DEVELOPMENT OF AN OFF-DESIGN PREDICTIVE
METHOD FOR SUPERCAVITATING PROPELLER PERFORMANCE

TETRA TECH, INCORPORATED, PASADENA, CALIFORNIA

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FINAL REPORT

DEVELOPMENT OF AN OFF-DESIGN PREDICTIVE METHOD FOR SUPERCAVITATING PROPELLER PERFORMANCE

Tetra Tech Contract TC-676
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FINAL REPORT

DEVELOPMENT OF AN OFF-DESIGN PREDICTIVE METHOD FOR
SUPERCAVITATING PROPELLER PERFORMANCE

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ABSTRACT

This is the Final Report, describing all the tasks accomplished in Phases A and B of Contract No. N00600-76-C-0790, including a listing of the computer program developed under the present contract and manual describing input and output data.

In order to incorporate the three-dimensional effect and cascade effect into the performance prediction of the supercavitating propellers, a two-dimensional supercavitating (2-D s/c) cascade theory and a lifting line theory were combined. The force coefficients obtained from the 2-D s/c cascade theory will account for the cascade effect whereas the three-dimensionality is incorporated in terms of effective flow incidence angles at each selected spanwise location of the blade for the 2-D program.

An inherent difficulty in applying the 2-D s/c cascade theory to three-dimensional flows arises due to the existence of the choking condition but was overcome by correcting the effective upstream velocities depending on the cavity thickness. Mathematical formulation combining the 2-D s/c cascade theory and a lifting line theory is described. The method proposed for the cavity thickness correction is explained, followed by numerical procedures to solve the problem.

Numerical results made with the 2-D s/c cascade program for a s/c NSRDC Model 3770 propeller geometry have shown a most significant cavitating cascade effect. These results seem to explain very well the
discrepancy existing between previous experimental data and design data. The propeller characteristics such as thrust, torque coefficients and efficiency have been calculated and compared with experimental data, having provided a good correlation over a supercavitating range of speed coefficient, J.

However, for J's beyond the above range, the present results quickly deviate from the experimental data because a part of the propeller near the hub is at partially cavitated condition to which the present theory is not applicable.
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NOMENCLATURE

\[a, b, c = \xi\text{-coordinates in two-dimensional cascade problem}\]
\[\tilde{A} = \text{scale factor of cascade mapping function}\]
\[c = \text{chord length of blade}\]
\[C_p = \text{power coefficient } (= \frac{P}{\frac{1}{2}\alpha V_a^3 \pi R^2})\]
\[C_T = \text{thrust coefficient } (= \frac{T}{\frac{1}{2}\rho V_a^2 \pi R^2})\]
\[d = \text{spacing between two blades}\]
\[D = \text{propeller diameter } (= 2R)\]
\[g = \text{number of blades}\]
\[G = \text{normalized circulation } (= \Gamma / 2\pi RV_a)\]
\[i_a, i_t = \text{induction factors for the axial and tangential induced velocities } w_a \text{ and } w_t\]
\[J = \text{speed coefficient } (V_a \pi / \omega R = \pi \lambda)\]
\[n = \text{propeller rotational speed}\]
\[P_l, P_c = \text{static pressures at upstream uniform flow and inside the cavity}\]
\[P = \text{power}\]
\[Q = \text{torque}\]
\[r = \text{radial position from the propeller axis}\]
\[r_h = \text{propeller hub radius}\]
\[R = \text{propeller radius}\]
\[s_{ol} = \text{arc length on the blade measured from the cavity separation point}\]
\[= \text{solidity } (= c/d)\]
\[S = \text{total wetted arc length of the blade}\]
\[T = \text{thrust}\]
\( U_1 \) = velocity at upstream infinity with downwash correction 
(but not including cavity correction) \( (= V_e + V_c) \)

\( U_2 \) = velocity at downstream infinity 

\( U_\infty \) = geometric mean velocity 

\( V \) = relative velocity to the blade \( (= ((\omega r)^2 + V_a)^{\frac{1}{2}}) \)

\( V_a \) = advance speed (axial flow speed) 

\( V_c \) = induced velocity due to the cavity thickness effect 

\( V_e \) = effective velocity including induced velocities 
\( (= \left\{ (\omega r - w_t)^2 + (w_a + V_a)^2 \right\}^{\frac{1}{2}} - V_c) \)

\( w_a, w_t \) = induced velocities in the axial and tangential directions, respectively 

\( x \) = normalized radial position \( (= r/R) \)

\( \alpha_2 \) = deflected flow angle referred to the nose-tail line 

\( \alpha_i \) = induced flow angle 

\( \alpha_e \) = effective incidence angle 

\( \alpha_g \) = geometric incidence angle 

\( \alpha_m \) = geometric mean flow angle 

\( \beta \) = pitch angle \( (= \tan^{-1} \frac{V_a}{\omega r} = \tan^{-1} \frac{J}{\tau x}) \)

\( \beta_i \) = pitch angle including downwash effect \( (= \tan^{-1} (w_a + V_a)/(\omega r - w_t)) \)

\( \bar{\beta} \) = local slope of blade 

\( \gamma \) = geometric stagger angle 

\( \Gamma \) = circulation 

\( \delta \) = stagger angle in potential plane \( (= \alpha_e + \gamma) \)

\( \zeta \) = transform potential plane 

\( \eta \) = propeller efficiency \( (= C_T/C_p) \)
\[ \lambda = \text{advance coefficient (} = \frac{V_a}{uR}) \]
\[ \zeta = \text{real axis of the } \zeta \text{-plane} \]
\[ \rho = \text{density of fluid} \]
\[ \sigma_e = \text{local cavitation number (} = \frac{p_1 - p_c}{\frac{1}{2} \rho V_e^2}) \]
\[ \omega = \text{angular velocity of propeller} \]
1. INTRODUCTION

The development of a prediction method for supercavitating propeller performance at off-design conditions is a difficult task due to an additional complexity of the cavity flow to that of three-dimensional propeller configurations.

Unlike the conventional propellers used at relatively low ship speeds, supercavitating propellers are expected to have strong cascade effects. The existence of blade cavities causes blocking or choking effects on the flow passages as the extent of cavity becomes large both in length and thickness. Some propeller designers already pointed out the importance of the cascade effect in s/c propeller design in their earlier papers such as [1] *

A similar effect was also found important even for subcavitating propellers: the paper by Kerwin and Leopold [2] showed that large incidence angle corrections are necessary due to blade thickness effect even if the thickness is small. The correction becomes particularly significant as the thickness ratio to the blade spacing becomes high, i.e., near the hub. This is considered to be exactly the same blocking effect as that for the cavity flow. It is now evident that the cascade effect of blocking effect must be correctly incorporated into the performance prediction of s/c propellers.

*Number in brackets designates Reference at end of paper
Although linearized s/c cascade theories in [3] have long existed, such theories are unable to accurately predict hydrodynamic characteristics of these highly nonlinear s/c cascade flows. It was not until just recently that a fully nonlinear 2-D s/c cascade theory [4] was developed, greatly facilitating the calculation of these cascade effects and providing a powerful engineering tool.

The method of solving the present problem is a combination of the 2-D s/c cascade theory with a lifting line theory. The procedure to be used is stated as follows. Specifying all physical and geometric conditions of the s/c propeller, 2-D solutions at several radial or spanwise locations of blades will be obtained. A difficult question however arises as to what effective flow incidence angles, $\alpha_e$, must be used for the 2-D analysis. The downwash effect in propeller flows are usually so strong that the geometric flow incidence angles, $\alpha_g$ (see Figure 1 for definition and also Table C2 for actual values of $\alpha_g$ for the 3770 supercavitating propeller), are completely different from the effective incidence angles, $\alpha_e$.

The present propeller problem is very much similar to that of a single airfoil of finite span for which an integral equation of a lifting line theory must be solved. The result determines $\Gamma$, the distribution of circulation, or equivalently lift over the blade span so as to provide a right amount of downwash effect everywhere for generating the above circulation, $\Gamma$. The evaluation of the downwash angle, $\alpha_d$, in this case is most simply made by a propeller lifting line theory but in a somewhat complicated form. The effective angle of flow incidence, $\alpha_e$, is then obtained by
subtracting $\alpha_i$ from $\alpha_e$. Applying this $\alpha_e$ to the lift curves calculated by the 2-D s/c theory, we can determine the circulation distribution, eventually ending up with an integral equation for $\alpha_e$ with the span location as a variable.

A different type of difficulty arises when applying a two-dimensional flow approach to a three-dimensional flow, although this type of approach has been well adopted for subcavitating propeller design. Contrary to the supercavitating propeller problem, the same method for subcavitating propellers creates no serious problems in determining the forces at any blade location for any given effective incidence angle, $\alpha_e$, since the lift and drag forces used for subcavitating propellers are continuous function of $\alpha_e$. However, in the present problem, due to the choking condition the force curves obtained by the 2-D s/c cascade theory are discontinued right at that point (see Figures 7(a) to (f) for choking conditions on the lift curves). The physical meaning of this is explained as follows. The cavity length and thickness increase as the incidence angle increases, and finally the cavity extends to downstream infinity with a maximum cavity thickness. This blocks or chokes the flow path of cascade. It therefore becomes impossible to increase the total mass flow going through a cascade beyond that point at the choking condition.

This type of 2-D choking condition never occurs in the three-dimensional (3-D) flow configuration even if the flow cavitates and locally chokes.

Consider a cascade of blades having finite span length. The 3-D cascade
can have a similar choking condition locally, but the amount of flow we can push from the upstream infinity with any incidence angle is unlimited since any mass flow in excess of that going through the cascade can go around the corners of cascade in the direction of span. Thus, the 'effective' flow velocity going through the propeller remains almost constant at each blade radial position. The terminal values depend on the cavity thickness but not depend on the upstream velocity. As long as the cascade span is finite this phenomenon holds true. However, once the span extends to infinity, going back to a totally two-dimensional configuration, the inherent problem mentioned above arises.

As a first step for resolving the present difficult situation, we use a simple, intuitive method with the above physical picture of 3-D cavity flow in mind. The upstream velocity is corrected at each spanwise location by distributing line sources in cascade configuration whose strengths are determined based on the cavity thickness. The effective velocity obtained with this method is always smaller than that of the original flow so that the cavitation number to be used for the 2-D analysis becomes larger thus being able to avoid the choking condition. It must be pointed out that the correction here is not on the incidence angle as downwash correction but on the upstream velocity or equivalently the cavitation number.

In this report, we present a mathematical formulation which combines the 2-D s/c cascade theory with a lifting line theory and a method for correcting the cavity choking effect, followed by numerical procedures.
to solve the problem. Computed 2-D s/c cascade results for a chosen NSRDC Model 3770 propeller geometry are then presented, showing a remarkable cascade effect. With these 2-D s/c force coefficients used, the propeller performance was calculated and compared with the experimental data [5].

The results correlate well in the supercavitating regions but quickly deviate as the speed coefficient, $J$, becomes larger due to the appearance of partial cavitation near the hub. This discrepancy is naturally expected since the present theory is only applicable to the fully supercavitating propellers.

Originally, the present work had been planned to incorporate the results of the above computations into a lifting surface theory (see Reference [6] for the detailed procedure of this method). However, it has been found that the present method combining the 2-D s/c cascade theory, lifting line theory and cavity thickness correction provides accurate results correlating well with existing experimental data. We describe several theoretical backgrounds why the method accounts for all supercavitating propeller characteristics as follows:

i) By having used the results of 2-D s/c cascade theory, we have accounted for a most important effect of supercavitating propeller flows, i.e. the existence of the cavity in cascade
geometry. The forces on the s/c cascade used as the basis for propeller calculations corrected by a lifting line theory are found to be much smaller than those of a single foil due to the influence of the low pressure region of the cavity on the pressure side of an adjacent blade. In addition, the blocking effect due to the cavity has also been incorporated in terms of a cavitation number correction.

ii) The finite aspect ratio correction to s/c propeller blades is usually smaller than that on fully wetted propeller blades (see [8]) and also limited by the choking condition. Therefore the lifting line theory used above is considered fairly accurate as this has been proven by a good correlation with experimental data.

iii) Although a lifting surface theory will give all boundary value corrections including both the wetted portion of the blade and cavity streamlines as mentioned in [6], no information about the correction for upstream flow velocity is obtained by the present method, it has been proven that a correction for cavitation number
is a most important feature in applying a 2-D s/c cascade theory to calculations of the propeller performance.

iv) For those cases in which the cavitation number, $\sigma$, is close to that of the choking condition, the 2-D s/c cascade theory itself fails to converge in the numerical iterative procedure as will be explained later. For such $\sigma$'s, the 2-D theory may not provide a convergent solution for new boundary values set by a lifting surface theory. In the present approach, however, this difficulty is overcome by an interpolation scheme as will be seen later.

Consequently, we believe that the present approach accurately accounts for all the hydrodynamic effects of supercavitating propellers which a lifting surface theory will provide and furthermore that the former is superior to the latter from the viewpoints of simplicity in concept and economy in computation.
2. MATHEMATICAL FORMULATIONS

A two-dimensional supercavitating cascade hydrodynamic problem has recently been solved by using the hodograph variables to satisfy the exact boundary conditions. In this method the blade and cascade geometry, the upstream flow conditions and the cavitation number are specified. A system of five nonlinear functional equations involving five unknown solution parameters was formulated and solved numerically using a functional iterative method combined with Newton's method. The details of the theory and numerical method are described in [4].

In order to incorporate the two-dimensional (2-D) cascade theory into the analysis of supercavitating (s/c) propellers, the effective angle of incidence, $\alpha_e$ (see Figure 1), must be determined. The geometric flow incidence $\alpha_g$, which is determined by the propeller blade pitch, rotational speed $\omega$, and axial flow speed $V_a$, is typically much larger than $\alpha_e$ due to the strong downwash effects generated by the propeller helical vortex sheets. For example, in some cases of s/c propellers, the downwash angle $\alpha_i = \alpha_g - \alpha_e$ can be as high as ten degrees although the effective angle of incidence is only four degrees. It has become clear that neglecting the downwash in two-dimensional cascade calculations can result in a solution far from the actual propeller flow situation. One of the ideas in capturing the three-dimensional effect is to incorporate vortex singularities into a lifting line theory.
With the geometry of cascade and propeller blades, \( \omega^* \) and \( V_a \) specified, the 2-D cascade problem can be solved if \( \alpha_e \) and effective upstream velocity, \( V_e \), are assumed known at each radial station \( r \) since \( \alpha_e \), \( \beta_i \) and \( \sigma \) are obtained:

\[
\alpha_i = \alpha_g - \alpha_e \\
\beta_i = \beta + \alpha_i \tag{2}
\]

\[
\sigma = \frac{p_1 - p_e}{\frac{1}{2} \rho V_e^2} \tag{3}
\]

Five equations in the 2-D problem are now rewritten:

\[
f_1 = \text{Re} \left\{ \omega(\zeta_1) \right\} - \alpha_e = 0 \quad \text{(Upstream flow angle condition)} \tag{4}
\]

\[
f_2 = \text{Im} \left\{ \omega(\zeta_1) \right\} + 2n U_2 = 0 \quad \text{(Upstream flow velocity condition)} \tag{5}
\]

\[f_3 = \gamma_3 - \alpha_2 = 0 \quad \text{(Downstream flow angle condition)} \tag{6}\]

\[f_4 = s(1) - S = 0 \quad \text{(Scaling between the physical and transform planes)} \tag{7}\]

\[f_5 = \gamma_5 - d \left( \sin(\alpha_e + \gamma) - U_2 \sin(\alpha_2 + \gamma) \right) = 0 \quad \text{(Continuity equation)} \tag{8}\]

*See Nomenclature and also the Blade Definition Figure 1 for the definition of each symbols.
where explicit expressions for \( \text{Re } w(\zeta_1) \), \( \text{Im } w(\zeta_1) \), \( g_3 \), \( s(-1) \) and \( g_5 \) are given in Appendix A (and also see 4). It is noted that in equations (4) thru (8) \( a_\epsilon \)'s simply replace \( a_1 \) in equations (7), (8), (9), (15) and (16) in 4. It must be mentioned that the upstream velocity, \( V_e \), used for these 2-D calculations is different from the velocity simply composed of the axial flow, \( V_a \), and the rotational velocity, \( \omega r \), as is shown in Figure 1. Due to the three-dimensional downwash effect and the cavity blocking effect, \( V_e \) is given by a following equation:

\[
V_e = U_1 - V_c
\]  

(9)

where

\[
U_1 = \left[ (\omega r - w_t)^2 + (V_a + w_a)^2 \right]^\frac{1}{2}
\]

\( w_a, w_t \) = propeller induced flow velocities in the axial and tangential directions

\( V_c \) = retarding flow velocity due to the cavity blocking effect.

Before describing the methods of determining \( w_a, w_t \) and \( V_c \), first look at how to obtain the circulation, \( \Gamma \), from the above two-dimensional calculations, which will be used in a lifting-line theory. Taking the control volume designated by ABCD shown in Figure 2, the differences in potential between the points A and D, and B and C are calculated respectively by

\[
\Delta \Phi_{DA} = V_e \, d \sin (\gamma + \alpha_e)
\]

\[
\Delta \Phi_{CB} = U_2 \, d \sin (\gamma + \alpha_2)
\]
thus the net change of the potential in this control volume, that is, 
\( \Gamma(x) \), is given by

\[
\Gamma(x) = \Delta \Phi_{DA} - \Delta \Phi_{CB} = V_e \, d \sin (\gamma + \alpha_e) - U_2 \, d \sin (\gamma + \alpha_2)
\]

(10)

where \( x \equiv r/R \).

This formula holds both for the finite and infinite cavity cascade flows, but for the former case a simpler form is obtained by using a continuity equation between the upstream and downstream flows, i.e.

\[
V_e \, d \cos (\gamma + \alpha_e) = U_2 \, d \cos (\gamma + \alpha_2);
\]

\[
\Gamma(x) = V_e \, \frac{\sin(\alpha_e - \alpha_2)}{\cos(\gamma + \alpha_2)}, \text{ for finite cavity flows.}
\]

(11)

\( \Gamma(x) \) calculated in Equation (10) or (11) connects the 2-D results with a three-dimensional lifting-line theory to find the propeller induced velocities.

