EXTENDED KALMAN FILTERING APPLIED TO THE POSITION LOCATING AND REPORTING SYSTEM (PLRS)

by

Bernard M. de Mahy, Jr.

December 1976

Thesis Advisor: H. A. Titus

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**Abstract:**
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This study has applied Extended Kalman Filtering techniques to that problem, evolving from a simple Extended Kalman Filter Observer to three moving observers, whose position is uncertain, estimating the position of another unit.
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The author, moreover, wishes to acknowledge the love and sacrifices that he received from his wife, Pam, and children, Marc and Jennifer, while he conducted this study and hereby dedicates this Thesis to them.
I. INTRODUCTION

The precise location of all assets in and about the battle area is of prime importance to the tactical Marine Commander. In the past locating has had to depend on the individual knowing his own position and being able to report it through radio links to higher command. This system suffered from the limitations of terrain, daylight, weather, and the volume of radio traffic during battle.

To alleviate these shortcomings the Marine Corps and Army are investigating a Position Locating and Reporting System (PLRS) to collect, process, and display the location of units, vehicles, and aircraft in and about the battle area.

The PLRS consists of field units and a master unit. The field unit is compact enough to be carried in the field by a man, vehicle, or aircraft. These units will determine the range to other field units in the area and report this information to the master unit for processing and display. The range information is determined by measuring the time required to send a signal from one unit to another and back again plus some "system" delay. When a unit's position is being updated it is referred to as the "Update" unit; and all others are referred to as "Ranging" units.

In a previous study in this area,[1], tests were conducted to investigate the use of the error ellipse in visually displaying the degree of uncertainty of the position of an update unit and the effect of numerous updates on reducing that degree of uncertainty. It was
found that the degree of uncertainty is reduced in the direction of the ranging unit with consecutive updates as shown in Fig 1 taken from that study.

That study also simulated one jet aircraft flying Mach 1 in a constant radius turn as an update unit being ranged on by two stationary ranging units to explore the proper random forcing excitation covariance necessary for adequate filter performance.

It is the intent of this study to further expand the simulation begun in the previous work by adding an additional ranging unit, allowing the movement of the ranging units, and considering the effect of ranging from a unit whose position is not known exactly.
Figure 1 - Consecutive updates will reduce the degree of uncertainty in the direction of the ranging unit.
II. EXTENDED KALMAN FILTERING

The Extended Kalman Filter is widely documented and no attempt at a development of that theory will be made in this work. A brief treatment has been included to establish nomenclature and formulas used. For a more complete development one is referred to reference [2] or similar texts.

As defined in this work, PLRS is described by a set of discrete, linear, cartesian system equations

\[ x(k+1) = \Phi(k) \cdot x(k) + \Gamma(k) \cdot w(k) \]  

(1)

and a set of discrete non-linear measurement equations

\[ z(k) = \underline{m}(x(k),k) + y(k) \]  

(2)

where \( \Phi \) and \( \Gamma \) are linear functions and \( \underline{m} \) is a nonlinear function of the state variables \( x(k) \); \( w(k) \) is the excitation noise and \( y(k) \) is the measurement noise of the system.

The plant noises are considered uncorrelated, zero-mean, and white.

The non-linear measurement equations can be linearized by expanding equation (2) around the best estimate at time \( k \) and using the first-order terms yielding
\[ z(k) = H(k) \hat{x}(k) + v(k) \]

where

\[ H(k) = \frac{\partial H}{\partial x} \mid_{x = \hat{x}(k/k-1)} \]

\( \hat{x}(k/k) \) is the estimated value of the state at \( k \) after the \( k \)th measurement and \( \hat{x}(k/k-1) \) is the predicted value of the state at time \( k \) before the \( k \)th measurement.

The state error vector is

\[ \hat{\epsilon}'(k/k) = \hat{x}(k/k) - \hat{\epsilon}(k) \]

and the predicted error vector is

\[ \hat{\epsilon}'(k/k-1) = \hat{x}(k/k-1) - \hat{\epsilon}(k) \]

The covariance of the state error matrix is

\[ P(k/k) = E[\hat{\epsilon}'(k/k) \hat{\epsilon}'(k/k)] \]

and the predicted covariance of the state error matrix is

\[ P(k/k-1) = E[\hat{\epsilon}'(k/k-1) \hat{\epsilon}'(k/k-1)] \]
The state excitation matrix is

\[ Q(k) = E \cdot \Gamma(k) \cdot w(k) \cdot \Gamma^T(k) \cdot \Gamma(k) \]

and the measurement noise covariance matrix is

\[ R(k) = E[\Sigma(k) \cdot \Sigma^T(k)] \]

The equations that made up the Kalman Filter used in this work are as follows:

\[ P(k/k-1) = \Phi(k) \cdot P(k/k) \cdot \Phi^T(k) + Q(k) \]

\[ G(k) = P(k/k-1) \cdot H^T(k) \cdot [H(k) \cdot P(k/k-1) \cdot H^T(k) + R(k)]^{-1} \]

\[ P(k/k) = [I - G(k) \cdot H(k)] \cdot P(k/k-1) \]

\[ \hat{x}(k/k) = \hat{x}(k/k-1) + G(k) \cdot [z(k) - H(k) \cdot \hat{x}(k/k-1)] \]

\[ \hat{x}(k/k-1) = \Phi(k) \cdot \hat{x}(k/k) \]

\[ z(k) = \mathbb{M}(\hat{x}(k/k-1), k) \]

Since the only observations in this system are ranges,
the observation equation is

\[ z(k) = \left[ x^2(k) + y^2(k) \right]^{1/2} \]

and from equation (3) we get

\[ H(k) = \begin{pmatrix} \frac{x(k)}{x^2(k) + y^2(k)} & 0 & \frac{y(k)}{x^2(k) + y^2(k)} & 0 \end{pmatrix} \]

The covariance of estimation error, \( P \), is an expression of the uncertainty in the estimation of the states. Considering only the estimation's position error, \( P_{\text{position}} \) can be expressed as

\[ P_{\text{position}} = \begin{pmatrix} \sigma_x^2 & \sigma_x \sigma_y \\\n\sigma_y \sigma_x & \sigma_y^2 \end{pmatrix} \]

Since the position estimation error is normally distributed, a curve of constant error probability can be defined by using the exponent of the normal distribution,

\[ x^2 \quad 2r_{xy} \quad xy \quad y^2 \]

\[ \frac{x^2}{\sigma_x^2} - \frac{2r_{xy}}{\sigma_x \sigma_y} + \frac{y^2}{\sigma_y^2} \]
This curve defines an ellipse. Graphically, for the given probability, the estimation may be anywhere in that ellipse.
III. CHOOSING THE BEST RANGER

To move from a simple Kalman Filter observer to the PLRS model the first problem encountered was to choose the best ranger from which to take the measurement. In the previous work,[13], it was shown that the most useful measurement, the one causing the most reduction in the error ellipse, is obtained by observing the update unit from a point aligned with the major axis of the error ellipse of the update unit.

