Coefficients as a Function of the Body Angle of Attack: All Ellipsoids
range available, there was no measurable effect on the body pitching moment coefficient due to Reynolds number.

In addition, it has been expected that using the common model support strut of Figure 15, for all the ellipsoids would result in varying degrees of model-strut interference on the pitching moment coefficients. This model-strut interference would have been expected to be a function of the model-to-strut diameter ratio, and to be evidenced by significant changes in the pitching moment coefficient slope, $C_{Ma}$, in the unrestricted medium tests of the three ellipsoids. However, this was not the case; the unrestricted medium $C_{Ma}$ being essentially constant for all three ellipsoids tested. These tests do not preclude a constant model-strut interference correction to $C_{Ma}$, evaluation of which would require additional testing with a second dummy strut to restore symmetry. It does appear that such a model-strut correction would be a constant multiplication factor for all three ellipsoids. As such, a multiplicative model-strut interference correction to the pitching moment coefficient slope, $C_{Ma}$, would not affect the experimental tunnel wall interference correction for $C_{Ma}$. Appendix C develops the reasoning behind this statement.

Table II collects the experimental pitching moment coefficient slope data and defines the experimental tunnel interference correction for $C_{Ma}$. In defining the experimental $C_{Ma}$ correction, it was assumed that a $\pm 1.0$ percent error in graphically defining $C_{Ma}$ from
### TABLE II

**TABULATED PITCHING MOMENT COEFFICIENT SLOPES AND TUNNEL INTERFERENCE CORRECTIONS**

<table>
<thead>
<tr>
<th>MODEL</th>
<th>MODEL/TUNNEL DIAMETER RATIO</th>
<th>$C_{M_{\alpha}}$ (MINIMUM)</th>
<th>$C_{M_{\alpha}}$ (AVERAGE)</th>
<th>$C_{M_{\alpha}}$ (MAXIMUM)</th>
<th>MINIMUM CORRECTION</th>
<th>AVERAGE CORRECTION</th>
<th>MAXIMUM CORRECTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALL MODELS</td>
<td>0.00</td>
<td>0.0175</td>
<td>0.0177</td>
<td>0.0179</td>
<td>0.978</td>
<td>1.00</td>
<td>1.022</td>
</tr>
<tr>
<td>2.0-INCH-DIAMETER ELLIPSOID</td>
<td>0.1567</td>
<td>0.0184</td>
<td>0.0186</td>
<td>0.0188</td>
<td>1.024</td>
<td>1.053</td>
<td>1.082</td>
</tr>
<tr>
<td>4.0-INCH-DIAMETER ELLIPSOID</td>
<td>0.333</td>
<td>0.0227</td>
<td>0.0229</td>
<td>0.0231</td>
<td>1.261</td>
<td>1.293</td>
<td>1.326</td>
</tr>
<tr>
<td>6.0-INCH-DIAMETER ELLIPSOID</td>
<td>0.500</td>
<td>0.377</td>
<td>0.0381</td>
<td>0.0385</td>
<td>2.106</td>
<td>2.153</td>
<td>2.203</td>
</tr>
</tbody>
</table>

**NOTE:** In defining $C_{M_{\alpha}}$ from Figures 19, 20, and 21 for the various models, it is assumed that there can be a $\pm 1.0$ percent error in graphical determination of the slopes. This $\pm 1.0$ percent slope error is used to define the minimum and maximum $C_{M_{\alpha}}$ corrections following Equation (52).
the plots, Figures 19, 20, and 21 was possible. Then, three experimental $C_{M_a}$ model/tunnel interference corrections were defined at each model-to-tunnel diameter ratio. They are:

1. The maximum correction

$$\frac{C_{M_a IT}}{C_{M_a FS}} (\text{Maximum})$$

2. The minimum correction

$$\frac{C_{M_a IT}}{C_{M_a FS}} (\text{Minimum})$$

and

3. The average correction

$$\frac{C_{M_a IT}}{C_{M_a FS}} (\text{Average})$$

These experimental $C_{M_a}$ tunnel wall interference corrections for $C_{M_a}$ are plotted with the theoretical predictions on Figure 9 for comparison.