The induced velocities in the axial and tangential directions \( w_a \) and \( w_t \), for the case where the blades extend from the hub at \( r = r_h \) to the tip \( r = R \), are obtained (see [7] for detailed derivations) from:

\[
\frac{w_a}{V_a} = \frac{1}{2} \int_{x_h}^{1} \frac{dG(x')}{dx'} \frac{1}{x-x'} \, i_a(\beta_i) \, dx'
\]

(12)
\[
\frac{w_t}{V_a} = \frac{1}{2} \int_{x_h}^{1} \frac{dG(x')}{dx'} \frac{1}{x-x'} i_t(\beta_i)dx' \tag{13}
\]

where \(G(x)\) is a normalized circulation:

\[
G(x) = \frac{\Gamma(x)}{2\piRV_a} \tag{14}
\]

and

\[
x_h = \frac{r_h}{R}. \tag{15}
\]

\(i_a\) and \(i_t\) are the induction factors obtained by Lerb [7] and detailed expressions are found in Appendix B. It must be mentioned that for s/c propellers, the propeller loading is expected to be moderate to heavy, thus the downwash effects (12) and (13) must be evaluated by taking into account the deflection of the vortex sheet location behind the bound vorticities. Lerb [7] showed from a discussion of energy balance BETWEEN the propeller disk and the ultimate wake that the location of vortex sheets should be on a helical surface having an angle \(\beta_i\) (instead of \(\beta\)), which is a function of \(r\). In the present calculations of \(w_a\) and \(w_t\), we use \(\beta_i\) to evaluate \(i_a\) and \(i_t\). The downwash angle \(\alpha_i\) is then obtained from the following equation to an accuracy of first order in \(\beta_i\):

\[
\beta_i = \tan^{-1} \left( \frac{1 + w_a/V_a}{\pi x J - w_t/V_a} \right)
\]

\[
\alpha_i = \beta_i - \beta. \tag{16}
\]
It has now come to a point of how we incorporate the choking or cavity thickness effect into the problem, which relates to determining $V_c$ in Figure 1. The physical picture of the choking phenomena in the 2-D and 3-D flows has already been discussed in the Introduction of this report. A rigorous treatment of this type of problem will require an enormous effort involving complicated mathematics, although it must be done some time in the near future. Meanwhile, we use a somewhat more intuitive method as a first step to avoid an inherent difficulty in applying the results of 2-D s/c cascade flow to the propeller problem.

In order to represent the cavity thickness, a row of source singularities of strength $m$ are placed with a distance $d$ in a uniform flow, the velocity of which is $U_1$ with a stagger angle, $\gamma + \alpha_e$, as depicted in Figure 3. The velocity potential of the flow is given by

$$W = U_1 Z e^{-i(\gamma + \alpha_e)} + m \ln \left\{ \sinh \left( \frac{\pi Z}{d} \right) \right\}, \quad (17)$$

thus the velocity potential is obtained:

$$\frac{dW}{dz} = U_1 e^{-1(\gamma + \alpha_e)} + \frac{mn}{d} / \tanh (\pi z/d). \quad (18)$$

As $x \to \pm \infty$, the $x$ - component of the velocity changes by $\pm mn/d$, respectively. If we know the thickness of the cavity, $d \cdot e$, the strength of source, $m$, is calculated by using the continuity equation
\[
\left\{ U_1 \cos(\gamma + \alpha_e) - \frac{mn}{d} \right\} d = \left\{ U_2 \cos(\gamma + \alpha_2) + \frac{mn}{d} \right\} (d - de)
\]

or
\[
\frac{mn}{d} = \frac{U_1 \cos(\gamma + \alpha_e) - U_2 \cos(\gamma + \alpha_2)(1 - e)}{2 - e}
\]

(19)

It means that although the mass flow, \( U_1 \cos(\gamma + \alpha_e) \cdot d \) per blade from the upstream infinity, comes into the cascade, the amount of \( mn \) is rejected to go through the blade passage due to the existence of cavity.

The rejected mass flow, \( mn \), should go normal to the paper plane, in reality, in the radial direction of the propeller. Therefore \( V_c \) is calculated from (19) by taking the component in the \( U_1 \) direction:

\[
V_c = \frac{mn}{d} \frac{1}{\cos(\gamma + \alpha_e)}
\]

(20)

The effective upstream flow velocity to be used in the 2-D analysis is now obtained

\[
V_e = U_1 - V_c
\]
\[
= \left\{ U_1 + U_2 \frac{\cos(\gamma + \alpha_2)}{\cos(\gamma + \alpha_e)} \right\} \cdot \frac{1 - e}{2 - e}
\]

(21)

where \( e \) is a function of \( \sigma \) and \( \alpha_e \), obtained from the results of the 2-D computations. Strictly speaking, the present method is only valid for the infinite cavity flow cases in which the cavity is fully developed. However, even for the finite cavity cases it is considered that the same cavity blockage evaluation holds true by taking the cavity thickness at the end
points of cavity as $e$ in Equations (19) and (21).

It must be noted here that the correction of the upstream velocity by (21) changes the cavitation number, $\sigma_e$, for which $e$ is obtained at $\alpha_e$. It needs an iterative scheme to satisfy the relationship in Equation (21) by starting with $e$ for $\sigma(U_1)$ and $\sigma_e$ as an initial step and then finding a new $e$ for a new $\sigma(V_e)$ where $V_e$ is just obtained from (21). It has been found in the actual computations that the convergence of the iteration is rather fast.

The problem to be solved is now fully defined. With the propeller geometry, $V_a$ and $\omega$ specified, one can determine a circulation distribution, $\Gamma(x)$, in such a way that the free vortex sheets associated with the $\Gamma$ distribution generate a correct amount of downwash velocity to have a sectional blade lift equal to $\rho U_\infty \Gamma$ where $U_\infty$ is the geometric mean velocity of the upstream and downstream velocities (see Figure 4).

It is immediately seen that the problem is completely nonlinear including integral equations and thus cannot be solved explicitly. Two numerical iterative methods are proposed to solve this type of situation and both procedures used here will be explained in the following section.
3. NUMERICAL PROCEDURES

Two numerical methods for solving the above nonlinear integral equations are proposed and have been tested in actual computations for their convergence. The first method is what is called a substitutional iterative method and the second one is Newton's iterative method similar to that used in the problem of three-dimensional supercavitating hydrofoils [8].

3.1 Substitutional Iterative Method

This method exactly follows the steps of the mathematical formulation, the flow chart being shown in Figure 5.

Assuming the effective incidence angles \( \alpha_e^{(n)}(x) \), \( n=0 \), at each spanwise location one can find downwash angles \( \alpha_1^{(n)}(x) \) and a cavitation number \( \sigma_0(x) \) from equations (1) and (3). The solutions of the two-dimensional s/c cascade problem provide \( \alpha_2 \), the deflected flow angle at downstream infinity. In actual computations it is convenient to establish a functional relationship of \( \alpha_2 \) as a function of \( \alpha_e \) and \( \sigma \) at each blade section. Since \( \alpha_2 \) is a smooth function of \( \alpha_e \) and \( \sigma \), the 2-D calculations for several values of \( \alpha_e \) and \( \sigma \)'s will be sufficient to represent \( \alpha_2 \) by functionally establishing the results at discrete points. By doing this one can save a considerable amount of computer time since the 2-D computations are the most time consuming part of the calculation. If this relation is not established at the beginning of the computation procedure, the 2-D program must be run for each iterative loop. This can
be seen in the flow chart, Figure 5, where the returned loop will now
go back to the 2-D calculation box instead of the $\alpha_2$-box. In some cases
in which the stagger angle and blade solidity are large, the 2-D s/c
cascade program becomes numerically unstable as was reported in [4].
This problem, however, is overcome if the 2-D features are completely
calculated at the initial stage of the numerical procedure.

The induced velocity, $V_e$, due to the existence of cavity and thus the
effective flow velocity, $V_e$, are obtained by a small iterative procedure
in Equation (21). The sectional circulation distribution $\Gamma(x)$ is then ob-
tained by Equation (10) or (11), thereby enabling us to calculate $w_a, w_t$
and $\alpha_i(n+1)(x)$. The values of $\alpha_i(o)(x)$ first assumed are now checked to
determine that they are corrected. If not, with a new $\alpha_i(n+1)(x)$ and
$\sigma_i(n+1)$, we proceed to the next iteration until a convergent solution is
obtained. In each iteration, $\beta_i(n)$, starting with an assumed value
$\left(\beta_i(o) = \beta + \alpha_i(o)\right)$, must be calculated and a new value of $\beta_i(n)$ must be used
in calculations of $w_a$ and $w_t$. It must also be noted that the cavitation
number $\sigma$ based on $V$ is used for the first iteration but $\sigma_e$ based on $V_e$
is used from the second iteration on.

If the test for the convergence of solution parameters, for example, $\alpha_i$, is passed, we proceed to calculate the propeller characteristics such
as thrust, power coefficients and efficiency.

When the method was applied to the present problem, we found that con-
verged solutions were obtained only if assumed starting values of $\alpha_e(o)$
were close to the actual solutions. It is for this reason that a second method using Newton's technique is proposed for seeking a better convergence.

3.2 Newton's Iterative Method

We incorporate Newton's method into the nonlinear integral equations for improving the convergence of iteration. This requires a new arrangement of the problem in order to identify the solution parameters.

From Equations (16), (B-16) and (B-17),

\[
f = \tan \left( \beta_g(x) - \alpha_e(x) \right) \left( \frac{\pi}{f} - \frac{1}{1 - x_h} \sum_{m=1}^{k} m G_m h_m^a (\varphi(x)) \right)
- \left\{ 1 + \frac{1}{1 - x_h} \sum_{m=1}^{k} m G_m h_m^t (\psi(x)) \right\} = 0
\]  

(22)

and from Equations (14) and (B-11),

\[
g = \sum_{m=1}^{k} G_m \sin m \varphi(x) - \frac{\Gamma(x, \alpha_e(x), \sigma_e)}{2\pi RV_a} = 0
\]  

(23)

Choosing discrete control points in the radial direction of the blade for which the computations will be made, say \( x = 0.4 \) to 0.9 by 0.1 increment, we have six independent equations in (22) so that the same number of \( G_k \)'s are chosen for the solution parameters, in this case \( k = 6 \). Since all other quantities in equations (22) and (23) are known except for \( \alpha_e(x) \)'s, they are naturally chosen as another six solution parameters,
called \( \alpha_{ek} \). We now have 2k solution parameters for a system of nonlinear integral equations having an order of 2k.

Rewriting these equations and parameters symbolically by

\[
\bar{F} = (f, g)
\]

\[
\bar{x} = (G_k, \alpha_{ek})
\]

we can describe the above set of equations as follows:

\[
\bar{F}(\bar{x}) = 0
\]

thus Newton's iterative loop is established by

\[
\bar{J}(\bar{x}^{(n)}) \cdot (\bar{x}^{(n+1)} - \bar{x}^{(n)}) = -\bar{F}(\bar{x}^{(n)})
\]

(24)

where \( \bar{J} \) is a Jacobian matrix whose component is given by

\[
\bar{J} = \frac{\partial \bar{F}_i}{\partial x_j}.
\]

(25)

In the present case each component of \( \bar{J} \) is either analytically or numerically calculated:

\[
\frac{\partial \bar{F}_i}{\partial x_j} = \begin{cases}
\frac{\partial f_i}{\partial G_j} = \tan(\beta g(x_i) - \alpha_{ei}) \left\{ - j h_j^a(\varphi(x_i)) \right\} / (1 - x_k) \\
eg j h_j^t(\varphi(x_i)) / (1 - x_n) : i = 1 \sim 6, j = 1 \sim 6
\end{cases}
\]

(26)
\[
\begin{align*}
\frac{\partial f_i}{\partial \alpha_{ek}} &= -A(x_1) \delta_{ik}/\cos^2(\beta g(x_1) - \alpha_{ek}) ; i = 1 \sim 6, \\
j &= 7 \sim 12, \ k = j - 6 \\
\frac{\partial F_i}{\partial x_j} &= \left\{ \begin{array}{ll}
\frac{\partial g_k}{\partial \alpha_{ij}} &= \sin \{j \varphi(x_k)\} ; i = 7 \sim 12, \ j = 1 \sim 6, \ k = i - 6 \\
\frac{\partial g_k}{\partial \alpha_{ek}} &= -\frac{\delta_{k\ell}}{2\pi R |V_a|} \cdot \frac{\partial}{\partial \alpha_{e\ell}} \Gamma(x_k, \sigma_{ek}, \sigma_e) ; i = 7 \sim 12, \\
j &= 7 \sim 12, \ k = i - 6, \ \ell = j - 6
\end{array} \right. 
\end{align*}
\] (27, 28, 29)

where all partial derivatives are analytically calculated except for \( \Gamma \)
for which a finite difference method is used.

Iterative numerical procedures for this case shown in Figure 6 are very
similar to those of the first method shown in Figure 5. Our experience
in using this Newton's method for the present problem indicated rather
slow but steady convergences for almost all cases. It has also been
found that the method is much less sensitive to the initial starting values
of solution parameters.
4. CALCULATIONS OF THRUST, TORQUE COEFFICIENTS AND EFFICIENCY

In the cascade flow the lift force acting on the blades is known to be normal to a geometric mean angle $\alpha_g$ (see References [4] and [9]) which is depicted in Figure 4:

$$\alpha_g = \cos^{-1} \left( \frac{1}{2U_g} \left( V_e \cos \alpha_e + U_2 \cos \alpha_2 \right) \right)$$  \hspace{1cm} (30)

where

$$U_g = \frac{1}{2} \left( V_e^2 + U_2^2 + 2V_e U_2 \cos (\alpha_e - \alpha_2) \right)^{\frac{1}{2}}$$

or

$$\frac{U_g}{V_a} = \frac{1}{2} \left( \frac{V_e}{V_a} \right) \left( \frac{U_2}{V_e} \right)^2 + 2 \frac{U_2}{V_e} \cos (\alpha_e - \alpha_2)^{\frac{1}{2}}.$$  \hspace{1cm} (31)

$V_e$ is taken to be unity in the 2-D calculations and from Figure 1 $V_e/V_a$ is calculated from

$$\frac{U_1}{V_a} = \frac{V_e + V_c}{V_a} = \left( \cot \beta - \frac{w_t}{V_a} \right) \frac{1}{\cos \beta_i}.$$  \hspace{1cm} (32)

Thus, a sectional thrust is obtained:

$$dT = g \left\{ \rho U_{\infty} \Gamma \cos (\beta_i + \alpha_e - \alpha_\infty) - (D_\infty + D_f) \sin (\beta_i + \alpha_e - \alpha_\infty) \right\} \, dr$$

where $D_\infty$ and $D_f$ are pressure drag on the cavitating blade parallel to the direction of $U_{\infty}$ and friction drag on the propeller blade:

$$D_\infty = C_{D \infty} \cdot c \, \frac{1}{2} \rho U_{\infty}^2$$

$$D_f = C_f \cdot c \, \frac{1}{2} \rho U_{\infty}^2$$

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\[ T = g \int_{r_h}^{R} \left\{ \rho U_a \Gamma \cos \left( \beta_i + \alpha_e - \alpha_\infty \right) - (D_\infty + D_f) \sin \left( \beta_i + \alpha_e - \alpha_\infty \right) \right\} dr' \]

\[ = g \frac{2\pi R^2}{2} \rho \frac{V_a^2}{a} \int_{x_h}^{1} \left\{ \frac{U_\infty}{V_a} (x') G(x') \cos \left( \beta_i + \alpha_e - \alpha_\infty \right) \right\} dx' \]

\[ - \frac{1}{4\pi} \frac{c(x')}{R} \left( \frac{U_\infty}{V_a} \right)^2 (C_{D\infty} + C_f) \sin \left( \beta_i + \alpha_e - \alpha_\infty \right) dx' \quad (33) \]

The thrust coefficient \( C_T \) is obtained by normalization:

\[ C_T = \frac{T}{\frac{1}{2} \rho V_a^2 \pi R^2} = 4g \int_{x_h}^{1} \left\{ \frac{U_\infty(x')}{V_a} G(x') \cos \left( \beta_i + \alpha_e - \alpha_\infty \right) \right\} dx' \]

\[ - \frac{1}{4\pi} \frac{c(x')}{R} \left( \frac{U_\infty(x')}{V_a} \right)^2 (C_{D\infty} + C_f) x \sin \left( \beta_i + \alpha_e - \alpha_\infty \right) dx' \quad (34) \]

Similarly the power coefficient \( C_p \) is calculated as:

\[ dP = rw \, dF \, dr \]

\[ = rwg \left\{ \rho U_a \Gamma \sin \left( \beta_i + \alpha_e - \alpha_\infty \right) + (D_\infty + D_f) \cos \left( \beta_i + \alpha_e - \alpha_\infty \right) \right\} dr \]

or

\[ P = g \frac{2\pi w R^3}{3} \frac{V_a^2}{a} \int_{x_h}^{1} \left\{ \frac{U_\infty(x')}{V_a} G(x') \sin \left( \beta_i + \alpha_e - \alpha_\infty \right) \right\} dx' \]

\[ + \frac{1}{4\pi} \frac{c(x')}{R} \left( \frac{U_\infty(x')}{V_a} \right)^2 (C_{D\infty} + C_f) \cos \left( \beta_i + \alpha_e - \alpha_\infty \right) dx' \quad (35) \]

and

\[ C_p = \frac{P}{\frac{1}{2} \rho V_a^3 \pi R^2} = 4g \int_{x_h}^{1} \left\{ \frac{U_\infty(x')}{V_a} G(x') \sin \left( \beta_i + \alpha_e - \alpha_\infty \right) \right\} dx' \]

\[ + \frac{1}{4\pi} \frac{c(x')}{R} \left( \frac{U_\infty(x')}{V_a} \right)^2 (C_{D\infty} + C_f) x \cos \left( \beta_i + \alpha_e - \alpha_\infty \right) dx' \quad (36) \]
where

\[ \lambda = \frac{V_a}{\omega R} . \]

It must be emphasized that \( U_\infty \) is calculated as a nondimensional number in the 2-D cascade theory problem, referring it to \( V_e \). However, \( U_\infty \) in Equations (33) to (36), need to be absolute values. They must be multiplied by \( V_e \) in these calculations. Similarly, \( \Gamma \) calculated in the 2-D problem by equation (10) must use 'd' which has a dimension since again in 2-D calculations \( d \) is normalized by the chord length \( c \). The propeller efficiency \( \eta \) is finally calculated as:

\[ \eta = \frac{C_T}{C_p} . \]  

Another definition of thrust and torque coefficients, using symbols \( K_T \) and \( K_Q \), is given by

\[ K_T = \frac{T}{\rho n^2 D^4} \]  
\[ K_Q = \frac{Q}{\rho n^2 D^5} , \]

thus the relations between these numbers and \( C_T \) and \( C_p \) are obtained

\[ K_T = \frac{\pi n^2}{8} C_T \]  
\[ K_Q = \frac{J^3}{16} C_p \]

and

\[ \eta = \frac{K_T}{K_Q} \frac{J}{2\pi} . \]
An alternative way to obtain these coefficients is that instead of using $\Gamma$ and $D_\infty$, we can directly use $C_L$ and $C_D$ which are normal and parallel to the direction of the upstream velocity, $V_e$. $C_T$ and $C_p$ are now expressed by the following formulae:

\[
C_T = 2 \int_{x_h}^{1} \text{sol}(x') \left( \frac{V_e(x')}{V_a} \right)^2 x' \left\{ C_L \cos \beta_{i} - C_D \sin \beta_{i} - C_f \sin \beta_{g} \right\} dx' \tag{43}
\]

\[
C_p = \frac{2}{\lambda} \int_{x_h}^{1} \text{sol}(x') \left( \frac{V_e(x')}{V_a} \right)^2 x' \left\{ C_L \sin \beta_{i} + C_D \cos \beta_{i} + C_f \cos \beta_{g} \right\} dx' \tag{44}
\]

where $\text{sol}(x')$ is a solidity of the blade at $x$. 
5. NUMERICAL RESULTS OF THE TWO-DIMENSIONAL SUPERCAVITATING CASCADE

As shown in the flow charts of Figures 4 and 5 the first step in the numerical procedure of the present method is the calculation of the two-dimensional hydrodynamic characteristics of supercavitating cascade.