To find the ranger most closely aligned with the major axis of the update unit's error ellipse the orientation of the error ellipse must first be found using the following equation.

\[ \theta = \frac{1}{2} \tan^{-1} \left( \frac{2 \text{ Cov}(x,y)}{\sigma_x^2 - \sigma_y^2} \right) \]

This angle(\(\theta\)) gives the angle between -90° and 90° that the x-axis of the ellipse makes with the x-axis of the co-ordinate system. Looking at the ellipse in this new posture one can find the new "Uncorrelated" variances that define the major and minor axes.

\[ \sigma_x^2 = \frac{\sigma^2_x + \sigma^2_y - \text{Cov}(x,y)}{2 \sin 2\theta} \]

\[ \sigma_y^2 = \frac{\sigma^2_x + \sigma^2_y + \text{Cov}(x,y)}{2 \sin 2\theta} \]
If $a_x^2$ is greater than $a_y^2$ the $x$-axis of the error ellipse is the major axis and $\theta$ is the angle we seek. If $a_y^2$ is greater than $a_x^2$ then the $y$-axis of the error ellipse is the major axis and the angle we seek is $\theta + 90^\circ$.

The bearing of the update unit from the ranger must then be found and it is simply

$$\beta = \tan^{-1} \frac{Y_u - Y_R}{X_u - X_R}.$$
A. TWO RANGING UNITS

In previous work\textsuperscript{[1]}, the PLRS simulation was setup for a jet aircraft flying Mach 1 in a constant 10 Km turn about the origin to act as the update unit for all measurements. Two stationary ranging units were placed at the origin and at 10Km north, 10Km east. Using a one second sample interval, the jet was described by the following matrices:

\[
\begin{align*}
\phi &= \\
&= \\
&= \\
&=
\end{align*}
\]

\[
\begin{pmatrix}
1 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

\[
\begin{pmatrix}
0.5 & 0 \\
1 & 0 \\
0 & 0.5 \\
0 & 1
\end{pmatrix}
\]

Its initial state was

\[
\begin{pmatrix}
0 \\
0.333 \text{ Km/s} \\
10 \text{ Km} \\
0
\end{pmatrix}
\]
Its initial covariance of error matrix, measurement noise covariance, and excitation forcing matrix were

\[
P(1/0) = \begin{pmatrix}
10^{-4} & 0 & 10^{-4} & 0 \\
0 & 10^{-4} & 0 & 0 \\
10^{-4} & 0 & 10^{-4} & 0 \\
0 & 0 & 0 & 10^{-4}
\end{pmatrix}
\]

and

\[
R = 10^{-4}
\]

with

\[
Q = \begin{pmatrix}
2.5 \times 10^{-5} & 5 \times 10^{-5} & 0 & 0 \\
5 \times 10^{-5} & 10^{-4} & 0 & 0 \\
0 & 0 & 2.5 \times 10^{-5} & 5 \times 10^{-5} \\
0 & 0 & 5 \times 10^{-5} & 10^{-4}
\end{pmatrix}
\]

Fig 2 is a display of its final runs. The filter tracked accurately and the error ellipses shown are twenty times their actual size to make them visible. Table 1 shows which was the ranging unit at each measurement time.
Figure 2 — PLRS SIMULATION - A JET IN A CONSTANT 10KM RADIUS TURN FLYING BETWEEN TWO STATIONARY RANGERS
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**Table 1** - The Ranger Chosen at Each Time for the PLRS Two Stationary Ranger Simulation
B. THREE RANGING UNITS

The first step of this study was to add a third ranging unit at 0 north, 10Km east. The algorithm was enlarged to include the additional unit and its comparison with the alignment indicators of the other ranging units.

It can be seen in Fig 3 that the size of the error ellipses were reduced in size in the mid-range area where the jet and the two original units were in line; and the third ranger provides the triangular measurement.

Table 2 shows the ranging unit chosen for the measurement at each time k.
Figure 3 - PLRS SIMULATION - A JET IN A CONSTANT 10KM RADIUS TURN FLYING AMONG THREE STATIONARY RANGERS
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**Table 2** - The ranger chosen at each time for the three stationary ranger simulation
C. RANGING UNITS IN MOTION

In the second step the rangers are given motion. The rangers at the origin and at 10Km north, 10Km east were to move north and south respectively at 3Kts as infantrymen. The ranger at 0 north, 10Km east was to move west at 120Kts as a helicopter. Again using one second sample intervals, their motion was defined using discrete linear state equations

\[ x(k+1) = \phi(k) x(k), \]

where

\[
\phi = \begin{bmatrix}
1 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

with the initial states shown below:

\[
x = \begin{bmatrix}
0 \\
0 \\
1.67 \times 10^3 \text{ Km/s}
\end{bmatrix} \text{ INFANTRYMAN}
\]

\[
x = \begin{bmatrix}
10 \\
0 \\
-1.67 \times 10^3 \text{ Km/s}
\end{bmatrix} \text{ INFANTRYMAN}
\]
It can be seen in Fig 4 that no system depreciation resulted from the motion of the rangers. Table 3 shows the ranging unit chosen for the measurement at each time.
Figure 4 - PLRS SIMULATION - A JET IN A CONSTANT 10KM RADIUS TURN FLYING AMONG THREE MOVING RANGERS
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**TABLE 3** - THE RANGER CHOSEN AT EACH TIME FOR THE THREE MOVING RANGER SIMULATION
D. SOURCE OF MEASUREMENT NOISE

In the above simulations the position of the ranging unit has been assumed to be exact; while in actual application the ranging units will have covariances of estimation error defining an error ellipse; and the ranging unit might be anywhere within that ellipse. To bring this position uncertainty into the simulation, the radius of the error ellipse along the bearing from the ranging unit to the update unit was defined as the covariance of measurement error.

The equation for the radius of an ellipse is a function of the major axis, the minor axis, and the angle at which the measurement is made. To find the measurement noise covariance, or the ellipse radius, $\sigma_x^2$ and $\sigma_y^2$ must be compared and the larger defined as $M_J$, the major axis, and the smaller defined as $M_n$, the minor axis. The angle, $\alpha$, at which the radius is determined is measured from the major axis and thus is calculated as the difference between $\theta$ and $\beta$. Fig 5 shows the geometry of the calculation of the covariance of measurement noise. The equation for $R$ and the radius squared of the ellipse is:

$$ R = r^2 = \frac{M_J M_n}{M_J \sin^2 \alpha + M_n \cos^2 \alpha} $$

It can be seen in Fig 6 that performance was improved slightly using the covariance of estimation error as the sole source of measurement noise.
Table 4 shows the ranger chosen at each time for the three moving rangers with position uncertainty simulation.
Figure 5 - r is the radius of the error ellipse - $r^2 = R$

$R$ is the covariance of measurement noise
Figure 6 - PLRS SIMULATION - A JET IN A CONSTANT 10KM RADIUS TURN FLYING AMONG THREE MOVING RANGERS WITH POSITION UNCERTAINTY
<table>
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</table>

TABLE 4 — THE RANGER CHOSEN AT EACH TIME FOR THE THREE MOVING RANGERS WITH POSITION UNCERTAINTY SIMULATION
V. CONCLUSION

The placement of the third ranger showed the value of triangulation of the rangers. The closer to normal the bearings of the rangers are to each other, the better the results of consecutive ranges.

The allowance for motion and the representation of the ranger's position uncertainty as the source of measurement error were important steps toward full simulation of the system; and they were accomplished without degradation of performance.

A better simulation may be to represent the measurement error as the ranger's position uncertainty plus some system measurement error.

Still to be accomplished is the ability to update all units at each ranging, and to provide a gating system that will demand more frequent updates for faster moving units and less frequent updates for slower units.

