Project Report
Discrete Address Beacon System

ATC-73

J. W. Andrews

Air-to-Air Visual Acquisition Performance
with Pilot Warning Instruments (PWI)

25 April 1977

Prepared for the Federal Aviation Administration by

Lincoln Laboratory
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
LEXINGTON, MASSACHUSETTS

Document is available to the public through
the National Technical Information Service,
Springfield, Virginia 22151.
Subject pilot flight tests conducted at the M.I.T. Lincoln Laboratory have produced new data characterizing the ability of general aviation pilots to visually acquire potential collision hazards when aided by Pilot Warning Instruments (PWI).

In this paper major issues in the design of Pilot Warning Instruments are reviewed. Visual acquisition performance is described in terms of a non-homogeneous Poisson process and results of previous experiments are reinterpreted in this light. It is shown that the major test results can be explained in terms of an acquisition rate which is proportional to the solid angle subtended by the target. Model parameters appropriate for Lincoln Laboratory flight test data are derived by maximum likelihood techniques. A statistical analysis of significance is performed for other factors which are not explicitly included in this model. Performance predictions for a wide variety of aircraft sizes, approach speeds, and visibility conditions are presented.
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1. **INTRODUCTION**

1.1 **Visual Acquisition as a Means of Separation Assurance**

In the first few decades of flight safe separation between aircraft was achieved almost entirely by visual means. However, as aircraft speeds and traffic densities increased and as aircraft flew more often under conditions of restricted visibility, the need for alternative means of separation assurance became apparent. As a result a system of airspace structure and active Air Traffic Control (ATC) began to evolve. Today an extensive ATC system employing thousands of controllers exists in the United States with the primary objective of preventing collisions (or interference) between aircraft. This ATC system has been highly effective in ensuring separation between aircraft which participate fully in the system by filing flight plans and flying under Instrument Flight Rules (IFR). Unfortunately, a large number of general aviation aircraft do not or cannot participate at this level (often because of the expense of required avionics or the lack of necessary pilot training). These aircraft must operate under Visual Flight Rules (VFR) with essentially no help from the ATC system (except for traffic advisories issued on a "workload permitting" basis which means that they are generally unavailable when most needed). When IFR and VFR aircraft must share the same airspace, then all pilots must exercise "see-and-avoid" techniques in order to ensure safety. The control problems which result from such mixing of controlled and uncontrolled aircraft have resulted in a trend toward exclusion of uncontrolled aircraft from more and more segments of airspace. A genuine concern exists that this trend will greatly erode the viability of VFR flight as a flexible
and economical means of air transportation. Issues such as these have created a need for understanding pilot visual see-and-avoid performance in order to evaluate current safety practices and to explore options for future ATC system development.

For many reasons to be discussed more fully later, see-and-avoid as currently practiced is far from totally reliable as a means of separation assurance. Various approaches such as improved pilot training, airspeed limitations, and aircraft conspicuity enhancement have been taken to improve performance. Recently serious efforts have been directed toward the development of a pilot warning (or proximity warning) instrument (PWI) which would alert the pilot to the presence of a collision hazard and assist him in visually locating the aircraft in question. In particular, PWI is an integral part of the Intermittent Positive Control (IPC) collision avoidance system which the FAA has proposed for implementation in the next decade.

In the development of the PWI concept several questions have emerged as being of major concern:

How effective is the current see-and-avoid doctrine in ensuring separation?

How effective would a PWI system be?

Is PWI compatible with other separation assurance services (e.g., IPC avoidance commands)?

A basic goal of research in this area has been the formulation of a mathematical model which reliably reflects the air-to-air visual acquisition capabilities of pilots. One approach to the construction of such a
model is to adapt laboratory data characterizing human visual performance at the neurological level into an appropriate higher level model. More direct approaches have involved ground-based experiments in which pilots attempt to acquire targets projected upon screens while flying aircraft simulators. Both of these approaches are useful, but the many discrepancies between the visual stimuli of the laboratory and the actual environment make model validation under field conditions essential before confidence can be placed in the results.

In this study two sets of data gathered under actual flight conditions are examined. The second set of data, that gathered during the IPC/PWI flight tests at the M.I.T. Lincoln Laboratory, is of particular interest in that it involves typical pilots utilizing an actual PWI instrument (as opposed to the first set of data which involves test pilots informed of traffic by other means). Our analysis will be directed toward finding the simplest model which adequately reflects the essential features of air-to-air visual acquisition performance. This model will then be used to derive predictions of visual acquisition performance for a variety of conditions.

1.2 The Nature of the Visual Acquisition Task

The success of avoidance by visual means depends upon the proper performance by the pilot of an extremely difficult series of tasks. In addition to his other cockpit duties, the pilot must detect approaching threats, evaluate the situation, and react in an appropriate manner. In most cases where see-and-avoid fails it appears that "seeing" (acquisition) either did not occur or occurred too late for effective pilot reaction*.

*There are, however, a few documented cases where the pilot chose ineffective avoidance maneuvers even though detection had occurred with adequate lead time. (e.g., Carmel, New York; December 1965).
Acquisition performance is primarily a function of the following major factors:

1. **Search.** The term search refers to the scanning of the pilot's line-of-sight through the angular field of possible target locations in an attempt to acquire the target. When occupied by other cockpit duties the pilot cannot give constant attention to the visual search for traffic. Unless provided with specific information a pilot may concentrate his search in directions other than the direction of the threat*. Under certain circumstances even a few seconds inattention can mean the difference between an adequate and inadequate acquisition time.

2. **Field of View.** The typical airplane cockpit allows unobstructed view of only a fraction of possible approach directions. For slower aircraft which may be overtaken from behind this is an important consideration.

3. **Detectability.** The term detectability refers to those visual factors which determine whether a target will be seen which is in the searching pilot's field of view. In optical terms, the major factors are the target apparent size and the contrast of the target with its background. Other factors such as target motion with respect to the background may be considered in this category.

4. **Speed of Approach.** The speed of approach determines the time available for detection, and it is convenient to treat it as a separate independent factor.

*One problem is that of fixation upon one nearby aircraft at the same time that another more serious threat is approaching from another direction.
Attempts to improve visual acquisition performance may be classified in terms of their impact upon the four factors listed above.

1.3 PWI as an Aid to Visual Acquisition

Pilot warning instruments (PWI) are intended to aid the pilot in the visual acquisition task. Typically the approaching traffic is detected by electronic means, and the pilot is presented with information concerning the approach bearing and/or altitude of the threat. Fig. 1-1 depicts the PWI display currently associated with the Intermittent Positive Control (IPC) system. The outer ring of 36 lights correspond to three possible altitude bands (below, co-altitude, above) at 12 clock positions (30° sectors). Lights may be displayed as steady or flashing depending upon the time available for avoidance. The central part of the display provides collision avoidance commands. The relationship of the PWI function to the collision avoidance function will be discussed in Section 6.