It is intended that results of trial computation will be compared with existing experimental data [5] and those being currently obtained at David Taylor Naval Ship Research and Development Center (DWTNSRDC). The geometry of a supercavitating propeller Model 3770 designed on the basis of the method developed by DWTNSRDC [1] has therefore been chosen. The profiles of the blades were designed based on a Tulin-Burkart two-term camber with an additional camber to account for a lifting surface correction [10]. Appendix C describes the equations of the two-term camber with some representative coordinates and K, correction factors, including other hydrodynamic design and geometric parameters.

In order to cover a complete matrix of possible effective incidence angles \( \alpha_e \) and local cavitation numbers \( \sigma_e \) with \( J \) in the 2-D computations, the following procedure is used.

The design cavitation number of Model 3770, based on the ship speed \( V_a \), is chosen to be 0.617. First of all, the local cavitation number \( \sigma_V \) based on \( V = \left( (\omega r)^2 + V_a^2 \right)^{\frac{1}{2}} \) can be calculated at each radial location:
\[
\sigma_v = \frac{P_1 - P_c}{\frac{1}{2} \rho V^2} = \sigma_{va} \left\{ 1 + \left( \frac{\pi x}{J} \right)^2 \right\}^{-1}.
\]

(45)

\(\sigma_v\)’s calculated this way are listed in Table C2 of Appendix C.

Strictly speaking, however, the cavitation number to be used for 2-D calculations must be based on the effective upstream flow velocity \(V_e\) so that

\[
\sigma_e = \frac{P - P_c}{\frac{1}{2} \rho V_e^2} = \frac{P - P_c}{\frac{1}{2} \rho V_a^2} \left( \frac{V_a}{U_1} \right)^2 \left( \frac{U_1}{V_e} \right)^2 = \sigma_{va} \left( \frac{V_a}{U_1} \right)^2 \left( 1 + \frac{V_c}{V_e} \right)^2
\]

where \(V_a/U_1\) and \(V_c/V_e\) are calculated from Equations (32) and (21), respectively. As a matter of fact, these \(\sigma_e\)'s have been used in the present propeller computations.

From the flow angles \(\beta\), blade setting angles \(\beta_g\) and speed coefficient \(J\), the geometric incidences angles \(\alpha_g\) can be calculated to estimate the initial values for \(\sigma_e\). Since \(\beta_g\) is shown in the Table C1 of Appendix C and \(\beta\) is calculated from:

\[
\beta = \tan^{-1} \frac{1}{J \pi x},
\]

(46)

\(\alpha_g\) is easily obtained and is tabulated in Table C3.
It is seen from Table C3 that the maximum and minimum values of \( \alpha_g \) are 21.11 degrees and 2.06 degrees at \( x = 0.3, J = 0.3 \) and \( x = 0.9, J = 0.7 \), respectively. The range of cavitation number based on \( V \) in Table C2 is found to be 0.0069 to 0.2194. Based on these values it was decided to calculate the 2-D s/c cascade characteristics at four different incidence angles, 2, 3, 4 and 6 degrees with a cavitation number ranging from the choking condition to about 0.08. For any other combination of an incidence angle and \( \sigma \) which will arise in the iterative procedure, the 2-D flow characteristics will be extrapolated or interpolated analytically. It is noted that the maximum value of \( \alpha_e \), i.e. 6 degrees does not seem to cover a value of \( \alpha_g \) of 21.11 degrees. However, the downwash effect near the hub is so large that the effective angle of attack will be near or within 6 degrees. It is also obvious that no supercavitation occurs at \( \sigma = .2194 \).

Figures 7(a) thru 7(f) show the calculated lifts \( C_L \) normal to the upstream flow as functions of cavitation number \( \sigma_e \) at normalized radius locations, \( x = 0.4, 0.5, 0.6, 0.7, 0.8 \) and 0.9. The 2-D calculations were left out for the point at \( x = 0.3 \) because the cavitation number is too large and the solidity is too high to obtain convergent solutions in the 2-D computations. In addition, the propeller performance can be accurately calculated without the information at that point by an interpolation scheme as will be seen later.

Two different computer programs (see [4]) were used, one for the choking condition at which the cavity extends downstream to infinity and the
other for the finite cavity case. In these figures we see that a significant cascade effect occurs in cavity flows. In Figure 7(a), for example, where the solidity is small, 0.244, near the tip (at \( x = 0.9 \)) with a stagger angle of 74 degrees, it is seen that the lift coefficient \( C_L \) increases as the incidence angle increases. This phenomenon is quite similar to that observed in single lifting foil cases. It means that the solidity of 0.244 at this location with \( \gamma = 74^\circ \) is yet too small to see much of a cascade effect and thus the flow is similar to that of a single foil except that the choking phenomenon appears. However, at \( x = 0.8 \) where the solidity becomes slightly larger, 0.365, with a stagger angle of 72.4 degrees, the lift coefficient \( C_L \) at \( \alpha_e = 6^\circ \) loses its value as \( \sigma \) becomes small (see Figure 7(b)), until finally its value becomes even smaller than that obtained at \( \alpha_e = 4^\circ, 3^\circ \) and \( 2^\circ \). The reason why this occurs at smaller \( \sigma \)'s is obvious; the smaller the cavitation number, the longer and thicker is the cavity (see Figures 10(a) thru 10(f)), so that the cavity boundary with a low cavity pressure is close to the pressure side of the neighboring blade, causing a loss in lift. It is also seen that the cavitation number \( \sigma_e \) at which this change in \( C_L \) occurs in Figures 7(a)-(f) checks quite well with the value of \( \sigma_e \) at which the length of cavity starts extending to infinity (choking conditions) as shown in Figures 10. One can also observe a similar behavior in \( C_L \) for \( \alpha_e = 4^\circ \), occurring here at a smaller \( \sigma_e \) than for the \( \alpha_e = 6^\circ \) case. This trend becomes even stronger (see Figures 7(c) thru 7(f)) since the solidity becomes larger increasing from 0.479 to 0.912. In Figure 7(f) where \( x = 0.4 \) and the largest solidity occurs, the relation between the lift and incidence angle completely flips over for a range of \( \sigma_e \)'s, i.e. the lift is the highest at the lowest incidence angle.
This peculiar behavior for cavitating cascade flow observed above never happens in the cases of single lifting foils. Physically it can be understood and explained as follows. When the solidity becomes large and thus the blades are more closely packed in the cascade configuration, the adjacent blades are strongly affected by the existence of cavity thus causing significant hydrodynamic effects. This effect becomes stronger as the cavity becomes thicker and longer or as the flow incidence angles become larger and the cavitation number becomes smaller as has been seen above.

To our knowledge the above highly nonlinear cascade effect have never been incorporated into supercavitating propeller design. If the lift curves of single supercavitating foils are used for such designs, large discrepancies between the design and experimental data are to be expected. For example, increasing flow incidence angles or, equivalently, increasing blade camber at a radial location of the blade having a relatively large solidity essentially decreases the sectional lift. This results in a smaller thrust coefficient in experiments than expected by design. A totally opposite situation must sometimes be taken; blade angles and camber must be decreased to increase lift depending on the solidity and cavitation number. This may be one of the reasons why the experimental thrust and torque coefficients of NSRDC Model 3770 were found far short of the design values based on single foil predictions. More detailed discussions about this point will be made in the
next section.

It is interesting to compare the lift coefficients obtained in the present nonlinear cascade theory and those of supercavitating single foils. The latter values at $\sigma_e = 0$ are easily computed from the design lift coefficients, correction factors listed in Table C1 and angles of attack:

$$C_{LS0} = C_{L_d} \cdot K + \pi a/2$$ (47)

where the subscripts $S$ and $0$ in (47) designate 'single foil' and 'zero cavitation number', respectively. $C_{LS0}$ calculated based on Equation (47) are plotted in each Figure 7(a) through 7(f). It is seen that a well known approximation for finite cavity length by a correction factor $(1 + \sigma)$, commonly used for a single foil flow, cannot be applied to the s/c flow in the cascade configuration whatsoever. It is also noted that $C_{LS0}$'s are much larger than those values extrapolated from the linear portions of $C_L$ curves, again showing a remarkable supercavitating cascade effect on lifting forces.

It is also seen that the choking conditions marked in Figures 7(a) through 7(f) vary to a great degree depending on the solidity. With small solidity and a small incidence angle, the choking flow does not occur until $\sigma_e$ becomes fairly small, say 0.007 (see Figure 7(a)), while a $\sigma_e$ of 0.041 is enough to cause the same condition for a large solidity and a large angle of attack (see Figure 7(f)). This behavior is also attributable to the increasing cascade blockage effects with increase in solidity and incidence angle.
Figures 8(a) through (f) show drag coefficients parallel to the upstream flow direction, each corresponding to Figure 7(a) through (f), respectively. It is seen that these drag forces also exhibit a trend similar to that of the lift coefficient.

The lift and drag coefficients shown in these figures will be used later in propeller analysis to calculate thrust and torque coefficients by using the formulae in Equations (43) and (44). The information which now connect the 2-D theory to the propeller lifting line theory are those about the circulation, $\Gamma$. Using the computer outputs of the 2-D s/c cascade theory, in particular the flow deflection angle, $\alpha_2$ and $U_2/V_e$, we calculated $\Gamma$'s and plotted them in Figures 9(a) to (f). Equations (10) and (11) are used for $\Gamma$'s of the infinite and finite cavity cases, respectively. The numbers read out from these figures are used as input data to a computer program for s/c propellers.

Finally, Figures 11(a) through 11(f) show lift-to-drag ratios ($L/D$) as a function of lift. It is interesting to see that the values of $L/D$ are less sensitive to cavitation number (or $C_L$ in the figures) and solidity as long as incidence angles and other parameters remain constant. This indicates that a blade section having a good $L/D$ value as a single foil with an infinitely long cavity is also guaranteed to have a good $L/D$ at finite cavity lengths in a cascade configuration. This fact also seems to explain the good correlation, obtained in the efficiency of the 3770 propeller, between experimental data and design data, while the thrust and torque coefficients are way off as already discussed.
The numerical computations presently carried out with the two-dimensional supercavitating cascade programs are a most time-consuming and difficult part during the present analysis. In particular, such flow configurations as having large solidity with large stagger angles and large incidence angles cause numerical instabilities in the functional iterative procedure as was pointed out in [4]. In the present case, for example at \( x = 0.4 \) and \( \alpha_e = 6^\circ \), where the solidity is 0.912, the next value of \( \sigma \) to the choking case which could be calculated was \( \sigma_e = 0.068 \). For any cavitation number between these two points, the numerical procedure failed to obtain a convergent solution. However, other points, where \( \sigma > 0.068 \), were calculated without any difficulties and these points have been smoothly connected by a curve as shown in Figure 7(f). The error incurred in this way is not considered to be significant in the present analysis. For all other cases, convergent solutions were obtained at almost equally spaced values of \( \sigma_e \). Execution time to obtain a convergent solution at each data point was about 150 seconds with CDC 6600 and about 40 seconds with CDC 7600. About sixty data points were computed to generate the present 2-D s/c cascade data, so that a total of 2400 seconds of computer time was used with the CDC 7600 or equivalently 9000 seconds with the CDC 6600.
6. CALCULATIONS OF PROPELLER PERFORMANCE FOR NSRDC MODEL 3770 SUPERCAVITATING PROPELLER

Based on the mathematical formulation and numerical procedures described in the precedent sections, a computer program has been written for calculating $K_T$, $K_Q$ and (thrust, torque coefficients and efficiency) of supercavitating propellers. (A complete listing of the computer program and input-output data manual are given in Appendix D).

The 2-D s/c cascade data for the 3770 propeller have been already prepared for propeller analysis. By taking five discrete radial points on the blade, i.e., $x = 0.4$, $0.6$, $0.7$, $0.8$ and $0.9$, the propeller hydrodynamic characteristics have been calculated. The cavitation number, $c_{Va}$ and speed coefficient, $J$, of the 3770 propeller at the design point are $0.617$ and $0.44$, respectively (see Table 1 of Reference 5). First of all we calculated $K_T$, $K_Q$ and $\eta$ at this point and show the results in Table 2 in comparison with design and experimental data taken from Reference 5 (also see Figure 12). It is clearly seen that the present method predicts them well, in particular, thrust coefficient $K_T$, and efficiency $\eta$ particularly, i.e. within 4 percent of the experimental data. There exists a large discrepancy in $K_T$ and $K_Q$ between the design data and experiment which use about 15 and 11 percent, respectively, although the $\eta$ there is close to others. The reason for this discrepancy has already been explained in the previous sections; the data basis for the present method depends on the supercavitating cascade theory whereas the design method depended on an infinite cavity, single foil theory.
This point will become even more clear when we compare some of blade sectional characteristics between the two methods. Four different parameters are shown in Table 3 for comparison. These include local cavitation number, effective flow incidence angle, downwash angle and lift coefficients. The local cavitation numbers of the present method shown in the table are those corrected on the basis of the cavity thickness data from the 2-D cascade calculations (see Equation (21)), whereas those of all design methods are simply based on \[ V = ((\omega r)^2 + V_a^2)^{\frac{1}{2}}. \]

The discrepancies shown in all these parameters of Table 3 are quite large and it is again considered that they are attributable to the differences in the force characteristics used by the two different methods as was mentioned before. Among others, it is seen that one of the most significant differences exists in downwash \( \alpha_i \) angles and thus effective flow incidence angles \( \alpha_e \), the latter in the design method are all about 2 degrees whereas the former range from 6.8 degrees to 4.6 degrees. The local lift coefficients near the hub predicted by the present methods are larger than those of the design method but become smaller as one proceeds toward the propeller tip. A question now arises why the thrust and power coefficients predicted here are smaller than those of the design method. It is simply answered that lift forces near the tip are more contributory to \( K_T \) and \( K_Q \) due to the smaller pitch and larger radius as will be readily seen from Equations (34) and (36).

It is interesting to investigate the boundary between the supercavitating and partial cavitating regimes of propeller operation. According to the
2-D s/c cascade data regarding the length of cavity, shown in Figures 10(a) to (f), it reads that the propeller blade at $x = 0.4$ is partially cavitated or fully wetted if $\alpha_e = 2.06^\circ$ and $\sigma = .0656$ obtained by the design method are true (see Figure 10(f)). With $\alpha_e = 6.8^\circ$ and $\sigma = .0798$ at $x = 0.4$ predicted by the present method, the length of cavity is about 1.1 chord length which means that the propeller barely stays in a supercavitating regime. Unfortunately, no clear photographic evidence of 3770 is available for us to check this feature. In any case, it is strongly recommended that not only force data but also photographic data at each measurement point be taken for any future supercavitating propeller experiments. The former will justify the overall accuracy of a prediction method used whereas the latter will play an important role in verifying a local flow phenomena.

Figure 12 shows $K_T$, $K_Q$ and $\eta$ over a range of speed coefficient, $J$, for the design cavitation number $\sigma_{Va} = .617$. As $J$ is taken close to about 0.5, the calculated $\alpha_e$ and $\sigma_e$ at $x = 0.4$ become $4.5^\circ$ and .097. Again from Figure 10(f), we can see that the blade near the hub (at $x = 0.4$) under the above conditions is in a partially cavitating region where the present supercavitating propeller theory becomes invalid. It seems this reason that all curves in Figure 12 start showing deflection around such a $J$.

In Figure 13 we compare the present results with experimental data for $\sigma_{Va} = .500$. It is seen that the correlation for $K_T$, $K_Q$ and $\eta$ at such $J$ of 0.44 to .5 is excellent but that the discrepancy starts growing
as J becomes outside the above range of J. The same reason as above explains the discrepancy for large J's at which a part of the propeller operates at partial cavitating condition.

The reason for the discrepancy for smaller J (say, less than 0.44) is not known at this point. Physically, a decrease of J increases the effective flow angle $\alpha_e$ and decreases local cavitation number $\sigma_e$, leading more and more to a local choking condition of supercavitating propeller. From certain values of $\alpha_e$ and $\sigma_e$ on, neither of these values cannot change. It means that the lift and drag coefficients should reach constant values thus $C_T$ and $C_p$ in Equations (43) and (44) also approach constant values. However, $K_T$ and $K_Q$ are proportional to $J^2$ and $J^3$ respectively (see Equations (40) and (41)) so that these values theoretically decreases as J decreases. Experimental data in Figure 13 do not show any of this type of trend at least before $J = 0.4$ whereas the results of the present theory clearly demonstrate the above theoretical argument.

It is also considered that the discrepancy in $K_T$, $K_Q$ and $\eta$ for smaller J may be suggestive even for incidence angle correction due to the choking condition of the 2-D cascade theory in addition to the $\sigma$-correction. For determining which argument is true, further comparisons of the present theory with existing and possibly new experimental data from supercavitating propellers are urgent.

In order to accurately predict propeller performance for a range of larger J, say 0.51 in this case, the 2-D cascade data based on partial cavitating
conditions must be used. Although a linearized theory for partial cavitation cascade is available, it cannot be applied for the propeller analysis in which the flow there is highly nonlinear. Development of a similar nonlinear theory to that for supercavitating cascade is now in order.

Figure 13 also shows results before we applied the correction method to cavitation numbers (by dotted lines). When J was set at a slightly smaller value than 0.46, in this case, a combination of such $\alpha_e$ and $\sigma$ put us beyond the choking condition where there exists no 2-D cascade data. By forcing as to find $\Gamma$ by extrapolation (which seems irrelevant, first of all) we managed to determine $K_T$, $K_Q$, and $\eta$, which are plotted in Figure 13. These values are of course not valid. Without using either cavitation number correction as does the present method or possibly incidence angle correction, it is impossible to avoid this inherent 2-D choking problem in three-dimensional flow applications. It is clearly seen that an adequate treatment of this problem is most crucial in the entire supercavitating propeller analysis as long as a cascade theory is intended to be used.

For the above computations, we have used Newton's iterative procedures which showed a slow but steady convergence. For each run, it took about 30 seconds for a CDC 7600.
7. CONCLUSIONS AND RECOMMENDATIONS

The present method has incorporated a 2-D s/c cascade theory into a propeller lifting line theory with downwash angle correction and cavitation number correction made.

The results obtained from the theory have shown an excellent correlation with experimental data within a certain range of speed coefficient, J, whereas some discrepancy exists outside that range. It is quite clear that the discrepancy for larger, J, is attributable to the appearance of the partial cavitating flow near the hub so that the present theory naturally becomes invalid. That for smaller, J, cannot be clearly explained at this stage as mentioned before.

Furthermore, it has been found that there exists a significantly large difference in local flow conditions between the present theory based on cascade data and design theory based on single foil data. These include downwash flow angles, effective flow angles, cavitation numbers and lift coefficients.

In order to clarify some of the uncertainties having just arisen, the following specific recommendations for future research works to be carried out are made.

