FINAL REPORT

ANALYSIS OF DATA ON WATER WAVES

VOLUME II MONO LAKE FIELD EXPERIMENT

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1. INTRODUCTION

The Mono Lake Field Tests provided much needed data on the generation, propagation, shoaling, breaking, and run-up of water waves produced by explosions in deep water. Although there were only ten shots, they provide one with two basic opportunities

1. to gain more data for charge weights an order of magnitude larger than most of the available data, and
2. to perform a valid evaluation of prediction techniques being applied prior to these tests.

The latter opportunity is the result of a study performed by Le Mehaute and Whalin (Ref. 1) where predictions of the deep water wave amplitude, shoaling characteristics, and run-up were made prior to the experiment.

Data obtained from this test series seemed to be compatible with that resulting from the DASA wave program. The application of the Kranzer and Keller type of theory as modified by Whalin and Sakurai produces good comparisons with the data. Energy computations are most easily performed by adjusting the cavity parameters of the mathematical model so the theoretical wave train envelope closely approximates the measured data. The energy of the theoretical model should then closely approximate that of the data. A reliable estimate of the energy can be obtained from measurements of the maximum envelope amplitude at any station and an approximate formula developed by Sakurai. More laborious computations of the energy can be obtained by computing the potential energy of the wave train from digitized magnetic tape data. All these methods of energy computation are discussed in Section 3.

A matching of two theories to the data for deep water wave trains is recommended in Section 3 and the mathematical analysis required is formulated. This should lead to a determination of the best mathematical model for representing deep water propagation characteristics.
2. DATA REVIEW AND ANALYSIS

This section is a discussion of the result of comparisons with data from the DASA wave program and presents the mathematical analysis necessary for constructing several computer programs recommended for further analysis of the 1965 Mono Lake results.

2.1 DISCUSSION

The data obtained at Mono Lake in 1965 provides for an analysis of all the following phenomena

1. deep water wave generation and propagation characteristics in water of constant depth
2. wave train propagation over a simulated continental slope
3. wave train propagation and shoaling on the continental shelf
4. breaking characteristics of the wave train, and
5. run-up from an explosion generated wave train on different beach slopes.

Only item 1 above is related to the DASA wave program whose objective is to perform measurements and analysis of the generation and propagation characteristics of explosively generated water waves in water of constant depth. The other phases of analysis (items 2 - 5 above) would be extremely beneficial and are recommended for further evaluation upon completion of this program.

Measurements on Radial 4 (the constant depth radial) were made at five stations and the analysis of this data showed good comparisons with the data from the DASA wave program. The periods associated with the maximum of the first wave envelope, $T_{\text{max}}$, agreed well with that predicted from extrapolation of the empirical formula derived during the DASA wave program. Periods were predicted to be approximately 5.7 sec. and measurements revealed the periods to be from 5.5 to 6.0 sec., yielding excellent agreement. Computation of the parameter $\eta_{\text{max}} r/W^{0.54}$ also compared well with the curve constructed during the DASA wave program. These values fell within the range of variation of the previous data. It is suggested
that all data points on the constant depth radial be plotted on the graph of \( \eta_{\text{max}} r/W^{0.54} \) vs. \( z/W^{0.3} \) resulting from the DASA program to illustrate the comparison between the two test series. The periods \( T_{\text{max}} \) should be shown on a similar plot.

2.2 COMPARISONS OF THEORETICAL AND EXPERIMENTAL WAVE TRAINS

Two theoretical models of the wave generation mechanism are presently being used for prediction purposes in deep water. Each represents an initial deformation which is given by

\[
\eta(r_0) = \begin{cases} 
\eta_0 \left[ \frac{r^4}{3R_0^4} - \frac{4r^2}{3R_0^2} + 1 \right], & r \leq \sqrt[3]{3} R \\
0, & r > \sqrt[3]{3} R
\end{cases} \tag{1a}
\]

\[
\eta(r_0) = \eta_0 \left[ 1 - 2 \left( \frac{r}{R_0} \right)^2 \right], \quad r \leq R \\
0, \quad r > R \tag{1b}
\]

\( \eta_0 \) is taken to be negative when a depression is assumed at \( r = 0 \).

A determination should be made of which mathematical model best reproduces the characteristics of the entire wave train. This can be accomplished by performing comparisons with the Mono Lake data where computer plots are made for various values of the empirical constants \( \eta_0 \) and \( R_0 \). The theoretical solutions for the wave train from each of the above deformations are given by

\[
\eta(r, t) = \frac{4\eta_0 h}{r^2} J_4 \left( \frac{\sqrt[3]{3} \sigma R_0}{h} \right) \sqrt{\phi(\sigma)} \cos \left( \frac{\sigma r}{h} - \sqrt{\frac{g}{h} \sigma \tanh \sigma t} \right) \tag{2a}
\]

\[
\eta(r, t) = \frac{\eta R_0}{r^2} J_3 \left( \frac{\sigma R_0}{h} \right) \sqrt{\phi(\sigma)} \cos \left( \frac{\sigma r}{h} - \sqrt{\frac{g}{h} \sigma \tanh \sigma t} \right) \tag{2b}
\]

where

\[
\phi(\sigma) = \frac{1}{2} \left\{ \sqrt{\frac{\tanh \sigma}{\sigma}}, + \frac{1}{\cosh \sigma} \sqrt{\frac{\sigma}{\tanh \sigma}} \right\} = \frac{r}{\sqrt{gh t}}
\]
\( \eta_0 = \text{depth of the initial deformation at } r = 0 \)

