A STUDY OF THE METHOD OF SURFACE FIT APPROXIMATIONS FOR AIRCRAFT OPTIMAL DESIGN

THESIS

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A STUDY OF THE METHOD OF 
SURFACE FIT APPROXIMATIONS 
FOR AIRCRAFT OPTIMAL DESIGN 

Master's thesis 

THESIS 

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by 
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Preface

This study was the first attempt at providing a usable set of computer routines (which employed current concepts for the selection of the design space, second order surface fit approximations, and a conjugate gradient minimization algorithm) to be used by the Air Force Flight Dynamics Laboratory Design Branch in the initial design phase of an aircraft. Several proposals and programs were available but the lack of information as to the use and application of these concepts led to this study. Computer routines that had been working were found not to be compatible with the Wright-Patterson Air Force Base computer system and had to be modified and rewritten. This study attempted to eliminate these problems and to demonstrate the usefulness of the surface fit approximation scheme.

I would like to thank Mr. Jim Parker and Captain Russ Morrison of the Air Force Flight Dynamics Laboratory Design Branch for their assistance in providing me with input information for the mission simulation used in this study.

I would also like to thank Dr. K. S. Nagaraja of the Air Force Flight Dynamics Laboratory Aerodynamics Branch and Major Steve Koob of the Aeromechanical Engineering Department of the Air Force Institute of Technology who provided the thesis topic and valuable aid during the course of this study.

Finally, I would like to express my appreciation to Major Gerald Anderson of the Mechanics Department of the Air Force Institute of Technology who agreed to sponsor this thesis topic.

Captain Martin L. Marler
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Abstract

A study was made to demonstrate the feasibility of using surface fit approximations in the design analysis of an advanced tactical fighter. Design variables were selected and a design space was defined based on a simple latin square method. The take-off-gross-weight (TOGW) and landing distance (DISTL) for each configuration were determined by use of a computer program which simulated the required mission. A regression analysis was performed on this data to obtain a quadratic surface fit representing TOGW and DISTL as functions of the design variables. A conjugate gradient technique was employed to find the minimum TOGW subject to constraints on DISTL within the design space. This value was compared to the minimum obtained by running all possible combinations of design variables through the mission simulator. It was concluded that the surface fit approximation based on the latin square design space did yield accurate results and required fewer computer simulations.
A STUDY OF THE METHOD OF
SURFACE FIT APPROXIMATIONS
FOR AIRCRAFT OPTIMAL DESIGN

I. Introduction

The Air Force Flight Dynamics Laboratory (AFFDL) Design Branch became interested in several proposals concerning the selection of an airplane engine by optimization of surface fit approximations. The original studies had been conducted by the Aero Propulsion Laboratory (APL) at Wright-Patterson Air Force Base. The results of these studies showed that considerable savings could be made in engineering and computer time by using limited quantities of simulation data (Ref 7:599). The AFFDL Design Branch was involved in a design scheme similar to those used in the aforementioned studies. The Design Branch wanted to investigate the use of surface fit approximations based on the latin square design selector and integrate these ideas into their design analysis.

In particular, the Design Branch wanted to apply the method to an aircraft that would be capable of accomplishing an interdiction mission that might be required of an advanced tactical fighter. The vehicle was to have a one man crew, carry 5500 lbs of stores, and be capable of a supersonic range of up to 350 nautical miles.

The purpose of this study was to demonstrate the feasibility of using surface fit approximations in the design analysis of an advanced tactical fighter. The design process that was considered consisted of six major steps. The first step required the selection of independent
design variables (engine, airframe, and/or mission) and dependent performance or response functions. The performance functions would be calculated based on the independent design variables. The independent variables selected were the wing loading (airframe variable), aspect ratio (airframe variable), vehicle thrust to weight ratio (engine variable), and the supersonic dash range (mission variable). The dependent performance functions were the take-off-gross weight and the landing distance. The next step was to choose values for the independent variables. This was done by establishing the upper and lower limits on each variable and then dividing that range into an equal number of parts (equal to the number of independent variables). The ranges of the independent variables were chosen so that the performance functions could be adequately represented by quadratic equations in the regions of interest. The third step was to select from all possible combinations of the independent parameters that subset which would still be representative of the complete design space. This reduces the number of data points needed and also reduces the computational time for deriving the performance function values. This was accomplished by a statistical technique called the simple latin square design selector. This technique joins together permutations of the various values of the independent variables in such a way that each variable is stepped through all its values once every "1" data points where "1" is the number of values for each variable. The fourth step was to compute the dependent performance parameters by inputting the selected design variables into a mission simulator and outputting the performance values. The available mission simulator was that used by the AFFDL Design Branch (the
Computerized Initial Size Estimate - CISE). This program is used to give initial values of the design parameters at the "zero level" of analysis, i.e., even before a drawing of the configuration is proposed (Ref 4:2). The program performs a weight oriented sizing process to produce the basic physical characteristics of the vehicle. The next step was to perform a regression analysis on the performance data to yield quadratic surface fits representing the performance functions in terms of the independent design variables. Finally, a constrained minimization of the performance function of primary interest was performed and located those values of the independent parameters that yielded the constrained minimum.

The number of independent variables in this study was taken to be four with each variable taking on five values each. The number of variables was limited so that all possible design combinations could be calculated and input into the mission simulation. The minimum design point from the complete set could then be compared to that found by using the surface fit approximations.
II. Design Space Selection

General

The first part of the study was to identify those variables that might be significant in the design of an aircraft given a general mission. A distinction is made between dependent and independent variables. The dependent variables are selected by considering the total design goal. The TOGW is generally used as one dependent variable since it can be calculated from the various other parameters by using a mission simulation. The independent variables are chosen based on the ability of the designer to change these parameters and their impact on the total design objective. These parameters (both dependent and independent) are typically engine variables (bypass ratio, thrust to weight ratio, etc.), airframe variables (wing loading, aspect ratio, leading edge sweep angle, etc.), or mission parameters (range, altitude, loiter ability, etc.).

In this study, it was desirable to have at least one variable from each of the three possible sets of airframe, engine, and mission. The mission parameter selected was the supersonic dash range (RN). It was chosen because it could be varied in the CISE program and is a parameter of interest in the advanced tactical fighter. The engine parameter that was chosen was the thrust to weight ratio (TW), i.e., the ratio of the installed engine thrust to the vehicle weight. Two airframe parameters were selected. The wing loading (WOS) and aspect ratio (AR) were chosen from this group. They represent two characteristics that are available for use in the CISE program.
The next step in the design space selection was to determine a reasonable range for each variable. The ranges were chosen so as to allow a second order polynomial in terms of the independent variables to approximate the dependent parameters (to be discussed in Chapter IV). The ranges in this study were based on previous aircraft of the same type having a comparable mission. The range of each variable was then divided into four equal segments so that all four variables would have five possible values. The possible values of the four parameters were chosen as follows:

- **(Airframe Variable) WOS**
  - 70
  - 80
  - 90
  - 100
  - 110

- **(Airframe Variable) AR**
  - 2.5
  - 3.0
  - 3.5
  - 4.0
  - 4.5

- **(Engine Variable) TW**
  - 0.5
  - 0.7
  - 0.9
  - 1.1
  - 1.3

- **(Mission Variable) RN**
  - 150
  - 200
  - 250
  - 300
  - 350

where WOS has units lbs/sq ft and RN has units of nautical miles. There are 5^4 possible combinations that made up the complete design space. These 625 design points were input to the CISE program which gave as output 625 TOGW's and DISTL's.