1) More comparisons between the theory and experiments for different types of propeller
configurations such as those in 11 are necessary for verifying the accuracy of the present prediction theory.

2) Slightly beyond a range of the largest efficiency or optimum design point, there exists a partially cavitating flow regime. We always have a possibility that s/c propellers sometimes operate at such a regime. In order to cover a full range of performance prediction for s/c propellers, it is advisable to develop a non-linear partially cavitating cascade theory and to incorporate the data from such a theory into the present method.

3) Although the two-dimensional supercavitating cascade theory [4] used here was compared well with experimental data before, such a comparison was very much limited due to the lack of experimental data. It is recommended that more experiments be conducted, in particular for a range of high solidity and high stagger angle.

4) A more rigorous evaluation of the effects of propeller cavity thickness on cavitation
number and possibly on local flow angle may be necessary.

Finally, it is concluded that cascade and three-dimensional effect of supercavitating propellers plays a most crucial role in their hydrodynamic performance. In cooperation of this effect into both prediction and design methods for such propellers is unavoidable.
Figure 1  Flow Configuration in Cascade Geometry of Propeller at a Constant Radius r with Velocity Diagram.
Figure 2  Change of Circulations $\Gamma$ in Cascade Flow.
Figure 3  Representation of Cavity Thickness by Distribution of Source Singularities in a Cascade Row.
Figure 4  Geometric Mean Angle $\alpha_\infty$, Force Normal to Geometric Mean Velocity $U_\infty$ in Cascade Flow and Thrust and Torque Components.
START

2-D S/C Cascade Program
- Propeller Geometry, $a$ & $V_0$ specified
- $a_e^{(n)}$ assumed ($n = 0$) $\rightarrow$ $a_1^{(n)}$, $b_1^{(n)}$ (Eq. 1 & 2)
- $c_1^{(n)}$ found ($n = 0$) (Eq. 3)

Obtain $a_2 = a_2$ (Geometry, $a_e^{(n)}$, $c_1^{(n)}, x$)

Find $\Gamma(x) = \frac{a_0 V_0 \sin (a_e - a_2)}{\cos (a_2 + \gamma)}$
(Eq. 10 or 11)

Lifting Line Theory
Find induced velocities $w_a$ & $w_{\frac{3}{2}}$
and then $c_1^{(n+1)}$ (Eqs. 12, 13 & 16)

New $a_e^{(n)}$, $b_1^{(n)}$, $c_1^{(n)}$

$\left| a_1^{(n+1)} - a_1^{(n)} / a_1^{(n)} \right| < \epsilon$
where $\epsilon$ is a desired value for convergence

NO

YES

Calculate $K_T$, $K_P$ & $\eta$
(Eqs. 37, 38 & 39)

STOP

Figure 5  Flow Chart of Substitution Procedures Iterative
2-D s/c Cascade Program
- Propeller Geometry, \( w \) & \( V_a \) specified
- Calculate s/c cascade characteristics for various \( \alpha \) and \( \alpha_e \)

- Obtain \( \alpha_2 \) from the 2-D results
- Calculate \( V_c \) and \( V_e \) from Eqs. (20) & (21)

- Assume \( \alpha_e^{(n)}(x_1) \) to find \( \Gamma \) from Eq. (11),
- Calculate \( G_j^{(n)} \) from Eqs. (14) and (B-11) by Fourier Transform

Newton's Iterative Method
Eqs. (24) - (29)

Calculate \( C_T, C_p \) and \( \eta \)
(Eqs. (37), (43) and (44))

STOP

Figure 6  Flow Chart of Newton's Iterative Procedures
Figure 7(a)  Lift Coefficient $C_L$ Normal to the Upstream Velocity vs. Cavitation Number $\sigma$ for Incidence Angles $\alpha_e = 2^\circ$, $3^\circ$, $4^\circ$, $6^\circ$ and $6^\circ$ at the Blade Span-wise Location at $x = 0.9$ where the Solidity is $0.244$ and geometric stagger angle $\gamma$ is $74.03^\circ$.

$\Delta$ $\bigcirc$ $\square$ are $C_L$ Values of $\alpha_e = 2^\circ$, $4^\circ$, $6^\circ$ Respectively Calculated from a Linearized Theory for a Single Foil (Equation 47).
Figure 7(b) The Same as Figure 7(a) Except That $x = 0.8$ where the Solidity is 0.365 and $\gamma$ is 72.40°.
Figure 7(c) The Same as Figure 7(a) Except That $x = 0.7$ where the Solidity is 0.479 and $\gamma$ is $70.33^\circ$. 

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Figure 7(d) The Same as Figure 7(a) Except That $x = 0.6$ where the Solidity is 0.594 and $\gamma$ is 67.61°.
Figure 7(e)  The Same as Figure 7(a) Except That x = 0.5 where the Solidity is 0.728 and $\gamma$ is 63.94°.
Figure 7(f) The Same as Figure 7(a) Except That $x = 0.4$ where the Solidity is 0.912 and $\gamma$ is 58.77°.
Figure 8(a)  Drag Coefficient $C_D$ Corresponding to Figure 7(a) ($x = 0.9$).
Figure 8(b) Drag Coefficient $C_D$ Corresponding to Figure 7(b) ($x = 0.8$).
Figure 8(c) Drag Coefficient $C_D$ Corresponding to Figure 7(c) ($x = 0.7$).
Figure 8(d)  Drag Coefficient $C_D$ Corresponding to Figure 7(d) ($x = 0.6$).
Figure 8(e) Drag Coefficient $C_D$ Corresponding to Figure 7(e) ($x = 0.5$).
Figure 8(f)  Drag Coefficient $C_D$ Corresponding to Figure 7(f) ($x = 0.4$).
Figure 9(a)  Normalized Circulation $\Gamma / dV_e$ vs. $\sigma_e$
Corresponding to Figure 7(a) ($x = 0.9$).
Figure 9(b) \( \frac{\Gamma}{dV_e} \) vs. \( \sigma_e \) Corresponding to Figure 7(b) 
\( (x = 0.8) \).
Figure 9(c) $\Gamma/dV_e$ vs. $\sigma_e$ Corresponding to Figure 7(c) ($x = 0.7$).
Figure 9(d)  \( \frac{\Gamma}{dV_e} \) vs. \( \alpha_e \) Corresponding to Figure 7(d)  
\( (x = 0.6) \).
Figure 9(e) \( \frac{\Gamma}{dV_e} \) vs. \( \sigma_e \) Corresponding to Figure 7(e) \\
(\( x = 0.5 \)).
Figure 9(f) $\Gamma/\text{d}V_e$ vs. $\sigma_e$ Corresponding to Figure 7(f) ($x = 0.4$).
Figure 10(a)  Length of Cavity vs. $\sigma_e$ for Incidence
Angles $\alpha_e = 2^\circ$, $4^\circ$ and $6^\circ$ at $x = 0.9$. 

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Figure 10(b) The Same as Figure 10(a) Except That $x = 0.8$. 

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Figure 10(c)  The Same as Figure 10(a) Except That \( x = 0.7 \).
Figure 10(d)  The Same as Figure 10(a) Except That $x = 0.6$. 
Figure 10(e)  The Same as Figure 10(a) Except That \( x = 0.5 \).
Figure 10(f) The Same as Figure 10(a) Except That $x = 0.4$. 
Figure 11(a)  Lift-to-Drag Ratio L/D vs. $C_L$ for Incidence Angles $\alpha_e = 2^\circ$, $4^\circ$ and $6^\circ$ at $x = 0.9$. 
Figure 11(b)  The Same as Figure 11(a) Except That $x = 0.8$.  

\[ L/D \]

\[ C_L \]
Figure 11(c) The Same as Figure 11(a) Except That
\( \alpha_e = 2^\circ \).
\( x = 0.7 \).
Figure 11(d)  The Same as Figure 11(a) Except That $\alpha = 0.6$. 

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Figure 11(e)  The Same as Figure 11(a) Except That
\( x = 0.5 \).
Figure 11(f)  The Same as Figure 11(a) Except That $x = 0.4$. 
Figure 12  Performance Prediction for 3770 at Design $\sigma_{Va} = .617$. 

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Figure 13  Comparison of 3770 Performance between the Present Theory and Experimental Data [5] at $\sigma_{Va} = 0.500$. 

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TABLE 1
Design Characteristics of NSRDC Model 3770
Supercavitating Propeller (from Reference [5])

<table>
<thead>
<tr>
<th>Blade Number (g)</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch/Diameter</td>
<td>0.786</td>
</tr>
<tr>
<td>EAR</td>
<td>0.508</td>
</tr>
<tr>
<td>Chord/Diameter at $x = 0.7$</td>
<td>0.351</td>
</tr>
<tr>
<td>Design $\alpha_e$ at $x = 0.7$</td>
<td>2$^\circ$</td>
</tr>
<tr>
<td>Design $C_L$ at $x = 0.7$</td>
<td>0.147</td>
</tr>
<tr>
<td>Section</td>
<td>TMB Modif.</td>
</tr>
<tr>
<td>$J$</td>
<td>0.440</td>
</tr>
<tr>
<td>$\sigma_{Va}$ at $x = 0.7$</td>
<td>0.617</td>
</tr>
</tbody>
</table>
TABLE 2

Comparison of $K_T$, $K_Q$ and $\eta$ between the Present Result, Design Data and Experimental Data for 3770 Supercavitating Propeller at Design Point, $\sigma_{\nu_a} = .617$ and $J = .440$.

<table>
<thead>
<tr>
<th></th>
<th>Design Data (Ref. 5)</th>
<th>Experimental Data (Ref. 5)</th>
<th>Present Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_T$</td>
<td>.1004</td>
<td>.085</td>
<td>.0819</td>
</tr>
<tr>
<td>$K_Q$</td>
<td>.0130</td>
<td>.0115</td>
<td>.0106</td>
</tr>
<tr>
<td>$\eta$ (%)</td>
<td>54.1</td>
<td>52.0</td>
<td>54.0</td>
</tr>
</tbody>
</table>
### Table 3
Comparison of the Detailed Flow Characteristics of 3770 between the Design Method [5] and Present Method at Design Point, \( \sigma_a = .617 \) and \( J = .440 \).

<table>
<thead>
<tr>
<th>Nondimensional Radius ( x )</th>
<th>Local Cavitation Number ( c_0 )</th>
<th>Effective Incidence Angle ( \theta_e )</th>
<th>Downwash Angle ( \alpha_q )</th>
<th>Lift Coefficient ( C_L )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( 0.4 )</td>
<td>0.0656</td>
<td>2.06(^o)</td>
<td>6.80(^o)</td>
</tr>
<tr>
<td></td>
<td>( 0.6 )</td>
<td>0.0302</td>
<td>0.0302</td>
<td>1.99(^o)</td>
</tr>
<tr>
<td></td>
<td>( 0.7 )</td>
<td>0.0223</td>
<td>0.0223</td>
<td>1.91(^o)</td>
</tr>
<tr>
<td></td>
<td>( 0.8 )</td>
<td>0.0170</td>
<td>0.0170</td>
<td>1.91(^o)</td>
</tr>
<tr>
<td></td>
<td>( 0.9 )</td>
<td>0.0134</td>
<td>0.0134</td>
<td>1.91(^o)</td>
</tr>
</tbody>
</table>

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REFERENCES


APPENDIX A

Two Dimensional Nonlinear Supercavitating Cascade Theory (Reference [4])
The detailed mathematical formulation is described in [4]. However some important features needed for the present calculations are repeated here for convenience.

\[ w(\zeta_1) = \text{Re}\{w(\zeta_1)\} + i \text{Im}\{w(\zeta_1)\} \]

\[ = \sqrt{(\zeta_1 + 1)(\zeta_1 - b)} \left\{ \frac{1}{2\pi i} \int_a^b - \frac{i2 \ln(\sqrt{1 + \sigma} U_2)}{\sqrt{(\xi_1 + 1)(\xi_1 - b)}} \frac{d\xi_1}{\xi_1 - \zeta_1} \right. \]

\[ + \frac{1}{2\pi i} \int_{a-1}^{b} \frac{2 \ln(\sqrt{1 + \sigma} U_2)}{i \sqrt{(1 + \xi_1')(b - \xi_1')}} \frac{d\xi_1'}{\xi_1' - \zeta_1} + \frac{1}{2\pi i} \int_0^b \frac{2\pi i}{\sqrt{(1 + \xi_1')(b - \xi_1')}} \frac{d\xi_1'}{\xi_1' - \zeta_1} \]

\[ + \frac{1}{2\pi i} \int_{b}^c \frac{i \cdot 2 \ln(\sqrt{1 + \sigma} U_2)}{\sqrt{(\xi_1 + 1)(\xi_1 - b)}} \frac{d\xi_1}{\xi_1 - \zeta_1} \} \]  

(A-1)

where

\[ \zeta_1 = \tilde{A} \exp\{i(\pi/2 - \delta)\}, \]

\[ g_3 = \frac{1}{\pi} \ln(\sqrt{1 + \sigma} U_2) \left\{ \ln \frac{-(b+1)}{2\sqrt{(a+1)(a-b)+(a+1)+(a-b)}} + \ln \frac{(b+1)}{2\sqrt{(c+1)(c-b)+(c+1)+(c-b)}} \right. \]

\[ + \left( \frac{\pi}{2} - \sin^{-1} \frac{1-b}{1+b} \right) + \frac{1}{\pi} \int_{-1}^{b} \frac{\bar{f}(\xi_1')d\xi_1'}{\sqrt{(1 + \xi_1')(b - \xi_1')}} \]  

(A-2)
\[ s(-1) = -\int_{\xi}^{1} h(\xi', a, b, c, U_2(\alpha_2)) \cdot k(\xi', \tilde{\alpha}) \, d\xi' \quad (A-3) \]

where

\[
h(\xi, a, b, c, U_2(\alpha_2)) = \left[ \exp \left\{ -\frac{\ln(\sqrt{1+\sigma}/U_2)}{\pi} \left( \pi + \sin^{-1} \frac{(1+\xi)(a-b)+(\xi-b)(1+a)}{(\xi-a)(1+b)} \right. \right. \right.
\]
\[
+ \sin^{-1} \left. \frac{(1+\xi)(c-b)+(\xi-b)(1+c)}{(c-\xi)(1+b)} \right) + \frac{\sqrt{(1+\xi)(b-\xi)}}{\pi} \right] \times \quad (A-4)
\]
\[
\left\{ \int_{\xi}^{b} \frac{\bar{3}(\xi')}{\sqrt{(1+\bar{\xi}')(b-\bar{\xi}')}} \, d\xi' \right\} \times 2\sqrt{b} \sqrt{(1+\xi)(b-\xi)+\xi(b-1)+2b} \frac{1}{U_2} \]

\[ k(\xi, \tilde{\alpha}) = \frac{d}{\pi} \frac{\xi \cos \delta}{(\xi - \tilde{\alpha} \sin \delta)^2 + (\tilde{\alpha} \cos \delta)^2} \quad (A-5) \]

and

\[ \delta = \alpha + \gamma, \quad (A-6) \]

\[ g_5 = \frac{d}{2\pi} \left\{ e^{-i\delta} \ln \frac{\xi_1 - c}{\xi_1 - a} + e^{i\delta} \frac{\bar{\xi}_1 - c}{\bar{\xi}_1 - a} \right\} \quad (A-7) \]

In these equations, \( a, b, c \) are \( \xi \)-coordinates in the mapped plane (see Figure A-2), \( \tilde{\alpha} \) is a parameter associated with the cascade mapping function and \( U_2 \) is the velocity at downstream infinity which is related to \( \alpha_2 \) through a continuity equation:
\[
U_2 = \frac{\cos(\alpha_e + \gamma)}{\cos(\alpha_2 + \gamma)}
\]

where \( V_e \) is taken to be unity in the 2-D calculations and all of these values are solution parameters to be determined by Equations (4) through (8) shown in the text. The potential plane from which the \( \zeta \)-plane is mapped and the definition of \( s \) and \( \beta \) are shown in Figures A1 and A3.
FIGURE A1: Potential Plane $w = \phi + i \psi$

FIGURE A2: Transform Plane $\zeta = \xi + i \eta$

FIGURE A3: Definition of the arc length and the body inclination for the wetted portion of the foil
APPENDIX B

Calculations of Induced Velocities $w_a$ and $w_t$ by a Lifting Line Theory $a$
The following calculation method of the induced velocities $w_a$ and $w_t$ is based on the work of Lerbs [7]. The equations for $w_a$ and $w_t$ are written again:

$$\frac{w_a(x)}{V_a} = \frac{1}{2} \int_{x_h}^{1} \frac{dG(x')}{dx'} \frac{1}{x-x'} i_a(\beta, x', x) \, dx'$$  \hspace{1cm} (B-1)$$

$$\frac{w_t(x)}{V_a} = \frac{1}{2} \int_{x_h}^{1} \frac{dG(x')}{dx'} \frac{1}{x-x'} i_t(\beta, x', x) \, dx'$$  \hspace{1cm} (B-2)$$

where

$$G(x) = \frac{r}{2\pi R V_a}$$  \hspace{1cm} (B-3)$$

$$i_a(\beta) = \begin{cases} 
  g \frac{x}{x' \tan \beta} \left( \frac{x'}{x} - 1 \right) (1+B_2), & x < x' \\
  -g \frac{x}{x' \tan \beta} \left( \frac{x'}{x} - 1 \right) B_1, & x > x' 
\end{cases}$$  \hspace{1cm} (B-4)$$

$$i_t(\beta) = \begin{cases} 
  g \left( \frac{x'}{x} - 1 \right) B_2, & x < x' \\
  -g \left( \frac{x'}{x} - 1 \right) (1+B_1), & x > x' 
\end{cases}$$  \hspace{1cm} (B-5)$$

B-1
\[ B_{1,2} = \left( \frac{1+y}{1+y^2} \right)^{1/2} \left[ \frac{1}{gA_{1,2}} - \frac{1}{2g} \frac{y^2}{(1+y^2)^{1.5}} \ln \left( 1 + \frac{1}{gA_{1,2}} \right) \right] \]  
(B-6)

\[ A_{1,2} = \pm \left( \sqrt{1+y^2} - \sqrt{1+y'^2} \right) \frac{1}{2} \ln \frac{\left( \sqrt{1+y^2} - 1 \right) \left( \sqrt{1+y^2} + 1 \right)}{\left( \sqrt{1+y'^2} + 1 \right) \left( \sqrt{1+y'^2} - 1 \right)} \]  
(B-7)

\[ y' = \frac{1}{\tan \beta_i} \]  
(B-8)

\[ y = \frac{x}{x' \tan \beta_i} \]  
(B-9)

where Nicholson's asymptotic formulae have been applied in obtaining \( B_1 \) and \( B_2 \) from the original integrals of vortex sheets.

By introducing \( \varphi \) for a change of variables:

\[ x = \frac{1}{2} (1+x_h) - \frac{1}{2} (1-x_h) \cos \varphi , \]  
(B-10)

thus \( x = x_h \) and \( 1 \) correspond to \( \varphi = 0 \) and \( \pi \).

