RAYLEIGH-WAVE DISPERSION TECHNIQUE FOR RAPID SUBSURFACE EXPLORATION

by

This paper reports an investigation of improved or new techniques to extend the depth and/or resolution capability of rapid shallow-depth (less than 500 ft) explorations that would quickly and economically provide an engineer with accurate information on substrate conditions. Current geophysical techniques for rapidly exploring construction sites and investigating foundations, i.e., electrical resistivity, seismic refraction, and vibratory techniques, were reviewed for relative merits and shortcomings. By combining various attributes of the refraction seismic and vibratory techniques, it was considered feasible that a study of surface-wave phenomena could measurably extend the depth limits of current investigation techniques. As a result, the so-called "Rayleigh-wave dispersion technique" was envisioned as a potentially promising, in situ test method. A test program was formulated to adapt conventional refraction seismic equipment and field procedures to acquire Rayleigh-wave data. Theoretical considerations, test procedures, interpretation of data, and comparisons with data obtained by standard methods are presented. Various advantages and limitations of the Rayleigh-wave dispersion technique are discussed.
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The findings in this report are not to be construed as an official Department of the Army position unless so designated by other authorized documents.
FOREWORD

This report is a paper that was presented to the 42nd Annual International Meeting of the Society of Exploration Geophysicists. The investigation that generated the information used in the paper was assigned as a work unit, "Techniques for Rapid Subsurface Exploration," of an RDT&E Project, "Permanent Construction Materials and Techniques for FY 70." This report is an interim report; a final report will be published in FY 74.

Engineers of the U. S. Army Engineer Waterways Experiment Station (WES) who were actively engaged in the investigation, analysis, and report phases of this study were Messrs. R. W. Cunny, Z. B. Fry, R. F. Ballard, and F. K. Chang, of the Soils and Pavements Laboratory (S&PL). The work was performed during the period December 1969 through March 1972 under the general supervision of Mr. J. P. Sale, Chief, S&PL. The paper was prepared by Messrs. Ballard and Chang.

COL Levi A. Brown, CE, and COL Ernest D. Peixotto, CE, were Directors of WES during the conduct of the investigation and publication of this report. Mr. F. R. Brown was Technical Director.
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CONVERSION FACTORS, BRITISH TO METRIC UNITS OF MEASUREMENT

British units of measurement used in this report can be converted to metric units as follows:

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<td>pounds</td>
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<td>pounds per cubic foot</td>
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Rayleigh-Wave Dispersion Technique

for

Rapid Subsurface Exploration

F. K. Chang* and R. F. Ballard, Jr.**

This paper reports an investigation of improved or new techniques to extend the depth and/or resolution capability of rapid shallow-depth (less than 500 ft) explorations that would quickly and economically provide an engineer with accurate information on substrate conditions. Current geophysical techniques for rapidly exploring construction sites and investigating foundations, i.e., electrical resistivity, seismic refraction, and vibratory techniques, were reviewed for relative merits and shortcomings. By combining various attributes of the refraction seismic and vibratory techniques, it was considered feasible that a study of surface-wave phenomena could measurably extend the depth limits of current investigation techniques. As a result, the so-called "Rayleigh-wave dispersion technique" was envisioned as a potentially

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promising, in situ test method.

A test program was formulated to adapt conventional refraction seismic equipment and field procedures to acquire Rayleigh-wave data. Theoretical considerations, test procedures, interpretation of data, and comparisons with data obtained by standard methods are presented. Various advantages and limitations of the Rayleigh-wave dispersion technique are discussed.
INTRODUCTION

Purpose and Scope

The purposes of the study from which this paper was compiled were to review rapid subsurface exploration techniques currently employed to provide engineering information on subsurface geological formations that may be used as foundation or construction materials, and to investigate alternate or supplemental techniques as improved tools for rapid subsurface exploration (Ballard and Chang, 1973). The scope of this paper involves a cursory review of existing rapid, reliable, low-cost geophysical methods for site investigation and location of construction materials, as well as a detailed description of a new Rayleigh-wave dispersion technique for improvement of existing rapid exploration capabilities.

Background

The strength, compressibility, and permeability properties of soils or rocks usually control the design of foundations. During preliminary site or foundation investigations, rapid means of indirectly determining these properties or indexes to the properties and general stratification over large areas are desired. During later phases of the investigations at particular sites, more detailed or specific descriptions of the foundation materials are desired. The subsurface exploration techniques used to determine the desired properties of the material can be divided into
indirect and direct techniques (Glossop, 1968).

