THEORETICAL STUDIES OF FIBER OPTICAL WAVEGUIDES
AND INTEGRATED OPTICAL CIRCUITS

C. YEH

UCLA-ENG-7671
JULY 1976
This report summarizes the results of the research carried out at the Electrical Sciences and Engineering Department of the University of California at Los Angeles under Contract N00123-73-C-1192-00003 with the Naval Electronics Laboratory Center, Department of the Navy. The report covers activities during the period March 1973 to February 1976. The research dealt in general with the theoretical studies of fiber optical waveguides and integrated optical circuits.
FINAL REPORT

THEORETICAL STUDIES OF FIBER OPTICAL WAVEGUIDES
AND INTEGRATED OPTICAL CIRCUITS

by

C. Yeh

Research sponsored by the Department of Navy
Under Contract No. N00123-73-C-1192-00003

Electrical Sciences and Engineering Department
School of Engineering and Applied Science
University of California
Los Angeles, California 90024

DISTRIBUTION STATEMENT A
Approved for public release; Distribution Unlimited
ABSTRACT

This report summarizes the results of the research carried out at the Electrical Sciences and Engineering Department of the University of California at Los Angeles under Contract N00123-73-C-1192-00003 with the Naval Electronics Laboratory Center, Department of the Navy. The report covers activities during the period March 1973 to February 1976. The research dealt in general with the theoretical studies of fiber optical waveguides and integrated optical circuits.
Table of Contents

Investigations .............................................. 1

1. MODES IN WEAKLY GUIDING ELLIPTICAL OPTICAL FIBRES .......... 1

2. SINGLE-MODE AND MULTI-MODE PULSE PROPAGATION ALONG RADially STRATIFIED OPTICAL FIBER ...................... 2

3. TRANSIENTS IN A PERIODIC SLAB: COUPLED WAVES APPROACH .......... 4

4. ARBITRARILY SHAPED INHOMOGENEOUS OPTICAL FIBER OR INTEGRATED OPTICAL WAVEGUIDES .................................. 4

5. MODE CONVERSION IN PERIODICALLY DISTURBED THIN-FILM WAVEGUIDES .. 5

6. SCATTERING OF ELECTROMAGNETIC WAVES BY ARBITRARILY SHAPED DIELECTRIC BODIES .................................... 6

7. SYNCHROTRON-DIFFRACTION RADIATION SPECTRA IN THE PRESENCE OF A PENETRABLE SPHERE ....................... 8

8. ON THE CERENKOV THRESHOLD ASSOCIATED WITH SYNCHROTRON RADIATION IN A DIELECTRIC MEDIUM ................. 9

9. STOP BANDS FOR OPTICAL WAVE PROPAGATION IN CHOLESTERIC LIQUID CRYSTALS ............................................ 10

10. DISTRIBUTION NETWORKS AND ELECTRICALLY CONTROLLABLE COUPLERS FOR INTEGRATED OPTICS ........................................ 11

11. LEAKY WAVES IN A HETEROEPITAXIAL FILM ..................... 11

COMPLETE LIST OF REPORTS AND PUBLICATIONS ...................... 13
Investigations

This final report summarizes the work performed under Contract N00123-73-C-1192-00003 with the Naval Electronics Laboratory Center, Department of the Navy awarded to the Electrical Sciences and Engineering Department of the University of California at Los Angeles. The work began in March 1973 and terminated in February 1976. During this period, the research investigations resulted in the publication of ten papers, two technical reports and three theses. Summary descriptions of the research investigations are given in the following:

1. MODES IN WEAKLY GUIDING ELLIPTICAL OPTICAL FIBRES

The announcement of the attainment of glass fibres with attenuation below 20 dB km⁻¹ initiated a major movement in the tele-communication communities throughout the world to study in earnest the implementation of optical fibres in high data rate communication systems. The basic configuration of an optical fibre transmission line consists of a circular cylindrical core of radius a and index of refraction n₁ coated with a cladding material of refractive index n₀ (n₁ > n₀). The problem of the propagation characteristics of different modes in such a circular guiding structure has been studied extensively. However, if the core of the fibre is somewhat deformed, say into an elliptical cross-sectional shape, a single mode in a circular fibre may split into two modes with different polarizations and different propagation velocities in the deformed elliptical fibres. These elliptical deformations will therefore introduce delay distortion to the transmitted signal in a single mode fibre and additional delay distortion in a multi-mode fibre. Although exact dispersion relations for various modes in terms of infinite determinants whose elements are given as functions of radial and angular Mathieu functions exist for the elliptical fibre, numerical computations have only been attempted
for the dominant \( HE_{11} \) and \( OE_{11} \) modes. Even for these lowest order modes, the task was very onerous. Approximate perturbation analysis have also been carried out for these dominant modes when the eccentricity of the deformed fibre is near unity. The need to have simplified formulas for the dispersion relations that are valid for all eccentricities and from which the propagation characteristics of higher order modes may be easily computed is quite apparent. Fortunately, the refractive index difference between the core and its cladding of optical fibres that are contenders for use as practical optical communication lines is very small; i.e., \( (n_1/n_0 - 1) \ll 1 \). Approximate analysis are carried out using this condition. It will be shown that very significant simplifications are achieved. The purpose of this study is to show these simplified dispersion relations for higher order modes and to provide some computed results for these higher order modes. It should be noted that elliptical optical dielectric waveguides are good approximations to many guide configurations encountered in the study of integrated optical circuits and so have additional applications in their own right. Results of this investigation have been published (Optical and Quantum Electronics 8, 43 (1967)).

2. SINGLE-MODE AND MULTI-MODE PULSE PROPAGATION ALONG RADIALY STRATIFIED OPTICAL FIBER

This task deals with characterizing radially inhomogeneous optical fiber in terms of its dispersion and Poynting flux properties, and in terms of its pulse transmission properties.

The fiber dispersion and Poynting flux properties are first investigated. The analytical approach utilized is normal mode theory and a piece-wise-homogeneous stratification of the fiber. This latter step allows the wave equations to be uncoupled. The theory is formulated such that any radially arbitrary refractive-index profile can be analyzed. Furthermore, the dispersion
and Poynting flux relations are formulated in terms of $4 \times 4$ matrix operations such that computer storage does not limit accuracy.

The above theory is applied to various refractive-index profiles of current interest such as the homogeneous, parabolic, "doughnut", and v-guide profiles.

The pulse propagation properties of optical fiber are then investigated. As preliminary considerations, the theory of pulse propagation is developed in a rigorous manner with notation of the approximations that are required to make the theory tractable to further analysis. Then, a unified discussion is presented of the available analytical techniques for describing fiber dispersion properties. After these considerations, the characterization of pulse transmission along single mode and multimode guide is undertaken.

The analytical approach utilized to study single mode fiber is to describe the fiber dispersiveness via normal mode theory. The guide is characterized with three values of dispersiveness; then, pulse distortion behavior is studied with three waveforms: Gaussian, rectangular, and triangular.

For multimode fiber, the analytical approach again is to characterize the fiber dispersiveness via normal mode theory. Rather than characterize every propagating mode, this study utilizes the properties of a representative number of modes to describe multimode pulse propagation.

The pulse distortion properties of homogeneous and parabolic profile fiber are investigated. Again, pulse distortion is studied for different input waveforms: Gaussian, rectangular, and triangular.

And finally, to test the validity of approximate theories, the relative time-delay predictions of normal mode analysis are compared against the predictions of geometric optics, scalar wave equation, and WKB theories.

Results of this investigation are given in a technical report.
3. TRANSIENTS IN A PERIODIC SLAB: COUPLED WAVES APPROACH

Most of the work in the field of electromagnetic wave propagation in periodic structures has been directed toward continuous monochromatic waves. However, it is of importance to study the transient response of these structures to finite pulses. In this task we study the reflection and transmission of a rectangular pulse and a Gaussian pulse impinging on a periodic structure of finite length \( L \). In our analysis, we consider the general case of transversely bounded structures where the coupled waves could be different modes and therefore have different longitudinal wave-vectors. For simplicity, and with no loss of generality, the numerical examples will correspond to the special case of transversely unbounded structures. The coupled wave approach is used, and we shall discuss briefly its limitations relative to the more general Floquet approach. There is no limitation on the type of periodicity as long as the coupling coefficient \( \chi \) is small enough to satisfy the coupled wave approach conditions.

The reflection and transmission of rectangular and Gaussian pulses impinging on a periodically stratified slab are studied and a number of examples are illustrated and analyzed. The coupled waves approach is used to derive the reflection and transmission coefficient, then the fast Fourier transform is used to illustrate the transient responses. Results have been published (IEEE Transactions on Antennas and Propagation AP-23, 352 (1975)).