**Latin Square Design Selector**

The computer time required to run 625 mission simulations was approximately 600 seconds. The need to reduce this time and yet maintain some degree of accuracy in the final optimal design point led to the use of a statistical design selector. The latin square design selector will select out of the entire design space an appropriate subset which yields a good representation of the complete space. The representation is good in the sense that approximating equations for the dependent variables based on the latin square subset will predict...
the actual or true value to within five percent. The simple latin square technique uses a method (to be discussed later) which yields a sequence of values for each variable. The sequence is formed by joining together permutations of the various values of the variables in such a way that each variable is stepped through all its values once every "1" data points, where "1" is the number of values for each variable.

The number of data points selected by the latin square technique depends on the number of independent variables. If there are "n" design variables then there will be \((n+1)^2(n+2)/2\) terms in the regression equation and the regression routine will require at least this many data points. The simple latin square will generate \((n+1)^2\) design points that have design values of each independent variable equally spaced on the normalized interval \((-1,1)\) (Ref 2:4-5). Accurate fits may not be obtained with this small number of points so the simple latin square technique was modified to generate more data points (this was done to gain more accuracy and at the same time the time for computer simulations of the mission was increased). Table I contains the number of data points required by the modified latin square for a given number of variables.

The simple latin square technique uses field algebras, that are derived from modulo (or residues class) arithmetic using the algebra of integers, to establish "n" matrices (where again "n" is the number of independent design variables) that are used to generate the design points (Ref 6:57). Table II contains the matrices for the four variable case.
<table>
<thead>
<tr>
<th>No. of Variables</th>
<th>No. of Design Points</th>
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<td>2</td>
<td>9</td>
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<td>3</td>
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<td>4</td>
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<td>16</td>
<td>561</td>
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<tr>
<td>17</td>
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Table II
Latin Square Matrices for Four Variables

<table>
<thead>
<tr>
<th>Variable No.</th>
<th>Matrix</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>0 1 2 -2 -1</td>
</tr>
<tr>
<td>1</td>
<td>1 2 -2 -1 0</td>
</tr>
<tr>
<td></td>
<td>2 -2 -1 0 1 2</td>
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<tr>
<td></td>
<td>-2 -1 0 1 2 -2</td>
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<td>-1 0 1 2 -2</td>
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<td>-1 0 1 2 -2</td>
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<td>1 2 -2 -1 0</td>
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<td>-2 -1 0 1 -2</td>
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<tr>
<td>2</td>
<td>0 1 2 -2 -1</td>
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<td>-1 0 1 2 -2</td>
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<td>-1 0 1 2 -2</td>
</tr>
<tr>
<td>3</td>
<td>0 1 2 -2 -1</td>
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<tr>
<td></td>
<td>-2 -1 0 1 2</td>
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<td>-2 -1 0 1 2</td>
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<td>1 2 -2 -1 0</td>
</tr>
<tr>
<td></td>
<td>-1 0 1 2 -2</td>
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</tbody>
</table>
The data space is obtained by finding the mid-point and an appropriate step size for each variable. The first element of the data space is obtained by multiplying each variable's step size by element (1,1) of that variable's matrix and adding to the mid-point. The second design point is obtained by multiplying each variable's step size by element (2,1) and adding to the mid-point. This procedure is continued until the space is complete.

Several problems are possible when using the simple latin square technique. The most significant problem is that the selected set of data points may not be independent as shown in Fig. 1 (Ref 6:59). The bad fit does not adequately represent the space, while the good fit does. The lack of independence allows the possibility of significant cross-correlations building up between the variables; however, these cross-correlations would be detected during the surface fit procedure. These problems did not appear in this study.

Inputs and outputs to the latin square computer program are described in Appendix A.

Fig. 1. Two Examples of Latin Square Design Space (2-Dimensional Cross-Section Views)
III. Mission Simulation

The mission profile used in this study is given in Fig. 2 from Ref. 5. This mission along with the independent design variables was used as input to a mission simulation program used by the AFFDL Design Branch. The CISE program was used to calculate the TOGW required to accomplish the specified mission given the set of design variables.

The mission was based on a typical interdiction mission which an advanced tactical fighter might be required to fulfill. The vehicle was to perform the mission with a one man crew and was to carry 5500 lbs of stores. The only requirement on the combat performance was the fuel allowance of fifteen percent of the total initial fuel weight (Ref 5). This requirement was met by changing the number of turns in the combat segment of the mission.

The CISE program was developed to be used as a design tool at the "zero level" of design, i.e., even before a drawing of the configuration is prepared (Ref 4:2). The program performs a weight oriented sizing process to produce the basic physical characteristics of the vehicle. The procedure was set up to use DO loops for the four design variables. An iterative process is accomplished for each combination of variables. The iteration assumes an initial TOGW and then calculates a geometry based on the guessed weight and input variables. The input mission is then "flown" and fuel needs are calculated to meet the requirements of takeoff reserves, combat penalties, and landing reserves. The final process in the iteration is to calculate the major weight components that include the wing, the body, the tail, landing gear, systems and
1. Warm up and takeoff
2. Climb to 45000 ft
3. Cruise at .8M for 200 NM
4. Accelerate to 1.6M
5. Climb to 50000 ft
6. Cruise at 1.6M for variable distances (150, 200, 250, 300, 350 NM)
7. Combat (15% of initial fuel for combat penalty)
8. Drop stores
9. Cruise at 1.6M at 55000 ft for same distance as Item 6
10. Decelerate to .8M and cruise back at 45000 ft
11. Loiter at sea level for 20 minutes and land

Fig. 2. Mission Profile
equipment, and payload. These group weights are summed to yield a TOGW at the end of the iteration cycle. When this weight is within a specified tolerance of the initial guess, then the process is terminated and another design point is initiated. If the weight is not within the tolerance, then it is used as the initial guess and the process is repeated.

The procedure that would normally be followed in the design process using surface fit approximations would be to input the Latin square design space of independent variables (in this particular study the 45 design points of Table III) to the mission simulator which would output the dependent variables (in this particular study the TOGW's and DISTL's of Table IV). These steps would be followed by a surface fit procedure and an optimization routine (both to be discussed in later chapters). However, this normal procedure was deviated from slightly in this study.

It was desired to have a basis for judging the "goodness and accuracy" of the surface fit approximations. In order to accomplish this goal, the entire design space consisting of all 625 design points were input to the CISE program which output 625 TOGW's and DISTL's. Those points which had been selected by the Latin square technique (Table III) were included in the 625 design points; hence, the 45 output functions (TOGW's and DISTL's) for the Latin square design space were already available from having run the simulation for the complete design space.

There would have been a slight difference in the values of the output functions for the 45 Latin square design points had they been the only data points input to CISE as is the case in the normal design procedure described above. This is due to the weight tolerances and initial guess in the iteration process. These differences were negligible.
The TOGW's for the selected design space are given in Table III in the results section.

Input and output requirements for the CISE program are given in Appendix B.
IV. Surface Fit Procedure

The next step in the process was to perform a regression analysis on the performance function data. A second order polynomial was to be generated relating the TOGW to the four design variables, i.e.,

\[
TOGW = A_0 + \sum_{i=1}^{n} A_i X_i + \sum_{j=1}^{n} \sum_{i=1}^{n} A_{ij} X_i X_j
\]  

(1)

where the A's are the coefficients and the X's are the four design variables. The design variables were input in a normalized form so that all values would lie on the interval (-1,1). This was done by using the following transformation:

\[
X' = \frac{X - \frac{1}{2} (X_{\text{MAX}} + X_{\text{MIN}})}{\frac{1}{2} (X_{\text{MAX}} - X_{\text{MIN}})}
\]  

(2)

This normalization was necessary to eliminate round-off errors which would have occurred due to the difference in magnitude of the variables. The normalized variables were also used in the optimization routine and transformation to real range values occurred when an optimal design point had been found.