Semi-empirical predictions (27) of the pitching moment coefficient slope, $C_{M_a}$, for the ellipsoids indicate a value of 0.0188/degree in the unrestricted medium. The experimentally determined value for $C_{M_a}$ averaged 0.0177/degree or approximately 5.8 percent below the empirical value. Aside from a possible constant model-strut interference correction, Appendix C provides an analysis of the possible errors in the measurements of $C_{M_a}$. 
As an aid to future researchers, attempting to duplicate the experimental results of this thesis, the experimental data used to generated Figures 19 to 22 are tabulated as Appendix D.
The results of the theoretical and experimental work contained in this thesis are summarized by Figures 9 and 10. One must conclude from Figure 9 that the present theory developed from slender body theory is inadequate for predicting corrections to $C_{M_a}$ and $C_{L_{q}}$ for tunnel wall interference. The possible exception to this statement might be the correction to experimental $C_{M_a}$ and $C_{L_{q}}$ data for models with a model-to-tunnel diameter ratio of less than approximately 0.20. Here, the uncertainties in the experimental data presented in Table 2 and Appendix C make a decision between the various theories difficult. However, based on the average experimental correction for $C_{M_a}$, the theories of Goodman or Peirce, appear to agree better with the experimental results than the present theory. Unfortunately, no similar statements regarding the applicability of any of the three theories discussed in this thesis for correcting the experimental acceleration stability derivatives $C_{L_a}$ and $C_{M_{q}}$, for tunnel wall interference can be made since no experimental data exists for comparison.

Goodman's theory appears to offer the best agreement with the present experimental $C_{M_a}$ interference correction data in a model-to-tunnel diameter ratio range of practical interest. The upper limit for this
range is a model-to-tunnel diameter ratio of approximately 0.40. However, for the differences between the interference corrections for $C_{M_{3}}$ as predicted by Goodman or Peirce, the computational complexity of Goodman's technique makes Peirce's simple theory look very attractive for rough or preliminary calculations. Also, the present experimental interference correction data for $C_{M_{3}}$ departs from Goodman's theory for the higher model-to-tunnel diameter ratios and approaches Peirce's theory. Further pitching moment measurements at model to tunnel diameter ratios between 0.40 and 0.50 are needed to establish the tunnel interference correction in this range of model to tunnel diameter ratios.

Since in the range of practical model-to-tunnel diameter ratios below approximately 0.40 Goodman's theory offers the best overall agreement with the limited experimental data of this thesis and does consider the effect of body geometry on the interference corrections, it is suggested that all future theoretical work involve this theory. However, the theory developed for this thesis may prove useful in indicating relative effects of body shape parameters such as prismatic coefficient or body cross-sectional area distributions on the tunnel interference corrections.

The concept of the interference correction developed for this thesis is uniquely suited for correcting the experimental stability derivative data obtained from testing with the Planar Motion Mechanism. For Planar Motion Mechanism tests, the flow-correcting liner technique developed by Peirce for correcting pressure distributions on the body
has several disadvantages. First, the liner is only good for cases where the body is axisymmetric in the tunnel. Obviously, for cases where the body is at an angle of attack or displaced from the tunnel centerline by the oscillations of the Planar Motion Mechanism, the flow-correcting liner is not correct. Indeed, to use the flow-correcting liner for bodies at an angle of attack would require a different liner for each angle of attack. The cost in time and materials make this approach impractical. Secondly, although the liner does provide an approximately correct pressure distribution over the body, the accuracy of the correction provided by the liner is not sufficient for drag measurements. This is evidenced by the need for further corrections to experimental drag measurements made in the 48.0-inch-diameter water tunnel at ARL.
This thesis has presented tunnel interference corrections for only one prismatic coefficient and body cross-sectional area distribution, namely the ellipsoid. It would prove interesting to examine the effect of varying the model cross-sectional area distributions on the body x-axis at the prismatic coefficient of the ellipsoid, using Goodman's theory for the tunnel interference corrections as a practical minimum. Similarly, varying the prismatic coefficient and the body cross-section area distribution would establish the theory sensitivity to body geometry. In the limit as the prismatic coefficient, $\gamma$ approaches 1.0 Goodman's results should approach Peirce's theory, Equation (2).

