APPLICATIONS OF HIGHER MODE SURFACE WAVES

TECHNICAL REPORT NO. 10
VELA NETWORK EVALUATION AND AUTOMATIC PROCESSING RESEARCH

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Prepared for
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Alexandria, Virginia 22314

Sponsored by
ADVANCED RESEARCH PROJECTS AGENCY
Nuclear Monitoring Research Office
ARPA Program Code No. 6F10
ARPA Order No. 2551

8 October 1976

Acknowledgment: This research was supported by the Advanced Research Projects Agency, Nuclear Monitoring Research Office, under Project VELA-UNIFORM, and accomplished under the technical direction of the Air Force Technical Applications Center under Contract Number F08606-76-C-0011.
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20. continued

In order to estimate the effect of the seismic source structure, the ratio of first higher to fundamental Love wave amplitudes was found experimentally and compared to the theoretical prediction. Agreement was better than for the spectral fitting method but was still short of being satisfactory.
ABSTRACT

Amplitudes of first higher mode Love waves from seismic events in North America were measured on the North American continent and corrected for attenuation, structure, and distance. The corrected amplitudes were fit to a source model to estimate the source depth. The high value of the attenuation coefficient and lack of a good seismic model prevented good agreement between known and calculated depths.

In order to estimate the effect of the seismic source structure, the ratio of first higher to fundamental Love wave amplitudes was found experimentally and compared to the theoretical prediction. Agreement was better than for the spectral fitting method but was still short of being satisfactory.

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SECTION I
INTRODUCTION

It is well known that earthquakes and underground nuclear explosions radiate energy in the form of surface waves which propagate at velocities which are the roots of equations relating frequency, seismic velocity, and propagation velocity (Harkrider, 1970). There are an infinite number of roots to this equation both for Love and Rayleigh waves, and in principle every seismic event is accompanied by energy propagating in each of the modes corresponding to these roots. In practice, only the fundamental mode (i.e., $R_0, L_0$) with lowest velocity is excited with large enough amplitude to be useful for most detection and discrimination problems. This report describes work attempting to obtain information about the source from hitherto unused higher mode surface waves.

Two uses for higher modes are examined here, both employing the first higher mode Love wave (denoted $L_1$). This mode was chosen because its amplitude is expected to be larger than that of any other higher mode Love wave, and because it varies more in amplitude with period than does the first higher mode Rayleigh wave, the other possible candidate for this study.

A number of studies (Turnbull, et al., 1973; Turnbull, 1976) have described a procedure for fitting measured seismic spectral amplitudes to theoretical source spectra. Seismograms recorded at stations with good azimuthal distribution about the source (if possible) are narrowband filtered to estimate the Fourier component attributable at that period to the arrival of interest (Alexander, 1963). These spectral amplitudes are corrected for the earth's attenuation, structure, and for distance, and the mean square difference between them and those predicted by a model is minimized as a function...
of the model parameters. The parameter most sensitive to this fitting process is the source depth, so that this method can be used to discriminate between deep and shallow events.

The radiation pattern for higher mode surface waves is similar to that of the fundamental mode, since the various modes are characteristic of the medium rather than the source (Turnbull, 1976). The amplitude response of the earth is different for different modes, due to their different wavenumbers, and this response can be calculated for a given earth model. Finally, the attenuation of the earth may be different for various modes, and must be found by experiment. Then higher mode amplitudes may be used to calculate source depth in the same way as are fundamental mode amplitudes.

It was attractive to look for another discriminant based on higher modes, and the amplitude ratio $L_1/L_0$ was investigated for this purpose. This ratio was calculated for source models at various orientations for a range of periods from 6 to 15 seconds, for several earth models. Under the appropriate models the ratio's level changed with orientation but its general features as a function of period did not, and the ratio displayed features below 15 seconds which occur at lower periods for lower velocity surface layers. Measurement of this ratio should then give an indication of the seismic velocity at the source.
SECTION II
EXPERIMENTAL PROCEDURE

The spectral fitting method outlined in the previous section was applied to data collected by instruments and events from the same continent, in order to simulate the counterevasion problem as closely as possible. The parameter determined by this method is the source depth, which is an important discriminant between underground nuclear explosions and earthquakes. This experiment is described in Subsection A.

A second use for the same kind of data is presented in Subsection B on the ratio \( L_1/L_0 \). This ratio is expected to yield information about the source structure given that the event is shallow. This information is supplementary to that required for discrimination.

