NAVIGATIONAL ACCURACY
REQUIREMENTS OF AEROMANEUVERING
SPACE VEHICLES

THESIS

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NAVIGATIONAL ACCURACY
REQUIREMENTS OF AEROMANEUVERING SPACE VEHICLES

Thesis

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by

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Preface

I became interested in the concept of the aeromaneuvering orbital transfer vehicle while doing research on Space Shuttle upper stages. The potential for increased capability over purely propulsive OTVs is so much greater as to warrant a detailed investigation.

This thesis is an extension of work performed by the Lockheed Missiles and Space Company under NASA contracts NAS8-28586 and NAS8-31452, and is based on the recommendations for future study contained in these reports. I felt qualified to examine the problems of navigational accuracy during the aeromaneuvering phase of flight since I had completed the course sequences in Navigation and in Guidance and Control at AFTT.

I would like to express my appreciation to my thesis advisor, Maj Richard M. Potter, for his advice and critical review of my work during the preparation of this report. I would also like to thank Mr J.P. Hethcoat of the Marshall Space Flight Center for his assistance during my research for this study.

Robert J. Chambers
## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preface</td>
<td>ii</td>
</tr>
<tr>
<td>List of Figures</td>
<td>iv</td>
</tr>
<tr>
<td>Notation</td>
<td>v</td>
</tr>
<tr>
<td>Abstract</td>
<td>vii</td>
</tr>
<tr>
<td>I Introduction</td>
<td>1</td>
</tr>
<tr>
<td>Background</td>
<td>1</td>
</tr>
<tr>
<td>Purpose of the Study</td>
<td>2</td>
</tr>
<tr>
<td>Previous Investigations</td>
<td>3</td>
</tr>
<tr>
<td>II Guidance Concept</td>
<td>5</td>
</tr>
<tr>
<td>Explanation of Method</td>
<td>5</td>
</tr>
<tr>
<td>Equations of Motion</td>
<td>6</td>
</tr>
<tr>
<td>Linear Regulator Problem</td>
<td>10</td>
</tr>
<tr>
<td>Weighting Matrices</td>
<td>13</td>
</tr>
<tr>
<td>III Description of Simulation</td>
<td>18</td>
</tr>
<tr>
<td>Purpose</td>
<td>18</td>
</tr>
<tr>
<td>Assumptions</td>
<td>18</td>
</tr>
<tr>
<td>Equations of Motion</td>
<td>19</td>
</tr>
<tr>
<td>Computer Program</td>
<td>20</td>
</tr>
<tr>
<td>IV Navigation System Errors</td>
<td>23</td>
</tr>
<tr>
<td>V Results</td>
<td>25</td>
</tr>
<tr>
<td>Constant Error</td>
<td>25</td>
</tr>
<tr>
<td>Sinusoidal Error</td>
<td>28</td>
</tr>
<tr>
<td>Random Error</td>
<td>33</td>
</tr>
<tr>
<td>Error Coupling</td>
<td>34</td>
</tr>
<tr>
<td>VI Conclusions and Recommendations</td>
<td>36</td>
</tr>
<tr>
<td>Conclusions</td>
<td>36</td>
</tr>
<tr>
<td>Recommendations</td>
<td>37</td>
</tr>
<tr>
<td>Bibliography</td>
<td>38</td>
</tr>
<tr>
<td>Appendix A: The AMOOS Concept</td>
<td>39</td>
</tr>
<tr>
<td>Appendix B: Program Listing</td>
<td>43</td>
</tr>
<tr>
<td>Appendix C: Sample Trajectory</td>
<td>52</td>
</tr>
<tr>
<td>Vita</td>
<td>64</td>
</tr>
</tbody>
</table>

iii
List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Inertial Coordinate Frame</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>Definition of Bank Angle</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>Weighting Coefficient R</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>Feedback Gain Vector C.</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>Flow Chart of Simulation</td>
<td>21</td>
</tr>
<tr>
<td>6</td>
<td>Constant Velocity Errors</td>
<td>26</td>
</tr>
<tr>
<td>7</td>
<td>Constant Flight Path Angle Errors</td>
<td>27</td>
</tr>
<tr>
<td>8</td>
<td>Constant Acceleration Errors</td>
<td>29</td>
</tr>
<tr>
<td>9</td>
<td>Sinusoidal Velocity Errors</td>
<td>30</td>
</tr>
<tr>
<td>10</td>
<td>Sinusoidal Flight Path Angle Error</td>
<td>31</td>
</tr>
<tr>
<td>11</td>
<td>Sinusoidal Acceleration Error</td>
<td>32</td>
</tr>
<tr>
<td>12</td>
<td>Random Velocity Errors</td>
<td>34</td>
</tr>
<tr>
<td>13</td>
<td>General Configuration of AMDOS Vehicle</td>
<td>40</td>
</tr>
<tr>
<td>14</td>
<td>Definition of Angle of Attack, Bank Angle, and Flight Path Angle</td>
<td>42</td>
</tr>
</tbody>
</table>
Nomenclature

The MKS system is used in this study.

AMOOS - Aeromaneuvering Orbit-to-Orbit Shuttle
A_r - aerodynamic reference area = 15.69 m^2
C_d - coefficient of drag = 2.5125
C_1 - coefficient of lift = 1.3457
C_{12} \{ \text{gain coefficients} \}
C_3
D - drag
GEO - geosynchronous equatorial orbit
h - altitude
h_0 - reference altitude = 7,315 km
I - lift
LEO - low earth orbit
m - mass of vehicle = 6804 kg
r - inertial position vector
u - unit vector or control variable
V - inertial velocity
V_r - relative velocity
V_e - inertial velocity of atmosphere
\phi - bank angle
\phi_c - commanded bank angle
\phi_0 - nominal bank angle = 90 degrees
\psi - flight path angle
$\theta$ - see Fig. 1

$u$ - gravitational parameter

$\rho$ - atmospheric density

$\rho_0$ - atmospheric density at sea level

$\Upsilon$ - see Fig. 1
Abstract

A preliminary investigation was conducted to determine the navigational accuracy required by the Aeromaneuvering Orbit-to-Orbit Shuttle (AMOOS) during the atmospheric phase of flight. The guidance scheme, which is the same as the one developed by Lockheed in the original AMOOS study, uses the parameters of velocity, flight path angle, and density altitude to correct to a nominal trajectory. Density altitude is obtained from atmospheric density, which is calculated from vehicle acceleration. Errors in velocity, flight path angle, and acceleration were introduced into a three-degree-of-freedom computer simulation of the vehicle trajectory using bank angle commands generated by the guidance equations. Deviations from standard atmospheric density were taken into account.