The quadratic approximation was chosen for two basic reasons. The first reason was that the quadratic polynomial is simple and easy to work with and makes possible the easy and economical calculation of partial derivatives used in the optimizer. Adequate representation can be obtained with second order surfaces thus avoiding higher order approximations (Ref 7:595). The second reason is more mathematically
rational. Basically, the second order assumption is equivalent to assuming that the performance function can be adequately approximated by the first few terms of its Taylor series expansion about some nominal design point \((x_1^*, x_2^*, x_3^*, x_4^*)\). The basic requirement is that the range of values for the design variables be kept reasonably small. If this is the case, then the quadratic polynomial derived from the regression analysis will provide adequate approximations within these ranges (Ref 2:4).

The number of unknown regression coefficients \((k)\) in the quadratic polynomial of Eq (1) is \((n+1)*(n+2)/2\) where "n" is the number of independent variables. The number of design data points must be greater than or equal to the number of unknown coefficients so that an overdetermined system of linear equations in the form of Eq (1) in terms of the unknown coefficients \(A_0, A_1,\) and \(A_{ij}\) can be solved by the method of least squares. The least squares method selects the terms so as to minimize the error (or residual) sum-of-square, \(SSE\),

\[
SSE = \sum_{i=1}^{N} \varepsilon_i^2
\]

where \(\varepsilon_i\) is the difference between the true value of the performance function and the approximated value and where \(N\) is the number of data points input to the regression analysis.

The goodness-of-fit of the regression surface is tested statistically by variance analysis. The regression program used in this study employed four tests to determine the goodness-of-fit. These were the standard \(F\)-statistic for regression, the multiple correlation coefficient squared, the significance ratio, and the standard error.
The F-statistic is defined as the ratio of the regression mean square,
\[
\text{MSR} = \frac{\sum_{k=1}^{N} (\text{TOGW}_k - \overline{\text{TOGW}})^2}{N}
\]
(4)
to the error mean square,
\[
\text{MSE} = \frac{\sum_{k=1}^{N} (\text{TOGW}_k - \overline{\text{TOGW}}_k)^2}{(N - K)}
\]
(5)
where \(\overline{\text{TOGW}}\) is the mean of the true TOGW's in the \(N\) data points, \(\text{TOGW}_k\) is the TOGW predicted by the polynomial at data point \(k\), and \(K\) is the number of coefficients in Eq (1). The F value calculated should exceed the F value found in the standard tables for \(K\) and \(N-K\) degrees of freedom at the 95% confidence level to ensure a good fit (Ref 6:595). If this test is met, then a significant regression is assumed.

The multiple correlation coefficient squared (\(\text{MCC}^2\) or \(R^2\)) is defined as
\[
\text{MCC}^2 = \frac{\sum_{k=1}^{N} (\text{TOGW}_k - \overline{\text{TOGW}})^2}{\sum_{k=1}^{N} (\text{TOGW}_k - \overline{\text{TOGW}}_k)^2}
\]
(6)
where the terms are as defined for Eqs (4) and (5). This quantity varies between 0 and 1 and a value close to 1 indicates that the regression equation tracks the data quite well. It is desirable to have a value close to 1.
The standard error is

\[
\text{STERR} = \sqrt{\text{MSE}}
\]  

(7)

and is a measure of goodness-of-fit. The smaller the value of the standard error the better the approximation.

Information on the significance ratio used in the regression program was limited to the fact that it should be as large as possible in order to ensure a good fit. The significance ratio was used to determine which step in the regression analysis would be used to select the coefficients for the quadratic approximation. A forward and backward regression analysis was performed, i.e., variables were entered and removed whenever a better fit could be obtained. The second order and cross product terms were considered as separate variables in this part of the problem; that is, \((WOS^2)\) could be entered at a step even though \(WOS\) was not in the equation. The step chosen to be the most representative was based on maximizing

\[
\text{FACT} = \sqrt{\frac{\text{SIG RATIO}}{\text{STERR}}} \times \text{MCC}^2
\]  

(8)

Input and output requirements for the regressor are found in Appendix C.
V. Surface Fit Optimization

The basic optimization problem considered was to minimize or maximize some performance function subject to constraints on other performance functions. A maximization problem can be converted to a minimization problem simply by taking a negative of the function to be maximized and then performing a minimization (i.e., RANGE is maximized by minimizing -RANGE). Similarly, inequality constraints can all be regarded as less than or equal to constraints. This is accomplished by taking the negative of a function that is to be greater than some value and making it less than the negative of this value (i.e., RANGE ≥ 150 would be rewritten -RANGE ≤ -150 where -RANGE would be defined as a new function). Thus, using these conventions, the optimum design problem can be rewritten as

\[
\begin{align*}
\text{Minimize} & \quad f_1(\mathbf{x}) \\
\text{Subject to} & \quad f_1(\mathbf{x}) \leq c_i \quad (i=2,\ldots,m) 
\end{align*}
\]

where \( \mathbf{x} \) is the vector of independent variables, \( f \)'s are the performance functions, and the \( c \)'s are the fixed numerical values defining the upper limits on the constraint functions. The independent variables are also constrained to lie within a region of interest, within which an optimal design is supposed to lie. This region is the area determined in the selection of the design space and is the region in which the quadratic approximation is valid.

The penalty function method has been shown to work on the minimization problem of Eq (9) (Ref 6:29-32). This method transforms the
inequalities out of the problem by establishing a penalized cost function of the form

\[ F(x) = f_1(x) + P_K \sum_{j=2}^{m} (CV_j)^2 \]  \hspace{1cm} (10)

where \( CV \) represents the violation of the constraint inequalities; if the constraint is satisfied then \( CV = 0 \). \( P_K \) is a weighting factor in Eq (10) which modulates the severity of violating the constraints. This weighting factor can be considered the tolerance level for constraint violations; hence, if the acceptable deviation from the constraints was one percent then \( P_K \) would be 100. It was assumed in this study that all constraints required this tolerance and \( P_K \) was taken to be a constant equal to 100.

The penalty function method has the advantage that the non-linear constraints are removed from the optimization problem by a simple transformation approach. It also has an advantage in that it is not necessary to locate a feasible starting point for the optimization search. The penalty function algorithm can always be started from a nonfeasible point (Ref 6:31).

There are, however, two major problems in the penalty function method. The first problem is that for large values of the weighting factor \( P_K \) in Eq (10), the penalty cost function can develop a long narrow valley containing the minimum point (Ref 1:40-41). Gradient procedures for finding the minimum tend to go back and forth between the sides of the valley instead of going down the long direction of the valley. Another problem is that the generation of the penalty
function may also create new pseudo minima that are not present in the original problem. Ref 2 (p. 31) states "These difficulties are numerical in nature, corresponding to large values of $P_K$ and/or a sharply decreasing cost function. These difficulties should not occur with the approximating functions which are used."