The primary intent of PWI is to improve the search performance of the pilot. The PWI alarm ensures that scanning will be given high priority when it is most critical and by directing the pilot's search to a particular sector, the area to be scanned is greatly reduced. Another effect discovered in the IPC/PWI flight tests is the tendency of PWI to reduce the effect of airframe obstruction. Not only do pilots shift their positions within the cockpit in an effort to scan a threat sector, but many pilots alter the aircraft attitude in order to achieve an unobstructed view in the threat direction. Thus, PWI favorably affects the first two elements of acquisition (search and field of view). It does not alter detectability or speed of approach in any direct way.
Fig. 1-1. IPC display consisting of an outer ring of 36 PWL lights and an inner circle containing collision avoidance command symbols.
2. AIR-TO-AIR VISUAL ACQUISITION

2.1 Approach to Modelling Visual Acquisition Performance

The model of visual acquisition performance which is developed in this study is intended to facilitate an understanding of flight test results and to provide a means for mathematical analysis of visual acquisition performance and PWI design. It will be shown that a highly simplified model of the acquisition process is adequate for these purposes and that such a model can be adapted to various flight conditions. The following discussion identifies the manner in which the essential elements of the actual process are incorporated into the model.

2.2 The Visual Search Process

Under normal daylight visual conditions the probability of acquisition decreases with angular distance of the target from the observer's line of sight. This is due to the fact that the acuity of human vision is greatest when the object image falls upon the portion of the retina known as the fovea where the density of visual cones is greatest. The visual search process may be viewed as the movement of the line of sight from one position to another in an attempt to bring this line near enough to the target to allow detection.

Two major questions which must be considered are the amount of time the pilot devotes to searching for traffic and the angular distribution of his search time.

Angular Distribution of Search

Howell (Reference 4) found that unalerted pilots tended to concentrate their glances in the forward direction (within 30° of straight ahead), but that pilots who were informed that they were on a collision course spread their glances more evenly over the visible area. If the pilot possesses
a PWI system which directs him to search a limited area, it is likely that his glances will be distributed uniformly over that region. In the calculations to be presented later, it is generally assumed that search effort is uniform. It should be kept in mind that this assumption may require modification when applied to unalerted search.

Size of Search Area

For PWI systems it is generally assumed that the size of the search area is directly related to the resolution of the PWI bearing information. In practice pilots must allow for error in the PWI information due to measurement errors, wind (crab angle), quantization, projection of sector boundaries upon the environment, etc., and the resulting search area may vary considerably from individual to individual.

Time Devoted to Search

The percentage of time which a pilot devotes to looking outside the cockpit has been found to vary from a low of 22% for air carrier operations to a high of 52% for certain categories of small aircraft flight (Reference 2). These figures represent only an upper bound on the proportion of time which is spent actively scanning for traffic, since much of the time spent looking outside may be devoted to observing the weather or just sightseeing. In flight tests at Lincoln Laboratory it was estimated that after PWI alarms were noted, pilots devoted approximately 95% of their time to the search for traffic.

A Search Model

A standard approach to modelling the search process is the following: consider a search area $S$ with a target of negligible angular extent located
within this area. The search process consists of the movement of the line of sight from place to place within the search area in a series of fixations, each of duration $\tau$. Such fixations are often referred to as "glimpses". The probability that the target will be detected in a given glimpse depends upon the properties of the target itself (size, contrast, etc.) and the angular distance of the target from the line of sight. For a given target the detection probability is usually visualized in terms of an equivalent area, $a$, defined by:

$$a = \int_{S} P(\theta) 2\pi \theta \, d\theta$$

Where $P(\theta)$ = single glimpse probability of detection for a target at an angle $\theta$ from the line of sight. The single glimpse probability of detection is then $a/S$. If glimpse directions are distributed at random over a search area large with respect to $a$, one may define an acquisition rate $a/S\tau$ which then totally characterizes visual acquisition performance. It is important to note that although the detection area may vary in a complicated manner as the target characteristics change, the effect of the search area $S$ is merely to alter the acquisition rate by a fixed fraction.

### 2.3 Effects of Target Size and Contrast

Laboratory studies have shown that visual detection thresholds are principally a function of the product of target apparent size and target contrast with background (Reference 3). Often other considerations such as target shape and color can be modelled as altering the effective area or contrast.
For a target presenting a visible area $A$ at a range $r$ the apparent size can be defined in terms of the solid angle subtended by the target, $A/r^2$. This apparent size is influenced by the following factors:

(a) **Range**— Solid angle varies inversely as the square.

(b) **Aspect angle**— The visible area is generally less when the aircraft is viewed head-on than when it is viewed broadside. The broad surface of the wings is not visible unless the target is viewed away from the horizontal or unless the target banks. In flight tests at Lincoln Laboratory, the target sometimes went undetected until it rolled into a turn, at which point it was immediately detected.

(c) **Aircraft size**— The visible area is generally proportional to the actual size of the aircraft.

Target contrast is affected by many factors and may vary in an unpredictable manner during a single encounter. The term "contrast" is employed in a general sense to include those effects which make the total luminance (brightness) of the target area differ from that of an equal area of background. In this general sense contrast is affected by the following factors:

(a) **Background luminance**— This is primarily a function of the brightness of the sun and the position of the sun with respect to the aircraft. The same aircraft may appear as light or dark depending upon the angle of approach with respect to the sun. During the IPC/PWI flight tests, the test pilots pointed out that a slight amount of haze behind the aircraft presented a white background which made acquisition easier than under the blue background conditions of unlimited visibility.
(b) Reflections – Sunlight glinting off aircraft may aid detection, especially if aircraft turns and thus rotates the directions of specular reflections.

(c) Background complexity – The aircraft is usually easier to detect against a uniform background of sky rather than a complex terrain background. Properties of terrain background may vary with the season from summer green to autumn gold to winter white.

(d) Atmospheric visibility – The presence of haze of fog between the pilot and target results in an effective lowering of contrast. The magnitude of the effect is normally assumed to vary according to \( e^{-3.92 \frac{r}{R}} \) where \( R \) is the atmospheric visibility and \( r \) is the range to the target.
3. ACQUISITION PERFORMANCE ANALYSIS AND MODELLING TECHNIQUES

3.1 Acquisition as a Nonhomogenous Poisson Process

The results of visual acquisition experiments are often presented in terms of the cumulative probability of acquisition by a given range, time, etc. Different approach speeds or visual conditions produce different curves, and it is usually difficult to perceive the nature of the common acquisition process which underlies all such curves. In this section a mathematical model of the visual acquisition process will be developed which allows data collected under a wide variety of conditions to be analyzed in a common framework. Cumulative acquisition probability curves will then be derived from this model.

A simple yet highly adaptable approach is to assume that acquisition is a random process which may be described in terms of the probability of acquisition per unit time, hereafter referred to as the acquisition rate. The acquisition rate is denoted $\lambda(x)$ where $x(t)$ is a vector whose components are $k$ variables upon which the acquisition rate depends. As defined here, acquisition is a nonhomogenous Poisson process in which the first arrival (event) corresponds to acquisition and terminates the trial.

Other authors sometimes present visual detection data in terms of a "single-glimpse probability of detection". The relation of this quantity to acquisition performance is dependent upon the time duration associated with a "glimpse". By modelling acquisition as a continuous process it is assumed that only the time-averaged acquisition effort is needed to characterize performance and one need not be concerned with defining the duration of a glimpse.

*In the classic homogenous Poisson process, the arrival rate is assumed to be constant. All equations derived herein for the nonhomogenous process reduce to the classic equations when $\lambda$ is a constant.*
3.1.1 Relationship Between Acquisition Rate and Cumulative Acquisition Probability

The manner in which various properties of the acquisition process can be derived from a knowledge of \( \lambda \) will now be demonstrated. For convenience the symbol \( \lambda(t) \) will be utilized in place of \( \lambda(x(t)) \) for trials in which the time history of \( x \) (and consequently of \( \lambda \)) are defined.