The maximum errors allowable that still permitted the vehicle to attain its phasing orbit apogee altitude of 720 km (±100 km) were determined. Three types of error were investigated: constant, sinusoidal, and random. For time-varying errors, the frequency dependence was examined, as was cross-coupling of errors.

It was concluded that the guidance scheme can tolerate fairly large errors and still guide to an acceptable apogee altitude. The navigational accuracy required for the atmospheric phase of flight is within the capability of present day astrionics.
I. Introduction

Background

There is considerable interest in geosynchronous equatorial orbits for a variety of civil and military applications, such as communication links and surveillance. Unfortunately, it will be beyond the capability of the Space Shuttle to reach these orbits. The Orbiter vehicle is limited to an altitude of 300 km, and an upper stage is required to reach geosynchronous altitude of 35,800 km. An upper stage vehicle that has been proposed for the round trip from low earth orbit (LEO) to geosynchronous equatorial orbit (GEO) is the Space Tur. However, because of the large velocity change (ΔV) required for the round-trip mission, about 8740 m/sec, the large propellant mass fraction does not allow the vehicle to carry much payload.

An attractive alternative that has been presented by Lockheed is the Aeromaneuvering Orbit-to-Orbit Shuttle (AMOOS) (Ref 1, Ref 2). The advantage of this concept over the purely propulsive Space Tur is that part of the ΔV required for slowing and returning to LEO is provided by making a braking pass through the atmosphere. After exiting the atmosphere, a short rocket burn at apogee will raise the vehicle periapsis out of the atmosphere and establish a phasing orbit for rendezvous with the Shuttle Orbiter. The total ΔV required in this case is 6413 m/sec, reducing the fuel load considerably.
and allowing for two to three times the payload. The basic idea is that for a small increase in structural strength and the addition of a thermal protection system, a large savings in the amount of fuel required can be realized.

One of the problems associated with the AMOOS concept is that a very accurate atmospheric flight path is required. If the vehicle is one kilometer low, it will reenter, and if it is one kilometer too high, it will miss the apoapse of its phasing orbit by thousands of kilometers.

The vehicle guidance problem is complicated by the fact that the density of the upper atmosphere varies widely at high altitudes. The density will vary from $+40\%$ to $-35\%$ from its nominal value (Ref 1:67, Ref 3:20). The AMOOS vehicle is able to compensate for off-nominal conditions by modulating its lift vector, and this aeromaneuvering ability is what differentiates this proposal from earlier ballistic trajectory concepts.

**Purpose of the Study**

One of the areas of the AMOOS proposal that has not been investigated so far is the accuracy required of the vehicle navigation system to successfully implement the guidance equations, and this is the subject of my thesis. I have taken the guidance scheme used in the original study and incorporated it into a trajectory simulation in which navigation system errors can be introduced.

The goal of the guidance system is to place the vehicle in an orbit with an apoapse of 720 km ($\pm 100$ km). The ability of the
A previous navigational accuracy study has been accomplished for the aerobraking return from GEO to LEO (Ref 4:33-45). However, this study examined a vehicle that followed an unguided trajectory through the atmosphere (it used aerobraking, but not aeromaneuvering). The results of this study indicated that a one-pass unguided braking maneuver was not feasible using forecast shuttle-era technology, due to the high accuracy requirements and unpredictable variations in atmospheric density.

A preliminary investigation by Lockheed of the effects of exoatmospheric navigation errors has determined that it will be necessary to position the AMOOS vehicle within a corridor of ± 3.5 km in depth prior to entry (Ref 2:73). The investigation did not include the effects of atmospheric navigation errors. These errors are the subject of this study, which solely concerns itself with navigational errors generated within the atmosphere and their effect on the guidance equations. Of course, the two studies are not independent, since the
same astronics will most likely be used in both cases. It is expected that the astronic equipment will include a combination of the following sensors: Inertial measurement unit (IMU), star tracker, horizon sensor, landmark tracker, and laser or conventional radar.

It is not the purpose of this study to actually design a navigation system and select the equipment used (each of which would have its own characteristic errors), but the results indicate to what tolerances the navigation system must supply information to implement the guidance scheme.

Chapter II will present the guidance law used in the study. Chapter III will be a description of the simulation, and Chapter IV covers the introduction of navigational errors. Results are presented in Chapter V, and conclusions are in Chapter VI.
II. Guidance Concept

Explanation of Method

The AMOOS vehicle is constrained to fit within the Shuttle Orbiter payload bay, and is basically a cylinder, with one end truncated into a lifting body shape. A detailed description of the vehicle is included in Appendix A. For the purpose of deriving the guidance law, it should be noted that the vehicle has a L/D ratio of from .5 to 1.0 at hypersonic velocities.

The guidance problem can be simply stated as maneuvering the vehicle so as to leave the atmosphere with just enough velocity to hit the proper apogee of the phasing orbit (720 km). This is a different problem than conventional reentry guidance, which is concerned with reducing velocity below some terminal value, with heating and dynamic pressures being the primary constraints.

The guidance method that has been proposed for AMOOS is controlling to a nominal trajectory, using a linear regulator. The lift vector is used to steer to the nominal trajectory by bank angle modulation. Bank angle, the orientation of lift, is used instead of angle of attack for lift control because the vehicle shape does not provide sufficient variation of the lift coefficient with angle of attack. This also permits the vehicle to present the same orientation into the relative wind, minimizing the surface area that must be covered by the thermal protection system.

Using bank control has an additional advantage. When the vehicle returns from geosynchronous orbit, it must also change
its inclination from zero to 28.5 degrees (the orbital inclination of the Shuttle Orbiter, and the latitude of Kennedy Space Center).

A nominal bank angle of 90 degrees produces a plane change capability of approximately 7 degrees. This reduces the propulsive requirements at synchronous altitude for plane change $\Delta V$. Changes in the bank angle from 90 degrees for steering corrections will reduce this plane change capability slightly.