4. ARBITRARILY SHAPED INHOMOGENEOUS OPTICAL FIBER OR INTEGRATED OPTICAL WAVEGUIDES

Since the attainment of glass fibers with attenuation below 3 dB/km, communication via optical fibers is no longer a dream but a reality. Understanding of the propagation characteristics of optical signals along guiding structures depends on the availability of analytic or numerical solutions of
the problems. Analytic solutions are known for only a few light-guiding structures with simple geometrical shapes, such as circular cylinders, elliptical cylinders, or planar layers. Approximate solutions are also available for homogeneous rectangular cylinders provided that the refractive index of the core region is only slightly higher than that of the surrounding region.

The need to have a method that is capable of providing the solutions to the problem of waveguiding along practical guiding structures, which may be inhomogeneous optical fibers with noncircular core cross-sections or may be inhomogeneous channel or embossed integrated optical waveguides, is quite apparent.

Significant advances in recent years in the successful application of the finite element method to the complicated structural and continuum mechanics problems provided the impetus for us to search for the possible application of the finite element technique to problems involving optical waveguiding. This paper describes our successful attempt in finding the solution to the problem of wave propagation along arbitrarily shaped inhomogeneous optical waveguides using the finite element technique (J. Appl. Phys. 46, 2125 (1975)).

5. MODE CONVERSION IN PERIODICALLY DISTURBED THIN-FILM WAVEGUIDES

Periodic structures have a large field of applications in active and passive optical thin-film structures. Their stop-band passband characteristic can be used for distributed feedback, filtering, and coupling. Their space-harmonic characteristic can be mainly used for phase-matched nonlinear interactions and for coupling to drifting electrons in a thin-film semiconductor.

In a recent paper, we have studied the coupling between identical modes in different types of periodic optical thin-film waveguides (surface periodicity, waveguide index periodicity, and substrate periodicity). In this paper we extend our previous work to the case of coupling between nonidentical modes.
and we evaluate the efficiency of the mode conversion and its use for mode generation, filtering, and distributed feedback. The last application consisting of the use of a higher-order mode to carry the feedback function in a DFB (distributed feedback) laser operating on a lower forward mode. As we will show in this paper, this scheme is sometimes more efficient than in the case where the feedback and direct waves are in the same mode. We also show that in a number of cases, the coupling coefficient (for mode conversion, feedback, or filtering) reaches a maximum in a specific frequency region thus allowing optimization of the system.

The approach used in this paper is slightly different than that given previously; however, for the sake of continuity and to avoid repetitions, our previous notations and a number of results and figures will be used here. Our study is limited to the case of TE waves, but the approach can be applied in a straightforward manner, for TM waves.

We shall study three important types of periodic structures: periodically inhomogeneous thin-film guides, periodically inhomogeneous substrate guide, and thin-film waveguide with period surfaces. All these structures are technologically feasible. Results have been given in a paper (J. Appl. Phys. 45, 3494-3494(1974)).

6. SCATTERING OF ELECTROMAGNETIC WAVES BY ARBITRARILY SHAPED DIELECTRIC BODIES

The calculation of the scattering of electromagnetic waves by dielectric objects is a problem that has received increased attention in recent years. Knowledge of the scattered field is required in many areas, such as in investigations of the scattering of microwaves by raindrops and the scattering of light by small chemical and biological particles. The solution of the scattering problem for spherical objects is well known (the Mie theory) and has been used to great advantage in the study of many physical systems. However,
many investigations are concerned with the scattering by nonspherical bodies, and the need for a method to determine rapidly the theoretical scattering by nonspherical objects is clearly indicated. Furthermore, a primary need is for methods that are applicable to objects whose physical size is on the order of the wavelength of the incident radiation (the so-called resonance region).

This investigation will be concerned with the solution of the scattering problem for nonspherical dielectric bodies. The method is most applicable to objects lying in the electromagnetics resonance region. The theoretical equations will be solved numerically for common solid geometric shapes to study the dependence of the scattering on the size, shape, and index of refraction of the scattering object.