Define the cumulative probability of acquisition, \( F(t) \), as the probability that acquisition occurs at a time less than or equal to \( t \). The probability density function for the acquisition time is then \( f(t) = \frac{dF}{dt} \). Consider a small time interval \( \Delta t \) centered at time \( t \). The probability that acquisition will occur in this interval is approximately \( f(t) \Delta t \). But this probability is also equal to the probability that acquisition did not occur before \( t \) multiplied by the probability that, given rate \( \lambda(t) \), acquisition will occur in a duration \( \Delta t \), i.e.

\[
f(t) \Delta t = [1 - F(t)] \lambda(t) \Delta t \text{ as } \Delta t \to 0 \quad (3-1)
\]

The resulting differential equation

\[
dF/dt = [1 - F(t)] \lambda(t)
\]

has solution

\[
F(t) = 1 - \exp \left[ - \int_{-\infty}^{t} \lambda(\xi) \, d\xi \right] = 1 - \exp \left[ -\eta \right], \text{ where } \eta = \int_{-\infty}^{t} \lambda(\xi) \, d\xi \quad (3-2)
\]

and consequently

\[
f(t) = \frac{dF}{dt} = \lambda(t) \exp \left[ -\eta \right] \quad (3-3)
\]
It is instructive to note that for the usual Poisson process the quantity $n$, the integrated acquisition rate, is the expected number of arrivals occurring by time $t$.

As will be seen in Section 4.1, it is possible in certain simple cases to plot $\lambda(x)$ directly from the data without postulating a specific parametric dependence upon $x$. However, a mathematical technique for finding a "best fit" expression is desirable for the following reasons:

1. Multi-variable dependences can be conveniently examined.
2. A mathematical expression for use in theoretical calculations is then available.
3. An error analysis can be performed upon the estimated parameters in order to evaluate the significance of differences between various sets of data.

3.1.2 A Linear Form for Acquisition Rate

Suppose that $k$ variables upon which the acquisition rate depends have been identified. Let $\lambda$ be modelled as a linear function of those variables, i.e.,

$$\lambda(x) = \sum_{i=1}^{k} \beta_i X_i = \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k$$

where the $\beta_i$ are parameters to be determined. By proper definition of the variables $X_i$, this model may be adapted to a variety of forms which are decidedly non-linear with respect to the original data. For example, if range is a given quantity, define $X_1 = 1/r^2$ and thus obtain a linear form which models a dependence upon the inverse square range. Furthermore, note the correspondence of the following forms:
where $X_1' = X_1/r^2$ and $X_2' = X_2/r^2$.

3.2 Maximum Likelihood Estimation of Model Parameters

In order to estimate the values of the parameters $\beta_j$ which best fit the data, the method of maximum likelihood will be utilized. This method attempts to find those values of $\beta_j$ which maximize the computed probability of obtaining the given set of experimental results.

Suppose that trial $i$ begins at time $t=0$ and continues until either acquisition occurs or until some "break-off" time $t=T_i$ is reached. The addition of the break-off time allows one to study experimental trials which were not pursued to acquisition or to study only specific periods within trials. It may be seen by reference to equation 3-2 that the probability of termination at $t=T_i$ (without acquisition) is

$$
\exp \left[ -\eta(T_i) \right] \text{ where } \eta(T_i) = \int_0^{T_i} \lambda(\xi) \, d\xi \tag{3-7}
$$

For those trials in which the termination time is within the interval $0$ to $T_i$ (with acquisition) the probability density (from equation 3-3) is

$$
\lambda(t) \exp \left[ -\eta(t) \right] \quad 0 \leq t < T_i \tag{3-8}
$$
Thus the probability density of termination times has both discrete and continuous components and is given by

\[
\begin{align*}
\exp[-\eta(t)] & \quad t = T_i \text{ (discretely)} \\
\lambda(t) \exp[-\eta(t)] & \quad 0 \leq t < T_i \text{ (continuously)} \\
0 & \quad \text{elsewhere}
\end{align*}
\]

(3-9)

For \( n \) independent trials the likelihood function which is to be maximized is just the product of the \( n \) individual functions evaluated at the observed termination times and may be written

\[
L(t_1, t_2, \ldots, t_n) = \prod_{i=1}^{n} \begin{cases} 
\exp[-\eta(t_i)] & \text{no acquisition} \\
\lambda(t_i) \exp[-\eta(t_i)] & \text{acquisition}
\end{cases}
\]

(3-10)

where \( t_i \) is the termination time for trial \( i \) (equal to \( T_i \) if the trial ended without acquisition but within the interval 0 to \( T_i \) otherwise).

In Appendix A the procedure for finding the \( \beta_j \) which maximize this expression is detailed. It is shown that for the simple case in which \( \lambda \) depends upon only one variable, i.e., \( \lambda(x) = \beta_1 x \), the maximum likelihood estimate of \( \beta_1 \) is

\[
\beta = \frac{N}{\sum_{i=1}^{n} \int_{0}^{t_i} X_i(\xi) d\xi}
\]

(3-11)

where \( N \) is the number of the \( n \) trials which ended in acquisition and \( t_i \) is the termination time for trial \( i \). This result will prove immediately useful in the next section.
4. EXAMINATION OF A PREVIOUS STUDY

4.1 Interpretation of CDC Flight Test Data

In early 1972 the Control Data Corporation conducted a series of photographic flights to obtain film for use in a visual detection simulation. During these flights the ranges at which the pilots in each aircraft detected the other was recorded. A table of detection ranges and approach speeds is given in Reference 5 and this data will be examined in order to compare these results to those obtained during the IPC/PWI flight tests. In presenting this data, the following experimental condition should be noted:

- For photographic reasons, the geometry of the encounter was controlled so that one of the aircraft being photographed would approach from the 9 to 12 o'clock position and did not approach from the sun.
- All encounters were flown at an altitude separation of 500 feet.
- The pilots were aware of the encounter geometry and were familiar with the appearance of the other aircraft.

One factor which is not known with regard to the CDC data is the visual search area which had to be scanned in order to detect the target. This area depends upon the precision with which the pilot could anticipate the approach bearing of the other aircraft. This point will be mentioned later when the CDC data is compared to the data gathered in the Lincoln flight tests.

The range dependence of the acquisition rate can be extracted from the available tabular data in the following non-parametric manner: divide the range axis into intervals of width $\Delta r$. For each interval determine
the total time during which an undetected target was in that interval and
the number of detections which occurred in that interval. Then the estimate
of the acquisition rate for the interval is given by

\[
\text{acquisition rate} = \frac{\text{total no. detections in interval}}{\text{total time in interval}}
\]

The choice of the interval width \( \Delta r \) is arbitrary, but should repre-
sent an appropriate balance between the granularity of the estimate and the
number of data points available for smoothing.

