Considerations Relative to Adapting TRANSIT Observations to Predicting Radar Range Correction

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<th>Air Force ionospheric refraction correction</th>
<th>Space Track Radar range error</th>
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Uncertainties in the output data of SPACE TRACK radars caused by the variability of the ionosphere can be reduced by using predictions of monthly median effects supplied by AWS/NC. For some ACOM missions, this degree of correction is inadequate, and a further reduction in error is necessary. This may be gained by using local measurements of ionospheric parameters, such as total electron content, which may be obtained from TRANSIT satellite passes, to adapt the model ionosphere.
An algorithm is proposed which uses data from sequential TRANSIT passes over a limited portion of the field of view of each ACOM radar, to adapt the AWS/GWC prediction for the entire field of view. From the available TRANSIT data, it is estimated that the rms refraction error may be reduced by a factor of 3 to 4 by this technique.

The principle problem which must be addressed by an adaptive technique is the inherent temporal and spatial variability of the ionosphere. The expected day-to-day variability is about 20 to 25 percent of the monthly median over a solar cycle for stations from the equator to auroral latitudes. In using TRANSIT data to adapt a local estimate of the ionospheric refraction correction, it is suggested that:

1. The adaptive techniques proposed, using TRANSIT data, should be tested against archive data.

2. Techniques using other sensors of the ionosphere, such as measurements of total electron content to synchronous satellites, should be developed and tested against archive data, and the results should be compared with the technique using TRANSIT data before a final method of correction is adapted.
1. INTRODUCTION

2. PRINCIPLE IONOSPHERIC PROBLEMS OF AVAILABLE CLIMATOLOGY
   2.1 Smoothed Prediction
   2.2 Day-to-Day Variability
   2.3 Small Scale Irregularities

3. ALGORITHM DERIVED TO MODEL THE MONTHLY AVERAGE CLIMATOLOGY OF IONOSPHERIC REFRACTION
   3.1 Off Line Program Provides Base Line Prediction
   3.2 Proposed Numerical Mapping and Radar Algorithms

4. USE OF TRANSIT DATA TO MODIFY THE MEDIAN
   4.1 Proposed Algorithm to Compare TRANSIT Observation to Base Line Prediction of TEC
   4.2 Use of Reduced TRANSIT Data Over the Field of View of the Radar
   4.3 Filtering TRANSIT Data
   4.4 Adapting the Base Line Prediction of TEC

5. CONSIDERATIONS IMPORTANT TO ANY TECHNIQUE USED TO ADAPT MONTHLY MEDIAN PREDICTIONS
   5.1 Percentage Coverage of Field of View
   5.2 Assigning TRANSIT Observations to a Specific Point in the Predicted Numerical Map
   5.3 Potential Improvement Over Prediction of Basic Climatology
   5.4 An Estimate of the Uncertainty in the TRANSIT Update
   5.5 Potential Areas for Further Study

REFERENCES
Illustrations

1. Diurnal Variation of Monthly Mean of Observations (O) of Total Electron Content Compared to Predictions (B) Made With AWS/GWC-Bent Program 9
2. Diurnal Variation of Monthly Mean of Observations (O) of Total Electron Content Compared to Predictions (B) Made With AWS/GWC-Bent Program 10
3. Diurnal Variation of Total Electron Content at Hamilton, Mass., a High Midlatitude Station 11
4. Diurnal Variation of Total Electron Content at Hamilton, Mass., a High Midlatitude Station 12
5. At Goose Bay, Labrador, Near the Southern Edge of the Auroral Region, the Monthly Mean (O) of Observation of Total Electron Content Often Have a Diurnal Variation That Departs Significantly From the Prediction of the Mean (B) Made With the AWS/GWC-Bent Program 13
6. At Goose Bay, Labrador, Near the Southern Edge of the Auroral Region, the Monthly Mean (O) of Observation of Total Electron Content Often Have a Diurnal Variation that Departs Significantly From the Prediction of the Mean (B) Made With the AWS/GWC-Bent Program 14
7. Overplot of Daily Observations of Total Electron Content for October 1967, at Hong Kong, Under the Peak of the Equatorial Ionsphere 16
8. Overplot of Daily Observations of Total Electron Content at Hong Kong for April 1968 16
9. Diurnal Variation of Monthly Mean Total Electron Count and of the RMS Variation About the Mean 16
10. Diurnal Variation of Monthly Mean Total Electron Content and of the RMS Variation About the Mean During an Equinox Month, March 1972 16
11. Variation of Monthly Mean Total Electron Content and of the Standard Deviation Over a Solar Cycle 18
14. Vertical Range Error, in Feet at Clear, Alaska, Over the Coverage Area of any 425 MHz Radar 24
15. Slant Range Error in Feet to a Target Above the F Region 25
16. Latitudinal Variation of Total Electron Content Along a Single TRANSIT Pass 27
17. Difference in Latitude Variation of Total Electron Content for 2 Sequential Passes of TRANSIT Satellites Over Lindau, FRG 28
18. Difference in Latitude Variation of Total Electron Content for 2 Sequential Passes of TRANSIT Satellites Over Lindau, FRG 28
Illustrations

19. Comparison of a Sample Latitude Gradient of Total Electron Content Derived From a TRANSIT Pass With First the Monthly Median Predicted by the AWS/GWC-Bent Program and Second That Prediction Normalized by a Scale Factor at the Point of Closest Approach to Millstone Hill, Mass. 29