\( R_0 = \text{radius of the initial deformation} \)

\( r = \text{radial distance of the measurement station} \)

\( t = \text{time after the detonation} \)

\( g = \text{acceleration of gravity} \)

\( h = \text{water depth} \)

The most efficient method of programming the above equations is to increment the computation for values of \( \Delta \sigma \) (steps of 0.1 from 0.1 to 10.0 are recommended), compute the time from the function \( \Psi(\sigma) \) and plot \( \eta(r, t) \) vs. \( t \) on the same plot of the digitized magnetic tape input from the measurement stations. Direct comparison of the wave trains can be made from these plots. The empirical constants for the cavity parameters of the theoretical model should be selected by the method formulated by Sakurai.

In order to modify the program so that the generated theoretical wave envelope agrees with the experimental data one only needs to compute \( R_0 \) such that the theoretical period at the envelope maximum coincides with that measured from data. The following formulas will yield this relationship

\[
R_0 \text{ (ft.)}^{\text{Adjusted}} = 2.26 T_{\text{max}}^2 \tag{3a}
\]

\[
R_0 \text{ (ft.)}^{\text{Adjusted}} = 3.42 T_{\text{max}}^2 \tag{3b}
\]

In order to fit the maximum amplitude of the first envelope the value of \( \eta_0 \) need only be adjusted to accomplish this.

\[
\eta_0^{\text{Adjusted}} = \left( \frac{\eta_{\text{max data}}}{\eta_{\text{max modified theoretical}}} \right) \eta_{0 \text{ theoretical}} \tag{4}
\]

where

\[
\eta_{\text{max modified theoretical}} = \max \left[ \frac{\eta_0 h}{r \sigma^2} J_4 \left( \frac{\sqrt{3} \sigma R_0^{\text{Adjusted}}}{h} \right) \cdot \sqrt{-\Psi(\sigma)} \right] \tag{5a}
\]
\[ \eta_{\text{max modified}} = \text{MAX} \left[ \eta_0 \frac{R_o \text{ Adjusted}}{r \sigma} J_3 \left( \sigma R_o \text{ Adjusted} \sqrt{\frac{\gamma \sigma}{\sigma'}} \right) \right] \] (5b)

The above formulas and relationships are all that is needed to construct a theoretical wave train and modify it to agree with the experimental data. Both theoretical models should be used for this purpose and upon completion of the comparison, a conclusion can be reached regarding the applicability of each model for prediction purposes.

2.3 ENERGY COMPUTATIONS

Energy computations can be performed by several different methods. The easiest approximate method is to use the modified theoretical cavity parameters of the previous section and compute the potential energy of the initial deformation. The energy equations are

\[ \text{Energy} = \frac{\pi \rho g \eta_0^2 \text{ Adjusted} R_o^2}{5} \] (6a)

\[ \text{Energy} = \frac{\pi \rho g \eta_0^2 \text{ Adjusted} R_o^2}{6} \] (6b)

The method of Sakurai presented in 2.5.3 of Volume I of this report can also be used to perform the energy computation. The digitized magnetic tape input needs only to be scanned to determine the maximum wave amplitude \( \eta_{\text{max}} \) for any one measurement station at a distance \( r \) from SZ. The energy is then represented by

\[ \text{Energy} = 66.2 \eta_{\text{max}}^2 r^2 \text{ ft.lbs.} \] (7a)

\[ \text{Energy} = 88.8 \eta_{\text{max}}^2 r^2 \text{ ft.lbs.} \] (7b)

The reason for the variation of the two above formulas is that they represent different wave envelopes and the one should be chosen which best reproduces the data with the adjusted cavity parameters discussed in the previous section.

More laborious computations can be performed by integrating the digitized
data directly, however, this method may lead to more error since the measurements only accurately represent the first or second wave envelope and the energy contained in all subsequent envelopes must be neglected. This point can be investigated further if desirable.
3. SUMMARY

The data from the 1965 Mono Lake test series agreed well with the analysis performed during the DASA wave program. It would be extremely beneficial to initiate further analysis of this data as pointed out in Section 2.1 with regard to propagation characteristics over a non-uniform slope, breaking, and run-up phenomena. Furthermore, the data should be analyzed in comparison to the predictions performed prior to the experiment in order to determine why the run-up predictions were consistently low by approximately 50%.

The mathematical analysis was formulated for constructing theoretical wave trains from two mathematical models using empirical constants as the input for any condition in the deep water surface wave case. The theoretical wave trains can be easily modified by changing the parameters $\eta_0$ and $R_0$ to agree with the experimental data.

Energy computations can readily be made by one of two approximate methods delineated. The method utilized should be the one corresponding to the mathematical model which best fits the data of the entire wave train after modification of the cavity parameters. Due to the additional time and cost of performing a direct integration from the digitized magnetic tape input to compute the potential energy, the error involved in neglecting subsequent wave envelopes should be investigated further.