**Equations of Motion**

The parameters used to generate bank angle commands are inertial velocity, flight path angle, and density, which is a function of altitude. The original derivation of the guidance law, from Reference 1, is presented here because it provides the basis for the remainder of the study. The equations of motion, in the osculating plane of the orbit, are derived as follows (Ref 1: D-1 - D-21):

$$\frac{d\vec{V}}{dt} = \vec{u}_r \frac{d\vec{V}}{dt} + \vec{u}_\perp \frac{d\theta}{dt}$$  \hspace{1cm} (1)

where $\vec{u}_r =$ unit vector in the direction of $\vec{V}$

$\vec{u}_\perp =$ unit vector perpendicular to $\vec{V}$

with the inertial reference frame as shown in Fig. 1.

Summing the forces in the direction of $\vec{V}$ gives

$$-D - \frac{\mu}{r^2} \sin \gamma = m \frac{dV}{dt}$$  \hspace{1cm} (2)

or

$$\dot{V} = - \frac{D}{m} - \frac{\mu}{r^2} \sin \gamma$$  \hspace{1cm} (3)

Summing the components perpendicular to $\vec{V}$ gives
Fig. 1 - Inertial Coordinate Frame

Fig. 2 - Definition of Bank Angle, $\beta$, ($Y_r$ lies in the vertical plane formed by the radius vector $r$ and $V_r$)
\begin{equation}
L - \frac{u}{r} m \cos \gamma = m V \dot{\theta}
\end{equation}

As can be seen from Fig. 1

\begin{equation}
\dot{\theta} = \ddot{r} + \dot{r}
\end{equation}
\begin{equation}
\ddot{r} = - \frac{V \cos \gamma}{r}
\end{equation}

Combining (5) and (6) gives

\begin{equation}
\dot{\theta} = \ddot{r} - \frac{V}{r} \cos \gamma
\end{equation}

Substituting (7) into (4) and solving for \( \gamma \) yields

\begin{equation}
\ddot{r} = \frac{L}{mV} - \frac{1}{V} \left[ \frac{u}{r^2} - \frac{V^2}{r} \right] \cos \gamma
\end{equation}

The derivative of radial distance \( r \) is given by

\begin{equation}
\dot{r} = V \sin \gamma = h
\end{equation}

If the states of the system are taken to be \( V, \gamma, \) and \( h \), then equations (3), (8), and (9) provide the state equations.

Although the lift and drag force directions are referenced to inertial velocity, they may be approximated by the force directions with respect to the vehicle velocity relative to the atmosphere since the directions of relative velocity and inertial velocity are almost parallel during the period of atmospheric flight.

Thus

\begin{equation}
D = \frac{1}{2} \rho \frac{V^2}{r} A_r C_d
\end{equation}
where bank angle, $\theta$, is defined in Fig. 2.

Control of the vehicle is accomplished by rotating the lift vector through a bank angle $\theta$, which changes the component of lift $C_1$ in the $Y_r$ direction, as shown in Fig. 2. The relation between $\theta$ and the vehicle orientation is shown in Fig. 14, Appendix A. The control variable, $u$, is defined as the deviation of the commanded value of $C_{1y}$ from some nominal value of bank, i.e.,

$$ u = C_{1y} - C_{1y_0} = C_1 \left( \cos \theta_o - \cos \theta \right) $$

where $\theta_o = 90$ degrees for maximum plane change capability.

Substituting (10), (11), (12), and (13) into equations (3) and (8) and assuming $\theta = \theta_o$ results in the revised equations of motion:

$$ \dot{V} = -\frac{1}{2m_0} \rho \left(V - V_e\right)^2 C_d A_r \frac{U}{r^2} \sin \gamma $$

$$ \dot{V} = \frac{1}{2m_0} \rho \left(V - V_e\right)^2 A_r \left[C_1 \cos \theta_o + u\right] - \frac{U}{r^2} \frac{V}{r} \cos \gamma $$

$$ \dot{\gamma} = V \sin \gamma $$

where density is an exponential approximation according to

$$ \rho = \rho_0 \ e^{-\left(h/h_0\right)} $$

These equations (with $u=0$) are integrated from initial conditions selected to result in an assumed altitude of 720 km. Since entry altitude is fixed (at 120 km), all possible initial
conditions consistent with the transfer orbit from GRO may be specified in terms of a vacuum perigee, which can be adjusted to give the desired apogee. This integration will provide a $V_{\text{nom}}$, $\gamma_{\text{nom}}$, and $h_{\text{nom}}$ that the vehicle can be controlled to, according to the linear state feedback control law

$$u = C_1 (V - V_{\text{nom}}) + C_2 (\gamma - \gamma_{\text{nom}}) + C_3 (h - h_{\text{nom}}) \quad (18)$$

The time-varying coefficients $C_1$, $C_2$, and $C_3$ are the solution to the linear regulator problem presented in the next section. The values of $C_1$, $C_2$, $C_3$, $V_{\text{nom}}$, $\gamma_{\text{nom}}$, and $h_{\text{nom}}$ can be precomputed for discrete time intervals and stored in the on-board computer.

**Linear Regulator Problem**

The development of the linear regulator problem involves a linearization of the equations of motion to the form of

$$
\begin{bmatrix}
\delta V \\
\delta \gamma \\
\delta r
\end{bmatrix} =
\begin{bmatrix}
A
\end{bmatrix}
\begin{bmatrix}
\delta V \\
\delta \gamma \\
\delta r
\end{bmatrix} +
\begin{bmatrix}
B
\end{bmatrix} u \quad (19)
$$

where

$$
\begin{bmatrix}
\delta V \\
\delta \gamma \\
\delta r
\end{bmatrix} =
\begin{bmatrix}
\frac{\partial V}{\partial V} & \frac{\partial V}{\partial \gamma} & \frac{\partial V}{\partial r} \\
\frac{\partial \gamma}{\partial V} & \frac{\partial \gamma}{\partial \gamma} & \frac{\partial \gamma}{\partial r} \\
\frac{\partial r}{\partial V} & \frac{\partial r}{\partial \gamma} & \frac{\partial r}{\partial r}
\end{bmatrix}
\begin{bmatrix}
\delta V \\
\delta \gamma \\
\delta r
\end{bmatrix} +
\begin{bmatrix}
B
\end{bmatrix} u \quad (20)
$$
and

\[
\begin{bmatrix}
\frac{\partial \dot{V}}{\partial u} \\
\frac{\partial \dot{V}}{\partial u} \\
\frac{\partial \dot{r}}{\partial u}
\end{bmatrix}
\]  

(21)