The form of the quadratic functions supplied by the regression analysis was

$$f_1(x) = A_{10} + A_{11}^T (x) + \frac{1}{2} x^T A_{12} x \quad (i=1,\ldots,m)$$

where $A_{10}$ is a constant term, $A_{11}$ is an "n" vector, and $A_{12}$ is an $n \times n$ symmetric matrix. The form of this equation suggests the use of a gradient type search procedure since the gradient of the cost function would be readily available. Indeed, the desire to use a gradient method was one of the reasons for attempting the surface fit. The gradient of the penalty cost function is

$$\xi(x) = A_{11} + A_{12} x + 2P_K \sum_{j=2}^{m} (C V_j) (A_{11} + A_{12} x)$$

A conjugate gradient technique was used in this study. The ordinary gradient has the drawback of slow convergence and methods have been developed to reduce the number of iterations required for convergence (Ref 9:10). These methods use a modified search direction to speed up the convergence rate.

The minimizing algorithm used was as follows: (a) an evaluation of the gradient $\xi_1(x)$ at a given point $x_1$ was accomplished, (b) the search direction $p_1$ was computed according to
\[ P_1 = H_1^T \tilde{x}_1 \]  

(13)

where "1" denotes the present point and \( H_1 \) was given by

\[
H_1 = H_{1-1} + \frac{(\Delta \tilde{x}_{1-1} - H_{1-1} \Delta \tilde{z}_{1-1}) (\Delta \tilde{x}_{1-1} - H_{1-1} \Delta \tilde{z}_{1-1})^T}{(\Delta \tilde{x}_{1-1} - H_{1-1} \Delta \tilde{z}_{1-1}) \Delta \tilde{z}_{1-1}} \]

(14)

where \( 1-1 \) denotes the previous point, \( H_0 \) was defined to be the unity matrix, \( \tilde{z}_{1-1} = z_1 - z_{1-1} \), and \( \tilde{x}_{1-1} = x_1 - x_{1-1} \), (c) a one dimensional search was performed on the function \( F(x_{1+1}) = F(x_1 + \Delta x_1) \) where

\[
\Delta x_1 = -t_1 P_1
\]

(15)

and the search was to minimize \( F(x_1 - t_1 P_1) \) with respect to \( t_1 \) (this one dimensional search will be discussed later), (d) having found the \( t_1 \) that yielded a minimum, the displacement \( x_1 \) was computed according to Eq (15), (e) the next point, \( x_{1+1} \) was obtained by

\[
x_{1+1} = x_1 + \Delta x_1
\]

(16)

The value of \( F(x_{1+1}) \) will always be less than the value of \( F(x_1) \) (Ref 8:271). These steps (a-e) were repeated until the stopping condition

\[
T \tilde{x}_1 \tilde{z}_1 \leq 10^{-6}
\]

(17)

was met. This stopping condition may correspond to either a minimal point or a nonminimal stationary point (Ref 8:275). For this reason, small perturbations about this point were investigated.
The entire algorithm may need to be restarted by setting $H_i = H_0$. This is required when the inequality

$$\left| \mathbf{g}_i^T \mathbf{p}_1 \right| \leq 10^{-6} \quad (18)$$

is satisfied. If the algorithm is not restarted on this condition then the starting point for the one-dimensional search will automatically satisfy the search stopping condition of Eq (22) so that $t_1 = 0$.

The linear search method was that proposed by Davidon and has been used by Fletcher and Powell (Ref 3). The method was (a) to estimate the order of magnitude of the stepsize $t_1$, (b) establish bounds on $t$ between which the minimum point lies, and (c) to perform a cubic interpolation to find $t_1$.

The determination of the order of magnitude of $t_1$ was based on an available estimate (est) of the value of $F(x)$ at the unconstrained minimum. This estimate was available in the problem at hand and would be expected to be around -1 when normalized variables were being used.

The step size was determined by

$$h = k = \frac{2(F(x_1) - \text{est})}{\mathbf{g}_1^T \mathbf{p}_1} \quad \text{if } 0 < k < \frac{1}{\sqrt{\mathbf{p}_1^T \mathbf{p}_1}} \quad (19)$$

or

$$h = \frac{1}{\sqrt{\mathbf{p}_1^T \mathbf{p}_1}} \quad \text{otherwise.}$$

The bounds on $t_1$ were established by examining

$$y(t) = F(x_1 - t \mathbf{p}_1)$$

$$y'(t) = -\mathbf{g}_1^T \mathbf{g} (x_1 - t \mathbf{p}_1) \quad (20)$$
at points where $t = 0, h, 2h, 4h, \ldots, a, b$ where $b$ is the first point at which either $y'$ is non-negative or $y$ has not decreased. It then follows that $t_1$ is bounded in the interval $(a, b)$.

Given these limits on $t_1$ a cubic interpolation estimating $t_1$ may be given by

$$t_e = b - \left( \frac{y'(b) + v - s}{y'(b) - y'(a) + 2w} \right) (b - a)$$

$$s = 3 \left( \frac{y(a) - y(b)}{b - a} \right) + y'(a) + y'(b)$$

$$w = \sqrt{s^2 - y'(a)y'(b)}$$

If neither $y(a)$ nor $y(b)$ is less than $y(t_e)$ then $t_e$ is accepted as the estimate of $t_1$ and the stopping condition is tested at $t_e$. If the condition is met, the one dimensional search is terminated. If the condition is not met, the cubic interpolation is repeated over the sub-interval $(a, t_e)$ if $y'(t_e)$ is positive and the cubic interpolation is repeated over the sub-interval $(t_e, b)$ if $y'(t_e)$ is negative. The stopping condition is determined by precision requirements and was

$$\left| \frac{T}{E_{i+1} E_1} \right| \leq 10^{-6}$$

where the constraint limit must be the same as in Eq (18).

Two different minimization problems were investigated. The first was to minimize the TOGW subject only to the constraints on the independent design variables. In solving this problem, the design variables were entered into the penalty cost function by creating
false performance functions consisting only of the variables and their limits. The second problem was to minimize the TOGW subject to a constraint on the landing distance which was a performance parameter available from CISE.

Input and output requirements for the optimization routine are described in Appendix D.
VI. Results

The design space selected by the Latin Square technique is given in Table III. Each design point was input to the mission simulator and the performance values (TOGW and DISTL) for each point are given in Table IV. These 45 points were then input to the regression analysis. The second order polynomial generated for each performance function was (equations are in terms of normalized values)

\[
\text{TOGW} = -0.31638 \times 0.06307 (\text{WOS}) + 0.04207 (\text{AR}) + 0.79413 (\text{TW})
+ 0.18533 (\text{RN}) + 0.03678 (\text{WOS}^2) - 0.04107 (\text{WOS})(\text{AR})
- 0.04316 (\text{WOS})(\text{TW}) - 0.03023 (\text{AR})(\text{TW}) - 0.31439 (\text{TW}^2)
+ 0.07082 (\text{TW})(\text{RN})
\]

(23)

\[
\text{DISTL} = 0.01156 + 0.87538 (\text{WOS}) - 0.10542 (\text{AR}) - 0.05551 (\text{TW})
- 0.08192 (\text{RN}) - 0.02251 (\text{WOS}^2) - 0.01018 (\text{WOS})(\text{AR})
- 0.01253 (\text{WOS})(\text{TW}) + 0.02614 (\text{WOS})(\text{RN}) - 0.00507 (\text{AR}^2)
+ 0.00966 (\text{AR})(\text{TW}) + 0.00488 (\text{AR})(\text{RN}) - 0.02229 (\text{TW}^2)
+ 0.00287 (\text{TW})(\text{RN})
\]

The statistical analysis for each equation is given in Table V. The value of the F-statistic from the standard tables for a numerator of 45 degrees of freedom and a denominator of 15 degrees of freedom is 2.19. The TOGW equation does meet this requirement so it has a 95\% confidence level that it represents the true data space. The other values for the TOGW equation also indicate a good fit. The F-value for the DISTL equation was lower than the required value; however, the
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<td></td>
</tr>
<tr>
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<td>41383</td>
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<td></td>
<td></td>
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<td>39</td>
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<td>6916</td>
<td></td>
<td></td>
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<tr>
<td>40</td>
<td>33546</td>
<td>7513</td>
<td></td>
<td></td>
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<tr>
<td>41</td>
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<td>4691</td>
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<td></td>
</tr>
<tr>
<td>42</td>
<td>33215</td>
<td>6805</td>
<td></td>
<td></td>
</tr>
<tr>
<td>43</td>
<td>43414</td>
<td>4703</td>
<td></td>
<td></td>
</tr>
<tr>
<td>44</td>
<td>37176</td>
<td>6124</td>
<td></td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>37297</td>
<td>7623</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
other statistical values are extremely good and the maximum difference that the equation predicts from the true data is only six percent.