In light of the magnitude of the uncertainties possible in the experimental correction factor data further static pitching moment experiments, attempting to distinguish subtle variations in body geometry appear futile. One exception to further testing would be further static pitching moment tests of an ellipsoid with a model to tunnel diameter ratio of approximately 0.42 to confirm the departure
of the experimental correction from Goodman's theory at the higher model to tunnel diameter ratios. Also, no experimental verification of the tunnel interference corrections for the acceleration stability derivatives of Figure 10 has been offered. Finally, further theoretical and experimental work yielding tunnel interference corrections for $C_{L\alpha}$ would be useful.
References


Equations (9), (26), (29), (30), and (31) are non-dimensionalized using the unblocked tunnel or free stream velocity, $U$, the body length, $\ell$; and the maximum cross-sectional area of the body, $S_M$. A dimensionless time $\tilde{t}$ is defined such that

$$
\tilde{t} = \frac{t}{\frac{U}{\ell}}
$$

(A-1)

and

$$
\frac{d}{dt} = \frac{U}{\ell} \frac{d}{d\tilde{t}}
$$

(A-2)

Then, it follows

$$
L = \frac{1}{2} \rho U^2 S_M C_L \quad h = a \quad b = b\ell
$$

$$
M = \frac{1}{2} \rho U^2 S_M C_M \quad a = a\ell \quad \frac{U}{x} = \frac{\bar{U}}{\bar{x}}
$$

$$
q = \frac{U}{\ell} \bar{q} \quad R = R\ell \quad \frac{\bar{h}}{\ell} = \frac{U^2}{\ell} \alpha
$$

$$
q = \frac{(U)}{\ell} \bar{q} \quad x = x\ell \quad \alpha = \frac{\bar{t}}{\ell}
$$

(A-3)
where an overbar indicates a non-dimensional quantity. $C_L$ and $C_M$ are the non-dimensional lift and pitching moment coefficients, respectively, and may be written in the form of Equation (1), in terms of $\alpha$, $\alpha$, $q$, or $q$, leading to the non-dimensional stability derivatives.
APPENDIX B

A GENERAL BERNOULLI EQUATION FOR A MOVING COORDINATE SYSTEM

The following development for the pressures produced in steady and unsteady translational motion in a two-dimensional field of flow is due to Wendel\(^{35}\). Consider a liquid particle whose center is assumed to lie at the point \(z, y\) at time \(t\). At time \(t + \delta t\), the same particle is located at the point \(z + w \delta t, y + v \delta t\), where the velocities in the \(z\) and \(y\) directions are designated by \(w\) and \(v\), respectively. Furthermore, let \(P\) equal the pressure, \(\rho\) the density, \(Z\) and \(Y\) the components of the external forces per unit length at the point \((z, y)\) at time \(t\) and \(\delta z\) and \(\delta y\) the lengths of the elements. In order to use the velocity potentials, Equations (18) and (19), which refer to a cylinder moving in a stationary medium, a coordinate system which moves with the cylinder is required. In that case, the change of position of the particle with respect to the moving system is \((w - Z_0)\) in the \(Z\) direction. Then, summing forces on the fluid element in the \(z\) and \(y\) directions and simplifying obtain:
\[ \frac{dw}{dt} + (w - Z_o) \frac{dw}{dz} + v \frac{dw}{dy} = Z - \frac{1}{\rho} \frac{dP}{dz} \]

and

\[ \frac{dv}{dt} + (w - Z_o) \frac{dv}{dz} + v \frac{dv}{dy} = Y - \frac{1}{\rho} \frac{dP}{dy} . \]

which are Euler’s equations referred to a coordinate system moving in translation with velocity \( Z_o \). The assumed ideal fluid has a velocity potential; hence, the following apply:

\[ w = \frac{d\psi}{dz}, \quad v = \frac{d\psi}{dy}, \]

and thus, also, \( \frac{dw}{dy} = \frac{d^2\psi}{dy \, dz} \), \( \frac{dv}{dy} = \frac{d^2\psi}{dy \, dz} \) \( \tag{B-2} \)

The external and hydrostatic forces need not to be considered any further, since the investigations concern only dynamic forces.

