STUDIES IN DIFFRACTION

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COLLEGE OF ENGINEERING
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Prepared by:

Professor Thomas B.A. Senior
Project Director

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A. D. BLOSE
Technical Information Officer
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The second major area was a consideration of material effects in scattering with particular reference to the use of the impedance boundary condition and resistive and conductive sheets in simulating the behavior of absorbers. The work was largely motivated by the need to develop materials and design criteria for suppressing such "non-specular" sources of scattering as edge and traveling waves that dominate the return from the wings and tail planes of aircraft at close to grazing angles of incidence.
STUDIES IN DIFFRACTION

This is the final report under Air Force Office of Scientific Research Grant No. 72-2262 entitled "Studies in Diffraction" covering the period 1 January 1972 through 31 December 1976.

The investigations have spanned a wide variety of topics ranging from the exact solutions of the boundary value problems for specific geometries with emphasis on the role played by the material properties of the body, to high and low frequency techniques for more general shapes. They have resulted in fifteen publications in the open literature, as well as two technical reports and several presentations at technical meetings. All were authored or co-authored by the project director with support provided by the Grant. To avoid unnecessary repetition we shall merely summarize here the main areas of investigation, citing the relevant publications at the appropriate places.

The initial focus was on techniques for high frequency scattering determination with particular emphasis on edged bodies. The two main methods are the geometrical theory of diffraction (GTD) and the physical or fringe wave theory (PTD). The former is an extension of geometrical optics through the inclusion of diffracted rays. It requires a knowledge of the diffraction coefficient associated with each local source of diffraction, but is a physically-based theory in which the various parts are actually observable (J-1). It is in principle capable of predicting second (and higher) order contributions to the scattering, but in common with all ray techniques it requires the incorporation of matching functions to ensure finite estimates of the fields at caustics where an infinity of rays come together. References J-2 and T-3 describe how this can be done even in regard to second order diffraction. The second method (PTD) is an extension of physical optics and though it does not suffer from the caustic problem it is much less developed than GTD, and there is, at the moment, no systematic procedure for obtaining higher order effects. However, it is possible to combine the generality of GTD with the caustic-correction advantage of PTD through
the use of equivalent (edge) currents, and we explored this third method in depth (T-2, T-4). Reference R-1 discusses in some detail the application to a circular edge (as of a disk) and the three methods are compared in J-5.

A necessary ingredient of GTD and the equivalent current method is a knowledge of the local diffraction coefficient. In previous experimental studies it had been observed that a curved edge could support a form of creeping wave similar to that supported by a surface both of whose radii of curvature are large, and a convenient canonical problem for investigating this type of wave motion is a disk of large radius at close to grazing angles of incidence. In at least the scalar (acoustical) case, the excitation, launch and decay coefficients have been determined (R-2), and the analysis also sheds light on the problem of the transition region when a diffraction point lies close to the shadow boundary.

The second major area was a consideration of material effects in scattering with particular reference to the use of the impedance boundary condition and resistive and conductive sheets in simulating the behavior of absorbers. The work was largely motivated by the need to develop materials and design criteria for suppressing such "non-specular" sources of scattering as edge and traveling waves that dominate the return from the wings and tail planes of aircraft at close to grazing angles of incidence. The appropriate canonical problems are wedges and half planes, and reference J-15 (see also J-6 and T-5) surveys the mathematical techniques available for the solution of a class of imperfect half planes. It has also been noted that resistive and conductive sheets are duals in the sense of Babinet's principle and a number of extensions of the classical principle have been developed (J-8, J-13). For these sheets as well as an impedance (boundary condition) half plane, the edge diffraction coefficients have been determined (J-10, J-11). Such a sheet provides a valid simulation of a dielectric layer of thickness small compared with the wavelength, thereby enabling us to replace what would otherwise be a volume distribution of polarization currents by a surface distribution. This is a great advantage in any numerical solution of a scattering problem, and we have applied some of the computer programs that have been written for resistive sheets of arbitrary configuration to determine (T-6) the scattering and absorptive properties of thin walled (sheath) ice crystals, as are found in cirrus clouds.
At frequencies such that the wavelength is much larger than all dimensions of a body, the scattering is attributable to the induced electric and magnetic dipoles. This is the Rayleigh region where the scattered field (amplitude) is inversely proportional to the second power of the wavelength, and here the polarizability tensors provide a particularly convenient means of characterizing the scattering. Programs have been developed to compute the tensor elements for a variety of rotationally symmetric perfectly conducting bodies and have been used to provide low frequency scattering information for a class of space objects, including those surrounded by a plasma sheath (J-9). We have also explored the applicability of these same tensors in acoustic scattering (J-4), as well as electromagnetic scattering by homogeneous dielectric bodies. Results have been obtained for rectangular parallelepipeds (J-14).

