CALCULATING THE SPEED WITH WHICH WATER-PERMEABLE GROUND IS FROZEN BY A ROW OF COLUMNS BEFORE FROZEN GROUND CYLINDERS JOIN

A.I. Pekhovich
### REPORT DOCUMENTATION PAGE

<table>
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<th>1. REPORT NUMBER</th>
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<th>3. RECIPIENT'S CATALOG NUMBER</th>
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<tbody>
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<th>10. PROGRAM ELEMENT, PROJECT, TASK AREA &amp; WORK UNIT NUMBERS</th>
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<th>15a. DECLASSIFICATION/DOWNGRADING SCHEDULE</th>
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<th>16. DISTRIBUTION STATEMENT (OF THIS REPORT)</th>
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<th>18. SUPPLEMENTARY NOTES</th>
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<th>19. KEY WORDS (CONTINUE ON REVERSE SIDE IF NECESSARY AND IDENTIFY BY BLOCK NUMBER)</th>
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<tbody>
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<td>FROZEN GROUND</td>
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<th>20. ABSTRACT (CONTINUE ON REVERSE SIDE IF NECESSARY AND IDENTIFY BY BLOCK NUMBER)</th>
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<tbody>
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ENGLISH TITLE: CALCULATING THE SPEED WITH WHICH WATER-PERMEABLE GROUND IS FROZEN BY A ROW OF COLUMNS BEFORE FROZEN GROUND CYLINDERS JOIN

FOREIGN TITLE: RASCHET SKOROSTI ZAMORAZHIVANIYA FIL'TRUYUSHCHEGO GRUNTA RYADOM KOLONOK DO SMYKANIYA LEDOGRUNTovyKH TSILINDROV

AUTHOR: A.I. Pekhovich


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In 1954 Volume 51 of the "Notes of the All-Union Scientific Research Institute of Hydraulic Engineering" contained an article which examined the method for calculating the freezing rate of water-permeable ground by a row of columns after frozen ground cylinders had joined. This article proposes a method for calculating the rate at which water-permeable ground is frozen by a series of columns before the frozen ground cylinders join.

1. Statement of the Problem

The problem under examination can be formulated as follows.

Let there be given a cofferdam of length L and thickness H (Figure 1). The cofferdam is built on a base which is permeable to water and heat. Water filters through the cofferdam under the action of a constant difference in head between the lateral boundaries of the cofferdam. Water does not filter around the cofferdam. The freezing columns with radius \( r_0 \), height h and total number L are arranged on the cofferdam on a plane which is parallel to the lateral (head) surfaces at uniform distances S from each other. Before the columns are implanted in the ground, the filtration flow under natural (normal) conditions has speed \( v_n \) and temperature \( T \). The ground freezes at temperature \( t_0 \). The thermophysical parameters of the frozen and unfrozen ground are known. It is necessary to find a dependency which determines the ground freezing rate before the cylinders join.

![Figure 1. a, Plan of Cofferdam; b, Transverse Cross-Section of Cofferdam.](image)

2. Heat Balance Equation

The heat balance equation of the frozen ground looks as follows:

\[
Q_F = Q_c - Q_{fi} - Q_g
\]

where \( Q_F \) is the amount of heat evolved per unit of time when water freezes on the surface of the frozen ground cylinders;
$Q_c$ is the amount of heat which is given off per unit of time by the freezing columns (the thermal absorption by the freezing columns);

$Q_{fi}$ is the amount of heat which the filtration flow gives off per unit of time to the frozen ground cylinders;

$Q_g$ is the amount of heat which the frozen ground loses per unit of time (cooling of the frozen ground cylinders).

The thermal flow $Q_g$, as is demonstrated in examining the freezing of water-permeable ground by a single column [1] can be taken into account by increasing the calculated value of the latent ice formation heat by the value

$$\frac{\gamma_1}{\lambda_1} \frac{Q_c}{\frac{1}{\pi r_1 h}},$$

where $\lambda_1$ is the thermal conductivity factor of the frozen ground;

$c_1$ is the thermal capacity of the frozen ground;

$\gamma_1$ is the specific weight of the frozen ground;

$h$ is the height of the frozen ground cylinder.