Three aircraft were involved in the CDC flights, and the resulting
detection rate curves for each are presented in Figs. 4-1, 4-2, and 4-3.
In each case a curve of form \( \beta/r^2 \) (\( \beta \) = constant) is presented. The cor-
respondence between the curves and the data is striking. Recall from sec-
tion 2.3 the assertion that detectability is mainly a function of the product
of size and contrast. These results suggest that the \( \beta/r^2 \) curve represents
this product. Note for instance that the Musketeer and Aztec are of length
25 and 30 feet, respectively, while the Gulf Stream is of length 80 feet.
If one assumes that the visible area is roughly proportional to the length
squared, then the ratio of visible areas is \((80/25)^2 = 10.24\). This is
approximately the ratio of the observed coefficient \( \beta \) for the Gulf Stream
vs. the smaller aircraft. Thus the data collected during the CDC photo-
graphic flights suggests that for fully informed and alerted professional
pilots searching continuously for known traffic, the acquisition rate
increases inversely as the square of the target range. Data concerning the
performance of subject pilots who were informed of approaching traffic by
PWI systems will now be examined.
Fig. 4-1. Acquisition rate for detection of Musketeer (derived from CDC flight test data, Reference 5).
Fig. 4-2. Acquisition rate for detection of AZTEC (derived from CDC flight test data, Reference 5).
Fig. 4-3. Acquisition rate for detection of Gulf Stream (derived from CDC flight test data, Reference 5).
4.2 Results of Photographic Simulation

The film produced by the photographic test flights was utilized in a visual detection simulation conducted at the DOT Transportation System Center in Cambridge, Mass. (Reference 6). Subject pilots selected from the aviation community flew a GAT simulator while slides depicting the view outside the cockpit were projected onto a set of screens surrounding the cockpit. The results of this study are presented in terms of a cumulative probability of detection for given time-to-closest approach, $t_c$. If it is assumed that the range between aircraft is roughly proportional to $t_c$, then one might expect to observe an acquisition rate proportional to $1/t_c^2$. The acquisition rate observed in the simulation study may be derived from the curve of cumulative probability of detection via equation (3-1). The rate so derived is shown in Fig. 4-4. Note that although the acquisition rate initially follows the inverse square form at larger $t_c$, the acquisition rate appears to drop at small $t_c$. This counter-intuitive behavior must be understood in terms of the differences between the ideal conditions for acquisition to which the inverse square performance corresponds and the non-ideal cases which may arise in other situations.

In order to illustrate this point, consider a case in which 90% of the pilots are able to make an effective search for traffic and 10% cannot. The reasons for being unable to search effectively are many. The pilot may be experiencing difficulty in flying the aircraft and may be concentrating upon flying to the virtual exclusion of scanning for traffic, or the direction of approach of the target may be obstructed. In the film-based simulation, the target sometimes passed out of the field of view of the camera before reaching the point of closest approach. At large $t_c$
Fig. 4-4. Acquisition rate for subject pilots in photographic simulation of visual acquisition.
when few acquisitions have yet occurred, the observed total acquisition rate is dominated by the 90% of the population which searches effectively. At small $t_c$, however, almost all of the effective searchers have acquired and the observed rate is dominated by those pilots who are having special difficulties in acquiring. In any statement concerning overall visual performance, it is, therefore, important to specify the proportion of cases in which acquisition is degraded from that corresponding to the ideal situation.
5.0 MODEL DEVELOPMENT BASED UPON IPC FLIGHT TESTS

5.1 Lincoln Laboratory IPC Flight Tests

During 1975 and 1976 a number of pilots selected from the general aviation community were invited to participate in the Intermittent Positive Control (IPC) flight test program conducted at the M.I.T. Lincoln Laboratory. Although the principal goal of these tests was validation of a particular collision avoidance system, the data collected and the IPC data base capability developed at Lincoln Laboratory can be used to obtain valuable insight into the ability of pilots to visually acquire traffic under actual flying conditions.

5.1.1 The IPC/PWI System

Intermittent Positive Control (IPC) is an automated ground-based collision avoidance system which utilizes the radar position reports of the Discrete Address Beacon System (DABS) in order to detect potential collision hazards. Collision avoidance messages are transmitted to aircraft via the DABS data link and displayed via the special IPC display shown in Fig. 1-1. Two levels of service are available: PWI and commands. PWI information is displayed in an outer ring of lights arranged in twelve groups of three which correspond to the threat clock position (12 o'clock straight ahead, 3 o'clock off right wing, etc.) and the threat relative altitude (top light for threat 500 to 2000 feet above, middle light for 500 feet above to 500 feet below, bottom light for 500 feet below to 2000 feet below). PWI lights at any position may be of two types: ordinary (steady) indicating traffic which is nearby but not urgent, and flashing indicating traffic which presents an imminent hazard. An aural alarm (tone) occurs whenever a PWI alert appears for the first time. Avoidance commands are displayed
via arrows (signifying the direction in which a pilot should turn or change altitude) and X's (signifying that a pilot should refrain from turning or changing altitude in the indicated direction).

The IPC algorithm (Reference 10) which resides in the ground computer issues alarms based upon violation of certain range and time-to-collision thresholds. These thresholds are chosen so that commands normally appear 15 seconds or more after the first PWI alarm.

5.1.2 **Flight Test Methodology**

Subject pilots were selected to represent a wide range of aviation backgrounds and levels of experience. Each pilot was given a brief period to familiarize himself with the test aircraft and then was asked to fly a simple cross-country course of about one hour duration. During this flight an interceptor aircraft conducted 6 - 7 intercepts upon the subject aircraft. The subject aircraft was either a Cessna 150 or a Piper PA-28 and the interceptor aircraft was either a Cherokee 180 or a Beechcraft Bonanza. Subject pilots were in voice contact with the test control room at all times and were asked to immediately report all traffic sightings. The time of each sighting of the interceptor was recorded by scan (one DABS antenna scan is approximately 4 seconds duration).

The following data were recorded on magnetic tape:
- precise times for all data-link transactions
- aircraft radar positions for each scan (including altitude reports)
- IPC/PWI messages for each scan
- aircraft position and velocity estimates from the IPC tracker for each scan
5.1.3 **IPC Data Base Capability**

In order to provide for the effective analysis of the large volume of data so produced, an IPC data base capability was developed so that data for all IPC encounters could be made available for analysis in a single processing environment. In order to accomplish this goal software was written which scanned the tapes from each IPC mission and selected data from periods of critical IPC activity. This selected data was then transferred to a single data base tape. Handwritten notes produced during the mission were coded and typed into a data file on magnetic disk. Software was then written which matched tape data and disk data and provided both to the user simultaneously under the IBM 370 Conversational Monitor System (CMS).

5.2 **Non-Parametric Presentation of IPC/PWI Flight Test Data**

It is possible to construct for the IPC/PWI flight test data a non-parametric plot of acquisition rate, $\lambda$, just as was done for the CDC flight test data and the simulation data. But rather than plot $\lambda$ against target range, $\lambda$ will be plotted versus the solid angle subtended by the target. Calculation of the solid angle requires an estimate of the visible area of the target as well as its range. An algorithm for the approximate calculation of target visible area has been developed for this purpose and is described in Appendix B. The resulting $\lambda$ plot is shown in Fig. 5-1. One-sigma estimation errors are shown for each data point. The expected linear relationship between $\lambda$ and the solid angle is indeed evident.
Fig. 5-1. Acquisition rate for alerted pilots from IPC flight test data.
At this point it is assumed that the usefulness of the proposed model which assumes a linear relationship between acquisition rate and target solid angle (i.e., $\lambda = \beta A/r^2$) has been demonstrated. Values of $\beta$ which are appropriate for different search conditions may now be derived from test data. This relationship may also be used for the study of the significance of factors which are not explicitly included in the model. A useful tool in this regard is the special statistic defined in the next section.