We also write \( G(x) \) and \( i \) in Fourier sine and cosine series, respectively:

\[ G(\varphi') = \sum_{m=1}^{\infty} G_m \sin m \varphi' \]  
(B-11)

\[ i(\varphi, \varphi') = \sum_{n=0}^{\infty} I_n(\varphi) \cos n \varphi' . \]  
(B-12)
The coefficients $G_m$ and $I_n(\varphi)$ in (B-11) and (B-12) are obtained by using the orthogonality of sine and cosine function:

$$G_m = \frac{2}{\pi} \int_0^\pi G(\varphi') \sin m \varphi' \, d\varphi'$$  \hspace{1cm} (B-13)

$$I_n^a(\varphi) = \frac{k}{\pi} \int_0^\pi i_a(\varphi, \varphi') \cos n \varphi' \, d\varphi' \quad \begin{cases} \text{k=1, n=0} \\ \text{k=2, n\geq1} \end{cases}$$  \hspace{1cm} (B-14)

$$I_n^t(\varphi) = \frac{k}{\pi} \int_0^\pi i_t(\varphi, \varphi') \cos n \varphi' \, d\varphi'$$  \hspace{1cm} (B-15)

where $x$ and $x'$ are replaced by $\varphi$ and $\varphi'$ by using the relation of equation (B-10).

Now $w_a$ and $w_t$ are written in the following forms:

$$\frac{w_a(\varphi)}{V_a} = \frac{1}{1-x_h} \sum_{m=1}^\infty m G_m h^a_m(\varphi)$$  \hspace{1cm} (B-16)

$$\frac{w_t(\varphi)}{V_a} = \frac{1}{1-x_h} \sum_{m=1}^\infty m G_m h^t_m(\varphi)$$  \hspace{1cm} (B-17)

where

$$h^a, t_m(\varphi) = \frac{\pi}{\sin \varphi} \left[ \sin m \varphi \sum_{n=0}^m i_n^a, t(\varphi) \cos n \varphi + \cos m \varphi \sum_{n=m+1}^\infty i_n^a, t(\varphi) \sin n \varphi \right]$$  \hspace{1cm} (B-18)
It must be noted that, at $\varphi=0$ and $\varphi=\pi$:

$$h_{m}^{a,t}(0) = \pi \left[ \sum_{n=0}^{m} I_{n}^{a,t}(0) + \sum_{n=m+1}^{\infty} I_{n}^{a,t}(0) \right]$$  \hspace{1cm} (B-19)$$

$$h_{m}^{a,t}(\pi) = -\pi \cos{\pi} \left[ \sum_{n=0}^{m} I_{n}^{a,t}(\pi) \cos{n\pi} + \sum_{n=m+1}^{\infty} I_{n}^{a,t}(\pi) \cos{n\pi} \right]$$  \hspace{1cm} (B-20)$$

where L'Hospital's rule has been used.
APPENDIX C

Geometric and Hydrodynamic Configurations of Supercavitating Propeller Model TMB(NSRDC) 3770
Sectional profiles of supercavitating propeller Model TMB(NSRDC) 3770 are made up by Tulin-Barkart two-terms camber sections at zero cavitation number, modified with a lifting surface correction factor. The equations of such profiles are given by:

\[
\frac{\overline{V}}{c} = \frac{8C_{Ld}}{5\pi} K \left\{ \frac{4}{3} \left( \frac{\overline{V}}{c} \right) + \frac{8}{3} \left( \frac{\overline{V}}{c} \right)^{3/2} - 4\left( \frac{\overline{V}}{c} \right)^2 \right\}
\]  

(C1)

where

\[ C_{Ld} = \text{design lift coefficient} \]
\[ \overline{c} = \text{chord length} \]
\[ K = \text{correction factor of a lifting line surface theory} \]

In Table C1 the \( y \)-coordinates of the blade pressure sides at several radial locations are shown with appropriate values of \( C_{Ld} \) and \( K \). Also shown in the same Table are the pitch-to-diameter ratio, \( P/D \), blade setting angle, \( \theta_g \), geometric stagger angle \( \gamma \) and solidity, including the number of propeller blades, diameter of propeller and hub-to-tip diameter ratio.

Table C2 shows the geometric flow angles \( \beta \) (see Figure 1) and the geometric flow incidence angles \( \alpha_g \) at various radius locations for various speed coefficients, \( J \). It is noted that the sum of \( \beta \) and \( \alpha_g \) is equal to the geometric blade setting angles \( \theta_g \) at any location for all \( J \)'s.

The local cavitation numbers based on \( V = \left\{ (\omega r)^2 + \left( \frac{V}{a} \right)^2 \right\}^{1/2} \) are calculated by equation (45), for various \( J \)'s and are shown in Table C3. This table
roughly shows a range of the cavitation number over which the super-
cavitating propeller hydrodynamics are to be calculated.
**Propeller Geometry of NSRDC Model 3770**

**Number of Blade = 3**
**Diameter of Propeller = 14.0" (0.424m)**
**Hub/Tip Diameter Ratio = 0.20**

### Y-Coordinates of 2-Term Camber of NSRDC 3770

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<th>X</th>
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<td>0.9000</td>
<td>0.9000</td>
<td>0.9000</td>
<td>0.9000</td>
</tr>
</tbody>
</table>

*DESIGN CL* | 2.050000 | 1.986000 | 1.798000 | 1.608000 | 1.467000 | 1.386000 | 1.328000 |
*K FACTOR* | 1.575943 | 2.120732 | 2.368084 | 2.507062 | 2.497577 | 2.117241 | 1.556000 |
*P/D* | 0.757 | 0.762 | 0.763 | 0.776 | 0.786 | 0.797 | 0.809 |
*BEITA (deg)* | 38.77 | 31.23 | 26.06 | 22.36 | 19.67 | 17.60 | 15.97 |
*STAG (deg)* | 51.23 | 58.77 | 63.94 | 67.64 | 70.33 | 72.40 | 74.93 |
*SOLIDITY* | 1.216 | 1.912 | 1.728 | 1.594 | 1.479 | 1.365 | 1.244 |

**NOTES**
- *K* is a correction factor for camber clp
- *P/D* is pitch to diameter ratio
- *STAG* is geometric blade angle
- *SOLIDITY* is geometric stagger angle

**Table C-1** Geometric and Hydrodynamic Configurations of Supercavitating Propeller Model TMB(NSRDC) 3770.
<table>
<thead>
<tr>
<th>J</th>
<th>Beta 3</th>
<th>Alpha 3</th>
<th>Beta 4</th>
<th>Alpha 4</th>
<th>Beta 5</th>
<th>Alpha 5</th>
<th>Beta 6</th>
<th>Alpha 6</th>
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<th>Alpha 7</th>
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<td>21.11°</td>
<td>23.00</td>
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<td>27.95</td>
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<td>17.80</td>
<td>17.66</td>
<td>13.57</td>
<td>21.70</td>
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<td>29.12</td>
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<td>14.29</td>
<td>11.77</td>
<td>17.66</td>
<td>6.69</td>
<td>20.91</td>
<td>5.15</td>
<td>24.02</td>
<td>2.04</td>
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<td>11.98</td>
<td>10.38</td>
<td>14.86</td>
<td>7.50</td>
<td>17.66</td>
<td>4.70</td>
<td>20.37</td>
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<td>11.90</td>
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<td>3.99</td>
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Table C-2 Geometric Flow Angles $\beta$ and Geometric Flow Incidence Angles $\alpha_g$ at Various Radial Locations of the Propeller 3770 for Various $J$’s.

<table>
<thead>
<tr>
<th>J</th>
<th>$\sigma$ Based on $V$</th>
<th>$\sigma$ Based on $V_a$</th>
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<td>.6</td>
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<td>.0087</td>
<td>.0152</td>
</tr>
<tr>
<td>.9</td>
<td>.0069</td>
<td>.0121</td>
</tr>
</tbody>
</table>

Table C-3 Local Cavitation Numbers $\sigma$ Based on $V = \left\{ \left( \frac{wr}{v} \right)^2 + \left( \frac{v_a}{v} \right)^2 \right\}^{1/2}$ for Various $J$’s.

for $\sigma_{V_a} = 0.617$ based on $V_a$
APPENDIX D

Computer Program Listing and
Input and Output Data Setup
1. INTRODUCTION

The computer program called 'SCSCREW' (listed below) calculates the hydrodynamic characteristics of supercavitating propellers with sectional two-dimensional s/c cascade data given as input data. Therefore, the computer program developed in [4] must be used to generate these 2-D s/c cascade data prior to the use of 'SCSCREW'. The method of preparing these input data will be explained later.

In what follows we describe the structure of the program 'SCSCREW' including functions of subroutines, input data set-up and type of output data obtained as the result of calculations.
2. STRUCTURE OF SCSCREW

SCSCREW consists of a main program and several subroutines, brief descriptions of which will be given as follows:

1) MAIN PROGRAM SCSCREW
   - Specify the dimensions for data.
   - Read input data.
   - Exercise Newton's iterative procedure (see Figure 6).
   - Calculate $C_T$, $C_p$, $K_T$, $K_Q$ and $\tau$ with and without drag forces.
   - Calculate local flow conditions including downwash flow angle, effective incidence angle, cavitation number, lift and drag coefficients.

2) FUNCTION CLCD (I, S, B, ILD)
   - Interpolate lift, drag and circulation with input data passed on to this program through common statement.

I: Index for radial or spanwise position on blade
S: Cavitation number for which CLCD to be calculated
B: Flow incidence angle for which CLCD to be calculated
ICLCD: Control Index

ICLCD = 0 for lift
ICLCD = 1 for drag
ICLCD = 2 for circulation
3) **SUBROUTINE FITCAV (MM1, SE, EE, MI, II, ACC)**

- Curve-fitting for cavity thickness by polynomials.

  **MM1:** Index for radial or spanwise position on blade

  **SE:** Cavitation number at which cavity thickness data are available

  **EE:** Cavity thickness data

  **MI:** Number of flow incidence angles for which cavity thickness data are available

  **II:** Number of cavitation numbers for which cavity thickness data are available

  **ACC:** Coefficients of polynomials fitted for cavitation number as a function of cavity thickness, calculated in FITCAV.

4) **SUBROUTINE CAVINO (NG, SC, AE, ACC, A2, II, SCN, EE, MI)**

- Correct a local cavitation number for the effect of cavity thickness (see Equation's (17) through (21)).

  **NG:** Index for radial or spanwise position on blade

  **SC:** Cavitation number before correction

  **AE:** Flow incidence angle

  **ACC:** Coefficients for polynomials with which cavitation number is fitted as a function of cavity thickness in Subroutine FITCAV

  **A2:** Flow incidence angles at which cavity thickness data are available

  **II:** Number of cavitation numbers for which cavity thickness data are available

  **SCN:** Corrected cavitation number
5) SUBROUTINE FANC (F, FE, SC, ACC, NG, KI, II)

- Provides a functional relation for cavitation number as a function of cavity thickness:

\[ f = (SC \cdot U_1^2/V_e^2) - (Polynomials\ in\ FITCAV) \]

where \( U_1/V_e \) is given in Equation (21) of the text.

F: \( f \) given above
EE: Cavity thickness \( (e \) in Equation (21))
SC: Cavitation number
ACC: Coefficients of polynomial in FITCAV
NG: Index for a radial position on the blade
KI: Index for an incidence angle
II: Dummy index (to be neglected)

6) SUBROUTINE MOSEC (A, B, ER1, ER2, X, J, NG, KI, SC, ACC, II)

- Find a root for \( f(x) = 0 \) where \( x \) must lie between \( A \) and \( B \) and \( f(A) > 0, \ f(B) < 0. \)

A, B: A root of \( f(x) = 0 \) lies between \( A \) and \( B \)
ER1, ER2: Accuracy control valuables with which \( |x_{\text{real}} - x| < ER1 \)
and \( |f(x_{\text{real}}) - f(x)| < ER2 \)
X: A root for \( f(x) = 0 \) found in this subroutine
J: Number of iterations executed in MOSEC
NG, KI, SC, ACC, II: The same as those in FANC.
7) SUBROUTINE DETERM \((A, N, D)\)

- Calculate determinant of a matrix \(A\) of rank \(N\).

\(A:\) Matrix input, requiring dimension \((50, 2N + 3)\)

\(N:\) Rank of the matrix

\(D:\) Calculated determinant of \(A\)

8) SUBROUTINE LSQUR (DATA, NUMBER, N, A, CHISQ, XM)

- Least square fitting for DATA with polynomials of order \(N\).

\(DATA:\) DATA(1, NUMBER) = \(x\)

\(DATA(2, NUMBER) = y\)

\(DATA(3, NUMBER) = \) Error in data

\(NUMBER:\) Number of input data

\(N:\) Order of polynomials

\(A:\) Coefficients of polynomials

\(CHISQ:\) Chi-square error to be specified

\(XM:\) Dimension of \((20, 43)\) needed, but neglect data in

\(XM\) (not used here)

9) FUNCTION FALF(X), FGAM(X) & FBET(X)

- Calculate \(\alpha, \beta, \gamma\) at \(x\) in Filon's integration formula (see Equations (25.4.47) to (25.4.57) in "Handbook of Mathematical Functions", National Bureau of Standard).

10) SUBROUTINE SPLINE \((X, Y, DY, S2, S3, T, SS, SS1, SS2, L1M, N, C1)\)

- Cubic spline curve fitting for \(Y(X)\) and evaluate \(SS(T)\)

\(Y(X):\) Dimensional Data of order \(N\)

\(T:\) Points for which \(Y\) to be evaluated

\(SS:\) Evaluated data at \(T\)

OTHER PARAMETERS: Disregard (not used)
3. **INPUT DATA SET-UP**

The following describes set-ups for input data cards. Typical input data are also listed at the end of the computer program listing.

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<th>Card No.</th>
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<th>Description</th>
<th>FORMAT</th>
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<tr>
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<td>MAXIT</td>
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<td></td>
<td>MM</td>
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<tr>
<td></td>
<td>MF, NF</td>
<td>Number of terms used for Filon integration method</td>
<td></td>
</tr>
<tr>
<td></td>
<td>NFILON</td>
<td>Number of increments in Filon integration method</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IOLD</td>
<td>If not equal to 0, VIVA of old calculations are fed in as input data. If 0, it is approximately calculated in the program (VIVA = $U_1/V_a$)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IWRITE</td>
<td>Number of the last iterations for which output printing is made.</td>
<td></td>
</tr>
<tr>
<td>Card No.</td>
<td>Symbol</td>
<td>Description</td>
<td>FORMAT</td>
</tr>
<tr>
<td>---------</td>
<td>----------</td>
<td>-------------------------------------------------------</td>
<td>----------</td>
</tr>
<tr>
<td>6</td>
<td>DIA</td>
<td>Propeller diameter in inches.</td>
<td>8F10.5</td>
</tr>
<tr>
<td></td>
<td>VASHIP</td>
<td>Propeller axial speed in feet/sec (= V_a)</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>XJJ</td>
<td>Advance coefficient (J)</td>
<td>8F10.5</td>
</tr>
<tr>
<td></td>
<td>ZZ</td>
<td>Number of blades (g)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SIGVA</td>
<td>Cavitation number based on V_a (= σ/V_a)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>XH</td>
<td>Propeller hub diameter/propeller tip diameter (= x_h)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>XXM</td>
<td>Weighting factor in iterative procedure</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>XX(I)</td>
<td>Nondimensional radial or spanwise position (= x) where I from 2 to MM-1</td>
<td>8F10.5</td>
</tr>
<tr>
<td>9</td>
<td>BETAG(I)</td>
<td>Geometric blade angle (see Figure 1) in degree (= θ_g)</td>
<td>8F10.5</td>
</tr>
<tr>
<td>10</td>
<td>ALFE(I)</td>
<td>Effective flow incidence angle in degree (= θ_e)</td>
<td>8F10.5</td>
</tr>
<tr>
<td>11</td>
<td>SOLI(I)</td>
<td>Solidity (= SOL)</td>
<td>8F10.5</td>
</tr>
<tr>
<td>12</td>
<td>VIVA(I)</td>
<td>U_1/V_a data only if IOLD ≠ 0</td>
<td>8F10.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Repeat for DXX from 2 to MM-1</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>XXX</td>
<td>Nondimensional radial position (= x) for lift, drag and circulation data</td>
<td>F10.3</td>
</tr>
<tr>
<td></td>
<td>MANGLE</td>
<td>Number of incidence angles (= θ_e) for which the data are available</td>
<td>I10</td>
</tr>
<tr>
<td></td>
<td>ISIG</td>
<td>Number of points for θ_e for which the data are available</td>
<td>I10</td>
</tr>
<tr>
<td></td>
<td>SIGMIN(IXX)</td>
<td>Not used, disregard</td>
<td>F10.5</td>
</tr>
</tbody>
</table>
Repeat MANG1 times

14 ANG1(IANG) Incidence angle at which the data are read-in. 8F10.5
15 SIGD(IXX, IANG, I) $\sigma_e$ for which the data are read-in at IXX & IANG 8F10.5
16 CL1(IXX, IANG, I) Data for lift coefficients ($= c_L$) 8F10.5
17 CDL(IXX, IANG, I) Data for drag coefficients ($= c_D$) 8F10.5
18 GGGI(IXX, IANG, I) Data for circulation ($= \Gamma/dV_e$) 8F10.5

Repeat for IXX = 2 to MM-1

19 XXX 8F10.5
MANG1 The same as before
19 ISIG1

Repeat for IANG = 1 to MANG1

20 ANG2(IANG) Incidence angle at which cavity thickness data are read-in 8F10.5
21 SIGE(IXX, IANG, I) Cavitation number at which cavity thickness data are read-in 8F10.5
22 EE(IXX, IANG, I) Cavity thickness at IXX and IANG 8F10.5
4. **TYPICAL OUTPUT DATA**

Typical output data are also listed at the end of the program listing. Most of them are self-explanatory. Those not explained in output data are described as follows:

- **XN(I)**: solution parameters
  \[
  XN(I) = \begin{cases} 
  a_e, & I = 1, \quad MM \\
  G_m, & I = MM + 1, \quad 2 \times MM
  \end{cases}
  \]
- **F(I)**: Residues of each function in Equation (22)
- **P(I, J)**: Partial derivatives of Jacobian \( J \) in Equation (25)
- **ALFG(I)**: \( a_g \)
- **BETA(I)**: \( \beta_i \)
- **ALFI(I)**: \( a_i \)
- **VIVA(I)**: \( U_1/V_a \)
- **SIGV(I)**: local cavitation number
PROGRAM SCSCREW(NINPUT,OUTPUT,TAPES=INPUT,TAPES=OUTPUT)
LIFTING LINE PROPELLER THEORY IN COMBINATION WITH 2-D S/C CASCADE THEORY.
PROGRAMMED BY O. FURUYA, S=27-76.
DIMENSION HETA(10), HETAI(10), ALFE(10), VIVA(I0), GGAM(10)
DIMENSION GGG(99), SML(10), ALFIN(10), XX(10), ALFEN(10)
DIMENSION PHI(12), T(100), W(100), SOUT(150), GM(100)
DIMENSION $1(150), UFDO(100), UFD(100), UFDP(100), ALFI(10)
DIMENSION CLS(10), COS(10), ALFG(10), FDO(10), FDP(10)
DIMENSION YALF(10), XHET(10), SINA(10:150), SINT(10:150), BETAG(10)
DIMENSION HMA(10,100), HMT(10,100), MAV(10), MVA(10), SIGV(10)
DIMENSION IVI(10), FCT(12), FCP(12), XCT(12), OFC(100), UIVA(10)
DIMENSION DY(150), S2(150), S3(150), S5(150), S8(150)
DIMENSION XIA(10,150), XIT(10,150), ALFINO(10)
DIMENSION GMU(20), P(50,20), R(50,20), F(20), XM(20), RGA(20), TRG(20)
DIMENSION GFT(20), PXJ(20), STM(20), VIVA(20), COLUM(10,10)
DIMENSION ANG(10), SIGE(10,5,5), EE(10,5,5), ACC(10,5,5)
DIMENSION SHT(10)
COMMON PAI, CONV, CONVI, SIGMIN(10)
COMMON SIGD(10,5,15), C1L(10,5,15), ANGI(5), CD1(10,5,15)
COMMON MANGLE, SIG, GGII(10,5,15)
PAI=3.141592654
CONV=PAI/180.
CONV=.186, /PAI