Indirect techniques

In geophysical techniques for shallow-depth exploration (maximum depth to bedrock = 500 ft), the seismic refraction and the electrical resistivity techniques are commonly used (Dorbrin, 1968, and Heiland, 1940). The depth as well as the velocity of subsurface materials can be directly determined by a seismic refraction sounding; the dynamic confined compression modulus of elasticity can be calculated from the velocity if the density of the material is known or can be estimated. The electrical resistivity technique is used to detect the variation in resistivity of earth materials, which is largely dependent upon the amount and salinity of the contained water; thus, indirect measurements of porosity, saturation, and permeability are possible. The seismic refraction and resistivity techniques each have certain advantages and disadvantages as exploration tools. Sometimes the two techniques complement each other for certain types of subsurface investigations, so that it may be very advantageous to use both as a means of achieving speed, economy, and reliability. Ten years ago, the U.S. Army Engineer Waterways Experiment Station (WES) adopted a combination of a vibratory technique in conjunction with the seismic refraction technique to determine the in situ shear and Young's elastic moduli of soils (Fry, 1963). The vibratory and seismic refraction techniques were used to measure the Rayleigh- and compression-wave velocities of soils, respectively. The
gravitational and magnetic techniques, generally used for the study of regional geological structure, are without depth control and have little significant engineering value.

The general advantages of the indirect techniques are:

a. Geophysical techniques of exploration identify gross changes in character of subsurface materials by surface measurement of the changes in certain physical properties of the earth at or near its surface.

b. Larger areas or projects of greater linear extent can be explored more rapidly and economically by use of geophysical techniques than by use of borings.

c. The surface measurements as determined by use of these techniques indicate average conditions within a limited area and not along a single vertical or inclined line, such as is the case when the boring technique is used.

d. Data acquired by use of these techniques can be used to detect the subsurface irregularities that often are missed by borings.

The disadvantages are:

a. Although the general character of subsurface materials can often be estimated, the materials cannot be definitely identified by geophysical techniques alone; therefore, these techniques must be supplemented by borings in which representative and/or undisturbed samples are obtained.
b. The interpretation of geophysical data requires trained and experienced personnel.

Geophysical techniques are, however, especially well suited for reconnaissance and preliminary determination of formation conditions of large areas to be used in the construction of missile sites, military underground structures, nuclear reactors, dams and reservoirs, tunnels, highways, airfields, large housing projects, etc. Geophysical techniques have also been used successfully for the determination of water tables and deposits of gravel or other engineering construction materials.

Direct techniques

The conventional or direct techniques utilized for subsurface exploration and determination of foundation properties include boring; sampling, measurements of penetration resistance, vane-shear strength, plate-bearing capacity, and California Bearing Ratio; and other special purpose tests and the conduct of desired laboratory tests on disturbed or undisturbed samples as the situation requires. The direct techniques of exploration are relatively slow and expensive and are not discussed further in this paper. The authors feel that the thrust of this paper should be directed toward a description of the Rayleigh-wave dispersion technique as a potential new tool for rapid subsurface exploration rather than discussing "yesterday's newspaper."
RAYLEIGH-WAVE DISPERSION TECHNIQUE

It is generally understood that the conventional seismic refraction technique will provide a compression-wave velocity and an accurate depth determination from the first arrival times of compression waves, and that the shear-wave velocity at depth can be determined by the vibratory technique, which has a limited depth of penetration of about 150-200 ft. Shear modulus, Poisson’s ratio, and Young’s modulus can be calculated by using the compression- and shear-wave velocities and the mass density of the soil or rock.

Through analyses of data acquired using the vibratory technique, it has been found that the waves produced by the vibrator are predominantly Rayleigh waves. It is considered feasible to explore shallow depth by means of Rayleigh waves generated by refraction seismic tests for the following reasons:

a. Rayleigh waves as well as compression waves are generated by the refraction seismic tests.

b. Rayleigh waves are more easily observed than shear waves.

c. A pulse traveling with Rayleigh-wave velocity should be a prominent feature on any seismic refraction record unless the instrumentation has been designed specifically to reject it.

d. Rayleigh waves generated by earthquakes have been used successfully as a tool for exploration of the earth’s
crust and mantle structure during the past three decades.

In view of the reasons mentioned above, a proposed seismic refraction test procedure and a plan of test to record the compression wave, shear wave, and Rayleigh wave on the same seismogram have been considered. Certain preliminary tests have been conducted for comparison of Rayleigh-wave velocities determined with both vibratory and refraction seismic techniques.