The matrices \( A \) and \( B \) are evaluated along the nominal trajectory. The partial derivatives are calculated from equations (14), (15), and (16) as follows:

\[
\frac{\partial \dot{V}}{\partial V} = - \frac{L}{m} \nu_r C_d A_r 
\]

(22)

\[
\frac{\partial \dot{V}}{\partial \gamma} = \frac{u}{r} \cos \gamma 
\]

(23)

\[
\frac{\partial \dot{V}}{\partial r} = - \frac{1}{2m} \nu_r^2 C_d A_r \frac{\partial \alpha}{\partial r} + \frac{2u}{r^2} \sin \gamma 
\]

(24)

\[
\frac{\partial \dot{Y}}{\partial V} = \frac{1}{mV} \left[ \nu_r - \frac{\nu_r}{2V} \right] \rho A_r \left[ C_1 \cos \alpha_0 + u \right] + \left[ \frac{u}{V^3} + \frac{1}{r} \right] \cos \gamma 
\]

(25)

\[
\frac{\partial \dot{Y}}{\partial \gamma} = \left[ \frac{u}{V^3} - \frac{v}{r} \right] \sin \gamma 
\]

(26)

\[
\frac{\partial \dot{Y}}{\partial r} = \frac{1}{2mV} \nu_r^2 A_r \left[ C_1 \cos \alpha_0 + u \right] \frac{\partial \alpha}{\partial r} - \left[ \frac{2u}{V^3} + \frac{v}{r^2} \right] \cos \gamma 
\]

(27)

\[
\frac{\partial \dot{r}}{\partial V} = \sin \gamma 
\]

(28)

\[
\frac{\partial \dot{r}}{\partial \gamma} = v \cos \gamma 
\]

(29)
\begin{align*}
\frac{\partial \hat{r}}{\partial r} &= 0 \\
\frac{\partial \hat{y}}{\partial u} &= 0 \\
\frac{\partial \hat{y}}{\partial u} &= \frac{1}{2mV \rho} \frac{V_r^2}{A_r} \\
\frac{\partial \hat{r}}{\partial u} &= 0 \\
\text{and} \\
\frac{\partial \hat{y}}{\partial r} &= -\frac{C}{h_0}
\end{align*}

Using standard optimal control techniques (Ref 5:209), a performance index is chosen to minimize the error in phasing orbit apogee. This index of performance (which is to be minimized) consists of the terminal miss which relates directly to the square of the apogee error, plus the integral of the square of the control variable $u$, over the nominal flight time. This last term tends to limit the control, hence limiting the amount of propellant required for the reaction control system.

Therefore,

\begin{equation}
J = \frac{1}{2} \delta x^T(t_f) H \delta x(t_f) + \frac{1}{2} \int_{t_0}^{t_f} (u^TR) \, dt 
\end{equation}

where

\begin{equation}
\delta x = \begin{bmatrix} \delta V \\ \delta y \\ \delta r \end{bmatrix}
\end{equation}

and $H$ and $R$ are weighting matrices, and $t_0 = t_{entry}$ and $t_f = t_{nom exit}$. 

12
The solution to the optimization problem is given by

\[ u = -R^{-1} H^T K \delta x = C^T \delta x \]  

(37)

where \( K \) is given by the Riccati differential equation

\[ \dot{K} = -A^T K - K A + K B R^{-1} B^T K \]  

(38)

which is integrated backwards through time from its final conditions, \( K(t_f) = H \).

**Weighting Matrices**

The matrix \( H \) is chosen to minimize the effect of the final state, \( \delta x(t_f) \), at atmospheric exit, on apogee altitude \( h_a \). For small variations, \( \delta h_a \) can be related to the final state by

\[ \delta h_v = \left[ \frac{\partial h_a}{\partial v} \delta v + \frac{\partial h_a}{\partial y} \delta y + \frac{\partial h_a}{\partial r} \delta r \right]_{t=t_f} \]  

(39)

The matrix \( H \) is determined by

\[ \delta h_v^2 = \delta x^T(t_f) H \delta x(t_f) \]  

(40)

which yields the elements \( h_{ij} \) of \( H \):

\[ h_{11} = \left( \frac{\partial h_a}{\partial v} \right)^2 \]  

(41)

\[ h_{22} = \left( \frac{\partial h_a}{\partial y} \right)^2 \]  

(42)

\[ h_{33} = \left( \frac{\partial h_a}{\partial r} \right)^2 \]  

(43)

\[ h_{12} = h_{21} = \frac{\partial h_a}{\partial v} \frac{\partial h_a}{\partial y} \]  

(44)
\[ h_{13} = h_{31} = \frac{\partial h_a}{\partial V} \frac{\partial h_a}{\partial r} \]  
\[ (45) \]

and

\[ h_{23} = h_{32} = \frac{\partial h_a}{\partial V} \frac{\partial h_a}{\partial r} \]  
\[ (46) \]

Evaluating the partial derivatives at the nominal conditions at the final time \( V=7980 \text{ m/sec}, \gamma = 1.75 \text{ deg}, \) and \( r=6,498,153 \text{ m} \) results in numerical values of

\[ \frac{\partial h_a}{\partial V} = 3.32 \text{ km/m.sec} \]

\[ \frac{\partial h_a}{\partial \gamma} = 74.73 \text{ km/deg} \]

\[ \frac{\partial h_a}{\partial r} = 3.116 \text{ m} \]

The function \( R(t) \) was selected to prevent the peaking of the gains at perigee, the period of maximum control effectiveness. The function selected is shown in Fig. 3.

The feedback gain vector derived from eq. (37) is shown in Fig. 4. In Figures 3 and 4, the point at which time \( t=0 \) is when the vehicle passes through 95.4 km upon entry. Above this altitude there is not sufficient atmosphere to permit control effectiveness.