20. Improvement of Prediction Using Information From a Previous TRANSIT Pass at 1120 Local Time to Scale the AWS/GWC-Bent Prediction as a Function of Latitude 30

21. Improvement Using Information From a Previous TRANSIT Pass at 1600 Local Time to Scale the Predicted Median 30

22. Scaling Factors Along the Latitudes for 6 Consecutive TRANSIT Passes Near Lindau, FRG for 10 December 1975 32

23. Scaling Factors Along 4 Consecutive Passes of TRANSIT Near Boulder, Colorado, October 13, 1974 32

24. Coverage Diagram for a Radar Located at Clear, Alaska 33

25. Function of the Radar Coverage Area for Targets at 1000 km and of the Ionosphere Which May be Specified by a Slant Observation to a TRANSIT Satellite. 34

26. Mean Height of the F Region Peak Determined From Incoherent Scatter Observations at Millstone Hill, Mass., 1968 Through 1971 35

27. Results When Observation of Total Electron Content at Hamilton, Mass. in 1972 are Used to Scale the 10 Day Predicted Median 38

28. Latitude-Time Variation of Total Electron Content Derived From ATS-3 Observations 39
Considerations Relative to Adapting TRANSIT Observations to Predicting Radar Range Correction

1. INTRODUCTION

Ionospheric refraction introduces uncertainties in the output data produced by radars which track space objects. The prediction of the monthly median of the ionospheric effects, used in the tracking algorithms within radar processors, substantially reduces these refraction errors. The day-to-day variability of the radar metric errors may be reduced to 25 to 50 percent of the uncorrected errors. In many present ADCOM radar systems, this residual day-to-day variability of the refraction effect about the predicted median is greater than the radar system goals. Tests have shown that measurements of the ionosphere, for example by TRANSIT satellites, may be used to adapt the predicted ionospheric refraction effects to the current local conditions. Reduction of the variability then depends on the spatial and temporal availability of the TRANSIT data in the pertinent region of the radar field of view. If the TRANSIT data is taken close enough to the target, the variability may be reduced to about 5 percent, which may meet some ADCOM mission goals as well as long-term average system goals.

2. PRINCIPLE IONOSPHERIC PROBLEMS OF AVAILABLE CLIMATOLOGY

The requirement, at the radar processor, is to provide a necessary description of the refraction effects of the ionosphere over the entire field of view of the

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radar. For single frequency radars, this requirement is first met by predicting a monthly average of ionospheric effects based on simple models which are supported by a source of ionospheric climatology. Millman1,2 has shown that radar range and elevation angle errors may be specified from an estimate of the integrated electron content (TEC) along the line of sight to the radar target. Second order effects (Savich and Vaslyev;3 Tucker and Fannin4) are generally much smaller than the day-to-day variability about the monthly median and hence can be neglected at this stage. The range correction for the ADCOM radars is inversely proportional to the square of the radar frequency and directly proportional to the TEC along the line of sight to the radar target. At 425 MHz, this results in a range error of 7.32 ft for each TEC unit of $10^{16}$ electrons/m².

2.1 Smoothed Prediction

There is never enough data to derive a worldwide time-dependent model to predict any ionospheric parameters. The accepted basis for predicting median values of some of the ionospheric parameters—the coefficient tapes for foF2 and M3000 supplied by NOAA—are the result of a great deal of interpolating and extrapolating of world data of variable quality. While there were data on the strong variation within the ionosphere for different longitudes, there was no choice but to extrapolate into major geographic areas where no ionospheric measurements existed. This is particularly true for radar sites in the Pacific. Ordering of the basic ionospheric measurements in the polar-subauroral region precluded displaying the details of the trough in electron density or of the enhancement in the active auroral region. Despite this, predictions of monthly median of TEC are very close to actual observed medians. Figures 1-6 show comparisons for stations at low, mid- and high-latitudes. The model used was the Air Weather Service – Global Weather Central (AWS/GWC) – Bent program with ionospheric parameters derived from the NOAA coefficient tapes in the same manner as would be used in an actual forecast condition.


Figure 1. Diurnal Variation of Monthly Mean of Observations (O) of Total Electron Content Compared to Predictions (B) Made With AWS/GWC-Bent Program. Hong Kong is situated under the peak of the equatorial anomaly. The largest systematic bias for March 1968 occurs during the afternoon decrease.
Figure 2. Diurnal Variation of Monthly Mean of Observations (O) of Total Electron Content Compared to Predictions (B) Made With AWS/GWC-Bend Program. Hong Kong is situated under the peak of the equatorial anomaly. The bias for June 1968 is very low over the entire day.
Figure 4. Diurnal variation of Total Electron Content at Hamilton, Mass., a High Midlatitude Station. The monthly mean of observations (O) is matched quite well by the AWS/GWC-Bent program. During summer months, here June 1972, the late afternoon changes in total electron content have large variations from year to year. Apparently the cause, such as winds or composition changes, are not specified by mean solar conditions.
Figure 5. At Goose Bay, Labrador, Near the Southern Edge of the Auroral Region, the Monthly Mean (O) of Observation of Total Electron Content Often Have a Diurnal Variation That Departs Significantly From the Prediction of the Mean (B) Made With the AWS/GWC-Bent Program. Here in March 1973 there is a large systematic bias at the peak of the day.
Figure 6. At Goose Bay, Labrador, Near the Southern Edge of the Auroral Region, the Monthly Mean (O) of Observation of Total Electron Content Often Have a Diurnal Variation that Departs Significantly From the Prediction of the Mean (B) Made With the AWS/GWC-Bent Program. Here in June 1972 the match is acceptable over most of the day but the diurnal shapes are quite different near midday.
Hong Kong (Figures 1 and 2) is under the peak of the equatorial anomaly; the afternoon gradient is often steep and its position is known to be variable over the solar cycle. The systematic bias over the whole day for these samples is not surprising since the measurement of the only parameter used in the model to specify the current state of the ionosphere is the estimated 12 month running mean of solar flux and/or sunspot numbers. Hamilton, Mass. (Figures 3 and 4) is at the auroral edge of the midlatitude ionosphere. The low systematic bias over most of the day and a prediction that is too low over the night hours, are typical of data taken 1968 through 1975.