<table>
<thead>
<tr>
<th></th>
<th>F-value</th>
<th>Significance Ratio</th>
<th>Multiple Correlation</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>TOGW</td>
<td>5.3352</td>
<td>922.4982</td>
<td>.99816</td>
<td>.04042</td>
</tr>
<tr>
<td>DISTL</td>
<td>.8058</td>
<td>25053.265</td>
<td>.99995</td>
<td>.00758</td>
</tr>
</tbody>
</table>

One problem was noted in the TOGW equation. The AR appeared only as a linear term indicating that a proper minimum would not exist. Also, the sufficiency condition for a minimum was not met in that the matrix of second partial derivatives was not positive, semi-definite.

To analyse this problem, graphs were made of the TOGW versus the AR for several values of WOS at constant RN and TW. These graphs are presented in Figs. 3-8. It was concluded from these graphs that the calculated curves do represent the actual data trends and yield values of TOGW that are within two percent of the actual data.

Three dimensional plots were made of TOGW versus AR and WOS at constant RN and TW. Once again the calculated plots exhibit the same shaped (although they are smoother as would be expected) as the true data and have a maximum difference of two percent. These surfaces are presented in Figs. 9-14.

Two minimizations were performed. The first problem was to find the minimum TOGW within the design space. The second problem was to
find the minimum TOGW subject to the constraint that the DISTL be less than 5000 feet and still be within the design space. These minimums were also found by manually analyzing the 625 data points that represented the complete data space. The minimum point for each range was found and the results are presented in Tables VI and VII.

Table VI indicates that the surface fit approximations yield minimum values of TOGW that are within two percent of the actual minimum values. The design parameters are identical with the exception of the AR. The graphs of Figs. 3-8 indicate that the TOGW is not significantly affected by changes in AR so the problem of different AR's is acceptable. Table VII shows similar results for the case when DISTL was constrained to be less than or equal to 5000 feet.

These results also indicate that the minimum design point is not inside the range of the design variables. In the unconstrained problem, the design point was at the lower limit of each of the variables. This does present a problem in that the minimum was assumed to lie within the design region. However, the approximation method did predict the same value as was found by looking at the complete space.
TOGW VS AR

RN = 150, TW = .5

ACTUAL DATA

LEGEND
- WOS = 70
- WOS = 80
- WOS = 90
+ WOS = 100
× WOS = 110

Fig. 3. TOGW vs. AR (RN = 150, TW = .5) - Actual Data
TOGW VS AR

RN - 150, TW - .5
CALCULATED VALUES

LEGEND

- WOS = 70
- WOS = 80
- WOS = 90
+ WOS = 100
x WOS = 110

Fig. 4. TOGW vs. AR (RN = 150, TW = .5) - Calculated Values
TOGW VS AR

RN - 250, TW - .9
ACTUAL DATA

Fig. 5. TOGW vs. AR (RN = 250, TW = .9) - Actual Data
TOGW VS AR

RN = 250, TW = .9
CALCULATED VALUES

LEGEND

- WOS = 70
- WOS = 80
- WOS = 90
- WOS = 100
- WOS = 110

Fig. 6. TOGW vs. AR (RN = 250, TW = .9) - Calculated Values
TOGW VS AR

RN - 350, TW = 1.3
ACTUAL DATA

LEGEND

- WOS = 70
O = WOS = 80
△ = WOS = 90
+ = WOS = 100
X = WOS = 110

Fig. 7. TOGW vs. AR (RN = 350, TW = 1.3) - Actual Data
TOGW VS AR

RN = 350, TW = 1.3
CALCULATED VALUES

LEGEND

□ - WOS = 70
○ - WOS = 80
△ - WOS = 90
+ - WOS = 100
× - WOS = 110

Fig. 8. TOGW vs. AR (RN = 350, TW = 1.3) - Calculated Values
Fig. 9. TOGW vs. WOS - AR (RN = 150, TW = .5) Actual Data
Fig. 10. TOGW vs. WOS - AR (RN = 150, TW = .5) Calculated Values
TOGW VS WOS - AR

RN - 250, TW - .9
ACTUAL DATA

Fig. 11. TOGW vs. WOS - AR (RN = 250, TW = .9) Actual Data
Fig. 12. TOGW vs. WOS - AR (RN = 250, TW = .9) Calculated Values
TOGW vs WOS - AR

RN = 350, TW = 1.3

ACTUAL DATA

Fig. 13. TOGW vs. WOS = AR (RN = 350, TW = 1.3) Actual Data
Fig. 14. TOGW vs. WOS - AR (RN = 350, TW = 1.3) Calculated Values
### Table VI
Results of the Minimization - DISTL Not Constrained

<table>
<thead>
<tr>
<th>Case</th>
<th>RN</th>
<th>WOS</th>
<th>AR</th>
<th>TW</th>
<th>TOGW</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>150</td>
<td>70</td>
<td>3.5</td>
<td>.5</td>
<td>29864</td>
</tr>
<tr>
<td>B</td>
<td>150</td>
<td>70</td>
<td>2.5</td>
<td>.5</td>
<td>30000</td>
</tr>
<tr>
<td>A</td>
<td>200</td>
<td>70</td>
<td>3.0</td>
<td>.5</td>
<td>30295</td>
</tr>
<tr>
<td>B</td>
<td>200</td>
<td>70</td>
<td>2.5</td>
<td>.5</td>
<td>30436</td>
</tr>
<tr>
<td>A</td>
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<td>70</td>
<td>3.0</td>
<td>.5</td>
<td>30774</td>
</tr>
<tr>
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<td>250</td>
<td>70</td>
<td>2.5</td>
<td>.5</td>
<td>30872</td>
</tr>
<tr>
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<td>300</td>
<td>70</td>
<td>3.0</td>
<td>.5</td>
<td>31282</td>
</tr>
<tr>
<td>B</td>
<td>300</td>
<td>70</td>
<td>2.5</td>
<td>.5</td>
<td>31308</td>
</tr>
<tr>
<td>A</td>
<td>350</td>
<td>70</td>
<td>3.0</td>
<td>.5</td>
<td>31818</td>
</tr>
<tr>
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<td>350</td>
<td>70</td>
<td>2.5</td>
<td>.5</td>
<td>31743</td>
</tr>
</tbody>
</table>

where Case A represents the complete data space analysis; Case B represents the surface fit approximation analysis.