A. SPECTRAL FITTING

Data for this experiment were collected by the Special Data Collection System stations. The stations are listed in Table II-2, which also gives their locations. Four events in North America were examined and are listed in Table II-1. Another event on the Burma-India border and given in Table II-1 was also examined using data recorded by the Seismic Research Observatory at Mashhad, Iran; the Alaskan Long Period Array; and at NORSAR.

First, transverse surface waves from the Nevada Test Site (NTS) explosion TYBO were examined for higher mode Love waves, using both narrowband filters and a maximum entropy filter. It was hoped that the increased resolution of the maximum entropy filter as compared to standard Fourier techniques would lead to better estimates of amplitudes.
### Table II-1

**EARTHQUAKE PARAMETERS**

<table>
<thead>
<tr>
<th>Event</th>
<th>Date</th>
<th>Coordinates</th>
<th>$M_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TYBO</td>
<td>14:00:02</td>
<td>37.3 N</td>
<td>4.4</td>
</tr>
<tr>
<td></td>
<td>14 May 1975</td>
<td>116.5 W</td>
<td></td>
</tr>
<tr>
<td>MAST</td>
<td>13:00:01</td>
<td>37.3 N</td>
<td>5.5</td>
</tr>
<tr>
<td></td>
<td>19 June 1975</td>
<td>116.3 W</td>
<td></td>
</tr>
<tr>
<td>California Earthquake</td>
<td>01:38:51.4</td>
<td>34.6 N</td>
<td>4.9</td>
</tr>
<tr>
<td></td>
<td>1 June 1975</td>
<td>116.4 W</td>
<td></td>
</tr>
<tr>
<td>Wyoming Earthquake</td>
<td>18:54:16.5</td>
<td>44.9 N</td>
<td>5.6</td>
</tr>
<tr>
<td></td>
<td>30 June 1975</td>
<td>110.3 W</td>
<td></td>
</tr>
<tr>
<td>Burma-India Border</td>
<td>05:08:02</td>
<td>25 N</td>
<td>5.6</td>
</tr>
<tr>
<td>Earthquake</td>
<td>12 Dec 1975</td>
<td>96 W</td>
<td></td>
</tr>
</tbody>
</table>
### TABLE II-2

**STATION DESCRIPTIONS**

<table>
<thead>
<tr>
<th>Site</th>
<th>Location</th>
<th>Coordinates</th>
</tr>
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<tbody>
<tr>
<td>CPSO</td>
<td>McMinnville, Tennessee</td>
<td>38.6 N</td>
</tr>
<tr>
<td></td>
<td></td>
<td>34.6 W</td>
</tr>
<tr>
<td>FN-WV</td>
<td>Franklin, West Virginia</td>
<td>38.5 N</td>
</tr>
<tr>
<td></td>
<td></td>
<td>79.5 W</td>
</tr>
<tr>
<td>HN-ME</td>
<td>Houlton, Maine</td>
<td>46.2 N</td>
</tr>
<tr>
<td></td>
<td></td>
<td>68.0 W</td>
</tr>
<tr>
<td>WH2 YK</td>
<td>White Horse, Yukon</td>
<td>60.7 N</td>
</tr>
<tr>
<td></td>
<td></td>
<td>135.0 W</td>
</tr>
</tbody>
</table>
However, it was found very difficult to identify higher modes given only the power as a function of time and period, which is the form of the maximum entropy filter's output. Therefore, this approach was abandoned in favor of narrowband filtering.

Next the fundamental mode energy was identified on traces filtered at periods between 6 and 35 seconds. The apparent velocity of each packet of energy so chosen was found by dividing the epicentral distance by the travel time, and plotted as a function of period in Figure II-1.

Also shown in this figure are the dispersion curves for the Hamilton-Healy and Gutenberg-Bullen earth models. While neither model fits the data well, the shape of the Hamilton-Healy curve is closer than that of the Gutenberg-Bullen curve. For this reason, and because it was developed for NTS, the Hamilton-Healy model was adopted for this study.

Filtered seismograms were then examined for pulses of energy arriving near the times predicted by the Hamilton-Healy model for higher modes. Such pulses, if they were clear and distinct, were identified as \( L_1 \) mode surface waves. Often, due to the low amplitude of the \( L_1 \) mode, a clear identification was not possible. In other cases the \( L_0 \) and the presumed \( L_1 \) energy overlapped, making the \( L_1 \) amplitude determination difficult. In these cases the identification and measurement of \( L_1 \) amplitudes became to some degree subjective, and this contributed to the experimental difficulties in this study.