In the region of atmospheric guidance, the vehicle acceleration is primarily affected by the drag terms. Therefore, neglecting the gravitational terms, and for small flight path angles, from eq. (14)

\[ m \ddot{V} = \frac{1}{2} \rho V_r^2 C_d A_r \]  
\[ (47) \]
Fig. 3 - Weighting Coefficient R (Ref 2: D-11)
Fig. 4 - Feedback Gain Vector C for AMOS Guidance (Ref 2: D-12)
or

\[ \rho = \frac{-2 \dot{v}_m}{V_r^2 C_d A_r} \]  \hspace{1cm} (48)

This density can be related to an altitude through the use of eq.(17)

\[ h = -h_o \ln\left(\frac{\rho}{\rho_o}\right) \]  \hspace{1cm} (49)

Thus, from measuring vehicle acceleration, a density altitude can be calculated. This is compared to the nominal density altitude to determine the vehicle altitude error.
III. Description of Simulation

Purpose

The validity of the guidance scheme is tested by a computer program which simulates the actual vehicle dynamics with guidance commands. Perturbations from the nominal trajectory are introduced to determine how well the guidance law will correct back to the nominal. In the case under study, perturbations are caused by deviations in atmospheric density and by errors in the calculated position and velocity information supplied by the vehicle's navigation system. Atmospheric density fluctuations are known to vary from -35% to 40% from normal in the region flown by AMOOS (from 120 to 70 km), and are easily input to the simulation. Navigational errors can also be easily input to the simulation and these error sources are described in the next chapter.

Assumptions

The following assumptions have been made in constructing the vehicle simulation:
1. The assumptions of a spherical rotating earth and atmosphere.
2. The atmospheric density can be modeled as an exponential function of altitude.
3. The vehicle is returning from GEO via a Hohmann transfer orbit with a 21.5 degree plane change, with a targeted vacuum perigee of 71.1 km. This value for perigee is the result of adjusting vacuum perigee to determine the initial entry conditions that result in an apogee of 720 km.
4. Vehicle flies constant angle of attack with lift, longitudinal vehicle axis, and relative wind always in the same plane. The orientation of this plane ($\alpha$) is the only vehicle control that affects translational dynamics.

5. The actual bank angle is equal to the commanded bank angle. It was originally intended to simulate an autopilot in the guidance loop to implement the bank commands, and to account for the vehicle's inability to instantly assume the commanded value. However, from the results of the simulation, the guidance commands are fairly continuous, and the value of ($\alpha - \alpha_c$) is small (about 2 - 3 degrees). This permits the vehicle to assume the value of $\alpha_c$ within the guidance cycle time (4 sec), assuming an angular acceleration of 1 deg/sec$^2$.

Equations of Motion

Since it was desired to examine the three-dimensional vehicle motion rather than just the motion in the osculating orbital plane, the equations of motion used in generating the guidance law could not be used. A different (although consistent) set of equations was used, based on a inertial geocentric-equatorial (TJK) coordinate system. The TJK reference frame was oriented with the $l$ direction in the direction of the vacuum perigee, to simplify the transformation from perifocal coordinates to TJK coordinates. These equations are the same as used in the original study (Ref 1:A-3).

The vector equation of motion is:

$$\ddot{r} = -\frac{\mu}{r^3}r + \frac{1}{2} \frac{\mathbf{V}_r^2}{m} \frac{A_r}{m} \ddot{l} + \frac{1}{2} \frac{\mathbf{V}_r^2}{m} \frac{A_d}{m} \ddot{n}$$

(50)

where $\dot{l}$ and $\dot{n}$ are the unit vectors in the directions of lift and drag, respectively.
\[ \hat{t} = \frac{V_r \times (r \times V_r)}{|V_r|^2} \cos \beta + \frac{(r \times V_r)}{|V_r|} \sin \beta \]  
(51)

and

\[ \hat{D} = -\frac{V_r}{|V_r|} \]  
(52)

The aerodynamic coefficients \(C_1\) and \(C_d\) and the vehicle specifications of mass, \(m\), and reference area, \(A_r\), are taken from the design of Reference 2. Relative velocity, \(V_r\), is defined by eq. (12). Bank angle, \(\beta\), comes from the guidance law (eq. (18)), and the density altitude, \(h\), is computed from eqs. (48) and (49).

The program uses a Runge-Kutta integration algorithm to integrate the equations of motion from atmospheric entry to exit (120 km).

**Computer Program**

A flow chart of the major elements of the computer program is shown in Fig. 5. A listing of the program is included in Appendix B.

The program first generates the nominal trajectory, using the equations of Chapter 11. The vector of time-varying guidance coefficients is input as data points, and an interpolating table look-up routine is used to reconstruct the curves of Fig. 4 and select values for a given flight time. The elements \(V\), \(V\), and \(r\) of the osculating orbit at \(r = 120\) km are converted to inertial position and velocity and are used as initial conditions for the integration of the equations of motion given in the previous section.

Simulated navigation errors in \(V\), \(V\), and acceleration are input to the guidance section, which then generates a bank angle. The integration proceeds until the vehicle leaves the atmosphere.
Fig. 5. Flow Chart of Simulation
(when it again reaches 120 km). At this point the angle of the phasing orbit is calculated, assuming two body dynamics.

The integration time step is 1 second, which was determined to give good accuracy without resulting in excessive computational time. New guidance commands are generated every 4 seconds, and bank angle commands are limited to ±30 degrees about the nominal 90 degree bank angle. The guidance cycle time and maximum bank angle represent the values that gave the best results in the original simulation work done by Lockheed. A sample trajectory is given in Appendix C.
TV. Navigation System Errors

It was attempted to model errors in a manner that would be typical of the outputs from the navigation system. Accordingly, the errors were input to the simulation as a constant (bias) error, a sinusoidal error, and as a random error. Each type of error was run with varying magnitudes to determine its relation with the output, apogee error. In addition, each error in velocity, flight path angle, and acceleration was considered for three different atmospheric density profiles. The first profile was the nominal density, and the second and third profiles were the worst-case situations of density being 40% greater than normal, and being 35% less than normal. This demonstrates the capability of the guidance system to correct for off-nominal atmospheric conditions, in addition to navigation errors.

The sinusoidal errors were run at varying frequencies to determine if there was any frequency-sensitivity in the guidance system. The range of frequencies to be examined was determined by the guidance cycle time and total time of flight. The high frequency limit was chosen to give a period equal to the guidance cycle time of 4 seconds. Any frequency greater than this would appear as a random error to the simulation. The low frequency limit was selected to give a period approximately equal to the time of atmospheric flight (400 seconds). A frequency much lower than this would appear to be almost a constant error.

The effects of random errors in velocity on apogee altitude were examined to determine if there was any significant difference between
the results for random errors and sinusoidal or constant errors.

The random errors were produced using the random-number utility subprogram of the computer system. A uniformly distributed random sequence of numbers between -1 and +1 was assigned to the elements of an array at a discrete interval. This interval, which could be varied, was called the correlation time. The elements of the array between these intervals were filled in using a linear function between the random end points of the interval. The elements of the array were multiplied by the appropriate error magnitude and then each element corresponded to an error value at each cycle of the guidance equations. The use of the correlation time provided a means of varying the rate at which the random errors were input to the simulation.