A new matrix formulation of scattering, which could be classified as an integral equation method, has recently been developed by Waterman. Originally published for conducting bodies, this method, called the Extended Boundary Condition Method, is exact and provides a general formulation for scattering from obstacles of arbitrary size and shape with size ratios from the Rayleigh region to the geometric optics limit. Waterman later extended the theory to dielectric obstacles using the vector Huygens's principle.

The present investigation begins with a theoretical development of the Extended Boundary Condition Method as it applies to scattering by dielectric objects. It will be shown that an alternate, but conceptually similar derivation, results using Schelkunoff's equivalence theorem, rather than the Huygen's principle.

The over-all goal is to determine the scattered field when an arbitrary dielectric body is illuminated by a plane electromagnetic wave. Specifically, we wish to compute the differential scattering, which is the complete scattering pattern in all directions due to an incident wave in one particular
direction. The dielectric body, assumed homogeneous and isotropic, is characterized by the constitutive parameters $\mu$, $\varepsilon$, where $\mu$ is the permeability, and $\varepsilon$ is the permittivity of the dielectric material ($\varepsilon$ may be complex to account for the lossy case). The surrounding medium is considered to be free space with parameters $\mu_0$, $\varepsilon_0$. The scattered field must be determined for all locations $P$ on a sphere surrounding the scattering object. Results have been published in a paper. (Appl. Optics 14, 2864 (1975)).

7. SYNCHROTRON-DIFFRACTION RADIATION SPECTRA IN THE PRESENCE OF A PENETRABLE SPHERE

Most previous studies on diffraction radiation were carried out for perfectly conducting (impenetrable) bodies, such as conducting half-plane, screens, or gratings, open ends of metallic waveguides, or conducting spheres. Very little work has been carried out for the corresponding problems of diffraction radiation due to the presence of penetrable (dielectric) bodies. This investigation deals with the case of a charged particle orbiting concentrically around a dielectric sphere. We wish to learn to what extent the synchrotron radiation is affected by the presence of a penetrable sphere. It is expected that due to the presence of multiple reflections within the sphere and diffraction phenomena, the radiation characteristics for this charged particle will be significantly modified. It should be noted that since an essential feature of diffraction radiation is that the sources excite a spectrum of frequencies, it is important that exact solution (valid for all frequency ranges) to the problems of diffraction radiation be obtained. Results have been given in a paper (J. Appl. Phys. 46, 643 (1975)).
ON THE ČERENKOV THRESHOLD ASSOCIATED WITH SYNCHROTRON RADIATION IN A DIELECTRIC MEDIUM

It is well known that a charged particle moving at superluminal velocity in a straight line through a dielectric medium will radiate electromagnetic energy known as Čerenkov radiation. Where the velocity is subluminal no radiation will occur. Thus, the radiation exhibits a "threshold" at the velocity of light in the medium. In an earlier publication we demonstrated how the Čerenkov threshold came about in the solution expressed in terms of a Green's function expanded in spherical vector harmonics in the case of a straight particle path. It was shown that the velocity dependence of the radiation was contained in the factor

\[ \int_0^\infty \left[ \frac{j_\ell(x)}{x} \right] \left[ \exp(i\alpha x) - (-1)^\ell \exp(-i\alpha x) \right] dx, \]

where \( \alpha = (n\beta)^{-1} \), \( \beta = v/c \), \( n = \varepsilon^{1/2} \) (the relative permittivity of the medium), and \( \ell \) is the order of the spherical harmonic \( \theta \) dependence. This integral is zero for \( n\beta < 1 \) and nonzero for \( n\beta > 1 \) regardless of the index \( \ell \). Thus, the threshold occurs in each term of the spherical vector harmonic series. The purpose of this paper is to discuss the Čerenkov threshold phenomenon in the case of a particle executing a circular motion in a homogeneous dielectric medium. Regardless of the velocity, such a particle radiates synchrotron radiation. However, above the velocity of light in the medium some manifestation of a Čerenkov effect should be discernable. The problem is formulated by means of the free-space dyadic Green's function expanded in vector spherical harmonics and appropriately modified to account for a homogeneous dielectric medium. We use this expansion to arrive at a result which explicitly shows the relative significance of each multipole comprising the radiation field. This gives an indication of the angular dependence of the radiation to be expected.
at each harmonic without detailed computation. The result of this formulation is an expression for the radiated energy as a function of the particle velocity and the index of refraction of the medium. The expression takes the form of a series of spherical radiative mode contributions at each harmonic of the orbital frequency of the charged particle. The series is computed for each of several harmonics at a number of values of $n$, where $n$ is the index of refraction of the medium and $\beta$ is the ratio of the particle velocity to the velocity of light in vacuum. Temporal dispersion must be included to yield a finite result. The computational results are plotted and the Cerenkov effect is manifest. Moreover, universal curves are obtained by plotting the total radiated energy multiplied by the index of refraction and divided by the square of the orbital angular velocity and the square of the charge as a function of $n\beta$. Results have been given in a paper (J. Appl. Phys. 45, 5251 (1974)).