Although the Bent program was not designed for use at high latitudes, its predictions for Goose Bay Labrador (Figures 5 and 6) are useful. The possibility of large bias is displayed by the early day portion of Figure 5. In general, TEC at Goose Bay is predicted about as well as at Hamilton; Goose Bay nighttime is predicted better than Hamilton nighttime, but in daytime the Bent model predicts Hamilton more accurately than Goose Bay in months when the differences in the observed means of Hamilton and Goose Bay differ by 20 percent or more. Note that at the peak of the day in March the vertical range correction for a 425 MHz radar would be 732 ft at Hong Kong, 256 ft at Hamilton, and 124 ft at Goose Bay. For targets near the horizon, these could increase by a factor of 3.

2.2 Day-to-Day Variability

When the monthly median refraction effect is predicted along the ray path to the target then the system error, on a monthly basis, is directly proportional to the day-to-day variability of observed values compared to the predicted median. If the bias of prediction is small enough, then that error may be estimated from the day-to-day variability of observed values compared to the observed median. A separate study (Mulkern5) of TEC from world stations from low (Hong Kong) to high (Narssarsuaq) latitudes suggests that the rms variability is about 20 percent of the mean value during the daytime and about 30 percent of the mean value during the night hours. Greatest variability appears near sunrise and after sunset, and during equinox transitions. Figures 7 and 8 are multiple plots of daily observations of TEC derived from Faraday rotation studies of the radio beacon on the geostationary satellite SYNCOM III during the last solar maximum. The rms variability with respect to the sample median is about 12 percent in the day and about 30 percent at night. Figures 9 and 10 show reduced data for Hamilton and Goose Bay. The rms variability is about 20 percent during the day, greater than the equatorial region, but comparable to it at night.

Figure 7. Overplot of Daily Observations of Total Electron Content for October 1967, at Hong Kong. Under the Peak of the Equatorial Ionosphere. The day-to-day variability is maximum at midday. There are often large increases of total electron content in late afternoon and late evening.

Figure 8. Overplot of Daily Observations of Total Electron Content at Hong Kong for April 1968. The large increase in late afternoon and evening may be 30 to 50 percent of the monthly median.

Figure 9. Diurnal Variation of Monthly Mean Total Electron Content and of the RMS Variation About the Mean. For June 1972 both Hamilton, Mass., at midlatitudes, and Goose Bay, Labrador at the edge of the auroral zone, have comparable behavior. The day-to-day variability is about 20 percent.

Figure 10. Diurnal Variation of Monthly Mean Total Electron Content and of the RMS Variation About the Mean During an Equinox Month, March 1972. Goose Bay, Labrador is at the edge of the auro rally disturbed ionosphere and Hamilton, Mass. is a subauroral station. Both have about the same behavior with a day-to-day variability of about 25 percent at midday.
The variability of TEC for each hour of each month over the descending phase of the solar cycle for 8 years of data taken at Hamilton, Mass. — including sunspot maximum to near sunspot minimum — is shown in Figure 11 (left). The pattern of variability is similar for each year; it is the absolute value that varies over the solar cycle, correlating with the variation in the mean value of TEC as shown in Figure 11 (right). The variability maximizes in daytime near the equinoxes, decreases somewhat in winter, and minimizes in summer. The nighttime variability follows the same seasonal behavior with the maximum values equivalent to the minimum values of the daytime variability. There is a rapid increase in variability after sunrise which peaks in midafternoon, then rapidly decreases after sunset. This seasonal, diurnal variation is also in close correspondence with the variation in the monthly mean TEC. Both the mean TEC and the day-to-day variability change by a factor of 4 over the solar cycle, thus maintaining a constant percentage variability which is approximately 20 to 25 percent of the mean.

Some of the variability of the TEC may be accounted for with variations in solar activity. While the principle solar agent may be EUV, there are no consistent long-term EUV measurements. Long-term studies have shown a useable correlation between solar radio flux measured at 2.8 GHz and the major ionospheric parameters. However, most of the variability of TEC has a period of less than 15 days. The fluctuations of 2.8 GHz solar radio flux is small for such small periods and therefore day-to-day variation of TEC does not correlate with daily observations of solar flux. Instead, the day-to-day variations are apparently reflections of changes in the loss rate caused by erratic motions of the neutral winds, discussed further in Section 2.4.

2.3 Small Scale Irregularities

If an actual sequence of observations is compared to what might be expected from some (normalized) smooth diurnal behavior of TEC, then any deviation can be considered an irregularity. Figure 12 shows very clearly the restless nature of the ionosphere at midlatitudes, as does Figures 7 and 8 for the equatorial zone. The synchronous satellite, Early Bird, was the source for the TEC data of Figure 12, the ionospheric intersection being slightly south of Hamilton, Mass. Note that some of the shorter period variations (the sample rate was 5 min) seem quasiperiodic in nature. For some of the literature, the short period variations are often selected to conform to what the analyst decides a priori is a "travelling disturbance"; therefore, quasiperiodic on the record. Synoptic studies (Kane) suggest that short period variations are about 2 percent or less of ambient.