Therefore thermal balance equation (1) can be represented in the following simpler form:

$$Q_f = Q_c - Q_{fi}.$$  \hspace{1cm} (3)

The thermal flux from the latent ice formation heat is equal to:

$$Q_f = 2\pi r_1 \sigma \frac{dR}{dr}.$$ \hspace{1cm} (4)

Here $R$ is the radius of the frozen ground cylinder;

$\sigma$ is the latent ice formation heat of a unit of ground volume;

$t$ is time.

Allowing for the thermal flux $Q_g$, instead of (4) it is necessary to write

$$Q_f = 2\pi r_t \frac{dR}{dr},$$

where

$$t = r_t.$$ \hspace{1cm} (5)

The absorption of heat by the freezing columns changes as the ground freezes. The law governing the change in heat absorption is determined by the characteristic of the refrigeration device and by the thermal resistance of the frozen ground.

The means for determining this dependency are analyzed in the technical literature [2]: here we will write this dependency with regard to the frozen ground cylinders in the following general form:

\[ Q_c = f(R). \]

This expression may be regarded as a known dependency; in doing this we keep in mind the fact that by properly selecting the refrigeration devices this dependency can be given any desired form.

If the freezing is conducted at a constant brine temperature \( \theta \), then

\[ Q_c = \frac{\pi h f_s (t_n - \theta)}{\ln \frac{R}{r_0}}. \]  

(7)

It remains for us to determine the value of the thermal influx from the filtration flow \( q_{fi} \). The determination of this component of heat balance equation (3) is the basic difficulty in solving the stated problem.

3. Calculating the Thermal Influx from the Unfrozen Water-Permeable Ground to the Frozen Ground Cylinders

We will determine the value of the thermal flux from the unfrozen ground to the frozen ground cylinders on the basis of the following equation:

\[ q_{fi} = c_w v_w S n I_z (T - t_{out}). \]

where \( c_w \) is the thermal capacity of the water;

\( v_w \) is the specific weight of the water;

\( I \) is the number of columns;

\( v_0 \) is the speed of the filtration flow in the cofferdam in the area where the frozen ground cylinders do not cover the entire cross-section;

\( S \) is the distance between the axes of the freezing columns;

\( T \) is the temperature of the filtration flow at the approach to the frozen ground cylinders (equal to the temperature of the filtration flow under natural conditions);

\( t_{out} \) is the average temperature of the filtration flow after passing through the frozen ground screen.
In order to make subsequent analysis convenient, we will introduce two parameters:

\[ v = \frac{v_1}{v_b}, \]

where \( v_b \) is the speed of the filtration flow before freezing begins and

\[ \Delta T = \frac{T - t_{out}}{T - t_0}; \]

where as before \( t_0 \) is the freezing temperature of the ground.

Then the equation for the thermal flux \( Q_{fl} \) acquires the following form:

\[ Q_{fl} = N \cdot \Delta T. \]

where

\[ N = c \cdot Sh_{g} \cdot \beta \cdot (T - t_0). \]

In order to use equation (10) it is necessary to know the values of \( v \) and \( \Delta T \).

In the problem which we are examining (a row of freezing columns, without filtration around the cofferdam) as the frozen ground cylinders increase in size, the rate of flow through the cofferdam decreases, and therefore the speed of the filtration flow \( v_0 \) and the value of \( v \) decrease. This problem was studied by G. S. Shadrin who used the graphic method of constructing motion grids [3]. An analytic examination of this problem has led us to the following relationship [4] which determines the desired parameter:

\[ \frac{v}{v_b} = \frac{z}{r = \sqrt{H - \frac{1}{2} \cdot \sin^{-1} \left( \frac{z}{r} \right)}}, \]

where

\[ z = \frac{2R}{S}; \]

\[ x = -H \cdot \frac{S}{S}; \]

\[ v = \frac{\pi}{1 - \frac{1}{2} \cdot \frac{z}{r}} - \frac{\pi}{2}. \]

A graph of the dependency \( v = f(\epsilon, \chi) \) is presented in Figure 2.
Figure 2. Graph for Determining the Relative Change in Filtration Flow Through the Cofferdam When the Frozen Ground Cylinders Increase in Size.

