SIGNAL-TO-INTERFERENCE PROTECTION RATIOS
FOR HF MARITIME MOBILE SERVICES

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General

HF maritime mobile services signals are characterized by widely fluctuating signal levels. The purpose of this report is to present a method of integrating the following factors into a model for the determination of a minimum acceptable signal-to-interference (S/I) ratio.

1. Minimum detectable signal (dBm)
2. Standard deviation ($\sigma_s$) of the desired signal (dB)
3. Standard deviation ($\sigma_i$) of the interference (dB)
4. Correlation coefficient between signal and interference (-1 to +1).
5. Acceptable minimum signal-to-interference threshold (dB)
6. Required probability of achieving the minimum (S/I) threshold

Report 264-2" presents a method for estimating the signal-to-interference ratio as explained below.

Consider a particular receiving location, at a distance $D_u$ from the wanted transmitter of power $P_u$ and at a distance $D_n$ from the interfering transmitter of power $P_n$, and consider an interval of one hour, the midpoint of which corresponds to the local times $H_u$ and $H_n$ of the mid-points of the path of the wanted and unwanted transmissions, then the ratio $R(T)$ in dB between the wanted hourly median signal level and the interfering hourly median signal level, exceeded for a percentage $T$ greater than 50%
of the hours of the year when the value \( R \) is exceeded, can be calculated for a non-directional receiving antenna from the following formula:

\[
R(T) = F_{H_a}(50) - F_{H_a}(50) - \sqrt{\delta_{H_a}^2(T) + \delta_{H_i}^2(100 - T)} + 2 \rho \delta_{H_a}(T) \delta_{H_i}(100 - T) 
\]

(1)

where \( \rho \) represents the correlation between the changes in hourly median values for the wanted and interfering signal propagation paths. In the absence of measurements of this factor \( \rho \), it is suggested that it be set equal to 0.5 in using equation (1).

It should be noted that \( \delta_{H_a} \) and \( \delta_{H_i} \) always have opposite signs and that the minus sign before the radical in (1) is associated with the practical situation normally encountered, where the time availability \( T \) of satisfactory service is greater than 50%.

Strictly speaking, equation (1) is applicable only to the extent that a log-normal distribution describes the data. However, for the distributions encountered in practice, the formula is an adequate approximation. It neglects the rapid variations of both the wanted and interfering signals, and also the noise level or required signal-to-noise ratio.

Consider a receiving system whose performance can be defined on the basis of input signal-to-noise and signal-to-interference threshold criteria. It is desired to specify the probability that such criteria will be met. Under these circumstances, acceptable system performance can be said to occur when

\[
\text{signal to noise} > R_1 \\
\text{signal to interference} > R_2
\]

(2)
Expressed in units of dB,

\[ S - N > r_1 \]
\[ S - I > r_2 \]

where

- \( r_1 \) and \( r_1 \) designate signal-to-noise ratio thresholds in numeric ratio and dB units, respectively;
- \( r_2 \) and \( r_2 \) designate signal-to-interference ratio thresholds in numeric ratio and dB units, respectively.

Both \( S \) and \( I \) are functions of the source levels of the respective signals and the gains and losses the signals will incur between the sources and the receiver in question. Thus, for nonmobile systems,

\[ S(dB) = P_d + G_d + G_i - L_d - D(I) \]
\[ I(dB) = P_i + G_i + G_i - L_i - F \]

where

- \( P_d \) and \( P_i \) desired signal and interference source levels, respectively;
- \( G_d \) and \( G_i \) desired signal and interference source antenna gains, respectively;
- \( L_d \) and \( L_i \) desired signal and interference path losses, respectively;
- \( G_a \) and \( G_b \) receiver antenna gains to desired signal and interference, respectively;
- \( D(I) \) desensitization of the desired signal by interference; and
- \( F \) receiver off-frequency rejection factor.

If the term \( D(I) \) is not significant, it has often been shown that, to a first-order approximation, \( S \) and \( I \) can be represented by

\[ S = \overline{P}_d + P_d + G_d + g_d + G_a + g_a - L_d - l_d \]
\[ I = \overline{P}_i + P_i + G_i + g_i + G_b + g_b - L_i - l_i - \overline{F} - f \]
Fig. 1 Joint probability distribution of S and I, assuming statistical independence.

Fig. 2. Conditions for meeting performance criteria, assuming statistical independence.
where the bars denote the expected values of the parameters of (5), and the corresponding lower case terms represent a sample from a normal distribution describing the variation of that parameter.