9. STOP BANDS FOR OPTICAL WAVE PROPAGATION IN CHOLESTERIC LIQUID CRYSTALS

Cholesteric liquid crystals can be represented by a structure consisting of molecules arranged in thin anisotropic layers, with the successive layers rotated through a small angle, leading to a spiral configuration. This configuration gives unique optical properties that can be used in many optical applications. These crystals have a very strong rotatory factor: $60,000^\circ/mm$ vs $300^\circ/mm$ for ordinary organic liquids. The periodicity of the structures leads to pass-band and stop-band characteristics for waves propagating through such structures; therefore they can provide a wide variety of optical-filtering functions. Furthermore, the fact that the pitch of the spiral configuration is a function of temperature, pressure, added chemicals, etc., may permit numerous other applications. Many authors have studied the propagation of optical waves in cholesteric liquid crystals, using various configurations and various methods of solutions. In this paper, we shall consider the case of an obliquely
incident wave upon a half-space of cholesteric liquids. The Brillouin diagram will be extensively used to obtain the properties of waves in the stop-band regions (relative bandwidth and attenuation factor) as functions of the incident angles. Results have been given in a paper (J. Opt. Soc. Am. 63, 840 (1973)).

10. DISTRIBUTION NETWORKS AND ELECTRICALLY CONTROLLABLE COUPLERS FOR INTEGRATED OPTICS

Recently, a number of researchers have reported the development of thin film and channel waveguide optical couplers for use in the emerging field of integrated optics. Applications of these couplers in optical networks, modulators, and multiplexers/demultiplexers, would be drastically increased if the coupling is dynamically controllable by an electric signal. Electro-optic substrates can be used to control the coupling coefficient between two waveguides, but such a scheme would be inefficient because of the upper limitation on the change of the refractive index of existing materials that could be achieved with reasonable voltage. In this communication, we study the power distributions as a function of the distance from the input plane in a network of N parallel guides. Then we discuss a number of functions that could be achieved using coupled optical waveguides, and we will study in detail a scheme for an electrically controllable coupler. Results are published (Appl. Optics 13, 1372 (1974)).

11. LEAKY WAVES IN A HETEROEPITAXIAL FILM

The development of low-loss fibers in recent years has spurred renewed interest in the study of optical-fiber communication lines as well as the possible construction of optical integrated circuits. One of the very basic configurations for optical integrated circuits is the deposition of a layer of material of thickness 2a and dielectric constant $\varepsilon_2$ on a substrate whose
dielectric constant is \(\varepsilon_3\). Optical waves may be supported along and within this layer. It is well known from electromagnetic theory that in order that this type of structure may support a surface guided wave, the dielectric constant of the three different regions must be such that \(\varepsilon_1 < \varepsilon_2\) and \(\varepsilon_3 < \varepsilon_2\). \(\varepsilon_1\) is the dielectric constant of the region above the thin layer. However, in many important physical situations such as the deposition of ZnS (\(\varepsilon/\varepsilon_0 = 5.48\)) or ZnSe (\(\varepsilon/\varepsilon_0 = 6.66\)) on GaAs (\(\varepsilon/\varepsilon_0 \approx 15\)) the dielectric constant of the substrate is higher than that of the layer. Hence, the ordinary surface guided wave cannot exist in this structure. The purpose of this paper is to investigate the problem of wave propagation along a structure where \(\varepsilon_1 < \varepsilon_2 < \varepsilon_3\).

It will be shown that when the thickness of the layer is large compared with a wavelength of light, low-loss leaky modes may exist. Experimental results verifying the existence of these modes will also be shown. Results have been given in a paper (J. Appl. Phys. 44, 2271 (1973)).
COMPLETE LIST OF REPORTS AND PUBLICATIONS