Let us now determine $\Delta \bar{f}$. For this purpose we will use the solution given by heat transmission theory for the problem of wall cooling [5]. The problem is formulated as follows.

An infinite, homogeneous wall of thickness $2X$ at the initial time ($r = 0$) has the same temperature $T$ at all points, and then ($r > 0$) both sides of its surface are exposed to a temperature of $t_0$. The temperature conductivity factor of the wall $a$ is known.

The solution to this problem has the following form:

$$\Delta \bar{f} = F_{\tau} \bar{s}(r, \tau). \quad (16)$$

where

$$F_{\tau} = \frac{T}{\tau}. \quad (17)$$

A graph of dependency (16) is presented in Figure 3, the lower left quadrant (see glued-in piece at the end of the book).

We will show how to use this solution in our case.

We will write the differential equation of heat conductivity in water-permeable ground (the arrangement of the coordinate axes is given in Figure 4):

$$\frac{\partial T}{\partial \tau} - \frac{\partial^2 T}{\partial x^2} - \frac{\partial^2 T}{\partial y^2} = \frac{\partial^2 T}{\partial x^2} - \frac{\partial^2 T}{\partial y^2}. \quad (18)$$
here $v_y$ is the projection of the filtration flow's velocity vector onto axis Y,

$v_x$ is the projection of the filtration flow's velocity vector onto axis X.
Ignoring the value of the thermal flux which arises due to physical heat conductivity and is directed along axis Y and the component of induced convection which is directed along X, we will convert equation (18) into the following form:

\[
\frac{\partial t}{\partial y} + \frac{a}{\lambda_2} \frac{\partial t}{\partial x} = 0
\]  

(19)

where

\[
a = \frac{\lambda_1}{\lambda_2}
\]  

(20)

\(\lambda_2\) is the thermal conductivity factor of unfrozen ground.

Equation (19) can be regarded as a one-dimensional thermal conductivity equation in which the time variable is replaced by the y coordinate variable.

Thus, the task of cooling a filtration flow which occurs in a gap 2X thick amounts to the task of cooling a wall, the solution to which is known (equation (16)). However, in the case which is of interest to us, cooling a filtration flow when it passes through a frozen ground stream, we are dealing with a gap of non-constant thickness.

We will show that the above-indicated solution can also be applied to our problem.
As is known, the temperature distribution in an infinite planar-parallel wall is determined by integrating the differential equation

$$\frac{d^2 T}{d\xi^2} = \alpha \frac{d^2 T}{d x^2}. \tag{21}$$

If we place the origin of the coordinate axis in the middle of the wall \((x = 0)\) and designate the wall thickness as \(2X\), then under boundary conditions

$$T(x, 0) = T_0, \tag{22}$$

$$T(-X, \tau) = T(X, \tau) = T_\infty; \tag{23}$$

the solution to equation (21), as is well known, will be as follows [5]:

$$T(x, \tau) = T_\infty - (T_0 - T_\infty) \sum_{n=1}^{\infty} A_n \cos (n \pi x / X) \exp (-n^2 \pi^2 Fo / X^2), \tag{24}$$

where

$$n = \frac{X}{L}; \tag{25}$$

$$Fo = \frac{4X^2}{\pi^2 \alpha}; \tag{26}$$

$$\zeta = \frac{(2n - 1) \pi}{2}; \tag{27}$$

$$A_n = \frac{4}{\pi^2 (2n - 1)^2} - \frac{1}{\pi^2}; \tag{28}$$

For any moment in time \((\tau = \tau_0)\) equation (24) can be regarded as an initial condition (instead of (22)) with equation (21) integrated. This initial condition is characterized completely by the value of criterion \(Fo\); in this case, from the viewpoint of the subsequent process it is completely irrelevant what the conditions are under which the process of heat transmission has occurred during the preceding period. The course of the wall's temperature, as before, will be described by equation (24), and the speed of increase of \(Fo\) will depend on the wall thickness at the given time. We will consider that the change in wall thickness and, consequently, the change in the area of the filtration flow's cross-section per se does not cause any temperature changes at compatible points, i.e., we will assume that when the stream of the filtration flow is instantaneously compressed or expanded, dependency \(t = f(\gamma_1)\) will remain unchanged.