A program listing of the three moving rangers with position uncertainty is included with an annotated data deck.
EXEC FORTCLGP,REGION.GO=200K

REAL*8 GAMMA,COVW,R PHI,H,TEMP,TEMP1,TEMP2,PKKM1,G,PKK,Q,EI,PR,PRR

COMMON E[4,4],Q[4,4],C[4,4],PKK[4,4],GAMMA[4,4],COVW[4,4]
ITEMP[4,4],TEMP1[4,4],TEMP2[4,4],H[4,4],PKKM1[4,4],R[4,4],PHI[4,4],MCSP0008
2VAR[4,4,60],GKS[4,4,60],PKKS[4,4,60],XM[4,4,60],ERR[4,4,60],CH200002
3GAMMA[4,4],PHIS[4,4],XS[4,4,60],HS[4,4],GK[4,4],SIGW[4,4],X[4,4],CH200003
4SIGXZ[4,4],XZMEAN[4,4],XHKK[4,4],XHKKM1[4,4],VTMP[4,4],Z[4,4],V[4,4],SIGV[4,4],CH200004
5XHATZ[4,4],X[60],Y[60],PX[10],PY[10],PR[4,4,4],CH200005
6N,NSAM,IQ,M,ITER,ITRK,IN,ISTAT,K,ITRO,IXZ,IV,IW,IEST,ND,NR

DIMENSION XP[80],YP[80],PRR[4,4]

N=ORDER OF SYSTEM MODEL AND FILTER (DIMENSION OF X,XHAT)
M=NUMBER OF MEASUREMENTS (DIMENSION OF THE VECTOR Z)
IN=NUMBER OF INPUT RANDOM FORCING FCNS (=DIMENSION OF W)
NSAM=NUMBER OF TIME SAMPLES
NENS=NUMBER OF MEMBERS IN ENSEMBLE
READ (5,81) N,M,IN,NSAM,NENS,NR

READ (5,82) ND
THE VALUE OF ND READ IN MUST EQUAL THE ROW (AND COLUMN) DIMENSION
SPECIFIED FOR THE SQUARE MATRIX "TEMP1", E.G. IF TEMP[3,3] IS
SPECIFIED IN THE COMMON STATEMENT "ND" MUST BE EQUAL TO 3.

IPRT=0 -- SOME OR ALL OUTPUT DATA IS PRINTED
IPLT=0 -- SOME OR ALL OUTPUT DATA IS PLOTTED
READ (5,84) IPRT,IPLT

CALL OVFLOW
IW = 6395217
IV = 1936748
IXZ = 135769

THE FOLLOWING SECTION READS THE SPECIFIED INPUT MATRICES
CALL MREAD (PHI,N)
C
DC 23 I=1,N
DO 23 J=1,N
23 PHI(I,J) = PHI(I,J)
WRITE (6,131)
CALL MWRITE (PHI,N,N)
C
CALL MREAD (H,M,N)
DO 25 I=1,M
DO 25 J=1,N
25 HS(I,J) = H(I,J)
WRITE (6,132)
CALL MWRITE (H,M,N)
C
CALL MREAD (R,M,N)
WRITE (6,133)
CALL MWRITE (R,M,N)
C
CALL MREAD (COVW,N,N)
WRITE (6,134)
CALL MWRITE (COVW,N,N)
CALL MREAD (GAMMA,N,N)
C
DO 30 I=1,N
DO 30 J=1,N
30 GAMMA(I,J) = GAMMA(I,J)
WRITE (6,136)
CALL MWRITE (GAMMA,N,N)
C
CALL MREAD (PKKM1,N,N)
WRITE (6,137)
CALL MWRITE (PKKM1,N,N)
C
DO 311 K=2,NR
CALL MREAD (PRR,N,N)
DO 310 I=1,N
DO 310 J=1,N
310 PR(I,J,K) = PRR(I,J)
311 CONTINUE
C
CALL VREAD (SIGV,M)
WRITE (6,138)
CALL WRITE (SIGV, M)

DO 340 I=1, NR
READ (5, 144) (XHATZ(I, J), J=1, N)
WRITE (6, 140)
340 WRITE (6, 146) (XHATZ(I, J), J=1, N)

DO 360 I=1, NR
READ (5, 144) (XS(I, J), J=1, N)
INITIAL CONDITION HAS BEEN READ
WRITE (6, 143)
360 WRITE (6, 146) (XS(I, J), J=1, N)

38 CALL TRACK

DO 390 K=1, NR
WRITE (6, 145)
WRITE (6, 146) (XS(K, I, J), I=1, N)
WRITE (6, 146) (XS(K, I, NSAM), I=1, N)
390 CONTINUE

THE FOLLOWING SECTION PREPARES FOR THE MONTE CARLO LOOP
FORM NXN IDENTITY MATRIX IN DOUBLE PRECISION

DO 41 I=1, N
DO 41 J=1, N
EIJ(I, J) = 0.00
41 IF (I .EQ. J) EIJ(I, J) = 1.00

GIVEN THE MATRIX GAMMA AND THE COVARIANCE OF W COMPUTE Q
USING DOUBLE PRECISION ARITHMETIC
CALL QMAT
WRITE (6, 135)
CALL MWRITE (Q, N, N)

SET UP ARRAYS FOR COMPUTING STATISTICS

DO 48 I=1, NR
DO 48 K=1, NSAM
DC 48 J=1, N
XM(I,J,K) = 0.
ERR(I,J,K) = 0.

DO 10 L = 1,N
48 VAR(J,L,K) = 0.

BEGIN MAIN ITERATION LOOP HERE

DC 54 ITER=1,NENS

49 DO 50 I = 1,N
50 XHKM1(I) = XHATZ(I,I)

DO 54 K = 1,NSAM
FORM NOISY MEASUREMENT FROM TRUE STATE VALUE

DO 51 I = 1,N
51 X(I) = XS(I,I,K)

CALL GAIN

DO 52 I = 1,N
DO 52 J = 1,M
52 GKS(I,J,K) = G(I,J)

UPDATE THE STATE ESTIMATE

53 CALL ESTIM

UPDATE RUNNING SUMS USED IN COMPUTING STATISTICS
CALL STAT

54 CONTINUE

DIVIDE RUNNING SUMS COMPUTED BY SUBROUTINE STAT BY ENSEMBLE
SIZE TO COMPUTE STATISTICS
ENS = NENS

DO 56 K = 1,NSAM

DO 56 J = 1,N
55 ERR(I,J,K) = ERR(I,J,K)/ENS

CH200017
CH200019
MCSP0364
MCSP0365
MCSP0366
MCSP0367
MCSP0368
MCSP0369
MCSP0370
MCSP0371
MCSP0372
MCSP0375
MCSP0376
CH200018
MCSP0378
MCSP0379
MCSP0380
MCSP0381
MCSP0382
MCSP0383
MCSP0384
CH200020
MCSP0386
MCSP0396
MCSP0397
MCSP0398
MCSP0400
MCSP0401
MCSP0402
MCSP0403
MCSP0404
MCSP0405
MCSP0406
MCSP0407
MCSP0408
MCSP0409
MCSP0411
MCSP0414
MCSP0417
MCSP0418
MCSP0419
MCSP0420
MCSP0421
MCSP0422
MCSP0423
CH200022
56 VAR(J,J,K) = VAR(J,J,K)/ENS-ERR(1,J,K)**2