Introducing Equations (B-2) into Euler’s equations, Equation (B-1) yields

\[ \frac{d^2\psi}{dz \, dz} + (w - Z_o) \frac{dw}{dz} + v \frac{dv}{dz} = -\frac{1}{\rho} \frac{dP}{dz} \]

and

\[ \frac{d^2\psi}{dy \, dt} + (w - Z_o) \frac{dw}{dy} + v \frac{dv}{dy} = -\frac{1}{\rho} \frac{dP}{dy} . \] \( \tag{B-3} \)

The Bernoulli Equation arises from the integration of Equation (B-3).

For the purposes of integration note that
\[ \frac{1}{2} \frac{d}{dz} \left( \omega^2 \right) = \frac{1}{2} \frac{d}{dw} \left( \omega^2 \right) \frac{dw}{dz} = \omega \frac{dw}{dz} \]  

(B-4)

and that Equation (B-3) may be written:

\[ \frac{d}{dz} \left\{ \frac{d\phi}{dt} + \frac{1}{2} \left( \omega^2 + v^2 \right) - \omega \frac{d\omega}{dz} \right\} = -\frac{d}{dz} \left[ \frac{p}{\rho} \right] \]

and

(B-5)

\[ \frac{d}{dy} \left\{ \frac{d\phi}{dt} + \frac{1}{2} \left( \omega^2 + v^2 \right) - \omega \frac{d\omega}{dz} \right\} = -\frac{d}{dy} \left[ \frac{p}{\rho} \right] \]

Performing the indicated integration, obtained from each of these Equations (B-5), the following relationship.

\[ \frac{d\phi}{dt} + \frac{1}{2} \left( \omega^2 + v^2 \right) - \omega \frac{d\omega}{dz} + \frac{p}{\rho} = F(t) \]

(B-6)

where the integration constant, \( F(t) \) may still be a function of \( t \).

Equation (B-6) may be designated as a general Bernoulli equation for a coordinate system moving with a velocity \( \mathbf{v}_0 \). In the polar coordinates of Figure 4, Equation (B-6) may be written:

\[ \frac{d\phi}{dt} + \frac{1}{2} \rho \left( \omega^2 + \frac{1}{r^2} \omega^2 \right) - 2 \omega \left( \frac{\omega}{r} \cos \theta - \frac{1}{r} \frac{\omega}{\rho} \sin \theta \right) + \frac{p}{\rho} = F(t) \]

(B-7)
where

\[ \phi_r = \frac{\partial \phi}{\partial r}, \]  
the radial velocity component

\[ \frac{1}{r} \phi_\theta = \frac{1}{r} \frac{\partial \phi}{\partial \theta}, \]  
the angular velocity component.
The errors in any measurement may be classified into two categories: constant and random. Constant errors, such as calibration and gain errors influence the magnitude of the data, while random errors, such as the reading fluctuations, reflect the precision and accuracy of the reading instrumentation.

Since a common measurement system was used for all measurements, the constant errors should be of no importance for the relative comparison of Table II. That is, the pitching moment coefficient, \( C_M \) may be written

\[
C_M = K (V_1/V_2), \quad (C-1)
\]

where \( K \) = combination of model cross-sectional area, body length, torque and pressure transducer calibrations,

\( V_1 \) = output from torque tube, and

\( V_2 \) = output of the pressure transducer.

Then, for the experimental interference correction for \( C_{M_a} \), defined as the ratio of \( C_{M_a} \) for the body in the tunnel so that for body in the unrestricted medium.
Therefore, magnitude errors will not affect the relative comparisons.