In this case

$$\frac{dFo}{d\gamma_1} = 0. \tag{29}$$
And, consequently,

\[ \frac{dx}{Fo} = \frac{\partial \psi}{\partial x} \, dx. \]  

(30)

Therefore the value of criterion Fo which has to be substituted into the solution of (24) is determined by the integral:

\[ Fo = a \int \frac{dz}{N_x^2}. \]  

(31)

In order to obtain this integral, we will express \( dx \) through \( X \), and in doing so we note that

\[ dx = \frac{dy}{S}. \]  

(32)

From Figure 4 it follows

\[ y = R + \int \sqrt{R^2 - \left( \frac{S}{2} - X \right)^2}. \]

by differentiation, we find:

\[ \frac{dy}{\sqrt{R^2 - \left( \frac{S}{2} - X \right)^2}} = \frac{X - \frac{S}{2}}{\sqrt{R^2 - \left( \frac{S}{2} - X \right)^2}} \, dX. \]  

(33)

Keeping in mind the fact that the average velocity of the filtration flow is inversely proportional to the area of the cross-section, we can write:

\[ v = \frac{S}{2N}. \]  

(34)

By substituting (32) into (31) and taking (33) and (34) into account, we find:

\[ Fo = \frac{2a}{S} \int_{X = \frac{S}{2}}^{X = \frac{S}{2} - R} \frac{X - \frac{S}{2}}{\sqrt{R^2 - \left( \frac{S}{2} - X \right)^2}} \, dX. \]

By integrating within the limits of \( X = \frac{S}{2} \) to \( X = \frac{S}{2} - R \), doubling the result (since the limits cover only half of the length of the slit) and by substituting expression (15), we find the value of the Fourier criterion after the filtration flow passes through the frozen ground screen:
and, keeping (12) in mind, we finally obtain:

$$F_0 - K_2(\alpha, \xi),$$

(35)

where

$$\Phi(x, z) = \frac{x - \Phi_{1, 2} - \xi}{z} \Phi_1;$$

(36)

$$K = \frac{5a}{\varepsilon S};$$

(37)

$\Phi$ -- see equation (15).

The results of calculating the Fourier criterion in terms of dependency (35) are also presented in Figure 3. By knowing $x$, $\varepsilon$ and $K$, this dependency makes it possible to find $\Delta t$.

Thus, by using the graph in Figure 2 it is possible to determine the values of parameter $\alpha$, and from the nomogram in Figure 3 it is possible to find the values of parameter $\Delta t$. This is sufficient to calculate the magnitude of the heat influx from the unfrozen water-permeable ground according to formula (10). Below is given a corresponding sample calculation for the purpose of illustration.

4. Sample Calculation of Heat Influx From Unfrozen Water-Permeable Ground

We will determine the thermal influx to one of the frozen ground cylinders which is in a row with others.

The following are given: $\lambda_2 = 1 \frac{\text{kcal}}{\text{m} \cdot \text{hr} \cdot \text{deg}}$;

$$c_w = 1 \frac{\text{kcal}}{\text{m} \cdot \text{deg}};$$

$$\gamma_w = 1000 \frac{\text{kg}}{\text{m}^3};$$

$$v_b = 0.04 \text{ m/hr}; \ T = 7^\circ C; \ t_0 = 0^\circ C; \ S = 1 \text{ m}; \ H = 3 \text{ m}; \ I = 1; \ h = 1 \text{ m}.$$

From equations (20), (14), (37), and (11), we find:

$$a = \frac{\lambda_2}{c_w} = \frac{1}{1000} = 0.001 \text{ kcal/hr};$$

$$x = \frac{H}{S} - \frac{S}{I} = 2;$$

$$K = \frac{4t}{v_b} = \frac{4 \times 0.001}{0.04} = 0.1;$$

$$N = \frac{\pi S h}{2} v_b (T - t_0) = \pi \times 1000 \times 1 \times 1 = 3.14 \text{ kcal/hr}.$$
According to computation equation (10):

\[ Q_f = \frac{k_0 w_2^2}{2\pi} h(T - t_f). \]  (38)

The rest of the computations are summarized in the table. For the sake of comparison the last graph in the table shows the results of calculating the heat influx to a single frozen ground cylinder, with these computations carried out according to formula of B. V. Proskuryakov [6]:

\[ Q_f = 8 \int \frac{k_0 w_2^2}{2\pi} h(T - t_f) \, dt. \]

### TABLE 1.

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<thead>
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Commas indicate decimal points.