C IF (IPRI.NE.0) GO TO 64
C CALL PRT
C 64 IF (IPLT.NE.0) GO TO 80
C CALL PLT
C 80 CONTINUE
C STOP
C 81 FORMAT (6(110))
C 82 FORMAT (12)
C 83 FORMAT (7(110))
C 84 FORMAT (2(15))
C 85 FORMAT (5(110))
C 130 FORMAT (4X,'N=',12,4X,'M=',12,4X,'IN=',12,4X,'NSAM=',13,4X,'NENS=',IMCS0616
C 15,4X'ND=',12,')
C 131 FORMAT (4X,'THE PHI MATRIX IS',/)
C 132 FORMAT (4X,'THE H MATRIX IS',/)
C 133 FORMAT (4X,'THE R MATRIX IS',/)
C 134 FORMAT (4X,'THE COVARIANCE OF W MATRIX IS',/)
C 135 FORMAT (4X,'THE Q MATRIX IS',/)
C 136 FORMAT (4X,'THE GAMMA MATRIX IS',/)
C 137 FORMAT (4X,'THE MATRIX P(0/-1) IS',/)
C 138 FORMAT (4X,'THE STD. DEVIATIONS OF MEASUREMENT NOISE ARE',/)
C 139 FORMAT (4X,'THE STD. DEVIATIONS OF INPUT FORCING W ARE',/)
C 140 FORMAT (4X,'THE VECTOR XHAT(0/-1) IS',/)
C 141 FORMAT (4X,'THE MEAN OF THE VECTOR X(0) IS',/)
C 142 FORMAT (4X,'THE STANDARD DEVIATIONS OF THE VECTOR X(0) ARE',/)
C 143 FORMAT (4X,'THE INITIAL STATE IS',/)
C 144 FORMAT (4F20.0)
C 145 FORMAT (4X,'THE FIRST AND LAST POINTS ON THE SINGLE TRACK TO BE USED ARE',/)
C 146 FORMAT (9(2X,1PE12.5),/)
C 156 FORMAT ('1')
C END

C SUBROUTINE QMAT
C THIS SUBROUTINE COMPUTES THE MATRIX Q FROM THE EQUATION
C \[ Q = \text{GAMMA} \left( \mathbf{E} \left( \text{W}^* \text{W} \right) \right) \text{ GAMMAT} \]
C DOUBLE PRECISION ARITHMETIC IS USED
C
REAL*8 GAMMA,COVW,R,PHI,H,TEMP,TEMPI,TEMP2,PKKM1,G,PKK,Q,E1,PR
COMMON EI(4,4),Q(4,4),G(4,4),PKK(4,4),GAMMA(4,4),COVW(4,4),
TEMP(4,4),TEMPI(4,4),TEMPP(4,4),H(4,4),PKKM1(4,4),R(4,4),PHI(4,4)
END

CALL PROD (GAMMA,COVW,N,N,IN,IN,TEMP)
CALL TRANS (GAMMA,N,N,TEMPI)
CALL PROD (TEMP,TEMPI,N,N,N)
RETURN

SUBROUTINE QON

IF Q IS TO BE COMPUTED ON-LINE (IFLQ.NE.0) IT IS DONE
IN THIS SUBROUTINE

REAL*8 GAMMA,COVW,R,PHI,H,TEMP,TEMPI,TEMP2,PKKM1,G,PKK,Q,E1,PR
COMMON EI(4,4),Q(4,4),G(4,4),PKK(4,4),GAMMA(4,4),COVW(4,4),
TEMP(4,4),TEMPI(4,4),TEMPP(4,4),H(4,4),PKKM1(4,4),R(4,4),PHI(4,4)
END

THE APPROPRIATE STATEMENTS FOR COMPUTING Q ON-LINE MUST
BE INSERTED HERE BY THE USER

END

SUBROUTINE RON

IF R IS TO BE COMPUTED ON-LINE (IFLR.NE.0) IT IS DONE
IN THIS SUBROUTINE

REAL*8 GAMMA,COVW,R,PHI,H,TEMP,TEMPI,TEMP2,PKKM1,G,PKK,Q,E1,PR
COMMON EI(4,4),Q(4,4),G(4,4),PKK(4,4),GAMMA(4,4),COVW(4,4),
TEMP(4,4),TEMPI(4,4),TEMPP(4,4),H(4,4),PKKM1(4,4),R(4,4),PHI(4,4)
END

CALL PROD (GAMMA,COVW,N,N,IN,IN,TEMP)
CALL TRANS (GAMMA,N,N,TEMPI)
CALL PROD (TEMP,TEMPI,N,N,N)
RETURN

SUBROUTINE QON

IF Q IS TO BE COMPUTED ON-LINE (IFLQ.NE.0) IT IS DONE
IN THIS SUBROUTINE

REAL*8 GAMMA,COVW,R,PHI,H,TEMP,TEMPI,TEMP2,PKKM1,G,PKK,Q,E1,PR
COMMON EI(4,4),Q(4,4),G(4,4),PKK(4,4),GAMMA(4,4),COVW(4,4),
TEMP(4,4),TEMPI(4,4),TEMPP(4,4),H(4,4),PKKM1(4,4),R(4,4),PHI(4,4)
END

THE APPROPRIATE STATEMENTS FOR COMPUTING Q ON-LINE MUST
BE INSERTED HERE BY THE USER

END

SUBROUTINE RON

IF R IS TO BE COMPUTED ON-LINE (IFLR.NE.0) IT IS DONE
IN THIS SUBROUTINE

REAL*8 GAMMA,COVW,R,PHI,H,TEMP,TEMPI,TEMP2,PKKM1,G,PKK,Q,E1,PR
COMMON EI(4,4),Q(4,4),G(4,4),PKK(4,4),GAMMA(4,4),COVW(4,4),
TEMP(4,4),TEMPI(4,4),TEMPP(4,4),H(4,4),PKKM1(4,4),R(4,4),PHI(4,4)
END
THE APPROPRIATE STATEMENTS FOR COMPUTING R ON-LINE MUST
BE INSERTED HERE BY THE USER

RETURN
END

SUBROUTINE STAT

THIS SUBROUTINE COMPUTES RUNNING SUMS USED IN DETERMINING
THE SAMPLE STATISTICS OF TRACK AND ESTIMATION ERRORS. IN THE DEFAULT
OPTION (ISTAT.EQ.0) THE STATISTICS TO BE COMPUTED ARE MEAN OF
TRACK, MEAN OF ESTIMATION ERROR AND VARIANCE OF ESTIMATION
ERROR. IF (ISTAT.NE.0) THE OFF-DIAGONAL TERMS IN THE COVARIANCE OF
ESTIMATION ERROR MATRIX ARE ALSO COMPUTED.