5.3 Scanwise Factor Analysis: The $z$ Statistic

Suppose that special search conditions prevailed for $m$ scans of data, and that it is desired to determine whether acquisition performance for those $m$ scans is better or worse than expected from the proposed model. Let the probability as calculated from the model that acquisition will occur on a given scan $j$ be $p_j$. When the acquisition rate $\lambda_j$ is small $p_j = \lambda_j T$. Otherwise write $p_j = 1 - \exp(-\lambda_j T)$. Define an acquisition indicator $y_j$ such that

$$y_j = \begin{cases} 0 & \text{if no acquisition occurred on scan } j \\ 1 & \text{if acquisition did occur on scan } j \end{cases}$$

For the $m$ selected scans consider the sum

$$s = \sum_{j=1}^{m} y_j = \text{number of acquisitions which occurred during the } m \text{ scans}$$

Since each $y_j$ is assumed to be a statistically independent random variable the variance of the sum is simply the sum of the individual variances,
\[
\sigma_S^2 = \mathbb{V}[S] = \sum_{j=1}^{m} p_j (1 - p_j)
\]

The expected value of \( S \) is likewise

\[
E[S] = \sum_{j=1}^{m} E[Y_j] = \sum_{j=1}^{m} p_j
\]

The \( z \) statistic is now defined as the deviation of the actual number of acquisitions from the number expected from the model (expressed in standard deviations of \( S \)), i.e.,

\[
z = \frac{S - E[S]}{\sigma_S}
\]

Since there are typically a large number of scans in each analyzed group, it may be assumed (by the central limit theorem) that the distribution of \( z \) approximates that of a standard normal random variable. This is true however only when the acquisition performance is well characterized by the model employed. When the \( z \) statistic differs from zero by several units a significant departure from the assumed model is indicated.

5.3.1 Analysis of Target Area Dependence

The basic model which has been adopted assumes that the acquisition rate is proportional to the solid angle subtended by the target and hence, for a given range, is proportional to the visible area of the target. Since the visible area varies greatly with the aspect angle, it then becomes necessary to incorporate visible area into the model for the acquisition rate. For this reason an algorithm for the approximate calculation of visible area was developed (see Appendix B). As a check upon
both the assumption of visible area dependence and the area calculation algorithm, the z statistic may be utilized for the analysis of visible area dependence. For this purpose the model must regress slightly to a simpler formulation which assumes that the acquisition rate is a function of range only, i.e.,

\[ \lambda = \frac{\beta}{r^2} \text{ where } \beta = 0.10 \text{ nm}^2/\text{sec}^* \]

All available scans of data are then divided into groups corresponding to 25 square foot intervals of calculated visible area, and one then finds the z-statistic for each group. The results of this analysis are given in Fig. 5-2. There is indeed a trend toward increasing z with increasing visible area. Thus, the z-analysis supports the assumption that the acquisition rate should include a dependence upon visible area and not range alone.

5.3.2 Analysis of Cockpit Visibility Effects

In Section 1.2 the visibility obstructions present in the aircraft cockpit were mentioned as factors in visual acquisition failures. In the analysis of visual acquisition performance it is helpful to identify those cases in which acquisition was hindered by the cockpit visibility limitations. Figure 5-3 is a plot of the visibility from the Piper PA-28 cockpit. Note the greater angular area which exists on the side of the aircraft on which the pilot is sitting and the decreased visibility for approaches rear of the opposite (right) wing. Unfortunately, such plots are valid for only a single position of the pilot within the cockpit. Since pilot position within the cockpit may vary considerably (especially if the pilot is

*This value of \( \beta \) was obtained from inspection of Figs. 4-1 and 4-2. Its exact value is not critical to the question under consideration.
Fig. 5-2. Statistical Analysis of dependence of acquisition rate upon visible area (based upon inverse square range model).
Fig. 5-3. Approximate cockpit visibility for pilot in left seat of PA-28.
alerted to an approaching threat within an obstructed sector) the recorded test data is inadequate to reliably determine whether the pilot's view is obstructed on a particular scan. However, the z-statistic may be utilized to examine the statistical dependence of the acquisition rate upon the approach bearing, and thus evaluate the severity of impact of obstructions upon acquisition performance in terms of performance variations with approach direction.

In Fig. 5-4 the z-statistic is plotted versus target approach bearing (in 30° intervals). Note that very little degradation in performance occurs for bearings between -160° and +90°. However, beyond +90° there is a significant trend toward poorer acquisition performance. This is understandable since subject pilots sat in the left seat of the test aircraft. When alerted a pilot could turn to search areas quite far to his left, but there was little he could do to overcome the visibility problems posed by aircraft approaching from his right.

5.4 Estimation of Model Parameter Values

The IPC flight test data will now be utilized to estimate the values of the constant \( \beta \) in the expression for the visual acquisition rate. \( \hat{\beta} \) will be estimated by applying equation (3-11) in the form

\[
\hat{\beta} = \frac{1}{n} \sum_{i=1}^{N} \left( \frac{A}{r^2} \right) \mathrm{d}x
\]  

(5-2)

Since the value of \( \beta \) may vary as the conditions of the search are changed, different values of \( \beta \) must be calculated for different search conditions. When a specific condition is specified then the value of N
Fig. 5-4. Statistical analysis of dependence of acquisition rate upon approach bearing (based upon solid angle model).
in Equation 5-2 must include only those acquisitions occurring while the condition was satisfied. Similarly the integral in the denominator is restricted to those time intervals in which the specified condition was satisfied.

Search conditions will be specified in the following terms:

1. Approach azimuth - As the analysis of Section 5.3.1 has shown the visibility of the pilot is severely restricted beyond +120° in azimuth. We will therefore provide an option for separate consideration of scans upon which the target azimuth exceeded +120°.

2. Alert status - A pilot is said to be unalerted if he has received neither a flashing nor non-flashing PWI alert. He is said to be in the PWI-only state if he has received a PWI alert but no command. He is in command state if he has received positive or negative IPC collision avoidance commands. For the IPC system, the presence of commands implies that a PWI indication is also present.

Derived β values for several sets of search conditions are displayed in Table 5-1. The values of cases 1 and 2 are of greatest interest since they correspond to "pure" alerted and unalerted and alerted search which is little modified by airframe obstruction. The lower value of β during command periods (case 3) reflects the fact that with commands the pilot was forced to devote a considerable fraction of his time to monitoring the display, acknowledging, and maneuvering the aircraft. The motion of the aircraft during turns also causes the PWI position to lag the actual target bearing
### TABLE 5-1

\( \beta \) Values for Selected Search Conditions

(Approach Azimuths from \(-160^\circ\) to \(+120^\circ\))

<table>
<thead>
<tr>
<th>Case</th>
<th>Alert Status</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Unalerted</td>
<td>( 1.1 \times 10^4 / \text{sec} )</td>
</tr>
<tr>
<td>2</td>
<td>PWI-only</td>
<td>( 9.0 \times 10^4 / \text{sec} )</td>
</tr>
<tr>
<td>3</td>
<td>Commands (with PWI)</td>
<td>( 3.4 \times 10^4 / \text{sec} )</td>
</tr>
</tbody>
</table>
with a consequent decreased usefulness of the PWI information. The acquisition rate during command periods is thus approximately one-third of the value for the PWI-only periods.