Examples of filtered seismograms are shown in Figure II-2. The upper trace is the explosion TYBO as recorded at CPSO. The outputs of filters centered at 10 and 6 seconds are shown in the next two traces. Group velocities of presumed arrivals are indicated on the figure. While the identification of \( L_0 \) and \( L_1 \) is fairly unambiguous at 6 seconds, the presence of \( L_1 \) can only be inferred at 10 seconds by the asymmetrical shape of the \( L_0 \) energy, and its amplitude is highly uncertain.
Those $L_1$ group velocities which could be determined with some certainty are plotted versus period in Figure II-3, along with the dispersion curves for $L_1$ under the Gutenberg-Bullen model and the Hamilton-Healy model. Neither model fits the data better than the other over the wide range of periods, but the shape of the Hamilton-Healy curve over the period range of interest, from 6 to 5 seconds, is closer to the data than that of the Gutenberg-Bullen curve. This provides some additional support for the choice of the Hamilton-Healy model to represent the propagation paths examined here.

Stations CPSO, FN-WV, and HN-ME are roughly in line with NTS, as required for the method of determining the energy attenuation coefficient of seismic waves described by Sun (1976), which assumes that the energy in a group of waves varies with distance $x$ as $e^{-k_x}$. Then a plot against $x$ of the quantity

$$2 \ln A + \ln \sin \Delta$$

where the term involving the epicentral distance $\Delta$ in degrees corrects for the curvature of the earth, has slope $-k_e$. Such a plot for $L_1$ amplitudes from TYBO for periods of 6, 7, 8, 9, and 10 seconds, where available, is shown in Figure II-4. There is no reason to believe from the data that the slope is a function of period, so the best straight line through all the points above 7 seconds was found. The slope of this line is an average value of $-k_e$ although its intercept is difficult to interpret.

The best least mean squares fit to these data gave $k_e = 4.8 \times 10^{-3}$/km. It is difficult to assign a probable error to this number, because methods for computing standard errors assume that fluctuations in the data are random, and the data of Figure II-4 appear to contain systematic errors. As a rough estimate we may suppose that the true slope lies somewhere between the value found for the path CPSO-HN-ME, where $k_e = 4.7 \times 10^{-3}$.
and the path FN-WV - HN-ME, where $k_e = 6.0 \times 10^{-3}$. Then our estimate of $k_e$ is $4.8 \pm 0.1 \times 10^{-3}$/km. This value is an order of magnitude higher than those found by other workers (Tryggvason, 1965), but is consistent with the fact that the group velocity for this model is relatively low. The $Q$, or quality factor, corresponding to this $k_e$ is about 20.

This large value of $k_e$ is responsible for a great deal of difficulty in using $L_1$ amplitudes in North America. It enters the expression for the amplitude corrected for attenuation through:

$$Y = Ae^{kx/2}$$

where $Y$ is the corrected amplitude, $A$ is the measured amplitude, and $x$ is the distance. The fractional error in $Y$ due to errors in $k_e$ is

$$\frac{dY}{Y} = \frac{k_e x}{2} \frac{dk_e}{k_e}$$

and for the value of $k_e$ found here $\frac{k_e x}{2}$ is about 5 for typical epicentral distances. Consequently, fractional errors in the corrected amplitude will be about five times those in the attenuation coefficient. Errors in the attenuation coefficient, in turn, are expected to be large due to the uncertainty in $L_1$ amplitudes.

Furthermore, the value quoted for $k_e$ is valid only for the path where it was measured, and can only be assumed to be valid for paths to other stations and from the source to the closest station. The use of such other stations is essential to take advantage of the information contained in the radiation pattern of the seismic energy.

This problem is not a serious one for earth structures where $k_e$ is small, since the correction factor given in Equation (II-1) is nearly one.
Accepting the value of $k_e$ quoted, amplitudes from CPSO and WH2YK were corrected for instrument response, attenuation, and distance over the range of 6 to 10 seconds, where $L_1$ energy from TYBO was observable. The corrected spectra were fit to a double couple source model at various strikes, slips, dips, and at various depths in a Hamilton-Healy earth structure, as described by Turnbull (1976). The model with the smallest mean square deviation from the observations at each depth was chosen, and this mean square deviation plotted as a function of depth in Figure II-5.

Good agreement between model and observations should be detectable by a minimum in Figure II-5, but no such dip is visible there. The model is nearly an equally bad (or good) fit to the data at every depth. Similar results were obtained when $L_1$ amplitudes from the NTS event MAST and the California earthquake of 1 July 1975, were fit to the model.