REAL*8 GAMMA,COVAR,R,PHI,H,TMP,TEMDE,TEMDE2,PKKM1,G,PKK,Q,E1,PR
COMMON E1(4,4),G(4,4),GAMMA(4,4),COVAR(4,4),
IH(4,4),GAMMA(4,4),COVAR(4,4),R(4,4),PHI(4,4),
VAR(4,4,60),KKS(4,4,60),PKK(4,4,60),XMP(4,4,60),ERRR(4,4,60),
SIEG(4,4),SIGM(4,4),XK(4,4),SIGV(4,4),
NSAM,10,ITER,ISTAT,K,ITRO,IXZ,IV,IN,TEST,ND,CR
DIMENSION EXH(3)

IF (ITR.K.NE.1) GO TO 2
IF (ITER.NE.1) GO TO 4

DO 1 J=1,N
1 XM(1,J,K) = XS(I,J,K)
GO TO 4

2 CONTINUE

3 XM(1,J,K) = XM(1,J,K)*XS(I,J,K)

4 CONTINUE

DC 5 J=1,N
EXH(J) = XHK(J)-XS(I,J,K)
ERR(J,J,K) = ERR(J,J,K)+EXH(J)
VAR(J,J,K) = VAR(J,J,K)+EXH(J)**2

IF (ISTAT.EQ.0) RETURN

DC 6 L=2,N
LM1 = L-1
DO 6 J=1,LM

6 VAR(L,J,K) = VAR(L,J,K) + EXH(L)*EXH(J)

RETURN
END

SUBROUTINE ZERO

IF THIS SUBROUTINE GENERATES THE INITIAL STATE VALUE FROM A NORMAL
RANDOM NUMBER GENERATOR. IT IS ASSUMED THAT THE INITIAL STATE
HAS COMPONENTS THAT ARE INDEPENDENT

REAL*8 GAMMA,COVW,R,PHI,H,TEMP,TEMP1,TEMP2,PKKM1,G,PKK,Q,EI,PR
COMMON E(4,4),Q(4,4),G(4,4),PKK(4,4),GAMMA(4,4),COVW(4,4).

2 VAR(4,4,60),GKS(4,4,60),PKKS(4,4,60),X(4,4,60),ERR(4,4,60),
3 GAMMAS(4,4),PHIS(4,4),XS(4,4,60),HS(4,4),GK(4,4),SIGM(4),X(4).
5 XHATZ(4,4),XZ(60),Y(60),PY(10),PR(4,4,4),
6 N,NNSM,NP,IPTR,N,ISTAT,K,ITRO,IXZ,IV,IV,IV,IV,IV,IV,IV,IV,IV,IV,IV,IV
CALL SNORM (IXZ,X,N)

DO 1 I=1,N
1 XS(I,1,1) = SIGXZ(I)*X(I)+XZMEAN(I)

RETURN
END

SUBROUTINE ADD (A,B,N,M,C)

THIS SUBROUTINE ADDS THE NXM MATRICES A AND B, STORING THE
RESULT IN C

REAL*8 A,B,C
DIMENSION A(4,4),B(4,4),C(4,4)

DO 1 I=1,N
1 C(I,J) = A(I,J)+B(I,J)

RETURN
END

SUBROUTINE MREAD (A,N,M)

801O.5. THE ENTRIES IN THE FIRST ROW OF A ARE READ FIRST, THEN
THIS SUBROUTINE READS AN NXM MATRIX A ACCORDING TO THE FORMAT
THE ENTRIES IN THE SECOND ROW, AND SO ON.

REAL*8 A
DIMENSION A(4,4)

DO 1 I=1,N
1 READ (5,2) (A(I,J),J=1,M)

RETURN
C
2 FORMAT (8F10.0)
END
SUBROUTINE WRITE (A,N,M)
C
THIS SUBROUTINE WRITES THE ENTRIES OF THE NXM MATRIX A
REAL*8 A
DIMENSION A(4,4)
C
DC 1 I=1,N
1 WRITE (6,2) (A(I,J),J=1,M)
C
RETURN
C
2 FORMAT (9(2X,1PE12.5))
END
SUBROUTINE PROD (A,B,N,M,L,C)
C
THIS SUBROUTINE COMPUTES THE MATRIX PRODUCT AB AND STORES THE
RESULT IN C
C
A = NXM, B = MXL, C = NXL
REAL*8 A,B,C,T
DIMENSION A(4,4),B(4,4),C(4,4),T(4,4)
C
DC 1 I=1,N
C
DO 1 J=1,L
1 T(I,J) = 0.0
C
DO 2 I=1,N
2 T(I,J) = T(I,J) + A(I,K)*B(K,J)
C
DO 3 I=1,N
3 C(I,J) = T(I,J)
C
RETURN
END
SUBROUTINE SUB (A,B,N,M,C)
C
THIS SUBROUTINE SUBTRACTS THE NXM MATRIX B FROM THE NXM MATRIX
A AND STORES THE RESULT IN C
REAL*8 A,B,C
DIMENSION A(4,4),B(4,4),C(4,4)
C  DC  1  I=1,N
C  DC  1  J=1,M
  1 C(I,J) = A(I,J)-B(I,J)
C  RETURN
END
SUBROUTINE TANS (A,N,M,C)
THIS SUBROUTINE FORMS THE MATRIX TRANSPOSE CF A STORAGE THE
RESULT IN C
A = MXN,  C = MXN
REAL*8 A,C
DIMENSION A(4,4),C(4,4)
DO 1 I=1,N
C  DC  1  J=1,M
  1 C(J,I) = A(I,J)
C  RETURN
END
SUBROUTINE VADD (X,Y,N,Z)
THIS SUBROUTINE COMPUTES THE SUM OF THE N-VECTORS X AND
Y AND STORES THE RESULT IN THE N-VECTOR Z
REAL*4 X(4),Y(4),Z(4)
DO 1 I=1,N
  1 Z(I) = X(I)+Y(I)
C  RETURN
END
SUBROUTINE VPROD (A,X,M,N,Y)
THIS SUBROUTINE COMPUTES THE PRODUCT OF THE MXN MATRIX
A AND THE N-VECTOR X AND STORES THE RESULT IN THE
M-VECTOR Y
REAL*4 A(4,4),X(4),Y(4),T(4)
DO 1 I=1,M
  1 T(I) = 0.00
C  DO 1 J=1,N
  1 T(I) = T(I)+A(I,J)*X(J)
C
DO 2 I=1,M
   Y(I) = T(I)
2 RETURN
RETURN
END

THIS SUBROUTINE READS THE N-DIMENSIONAL SP. VECTOR V

DIMENSION V(N)
READ (5,1) (V(I),I=1,N)
RETURN
1 FORMAT (8F10.0)
END

SUBROUTINE VSUB (X,Y,N,Z)
REAL*4 X(4), Y(4), Z(N)
DO 1 I=1,N
   Z(I) = X(I) - Y(I)
1 RETURN
END

SUBROUTINE VWRITE (V,N)
DIMENSION V(N)
WRITE (6,1) (V(I),I=1,N)
RETURN
1 FORMAT (9(2*X,1PE12.5))
END

SUBROUTINE TRACK
IF TRACK IS TO BE GENERATED ON-LINE IT IS DONE IN THIS SUBROUTINE
IN THE DEFAULT OPTION (ITRK,EQ.0) THE TRACK IS GENERATED
FROM THE STANDARD LINEAR DIFFERENCE EQUATION

X(K+1) = PHI*X(K) + GAMMA*W(K)