L DATA FOR CUBIC SPLINE METHOD AND GAUSS QUADRATURE. ARE ALREADY IN.
READ(5,160) NGAUS
NGAUSS1=NGAUSS+1
NGAUSS2=NGAUSS/2
READ(5,500) (T(I),I=NGAUSS2,NGAUSS)
READ(5,500) (W(I),I=NGAUSS2,NGAUSS)
M25 1001, N2
T(I)=T(I)-N2
25 M(I)=M(I)-N2
WRITE(4,61) (T(I),I=NGAUSS2,NGAUSS)
WRITE(4,62) (W(I),I=NGAUSS2,NGAUSS)
60 FORMAT(4F20.10)
61 FORMAT(4F10.8,1X)
62 FORMAT(4F10.8,1X)

L READ_IN DATA******************************************************************************************
L N=NUMBER OF CONTROL POINTS ON THE BLADE RADIAL LOCATIONS TO BE EVALUATED.
L (MUST BE AN EVEN NUMBER)
L MAXI=MAXIMUM NUMBER OF ITERATIONS AT WHICH THE ITERATION IS STOPPED.
L XM=MAXIMUM BLADE SPAN.
L X1=F LEST SPEED COEFFICIENT (WAM/NC).
L Z=NUMBER OF BLADES.
L TGY=CAVITATION NUMBER BASED ON THE SHIP SPEED VA.
L XY=NUMBER OF RADIAL POINTS TO BE EVALUATED.
L BETA(I)=GEOMETRICAL BLADE SETTING ANGLES IN DEGREES.
L ALFE=ASSUMED EFFECTIVE FLOW INCIDENCE ANGLES.
L GM=NUMBER OF TERMS IN FOURIER SINE SERIES FOR GM, MF TO BE GREATER THAN MF.
L MF=NUMBER OF TERMS IN FOURIER COSINE SERIES FOR IN.
L MPIL=NUMBER OF TERMS (EVEN NO.) FOR FILLING FORMULA.
L VS=ADVANCED SPEED IN FEET/SEC.
L DTA=PROPELLER DIAMETER IN INCH.
L INLD=NUMBER OF DATA FOR VIVA(I), USE .W.D. FOR USING PREVIOUS DATA.
L IMRT=NUMBER OF LAST ITERATIONS FOR WHICH THE RESULTS ARE PRINTED.
L EP=INCREMENT RATIO FOR PARTIAL DERIVATIVES FOR CL/CD(ALFE).

Preceding page blank
C GGSI(J,XX,K) IS NORMALIZED CIRCULATION AS FUNCTIONS OF XX, SIGMA, AND ALFA
L ---(# GAMMA/(1+D)).
L CGG=GGSI(2,4PAI)*REVA
READ(5,101) EP
READ(5,100) MAXIT,MM,MF,NF,NFILUN,IOLD,WRITE
READ(5,101) GIA,YASHIP
READ(5,101) AJJ,JZ,SIGMA,XH,XX
MM=MM+1
MM2=MM+2
MMX=MM2
READ(5,101) (XX(I),I=2,MM1)
READ(5,101) (BETAG(I),I=2,MM1)
READ(5,101) (ALFC(I),I=2,MM1)
READ(5,101) (SOLI(I),I=2,MM1)
IF(IOLD.EQ.0) GO TO 722
READ(5,101) LVIVA(I),I=2,MM1
722 CONTINUE
100 FORMAT(81I0)
101 FORMAT(8F10.5)
C LIST THE READ-IN DATA.
WRITE(6,180) MAXIT,MM,MF,NF,JULO
WRITE(6,170) GIA
WRITE(6,170) YASHIP
WRITE(6,330) NFILUN
WRITE(6,110) ZZ
WRITE(6,180) XH
WRITE(6,110) AJJ
WRITE(6,110) SIGMA
WRITE(6,110) (XX(I),I=2,MM1)
WRITE(6,110) (BETAG(I),I=2,MM1)
WRITE(6,110) (ALFC(I),I=2,MM1)
WRITE(6,110) (SOLI(I),I=2,MM1)
IF(JULO.EQ.0) GO TO 724
WRITE(6,725) (VIVA(I),I=2,MM1)
724 CONTINUE
110 FORMAT(15X,NUMBER OF BLADES=*,F3.0)
111 FORMAT(15X,NUMBER OF FLOW CASSE=*,F3.4)
112 FORMAT(15X,NUMBER OF CASSE=*,F3.5)
113 FORMAT(15X,NUMBER OF CASSE=*,F3.5)
114 FORMAT(15X,NUMBER OF CASSE=*,F3.5)
115 FORMAT(15X,NUMBER OF CASSE=*,F3.5)
116 FORMAT(15X,NUMBER OF CASSE=*,F3.5)
117 FORMAT(15X,NUMBER OF CASSE=*,F3.5)
118 FORMAT(15X,NUMBER OF CASSE=*,F3.5)
399 FORMAT(15X,NFILUN=*,I3)
715 FORMAT(5X,PROPPELLER DIAMETER IN INCH=*,F10.5)
716 FORMAT(5X,ADVANCE SPEED IN FEET/SEC=*,F10.5)
735 FORMAT(4X,VIVA(I)=*,F10.5)
C THIS IS A ROUTINE FOR FINDING CIRCULATION FROM 2-D CASCADE DATA.************
L NIG=NUMBER OF DATA POINTS IN ONE CL-SIGMA CURVE(LESS THAN 10).
L SIGM=SIGMA AT WHICH ALFA=0, IN THIS CASE IS CALCULATED.
L MANGE=NUMBER OF INCIDENCE ANGLES AT ONE BLADE SECTION(LESS THAN 5).
L THESE NUMBERS MUST BE THE SAME FOR ALL XX(I).
NO 120 IXX=2,MM1
READ(5,102) XXH,MANGLEJSIGSIGMIN(Ixx)
102 FORMAT(F10.3*2,XX,F10.5)
WRITE(6,110) XXH,MANGLEJSIG
WRITE(6,665) SIGMIN(XX)
16 FORMAT(/5X,*=*,F5.3,*=*,F5.3,*=*,F5.3)*MU.
OF INCIDENCE ANGLES ==12,5X,*NO.
OF X DATA POINTS ON ONE CL CURVE==12)
665 FORMAT(5X,*=*,MIN. SIGMA FOR USE (IF THE MAX. ALFA (IN THIS CASE & DEGR
X=ES) == F7.4)
C SIGD(1,J,K)= DATA POINTS FOR SIGMA WHERE CL DATA ARE READ IN.
C IN ORDER FROM SMALL SIG TO LARGE ONES.
C ANG1(I)=INCIDENCE ANGLES USED IN 2-D.
C CL1(I,J,K)=LIFT COEFFICIENTS NORMAL TO VI.
C CL1(I,J,K)=DRAG COEFFICIENTS PARALLEL TO VI.
NO 120 IANG=1,MANCE
READ5(1,0) ANGI(IANG)
READ5(1,1) SIGD(IXX, IANG, I),I=1,1,1
READ5(1,1) CL1(XX, IANG, I),I=1,1,1
READ5(1,1) CD1(IXX, IANG, I),I=1,1,1
WRITE(6,110) ANGI(IANG)
WRITE(6,117) SIGD(XX, IANG, I),I=1,1,1
WRITE(6,118) CL1(IANG, I),I=1,1,1
WRITE(6,115) CD1(IANG, I),I=1,1,1
WRITE(6,810) (SIGG(IANG, I),I=1,1,1)
810 FORMAT(1X,*=*,G8.4,2)*0.
117 FORMAT(1X,*=*,SIGD(I,J,K)==,8F10.5)
118 FORMAT(1X,*=*,CL1(I,J,K)==,8F10.5)
119 FORMAT(5X,*=*,F5.2)
173 FORMAT(1X,*=*,8F10.5)
120 CONTINUE
C READ IN THE DATA FOR CAVITY THICKNESS.
NO 360 IXX=2*MH1
TF(IXX,IXX,2) WRITE(6,364)
READ5(1,2) XXH,MANG, I1, SIG1
NO 360 IANG=1,MANCE
READ5(1,1) ANGI(IANG)
READ5(1,1) SIGE(IXX, IANG, I),I=1,1,1SIGI
READ5(1,1) EF(IXX, IANG, I),I=1,1,1SIGI
WRITE(6,361) ANGI(IANG)
WRITE(6,362) (SIGE(XX, IANG, I),I=1,1,1SIGI)
360 WRITE(6,363) (EF(IXX, IANG, I),I=1,1,1SIGI)
361 FORMAT(5X,*=*,F5.2)
362 FORMAT(1X,*=*,SIG1(I,J,K)==,8F10.5)
363 FORMAT(1X,*=*,EF(I,J,K)==,8F10.5)
364 FORMAT(1X,*=*,DATA FOR CAVITATION THICKNESS==,8F10.5)
C CURV FLITING OF CAVITY THICKNESS BY POLYNOMIAL.
CALL FITCAY(MH1, SIGE, EF, I1, SIG1, I1, ACC)
NO 731 I1=2,MH1
NO 731 J1=1,4
731 WRITE(6,731) (ACC(I,J,K),K=1,3)
750 FORMAT(I10, I10, I10, I10, I10)
2.5 DATA READ IN ARE PASSED TO FUNCTION CLCD(MH1, SIGE, ANGLE, ICLCD).
C CALCULATE BETan=ATAN(VA/OMEGASMALLR)=ATAN(XJJ/(PAl*XX(I)))
C CALCULATE ALFA=BTAG-ALFC-BETA.
C DIRIFILED GRAM=00=BTAG.
NO 125 K1=2,MH1
GRAM(K1)=00=BTAG(K1)
PSI=KJJ/(PAI*X(K1))
BET=ATAN(PSI)
BET(K1)=PRE=PERL
ALFC(K1)=BTAG(K1)-BET(K1)
D-15
ALFI(K1)=RETAG(K1)-BETA(K1)-ALFC(K1)
WRITE(*,156) (BETA(I),I=2,MM1)
156 FORMAT(/3X,#12E15.6*)&,(F10.5,1X))
WRITE(*,157) (ALFI(I),I=2,MM1)
157 FORMAT(3X,#12E15.6*)&,(F10.5,1X))

C CALCULATE SIGV FROM AN ASSUMPTION FOR THE FIRST ITERATION.
C VVAV(I) FOR ITERATIONS GREATER THAN AND EQUAL TO 2 WAS CALCULATED.
C AFTER EACH ITERATION.
C VVAV(I) FOR ITERATION I IS CALCULATED BY AN ASSUMPTION OF VI=V*COX(ALFI).
C
DO 126 IS=2,MM1
  XALF(IS)=CONV*XALFI(IS)
  XRET(IS)=CONV*BETA(IS)
  VVAV(IS)=COX(XALF(IS))/SIN(XRET(IS))
  VV1=1./VVAV(IS)
  VVI2=VV1**2

126 SIGV(IS)=SIGV+(VV1+VV1)**2
WRITE(*,128) (SIGV(I),I=2,MM1)
128 FORMAT(3X,#12E15.6*)&,(F10.5,1X))

C FIND GAMMA(GG(I)) DISTRIBUTION NORMALIZED TO GG/(0*VI)).
C FOR THE FIRST ITERATION.
MAM=MANGLE
MAM=MAM-1
MAM=MAM-2
DO 130 NG=2,MM1
  AL=ALFE(NG)
  SC=SIGV(NG)
  WRITE(*,130) NG,SC

130 CALL CAVINNG(SC,AE,ACC,ANG2,ISIG1,SCN,EE,MANG1)
WRITE(*,130) SCN
S=SC/SCN
S=SQRT(S)
SC=SCN
TCLCD=0
CLS(NG)=CLCD(NG,SC,AC,ICLCD)
TCLCD=1
CLS(NG)=CLCD(NG,SC,AC,ICLCD)

C SUMMATION TIME CLCD ALSO INTERPOLATES NORMALIZED GAMMA(CIRCULATION).
TCLCD=2
130 RGGG(NG)=CLCD(NG,SC,AC,ICLCD)*XX(NG)*VIVA(NG)*SR/ZZ
WRITE(*,130) (CLS(I),I=2,MM1)
WRITE(*,130) (CDS(I),I=2,MM1)
WRITE(*,130) (GGG(I),I=2,MM1)
131 FORMAT(2X,#12E15.6*)&,(F10.5,1X))
CONTINUE
GGG(I)=0.
GGG(I)=0.

C FIND COEFFICIENTS OF FOURIER SINE TRANSFORMATION FOR GAMMA DISTRIBUTION.
C PHI(I)=TRANSFORMED COORDINATES FOR XX(I).
PHI(I)=0.
#1(MM2)=PAI
XAL=1.-XH
XH1=XH
XH2=XH
MM=5.*XH
MM=12.*IPH=2.*MM1
AL=(XH1+4.*XH1)/XH1

D-16
**FOR INTEGRATION**

**NFIL=NFIL/N2+1**

**gPAACL=PAI/NFILON**

**NF11=NFILON+1**

**NO=145 ICH=1, NF11**

**g(ICH)=SPACG*(ICH-1)**

**chi=1.E-7**

**CALL SPACI((PH1,...,GG2,YY,2,5,3,3,SOUT,591,892,MM2,NFI1,1,C1)**

**nD=145 ISK=1,MM**

**4AG=ISEK*SPACG**

**4AHI=FALF(ARG)**

**MM=FRE(I4]**

**CM=FOAM(ARG)**

**AFF=AMHI*(SOUT(1)-SOUT(NF11)/COS(ISEK*PAI))**

**$2K=0,$**

**$2K=0,$**

**NO=510 $2K=1,NFI1**

**T3A=K2+2**

**T3=K2+2**

**$2K=S2K+SOUT(12)*SIN(ISEK*PAI)**

**$T2=(K2,EO,NFI1) GO TO 510**

**$2K=S2K+SOUT(12)*SIN(ISEK*PAI)**

**510 CONTINUE**

**$2K=81H=S2K**

**$2K=GMH=S2K**

**$M1(ISEK)=SPACG*(AFF+B32K+G32K)**

**146 CONTINUE**

**$M1(ISEK)=GM1(ISEK)+Z PAI**

**WRITE(6,1A7) ISK,GH(ISEK)**

**147 FORMAT(5X,3HGM(12,J2M)=,E14.7)**

**225 CONTINUE**

**145 CONTINUE

**C CALCULATE INDUCTANCE FACTORS.**

**C CALCULATE SMALL IA AND IN.**

**C THEN CALCULATE INA AND INT.**

**NO=150 IM=9,NM**

**XEQ=(BLTAG(ID)+ALFE(IU))*CONV**

**TR11/TAN(XEQ)**

**TR12=TR1**

**C PHI(I)=XX(I) HAS BEEN DONE BEFORE.**

**NF1=NF1+1**

**NO=151 NX=1,NF1**

**XAMX=NX**

**TF(NX,GE,2) GO TO 320**

**NO=326 NF1=1,NFI1**

**C CHANGE OF VARIABLES FROM S(I) TO XP(I).**

**XP=XHI-XHA*COS(S(NF1)**

**XIN=XXP/X(XIN)**

**XI=1./XINO**

**AB1=ZI*(XINO,1)**

**AB2=AB1*TR1**

**C CALCULATE AL AND AZ, THEN B1 AND B2.**

**YPP=XI2=TH1**

**YPP2+YPP**

**$C=1.+YPP2**

**AN=1.*TR12**
ACS=SOMT(AC)
ROX=SOMT(DP)
NK1=BCS=BG0
NK2=(BDS+1)*(RCS+1)
NK3=(BDS+1)*(BCS-1)
ADK=N5*ALNG(NK2/NK3)
AA1=DX1=ADK
AA2=AA1
NF1=80/8C
NF2=6/RP+/L+
FGA1=EXP(ZZ+AA1)
FGA2=EXP(ZZ+AA2)
E1=1./FGA1-1.
E2=1./FGA2-1.
$IF(X>4.5,L1)GO TO 327$
AE2=ALUG(1.0+E21)
AB1=DF10*(E21+0.5*DF2*AE2/ZZ)
XI4(AID,NT)=AB1*(1.+BB2)
YIT(AID,NT)=AB1*BBP
$GO TO 326$
$327AE2=ALUG(1.0+E11)$
AB1=DF10*(E11+0.5*DF2*AE1/ZZ)
YIT(AID,NT)=AB1*BB1
$326 CONTINUE$
$329 CONTINUE$
C2KA=0.
C2KAEE=0.
C2KA=0.
C2KAT=0.
C2KAA=0.
C2KAT=0.
ON 350 (CH1=NF1H1
T2=ICH1+2+1
T2A=C80E2
C1T1=COS(NXAS(12))
C2K1=C2KA+XIA(AID,12)+C1T1
C2K1=C2K1+XIT(AID,12)+C1T1
$IF(XC00,E=.NF1HJ GO TO 350$
C2K1=COS(NXAS(12A))
C2KAA=C2KA+XIA(AID,12A)+C1T2
C2KAT=C2KAT+XIT(AID,12A)+C1T2
$340 CONTINUE$
C2K2=C2K2+5*(XIA(AID,NFIH1)+COS(NXASPAI)+XIA(AID,1))
C2K2=C2K2+5*(XIT(AID,NFIH1)+COS(NXASPAI)+XIT(AID,1))
AR1=NXASPACE
ARN=LRT(AR1)
GNN=FGAM(ARN)
SIN(AID,NX)=SPACE+(BNM*C2KAA+GNN*C2K1)/PAI
SIN(AID,NX)=SPACE+(BNM*C2K1+GNN*C2KAT)/PAI
$IF(X<.62,E=)SINA(AID,NX)=2.*SINA(AID,NX)$
$141 IF(X>.62,E=)SINT(AID,NX)=2.*SINT(AID,NX)$
$140 CONTINUE$

C FIND HA1(A) ANH MMT1(A)