We may now compare these values with the corresponding values for the CDC flight test data of Figs. 4-1, 4-2, and 4-3. The comparison cannot be exact since we must assume a value for the visible area for the CDC curves. For a target visible area of 75 square feet the value of $\beta$ would be:

$$\beta = \frac{0.15 \text{ nmi}^2/\text{sec}}{75 \text{ ft}^2} = 7.4 \times 10^4/\text{sec}$$

This value is consistent with the $\beta$ value of subject pilots alerted by PWI ($\beta = 9 \times 10^4/\text{sec}$).
6. VISUAL ACQUISITION PERFORMANCE PREDICTIONS

6.1 Techniques for Adapting Model to Various Search Conditions

The results of the preceding section will now serve as a basis for the construction of a model of visual acquisition which will be used to predict acquisition performance for a wide range of conditions. The basic observation upon which the model is based is that visual acquisition is characterized by an acquisition rate, \( \lambda \), which is proportional to the solid angle of the target, i.e.,

\[
\lambda = \beta \frac{A}{r^2}
\]  

(6-1)

where

\( \beta \) = a model parameter depending upon the search conditions

\( A \) = the visible area of the target

\( r \) = the range from observer to target

Since the value of \( \beta \) depends upon search conditions, a complete computational model must alter \( \beta \) when search conditions change (e.g., when the pilot moves from unalerted to alerted status). It is also desirable to specify certain modifications which allow the model to be adjusted for search conditions which differ slightly from those for which test data is available. In the paragraphs which follow several such modifications are suggested. Although caution must be exercised in modifying the model for conditions which differ greatly from those for which it has been validated, the results so obtained still prove useful for a first investigation of parametric dependencies.

Modification Due to Fractional Search Time

Suppose that the value of the model parameter corresponding to constant search is \( \beta_0 \). If the pilot spends only a fraction \( k \) of his time searching
then the effective value of $\beta$ will be lower than $\beta_0$ by the factor $k$, i.e., $\beta = \beta_0 k$. This simple correction is valid when the pilot frequently glances from inside the cockpit to outside, and thus spends a relatively uniform amount of time searching at each target range interval. If the pilot spends long intervals looking inside the cockpit, then the distribution of durations of these intervals must be taken into account.

**Modification for Search Area Size**

As argued in Section 2.2, the acquisition rate should be inversely proportional to the size of the angular area which the pilot is searching. In the IPC/PWI flight tests, the PWI instrument resolution was $30^\circ$ and the pilots were briefed to expect targets to appear occasionally in sectors adjacent to the alarm sector due to normal system errors. Although the actual area searched varied from pilot to pilot, most pilots seemed to search a sector of $60^\circ$ to $90^\circ$ centered upon the sector of the PWI alarm. If we denote the area of the search for the IPC/PWI system as $S_0$, then the acquisition rate for a system with search area $S$ must be modified by the ratio $S_0/S$.

**Modification for Atmospheric Visibility**

In Section 2.3 data was quoted which indicated that targets with similar contrast-size products possessed similar detection properties. Since the effect of atmospheric visibility is to decrease contrast the net effect upon the acquisition rate should be to reduce the rate by the same proportion. A corresponding increase in target size is then required to restore the original contrast-size product. Thus, one may utilize the contrast reduction, $\exp\left[-\frac{3.92r}{R}\right]$, as a factor in modifying Equation 6-1.
Conditions Requiring Zero Acquisition Rate

In applying the Equation 6-1 there are two conditions under which the acquisition rate must be set to zero. The first and most obvious condition is when the target is obstructed by the airframe, and therefore is not within the pilot's field of view. The second condition is when the size of the target is less than the resolution limit of the human eye. For conditions of daylight brightness (1000 foot lamberts) this limit is approximately one minute of arc.

The values to be used for the model parameters may be based upon Table 5-1. Cumulative probabilities of acquisition can then be calculated via Equation 3-2 using numerical integration. The model described above is summarized in Table 6-1. It should be noted that no attempt has been made to specify the precise value of $S_0$ to which the $\beta_0$ values correspond. This is of no consequence if one wishes to predict performance for a system with the same resolution as the IPC/PWI system (then $S/S_0 = 1$). For a system with a different search sector size however, a value for the ratio $S/S_0$ must be postulated.

**TABLE 6-1**

<table>
<thead>
<tr>
<th>Acquisition Rate</th>
<th>Modified acquisition model for use in performance calculations (see text for explanation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda = \frac{k}{(S/S_0)} \exp \left( -\frac{3.92r}{R} \right) \frac{\beta_0 A}{r^2}$</td>
<td></td>
</tr>
<tr>
<td>Cumulative Probability of Acquisition:</td>
<td></td>
</tr>
<tr>
<td>$P_a(t) = 1 - \exp \left[ -\int_0^t \lambda (\xi) , d\xi \right]$</td>
<td></td>
</tr>
<tr>
<td>Parameter Values Derived from IPC/PWI Data</td>
<td></td>
</tr>
<tr>
<td>$\beta_0 k = 1 \times 10^4$/sec for unalerted pilots</td>
<td></td>
</tr>
<tr>
<td>$\beta_0 = 9 \times 10^4$/sec for alerted pilots $(k = 1)$</td>
<td></td>
</tr>
</tbody>
</table>
For the following series of calculations it is assumed that aircraft are flying on collision courses (zero miss distance) at constant altitudes with constant headings. The closure rate and the visible areas presented by each are then constant. It will be assumed that the PWI system alerts the pilot whenever the range or the time until collision drops below specified thresholds. (For the IPC/PWI system typical thresholds are 2 nautical miles and 45-75 seconds. Because of the delay required for message transmission and display, it is assumed that the period of alerted search begins at 40-70 seconds before collision).

6.2 Single Pilot Search

Acquisition performance for encounters between two single engine general aviation aircraft with airspeeds of 100 knots (a typical cruising speed) will be considered first. For zero miss distance trajectories the closure speed and the visible area can be determined as a function of the crossing angle (difference in headings) of the aircraft. Figure 6-1 gives the cumulative probability of acquisition for an unalerted pilot for encounters at various crossing angles. The increased closure rates and decreased visible areas which are associated with the larger crossing angles dramatically decrease the probability of acquisition with sufficient lead time. These curves indicate that separation by unaided visual means can be highly effective only for the lowest crossing angles.

Figure 6-2 provides acquisition performance curves for the same encounter trajectories except that now it is assumed that the pilot is alerted by a PWI system so that alerted search begins at 40 seconds before collision. (Thus, from Table 5-1, the acquisition rate increases
Fig. 6-1. Predicted acquisition performance for unalerted search by single pilot.
Fig. 6-2. Predicted acquisition performance for PWI alert at 40 seconds to collision (single pilot).
by a factor of nine at \( t = 40 \). For the larger crossing angles the probability of acquisition with sufficient lead time is increased severalfold. However, it still appears that acquisition performance is adequate only for lower crossing angles (below 90°).