REAL*8 GAMMA, COV, R, PHI, H, TEMP, TIME, TEMP1, TEMP2, PKKM1, G, PKK, Q, E, PR
COMMON E(4,4), Q(4,4), G(4,4), PKK(4,4), GAMMA(4,4), COV(4,4)
TIME(4,4), TEMP1(4,4), TEMP2(4,4), H(4,4), PKKM1(4,4), R(4,4), PHI(4,4)
2VAR(4,4,60), GKS(4,4,60), PKKS(4,4,60), XM(4,4,60), ERR(4,4,60)
3GAMMA(4,4), PHIST(4,4), X(4,4,60), HS(4,4), GKS(4,4,60), SIGV(4,4,60)
4GAMMA(4,4), XZMEAN(4,4), XHKK(4,4), XHKKM(4,4), VTEMP(4,4,60), Z(I,4), V(4,4,60)
5XHAT(4,4), Z(4,60), PX(10), PY(10), PR(4,4)
6N, NSAM, M, IT, ITRK, IN, ISTAT, K, ITRO, IZ, IV, IW, IEST, ND, NR
COMMON W(3), M, N, IT, ISTAT, K, ITRO, IZ, IV, IW, IEST, ND, NR
DIMENSION W(3)
TO GENERATE A SINGLE TRAJECTORY AND STORE IT IN THE ARRAY
X(1,1), I=1, N, K=2
NSAM (NOTE THAT IF A SINGLE TRAJECTORY IS TO BE
GENERATED, THE INITIAL CONDITION HAS BEEN READ IN AND STORED
IN X(1,1), I=1, N)
TPI = 2.*3.14159265
DC 5 K=2, NSAM
EKM1 = K-1
T = 1.0*EKM1
A=0.03333*T
IF(A.LT.TPI) GO TO 10
MM=A/TPI
FM=MM
A = A - FM*TPI
10 CONTINUE
XS(1,1,K) = 10.*SIN(A)
XS(1,2,K) = .3333*COS(A)
XS(1,3,K) = 10.*COS(A)
XS(1,4,K) = .3333*SIN(A)
DO 7 I=2, NR
EKM1 = K-1
XS(I,1,K) = XS(I,1,EKM1) + XS(I,2,1)
XS(I,2,K) = XS(I,2,1)
XS(I,3,K) = XS(I,3,EKM1) + XS(I,4,1)
7 CONTINUE
C RETURN
END
SUBROUTINE MEAS
THIS SUBROUTINE STARTS WITH THE TRUE STATE VALUE XS
AND ADDS ZERO-MEAN WHITE GAUSSIAN NOISE TO XS TO
GENERATE A NOISY VECTOR OF MEASUREMENTS Z.
C
REAL*8 GAMMA, COVW, R, PHI, H, TEMP, TEMP1, TEMP2, PKKML, G, PKK, Q, EI, PR
COMMON EI(4,4), GI(4,4), GI(4,4), PKK(4,4), GAMMA(4,4), COVW(4,4),
1 TEMP1(4,4), TEMP2(4,4), H(4,4), PKKML(4,4), R(4,4), PHI(4,4), PR(4,4),
2VAR(4,4,60), GKS(4,4,60), PKKS(4,4,60), XM(4,4,60), ERR(4,4,60),
3GAMMAS(4,4), PHIS(4,4), XS(4,4,60), HS(4,4), G(4,4), SIGW(4,4), X(4),
4SIGXZ(4,4), XZMEAN(4), XHKK(4), XHKKM(4), VMP(4,4,4), Z(4,4,4), SIGV(4,4),
5XHATZ(4,4), XZ(60), YZ(60), PXL(10), PYL(10), PR(4,4,4),
6N, NSAM, IQ, M, ITER, ITRK, IN, ISTAT, K, ITRO, IXZ, IV, IW, IEST, ND, NR
C
ALPHA = XS(1,3,K)
BETA = XS(1,1,K)
Z(I) = SQRT(ALPHA**2+BETA**2)
Z(2) = ATAN2(ALPHA, BETA)
CALL SNORM (IV, V, M)
C
DO 1 I=1, M
1 VI = SIGV(I)*V(I)
C
CALL VADD (Z, V, M, Z)
ALPHA = Z(1)*COS(Z(2))
BETA = Z(1)*SIN(Z(2))
Z2(K)=ALPHA
Z2(K)=BETA
RETURN

C

SLOSEROUTINE GAIN
C

REAL*8 GAMMA,COVW,R PHI,H,TEMP1,TEMP2,PKMK1,G,PKK,Q,E1,PR
COMMUN E(4,4),Q(4,4),G(4,4),PKK(4,4),GAMMA(4,4),COVW(4,4),
1*TEMP(4,4),TEMP(4,4),TEMP2(4,4),H(4,4),PKKM(4,4),R(4,4),PHI(4,4),PR(4,4),PR(4,4)
2*VAR(4,4,60),GKS(4,4,60),PKKS(4,4,60),XM(4,4,60),ERR(4,4,60)
3*GAMMAS(4,4),PHI(4,4),X(4,4),HS(4,4),E(4,4),SIGN(4,4),X(4,4),
4*SIGXZ(4,4)*XZ(4,4),XZ(4,4),XZ(4,4),XZ(4,4)
5*HHATXZ(4,4),Y(4,4),Y(4,4);
6*NSAM,10,M,ITER,1,TRK,IN,ISTAT,K,ITRO,1X,1W,1H,1EST,ND,MR
DIMENSION BE(4), ER(4)

C

C G(K) = P(K,K)*HT+(H*P(K,K)*HT + R)
DC 300 I=1,4
DO 300 J=1,4
300 PKKS(I,J,K)=PKKM(I,J)
C

C IF(DABS(PKMM(1,1)-PKMM(3,3))>.GT.0) GO TO 11
PKMM(1,1)=PKMM(3,3)+.00001
11 CONTINUE
C

C FINE UPDATE UNIT’S ELLIPSE ORIENTATION
C
THE=.5*DATAN(.2*PKMM(1,1)/(PKMM(1,1)-PKMM(3,3)))
IF(DABS(THE)>.GT.0) GO TO 10
THE=.00001
10 CONTINUE
C

C FINE UNCOUPLED VARIANCES
C
SIGX=(PKMM(1,1)+PKMM(3,3))/2.*PKMM(1,1)/SIN(2.*THE)
SIGY=(PKMM(1,1)+PKMM(3,3))/2.*PKMM(1,1)/SIN(2.*THE)
C

C ADJUST THETA
C
IF(SIGX2.GE.SIGY) GO TO 63
THE=THE+.14159265/2.
C

C CALCULATE BEARING
DC 9 IN=1,3
IO = IN + 1
IF(ABS(XHKKM1(1)-XS(IO,1,K)).GT.0) GO TO 9
XHKKM1(1) = 0.0000001*XS(IO,1,K)
9 BE(IN)=ATAN((XHKKM1(3)-XS(IO,3,K))/(XHKKM1(1)-XS(IO,1,K)))
63 CONTINUE
66 DO 4 IN=1,3
4 ER(IN) = ABS(THE-BE(IN))

CHOOSE BEST RANGER

IF(ABS(COS(ER(1))).LE.ABS(COS(ER(2)))) GO TO 7
IF(ABS(COS(ER(1))).LE.ABS(COS(ER(3)))) GO TO 70
IN=1
GO TO 8
7 IF(ABS(COS(ER(2))).LE.ABS(COS(ER(3))))GO TO 70
IN=2
GO TO 8
70 IN=3
8 XIN=IN
IE=IN +1