### Table VII
Results of the Minimization - DISTL Constrained

<table>
<thead>
<tr>
<th>Case</th>
<th>RN</th>
<th>WOS</th>
<th>AR</th>
<th>TW</th>
<th>TOGW</th>
<th>DISTL</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>150</td>
<td>70</td>
<td>3.5</td>
<td>.5</td>
<td>29864</td>
<td>5007*</td>
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<td>150</td>
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<td>3.5</td>
<td>.5</td>
<td>30223</td>
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</tr>
<tr>
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<td>70</td>
<td>3.5</td>
<td>.5</td>
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<td>4947</td>
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<td>B</td>
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<td>70</td>
<td>3.5</td>
<td>.5</td>
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<td>4941</td>
</tr>
<tr>
<td>A</td>
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<td>70</td>
<td>3.0</td>
<td>.5</td>
<td>30773</td>
<td>4965</td>
</tr>
<tr>
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<td>3.0</td>
<td>.5</td>
<td>30983</td>
<td>4974</td>
</tr>
<tr>
<td>A</td>
<td>300</td>
<td>70</td>
<td>3.0</td>
<td>.5</td>
<td>31282</td>
<td>4905</td>
</tr>
<tr>
<td>B</td>
<td>300</td>
<td>70</td>
<td>2.5</td>
<td>.5</td>
<td>31308</td>
<td>5000</td>
</tr>
<tr>
<td>A</td>
<td>350</td>
<td>70</td>
<td>3.0</td>
<td>.5</td>
<td>31818</td>
<td>4844</td>
</tr>
<tr>
<td>B</td>
<td>350</td>
<td>70</td>
<td>2.5</td>
<td>.5</td>
<td>31744</td>
<td>4951</td>
</tr>
</tbody>
</table>

where Case A represents the complete data space analysis; Case B represents the surface fit approximation analysis.

*This number is within the constraint tolerance.
VII. Conclusions and Recommendations

Conclusions

1. The surface fit approximations can be used in the design process to predict designs that agree with more detailed and time consuming processes.

2. The fact that the independent variables are constrained to lie in the selected region requires that care be taken during the definition of the ranges of these variables. The ranges must be chosen small enough so that the quadratic approximations are valid; however, they must be large enough to allow the minimum point to lie within the design space. This will prevent a minimum design point which satisfies the performance function constraints from not being found.

3. The simple latin square technique as modified will yield a representative design space. This reduces significantly the number of mission simulations that must be performed; hence, computer time and engineering analysis are reduced.

Recommendations

1. Further studies should be made to demonstrate the capabilities with an expanded number of variables and performance functions. These studies should be made in close coordination with the AFFDL Design Branch using a design problem that they are considering.

2. Other design space selection techniques such as the orthogonal latin square and the $D$-optimal should be considered. These techniques may further reduce the number of mission simulations.
3. It is appropriate to incorporate the surface fit approximations in the design procedures at AFFDL. These techniques are currently being used by some aerospace companies and in particular at Boeing, enabling them to save many computer and engineering hours.
Bibliography


Appendix A

**Latin Square Computer Program Input/Output**

**Input**

<table>
<thead>
<tr>
<th>Card No.</th>
<th>Col. No.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12-13</td>
<td>Number of independent design variables (Format 12)</td>
</tr>
<tr>
<td>2-N</td>
<td>1-6</td>
<td>Alphanumeric label for the variable (Format A6)</td>
</tr>
<tr>
<td>11-20</td>
<td></td>
<td>Lower limit for the variable (Format E10.4)</td>
</tr>
<tr>
<td>21-30</td>
<td></td>
<td>Upper limit for the variable (Format E10.4)</td>
</tr>
</tbody>
</table>

"N" is the number of independent design variables plus one which allows the range of each variable to be input.

Input is assigned to Tape 5.

**Output**

The format of the output is (I8, 10F12.4/14X,7F12.4) which yields the data point number and the values of the design variables for that data point.

Output is assigned to Tape 6.
Appendix B

Computerized Initial Sizing Program (CISE) Input/Output

All input is in the standard Fortran Namelist format with the first character in each record appearing in the second column. The title of the design under study appears on the first data record and is in Hollerith format. The input is divided into five groups using the following names:

DESIGN, MISSION, WEIGHTS, GEOM, PROP

The items that can be input by each Namelist are:

$DESIGN

LF = Load Factor
VMAX = Maximum Equivalent Airspeed
AMMAX = Maximum Mach No.
ALTX = Altitude for AMMAX
PS = Energy Level Required at Flight Design Gross Wt.
GPS = G Level for PS
ALTPS = Altitude for PS
AMNPS = Mach No. for PS
NCREW = No. of Crew Members
NTANK = No. of 300 Gal External Tanks
NSTORX = No. of External Stores
NPYL = No. of Pylons to Carry External Stores and Tanks
NSTORI = No. of Internal Stores
CLMAX = Maximum Lift Coefficient for Takeoff and Landing
NCIX = $C_d$ Calculation Cue; When = 0, CDOSF = $C_e$; When = 1, 
$C_d$ is calculated by Program and CDOSF is the Modification Factor

CDOSF = Equivalent Skin Friction Coefficient or $C_d$ Modification Factor (depending on NCIX value input)

CDSTX1 = Subsonic Store Drag Modification Factor

CDSTX2 = Supersonic Store Drag Factor

ALTTOL = Altitude for Takeoff and Landing

DTEMP = Takeoff and Landing $F$ from Standard Day at ALTTOL

IPROP = 1 for Turbofan (P&W 401)
  = 2 for Turbojet (GE J79)
  = 3 for Reciprocating (Lycoming IGO-540)

AB = 1 for afterburner

AB ≠ 1 for no afterburner

TOWE = Engine Thrust to Weight Ratio

FDGW = Ratio of Total Fuel on Board to Define Flight Design Gross Weight (FDGW) where, in the Program $FDGW = TOGW - (1.0 - FDGW) * WFUEL$ (Initialized at .60)

LDGWFS = Same as FDGW, except for Structural Design Landing Gross Weight (LDGW) where, in the Program

LDGW = TPGW - (1.0 - LDGWFS) * WFUEL - WSTOR

(Initialized at .90)

LDGWF = Same as LDGWFS, except for Landing Distance Calculations, Using Design Landing Distance Gross Weight (LDGWLD) where, in the Program $LDGWLD = TOGW - (1.0 - LDGWF) * WFUEL - WSTOR$

(Initialized at .70)
IPRINT = 1 for Error Checkout Messages to be Printed
        = 0 for Error Checkout Messages not to be Printed
TOTHF = Thrust Modification Factor for Non-standard Day (Altitude and
        Temperature Adjustment) (Initialized at 1.0)
CDF = Total CD Modification Factor, May be Used to Adjust L/D by
        Modifying CD
CDSF = Factor to Adjust Supersonic Drag Contribution to CD_o

*MISSION*

N = Total Number of Mission Legs
R(I) = 0 for Warmup Fuel Allowance
         = -1 to Drop Stores
         = -2 to Turn at Fixed Gee
         = -3 to Accelerate
         = -4 for Energy Altitude Combat Fuel Allowance
         = -5 for Loiter Performance
         = -6 for Loiter at Fixed Altitude and Mach No.
         = -7 to Turn at Maximum Possible Gee
> 0 for Climb Performance
 = 1.0E6 to Climb on Intermediate (Military Power)
 = 1.1E6 to Climb with Afterburner with Distance Credit

GEE(I) = G Level for Turns (no 1g turns), When RANGE(I) = -7
         = Time (min) on Afterburner Power, When RANGE(I) = 1.0E6
         = No. of External Stores to be Dropped, When RANGE(I) = -1