Figure 11. Variation of Monthly Mean Total Electron Content and of the Standard Deviation Over a Solar Cycle. Hamilton, Mass. is a subauroral station. The ratio of the two is nearly constant over these years.
Figure 12. Small Scale Irregularities in Total Electron Content in the Ionosphere Slightly South of Hamilton, Mass., June 1965. Derived from Faraday measurements each 5 minutes of the 137 MHz beacon on the Early Bird synchronous satellite. Note that irregular variations from a smooth diurnal curve may be 2 to $5 \times 10^{16}$ electrons/m² for periods of one to two hours.

It is the nonperiodic variations from an expected smooth diurnal behavior that carry most of the short period energy. This of course means that variations having irregular forms, found in the daily records for almost any station, have not been given the detailed study accorded "TIDs."

Individual observations can easily depart 10 to 50 percent from the predicted values. If an attempt is made to normalize a monthly median curve to each daily
observation, then, in the general case, the departures from the median are sufficiently large and irregular, so as to make the possible normalization procedures highly subjective. That is, there are no a priori grounds for normalizing over specific time segments.

In operationally defining irregularities, first a prediction is made of the smoothed behavior of the ionosphere over some period of time and then the radar system defines any departure from that prediction as an irregularity. This is quite different from the experimental approach behind most research reported in the literature, where the definition can be classified in several general ways:

1. Most popular for verification of analysis, an a priori mental model of a particular deviation, such as a quasiperiodic amplitude variation, is used to sift examples from experimental observations;
2. A segment of some analytic function (say a third order polynomial) may be fitted in an rms sense to a short time sequence of observations and then the deviations from the smooth function are considered as irregularities;
3. Additional data such as simultaneous observations from three stations may be used to select a particular class of irregularities, such as travelling ionospheric disturbances. In any case, the results reported in the literature can only be used in a qualitative sense in the radar prediction.

If only a monthly median prediction is used at the radar, then the main source of error arises from median scale local features. Kane has shown that these features have correlation sizes less than 3000 km in both longitude and latitude, and that they may easily fluctuate ±50 percent on quiet as well as disturbed days. Figure 13 from Hamilton, Mass., is a sample which illustrates the severity of these disturbances. Noting that a 7 day centered mean is nearly coincident with a monthly median predicted from climatology, it is apparent that even an adaptive update of a prediction will be seriously affected by these large scale variations. For times as short as 1 hr, the error may be $2 \times 10^{16}$ electrons/m$^2$ at the local vertical. At 425 MHz, for targets near the horizon, the range errors would be 40 to 100 ft.

Cole has shown that a heating in the auroral electrojet regions during magnetically disturbed periods could cause a flow of the atmosphere away from the electrojet in the upper thermosphere, forming large convection cells. Kane has suggested that this may occur at any time, causing the neutral wind structure to always be erratic so that large scale ionospheric turbulence appears randomly in different geographic regions with slow motion in both latitude and longitude. This would then change loss rates, both by changing the atmospheric chemistry and by redistributing existing ionization in height. If some feature of the erratic neutral

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Figure 13. Observations of Total Electron Content at Hamilton, Mass. Compared to the Monthly Mean for June 1972. Large deviations from an expected smooth behavior could produce large range errors.
Figure 13. Observations of Total Electron Content at Hamilton, Mass. Compared to the Monthly Mean for June 1972. Large deviations from an expected smooth behavior could produce large range errors (Cont.)

wind structures could be observed systematically, then perhaps it could be used to account for some of the large day-to-day variability of TEC which cannot be accounted for by the small daily variations of the solar radio flux.

3. ALGORITHM DERIVED TO MODEL THE MONTHLY AVERAGE CLIMATOLOGY OF IONOSPHERIC REFRACTION

Basic programs, such as the ITS-78 computer program available from NOAA, predict the monthly average statistics for the principle ionospheric parameters.
Designed to cover the entire world and all temporal and solar conditions, they are inherently large in computer storage and in production time. One intent of this study is to provide a more restrictive model for predicting the local ionosphere.

3.1 Off Line Program Provides Base Line Prediction

From the viewpoint of the radar processor, the ITS-78 is excessively large. This is not a fault, but rather a description. The FORTRAN program requires a large storage and work area, and an additional large amount of data are stored on a coefficient tape. While ITS-78 could be tailored to a smaller, faster program for each radar, it is very doubtful if, even then, it could be used on a hit-to-hit basis. The fundamental mapping routines are based on orthogonal expansions of ionospheric data covering the whole world. To make a prediction at a single point, a large number of terms in the expansion must be computed and summed. Even on the fastest computers this requires more analysis time than the inter-pulse period. The ITS-78 program is, in effect, an off-line or a research tool, which is exactly its intent.

3.2 Proposed Numerical Mapping and Radar Algorithms

In order to meet the metric requirements of the SPADATS mission of the ADCOM radars, it is desired to make a correction for ionospheric refraction effects at the time when the radar echo is first being processed. For tracking radars, the correction might be made on average over several returns, but for phased arrays, such as Cobra Dane, this is not possible. Moreover, while the basic technique need only deliver a monthly median correction with acceptance of about 20 to 30 percent rms error, it is expected that in the future it can be adapted to use as yet unspecified local measurements to make a localized correction. In that case the error could be much smaller, possibly 5 percent rms over a month.