CALCULATE H

RR =((XHKKM1(1)-XS(IO,1,K))**2+(XHKKM1(3)-XS(IO,3,K))**2)**.5
WRITE (6,22)
22 FORMAT(6X,'THE',12X,'SIG2X',8X,'SIG2Y',10X,'BE(1)',8X,'BE(2)',CH3****
18X,'XIN',10X,'ER(1)',9X,'ER(2)',10X,'RR')
WRITE (6,146) THE,SIG2X,SIG2Y,BE(1),BE(2),XIN,ER(1),ER(2),RR
146 FORMAT (9(2X,1PE12.5),/)
H(1,1) = (XHKKM1(1)-XS(IO,1,K))/RR
H(1,3) = (XHKKM1(3)-XS(IO,3,K))/RR
IF(OABS(PR(1,1,IN)-PR(3,3,IN)).GT.0) GO TO 20
PR(3,3,IN) = PR(1,1,IN)*0.000001
CH3****
20 CONTINUE

FIND RANGER'S ERROR ELLIPSE ORIENTATION (THER)

THER = 0.5*DATAN(2.*PR(1,3,IN)/(PR(1,1,IN)-PR(3,3,IN)))
IF(ABS(THER)).GT.0 GO TO 19
CH3****
THER = 0.0000
CH3****
19 SIG2XR = (PR(1,1,IN) + PR(3,3,IN)) / 2. + PR(1,3,IN) / SIN(2.*THER)
CH3****
1

CALCULATE RANGER'S UNCORRELATED VARIANCES

SIG2YR = (PR(1,1,IN) + PR(3,3,IN)) / 2. - PR(1,3,IN) / SIN(2.*THER)
CH3****
1
IF(SIG2YR.GE.0.1) GO TO 21
CH3??????
SIG2YR = - SIG2YR
21 CONTINUE
   ERP = BE(IN) - THER
C
C
   CIND THE MAJOR AND MINOR AXES
IF(SIG2XR.GE.SIG2YR)GO TO 24
   SIGMJ = SIG2YR
   SIGMN = SIG2XR
   GO TO 25
24 SIGMJ = SIG2XR
   SIGMN = SIG2YR
25 CONTINUE
C
C
   CALCULATE THE NOISE COVARIANCE
   R(1,1) = SIGMJ*SIGMN/SIGMJ*(((SIN(ERP))**2) + SIGMN*(((COS(ERP))**2))PLR06500
   WRITE (6,146) THER, SIG2XR, SIG2YR, ERP, R(1,1), BE(3)
23 FORMAT (6X,'THER',12X,'SIG2XR', 8X,'SIG2YR', 9X,'ERP',10X,'R(1,1)')CH3****
   WRITE(6,146)THER, SIG2XR, SIG2YR, ERP, R(1,1), BE(3)
   CALL TRANS (H,M,N,TEMP2)
   CALL PROD (PKM1,TEMP2,N,N,TEMP)
   CALL PROD (H,TEMP,M,N,TEMP1)
   CALL ADD (TEMP1,R,M,M,TEMP1)
   IF (M.EQ.1) GO TO 2
   MG = ND
   CALL GAUSS3 (M,EPS,TEMP1,TEMP2,KER,MD)
   CALL PROD (TEMP,TEMP2,N,M,M,G)

C
C
   NOTE HERE: PKK(I,J) = P(K/K) WHERE
   P(K/K) = (I-G(K)+H)**P(K/K-1)
   CALL PROD (G,N,N,M,TEMP)
   CALL SUB (EI,TEMP,N,N,TEMP2)
   CALL PROD (TEMP2,PKM1,N,N,N,PKK)
C
C
   NOTE HERE: PKKM1(I,J) = P(K/K-1) WHERE
   P(K/K-1) = PHI*P(K-1/K-1)*PHI + Q
   CALL TRANS (PHI,N,N,TEMP2)
   CALL PROD (PK,TEMP2,N,N,N,TEMP)
   CALL PROD (PHI,TEMP,N,N,TEMP1)
   CALL ADD (TEMP1,Q,N,N,PKKM1)
   RETURN
C
C
   DO 3 I=1,N
   3  G(I,1) = TEMP(I,1)/TEMP1(I,1)
C
C
   GS TO 1
END

SUBROUTINE ESTIM
THIS SUBROUTINE UPDATES THE STATE ESTIMATE. IN THE DEFAULT
CONDITION TEST.EQ.0 THE STANDARD EQUATIONS
XHAT(K/K)=XHAT(K/K-1)+G(K)*(Z(K)-H(K)*XHAT(K/K-1))
XHAT(K+1/K)=PHI*XHAT(K/K)

ARE EVALUATED
REAL*8 GAMMA,COVM,R,PHI,H,TEMP,TEMP1,TEMP2,PKKM1,G,PKK,Q,E1,PR
COMMON E1(4,4),Q(4,4),G(4,4),PKK(4,4),PHI(4,4),H(4,4),PKKM1(4,4),R(4,4)

CALL VPROD (HS,XHKKM1,M,N,VTMP)
VTMP(1)=(XHKKM1(1)-X(1,1))**2+(XHKKM1(3)-X(1,3))**2 5
Z(1)=((XZ(K)-X(1,1))**2+(YZ(K)-X(1,3))**2 5

CALL VSUB (Z,VTMP,H,VTMP)
CALL VPROD (OK,VTMP,N,M,VTMP)
CALL VADD (XHKKM1,VTMP,N,XHKK)

XHAT(K/K) HAS BEEN COMPUTED AND STORED IN THE ARRAY XHKK
CALL VPROD (PHI,XHKK,N,N,XHKKM1)

XHAT(K+1/K) HAS BEEN COMPUTED AND STORED IN THE ARRAY XHKKM1
RETURN
END

SUBROUTINE PRT
REAL*8 GAMMA,COVM,R,PHI,H,TEMP,TEMP1,TEMP2,PKKM1,G,PKK,Q,E1,PR
COMMON E1(4,4),Q(4,4),G(4,4),PKK(4,4),PHI(4,4),H(4,4),PKKM1(4,4),R(4,4)

CALL VPROD (OK,VTMP,N,M,VTMP)
CALL VADD (XHKKM1,VTMP,N,XHKK)

XHAT(K/K) HAS BEEN COMPUTED AND STORED IN THE ARRAY XHKK
CALL VPROD (PHI,XHKK,N,N,XHKKM1)

