High Power Microwaves from a Non-Isochronic Reflecting Electron System (NIRES)

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May 1977
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High-power microwaves
Reflex triode
Non-isochronic electron systems

High-power microwave radiation of frequency 11 GHz and 36 GHz is observed when a 300 kV positive pulse is applied to an aluminized mylar anode of a reflex triode. The frequency of radiation varies as \( \nu \propto \left( V^{1/2}/d^{n} \right) \), where \( V \) is the anode voltage, \( d \) the anode-cathode spacing and \( 0 < n < 1 \). The basic features of the radiation are explained by a theoretical model based on the energy dependence of the electron oscillation frequency.

\[ \text{(square root of } V)/d \text{ to the } n\text{-th power} \]
HIGH POWER MICROWAVES FROM A NON-ISOCRONIC REFLECTING ELECTRON SYSTEM (NIRES)

Introduction

Reflecting electron systems in the form of a low inductance, coaxial reflex triode have been recently used successfully to produce MeV proton pulses of peak current in excess of 200kA.

In this letter, we report results which demonstrate that, under suitable conditions, reflecting electron systems can also generate high power microwaves with or without an applied magnetic field \((B)\). As will become apparent later on, reflecting electron systems have some distinct advantages over several of the previously reported devices that also use relativistic electron beams for the generation of microwaves.

As shown in Fig. 1, in its simplest form, a reflecting electron system consists of a positively biased, semi-transparent anode and a grounded cathode. Electrons emitted from the cathode are accelerated by the positive pulse that is applied to the anode, pass through it and form a virtual cathode. In general, the virtual cathode is formed at a distance from the anode that is different from the anode-cathode opening \((d)\). As a result of the positive potential on the anode, the electrons do not leave the system but rather oscillate between the real and virtual cathodes.

The microwave emission is attributed to the phase bunching of the oscillating electrons inside the potential well of the system. This bunching is due to the energy \((e)\) dependence of the electron oscillation frequency \(\omega_0\). For the idealized parabolic potential well of Fig. 1b, \(\omega_0\) is a function of \(e\) only for relativistic electrons. In the presence of an oscillatory electric field \(E = E_0 \cos(\omega t)\) of frequency \(\omega \approx \omega_0\), a sample of initially uniformly distributed electrons will be

Note: Manuscript submitted April 22, 1977.
bunched as shown in Fig. 1c. The reason for this bunching is that 
\( \frac{\partial \omega}{\partial t} < 0 \) and thus those electrons located in the upper half plane at 
t=0 gain energy (\( \omega \) decreases) and their phase slips behind the wave, 
while those at the lower half plane loose energy (\( \omega \) increases) and 
their phase advances ahead of the wave. This non-isochronic mechanism 
is similar to that of the electron cyclotron maser 8,9.

The experimental set-up has been described previously in connection 
with the generation of intense, pulsed ion beams.10,11 Briefly, the 50 
nsec duration, 300–350 kV positive pulse from a 7-ohm generator is applied 
to the anode of a reflex triode. The anode is made either from 6 \( \mu \)m thick 
aluminized mylar or bare, parallel copper wires spaced 2 cm apart. It 
has been determined in previous experiments that with these sort of anodes 
the ion flux generated by the device is very small. Both the power and 
the frequency of the emitted microwaves appear to be sensitive to the 
shape of the cathode. The maximum power was obtained at \( B_0 = 0 \) with a 
8.4 cm OD solid cathode facing an aluminized mylar anode for \( d = 1.2 \) cm.

The emitted microwave radiation is monitored in both the X-band 
\( f = (8.2 - 12.4 \) GHz) and the Ka-band (\( f = 26.5 - 40 \) GHz). The horn-antennas 
are situated about 26 cm from the anode with their axis either parallel 
to the direction of the accelerating electric field (end-on) or with 
their axis perpendicular to it (side-on). A very small portion of the 
microwave signal taken from the output of a directional coupler which is 
located immediately after the horn is fed almost undispersed to an 
oscilloscope and is used as a time marker. The rest of the microwave 
signal is transmitted through a long (323 m in the X-band; 60 m in the 
Ka) dispersive line and is also fed to the same oscilloscope. The time
delay between the two signals gives information about the frequency spectrum of the radiation. Typical dispersed signals are shown in Fig. 2 together with the voltage pulse applied to the anode and the current flowing in the device. Since the power of the X-band radiation is about equal at the only two accessible positions of the system that are located about 90° apart, it is reasonable to assume that the emitted radiation is isotropic. Extrapolating the power measured at the horn under the above assumption the power emitted by the device at a single frequency over a 4π solid angle is between 90-100 MW, corresponding to an efficiency of about 1.5%.