Field Procedure

The field procedure for recording the Rayleigh or R-waves during refraction seismic tests is as simple as that for recording the compression or P-waves. Actually, the P- and R-waves can be recorded on the same seismogram following one shot if the instrumentation incorporates specially designed automatic gain control (AGC) circuitry. Considering that a sharply breaking, large-amplitude, first arrival P-wave is necessary, two shots must be made with less sophisticated instrumentation: one using a large amount of TNT (3 to 5 lb) for recording P-waves, and one using a lesser amount of TNT (0.25 to 2 lb) for recording R-waves. The size of the charge depends on the shot-to-detector distance. At a close shot distance of 10 ft, an electrical cap may be sufficient.

For routine dynamic foundation studies, a twelve-geophone seismic cable with 25-ft spacing of the geophones is sufficient.
In practice, shots are fired at increasing distances from each end of the cable spread until seismic waves from the bedrock are recorded on an oscillograph. For rapid reconnaissance in a large area, a twelve-geophone seismic cable with geophones at 50-ft intervals can be used. For rapid subsurface exploration, two seismic profiles of fairly long length perpendicular to each other can usually be completed within two days.

The instrumentation system employed in field explorations is similar in most respects to that used in shallow refraction explorations. The band of frequency response including the geophone amplifier and galvanometer is wider (around the range of 2 to 200 Hz) than in standard seismic equipment. The geophones are of the velocity type and especially designed for low-frequency measurement. Present WES recording seismic instruments were adapted for the purpose of conducting the field experiments.

Data Interpretation

Definitions of the group and phase velocities

To analyze the dispersion characteristics of an R-wave, the group or phase velocities of several frequency components of the R-wave must be measured. The group velocity can be measured by assuming that the wave originates as a sharp pulse at the instant of the shot and at the surface directly above the shot point. On this assumption, Grant and West (1965) determined what the group
velocity $v_g$ for a travel time $t$ of a particular cycle of period $T$ observed at the minimum horizontal distance $X$ from the shot is given by

$$v_g = \frac{X}{t}. \quad (1)$$

In the case of soil deposits, the seismic-wave velocities increase with depth, and the group velocity is generally less than the phase velocity. The phase velocity can be found by numbering the peaks and troughs of the oscillations and following them from one adjacent trace to the next as will later be described in detail. Assuming that the period of a given cycle in the sequence does not change drastically from one trace to the next, the Rayleigh phase velocity or Rayleigh-wave velocity $v_r$ can be approximated by the relationship given by Grant and West (1965),

$$v_r = \frac{\Delta X}{\Delta t}, \quad (2)$$

where $\Delta X$ is the distance between adjacent seismometers and $\Delta t$ is the apparent time required for the R-wave to travel this distance. (See Figure 1a for graphic representation of the quantities.)

**Seismogram Analyses**

Figure 1b is a seismic record which has been idealized to show the prominent wave arrivals at two adjacent detector locations so that the R-wave analysis could be better illustrated.
Fig. 1. Layout of a typical seismic line and an idealized seismic record
To construct R-wave dispersion curves of phase and group velocity, the following steps must be undertaken:

**Phase velocity**

a. Number the crests or troughs of the Rayleigh-wave train for all traces.
b. Read time of arrival for each crest (or trough) and the average period $T$ for each trace.
c. Compute the incremental time $\Delta t$ between adjacent traces.
d. Determine the Rayleigh phase velocity $v_r$ for each wave number by $v_r = \frac{\Delta X}{\Delta t}$ where $\Delta X =$ distance between adjacent seismometers.
e. Compute the wavelength $L$ for each wave number by $L = v_r T$.
f. Tabulate data.
g. Plot $v_r$ as a function of $L$ or approximate depth, $L/2$.