Errors over which the experimenter has no control or random errors such as temperature and pressure fluctuations, calibration precision, and instrumentation accuracy and linearity, affect the degree of confidence which can be placed in the experimental data.

The first random error considered is that of the torque tube calibration. In the calibration apparatus, the moment arm had a precision of ± 0.005 in. while the weights were accurate to ± 0.10 percent. Thus, the uncertainty in applying the calibration torques was ± 0.18 percent. In reading the calibration torques from the voltmeter, the maximum reading was 398 MV with an uncertainty of ± 0.50 MV. Then, the total torque tube calibration uncertainty was ± 0.30 percent.

Similarly, the uncertainty in the pressure transducer calibration was ± 0.14 percent, thus, the total calibration error or uncertainty shown as column 1 of Table III, was ± 0.44 percent.

For the rotary table, i.e., the wind tunnel balance used to set the body angles of attack, the finest readable increment was ± 0.10 degree. Thus, the 4.0 degree maximum angle of attack, and an assumed uncertainty of ± 0.05 degree yeilds the ± 1.25 percent precision error presented as column 2 of Table III.
Column 3 of Table IV combines the uncertainties in the torque cell and pressure transducer readings as was done for the calibration precision errors. The total precision error or uncertainty in the pitching moment coefficient data, column 4 of Table III is then the sum of all the component precision estimates.

In all cases, the uncertainty in the experimental pitching moment coefficient $C_m$ is less than ± 2.6 percent.

The uncertainty or precision error in the experimental $C_{M_a}$ interference correction could be obtained in a manner similar to that expressed in Equation (53). Column 5 of Table III gives the precision limits on the experimental $C_{M_a}$ interference correction. Note that this precision estimate compares favorably with the maximum and minimum corrections derived by assuming a ± 1.0 percent error in obtaining $C_{M_a}$ from a graphical determination of the slopes in Figures 19, 20, 21 and 22.
### TABLE III

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<th>MODEL/TUNNEL DIAMETER RATIO</th>
<th>CALIBRATION ERRORS, PERCENT</th>
<th>ANGLE SETTING ERRORS, PERCENT</th>
<th>VOLTOMETER READING ERRORS, PERCENT</th>
<th>TOTAL ERROR PERCENT</th>
<th>C, CORRECTION ERRORS, PERCENT</th>
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**NOTE:**

1. Calibration error is the total error involved in torque tube and pressure transducer calibration and includes possible errors in applying torques or pressures, and uncertainties in reading the voltmeter during calibration.

2. Voltmeter reading error is the total error involved in torque tube and pressure transducer reading during wind tunnel tests.
APPENDIX D

TABULATED PITCHING MOMENT COEFFICIENT DATA

As an aid to future researchers attempting to duplicate the results of this thesis, the experimental $C_M$ data used to generate Figures 19 to 22 for the various ellipsoids are tabulated.

TABLE IV

PITCHING MOMENT COEFFICIENT DATA: 2.0-INCH DIAMETER ELLIPSOID

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### TABLE VI

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Tunnel Wall Interference for Bodies of Revolution in Non-Steady Motion

This thesis considers the problem of determining the effects of tunnel wall interference on the measured values of the stability derivatives for axisymmetric bodies. The tunnel wall interference corrections developed apply equally well to both subsonic wind tunnel or water tunnel testing in tunnels with circular test sections.

A linearized slender body theory is developed which can be used to predict corrections for:

1) the static lift and pitching moment derivatives, $C_L$ and $C_M$
2) the rotary damping derivatives, $C_{Lq}$ and $C_{Mq}$
3) the "Add Mass" derivatives, $C_{L\alpha}$ and $C_{M\alpha}$
4) the "accession-to-inertia" derivatives, $C_{L\beta}$ and $C_{M\beta}$

Limited evaluation of the developed theory is presented in the form of static pitching moment tests of three ellipsoids-of-revolution. In addition, comparison is made with two alternative theories developed by Peirce[1] and Goodman.[2]
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