The problem then is not straightforward. The correction supplied the radar processor has involved dependence on the interaction of spherical geometry and the ionosphere over the field of view. The method in which a solution was derived has been detailed in a previous report (Allen8).

In brief the problem was separated into two parts, admittedly dependent. First, a technique was developed to provide a grid of TEC for all hours of the day for a single month and to cover an area somewhat greater than the radar field of view. Then this grid was expanded in orthogonal functions of time and latitude chosen to map the TEC in a least square error sense. Second, simple real time

algorithms were developed to take point values from the TEC map and convert them into slant corrections for each radar hit. These were chosen to minimize the maximum error. Figures 14 and 15 show schematically the correction that would be computed at each point of the radar field of view at a particular UT for a target at 1000 km. The radar algorithm reduces this correction for targets at lower altitudes. The following requirements were satisfied:

Figure 14. Vertical Range Error, in Feet at Clear, Alaska, Over the Coverage Area of any 425 MHz Radar. Median values predicted by the AWS/GWC-Bent program for an average solar cycle maximum, at 1000 local time, at Clear, during March
Figure 15. Slant Range Error in Feet to a Target Above the F Region. Vertical values from the previous figure have been corrected by the slant increase depending on elevation angle from the radar at Clear.

(1) The numerical map could be run off-line in low priority time at the Global Weather Central.

(2) As prediction techniques advanced, they could automatically be incorporated in the monthly median maps sent each radar.

(3) Only one program need be used by the Global Weather Central to service existing and proposed radars.

(4) The correction is made on each single radar return.

25
(5) The correction could be updated by a numerical update derived from worldwide data collected by the Global Weather Central.
(6) The mapping techniques would not limit adaptive correction using local measurements.

4. USE OF TRANSIT DATA TO MODIFY THE MEDIAN

In previous studies (Allen et al., Dulong and Allen) we have shown that an improvement in prediction can be made if the median TEC (hence the median refraction correction) is adapted to local conditions through use of a local ionospheric measurement. Those radars having TRANSIT receivers as part of their calibration equipment (SPADATS Improvement Program) can substantially reduce their metric error by using these receivers for TEC measurements. With these measurements, for targets in the upper ionosphere, the day-to-day variability with respect to the monthly median may be reduced by a factor of 4 to 10.

4.1 Proposed Algorithm to Compare TRANSIT Observation to Base Line Prediction of TEC

An algorithm which could be used with the TRANSIT data is to smooth the original reduced data to provide estimates of equivalent vertical content at equal increments of subionospheric latitude over the restricted portion of the TRANSIT pass. As the TRANSIT sweeps across the radar field of view, the individual slant measurements along the viewing ray path will map features of the near instantaneous ionosphere into one dimension (as seen in Figure 16) for a TRANSIT pass on 23 March 1973 observed at Millstone Hill, Mass. Note the wave-like variations over the whole pass.

To our knowledge, there is no hard evidence for or against mapping these features into the prediction, but an intuitive argument may be made against it.

The argument is as follows: postulate that the characteristic life time \( \Delta \tau_i \) of an irregularity measured at a given position is directly proportional to its characteristic size \( \Delta \chi_i \):

\[ \Delta \tau_i = \kappa \Delta \chi_i \]


Figure 16. Latitudinal Variation of Total Electron Content Along a Single TRANSIT Pass. Observation plotted at the location of the 420 km height along the slant path from Millstone Hill, Mass. to the satellite

Then the irregularity either decays or moves a distance of a characteristic width faster than the time between observations. If the irregularity is of acoustic-gravity wave origin, then this follows from classical analysis. If it is a structure of some production source, such as an auroral or plasmapause feature, then the smaller scale structure is generally considered to be transient in space and time.

Lacking test data to check the usefulness of mapping irregularities, we suggest the data be smoothed to prevent aliasing. The least amount of smoothing that would be justified will depend on the confidence in the TRANSIT measurements and on results of tests to determine the degree of curvature that may be specified and used profitably over the field of view. This will depend on the time interval between TRANSIT passes.

We suggest a study of sequential TRANSIT passes to provide a basis for defining the required choices for smoothing the data. While isolated passes can show the nature of an irregularity, sequential passes are needed to test its stationarity. Figure 17 is an example of sequential TRANSIT passes taken at Lindau/Harz, FRG on 10 December 1975. There is a pronounced local feature in the first pass at 1120 LT between 35° and 43° latitude. In the following pass which occurred at 1240 LT, this feature has changed considerably. Obviously a smoothing of this feature would be preferable for specification purposes.

27
An example of a stationary feature is shown in Figure 18 for the same day. The first pass occurred at 1600 LT with a local feature between 47° and 54° latitude. The following pass exhibits the same feature relatively unchanged. Lacking sufficient data for a comprehensive study, we would recommend smoothing the data to remove components of a few degrees of latitude.

4.2 Use of Reduced TRANSIT Data Over the Field of View of the Radar

Smoothing the data can be done with a discrete filter approximating a gaussian. This smoothed data is then filtered to obtain the discrete Chebyshev sequencies. These are the unique fit to the data of Section 4.1 in a least mean square error sense. If the smoothed data is output at equal intervals of subionospheric latitude, then the FORTRAN form of the Chebyshev filter is the same as the one in the July 1976 report, with merely the latitude interval and the latitude range redefined.