D-18
\begin{verbatim}
\textbf{L NEWTON ITERATION LOOP} \textbf{***************}
\textbf{READY TO EXECUTE NEWTON METHOD.}
\textbf{L REMARK: ALFE(I) AND GM(I) BY A NEW SET OF VARIABLES,}
\textbf{L XH(I)=ALFE(I), I=1, MM AND XN(I)=GM(I), I=MM1, MMX.}
\textbf{ITERA=1}
\textbf{ITERA=MAXI}=1=WRITE
\textbf{CONTINUE}
\textbf{997 WRITE(6,661) ITERA}
\textbf{CONTINUE}
\textbf{661 FORMAT(///,10X,\ldots,\ldots,\ldots,13)}
\textbf{NO 525 INE=1,MM}
\textbf{INE1=INE1+1}
\textbf{ALFE(INE1)=ALFE(INE1)}
\textbf{CHU(INE)=CHU(INE)}
\textbf{SIGO(INE)=SIGO(INE)}
\textbf{VIVA(INE)=VIVA(INE)}
\textbf{525 XH(INE)=ALFE(INE1)}
\textbf{NO 526 INE=1,MMX}
\textbf{INEP=INEP+1}
\textbf{526 WRITE(4,531) (XH(I),I=1,MMX)}
\textbf{530 FORMAT(1X,2(N9.5,1X))}
\textbf{NO 170 L4=2,MM}
\textbf{WAVA(L4)=0.}
\textbf{WTV(L4)=0.}
\textbf{NO 171 L4=1,MM}
\textbf{XPAG=L4=1,MM}
\textbf{WAVA(L4)=WAVA(L4)+XPAG*HMA(L4,L4+LP)}
\textbf{171 WTV(L4)=WTV(L4)+XPAG*HMT(L4,L4+LP)}
\textbf{170 WTV(L4)=WTV(L4)/XAL}
\textbf{TF(ITERA LT1,ITERA} \textbf{CONTINUE) GO TO 224}
\textbf{WRITE(6,254) (WAVA(I),I=1,MM1)}
\textbf{WRITE(6,255) (WIVA(I),I=2,MM1)}
\textbf{CONTINUE}
\textbf{326 CONTINUE}
\textbf{299 FORMAT(1WX,*WAVA(I)=,8(E10.5,1X))}
\end{verbatim}
295  FORMAT(1UX,*MTV=(1)=*8(E10.3,1X))

L F(I) IS AN ARRAY OF FUNCTION AND P(I,J) IS PARTIAL DERIVATIVES,************
   00 530 IFM=1,MM
   01 IFN=IN+1
   02 RGA=0(IFM)*ALFE(IFM)*ACUNV
   03 T8G(IFN)=TAN(BGAE(IFN))
   04 P(XJ(IFN)=PA*X*(IFN)/XJ)
530  P(IFN)=T8G(IFN)*P(XJ(IFN)-MTVA(IFN))-(1.+NAYA(IFN))
   05 531 IFN=IN+1,MM
   06 CM=CMG+GM(IFN)*SIN(AG)
   07 CM=CMG+GGG(IFN)
   08 WRTEG(533)(F(I),I=1,MM)
533  FORMAT(1X,*F(I)=12(F9.5,1X))

L COMPUTATION UP PARTIAL SCRIVATIVES,************
   00 535 IPM=1,MM
   01 535 IPM=1,MM
   02 536 IPM=1,MM
   03 TF(IPM,IPB)=GO TO 536
   04 P(IPM,IPB)=0
   05 GO TO 535
536  APA=IPM+1
   01 CBGE=CBGE(IPM)
   02 CRGAE=CBGAE(IPM)
   03 A=IPM(IPM)*MTVA(IPM)/CRGAE
   04 P(IPM,IPB)=A
   05 GO TO 535
535  CONTINUE
   00 535 APA=1,MM
   01 535 APA=1,MM
   02 537 APA=1,MM
   03 536 APA=1,MM
   04 537 APA=1,MM
   05 TF(IPM,IPB)=GO TO 536
   06 P(IPM,IPB)=0
   07 GO TO 535
536  APA=IPM+1
   01 CM=CMG(IPM)
   02 SSRM=HMT(IPM)+1,MM
   03 IPM*IPB/IPB/XAL
537  P(IPM,IPB)=SSRM=HMA(IPM,IPBS)*IPBS/XAL

L TRG(T) AND P(XJ(I) ARE ALREADY CALCULATED,
   00 535 APA=1,MM
   01 535 APA=1,MM
   02 537 APA=1,MM
   03 536 APA=1,MM
   04 537 APA=1,MM
   05 TF(IPM,IPB)=GO TO 536
   06 P(IPM,IPB)=0
   07 GO TO 535
536  APA=IPM+1

L D(KL) ALF(A) IS A FINITE DERENCE METHOD.
   00 539 ICL=2,MM
   01 NEP=ABS(ALFE(NL))
   02 540 IF=ALFE(ICL)+IF
   03 540 IF=ALFE(ICL)-IF
   04 ACG=CC(YNZ(ICL))
   05 CALL CAV(NN(IL,SC,ACX,ACG,ANG2,ISIG1,SCNX,EQ,MANG))
   06 CALL CAV(NN(IL,SC,ACY,ACG,ANG2,ISIG1,SCNY,EQ,MANG))
   07 SCX=SCNX
   08 SCY=SCNY
   09 SRX=SCX/SCY
   10 SYR=SCY/SCY
   11 SRX=SH(M(SR))
   12 SYR=SH(M(SR))
   13 TLCD=2
   14 CCGX=CLCD(TL,SCX,ACX,ICLCO)*SHX

D-20
VIVA(KF)=CH*MTYA(KF)*CBI
180 SIGV(KF)=SIGVA/VIVA(KF)**2
181 IF (ITERA.LT.ITERAW) GO TO 227
182 WRITE(6,191)
183 WRITE(6,194) (ALFG(I),I=2,MM1)
184 WRITE(6,197) (8BT(I),I=2,MM1)
185 WRITE(4,198) (ALF(I),I=2,MM1)
186 WRITE(4,199) (SIG(I),I=2,MM1)
187 WRITE(4,191) (SIGV(I),I=2,MM1)
227 CONTINUE
191 FORMAT('1UX,****-NEW VALUES OF ALFE, VIVA,ETC.,-------')
192 FORMAT(1X,A16V(2I8,1E16.5))
193 FORMAT(1X,*ALF(I)=,8(E10.5,1X))
194 FORMAT(1X,*SIG(I)=,8(E10.5,1X))
195 FORMAT(1X,*SIGV(I)=,8(E10.5,1X))
196 FORMAT(1X,*SIGF(I)=,8(E10.5,1X))
197 FORMAT(1X,*RETAIL(I)=,8(E10.5,1X))

C CALCULATION OF PROPELLER FORCES AND EFFICIENCY.**************
C CALCULATE UNIF/VA, ALFINF AND GGG.
NO 190 KF=2,MM1
AE=ALF(KT)
SC=SIG(KT)
CALL CAVINO(KT,SC,AE,ACC,ANG2,ISIG1,SCN,EE,MANG1)
RI(KT)=SC/SCN
SRI(KT)=SQRT(SRI(KT))
IF (ITERA.LT.ITERAW) GO TO 775
WRITE(6,365) SC
WRITE(4,366) SCN
365 FORMAT(5X,*CAVITATION NO. BEFORE CORRECTION=,F10.5)
366 FORMAT(5X,*CAVITATION NO. AFTER CORRECTION=,F10.5)
775 CONTINUE
SC=SCN
TCLCD4=SC
CLS(KT)=CLCD4(KT,SC,AE,ICLD)
TCLCD4=1
CS(KT)=CLCD4(KT,SC,AE,ICLD)
TCLCD=2
GGG(KT)=XX(KT)*VIVA(KT)*CLCD(KT,SC,AE,ICLD)*SRI(KT)/IZ
100 CONTINUE
7F(ITERA.LT.ITERAW) GO TO 228
WRITE(4,198) (GGG(I),I=2,MM1)
WRITE(4,199) (CS(I),I=2,MM1)
WRITE(4,194) (CUG(I),I=2,MM1)
193 FORMAT(5X,*CUG(I)=,8(E10.5,1X))
194 FORMAT(5X,*CS(I)=,8(E10.5,1X))
195 FORMAT(5X,*GGG(I)=,8(E10.5,1X))
228 CONTINUE
C DATA FOR CUBIC SPLINE METHOD AND GAUSS QUADRATURE.
XT(1)=1.
YCT(1)=1.
XT(MM2)=1.
YCT(MM2)=1.
FCP(1)=0.
FCP(MM2)=0.
FDP(1)=0.
FDP(MM2)=0.
RE=5/13.*YASHIP*1.5
RE1L=1.6
C Reynolds Number Based on Propeller Radius and Advance Speed

\[ C = \frac{V}{D} \]

\[ \text{Reynolds Number} = \frac{\rho D V}{\mu} \]

**Friction Drag**

\[ D = \frac{C_D \cdot V^2}{2} \]

\[ C_D = \frac{F_D}{\frac{1}{2} \rho V^2 A} \]

\[ A = \pi D^2 / 4 \]

\[ F_D = C_D \cdot \frac{1}{2} \rho V^2 A \]

\[ \rho = \text{Density} \]

\[ V = \text{Velocity} \]

\[ D = \text{Diameter} \]

\[ \mu = \text{Viscosity} \]

\[ \pi = \text{Pi} \]

\[ A = \text{Area} \]

\[ C_D = \text{Drag Coefficient} \]

\[ F_D = \text{Drag Force} \]
&FRICTION DRAG=%F10.5,2X,%CPo=%F10.5,5X,%EFFICIENCY WITH FRICTION
&DRAG=%F10.5

545 CONTINUE
   IF(ITERA.EQ.MAXIT) GO TO 999
   ITERA=ITERA+1
   GO TO 755
   IUK=1
   GM(IUK)=XH*GMU(IUKA)+(1.-XH)*GM(IUKA)
   ALF(IUK)=XH*ALF(IUKA)+(1.-XH)*ALF(IUK)
   ALF(IUK)=BETA(IUK)-ALF(IUK)
   RALF(IUK)=BETA(IUK)-ALF(IUK)
   IF(ITERA.LE.ITERAH) GO TO 230
   WRITE(6,182) (ALF(I),I=2,MM1)
182 FORMAT(1X,*ALF(I)=,8(F10.5,1X))
   CONTINUE
   GO TO 987
999 STOP

END
FUNCTION CLDCL(1:S:Y:ILD)

C THIS SUBROUTINE INTERPOLATES CL,CD AND CIRCULATION FROM INPUT DATA AT X, SIGMA

C AND ALFA, DEPENDING ON ILCD=0,1,2.

COMMON PAI, CONV1, CONV2, SIGM1(10)
COMMON SIGC(10:5:15), CD1(10:5:15), AM1(5:CD1(10:5:15)
COMMON MANGE, SIGS, GGCI(10:5:15)
DIMENSION P(3,10), G(3,10), STUN(10:23), ALE2(10), A(10)

C I SPECIFIES XX(I).
C $ IS CAVITATION NO. FOR ALG2 TO BE EVALUATED.
C $ IS EFFECTIVE FLOW INCIDENCE ANGLE FOR ALG2 TO BE EVALUATED.

CHISSQ=0.0
ISIGA=ISIG-1
MANGE=MANGE-1
NAM=MANGE-1

IF(SIGM1(I)) NAM=M1
DI=1 K=1
NAM=MANGE
L=5
IF(SIGM1(I,K)) GO TO 10
IF(SIGC(I,K)) GO TO 11
IF(SIGM(I,K)) GO TO 10
IF(SIGC(I,K)) GO TO 11

20 SIGC(I,K)=S
IF(GE.X=IT) GO TO 20
IT=1
IF(SIGC(I,K)) STOP
GO TO 21

20 SIGC(I,K)=S
SIGM1(I,K)=1
IT=IT+1

22 P(1,1)=SIGC(I,K,MSC)
P(1,2)=SIGC(I,K,ISB)
P(1,3)=SIGC(I,K,ITT)
P(1,4)=SIGC(I,K,131)
IF(IKD.EQ.1) GO TO 30
IF(IKD.EQ.2) GO TO 32
IF(IKD.EQ.3) GO TO 34
GO TO 51

30 P(2,1)=CL1(I,K,MSC)
P(2,2)=CL1(I,K,ISB)
P(2,3)=CL1(I,K,ITT)
P(2,4)=CL1(I,K,131)
GO TO 51

32 P(2,1)=GGCI(I,K,MSC)
P(2,2)=GGCI(I,K,ISB)
P(2,3)=GGCI(I,K,ITT)
P(2,4)=GGCI(I,K,131)
GO TO 51

34 CONTINUE
DO 55 KA=1,4
55 P3(K,K)=P3.02
NM=
MM=3
CALL LSQULR(P,N,K,CHISSQ,STNR)
ALE2(K)=A(1)
MM=MM-1
DO 56 K=1,MM
56 K=K+1

D-25
56 ALE2(K)=ALE2(K)+A(KM1)*S**6
   GO TO 1
10 ISC=1
    IS=2
    ITT=3
    IS=4
   GO TO 22
11 ISC=ISIG-3
    IS=ISIG-2
    ITT=ISIG-1
    ISIG=ISIG
   GO TO 22
1 CONTINUE
C NOW FOUND ALE2(MANGLE) CORRESPONDING TO ANG1(MANGLE).
C THUS DETERMINE LCLD AND CIRCULATION CORRESPONDING TO A.
   DO 57 KL=1,3
      DO 57 KN=1,MAN
         IF(KL.EQ.1) U(KL,KN)=ANG1(KN)
         IF(KL.EQ.2) U(KL,KN)=ALE2(KN)
      57 IF(KL.EQ.3) U(KL,KN)=0.002
      MANA=MANA+1
      MANB=MANB+2
      CALL LSQUAR(U,MANA,MANB,A,CHISQ,STOK)
      CLEOD=1
      DO 58 KT=1,MANB
         XT=XT+1
      58 CLEOD=CLEOD+A(AXT)*B**XT
      RETURN
   END
SUBROUTINE FITCAV(M1,SE,EE,M1,I1,ACC)
DIMENSION GE(10,5,5),EE(10,5,5),ACC(10,5,5),P(50,5),Q(50,5)
NO 1 I=2,M1
NO 2 J=1,M1
NO 2 K=1,I1
TF(L,EE,1) P(K,L)=EE(I,J,K)
TF(L,EE,2) P(K,L)=I1.
2 TF(L,EE,3) P(K,L)=-3C(I,J,K)
CALL DETERM(P,I1,OR)
NO 3 L=1,I1
NO 4 LP=1,I1
q(LP,LP)=P(LP,LP)
q(LP,LP)=SC(I,J,LP)*EE(I,J,LP)
CALL DETERM(P,I1,DC)
ACC(I,J,LP)=UL/OR
NO 5 LP=1,I1
5 CONTINUE
1 CONTINUE
RETURN
END
SUBROUTINE CAVINO(NG,SC,AE,ACC,A2,I1,SCN,EE,M1)
DIMENSION ACB(5)
DIMENSION ACC(10,5,5),A2(10),EE(10,5,5),XX(5),P(50,5),Q(50,5)
FR1=1.16-3
FR2=ER1
IPL=0
A=[A+1-1
I=1
2 IF(AE.LE.A2(I)) GO TO 1
IF(I.EQ.M1) GO TO 1
I=I+1
1 CONTINUE
P=I-2
IF(I.EQ.1.1.OR.I.EQ.2) K=1
IF(I.EQ.M1) K=M1-2
30=K+2
6 FIND EEX
DO 3 K=K,K3
L A MUST BE LESS THAN ONE.
A=1.5
10 A=A-0.001
IF(L.EQ.1.1) WRITE(6,40)
40 FORMAT(1X,16=F5.3)
IF(L.EQ.1.1) STOP
CALL FANC(PF,B,SC,ACC,NG,KI,I1)
IF(PF.GE.0.) GO TO 11
A=B
GO TO 10
11 CONTINUE
A=AA
A=B
CALL MSEC(A,B,FR1,ER2,X,J,NG,KI,SC,ACC,I1)
C CONVERT C(U+R X) INTO SIGMA.
S1=+X
S2=X**2
S3=S1/S2
S4=S3**2
SIGMA=SC*S4
3 X(I(T))=SIGMA
C We Have CAVITATION NO. As A Func. Of ANG2.
C Put IN A POLYNOMIAL FORM.
NO 20 IL=1,3
NO 20 IT=1,3
20 P(IS,IT)=A2(K+IS-1)**(IT-1)
CALL OTERM(P,3,08)
NO 21 ID=1,3
NO 22 IE=1,3
O(IE,ID)=P(IE,ID)
22 P(IE,ID)=X*X(K+IE-1)
CALL OTERM(P,3,0C)
CH(ID)=OC,08
NO 25 IE=1,3
23 P(IE,ID)=Q(IE,ID)
21 CONTINUE
CNE=ACB(I1)
NO 24 IL=1,2
I1=I1+1
24 CNE=SCN+ACR(I1)*AE**IL
RETURN
END
SUBROUTINE FANC(F, CE, SC, ACC, NG, KI, II)
DIMENSION ACC(10, 5, 5)
RHS1 = ACC(NG, KI, 1) * CE + ACC(NG, KI, 2)
RHS2 = EL + ACC(NG, KI, 3)
RHS = RHS1 / RHS2
AA1 = 2. - CE
AA2 = 2. - CE
AA3 = AA1 / AA2
AA4 = AA3 # 2
XLHS = SC # AA4
F = XLHS - RHS
F = F
RETURN
END
SUBROUTINE MOSLC(A,B,ER1,ER2,X,J,NG,KI,SC,ACC,II)
DIMENSION ACC(I0,5,5)
J=0
XI=A
XZ=B

TF(J,GE,800) GO TO 8
CALL FANC(PFX1,X1,SC,ACC,NG,KI,II)
CALL FANC(PFX2,X2,SC,ACC,NG,KI,II)
XZ=X1+(X2-X1)*PFX1/(PFX1-PFX2)
CALL FANC(PFX3,X3,SC,ACC,NG,KI,II)
TF(PFX3)1,2,3
1 X2=X3
X3=X1
TF(A-E)10,10,11
10 Y=X3-ER1
TF(Y,LE,0) Y=0.
GO TO 12
11 Y=X3+ER1
12 CALL FANC(PFY,Y,SC,ACC,NG,KI,II)
TF(PFY) 5,2,2
3 XI=X3
X2=X2
TF(A-A)20,20,21
20 Z=X3+EM1
GO TO 22
21 Z=X3-EM1
22 CALL FANC(PFZ,Z,SC,ACC,NG,KI,II)
TF(PFZ)2,2,5
GO TO 4
2 PP=ABS(PFX3)
TF(PP=ER2) 6,6,4
6 X=X3
GO TO 7
8 WRITL(4,9) J
9 FORMAT(1X,2M10.4,E3)
STOP
7 RETURN
SUBROUTINE DETERM (A,N,0)

DETERM REVISID 02-28-73

REAL M
DIMENSION A(50,50),SAVEA(50,50)