The correlation time was analogous to the period of the sinusoidal errors, and so the range of correlation times was chosen from 4 to 400 seconds, using the same reasoning as was used to determine the frequency of sinusoidal errors.

Errors in each parameter were first examined independently, but the effects of cross-coupling of errors were also examined. This was to determine if the principle of superposition would be applicable to the error sources.

The results of the study are presented in the next chapter.
V. Results

Constant Error

The nominal trajectory produced by eqs. (14), (15), and (16) resulted in an apogee altitude of exactly 720 km. However, when this trajectory was "flown" in the three-dimensional simulation using eqs. (50), (51), and (52), with no errors input, it consistently yielded an apogee altitude of 130 km. This 10 km bias probably results from the different methods of calculating $v_e$ in each case. In the case of the nominal trajectory, it is approximated, since the orientation of the osculating orbit is not known, while in the 3-D simulation it is calculated exactly. This bias was taken into account when determining the results of the navigational errors.

Errors in apogee due to errors in measured inertial velocity exhibit a linear relationship, as shown in Fig. 6. The horizontal lines show the permissible variation ($\pm 100$ km) in apogee altitude, and the three diagonal lines show the three cases of atmospheric density being equal to nominal, and $+40\%$ and $-35\%$ from nominal.

From the figure, taking into account the 10 km bias and the possible variation in atmospheric density, it can be determined that the allowable error in measuring velocity is $\pm 22$ m/sec. A sensitivity coefficient of $-4.25$ km/m/sec can be calculated for constant errors (meaning that a $+1$ m/sec error in measuring velocity will result in an apogee error of $-4.25$ km).

Figure 7 shows the effects of a constant error in measured flight path angle. This time, however, the effects are seen to be
Fig. 6. Constant Velocity Errors
Fig. 7. Constant Flight Path Angle Error
non-linear. The maximum allowable errors, again allowing for the bias error, can be seen to be -.0045 and .0035 (-.26 deg and .20 deg). A linear sensitivity coefficient cannot be determined for these curves.

Errors in measuring acceleration result in the error shown in Fig. 8. The curves indicate that acceleration errors must be kept between ±.6 m/sec². The curves can reasonably be approximated by a straight line in a small region, permitting the calculation of an approximate sensitivity coefficient. For the region of ±.4 m/sec² error, the sensitivity coefficient is 120 km/m/sec².

**Sinusoidal Error**

Sinusoidally-varying errors in velocity, flight path angle, and acceleration were input to the simulation at several different amplitudes for each of the three atmospheric density profiles. Typical results are presented in Figures 9, 10, and 11. These are for the nominal density profile, with amplitude of the errors being respectively 20 m/sec, .004 radians, and .4 m/sec².

The figures will be discussed together, since they all indicate the same trend. The data is plotted as discrete points because it did not lend itself to fitting a smooth curve between the points. It can be seen that at frequencies greater than .1 rad/sec the apogee deviations are small compared to the deviations produced with constant errors of the same magnitude. Below .1 rad/sec the deviations increase erratically until they are approximately the same as for constant error. This is most likely due to the guidance scheme being more sensitive to errors at one point in the flight path (probably engine, since this is the region of greatest
Fig. 10. Sinusoidal Flight Path Angle Error (amplitude = .004 radians)
control effectiveness). The high frequency errors tend to cancel each other out, but at the lower frequencies there is a larger error in the sensitive region that is not cancelled out. The net effect is that the system, like most physical systems, acts like a low-pass filter. The guidance scheme did not show any instability in the frequency range examined.

The results using the off-nominal density conditions (-35%, +40%) were the same, with the data points being shifted up or down as in the constant error cases. Varying the amplitude of the errors only changed the deviation from the nominal 730 km apogee, but did not alter the shape of the figures.

**Random Error**

Randomly generated errors in navigational information were investigated to determine if there was a significant difference from the results obtained with constant and time-varying sinusoidal errors. It was also desired to find out if the correlation time affected the results appreciably. Correlation time, it will be recalled from the previous chapter, is the spacing between the randomly generated error parameters. Accordingly, the program was modified to run 10 randomly generated velocity error profiles (uniformly distributed between ± 20 m/sec) at each of 10 correlation times. The results are shown in Fig. 12. The correlation time is plotted versus the arithmetic mean and the standard deviation of the apogee deviations from 730 km for the 10 trajectories.

It can be seen that the standard deviation of random error is less than the apogee deviation due to constant errors, again
indicating that plus and minus errors tend to cancel each other out. As might be expected, the random errors produce greater apogee deviations than the sinusoidal errors.

The relationship of correlation time to the standard deviation is ambiguous. Although the standard deviation seems to increase with correlation time (as did the apogee deviations with increasing period length in the sinusoidal case), there is a minimum at 25 seconds. At any rate, there does not seem to be a strong relationship between correlation time and apogee error.

Error Coupling

So far, in the course of this study, errors in velocity, flight path angle, and acceleration have been input singly to the simulation. It was desired to determine if it would be possible to input combinations of the error parameters and predict the apogee deviation. Using the principle of superposition, it would be possible to determine the deviation for each individual error, then sum them to find the total apogee deviation. Unfortunately, this did not prove to be the case. The results of several tests indicate that the errors are strongly coupled and superposition does not hold. This can be seen from the sample trajectory of Appendix C. Constant errors of -10 m/sec, .002 radians, and .1 m/sec² were input to the simulation. From Figures 6, 7, and 8, it can be determined that the apogee deviations for each case are, respectively, +40 km, -15 km, and +12 km, totaling +37 km apogee error. However, the actual apogee altitude is 727 km, or -3 km error. No further investigation of the error coupling was attempted, although the interrelationship of error parameters can be seen from the partial derivatives of eq. (20).
VT. Conclusions and Recommendations

Conclusions

This study has shown to what accuracy the AMOOS vehicle navigation system must provide velocity, flight path angle, and acceleration information to successfully implement the guidance scheme. This information can be used to select and design the navigation sensors for the vehicle.

Velocity information must be supplied to an accuracy of ±22 m/sec. Constant errors in velocity relate to apogee deviations according to the sensitivity coefficient of -4.25 km/m/sec. Flight path angle information must be supplied within the accuracy range of -0.26 deg to +0.20 deg. Errors in measuring acceleration along the flight path (which is used to determine atmospheric density and then a density altitude) must be within ±0.6 m/sec^2. Errors in apogee altitude are related to errors in sensing acceleration by the approximate sensitivity coefficient of 120 km/m/sec^2.