From Table 1 it is evident that the magnitude of the heat influx from the unfrozen ground to a cylinder which is in a row with another reaches its peak value when \( \varepsilon = 0.8 \), and then rapidly decreases. A comparison with the data for a single cylinder shows that in the cited case conditions for ground freezing by a row of columns are more favorable. The corresponding calculations, however, show convincingly that under these circumstances, for instance at higher filtration flow speeds or when the cofferdam is quite thick, cases are possible where the thermal influx \( Q_f \) to the frozen ground cylinders in a row with other cylinders is greater than the flow to a single cylinder.

5. Solution to the Heat Balance Equation

We will substitute into heat balance equation (3) the expressions of its components (5) and (10), and by taking (13) into account we find:

\[ \frac{\pi}{2} \int_0^{t_f} \frac{z}{z} e^{-x} \, dx = \frac{\pi}{2} e \sqrt{\frac{k_0}{h}}. \]

The solution to the posed problem, determining the ground freezing time before the frozen ground cylinders join, is found by integrating this latter equation within the limits from \( \varepsilon = 2r_0/S \) to \( \varepsilon = 1 \) and from \( \tau = 0 \), to \( \tau = t_f \):
The sub-integral function is complex in form, and therefore the value of the integral must be calculated by the graphic or tabular method, and then determining the freezing time according to formula (39) presents no problem.

6. Sample of Calculating Freezing Rate

We will determine the period of ground freezing which occurs before the frozen ground cylinder join under the following initial conditions:

- length of cofferdam: \( L = 100 \) m
- thickness of cofferdam: \( H = 6 \) m
- height of column: \( h = 8 \) m
- distance between column axes: \( S = 1 \) m
- radius of freezing columns: \( r_0 = 0.05 \) m
- speed of filtration flow: \( v_b = 0.08 \) m/hr
- temperature of filtration flow: \( T = 6^\circ\text{C} \)
- latent ice formation heat per unit of ground volume: \( \sigma = 24,000 \) kcal/m\(^3\)
- thermal capacity of water: \( c_w = 1 \) kcal/kg/hr
- specific weight of water: \( \gamma_w = 1000 \) kg/m\(^3\)
- thermal conductivity factor of unfrozen ground: \( \lambda_2 = 1 \) kcal/m·hr·deg
- thermal conductivity factor of frozen ground: \( \lambda_1 = 2 \) kcal/m·hr·deg
- thermal capacity of frozen ground: \( c_1 = 0.34 \) kcal/kg\(\cdot^\circ\text{C}\)
- specific weight of frozen ground: \( \gamma_1 = 1600 \) kg/m\(^3\).

We find:

\[
\begin{align*}
\alpha &= \frac{1}{c_w} \times \frac{1}{1 \cdot 1000} = 0.001 \text{ m}^2 \cdot \text{hr} \cdot \text{kcal}^{-1}; \\
K &= \frac{4 \alpha}{c_b S} = \frac{4 \cdot 0.001}{0.08 \cdot 1} = 0.05; \\
\tau &= \frac{H}{S} = \frac{6}{1} = 6; \\
I &= \frac{L}{S} = \frac{100}{1} = 100; \\
\bar{N} &= \frac{c_w}{c_b} \cdot Shl_{t_b} (I - t_a) = 0.001 \cdot 1000 \cdot 1 \cdot 8 \cdot 100 \cdot 0.08 \cdot 6 = 384000 \text{ kcal/hr.}
\end{align*}
\]
The refrigeration device ensures the constant brine temperature \( \theta = -20^\circ C \); according to formula (7) and taking into account the fact that \( R = \frac{2\ell}{S^2} \), we will determine how much heat the freezing columns absorb:

\[
Q_e = \frac{\pi n F h_0 (t_0 - b)}{\ln \left( \frac{S^2}{2\ell} \right)} - \frac{2\pi \cdot 8 \cdot 100 \cdot 2 \cdot 20}{\ln \left( \frac{1}{2 \cdot 0.05} \right)} = 201 \cdot 10^5
\]