Consider a sample of TRANSIT data (Figure 19) taken 23 March 1972 at Millstone Hill. When enough terms in the Chebyshev expansion are used, about 30 for this case, the data points are reproduced exactly. If we assume that the wave-like variations between 28.7 and 40.7 degrees latitude are gravity waves
with a characteristic half width of 330 km ($3^\circ$LAT) and that they are moving through the upper ionosphere with velocity about 200 m/sec, they will have moved a half width in about 30 min. In this case, the wave-like structure should not be included in an adapted prediction used at time delays greater than about 15 min.

4.3 Filtering TRANSIT Data

The TRANSIT information needed to adapt the predicted median ionosphere should be reconstituted using only as many sequences as tests on real data prove useful. Using a small sample of TRANSIT passes, but extrapolating from a detailed study of the time variability of TEC at a single station, (DuLong and Allen, 10) it is concluded that using just the D.C. term alone to adapt the median prediction produces a significant mission improvement. Day-to-day variability could be reduced by a factor of 4 to 10. Figure 19 shows how well the predicted median fits the pass of 23 March when it is adapted in this fashion. The variability along the pass is very much smaller than the day-to-day variability indicated by
the ATS-3 observations, and therefore is much smaller than the variability that would be expected when the radar is used between TRANSIT passes. This is apparent in Figures 20 and 21 where the first pass in each of Figures 17 and 18 was used to scale the prediction for the time of the second pass. The observation for the second pass is shown with the scaled and the unscaled predictions. Using the fact that the prediction at the time of the first pass was scaled to exactly produce the observations at that instant, which is equivalent to using higher order sequencies, the error between the observations and scaled prediction shown in both cases is an example of the decay in improvement that could occur between passes. There is no indication that using higher order sequencies, rather than just the linear gradient, would improve this prediction at any time other than at the instant of the pass. A study of sequential TRANSIT passes is a necessity to determine the extent to which the observations can be used to adapt predictions with higher order sequencies.

Figure 20. Improvement of Prediction Using Information From a Previous TRANSIT Pass at 1120 Local Time to Scale the AWS/GWC-Bent Prediction as a Function of Latitude. The major differences in the basic latitude variation have been removed but small local features have been specified.

Figure 21. Improvement Using Information From a Previous TRANSIT Pass at 1600 Local Time to Scale the Predicted Median. The basic latitudinal variation and some long lived local features have been specified.
4.4 Adapting the Base Line Prediction of TEC

The filtered observations may be used to adapt the numerical maps which provide ionospheric parameters over the field of view of the radar. Note the qualifications in Section 5. A parsimonious algorithm would be to scale the filtered TRANSIT observations (defined in the algorithm of Section 4.1) against similar values derived from the numerical map of median TEC. The variation of scaling factor over the restricted latitude of the TRANSIT pass is used to modify predicted values over the equivalent latitude portion of the field of view, that is, for all longitudes. This may be done hit-to-hit outside of the numerical mapping algorithm. The scaling factors would be changed with each new TRANSIT pass.

Extrapolation of the TRANSIT data into latitudes not reduced from the original TRANSIT data may have higher risk than extrapolation in longitude across the field of view. However, both need to be tested against actual field observations. A preliminary study of 10 TRANSIT passes (DuLong and Allen) compared with single station observations of TEC, indicates that the error grows to the same order in 15 degrees of latitude as it does in about an hour displacement in time.

Many questions need to be answered before TRANSIT data can be effectively used in adapting numerical models. From the available TRANSIT data, it appears that the effectiveness of scaling a numerical map is dependent on the diurnal variation. A clear dependence cannot be determined from the analyzed data, but examples are apparent from the curves of Figure 22. Each curve represents the scaling that would be necessary at each point in latitude to reproduce the TRANSIT observations from the prediction for that time and latitude. The passes represented here are 6 consecutive TRANSIT passes observed at Lindau/Harz, FRG. (Leitinger) on 10 December 1975. The difference between scaling factors from one pass to the next is the error that would occur in using one pass to scale another. The difference from 1 on the scale for each curve, simply represents the difference between observation and prediction. It is obvious that using any of the first three passes to scale the latter three, would introduce new errors at midlatitudes. A similar example for 4 consecutive passes observed at Boulder, Colorado on 13 October 1974, covering a comparable span of time shows (Figure 23) that in nearly all instances the scaling would produce an improvement in the prediction. The factors on which these variations depend can only be determined through a study of a statistically valid data sample.

11. Leitinger, R., and Hartmann, G. K. (1976) Time and Latitude Dependence of Ionospheric Electron Content from the Combination of NNSS and ATS-6 Data, the Geophysical Use of Satellite Beacon Observations, COSPAR Symposium Proceedings, Boston University.
5. CONSIDERATIONS IMPORTANT TO ANY TECHNIQUE USED TO ADAPT MONTHLY MEDIAN PREDICTIONS

Tests have shown that many different ionospheric measurements may be used to adapt the predicted median to current conditions. Each procedure will have a residual error of prediction arising from the inherent limitation of the procedure coupled with the variability of the ionosphere. Klobuchar and Allen 12 have shown that point measurements of TEC or of the peak density of the F region may be used to significantly reduce the monthly rms error. This discussion will be concerned with utilization of TEC data from TRANSIT passes.