**Group velocity**

a. Number the crests or troughs of the Rayleigh-wave train for at least one trace.
b. Read time of arrival $t_o$ for each wave number and its period.
c. Determine group velocity $v_g$ for each wave number by $v_g = \frac{X}{t_o}$, \hspace{1cm} (3)

where $X =$ shot-to-detector distance.
d. Plot $v_g$ as a function of $T$ or $L$ (since $L = v_r T$)
Data presentation and reduction

Providing that terrain is relatively flat and background noise levels are sufficiently low, the oscillographic data presentation can be controlled in the field so that office interpretation time will be minimized. This can be accomplished by simply adjusting the galvanometer alignment so that each trace will be spaced in proportion to the distance between adjacent geophones. In this way, the interpreter may work directly from the field records if he so desires. The field data obtained at WES test site 2 (Figure 2) can be used to illustrate this and several other interesting points. As will be observed, data were recorded in four 225-ft, 12-trace segments from geophones that were spaced at 25-ft intervals. The size of the seismic impulse source ranged from electrical caps to \( \frac{1}{3} \) lb of C-4 explosive charges. (The size of the charge was kept to an effective minimum so as not to deface the landscape.) As a means of positive R-wave identification, traces 1, 2, and 3 were the registrations of a transverse, radial, and vertical geophone, respectively, located at the first station of each of the four lines.

Utilizing the simultaneous relationship of the vertical and horizontal (radial) geophones, one can plot the trajectory of particle motion. Figure 3 shows the construction of these diagrams for far-field stations located from 250 to 700 ft from the seismic source. If we note that both upward vertical motion and outward radial motion will produce a downward deflection of the
Fig. 3. Particle-motion diagrams for R-waves (WES test site 2)
seismogram trace, one should expect oscillations on the radial trace to lead those on the vertical trace by 90 degrees in phase. This is seen to be approximately true for those events labeled Rayleigh waves on the record shown in Figure 2. Earlier portions of the record do not show this phase shift due to direct, reflected, and refracted body waves causing distortion (Grant and West, 1965).

In each of the three cases, the particle motion roughly takes the shape of a retrograde elliptical orbit, thus fulfilling the definition requirements for classical R-waves.

Observing the first or near-field record shown in Figure 2, one can readily see that the trace signature characteristics are completely different from the far-field records. This can possibly be explained by the fact that the distance between the shot and the geophones was less than the longest wavelengths that were observed at the far-field stations; therefore, the geophone spread is too close to observe dispersion. However, a second factor must also be considered; the seismic source for the near-field record was a single electrical cap buried only 0.5-ft deep. This source very likely possessed insufficient energy to excite the lower frequencies associated with depth penetration and consequently greater soil masses.

As previously stated, interpretation can be performed directly on the records if so desired. For example, consider the second record in Figure 2 (250-475 ft) for illustrative purposes. Beginning with trace 3, some 11 distinct oscillations can be
observed, numbered, and followed through the succeeding traces on
the record. A line connecting adjacent troughs can then be con-
structed in a manner analogous to a conventional time-distance
plot. Essentially, the reciprocal slope of this line will yield
the wave velocity. An average period should then be determined so
that the wavelength can be computed and plotted in the manner pre-
viously described.

Conversion of wave velocities to soil properties

To become a useful tool in terms of conventional soil prop-
erties, the P- and shear- or S-wave velocities determined by the
above method must be related to elastic parameters such as shear
modulus $G$, Young's modulus $E$, and Poisson's ratio $\nu$. This
conversion can readily be accomplished knowing that the wave
velocity is dependent on the ratio of the elasticity of the medium
to its mass density $\rho$ and on the wave type. The relationship of
shear modulus $G$ and shear-wave velocity $v_s$ and density $\rho$ is
as follows:

$$G = \frac{v_s^2}{\rho}$$

(4)

S-wave velocity and surface R-wave velocity are related by Poisson's
ratio. For a homogeneous medium and for Poisson's ratios ranging
from 0.2 to 0.5, the difference in velocities is less than 9 percent.
Therefore, for practical purposes, S-waves can be considered to have
the same velocity as R-waves.

Poisson's ratio $\nu$ of a soil can be determined by the rela-
tion of S-wave velocities $v_s$ and P-wave velocities $v_c$.  

15
Young's modulus \( E \) is related to shear modulus \( G \) and Poisson's ratio of the soil by

\[
v = \frac{\left(\frac{\nu}{\nu_c}\right)^2}{2 - \left(\frac{\nu}{\nu_c}\right)^2}
\]

for isotropic, linearly elastic materials. From these equations, values of \( E \), \( G \), and \( \nu \) can be determined through the measurement of \( S \)- and \( P \)-wave velocities, provided the density of the soil is known or can be estimated.

Heukelom and Foster (1960), in their dynamic testing of pavements using the vibratory technique, found that excellent correlations could be estimated with known depths of layers if they empirically assumed a measured velocity applicable at a depth equal to half its associated wavelength. During the past ten years, WES has extensively employed this empirical relationship for investigations where boring data were also available to determine depths of interfaces for different materials and agreement has generally been good; consequently, this relationship is assumed valid for the \( R \)-wave dispersion method.