A striking feature of the experimental results is the variation of the microwave frequency (f) with applied voltage (V) shown in Fig. 3. Clearly, the central frequency $f_c$ of the spectrum varies as $f_c \approx \tilde{V}$, where $\tilde{V}$ is the time averaged anode potential. In addition, the results of Fig. 3 show that the frequency depends upon the shape of the cathode. It can be shown that the distance between the virtual cathode and anode decreases as the thickness of an annular cathode increases. Since the frequency of radiation $f \approx \mathcal{L}^{-1}$, where $\mathcal{L}$ is an integer and $\tau_c$ is the transit time of a typical electron in the potential well, higher frequencies are expected with higher voltages and shorter openings between the real and virtual cathodes. As the anode is moved closer to the real cathode, the spacing between the anode and the virtual cathode is also reduced and thus higher frequencies are expected. This is shown in Fig. 4.

The voltage dependence of the microwave frequency can also explain the frequency spread of the signal shown in Fig. 2. For this particular
shot the voltage pulse drops by 150 kV during its duration, corresponding to a $\Delta V/V = 0.4$ or to $(\Delta V/V)^2 = 0.2$, which is about equal to the observed $\Delta f/f$.

At $B_0 = 0$, with an aluminized mylar anode located 1.2 cm away from a solid carbon cathode, the Ka-band power of the microwave radiation at the horn located about 26 cm from the anode with its axis parallel to the accelerating field is about 5 kW. This corresponds to a total power emitted over the $4\pi$ solid angle of more than 10 MW. Prior to detection, the signal is passed through a 60 m long dispersive line and the observed time delay of 230 nsec corresponds to a central frequency of 36.6 GHz.

A drastic reduction in the X-band microwave power is observed when the reflex triode is immersed in an external magnetic field. The emitted power shows a resonance-like behavior with the applied field. The power at the peak of the resonance is about two orders of magnitude lower than the corresponding power at $B_0 = 0$. In addition, the frequency of the emitted radiation increases approximately linearly with the value of applied magnetic field.

To gain insight on the generation of the microwave radiation we have studied the dynamics of an ensemble of collisionless electrons that initially are distributed uniformly in phase inside an arbitrary potential well described by the electric field $E^{(0)}(z)$. It is assumed that at $t = t_0$ the system is perturbed by a small amplitude homogeneous electric field $E^{(1)} = \hat{E} \exp(\text{i} \omega t)$. The equilibrium orbit of an electron ($E^{(1)} = 0$) is written in the form

$$z^{(0)}(t) = \sum_{k=-\infty}^{\infty} z_k^{(0)}(z) \exp \left\{ \text{i} \hat{L} \left[ \omega_0(z) t + \gamma_0 \right] \right\},$$

4
where $\hat{z}_l$ is the $l$th Fourier amplitude, $\omega_0$ is the fundamental oscillation frequency and $\varphi_0$ is the initial phase of the particle. Note that $\omega_0$ and $\hat{z}_l$, are in general functions of the total equilibrium energy $\varepsilon = (\gamma^{(0)} - 1) m_0 c^2 - |e| \varphi^{(0)} = -\partial \varphi^{(0)}/\partial z^{(0)}$. The change of the system's kinetic energy density due to $E^{(1)}$, averaged over $\varphi$ for the $l=1$ mode is

$$
\Delta W_{k,E} W_{k,E}(t) = W_{k,E}(t_0) = \frac{|e| n_0 \hat{E}_{\omega}}{2(\Delta \omega)_1^2} \left[ \frac{\partial}{\partial \varepsilon} \hat{z}_1 \right]^2 (1 - \cos \omega_1 \tau)
+ |\hat{z}_1|^2 \frac{\partial w_0}{\partial \varepsilon} \left( 1 + \frac{2 \omega_0}{\Delta \omega_1} \right) (1 - \cos \omega_1 \tau - \omega_0 \tau \sin \omega_1 \tau),
$$

(1)

where $n_0$ is the average electron density, $\Delta \omega_1 = \omega - \omega_0$, and $\tau = t - t_0$.