The calculated value of the latent ice formation heat is determined from equations (2) and (6):

\[
s_1 = 2 - \frac{Q_e}{4 \pi k, \ell} \cdot \frac{24000}{2 \cdot 2 \cdot 100 \cdot 8} = 24000 + 27 \cdot 10 \quad Q_e
\]

The rest of the calculations are summarized in Table 2, and in this process, in accordance with equation (39) we utilize the following expression:

\[
2s = \frac{\pi}{2} \cdot S^2 h_0 \ell_2 - \frac{\Delta t}{Q_e N \cdot \Delta t^2} = \frac{\pi}{2} \cdot 8 \cdot 100 \cdot 8 \cdot 24 \cdot 10 \quad \frac{\Delta t}{Q_e N \cdot \Delta t^2}
\]

TABLE 2.

<table>
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<th>( n )</th>
<th>( Q_e ) kcal/hr</th>
<th>Found from Figure 2</th>
<th>( t_2 ) kcal/hr</th>
<th>( t_1 ) kcal/hr</th>
<th>( t_1 - t_2 ) kcal/hr</th>
<th>( A_t ) hrs</th>
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</table>

Commas indicate decimal points.

Thus, under the given conditions joining will not occur; the frozen ground cylinders will cease to grow as early as \( \varepsilon \leq 0.64 \). In the given case it is advisable to shift to freezing at a lower brine temperature. Below Table 3 shows a calculation of freezing time which demonstrates that if we assume that the brine temperature is \( \theta = -40^\circ C \), then joining will occur, for instance, on the 16th day.
TABLE 3.

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<tr>
<th>$r$</th>
<th>$Q_c$ kcal/hr</th>
<th>$Q_{fi}$ kcal/hr</th>
<th>$Q_{f1}$ kcal/hr</th>
<th>$Q_{j1}$ kcal/hr</th>
<th>$Q_{j2}$ kcal/hr</th>
<th>$Q_{j2}$ kcal/hr</th>
<th>$Q_{j2}$ kcal/hr</th>
<th>$Q_{j2}$ kcal/hr</th>
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</table>

Commas indicate decimal points.

7. Determining the Joining Conditions

The condition for the cylinders to join into a single frozen ground mass consists of the fact that the heat absorbed by the columns $Q_c$ at all values of $\varepsilon$ must exceed the thermal influx from the filtration flow, i.e., the following condition must be fulfilled:

$$Q_c > Q_{fi} \quad (40)$$

when

$$\frac{2r}{5} \leq \varepsilon \leq 1.$$

Instead of ensuring that condition (40) is fulfilled from the beginning of freezing until the frozen ground cylinders join, as a rule, it is sufficient to ensure that this equality is observed at frozen ground cylinder dimensions which correspond to the maximum $Q_{fi}$ value. Figure 5 shows the graph which makes it possible to approximately determine the value of parameter $\varepsilon$ at which $Q_{fi}$ reaches its peak. This graph was plotted as follows.

By utilizing Figures 2 and 3, dependencies such as $\Delta T \cdot \psi = f(\varepsilon)$ were found for various values $K = \text{const}$ and $\chi = \text{const}$, and each of these curves was used to determine $\varepsilon$ values which correspond to a maximum value of $\Delta T \cdot \psi$, and consequently to the maximum $Q_{fi}$ value (at given values $K$ and $\chi$). These additional calculations and structures are not given here, but only their results are shown in Figure 5.

We will cite an example of ensuring that the solidification conditions are fulfilled.
Assume that it is necessary to determine the distance between the freezing columns at which joining of the frozen ground cylinders will be ensured, if it is known that:

- length of cofferdam: \( L = 50 \text{ m} \)
- thickness of cofferdam: \( H = 8 \text{ m} \)
- height of freezing columns: \( h = 10 \text{ m} \)
- radius of freezing columns: \( r_0 = 0.05 \text{ m} \)
- speed of filtration flow: \( v_b = 0.05 \text{ m/hr} \)
- temperature of filtration flow: \( T = 5^\circ\text{C} \)
- thermal capacity of water: \( c_w = 1 \text{ kcal/kg\cdotdeg} \)
- specific weight of water: \( \gamma_w = 1 \text{ kg/m}^3 \)
- thermal conductivity factor of frozen ground: \( \lambda_1 = 2 \text{ kcal/m\cdothr\cdotdeg} \)
- thermal conductivity factor of unfrozen ground: \( \lambda_2 = 1 \text{ kcal/m\cdothr\cdotdeg} \)
- brine temperature in columns: \( \theta = -22^\circ\text{C} \)
- freezing temperature of ground: \( t_0 = 0^\circ\text{C} \).