Figure 4 is the \( R \)-wave velocity for WES test site 2 plotted as a function of depth \((L/2)\) prepared using the \( R \)-wave dispersion technique. In addition, data obtained by using the surface vibratory
Fig. 4. R-wave velocity as a function of depth (WES test site 2)
technique is shown for comparative purposes. It is interesting to note that excellent agreement exists between the vibratory and seismic methods to depths of about 40 or 50 ft; then the R-wave dispersion data diverges to indicate higher apparent velocities. This can possibly be explained by the fact that the earliest wave arrivals (generally the longest wavelengths) are a mixture of reflected P-, S-, and R-waves that tend to propagate at a higher apparent velocity. Unfortunately, the manual method of determining the phase velocity is seldom accurate enough, and it is generally necessary to make a Fourier analysis of each adjacent pair of traces and measure $\Delta t$ from the phase shift of the Fourier element whose period is $T$ (Grant and West, 1965). Such an analysis can be performed with available computer codes if it is considered necessary by the data user. A second possible explanation is the difference in stress levels generated by the two methods of testing. Though not as plausible as the first explanation, it must be considered as a possibility and further study must be made to document possible velocity changes as a function of source stress level.

Once the elastic moduli have been determined, they may also be conveniently displayed as a function of depth. These data, which were obtained at the same WES test site, are shown in Figure 5. Once these parameters are supplied to a knowledgeable designer, a structure with predictable performance can be designed (OCE, 1967). Another application for use of these soil parameters would be for a dynamic analysis of existing earthen structures such as dams.
Fig. 5. Elastic and shear moduli as a function of depth (WES test site 2)
potentially exposed to earthquake hazards.

**Theoretical calculations**

As a means of supplementing the interpretation of data obtained by the R-wave dispersion technique, comparisons with theoretical dispersion curves based on soil layering, P-wave velocity, S-wave velocity, and density of the media can be made. These parameters can be determined by borehole sampling, conventional seismic refraction, and some engineering estimates of material properties. Referring again to Figure 4, one can see the comparison of theoretical and experimental values. In this case, all three methods were used as an aid in establishing layer thickness and properties of the WES test site. The model chosen to represent this site was representative of a four-layer system. The following characteristics were assigned to each succeeding layer.

<table>
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<th>Depth (ft)</th>
<th>Velocity, fps</th>
<th>Wet Density (pcf)</th>
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<tr>
<td>0-12</td>
<td>1,110</td>
<td>350</td>
</tr>
<tr>
<td>13-56</td>
<td>5,000</td>
<td>650</td>
</tr>
<tr>
<td>57-76</td>
<td>5,000</td>
<td>980</td>
</tr>
<tr>
<td>77-∞</td>
<td>10,000</td>
<td>4550</td>
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</table>

Utilizing these assigned descriptors in a R-wave dispersion computer code developed by Watson (1970), the theoretical fundamental mode for this test, which was treated as a four-layer problem, can be calculated. It is apparent from Figure 4 that
some differences in absolute velocities do exist. Future research will seek to reconcile these differences.

Evaluation

The advantages and limitations of the R-wave dispersion technique are:

Advantages

a. R-waves are in most instances easily observed and should be prominent features on any seismic refraction record.

b. The computation of phase velocity of an R-wave is easy.

c. The R-wave dispersion technique is a combination of seismic refraction and vibratory techniques; thus, half of the field working time could be saved by using this technique. However, it must be stated that additional interpretation time spent in the office tends to minimize the overall cost savings.

Limitations

a. The investigation area must be large enough that adequate information can be obtained. For example, if the bedrock is 100 ft deep, the length of the profile line (seismic-cable spread) should not be less than 500 ft.

b. Interpretation of the R-wave data requires the services of a competent, trained geophysicist.

c. The R-wave dispersion technique yields velocities that
are consistently higher than those obtained by the surface vibratory technique and laboratory test methods.
CONCLUSIONS

Preliminary field experiments indicate that the R-wave dispersion method for determining shear-wave velocity by surface measurements is indeed feasible, but certain questions need to be resolved. Most apparent is the lack of absolute correlation between data from that method and surface vibratory data. At certain test sites examined at this time, the R-wave dispersion method yielded consistently higher velocities at depths greater than 50 ft. Indications are that the discrepancy may be due to wave interference (model superposition on the actual seismograms) or possibly stress conditions at the source. However, additional study is needed to resolve these observed differences.
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