These accuracy tolerances do not seem to be severe in view of the present capability of space navigation systems. The exoatmospheric navigation problem imposes more stringent accuracy requirements on navigational hardware, so there should not be a major problem in obtaining the accuracy required for the atmospheric flight.

The guidance scheme has demonstrated its capability to handle time-varying errors, both sinusoidal and random. There is no frequency instability in the guidance equations, and the apogee deviations for error frequencies above 0.1 rad/sec are minimal.
Recommendations

The next step in designing the navigation and guidance system for the AMOOS vehicle would be to select navigational sensors which can provide the required accuracy, both for the exoatmospheric and the atmospheric phases of the mission. Information from these sensors would have to be processed by the navigation computer before being input to the guidance system.

Existing hardware would have to be evaluated to determine if it would be suitable for use on the vehicle, or if new hardware would have to be designed.
Bibliography


APPENDIX A

The AMOOS Concept

The Aeromaneuvering Orbit-to-Orbit Shuttle (AMOOS) is designed to be carried to and from low earth orbit inside the Shuttle Orbiter payload bay. Basically, AMOOS differs from the Space Tug in that it returns to low earth orbit by making a braking pass through the atmosphere. The added vehicle weight due to increased structural strength and the addition of a thermal protection system is more than offset by the weight of fuel required for the same propulsive braking. Thus, AMOOS can carry 2 to 3 times the payload to GEO for the same weight vehicle.

The AMOOS concept has been proposed for both manned and unmanned missions. The mission of primary interest is to reach synchronous altitude, and that is the mission profile examined in this study. However, the concept is not limited to high earth orbits. An AMOOS vehicle could be used upon returning to earth from lunar missions, or it could be used to provide aeromaneuvering in any planetary atmosphere.

In order to provide for the maximum efficient use of the Shuttle, AMOOS is sized to the maximum Shuttle payload weight. The dimensions of the payload bay require that AMOOS be shaped like a cylinder with one end modified to an aerodynamic nose cap. The general configuration is shown in Fig. 13.

The nose cap is hinged to permit the rocket engine to fire forward. This aft cargo bay arrangement permits the vehicle to be adapted to modular configuration. A hinged flap at the rear provides the
Fig. 13. General Configuration of the AMOOS vehicle (Ref 1:155)
required aerodynamic trim characteristics. An ablative thermal protection system is used on the vehicle, and it must be replaced after each mission.

The possibility exists of placing heavy payloads into GEO by staging 2 AMOOS vehicles (using 2 Shuttle flights). This would permit the placing of a 15,000 km payload into GEO. A single stage AMOOS can carry a round trip payload of 2860 km, or deliver 5200 km. This compares to 1100 km and 3600 km, respectively, for the all-propulsive Space Tur. Thus it can be seen that AMOOS enjoys a definite performance advantage over the Space Tur.

Estimated costs (Ref 134) for the design, development, testing, evaluation, and production of AMOOS are as follows (in 1970 United States dollars):

- Development (DOE) 511 million
- 1st unit production 28.7 million
- Refurbishment (per flight) 1.1 million

These estimates do not include engine costs, estimated at $130 million for DOE, and $7 million unit cost; facility and operations costs; and the prime contractor's fee.

Figure 14 shows the orientation of the vehicle with respect to angle of attack (α), bank angle (β), and flight path angle (γ). Note that bank angle, as defined, is not the angle produced by a simple roll about the vehicle centerline. The autopilot must orient the vehicle to change the direction of the lift vector by angle α while keeping the lift vector, vehicle centerline, and the relative wind vector in the same plane. This allows the vehicle to keep the same side (covered with ablative material) flying into the relative wind.
Fig. 14. Definition of Angle of Attack, Bank Angle, and Flight Path Angle (Ref 1:115)
APPENDIX B

Program Listing

The computer listing which follows was used to simulate the vehicle trajectories. The program uses the following input information:

1. ERRVEL - velocity error - in this example, -10 m/sec constant error
2. ERRGAM - flight path angle error - in this example, +0.002 radians constant error
3. ERRACC - acceleration error - in this example, +1 m/sec² constant error
4. TFOR - format parameter - 0 for short output, 1 for full output
5. TTRAJ - trajectory designator - in this example, #1
6. VAP - vacuum periapse = 71,113 km
7. DX - guidance cycle time = 4 sec
8. DT - simulation time step = 1 sec
9. C₁(t) - feedback gain vector (see Fig. 4)

The output from this program is given in Appendix C.
**SECTION GENERATES A NOMINAL TRAJECTORY**

J=100

**ENTRY CONDITIONS (120 KMH)**

**INITIAL VELOCITY** $\gamma = \frac{1}{2\pi}$

**FLIGHT PATH ANGLE (GAMMA)**

**GUIDANCE CYCLE**

**SIMULATION TIME STEP, SEC = 10.02**

**CONTINUOUS**

**CONTINUE**

**END**
POINTER 35, APOGEE, TOPLAY
4C = RTY(/"" TARGET APPROX ALTITUDE = "", F10.2,
*" TIME DELAY = ", F9.1,
DEF = .00
5G FORMAT(T3,""""TABLE OF GUIDANCE CONSTANTS""""/"" TIME", T15,""C1", T25,""C2")
4C = 0G
9G FORMAT(F4.1, F10.4, F10.1, F10.5)
END

***/***/ THIS SECTION COMPUTES INERTIAL INITIAL CONDITIONS
5G = (VP2 + EPSI*NSH)/2.
1G = (NSA - VP2)/NSA
2G = ?L. 90
3G = VP2*(1. + NSA)
4G = ASP((NSA/PI - 1.)/180)
5G = ?1*CO8(NU)
6G = ?1*SI8(NU)
7G = G8(T1*P)*SI8(NU)
8G = G8(T2*P)*T(20 + COS(NU))
9G = T(2)*P
1G = G8(S1*P)*?22
2G = Sin(T1)*?22
3G = VP1
4G = COS(T2)+100
5G = SI1(T2)*?22
6G = TIK = TUR*TD 2 0NU=NU*TD
7G*=
8G*="TRANSFER ORBIT ELEMENTS"
9G*=""IMPLIKATION = ""POTH,"" REG"
10G*=""SPHENODISPLAY = "",EGC"
11G*=""TRUE ANOMALY AT ENTRY = ",MNU," DEG"
12G*=""TIME, ""INERTIAL POSITION AND VELOCITY AT ENTRY""
"CONTINUE" THIS SECTION GENERATES A RANDOM SEQUENCE