If $\Delta W_{k,E} > 0$ the perturbing wave is absorbed by the oscillating particles, while if $\Delta W_{k,E} < 0$ the particles lose kinetic energy to the wave resulting in wave growth. The term on the right hand side of Eq. (1) which contains the quantity $\delta |\hat{z}_1|^2 / \partial \varepsilon$ is always positive and hence is a stabilizing term. The remaining term which is proportional to $\omega_0 / \partial \varepsilon$, however, can be negative depending on the sign of $\omega_0 / \partial \varepsilon$ and $\Delta \omega_1$. It is this term which gives rise to the growth of the wave. It can be shown from Eq. (1) that the condition for the initial growth of the wave is that

$$
(\Delta \omega_1)^{-1} \frac{\partial \omega_0}{\partial \varepsilon} < 0,
$$

(2a)

together with

$$
\frac{\delta |\hat{z}_1|^2}{\partial \varepsilon} < - \frac{\hat{z}_1^2}{\Delta \omega_1^2} \frac{\partial \omega_0}{\partial \varepsilon}.
$$

(2b)
The particle oscillations are non-isochronous, \( \frac{\partial \omega}{\partial \varepsilon} \neq 0 \), if the potential well is non-parabolic and/or the particles are relativistic. In the present experiment both of these situations are satisfied. Using the wave equation for \( E^{(i)} \) an approximate dispersion relation has been derived, which in the neighborhood of the unstable frequency takes the form

\[
(\Delta \omega)^3 - w_b^2 \omega_o^2 m_0 \frac{3}{\beta c} (w_o |z_1|^2) \Delta \omega / 2 = (w_b^2 \omega_o^2 m_0 |z_1|^2 / 2) \partial \omega / \partial \varepsilon,
\]

where \( w_b^2 = \frac{4\pi |e|^2}{m_0} \).

As an illustration we consider the simplified situation where the particles are mildly relativistic and the potential is parabolic given by \( E^{(o)}(z^{(o)}) = \xi_o z^{(o)} \) where \( \xi_o \) is a constant. To lowest order in the small parameter \( \varepsilon / m_0 c^2 \) we find that

\[
\omega_o = c_o \left( 1 - \frac{3}{8} \frac{\varepsilon}{m_0 c^2} \right),
\]

and

\[
|z_1|^2 = \frac{c^2}{2c_0} \frac{\varepsilon}{m_0 c^2},
\]

where \( c_o = |e|^2 \xi_o / m_0 \) is the non-relativistic particle oscillation frequency in a parabolic potential. Using Eq. (4) in the expression for the dispersion relation we find that a threshold condition for instability exists and is

\[
\varepsilon / m_0 c^2 > b \frac{\beta}{\varepsilon} \frac{w_b}{c_o}.
\]

If the inequality in (5) is satisfied, Eq. (3) can be solved for the linear growth rate \( \Gamma = -\text{Im}(\Delta \omega) \) and frequency shift \( \delta \omega = \text{Re}(\Delta \omega) \), which are:
\[ \Gamma = \frac{\sqrt{3}}{4} \left( \frac{3}{\ell} \right)^{1/3} \left( \frac{\omega_0^2 c_0}{\omega} \right)^{1/3} \left( \frac{e}{m_0 c^2} \right)^{2/3} = \sqrt{3} \delta \omega. \]

It is apparent from the previous discussion that NIRESES have four interesting features: (i) the emitted power is maximum when \( B_0 = 0 \); (ii) compactness; (iii) tunability and (iv) monochromaticity. Thus, it is conceivable that they will lead to the development of practical, high frequency microwave sources in the future.

It is very likely the mechanism responsible for the generation of microwave radiation in our system is also responsible for the high power (more than 1 GWatt) X-band radiation observed by Doucet and Buzzi\(^{12}\), when a 400 kV, 200 kA relativistic electron beam is injected into a vacuum through a foil anode.
References

Fig. 1 — (a) Reflex triode and idealized potential well, and (b) phase bunching of electrons after several periods when $\omega \geq \omega_0$. 

Fig. 2 — Typical waveforms: (a) voltage at 90 kV/div, (b) x-band microwaves, (the reference signal has been delayed by 554 nsec), and (c) electron current at 7 kA/div.
Fig. 3 — Average frequency vs. the square root of the averaged applied voltage for three different cathodes of area: 55 cm$^2$ (solid); 89 cm$^2$ (thick annular); and 51 cm$^2$ (thin annular).
Fig. 4 — Average frequency vs. the square root of the averaged applied voltage for three anode-cathode spacings.