According to (11) and bearing in mind the fact that \( I = L/S \), we can write:

\[
N = c_w \cdot h \cdot v_b \cdot (T - t) = 1 \cdot 1000 \cdot 0.05 \cdot 5 = 12500 \text{ kcal/hr.}
\]

If we assume that \( S = 1.5 \text{ m} \), then according to expression (37) and (14) [with allowance for (20)]:

\[
X = \frac{4 \cdot \lambda_2}{c_w \cdot \gamma_w \cdot \beta} = \frac{1}{1000 \cdot 0.05 \cdot 1.5} = 0.053;
\]

\[
x \cdot \frac{H}{S} = \frac{8}{1.5} = 5.3.
\]

From Figure 5 we determine that the maximum heat influx of the filtration flow occurs when \( \epsilon \approx 0.94 \).

From Figures 2 and 3 we find that when \( \epsilon \approx 0.94 \):

\[
v = 0.48 \text{ and } \Delta \epsilon = 0.86,
\]

and consequently according to (10):

\[
Q = N \cdot 2\epsilon = 12500 \cdot 0.48 \cdot 0.86 = 51600 \text{ kcal/hr.}
\]

On the other hand, the heat absorbed by the columns is equal to:

\[
Q = \frac{2\pi \cdot 10 \cdot 50 \cdot 2.0 \cdot 22}{1.5 \times 0.94 \times 1.5} = 34 \times 50 \text{ kcal/hr.}
\]
Thus, it has been found that when \( S = 1.5 \text{ m} \) \( Q_{fi} > Q_c \) and consequently joining does not occur; therefore it is necessary to set the columns closer together. If we assume that \( S = 0.8 \text{ m} \) and repeat the course of the above-cited calculation, we find that the position \( \varepsilon = 0.92 \) will be most difficult and in this case: \( Q_{fi} = 75,000 \dfrac{\text{ kcal}}{\text{ hr}} \)
and \( Q_c = 86,500 \dfrac{\text{ kcal}}{\text{ hr}} \). Therefore this version is technically feasible since condition (40) is maintained.

![Graph for Determining the Value of Parameter \( \varepsilon \) at Which the Thermal Influx From the Unfrozen Water-Permeable Ground to the Frozen Ground Columns Reaches Its Peak.]

Conclusions

In this article we have examined the problems of making a thermal engineering calculation of water-permeable ground being frozen by a row of columns before the frozen ground cylinders join where the filtration flow arises through the cofferdam under the action of a constant head.

As a consequence a method corresponding to this case has been developed for calculating the period of freezing. In this process it was found that as the frozen ground cylinders increased in size, the thermal influx from the unfrozen ground first increased and then, due to the reduction in the flow rate of the filtration flow, decreased. Therefore the most difficult period of freezing occurs not directly prior to the joining of the frozen ground cylinders, but somewhat beforehand. If by the time the maximum thermal influx from
the filtration flow occurs the frozen ground cylinders are still growing in size, then it is considered that their joining into a unified frozen ground mass is ensured.

It should be noted that the method proposed in this work for a thermal calculation of the cooling of a filtration flow flowing between two frozen ground cylinders may also be used to solve a number of other thermal problems in which the temperature conductivity of a body and its dimensions vary according to a known law (for instance, in the problems of water course thermics). In this process, however, it must be kept in mind that this method is correct only when there are constant temperatures at the boundaries and if the physics of the event under consideration makes it possible to consider that the change in wall thickness (or water course depth, diameter of the penstock, etc.) does not per se cause temperature changes at compatible points.

BIBLIOGRAPHY