5.1 Percentage Coverage of Field of View

The percentage, \( \Phi \), of the total radar surveillance area (Figure 24) which can be specified by real time TRANSIT observations, depends on the zenith angle to which reliable measurements can be made. We may define

\[
\Phi(\%) = \left(\frac{\theta_z}{\theta_0}\right)^2 \times 100
\]

Figure 24. Coverage Diagram for a Radar Located at Clear, Alaska. The field of view is defined by the most distant observation of TRANSIT at 1000 km and zero degrees elevation. If the observation of total electron content to TRANSIT can be used down to 60 degrees zenith angle then the coverage area within the inner circle can be specified. The ionospheric region which can be adapted by such measurements is indicated by the dashed circle.
where $\theta_o$ is the earth angle distance between the radar position and the subsatellite position of TRANSIT at the horizon, and $\theta_z$ is the subsatellite position at the largest acceptable zenith angle. Figure 25 shows the percentage coverage, $\Phi$, as a function of useable zenith angle when the TRANSIT satellite is at the nominal 1000 km altitude. It also shows the percentage of an ionospheric map, whose centroid is at 420 km, and which may be specified at the same time. Note, for instance, if the reduced TRANSIT data may be used reliably down to 60 degrees zenith angle, then at the height of TRANSIT, 1000 km, the percentage of the surveillance area which can be specified is

$$\Phi = \left( \frac{11.5}{30.2} \right)^2 \times 100 = 15\%$$

Since the centroid of the ionosphere is about 420 km, then only about 8 percent of the ionospheric map can be specified. Conceptually, $\Phi$ is the percentage of the potential TRANSIT information which may be used by the radar processor without extrapolation, and is therefore, a basic parameter which can be used to compare different specification techniques.

5.2 Assigning TRANSIT Observations to a Specific Point in the Predicted Numerical Map

It is assumed that the algorithms designed to reduce the raw TRANSIT observations to estimates of slant and vertical electron content use a fixed value, such as 420 km, for the height of the centroid of the slant distribution of electron density. From monthly median climatology, and from a model study based on profile parameters derived from incoherent scatter measurements at Millstone Hill, we expect that the actual height of the centroid is dependent on both location and time. For instance, at midlatitudes and for ionospheric conditions representative of an average solar maximum, the expected height of the F region during the daytime would be about 290 km, based on the Millstone Hill observations for the years 1968, 1969, 1970, and 1971 (Figure 26). Using profile parameters, also derived from
those observations (Allen et al\textsuperscript{13}) a slant profile of electron density would have an expected daytime height near 335 km. This suggests that using 420 km would produce a mean bias of about 85 km over the year at solar maximum. The Millstone Hill measurements suggest that the average bias computed at a given hour-month would have an expected range from -180 to +110 km. Individual days would, of course, exceed these values.

The uncertainty in the value of centroid height may be translated into the uncertainty in the reduced TRANSIT data in two steps: the uncertainty of the sub-ionospheric location ($\Delta T_L$) and the uncertainty of the equivalent vertical electron content ($\Delta T_V$). Using the correlated profile parameters derived from the Millstone Hill observations, we computed the expected variations of subionospheric position with elevation angle and height of the centroid. Over the region 0 to 30 degrees elevation by 300 to 700 km height of centroid, we find

$$0.015 \leq \frac{\Delta T_L}{\Delta h_{\text{cent}}} \leq 0.025$$

or for simplicity

$$\Delta T_L (\text{earth angle}) \geq 0.022 \Delta h_{\text{cent}} (\text{km}) \ .$$

Therefore, for 85 percent of the radar field of view, that is for zenith angles greater than 60 degrees, the uncertainty of subionospheric locations, averaged over the year, could be

$$\Delta T_{LYR} \approx 0.022 \times 85 \approx 1.9 \text{ earth degree}$$

while the expected range of monthly average could be

$$\Delta T_{LMO} \approx 0.022 \times 180 \approx 4.0 \text{ earth degrees}.$$ 

Again, it is worth noting that individual days could be displaced by much more than this.

This uncertainty in location of the TRANSIT measurements may be reduced if an expected height of the centroid of the monthly median profile is used for the time and the location of the TRANSIT measurements. In that case, there will still be a day-to-day variability about the expected monthly value. Considering that $h_{F2}$ is an expected variability of about 7 percent, then

$$\Delta T_{LT}^{rms} \approx 0.022 \times 0.07 h_{max} \approx 0.44 \text{ earth degrees}.$$ 

The error that this uncertainty in the height of the centroid introduces into any technique for adapting monthly median refraction correction will depend on radar location, time of day, and solar conditions. In general, it will maximize during the afternoon decrease of TEC, especially during the equinoctial periods. On average, it is estimated that for each degree of earth distance, there will be introduced an uncertainty of about 1 percent of the daily maximum TEC. This could be serious for individual observations.

5.3 Potential Improvement Over Prediction of Basic Climatology

The day-to-day variability about the predicted monthly mean values for the refraction corrections will be at least as large as the day-to-day variability of TEC. For the sample shown previously, Figures 7-11, this was about 20 percent of the daytime values. When TRANSIT observations are used in an adaptive technique, such as that discussed in Section 4.2, the improvement will be limited by several factors.

A basic limitation is the accuracy which may be obtained along the satellite track, discussed, in part, in Section 5.2. For targets at the same altitude and in the same region as the TRANSIT data, the improvement immediately after the measurement may be a factor of 4 to 10. This improvement decays in that region with time.
An examination of the decay in time of any improvement in prediction was made using single station measurements of TEC. Some of the results are shown in Figures 27a, b, and c, where measurements of TEC were used to update the prediction of TEC for the station at 30 min, 1 hr, and 2 hr, after the measurement. The results for the year 1972 for measurements at Hamilton, Mass. show the decay in time in terms of percentage error. Within 30 min the error grows from 0 to 5 percent in daytime and up to 10 percent or more near sunrise. After an hour the error is about twice the 30 min error, and in 2 hr time it is about 3 times the 30 min error. Except for the hours near sunrise, the error is still less than that expected using the prediction without an update.