A general set of curves which can be readily applied to a range of encounter situations can be derived if we assume that acquisition does not occur before the pilot is alerted. (Otherwise, the PWI alert is superfluous). If the aircraft are on a collision course so that the range is \( r(t) = -\dot{r} t \) where \( \dot{r} \) is the range rate and \( t \) is the time until collision and if search begins at \( t_0 \), then the probability of no acquisition by time \( t \) is

\[
\exp \left[ - \int_{t_0}^{t} \frac{\beta A}{(\dot{r} t)^2} \, dt \right] = \exp \left[ \frac{T_a}{t_0} \right] \exp \left[ -\frac{T_a}{t} \right]
\]

where \( T_a = \frac{\beta A}{\dot{r}^2} \) is a constant for a particular encounter. Two sets of curves depicting the resulting probabilities of acquisition for different alert times are presented in Figs. 6-3 and 6-4. The constant \( T_a \) is a characteristic acquisition time since it represents the time-to-collision at which the probability of no acquisition drops to \( e^{-1} \) (36.8%) for a search begun at infinity \( (t_0 = \infty) \). The quantity \( e^{T_a/t_0} \) is the factor by which the probability of no acquisition has been increased by failure to begin searching at infinity. This expression is relevant to the determination of suitable PWI warning thresholds. Suppose for instance that acquisition failure is said to occur whenever the pilot fails to acquire before some critical time-to-collision, \( t_1 \). Let the PWI system alert the
Fig. 6-3. Generalized acquisition performance curve for alerted search beginning at 40 seconds to collision.
Fig. 6-4. Generalized acquisition performance curve for alerted search beginning at 70 seconds to collision.

\[ T_a = \frac{\beta A}{r^2} \]
Fig. 6-5. Probability of late acquisition as a function of warning time ($t_1 =$ minimum adequate time-to-collision of acquisition, $T_a = 8A/f^2$).
pilot at time $t_0 = kt_1$. The probability of failure to acquire by $t_1$ is then

$$\exp \left[ -\frac{T}{t_1} \begin{bmatrix} -k \frac{1}{k} \end{bmatrix} \right]$$

The resulting failure rate is plotted in Fig. 6-5. It is seen that no matter how early the pilot is alerted, it is impossible to decrease the failure rate below $e^{-T/a}$. Furthermore, as $k$ is increased beyond 3 or 4, very little decrease in the failure rate is to be expected for critical cases. Thus, a reasonable objective for a PWI system which wishes to achieve near maximum effectiveness without excessively early alarms is to provide a search time which is about three times the minimum acceptable acquisition lead time. If acquisition must occur by 15-20 seconds before collision, then a warning threshold of 45-60 seconds is appropriate.

6.3 Two Pilot Search

Up to this point visual acquisition performance has been discussed in terms of a single searching pilot. However, at least two pilots are involved in every collision, and it might be assumed that if either pilot acquires his traffic he will act to avert a collision. In that case, one must consider the cumulative probability for acquisition by at least one of two pilots. In order to do this, note that the probability of no acquisition by pilot $#1$ at by time $t$ may be written

$$P_1(t) = \exp \left[ -\int_0^t \lambda_1 (\xi) \, d\xi \right] \text{ where } \lambda_1$$

is the acquisition rate for pilot 1 against aircraft 2. Similarly, for pilot 2 against aircraft 1 write

$$P_2(t) = \exp \left[ -\int_0^t \lambda_2 (\xi) \, d\xi \right]$$
Fig. 6-6. Probability of acquisition by either pilot (two small aircraft, alerted search beginning at 40 seconds to collision).
Then, under the assumption that the acquisition probabilities are statistically independent, the probability of no acquisition by either aircraft becomes

\[ P_1 P_2 = \exp \left[ -\int_0^T \left( \lambda_1 (\xi) + \lambda_2 (\xi) \right) \, d\xi \right] \]

Thus, the effective rate for acquisition by either of two pilots is just the sum of the acquisition rates for each pilot separately. Acquisition probabilities for two pilot search are presented in Fig. 6-6 for two 100 kt aircraft. The marginal performance at higher crossing angles that was noted in previous cases is still evident.

Consider now two-pilot curves for encounters between a large jet (e.g., Boeing 727) and a single engine general aviation aircraft (e.g., Piper PA-28). Let the small aircraft have an airspeed of 100 kts and the larger an airspeed of 250 kts. The resulting cumulative probabilities of acquisition are given in Fig. 6-7. Note that although the closure rates are now greater due to the speed of the jet aircraft, the increased size of the faster aircraft more than compensates for this (e.g., head-on the closure rate is a factor 350/250 = 1.75 greater, but the visible area is a factor 330/20 = 16 greater).

Before becoming too encouraged by this trend one should note that the disparate speeds of the aircraft can lead to special difficulties. For instance, in the previous case of equal speed aircraft, the approach bearings of the threat tended to be within 90° of the aircraft nose, thus allowing a high probability of unobstructed viewing by both pilots. In the current case, as crossing angles drop below 45°, the faster aircraft tends to approach from behind at bearings for which obstruction is a
Fig. 6-7. Probability of acquisition by either pilot (small slow aircraft vs. large fast aircraft, alerted search beginning at 70 seconds to collision, unobstructed approach).
problem. Obstruction is especially likely if the faster aircraft approaches from the side opposite to the pilot. When such obstruction exists, the pilot who is attempting to acquire the large target (330 sq. ft.) cannot search effectively and the responsibility for acquisition rests upon the pilot who must search for a small target (20 sq. ft.). In this situation one must employ single-pilot acquisition curves and utilize a small value for the visible area. As an example, for the "tail chase" overtake rate of 150 kts, we may apply the curves of Fig. 6-5 to find that the overtaking pilot has only a 77 per cent chance of acquiring the small aircraft before 15 seconds to collision.

A related consideration which applies to the unobstructed cases is that because of the size differences the small aircraft is much more likely to acquire the larger than vice-versa (range and range-rate being necessarily equal for the two.) Experience obtained during the IPC flight tests indicates that it is more difficult to evaluate and avoid a threat that is faster than oneself than it is to evaluate and avoid threats of equal or slower speed. Whether or not this consideration is significant in offsetting the benefits of the earlier acquisition times achieved against larger aircraft is beyond the scope of this investigation.

6.4 PWI Compatibility Considerations

The IPC system provides PWI service and collision avoidance* service via a common display. Both services are intended to aid the pilot in avoiding other aircraft, but each achieves its goal in a quite different manner. The two services can be complementary, but they can also interfere with each

*The term collision avoidance system is used to designate a system which provides instructions (commands) to the pilot which tell him to maneuver in specific directions to avoid a collision.
other. In Section 5.4 it was indicated that the issuance of collision avoidance commands decreased the rate of visual acquisition by returning the pilot's attention to the instrument panel and forcing turns which induce bearing lag in the PWI indication. Collision avoidance commands can also make visual avoidance redundant if commands are consistently issued at ranges which are too great for visual avoidance to play a role. On the other hand, pilots may be reluctant to follow commands once the PWI has allowed them to visually acquire their traffic. This is especially true if the commands turn them in a way that breaks visual contact.