NTURNS(I) = No. of Turns, When RANGE(I) = -7
         = Mach, When RANGE(I) = -3
         = Energy Altitude for Combat Fuel Allowance (ft), When
         RANGE(I) = -4
TIME(I) = Loiter Time (min), When RANGE(I) = -6, or -5
   = No. of Internal Stores to be Dropped, When RANGE(I) = -1
ITANK(I) = No. of External Fuel Tanks to be Dropped, When RANGE(I) = -1
AMACH(I) = Leg Mach No., Except When RANGE(I) = -1
   = Weight of Cargo to be Dropped, When RANGE(I) = -1
ALT(I) = Leg Altitude
RG(I) = Distance Covered in a Particular Segment, When RANGE(I) = 1.1E6

$WEIGHTS$
WAVUN = Weight of Avionics Equipment
WNA = Weight of AMMO, Guns, etc. That Does Not Require Installation
   Weight in Addition
WSTORX = Weight of External Stores
WSTORI = Weight of Internal Stores
WINGF = Wing Weight Modification Factor
TAILF = Tail Weight Modification Factor
BODYF = Body Weight Modification Factor
GEARF = Gear Weight Modification Factor

$GEOM$
NWDS = No. of Wing Loadings to be Cycled, Maximum of 5
WOS(I) = Wing Loadings (Max of 5) #/sq. ft.
NAR = No. of Aspect Ratios to be Cycled, Maximum of 5
AR(I) = Aspect Ratios (Max of 5)
NSWP = No. of Quarter Chord Sweep Angles to be Cycled, Maximum of 5
SWP(I) = Sweep Angles (Quarter Chord) in Degrees (Max of 5)
NTROOT = No. of Root Thickness Ratios to be Cycled, Maximum of 5
TROOT(I) = Root Thickness Ratios (Max of 5)
TAPER = Wing Taper Ratio
SFVET = Fuselage Wetted Area
SHT = Horizontal Tail Volume Coefficient
LFUS = Fuselage Length
CWRT = Root Chord of the Wing in Inches
MAC = Wing Mean Geometric Chord in Feet
SWVET = Wing Wetted Area

$PROP
IPROP = 1 for Turbofan (P&W 401)
    = 2 for Turbojet (GE J79)
    = 3 for Reciprocating (Lycoming GO-540)
AB = 1 for afterburner
AB # 1 for no afterburner
SLTH = Thrust to Weight Ratio of the Vehicle
TOWE = Thrust to Weight Ratio of the Engine
SFCF = Specific Fuel Consumption Modification Factor

To end the input and begin the mission simulation, an 'E' is placed in column 2.

The input used in this study was:

THIS IS THE FIRST TEST CASE FOR ADCA
$DESIGN
STOREX = 2., IPRINT = 0., LF = 9.75,
AMMAX = 2., VMAX = 794.,
INITER = 50.,
$END OF DESIGN
$WEIGHTS
WAVUN = 1350., WSTORX = 5500., WMA = 600.,
GEARF = .97, WINGF = .83, BODYF = .85, TAILF = .83,
$END OF WEIGHTS
$END OF GEOM
$PROP
IPROP = 1, AB = 1., SLTH = .70, TOWE = 8., SFCF = 1.2,
$END OF PROP
$MISSION
N = 11, GE(7) = 6.5, NTURNS(7) = .8, DEIM = .4, TIME(1) = .75,
   TIME(11) = 20., GMIN = 2.0,
ALT(1) = 0., 40000., 40000., 40000., 50000., 50000.,
   50000., 50000., 55000., 45000., 0.,
AMACH(1) = 0., .8, .8, .8, 1.6, 1.6, 1.6, 1.6, .8, .4,
RANGE(1) = 0., 1.E6, 200., -3., 1.1E6, 150., -7., -1., 200., 200., -5.,
$END OF MISSION
E

Output

The values of the design variables and the corresponding performance values were output to Tape 2 and each value was of the format F10.2.

It was also possible to have a mission summary printed out which included an analysis of each leg in the mission along with a weight
summary. The component weights are also output but because of the many approximations utilized to derive these weights, only the weight empty and TOGW values can be considered statistically valid (Ref 4:7). This summary data is available on Tape 6.
Appendix C

Regression Analysis Computer Program Input/Output

Input

All numeric data requires a decimal point with the exception of card number three.

<table>
<thead>
<tr>
<th>Card No.</th>
<th>Col. No.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1-6</td>
<td>The word PROGRM must appear; this is a required card</td>
</tr>
<tr>
<td></td>
<td>12-17</td>
<td>A six character identification word in alphabetic format</td>
</tr>
<tr>
<td></td>
<td>22-31</td>
<td>Number of input variables per data point including dependent variables</td>
</tr>
<tr>
<td></td>
<td>32-41</td>
<td>Number of variables to be added by synthesis</td>
</tr>
<tr>
<td></td>
<td>42-51</td>
<td>Number of variable synthesis cards</td>
</tr>
<tr>
<td></td>
<td>52-61</td>
<td>Number of labeled variables</td>
</tr>
<tr>
<td></td>
<td>62-71</td>
<td>Number of data points</td>
</tr>
<tr>
<td>2</td>
<td>1-6</td>
<td>The word PROGOPT must appear; this is a required card</td>
</tr>
<tr>
<td></td>
<td>12-21</td>
<td>Number of Format cards if the default Format of (11X,6F10.0) is not used</td>
</tr>
<tr>
<td></td>
<td>22-25</td>
<td>TRUE if input data is on magnetic tape otherwise leave blank</td>
</tr>
<tr>
<td></td>
<td>32-35</td>
<td>TRUE rewinds auxiliary input data tape otherwise leave blank</td>
</tr>
</tbody>
</table>
52-55 TRUE prints covariance and correlation matrices otherwise leave blank

62-65 TRUE calculates the equation based on a curve through the origin (zero intercept) otherwise leave blank

3 1-3 If variable synthesis data is on cards then the integer 0 must appear; if the data is assigned to Tape 8 then a 1 must appear; this is a required card; the number must be in column 3 as needed 1-6 The word VARSYN must appear; these are the variable synthesis cards which form the second order terms in the equations; in general these cards are optional but were required in this study

8-10 Synthesis operation code which can take on the following values:

1.0 VAR(I) = Constant
2.0 VAR(I) = VAR(J)
3.0 VAR(I) = (VAR(J))\(\frac{1}{2}\)
4.0 VAR(I) = EXP(VAR(J))
5.0 VAR(I) = SIN(VAR(J))
6.0 VAR(I) = COS(VAR(J))
7.0 VAR(I) = TAN(VAR(J))
8.0 VAR(I) = Natural Log (VAR(J))
9.0 VAR(I) = ARCSIN(VAR(J))
10. VAR(I) = ARCCOS(VAR(J))
11. VAR(I) = VAR(J) + VAR(K)
12. VAR(I) = VAR(J) - VAR(K)
13. VAR(I) = VAR(J) * VAR(K)
14. VAR(I) = VAR(J)/VAR(K)
15. VAR(I) = VAR(J)**VAR(K) i.e. VAR(J) raised to the power = VAR(K)
16. VAR(I) = ARCTAN(VAR(J)/VAR(K))
17. VAR(I) = (VAR(J)**2 + VAR(K)**2)\(\frac{1}{2}\)
18. VAR(I) = MAX(VAR(J), VAR(K), Constant)
19. VAR(I) = MIN(VAR(J), VAR(K), Constant)
12-21 Assigns a value to I in the above operations
22-31 Assigns a value to J in the above operations
32-41 Assigns a value to K in the above operations
42-61 Assigns a value to the Constant in the above operations

as needed 1-6 The word LABELS must appear; is used to assign alphanumeric titles to each of the variables, both input and synthesized; is an optional item in the program