In most cases the location of the target will not be close to the location of the latest TRANSIT pass and the ionospheric model will have to be adapted by extrapolation in longitude and/or latitude. From the study of TEC at midlatitude stations, it is known that sunrise and sunset gradients are very variable, but there has not been enough TRANSIT data available to determine if they can be used across these longitude gradients.

5.4 An Estimate of the Uncertainty in the TRANSIT Update

It is assumed that the principle sources of variability of the update are uncorrelated and that therefore the total system variability ($\sigma_R$) is the root of the sum of the squares of all variables which can be identified as

$$\sigma_R^2 = [\delta_{TM}^2 + \delta_{TL}^2 + \delta_{Thm}^2 + \Psi(L, T) + \delta_{CF}^2 + \delta_{SY}^2 + \delta_{AA}^2$$

where

- $\delta_{TM}$ is the fundamental TRANSIT measurement error,
- $\delta_{TL}$ is the variability due to assignment of location,
- $\delta_{Thm}$ is the variability due to position for targets at heights different from TRANSIT,
- $\delta_{CF}$ is the error of truncation of the mapping of the median ionosphere,
- $\delta_{SY}$ is the error in the radar refraction algorithm,
- $\delta_{AA}$ is the error introduced by the adaptive algorithm, and
- $\Psi(L, T)$ is the increase in variability as the target is different from TRANSIT in location and time.

The amount of available TRANSIT data was not adequate for a thorough evaluation. It is estimated that $\delta_{TL}$ is about 2 percent of the total ionospheric correction (Section 5.2). In a previous in-house study for the SPADATS Improvement Program, it was estimated that $\delta_{Thm}$ would be controlled by the variability of
Figure 27. Results When Observation of Total Electron Content at Hamilton, Mass. in 1972 are Used to Scale the 10 day Predicted Median. The RMS percentage error represents the difference between the actual observations compared to the median scaled with a previous observation (a) 30 minutes, (b) 1 hour, (c) 2 hours previous). The interval around sunrise (dashed curve) has the largest percentage variability.
hmF2 and could be about 7 percent of the total ionospheric correction for targets near hmax. Since only about 1 percent of the SPADATS targets are below 500 km, the system variability $\delta_{Thm}$ will be less than this. Profiles of electron density at midlatitude taken by the incoherent scatter radars generally have at least two thirds of their TEC below 500 km, so it is estimated that $\delta_{Thm}$ will be nearer 2 percent than 7 percent.

5.5 Potential Areas for Further Study

The interim suggestion, that TRANSIT data may be used to adapt a prediction of the monthly median ionosphere to provide a significant reduction of the day-to-day variability of the refraction correction, should be verified.

(1) A verification could be made with archive measurements of TEC at fixed locations. Figure 28 shows such measurements made at four stations with nearly the same longitude. An analysis of groups of local measurements, such as these,

![Figure 28](image)

**Figure 28.** Latitude-Time Variation of Total Electron Content Derived From ATS-3 Observations. Continuous data at these fixed locations near 60°W longitude could verify predictions extrapolated in space and time from TRANSIT observations. Note the differences in the basic shapes of the diurnal curves. Curves for stations north of Hamilton displaced by multiples of 10 total electron content units.
would determine the usefulness of adapted predictions which are extrapolated in space and time between TRANSIT calibration passes. The period between passes may be as long as several hours and it is unresolved at present for what period of time the adaptive prediction should be used between distant passes and when it may be more useful to return to a simple baseline prediction.

(2) A second verification could be made with the test data from the calibration program planned for Clear Alaska during 1977. The result of using TRANSIT passes to adapt a prediction of radar refraction error should be compared to results when only the prediction is used. In addition, field data from local ionosondes and from local measurement of TEC to geostationary satellites could be used to adapt the same prediction. A comparison should then be made between the day-to-day variabilities of each of the predictive techniques. Special emphasis should be given to separating out gross effects, such as sudden ionospheric storms which can not be specified in advance with any of these simple adaptive techniques, since the major variations during storm periods could overwhelm the statistical results.

(3) A large number of sequential TRANSIT passes obtained at Lindau, FRG and reduced to absolute TEC are available spanning November 1975 to the end of February 1976, including nearly all passes which had zenith angles down to 60 degrees. These would permit the study of: the persistence of features in the latitudinal variation; optimizing the number of terms in the filtered TRANSIT calibration data discussed in Section 4, the time delay of the adaptive prediction.

(4) Finally, all observers agree on the presence of large (5 to 10 percent amplitude) wave-like disturbances, indicators of large localized cells covering several hundred kilometers. The specification of these, particularly the extrapolation of such major features in latitude, longitude and time, needs serious attention. The afternoon "bite-out" in Figure 28 may be such a feature. If it is either neutral winds moving ionization or an increase in the chemical loss rate, it is stronger at low latitudes than at high latitudes. The predictable features of such an effect can be examined from archive data. This is necessary to any program to implement TRANSIT adaption of a refraction correction since such cell-like disturbances, with major amplitude changes, are the principle source of day-to-day variability.
References


11. Leitinger, R., and Hartmann, G. K. (1976) Time and Latitude Dependence of Ionospheric Electron Content from the Combination of NNSS and ATS-6 Data, the Geophysical Use of Satellite Beacon Observations, COSPAR Symposium Proceedings, Boston University.