The visual acquisition model developed in the study may be utilized to allow a quantitative evaluation of the relationship between collision avoidance system parameters and PWI utility. A further discussion of these issues is available in the IPC flight test documentation (References 11 and 12).
7. CONCLUSIONS

It has been shown that despite the inherent complexity of the air-to-air visual acquisition task, the available test data can be well modeled statistically as a Poisson process in which the acquisition rate is proportional to the solid angle subtended by the target for which the pilot is searching. The constant of proportionality as determined from the IPC flight tests is 10000/steradian-sec for unalerted search and 90000/steradian-sec for alerted search (thus the presence of a PWI alert increases the acquisition rate by a factor of nine). Consideration of the nature of the acquisition process suggests that this basic result can be modified to account for the effects of other variables such as fraction of time devoted to search, angular area of search, and atmospheric visibility.

The model has been applied to the prediction of visual acquisition performance for typical encounter situations and suggests the following conclusions:

(1) Unaided visual acquisition is effective as a means of separation assurance only for lower values of crossing angles (relative heading). At higher values of crossing angle the increased closure speeds and decreased visible areas reduce performance considerably.

(2) PWI alarms increase the probability of acquisition by several-fold for the most adverse conditions, but fail to achieve more than a 50-80% cumulative probability of acquisition with adequate warning time.

(3) In typical cases the increased size of jet transport aircraft more than compensates for their increased speed, resulting in their being acquired at greater ranges than smaller, slower
aircraft. However, large speed differences increase the probability of a tail-chase encounter in which the faster aircraft overtakes the slower from a bearing in which pilot view is obstructed.

(4) Increasing PWI warning times beyond 40-60 seconds to collision has little effect upon the ultimate probability of acquisition since the angular size of the target at earlier times is much less.

(5) Issuance of IPC commands appears to interfere with the visual acquisition process, lowering the acquisition rate by a factor of three. This is probably due to the fact that commands return the pilot's attention to the instrument panel and result in turns that create PWI bearing lag.
APPENDIX A

MAXIMUM LIKELIHOOD ESTIMATION OF MODEL PARAMETERS

In Section 3-2 the dependence of the acquisition rate \( \lambda(x) \) upon the vector \( x \) was written in the parametric form

\[
\lambda(x) = \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_m x_m
\]  

(A-1)

The proper values of the parameters \( \beta_j \) for a given set of data is to be found by the method of maximum likelihood. The maximum likelihood estimates of the \( \beta_j \) are those values which maximize the likelihood function of Equation 3-10. This function may be written in the following form:

\[
L(t_1, t_2, \ldots, t_n) = \prod_{i=1}^{n} \left[ 1 - Y_i + Y_i \lambda(t_i) \right] \exp \left[ -\eta(t_i) \right] \tag{A-2}
\]

where

\( Y_i = \begin{cases} 
0 & \text{if the trial terminated at } t_i \text{ without acquisition.} \\
1 & \text{if the trial terminated at } t_i \text{ with acquisition.}
\end{cases} \)

Since log \( L \) is a monotonic function of \( L \) and thus obtains its maximum at the same \( \beta \) value as \( L \), one may choose to maximize

\[
\log L = \sum_{i=1}^{n} \left[ \log \left( 1 - Y_i + Y_i \lambda(t_i) \right) - \eta(t_i) \right] \tag{A-3}
\]

The \( m \) stationary conditions necessary for obtaining a maximum of log \( L \) are given by

\[
\frac{\partial \log L}{\partial \beta_j} = \sum_{i=1}^{n} \left[ \frac{Y_i}{1 - Y_i + Y_i \lambda(t_i)} \frac{\partial \lambda(t_i)}{\partial \beta_j} - \frac{\partial \eta(t_i)}{\partial \beta_j} \right] = 0 \quad j = 1, 2, \ldots, m \tag{A-4}
\]
When the derivatives on the right-hand side are evaluated using the definition of $\lambda$ of equation (A-1) and the definition of $n$ of Equation (3-7) one obtains

$$\frac{3\log L}{3\beta_j} = \sum_{i=1}^{n} \left[ \frac{Y_i}{1 - Y_i + Y_i \lambda(t_i)} \right] x_j(t_i) - \int_{0}^{t_i} x_j(t_i)d\xi = 0$$

\[ j = 1, 2, \ldots, m \quad (A-5) \]

These $m$ equations may be solved iteratively using a digital computer. But consider the simplest case in which only one variable, $x_1$, is present. Then

$$\frac{Y_i}{1 - Y_i + Y_i \lambda(t_i)} x_j(t_i) = \begin{cases} 0 & \text{if } Y_i = 0 \\ \frac{1}{\beta_1} & \text{if } Y_i = 1 \end{cases} \quad (A-6)$$

and the maximum likelihood estimate of $\beta_1$ is thus

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} Y_i}{\sum_{i=1}^{n} \int_{0}^{t_i} x_1(\xi)d\xi} = \frac{N}{n} \sum_{i=1}^{n} \int_{0}^{t_i} x_1(\xi)d\xi \quad (A-7)$$

where $N$ is the number of the $n$ trials which terminated in acquisition.
APPENDIX B

A TECHNIQUE FOR THE DETERMINATION OF VISIBLE AREA

The calculation of the visible area presented by a complex three-dimensional object such as an airplane when seen from an arbitrary direction is complicated by the fact that the various surfaces are seen at different angles and often shield one another. However, a rather simple but apparently adequate approximation technique for determination of visible area will now be described. Note first that the direction (bearing and elevation) of aircraft A as seen from aircraft B also defines the direction at which the line of sight from A strikes B. Thus the routine used for calculating traffic direction also yields the return line-of-sight. Consider now an aircraft centered coordinate axis as shown in Fig. B-1. Let the visible areas seen from the x (head-on) y (broadside) and z (above) axes be \( A_x \), \( A_y \), and \( A_z \) respectively.

Fig. B-1. Visible areas as viewed from three principal coordinate axes.
The aircraft may now be treated as if it were an object consisting of only three perpendicular planar surfaces $A_x$, $A_y$, and $A_z$. When viewed from an arbitrary direction the visible area presented by a single planar surface is simply the area of that surface multiplied by the cosine of the angle between the line-of-sight and the normal to the surface. Denote the three individual areas so calculated as $A'_x$, $A'_y$, and $A'_z$. Due to shielding the actual visible area is less than the sum of the three individual areas. We will approximate the shielding effect as follows:

$$\text{visible area} = \max(A'_x, A'_y, A'_z) + \frac{1}{3} \text{ of remaining areas}.$$ 

The $1/3$ factor compensates in an approximate manner for the effects of shielding. Note that this approximation is errorless when the aircraft is viewed along any of the principal coordinate axes. Principal areas for typical aircraft are given in Table B-1. Insofar as aircraft shape is invariant with wingspan, these figures may be extended to other aircraft by assuming that each area increases as the square of the wingspan of the aircraft.

<table>
<thead>
<tr>
<th>type aircraft</th>
<th>wingspan</th>
<th>head-on area, $A_x$</th>
<th>broadside area, $A_y$</th>
<th>above area, $A_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>single-engine general aviation (Piper PA-28)</td>
<td>32 ft</td>
<td>20 ft</td>
<td>100 ft</td>
<td>220 ft</td>
</tr>
<tr>
<td>multi-engine jet transport (Boeing 727)</td>
<td>108 ft</td>
<td>330 ft</td>
<td>1650 ft</td>
<td>3100 ft</td>
</tr>
</tbody>
</table>
REFERENCES