In addition to the above main areas, a number of isolated topics have been investigated, and some of these resulted in publications. The general topic of transient radiation and scattering was surveyed in an invited talk at the XVIIth General Assembly of URSI (T-1), and closed form results were obtained for the specific problem of an impulse incident on a half plane (J-3). The author also participated in a general review of progress in radio science during the three year period 1972-74 (J-7), and has treated a problem concerning cavity-aperture interaction (J-12).

A listing of the journal articles, reports and oral presentations that have been supported by the Grant is as follows:

**Journal Articles**


Abstract: The cone of diffracted rays, which is produced when an optical ray is incident on the edge of a reflecting wedge, is detected by illuminating the edge of a razor blade with a laser beam. All experimentally observed features are in agreement with the predictions of the geometrical theory of diffraction.

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Abstract: Certain errors in the second-order contributions to the scattered field derived in an earlier study are pointed out and the corrected formulae are presented. The results are compared with experimental data and an explanation is suggested for the remaining small discrepancies.

Abstract: Simple expressions are presented for the fields diffracted by a metallic half plane when illuminated by an impulsive plane, cylindrical or spherical (scalar or vector) wave.

Abstract: The leading terms in the low frequency expansions for acoustically soft and hard bodies are examined and the relevance of the magnetic polarizability tensor is discussed. For a hard, rotationally symmetric body, two tensor elements, functions only of the geometry, are now sufficient to specify the entire low frequency scattering behavior in just the same way as the electrostatic capacity suffices for a soft body. Even these quantities are subject to known constraints and computed data for a variety of bodies are presented.

Abstract: Three high-frequency methods of calculating the scattering from metallic edged bodies are compared. The first two are the physical and geometrical theories of diffraction, which have been well established since the late 1950's, and the third is the method of equivalent currents. It is shown that the three share remarkably similar features, although each has its particular virtues and limitations in practical applications.

The method of edge waves (MEW) is an important tool for estimating high frequency diffraction effects, and the above presentation by the originator of this method will be of interest to all workers in this field. As a result of an extended and detailed correspondence with the undersigned, Professor Ufimtsev has now corrected several of his more widely used formulas as indicated in his article, and it is pertinent to remark that those formulas which were criticized in [2] - [4] have now been amended or had their validity changed from that which was originally claimed. That the original shortcomings are not an inherent feature of MEW was fully appreciated by all who have studied the method, and was the reason why the phrase "if PTD (MEW) is to be judged by the description given by Ufimtsev [1]..." was used in conjunction with our criticisms [4]. It is therefore surprising that Ufimtsev should feel his method was misunderstood.


Abstract: Two different extensions of Babinet's principle are discussed. In the first of these, an aperture in a soft or hard screen is covered with a membrane characterized by a jump discontinuity in either the normal component of the fluid velocity or the pressure, and the complementary problem is then a membrane in isolation. In the second, a boundary condition is imposed in the aperture, the solution for which can be obtained from the solution for the complementary disk.


Abstract: The relevance of the polarizability tensors in low frequency scattering is discussed. Data are presented for a variety of perfectly conducting,
rotationally symmetric bodies simulating simple aerospace objects, and it is shown how the presence of any coating influences the results.


Abstract: The concept of nonmetallic, resistive, and "conductive" sheets is discussed, and for a plane electromagnetic wave incident on a half plane composed of any one of these materials, the exact solution of the boundary value problem is obtained for incidence in a plane perpendicular to the edge. In each case the edge diffraction coefficient is proportional to a product of split functions arising from the Wiener–Hopf method of solution. Since each function depends on only one angular variable, the coefficient for arbitrary directions of incidence and observation is expressible in terms of the backscattering coefficient for edge-on incidence on a half plane with an equivalent impedance. This last is rather easily computed, and data are presented to illustrate the scattering behavior.


Abstract: The imperfectly conducting edges considered are those of nonmetallic, resistive and "conductive" half planes illuminated by a plane electromagnetic wave at arbitrary (oblique) incidence. By expressing the boundary conditions at the surface in terms of the normal components of the field and their normal derivatives, each problem is reduced to a combination of two scalar ones analogous to those for a plane wave incident in a plane perpendicular to the edge. From the explicit and exact solutions thus obtained, the edge diffracted fields are derived and expressed in terms of edge diffraction tensors. The tensors are relatively simple and compact. Their computation is discussed and data are presented to illustrate the behavior of the field.