This lack of depth discrimination in the method is due to two causes. First, the large value of $k_e$ found above means that $L_1$ will have a low signal-to-noise ratio at teleseismic distances, and this makes it difficult to measure $k_e$ accurately. Consequently, large errors will appear in the corrected amplitudes, which will then fit no model well.

Another possible source of difficulty is that the model may not contain enough details of the earth structure at the source region. However, such fine structure would presumably lead to a sharpening of any minimum in the residuals of Figure II-6, whereas no minimum was found. Consequently, we conclude that problems with the data are great enough that the use of such fine structure is unjustified.

An event on the Burma-India border was examined at Mashhad (Iran), ALPA, and NORSAR in the hope that the paths to these stations would display less attenuation than paths across North America. Unfortunately, no $L_1$ energy was observed in the filtered seismograms. Presumably, the relatively complicated travel paths obscured higher modes at these stations.
FIGURE II-5
RESIDUALS FOR FIT TO TYBO $L_1$ AMPLITUDES
B. \[ \frac{L_1}{L_0} \text{ RATIO} \]

Use of the ratio \[ \frac{L_1}{L_0} \] should eliminate problems associated with inaccurate determination of \( k_e \) if \( k_e \) is the same for \( L_1 \) and \( L_0 \). The data of Figure 11-6 for TYBO \( L_0 \) amplitudes lead to a value of \( k_e \) above 7 seconds of \( 4.9 \times 10^{-3} \)/km. The data are somewhat better clustered about the regression line than those for \( L_1 \), and we estimate the range of \( k_e \) to be from \( 4.8 \times 10^{-3} \) to \( 5.7 \times 10^{-3} \) per kilometer. Consequently we conclude that the value of \( k_e \) is the same for fundamental and first higher mode Love waves for this transmission path.

The ratio \[ \frac{L_1}{L_0} \] was calculated according to a double couple model at 0.5 km depth for a number of fault plane orientations for both the Gutenberg-Bullen and Hamilton-Healy models. The absolute level of the ratio between 6 and 15 seconds depended on fault plane orientation, but the location of a peak at 7 seconds for the Hamilton-Healy model and a shoulder near 9 seconds for the Gutenberg-Bullen model was independent of orientation. This ratio for two fault plane orientations is shown in Figure 11-7.

Experimental tests of this method were undertaken by examining this ratio for the events TYBO and the Wyoming earthquake measured at stations CPSO and WH2YK. The results are plotted in Figures 11-8 and 11-9, respectively. The results for MAST are similar to those for TYBO.

For both explosions there is a peak in \[ \frac{L_1}{L_0} \] at 7 seconds period as predicted by the theory. For the Wyoming earthquake the results are less clear. The absence of \( L_1 \) energy below 9 seconds, and its presence above 15 seconds, are in contradiction to the model prediction. Consequently, while these results are more encouraging than those of the spectral estimation method, they are not conclusive proof that the shapes of the curves in Figures 11-8 and 11-9 are controlled by the structure at the source.
Figure II-6

$L_0$ ENERGY ATTENUATION
FIGURE II-7

$L_1/L_0$ FOR TWO EARTH MODELS AND TWO FAULT PLANE ORIENTATIONS

Hamilton-Healy
Gutenberg-Bullen
FIGURE II-8
$L_1/L_0$ FOR TYBO
FIGURE II-9

$L_1 / L_0$ FOR WYOMING EARTHQUAKE
Good signal-to-noise ratio and a travel path within North America from a source region with high seismic velocity are required for a fair test of the method.
SECTION III
CONCLUSIONS

Surface waves from a number of events have been examined for the presence of higher mode energy. Amplitudes of the first higher mode Love wave have been measured, corrected for distance, instrument response, and attenuation, and fit to a model of the fault plane and propagation path. The results suggested that this mode, the most promising for the purpose, has no capability for depth discrimination, at least in North America. The reasons for this failure are the high value of the energy attenuation coefficient and the lack of a good source model.

In order to discriminate between different source structures, the ratio $L_1/L_0$ was calculated as a function of period both experimentally from the observed data and theoretically from two source models. Agreement between observation and theory was more encouraging than for the spectral fitting method, but was still short of satisfactory.

It is recommended that any further study of the utility of higher modes for discrimination be directed toward data from a region of low attenuation coefficient and well known simple structure.
SECTION IV
REFERENCES