IF ( N .EQ. 1) GO TO 46
C = 1.
MM = N
DO 9 J = 1,MM
DO 9 I = 1,MM

9 SAVEA(I,J) = A(I,J)
K = 1
GO TO 13

12 K = K + 1
13 I = I + 1
L = K
GO TO 17

16 I = I + 1
17 IF (ABS(SAVEA(I,K)) .GT. ABS(SAVEA(L,K))) L = I
IF (I .NE. MM) GO TO 16
IF (L .EQ. K) GO TO 28
J = K

ROW INTERCHANGE
GO TO 23

22 J = J + 1
23 SAVEA(J,J) = SAVEA(K,J)
SAVEA(K,J) = SAVEA(L,J)
SAVEA(L,J) = SAVEA(J,J)
IF (J .NE. MM) GO TO 22
C = -C

28 I = K + 1
GO TO 31

30 I = I + 1
31 CONTINUE
IF (SAVEA(K,K) .EQ. 0.) GO TO 48
M = SAVEA(I,K) / SAVEA(K,K)
SAVEA(I,K) = 0.
J = K + 1
GO TO 36

35 J = J + 1
36 SAVEA(I,J) = SAVEA(I,J) - M * SAVEA(K,J)
IF (J .NE. MM) GO TO 35
IF (I .NE. MM) GO TO 30
IF (K .NE. (MM+1)) GO TO 12
N = 1.
ON 43 I = 1,NN
J = I
D = D * SAVEA(I,J)
IF (ABS(D) .LT. 1.E-36) GO TO 48
CONTINUE
D = D * C
RETURN
46 D = A(1,1)
RETURN
48 D = 0.
WRITE (0,51)
RETURN

51 FORMAT(/'Sx'#ERROR MESSAGE FROM DETERM, S/
'Sx#MATRIX IS SINGULAR. DETERMINANT SET = 0.# //)

END
SUBROUTINE LSQUR(DATA, NUMBER, N, A, CHISQ, XM)

L S Q U R  DATE OF OBJECT DECK 06-09-72
L S Q U R  PROVIDES A POLYNOMIAL FIT (OF DEGREE N=1) TO THE FUNCTION
L Y = \( f(x) \). THE \( 3 \times N \) DATA MATRIX IS A MATRIX OF DATA VECTORS OF
L THE FORM \( x(1) \), \( y(1) \), \( \sigma(1) \).
L CALLING SEQUENCE CALL LSQUR(DATA, NUMBER, N, A, CHISQ, XM)
L NUMBER= NUMBER OF DATA POINTS
L N= DEGREE OF POLYNOMIAL + 1
L A= ARRAY; DIMENSION N, CONTAINING THE COEFFICIENTS OF THE POLY-
L NOMIAL DEFINED AS \( a(1) \times x^{0} + \ldots + a(n) \times x^{n-1} \)
L CHISQ= REAL VARIABLE. IF CHISQ=0, WHEN ENTERING LSQUR, ERROR
L MESSAGES IF ANY, WILL BE PRINTED. UPON NORMAL RETURN TO THE CALLER
L CHISQ CONTAINS SOME POSITIVE NUMBER. IF DURING INVERSION, AN ERROR
L HAS BEEN ENCOUNTERED, CHISQ IS SET TO A NEGATIVE VALUE,
L -1. IF THE MATRIX WAS SINGULAR,
L -2. IF AN OVERFLOW OR DIVIDE CHECK OCCURRED.
L COMM/LSQUR/TMP(100)
L COMM/LSQUR/SING/NORM
L LOGICAL R NIT E
L THE ARRAY XM IS FOR WORKING STORAGE. IT SHOULD BE DIMENSIONED IN THE
L MAIN PROGRAM BY THE FOLLOWING STATEMENT
L WHERE NMAX IS THE MAXIMUM VALUE OF N IN THE CALLING PROGRAM.
L DIMENSION DATA(3, NUMBER), A(N), XM(N), I1
L SAVE SPACE
L EQUIVALENCE (N2, KK), (HR, R), (XX, S)
L MPS=1.0E-6
L R N I T E=. F A L S E
L IF THE INPUT VALUE OF CHISQ IS 0. ALLOW PRINTING OF ERROR MESSAGES.
L TF (CHISQ=0.0, R N I T E=. T R U E
L TF R N I T E=. F A L S E
L TF R N I T E=. T R U E
L N2I=S2*I+1
L N2I,S2*I+1
L N2I,S2*I+1
L TF (NRMH) S1=I
L NO 2 I=1, NUMBER
L TEMP(I)=DATA(1, I)
L TF (NRMH=1) I=1, S
L AVE=0.0
L SIGMA=S0
L NO 210 I=1, NUMBER
L AVE=AVE+DATA(1, I)
L SIGMA=SIGMA+DATA(1, I)*S
L 210 CONTINUE
L AVE=AVE/NUMBER
L SIGMA=SQRT(SIGMA/NUMBER-AVE*AVE)
L NO 220 I=1, NUMBER
L DATA(1, I)=(DATA(1, I)-AVE)/SIGMA
L 220 CONTINUE
L SIGMA=SIGMA/SIGMA
L AVE=AVE/SIGMA
L 1 N0 12 I=1, N
L NO 12 J=2I, N23
L XM(J)+E0
L COMPUTE THE NUMNITS IF THE DATA
L N2I=S2*I
L NO 26 I=1, NUMBER
L PR=(1.0/DATA(3, I))*SIGMA
L XM2I2=XM(I, N2I)+PR

D-32
XX = DATA(2, I)*RR 
XM(1, N23) = XM(1, N23) + XX 
TF (N, EQ=1) GO TO 26 
NO 21 J = 3, M2 
RR = R*DATA(1, I) 
TF (J, GT=20) GO TO 22 
XM(J, N21) = XM(J, N21) + RR 
NO TO 21 
22 XM(J, N22) = XM(J, N22) + RR 
21 CONTINUE 
NO 25 J = 2, N 
XM = R*DATA(1, I) 
25 CONTINUE 
XM(J, N23) = XM(J, N23) + XX 
26 CONTINUE 
42 XM(J, J) = XM(K, N21) 
NO TO 31 
52 XM(J, J) = XM(K, N22) 
51 CONTINUE 
CALL DOUBLE PRECISION MATRIX INVERSION ROUTINE 
TF (N, NE=1) GO TO 35 
XM(1, I) = CI(0: 60) / XM(1, I) 
TF (N, NE=0) GO TO 89 
A(I) = XM(1, I) * XM(1, I) 
GO TO 37 
35 CALL MLHSHAR(N, XM, XM(1, N21), ITER, EPS, A, ITEST, 0, XM(1, N+1)) 
TF (IEST = 0) GO TO 80 
C COMPUTE CHISQUARE FOR RESULTING FIT 
37 CHISQ = 0.0 
NO 70 I = 1, NUMBER 
R = A(I) 
TF (N, NE=1) GO TO 69 
R = R*DATA(I, I) 
NO 68 J = 2, N 
R = R*DATA(I, I) 
68 CONTINUE 
CHISQ = CHISQ + ((8 - DATA(2, I))/DATA(3, I)) ** 2 
TF (IEST = 0) GO TO 80 
C PRIMARY MESSAGES AFTER INVERSION OF THE MATRIX XM (H IN THE WRITE-UP) 
30 CONTINUE 
SING = S. 
TF (RITE) WRITE (6, 800) I 
FORMAT (/1X,NU CONVERGENCE?, I5, ITERATIONS?) 
CHISQ = 2.0 
79 TF (NUM, EQ=0) OR (NUM, EQ=1) RETURN 
I = 1 
NO 6 G = I**L 
XM(I, I) = AV**I 
6 XM(I, J) = 0.0 
XM(I, J) = I, EQ 
XM(N, N) = 0.0 
NO 7 I = 1, L 
XM = I**1 
NO 8 J = 2, K 
8 XM(J, J) = XM(J, J) + XM(J-1, J) 
X = I**1
DO 9 J=K,N
9 A(I)=A(I)+A(J)*XM(J-I+1,2)*XM(J-I,1)

DO 7 CONTINUE
7 A(I)=A(I)*SIGMA*(I-1)
RETURN
END

DATA NUM/*0/ NEND

SUBROUTINE ML3RAR(N, A, MTX, V, ITER, EPS, F, IT, INCH, A)
LOGICAL KIF
COMMON /LQUMP/ SING
N = UNDER OF MATRIX
M = DIMENSION ARRAY OF CUERFICIENTS
V = RIGHT-HAND VECTOR
ITER = MAXIMUM NUMBER OF ITERATIONS DESIRED
EPS = TOLERANCE FOR CONVERGENCE (.GE. 1.E-7)
F = RESULTING VECTOR
IT = UUPID FROM ROUTINE SPECIFYING NUMBER OF ITERATIONS ACTUALLY D
INEM (FIRST CALL) SET INEM = NE. 1
(LATER CALLS) IF THE MATRIX IS UNCHANGED AND ONLY THE COLUMNS VECTOR B IS CHANGED, THEN SET INEM = 1
DIMENSION A(N,N), V(N), F(N), A(N,N), X(50), IDX(50), XT(50)
INEW = INEM = FALSe.
IF (ITNEQ(LT,0)) RITE = TRUE.
ITERABS(ITER)
IT = 0
DO 9 I = 1,N
X(I) = V(I)
F(I) = 0.0
9 CONTINUE
XI = N = 2.
IF (INEM .EQ. 1) GO TO 101
10 N = 1, N
10 J = 1, N
A(I,J) = AMTX(I,J)
CONTINUE
DO 12 J = 1,N
12 YDX(I) = 1
SIG = 0.
DO 60 I = 2, N
PARTIAL PIVOTING, CHECK FOR MAX ELEMENT IN (I-1)ST COLUMN.
IMJ = 1, I
AMJ = ABS(A(I,J,IMJ))
JX = IMJ
16 J = J
AMJ = ABS(A(J,IMJ))
YDX(J) = AMJ
IF(YDX(J) .LE. AMJ) GO TO 16
AMJ = AMJ
JX = J
CONTINUE
10
YDX(J) = 0.0
101 GO TO 10
102 GO TO 20
MOVE THE VX WITH MAX A(J,IMJ) TO (IMJ)ST ROW.
DO 18 K = 1,N
IF(J=K) GO TO 20
T = A(IMJ, K)
\[
\begin{align*}
A(i,m,k) &= A(j,m,k) \\
10 \quad A(j,m,k) &= T \\
15 \quad T = I \times (i,m) \\
20 \quad I = I \times (i,m) \\
25 \quad I = I \times (j,m) \\
30 \quad S = 1.0 \\
35 \quad CONTINUE \\
40 \quad IF (A(i,m) .EQ. 0.0) GO TO 200 \\
45 \quad NO 55 \quad J = i, M \\
50 \quad CX = A(j,m) / A(i,m) \\
55 \quad NO 60 \quad K = i, N \\
60 \quad A(j,k) = A(j,k) - CX \times A(i,m,k) \\
65 \quad CONTINUE \\
70 \quad CONTINUE \\
75 \quad CONTINUE \\
80 \quad CONTINUE \\
85 \quad CONTINUE \\
90 \quad CONTINUE \\
95 \quad CONTINUE \\
100 \quad FINISHED \\
105 \quad RETURN \\
110 \quad CONTINUE \\
115 \quad CONTINUE \\
120 \quad CONTINUE \\
125 \quad CONTINUE \\
130 \quad CONTINUE \\
135 \quad CONTINUE \\
140 \quad CONTINUE \\
145 \quad CONTINUE \\
150 \quad CONTINUE \\
155 \quad CONTINUE \\
160 \quad CONTINUE
\end{align*}
\]
\( y(i) = v(i) = r \)

170 CONTINUE

181 IF (IG1 .LE. 0.0) GO TO 62

C PERMUTE X BEFORE PERFORMING FORWARD PASS.

182 IF (IG1 .LT. 0) PERMUTE X

GO TO 62

182 \( x(l) = x(i) \)

GO TO 62

\( x(i) = x(k) \)

GO TO 62

200 IF (IITE) WRITE (6,510) IM1

510 FORMAT(/I6, 'ERROR RETURN FROM MLSRARE DIAGONAL TERM t,12,

1 REDUCED TO ZERO.+//')

RETURN

END
FUNCTION FALF(X)
   IF(X.EQ.0.) GO TO 1
   RX=SIN(2.*X)
   X2=X**2
   X3=X**3
   RX=SIN(X)**2
   FALF=1./X**3/(X2+2.*X3)-2.*X2/X3
   GO TO 2
1   FALF=0.
2   RETURN
END
FUNCTION FgAM(x)
  IF(x.EQ.0.) GO TO 1
  s=sin(x)
  c=cos(x)
  x2=x*x
  y1=s*x2
  y2=c*x2
  FgAM=4.*((y2/y1-y1/cx/x2)
  GO TO 2
  1 FgAM=1.333333333
  2 RETURN
END
FUNCTION FAFT(X)
1 IF(X.EQ.0.) GO TO 1
2 C2=CUB(X)
3 X2=X**2
4 S3=SIN(2.*X)
5 X3=X**3
6 Y3=X3*X
7 P3=F2*(1./C2+X2/12)-92*X3/13
8 GO TO 2
9 FAF=0.60606060606067
10 RETURN
11 END
SUBROUTINE SPLINE(X,Y,DY,S2,S3,T,SS,SS1,SS2,LIM,N,C1)

SPLINE INTERPOLATION

DIMENSION X(LIM),Y(LIM),DY(LIM),S2(LIM),S3(LIM),T,LIM

DO 10 J=1,LIM
10 DY(J)=Y(J+1)-Y(J)/(X(J+1)-X(J))

DO 20 J=2,LIM
20 S2(J)=S2(J-1)+SS(J-1)*DY(J)

DO 30 J=3,LIM
30 S3(J)=S3(J-1)+SS(J-1)*DY(J)

S2(LIM)=S2(2)
S3(LIM)=S3(2)

OMEGA=1.017968
DATA MAXITER/20/
ITER=0

ETA=0.

ITER=ITER+1

DO 40 J=2,LIM
40 S2(J)=S2(J-1)+(S2(J-1)-S2(J-2))*OMEGA

ETA=MAX1(ETA,ABS(OMEGA))

IF(ETA GT C1.AND.ITER LT MAXITER) GOTO 25

DO 50 J=1,LIM
50 Y(J)=Y(J+1)-Y(J)*DY(J)

ENTRY ABOCD

DO 60 J=1,N
60 IF(T(J)=T(1)) 58,17,55

IF(T(J)=T(N)) 80,59,5A

IF(T(J)=T(1)) 60,17,57

GOTO 56

PRINT 44,J

FORMAT(1X,5X,4,ARGUMENT INIT IIF OF RANGE*)

GOTO 61

I=LIM

56

D-40
\begin{verbatim}
60  I=I-1
17  HT1=T(J)-x(I)
    HT2=T(J)-x(I+1)
    PRUD=HT1*HT2
    S32(J)=S2(I)+HT1*S3(I)
    DSQ5=(S2(I)+S3(I)+S2(J))*1.666666667
    S31(I)=DY(I)+(HT1+HT2)*DSQ5*PROD5S(1)*1.666666667
    S3(I)=Y(I)+HT1*DY(I)+PRUD*DSQ5

61  CONTINUE

RETURN

END
\end{verbatim}
| 64 | 0243502926 | 0729931218 | 1214628193 | 1696444204 |
| 63 | 2174236427 | 2664871622 | 3113287220 | 3577201583 |
| 62 | 4022701579 | 4463660173 | 4894031457 | 5312794640 |
| 61 | 5718954632 | 6115535952 | 6489654713 | 6852361311 |
| 60 | 7198815922 | 7528549873 | 7839725589 | 8132653151 |
| 59 | 8406292963 | 8659993982 | 8893154460 | 9105221371 |
| 58 | 9295691721 | 946413749 | 9610087997 | 9733268278 |
| 57 | 9833362539 | 9910133715 | 9963401168 | 9995050417 |
| 56 | 0486906707 | 0485754674 | 0485097250 | 0487993886 |
| 55 | 0475980557 | 0469618128 | 0462847966 | 0454916279 |
| 54 | 0445906582 | 0435373245 | 0424735151 | 0412656332 |
| 53 | 0390537411 | 0385301352 | 0370512805 | 0354722133 |
| 52 | 0338651618 | 0320579284 | 0302346571 | 0283967262 |
| 51 | 0267766979 | 0243527026 | 0222701738 | 0201348231 |
| 50 | 0179515158 | 0157260395 | 0134630479 | 0111681395 |
| 49 | 0088675982 | 0065044580 | 0041470333 | 0017832807 |

| 150 | 5 | 20 | 30 | 10 | 1 | 10 |
| 14 | 29 | 17 | 2 | 0 | 0 |
| 13 | 7 | 6 | 2 | .9 |
| 12 | 19.67 | 17.60 | 15.97 |
| 11 | 4.65536 | 4.55185 | 4.26112 |
| 10 | 4.57775 | 5.53333 | 6.21720 |
| 9 | .5120 | .5120 | .5120 |
| 8 | .5000 | .5000 | .5000 |
| 7 | .4900 | .4900 | .4900 |
| 6 | .4800 | .4800 | .4800 |
| 5 | .4700 | .4700 | .4700 |
| 4 | .4600 | .4600 | .4600 |
| 3 | .4500 | .4500 | .4500 |
| 2 | .4400 | .4400 | .4400 |
| 1 | .4300 | .4300 | .4300 |

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D-45
7 TYPICAL OUTPUT DATA

MAX1=150 MM= 5 NF=30 TORD= 1

PROPPELLER DIAMETER IN INCH= 14.00000
ADVANCE SPEED IN FT/SEC. = 25.00000

NFILON=100

NUMBER OF BLADES = 3.

MID/TIP = .200

SPEED COEFF. = .4400

CAVITATION N 0. BASED ON VA = .61700

XX(I) = .40000 .60000 .70000 .80000 .90000

BETA(I) = 31.23000 72.36000 19.67000 17.60000 15.97000

ALF(I) = 6.05306 5.67879 4.83473 4.72926 4.41962

SOL(I) = .91200 .59400 .47900 .36500 .24400

VTV(I) = 2.81680 4.18272 4.85920 .5.3907 6.21803

---NEW VALUE OF ALF---

---CAVITATION NO. BEFORE CORRECTION---

---CAVITATION NO. AFTER CORRECTION---

---CAVITATION NO. BEFORE CORRECTION---

---CAVITATION NO. AFTER CORRECTION---

---CAVITATION NO. BEFORE CORRECTION---

---CAVITATION NO. AFTER CORRECTION---

---CAVITATION NO. BEFORE CORRECTION---

---CAVITATION NO. AFTER CORRECTION---

ECC= .05275 .03951 .03756 .03771 .03267

CD(I) = .26114 .16011 .14141 .11705 .09421

CD(I) = .28204 .01460 .01118 .00804 .00693

EFFICIENCY   = .54059 EFFICIENCY WITH FRICTION DRAG = .54059
# Development of an Off-Design Predictive Method for Supercavitating Propeller Performance

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**Abstract:**
The present method incorporates a two-dimensional supercavitating cascade theory into a propeller lifting line theory for downwash induced angle corrections. In addition, cavitation number corrections are made in order to account for choking conditions which occur in flow passages of propeller blades having cascade configuration. The results are compared with experimental data for JNB Model 3770 supercavitating propeller.