Abstract: For an E-polarized plane wave incident on a perfectly conducting cylindrical shell having a longitudinal slit aperture, the fields inside the cavity are determined by a numerical solution of the E field integral equation. Selected data are presented and the first few complex frequency (SEM) singularities are determined for a variety of aperture sizes.

J-13 "Some extensions of Babinet's principle in electromagnetic theory", by T.B. A. Senior, accepted by IEEE Trans. AP.

Abstract: The concept of resistive and conductive sheets provides a meaningful extension of Babinet's principle to surfaces which are no longer perfect. The complementary problems are described and the appropriate field relations derived.

J-14 "Low frequency scattering by rectangular dielectric particles", by D.F. Herrick and T.B.A. Senior, accepted by J. Appl. Phys.

Abstract: The field scattered by a homogeneous isotropic dielectric particle illuminated by a low frequency plane electromagnetic wave is expressed in terms of a single polarizability tensor which is a function of only the geometry of the particle and a material parameter representing either the relative permittivity or permeability of the dielectric. The mathematical formulation is specialized to the case of a rectangular parallelepiped and numerical techniques are developed for computing the tensor elements. Specific data are presented for the tensor elements of rectangular parallelepipeds having square cross sections and are compared to the results obtained for spheroids and right circular cylinders of similar dimensions.


Abstract: As part of a continuing study of how the material of a body affects its scattering, we consider here the influence of material properties on edge diffraction. An appropriate canonical problem is the scattering of an electromagnetic wave by a half plane whose boundary condition is chosen to simulate the material in question. The half planes considered fall into
two categories: impedance sheets subject to a Leontovich boundary condition on both faces, and resistive and/or conductive sheets characterized by a jump condition. In each case the material parameter (impedance, resistivity or conductivity) may be a tensor, but is assumed to have the same value at all points on a given face, corresponding to a homogeneous, anisotropic sheet. The various forms of the boundary conditions are discussed for a wave incident in a plane perpendicular to the edge and for oblique incidence when the conditions couple two components of the field. We then consider the two basic methods which are available for solving boundary value problems of this type and examine their application to the different half planes. With the advances that have taken place recently, the two methods now have the same capability, but neither works when the material is anisotropic except in those situations where the anisotropy has no effect.

Technical Reports


Abstract: For a ring discontinuity in slope as at the base of a right circular cone, the second order (re-) diffracted field is examined in the general case of bistatic scattering. It is shown that the ray paths are specified by a quartic equation whose solution is discussed. Selected results are presented, and an expression for the field contribution of any one such path is derived. An alternative formulation of the problem using equivalent currents leads to a compact expression of the complete second order field as a double line integral which, when evaluated by the stationary phase method, gives precisely the wide angle contributions previously obtained. However, the integral expression is also finite in the direction of the axial caustic and can be used to find the caustic matching function in second order GTD. These take the form of complementary Fresnel integrals whose practical
effectiveness is verified by a comparison of the results of a numerical evaluation of the integral with the caustically-matched expression for the field in the particular case of backscattering.


Abstract: Experimental measurements of the surface current on an electrically large, perfectly conducting thin disk indicate that under certain conditions creeping waves can exist on the disk surface. Though theoretical verification of this effect is possible for the electromagnetic problem, the solution of a similar scalar problem is expected to exhibit the same type of behavior but be much simpler to treat mathematically. To this end, the surface field on a soft thin disk of large radius due to the presence of a point source far from the disk is found.

Since the disk is a complete co-ordinate surface in the oblate spheroidal system where the scalar wave equation is separable, the surface field can be expressed as an infinite sum of Resolvent Green's functions. The functions are formed of solutions of the separated differential equations and these solutions are constructed using the theory of differential equations containing a large parameter. Except for edge-on incidence, the series is valid only on the shadow side of the disk.

The series expression for the surface field is evaluated for various angles of incidence, though the results are mathematically simple for only two: broadside and edge-on. In the broadside case, the disk edge field is found to be the same as that of a soft half-plane for normal incidence. Away from the edge, the surface field behavior is more complicated, but can still be characterized as an edge wave behavior. For edge-on incidence, things are markedly different. In this case, the edge field consists of optics and creeping wave terms. The optics term at the specular point is the same as the soft half plane edge field for edge-on incidence. The creeping waves
are similar in form to those found on electrically large cylinders. Finally, the expected correspondence between the electromagnetic and scalar analysis of the problem for edge-on incidence is verified.

Papers Presented at Technical Meetings


T-6 "Electromagnetic scattering by columnar sheath ice crystals and other dielectric cylinders", presented at the URSI/AP-S Symposium, Amherst, Massachusetts (October 1976) (with H. Weil).