CONTINUE
CALL MULTIST(3,0)
DO 13 I=1,300,10
13 \( i(i) = (3 * \text{MULTIST}(i, 1) - 3 * \text{MULTIST}(i, 2)) * \text{FLOAT}(\text{NN}) + \text{MULTIST}(i, 1) \)
CONTINUE

"VARIETY" "VELOCITY ERROR = " \text{CRVEL}
"VARIETY" "FLIGHT PATH ANGLE ERROR = " \text{CRPGM}
"VARIETY" "FORCED ACCELERATION ERROR = " \text{CRACC}
L=1 \text{ NN} = 4 \text{ TR}=1
\text{SP(3)} = 0 \text{ SP(4)} = 0 \text{ SP(5)} = 0 \text{ SP(6)} = 0
\text{FOR}(1) = 0 \text{ FOR}(2) = 0 \text{ FOR}(3) = 0
\text{VSI}(1) = 0 \text{ VSI}(2) = 0 \text{ VSI}(3) = 0
\text{Z}=0
\text{CPO}=3400 \text{ PhO=40}
\text{CD}=30 \text{ CM?}
\text{CT}=30 \text{ COP}
\text{MT}=10 \text{ MTP}
\text{FA}=30 \text{ FA2}
\text{ALT}=0 \text{ ALT2}=0
\text{Z}=0 \text{ ZT}=0
0100
100 CONTINUE

101 CONTINUE

102 DO=1,4

103 IF (DO.EQ.1) THEN

104 PRINT*, "SECOND","TIME","ALTITUDE", "LATITUDE", "LONGITUDE","VELOCITY", "ACCELERATION", "MASS","MOMENTUM","TIME", "ANGLE", "DEGREES"

105 ELSE IF (DO.EQ.2) THEN

106 PRINT*, "SECOND","TIME","ALTITUDE", "LATITUDE", "LONGITUDE","VELOCITY", "ACCELERATION", "MASS","MOMENTUM","TIME", "ANGLE", "DEGREES"

107 ELSE IF (DO.EQ.3) THEN

108 PRINT*, "SECOND","TIME","ALTITUDE", "LATITUDE", "LONGITUDE","VELOCITY", "ACCELERATION", "MASS","MOMENTUM","TIME", "ANGLE", "DEGREES"

109 ELSE IF (DO.EQ.4) THEN

110 PRINT*, "SECOND","TIME","ALTITUDE", "LATITUDE", "LONGITUDE","VELOCITY", "ACCELERATION", "MASS","MOMENTUM","TIME", "ANGLE", "DEGREES"

111 END IF

112 CONTINUE

113 END
The sample trajectory contained in this section was produced by the program listed in Appendix B. The output consists of the following:

1. Echo print of input data, plus calculated entry conditions.
2. Nominal trajectory which the guidance system will try to control the vehicle along. Total time of flight is 420 seconds, and pericentre (when flight path angle changes from - to +) is 70.113 km.
3. Desired apogee altitude and time required to descend to 95.4 km.
4. Echo print of feedback gain vector.
5. Transfer orbit elements and inertial position and velocity at beginning of atmospheric phase of flight.
6. Echo print of error inputs.
7. Actual vehicle trajectory, giving actual altitude, velocity, and flight path angle; calculated density; commanded bank angle; and inclination of the osculating orbit.
8. Density profile which was used in this trajectory, and the actual apogee altitude the vehicle reached.
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### Transfer Orbit Elements

- **Inclination** = 72.900\(^\circ\) [deg]
- **Eccentricity** = 0.5949827176
- **True Anomaly at Entry** = -10.2048407894 deg

### Theoretical Impact and Velocity at Entry

- **R(I) = 6.270562, 6.4526 [miles]**
- **R(K) = 14.10723399, 14.56900 [miles]**
- **V(I) = 11714.6264 [miles/sec]**
- **V(K) = 9461443756 [miles/sec]**

### Velocity Drop = -10.

### Flight Path Angle Drop = 0.018

### Mass Percent Decomposition Drop = 0.1

### Actual Vehicle Trajectory

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Vita

Robert J. Chambers, Jr., was born on May 30th, 1948 in Orange, New Jersey. He graduated from Mountain High School, West Orange, New Jersey in June, 1966 and entered the United States Air Force Academy, Colorado. He graduated in June, 1970, receiving a commission as a Second Lieutenant and a Bachelor of Science Degree, with majors in Astronautics and in Engineering Sciences.

After attending pilot training at Craig AFB, Alabama he was assigned to Rhein-Main Air Base, Federal Republic of Germany. While stationed here he flew as a squadron pilot in the C-130 aircraft, and later the C-131/P-29 aircraft. In 1975 he was selected to attend the Air Force Institute of Technology and entered in August of that year.

Permanent Address: 2105 Cherry Hill Drive
Arlington Heights, Illinois 60004

This thesis was typed by the author.
# REPORT DOCUMENTATION PAGE

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**Naviational Accuracy Requirements of Aeromaneuvering Space Vehicles**

A preliminary investigation was conducted to determine the navigational accuracy required by the Aeromaneuvering Orbit-to-Orbit Shuttle (AMCOS) during the atmospheric phase of flight. The guidance scheme, which is the same as the one developed by Lockheed in the original AMCOS study, uses the parameters of velocity, flight path angle, and density altitude to correct to a nominal trajectory. Density altitude is obtained from atmospheric density, which is calculated from vehicle acceleration. Errors in velocity,
flight path angle, and acceleration were introduced into a three-degree of freedom computer simulation of the vehicle trajectory using bank angle commands generated by the guidance equations. Deviations from standard atmospheric density were taken into account.

The maximum errors allowable that still permitted the vehicle to attain its phasing orbit apoee altitude of 720 km (4400 km) were determined. Three types of error were investigated: constant, sinusoidal, and random. For time-varying errors the frequency dependence was examined, as was cross-coupling of errors.

It was concluded that the guidance scheme can tolerate fairly large errors and still guide to an acceptable apoee altitude. The navigational accuracy required for the atmospheric phase of flight is within the capability of present day astrionics.