XHAT(K+1/K) HAS BEEN COMPUTED AND STORED IN THE ARRAY XHKKM1
RETURN
END

END
WRITE GAINS, THEORETICAL COVARIANCES OF ESTIMATION ERROR
WRITE (6,148)
DC 59 K=1,NSAM
WRITE (6,149) K
DO 59 I=1,N
59 WRITE (6,146) (GKS(I,J,K),J=1,M)
WRITE (6,150)
DO 60 K=1,NSAM
WRITE (6,151) K
DO 60 I=1,N
60 WRITE (6,146) (PKKS(I,J,K),J=1,N)
WRITE (6,156)
WRITE (6,152)
WRITE (6,153)
DO 62 K=1,NSAM
WRITE (6,155)
DO 62 I=1,N
62 WRITE (6,154) K, I, XM(I,K), ERR(I,K), VAR(I,K)
WRITE (6,156)
WRITE (6,156)
146 FORMAT (9(2X,1PE12.5),//)
147 FFORMAT ('1%',20X,'OUTPUT DATA',//)
148 FORMAT (10X,'THE GAIN MATRICES ARE',//)
149 FORMAT (5X,'K=',13,/,10X,'G(K)=',//)
150 FORMAT (1X,/,10X,'THE THEORETICAL COVARIANCE MATRIX IS',//)
151 FORMAT (5X,'K=',13,/,10X,'P(K/K)=',//)
152 FFORMAT (T5,'TIME',T16,'VECTOR COM-',T34,'SAMPLE MEAN',
     1 T51,'SAMPLE MEAN OF',T71,'SAMPLE VARIANCE CF',
153 FORMAT (T5,'INDEX',T16,'PONENT INDEX',T34,'CF TRACK',
     1 T51,'ESTIMATION ERROR',T71,'ESTIMATION ERR'})
154 FORMAT (6X,13,13X,11,10X,1PE14.7,2(6X,1PE14.7))  
155) FORMAT ('//')  
156) FC1RAT("10")  
157) FORMAT (10X,'THE SAMPLE COVARIANCE OF EST. ERROR MATRIX IS',//)  
158) FORMAT (//,2X,'K=\',13,//)  
RETURN  
END  
SUBROUTINE PLT  
REAL*8 GAMMA,COVW,R,PHI,H,TEMP,TEMP1,TEMP2,PKKM1,G,PKK,Q,EI,PR  
COMMON E1(4,4),Q1(4,4),G(4,4),PKK(4,4),GAMMA(4,4),COVW(4,4),  
TEMP(4,4),TEMP1(4,4),TEMP2(4,4),H(4,4),PKKM1(4,4),RE1(4,4),PHI(4,4),  
2VAR(4,4,60),GKS(4,4,60),PKKS(4,4,60),XM(4,4,60),ERR(4,4,60),  
3GAMMAS(4,4),PHIS(4,4),XS(4,4,60),HS(4,4),CK(4,4),SIGN(4),X(4),  
4SICXZ(4),XZMEAN(4),XHKK(4),XHKKM(4),VTMP(4),Z(4),V(4),SIGV(4),  
5XHATZ(4,4),XZ(60),Y(60),PX(10),PY(10),PR(4,4,4)  
6N,NSAM,IP,M,ITER,ITRK,IN,ISTAT,K,ITRO,IXZ,IV,WT,EST,ND, NR  
C INTEGER*4 ITB(12)/12/0  
REAL*4 RTB(28)/28*0.0/  
DIMENSION XP(60),YP(60)  
EQUIVALENCE (TITLE,RTB(5))  
REAL*8 TITLE(12)/*X = TRUE, + = FILTER, SQUARE = NOISY*/  
C IGRLT=1  
I1HVPLT=1  
IMTPTLT=1  
ISPLT=1  
ISVPLT=1  
DO 500 KY=1,NR  
KX=NR+1-KY  
DO 50 K=1,NSAM  
XP(K) = XM(KX,1+K)  
YP(K) = XM(KX,3+K)  
CALL PLOTP(XP,YP,NSAM,0)  
C 500 CONTINUE  
ITB(1)=1  
ITB(2)=1  
CALL DRAWP(60,XP,YP,ITB,RTB)  
DO 51 K=1,NSAM  
XP(K) = XS(1,1,K)+ERR(1,1,K)  
YP(K) = XS(1,3,K)+ERR(1,3,K)  
CALL PLOTP(XP,YP,NSAM,0)  
ITB(1)=2  
ITB(2)=2  
CALL DRAWP(60,XP,YP,ITB,RTB)  
ITB(2)=0  
DO 2 J=1,60  
IF(ABS(PKKS(1,1,J)-PKKS(3,3,J)).GT.0) GO TO 11
PKKS(1,1,J)=PKKS(3,3,J)+0.000001
11 CONTINUE
TFE=0.5*ATAN(2.*PKKS(1,3,J)/(PKKS(1,1,J)-PKKS(3,3,J)))
IF(ABS(THE).GT.0) GO TO 10
TFE=0.000001
10 CONTINUE
SIG2X=(PKKS(1,1,J)+PKKS(3,3,J))/2.*PKKS(1,3,J)/SIG2Y
SIG2Y=(PKKS(1,1,J)+PKKS(3,3,J))/2.-PKKS(1,3,J)/SIG2Y
WRITE (6,146) THE,SIG2X,SIG2Y
146 FORMAT (9(2X,1PE12.5),/)
SX=(SIG2X)**.5**20.
SY=(SIG2Y)**.5**20.
PT=3.14159265/12.
CT=COS(THE)
ST=SIN(THE)
DC 1 I=1,25
XI=I
XP(I)=SX*COS(PT*XI)*CT-SY*SIN(PT*XI)*SI+XS(1,1,J)
YP(I)=SX*COS(PT*XI)*CT+SY*SIN(PT*XI)*SI+XS(1,3,J)
2 CALL DRAWP(25,XP,YP,ITB,RTB)
DO 200 J=2,NR
DO 200 K=1,NSAM
XP(K)=XS(J,1,K)
200 YP(K)=XS(J,3,K)
IT=(J/2)+3
ITB(2)=IT
201 CALL DRAWP (60,XP,YP,ITB,RTB)
ITB(1)=3
ITE(2)=3
CALL DRAWP(60,XZ,YZ,ITB,RTB)
65 XP(K) = K
C IF (IGPLT.NE.1) GO TO 68
C DO 67 I=1,N
C DO 67 J=1,M
C DO 66 K=1,NSAM
66 YP(K) = GKS(I,J,K)
C WRITE (6,156)
CALL PLOTP (XP,YP,NSAM,0)
67 WRITE (6,159) I,J
C 68 IF (ITHVPL.NE.1) GO TO 71
C
DO 70 I=1,N
   DO 69 K=1,NSAM
   YP(K) = PKKS(I,I,K)
C
   WRITE (6,156)
   CALL PLOTP (XP,YP,NSAM,0)
   WRITE (6,160) I,I
C
   71 IF (IMPLT.NE.1) GO TO 74
C
   DO 73 I=1,N
   DO 72 K=1,NSAM
   YP(K) = XM(I,I,K)
C
   WRITE (6,156)
   CALL PLOTP (XP,YP,NSAM,0)
   WRITE (6,161) I
C
   74 IF (ISVPLT.NE.1) GO TO 77
C
   DO 76 I=1,N
   DO 75 K=1,NSAM
   YP(K) = ERR(I,I,K)
C
   WRITE (6,156)
   CALL PLOTP (XP,YP,NSAM,0)
   WRITE (6,162) I,I
C
   77 IF (ISVPLT.NE.1) GO TO 80
C
   DO 79 I=1,N
   DO 78 K=1,NSAM
   YP(K) = VAR(I,I,K)
C
   WRITE (6,156)
   CALL PLOTP (XP,YP,NSAM,0)
   WRITE (6,163) I
   80 CONTINUE
C
   WRITE (6,156)
   156 FORMAT (111)
   159 FORMAT (12X,'G(',V1,,',V1,,',V1,,') VS. K')
   160 FORMAT (12X,'PKK(',V1,,',V1,,',V1,,') VS. K')
   161 FORMAT (12X,'MEAN OF X(',V1,,',V1,,') VS. K')
162 FORMAT (12X,'XHATKK','IL,Il' ) -X('IL, Il') VS. K
163 FORMAT (12X,'ERROR VARIANCE('IL,Il') VS. K')
RETURN
END

GO.FT06F001 DD SYSOUT=A,SPACE=(CYL, (4, 1))
GO.SYSIN DD *

$$$$$$$$$$DATA DECK$$$$$$$$$$$$$

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