Equation (5) can be simplified to the forms

\[
S = \bar{S} + e_s \\
1 = 1 + e_1
\]

where

\[
\begin{align*}
S &= \bar{F}_d + \bar{G}_d + \bar{G}_a - \bar{L}_d \\
1 &= \bar{F}_1 + \bar{G}_1 + \bar{G}_b - \bar{L}_1 - \bar{F}
\end{align*}
\]

and \(e_s\) and \(e_1\) are samples from another normal distribution such that

\[
\begin{align*}
\sigma_{s_1}^2 &= \sigma_{s_1}^2 + \sigma_{s_1}^2 + \sigma_{s_1}^2 + \sigma_{s_1}^2 \\
\sigma_{s_1}^2 &= \sigma_{s_1}^2 + \sigma_{s_1}^2 + \sigma_{s_1}^2 + \sigma_{s_1}^2
\end{align*}
\]

Substituting (6) and (3) gives

\[
\begin{align*}
S - N + e_s &\geq r_s \\
S + e_s - (1 + e_1) &\geq r_s
\end{align*}
\]

Initially assume that the statistics associated with the desired signal and the statistics associated with the interference signal are uncorrelated. If that is the case, then a plot of equal probability contours of \(S\) versus \(I\) would look something like that shown in Figure 1. The joint probability function would be centered at \((S, I)\), and would be described by the relationship

\[
\rho(x,y) = \frac{1}{2\pi\sigma_x\sigma_y} \exp \left( -\frac{1}{2} \left[ \frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2} \right] \right)
\]
where $x$ and $y$ are displacements from $S$ and $I$, respectively. The plot is actually a three-dimensional one, with the dimensions being $S$, $I$, and $p(x,y)$. Now refer again to (2) rewritten as follows:

$$S \geq r_1 + N - r_3$$  \hspace{1cm} (10)

$$S \geq I + r_3$$  \hspace{1cm} (11)

where $r$ is the minimum acceptable signal level. These equations can be superimposed on Figure 1 to give Figure 2. Only that portion of the graph left unshaded in Figure 2 meets the required performance criteria. The total probability of the criteria being met is equal to the volume under the unshaded curve and bounded by the $p(x,y) = 0$ plane and the planes denoted by (9) and (10). This volume can be expressed as

$$P(r_3, r_3) = \frac{1}{2\pi \sigma_S \sigma_I} \int_{r_3}^{s} \exp \left[-\frac{(S - S)^2}{2\sigma_S^2} \right] \cdot \int_{r_3}^{I} \exp \left[-\frac{(I - I)^2}{2\sigma_I^2} \right] dI \, dS. \hspace{1cm} (12)$$

Sachs goes on to derive the relationship for correlated signal and interference as follows:

$$P(r_3, r_3) = \frac{1}{\sqrt{2\pi \sigma_S \sigma_I}} \int_{-\infty}^{\infty} \exp \left(-r^2\right) \cdot \Phi \left[ \frac{\sqrt{2}(I - I - r) \sigma_I}{\sigma_S \sqrt{1 - \rho^2}} \right] dx. \hspace{1cm} (13)$$

The graphical representation of this situation is shown in Figure 3.

Results

The model was computer programmed and exercised to determine the probability of successful communication for a typical maritime coast-to-ship communications link. Available data indicates that standard deviations of desired signal and interference signal links are of the order of 8-10 dB for this situation.
Fig. 3 Conditions for meeting performance criteria, assuming statistical dependence.
There was no data found concerning the correlation coefficient for the signal and interference levels.

Minimum acceptable signal-to-interference ratio thresholds were set at 8 and 10 dB based on Articulation Index scoring results reported elsewhere. The required probability of successful communication was set at .84. In summary the situation established was that the signal-to-interference ratio should equal or exceed 8-10 dB, 84% of the time. The results are illustrated in Figures 4 and 5.

Conclusions

The results are quite sensitive to the correlation coefficient. Significantly lower S/I ratios can be tolerated when the signal and interference level fluctuations show a high positive correlation. No data has been found to provide a basis for establishing a typical correlation coefficient.

Since there appears to be a significant difference in the S/I ratio requirement between "strong" and "weak" desired signal levels, the establishment of minimum S/N ratios for which given S/I ratios are acceptable protection should be specified.
FIGURE 5
Signal-to-Interference Threshold = 10 dB
Probability of Acceptable Communication = .84
Noise Level = -104 dBm
$\rho = \text{Correlation coefficient}$
FIGURE 4

Signal-to-Interference Threshold = 8 dB

Probability of Acceptable Communication = .84

Noise Level = -104 dBm

\( \rho = \text{Correlation Coefficient} \)
BIBLIOGRAPHY