12-15 Index number for the first labeled variable
16-21 A six character label for the first labeled variable

22-25 Index number for the second labeled variable
26-31 A six character label for the second labeled variable

32-35 Index number for the third labeled variable
36-41 A six character label for the third labeled variable

62-65 Index number for the sixth labeled variable
66-71 A six character label for the sixth labeled variable

as needed 1-6 The word FORMAT must appear if the input data points are in a Format other than the default value of (L1X, 6F10.0)

12-71 Valid Format specification for the input data points; must begin with a left parenthesis and end with a right parenthesis
(Data points are input at this point)

as needed  1-6  The word PROBLM must appear; this is a required card and initiates the analysis for the selected independent variable

12-21  The index number of the variable chosen to be the dependent variable

22-31  Number of input or synthesized variables not allowed in the regression equation

32-41  Limit the number of steps for calculation

42-45  TRUE deletes the detailed printout of steps otherwise leave blank

52-55  TRUE deletes the summary printout otherwise leave blank

62-65  TRUE deletes the residual printout otherwise leave blank

as needed  1-6  The word DELETE must appear if this optional routine is used; is used in conjunction with the PROBLM card columns 22-31

12-21  The index number of the first deleted variable

22-31  The index number of the second deleted variable

******

62-71  The index number of the sixth deleted variable

(Additional PROBLM cards can be input at this time to restart the regression analysis using a new dependent variable or deleting different variables from the analysis)

Last  1-6  The word FINISH must appear; this is a required card which causes proper termination of the program
The only variable synthesis operation code used in this study was code 13., \( \text{VAR}(1) = \text{VAR}(J) \times \text{VAR}(K) \). This was used to form the squared and cross product terms.

The input for this study was set up using seven input variables, the four independent design variables \((\text{VAR}(1) = \text{WOS, VAR}(2) = \text{AR, VAR}(3) = \text{TW, VAR}(4) = \text{RN})\) and three performance function parameters \((\text{VAR}(5) = \text{TOGW, VAR}(6) = \text{DISTL - landing distance}, \text{VAR}(7) = \text{DISTO - takeoff distance})\). Regression analysis was done on all three performance functions. See Fig. for the input. Note that spacing is to be as described above.

**Output**

The output from the regression analysis will depend on the options selected in the input. A complete listing of the means and standard deviations for the input parameters is possible and a printout containing the covariance and correlation matrices may also be obtained. A detailed printout of the variables entered or deleted at each step of the regression analysis along with regression statistics will be received unless otherwise specified. A point by point comparison of the actual data with the calculated data will be made and yields information indicating the percent error at each point (residual information). The coefficients for the selected equation are printed and also appear as punched cards. The coefficients are available on Tape 7 with the first record having a Format of \((16X, 4E16.8)\) and the following records have a Format of \((5E16.8)\).
Card No.

1   PROGM   TEST11  7.  10.  10.  14.  45.
2   PROOPT  1.
3
4   VARSYN  13.  8.  1.  1.  (note: this sets VAR(8) = WOS²)
5   VARSYN  13.  9.  1.  2.  (note: this sets VAR(9) = WOS*AR)

13  VARSYN  13.  17.  4.  4.  (note: this sets VAR(17) = RN²)
14  LABELS  1.00WOS  2.00AR  3.00TW  4.00RN  5.00TOGW  6.00DISTL
15  LABELS  7.00DISTO  8.00WOS*²  9.00WOS*AR  10.00WOS*TW  11.00WOS*RN  12.00AR*²
16  LABELS  13.0AR*TW  14.0AR*RN  15.0TW*²  16.0TW*RN  17.0RN*²
17  FORMAT  (10X,7F10.2)

45 Data Points

64  PROBLM  5.  2.  30.  TRUE  (note: this sets the dependent variable to be TOGW)
65  DELETE  6.  7.  (note: this deletes the DISTL and DISTO variables from the
              regression analysis)
66  PROBLM  6.  2.  30.  TRUE  (note: this sets the dependent variable to be DISTL)
67  DELETE  5.  7.  (note: this deletes TOGW and DISTO from the regression)
68  PROBLM  7.  2.  30.  TRUE  (note: this sets the dependent variable to be DISTO)
69  DELETE  5.  7.  (note: this deletes TOGW and DISTL from the regression)

Fig. 15. Input to the Regression Analysis
### Appendix D

#### Optimization Program Input/Output

<table>
<thead>
<tr>
<th>Card No.</th>
<th>Col. No.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1-5</td>
<td>The number of independent design variables (Format I5)</td>
</tr>
<tr>
<td></td>
<td>6-10</td>
<td>The number of performance equations to be used (Format I5)</td>
</tr>
<tr>
<td></td>
<td>11-15</td>
<td>The number of constraint equations to be used (Format I5)</td>
</tr>
<tr>
<td>as needed</td>
<td>1-80</td>
<td>The coefficients for each performance equation are input in the same format as they were output from the regression analysis. The first set of coefficients must be the function to be minimized. The second set of coefficients must be the first inequality constraint function. When all the constraints have been read in, the remaining functions are input.</td>
</tr>
<tr>
<td>as needed</td>
<td>1-5</td>
<td>Alphanumeric character for the label of the inequality constraint (Format A5)</td>
</tr>
<tr>
<td></td>
<td>7-8</td>
<td>The inequality characters LE or GE</td>
</tr>
<tr>
<td></td>
<td>10-30</td>
<td>The constraint value (Format G20.8)</td>
</tr>
<tr>
<td>as needed</td>
<td>1-80</td>
<td>The initial values of the design variables; provides the starting point for the search routine (Format G20.8)</td>
</tr>
</tbody>
</table>
Output

The output will indicate the values of the design variables and the performance functions at the minimum point. All numeric output is of the Format G16.8 and is on Tape 6.
Vita

Martin L. Marler was born 17 November 1949 in St. Louis, Missouri. He graduated from Northwest High School in St. Louis, Missouri in 1967, attended Purdue University from which he received the degree of Bachelor of Science in Aeronautical and Astronautical Engineering and a commission in the USAF in 1971. He was elected to Phi Kappa Phi while attending Purdue University. He served on a Minuteman Missile Crew at Ellsworth Air Force Base, South Dakota, from September 1971 until May 1975. During this same time period, he received the degree of Master of Arts with a major in business from the University of Northern Colorado. He entered the Air Force Institute of Technology in May 1975.

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This thesis was typed by Mrs. Frances Jarnagin.
A study was made to demonstrate the feasibility of using surface fit approximations in the design analysis of an advanced tactical fighter. Design variables were selected and a design space was defined based on a simple Latin square method. The take-off-gross-weight (TOGW) and landing distance (DISTL) for each configuration were determined by use of a computer program which simulated the required mission. A regression analysis was performed on this data to obtain a quadratic surface fit representing TOGW and DISTL as functions of the design variables. A conjugate gradient technique was employed to find the...
minimum TOGW subject to constraints on DISTL within the design space. This value was compared to the minimum obtained by running all possible combinations of design variables through the mission simulator. It was concluded that the surface fit approximation based on the latin square design space did yield accurate results and required fewer computer simulations.