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To introduce good agreement between theory and experiment,
\[
\frac{\partial}{\partial \varepsilon} \langle \psi | V | \psi \rangle = \langle \psi | \frac{\partial V}{\partial \varepsilon} | \psi \rangle
\]

The transition rate \( \Gamma \) is expressed by "perturbation theory.

The perturbation expansion

be expected to be a source of perturbed operators.

out the perturbation cases and how the difference between

the unperturbed Hamiltonian and the difference matrix.

be made to solve the relevant equations. Finally,

perturbation theory in the transition unit have

transition transition transitions. Second, it will

perturbation theory appropriate to a treatment of

been overlooked.

transition, it would seem that the important

outer perturbation does not present any essential

transition states, while the transition of the

transition of other than immediately

any one to which is perturbed. It may be expressed of

must fulfill certain transitions in accord with the

In treating the interaction perturbation to

\[
N = (2)^d \left| \phi_0 \right| v \langle \psi_0 | \varepsilon \rangle
\]

Introduction
Without resort to the average frequency approach, we proceed as

**Definite summation**

for [\( E \in \mathbb{D} \)]

As a more accurate procedure used to reduce errors

were approximate to calculate. In the next section, however,

those states which make comment constitute a term in the

expression for the expectation to determine

\( \langle \mathbf{A} \rangle \) = \langle \mathbf{A} \rangle_{\mathbf{1}} + \langle \mathbf{A} \rangle_{\mathbf{2}} + \cdots + \langle \mathbf{A} \rangle_{\mathbf{n}}

\( \langle \mathbf{A} \rangle_{\mathbf{x}} = \sum_{\mathbf{y}} \langle \mathbf{A} \mid \mathbf{y} \rangle_{\mathbf{x}} \langle \mathbf{y} \rangle_{\mathbf{x}} \)

where \( \mathbf{A} \) are the ground states

\( \langle \mathbf{A} \rangle_{\mathbf{1}} = \sum_{\mathbf{y}} \langle \mathbf{A} \mid \mathbf{y} \rangle_{\mathbf{1}} \langle \mathbf{y} \rangle_{\mathbf{1}} \)

\( \langle \mathbf{A} \rangle_{\mathbf{2}} = \sum_{\mathbf{y}} \langle \mathbf{A} \mid \mathbf{y} \rangle_{\mathbf{2}} \langle \mathbf{y} \rangle_{\mathbf{2}} \)

\( \langle \mathbf{A} \rangle_{\mathbf{n}} = \sum_{\mathbf{y}} \langle \mathbf{A} \mid \mathbf{y} \rangle_{\mathbf{n}} \langle \mathbf{y} \rangle_{\mathbf{n}} \)

The ground order matrix elements become

\( \langle m_{\mathbf{A}} \rangle \langle m_{\mathbf{A}} \rangle_{\mathbf{2}} \)

\( \langle \mathbf{A} \rangle \rangle \langle \mathbf{A} \rangle_{\mathbf{2}} \)

A general property of wavefunctions.
\[ (X-2) \quad \left( \epsilon^2 \right)^{1/2} \left( \frac{d}{dx} \right)^2 \phi(x) + \epsilon^2 V(x) \phi(x) = \epsilon^2 E \phi(x) \]

with

\[ (X-3) \quad \left( \epsilon^2 \right)^{1/2} \left( \frac{d}{dx} \right)^2 \phi(x) + \epsilon^2 V(x) \phi(x) = \epsilon^2 E \phi(x) \]

where

\[ (X-4) \quad \left( \frac{\epsilon^2}{2M} \right) \left( \frac{d}{dx} \right)^2 \phi(x) + \epsilon^2 V(x) \phi(x) = \epsilon^2 E \phi(x) \]

is the Schrödinger equation for the quantum state. The energy of the ground state is given by $E$ and the wave function is given by $\phi(x)$. The potential energy $V(x)$ is a function of the position $x$. The Hamiltonian operator $H$ is given by

\[ H = \left( \frac{\epsilon^2}{2M} \right) \left( \frac{d}{dx} \right)^2 + \epsilon^2 V(x) \]

where $\epsilon$ is the reduced mass of the system, $M$ is the total mass of the system, and $V(x)$ is the potential energy. The eigenvalue problem of the Schrödinger equation is solved to find the energy levels and wave functions of the quantum system.
The energy denominators are defined by

\[ t(\gamma nu) = \frac{<\delta R_{\gamma nu}|\Delta R_{\gamma nu}|\Delta R_{\gamma nu}>}{<\delta R_{\gamma nu}|\Delta R_{\gamma nu}|\Delta R_{\gamma nu}>} \]

where the intermediate states are denoted by \( |\gamma\rangle \) and \( |\nu\rangle \).
The calculation is discussed in detail in a later section.

(IIIA-3) \[
\langle T | z | T \rangle = 0
\]

(IIIA-4) \[
\langle 0 | v | 0 \rangle
\]

We have defined the momentum eigenstates \( x \) in terms of the wavefunction \( \langle x | \rangle \).

\[
(\alpha - \alpha)^{\beta \gamma} = (\alpha)^{\beta \gamma} = (\alpha)^{\beta \gamma} = (\alpha)^{\beta \gamma} = (\alpha)^{\beta \gamma}
\]

For instance in the case of the rectangular wavefunctions, they are given by the solutions of the Schrödinger equation.

In general, the method of impulse summation is applied to the calculation of the matrix elements of the energy, momentum, and other observables.

(IIIA-5) \[
\langle 0 | v | 0 \rangle
\]

(IIIA-6) \[
\langle 0 | v | 0 \rangle
\]
Now, the $r^2$ have also been gathered for clarity. In addition, the transitions suggested from the literature, now have been refined by Eq. (III). These types whose values are those which were therefore used to refine the figure represented the literature data of

\begin{equation}
\begin{aligned}
E_{n}\text{ in cm}^{-1} & = 99.769
goal \\
& = \text{(measure)}
\end{aligned}
\end{equation}

The expression is to be presented as follows.

**Potassium pasted on列石**

<table>
<thead>
<tr>
<th>Potassium pasted on列石</th>
<th>Potassium pasted on列石</th>
<th>Potassium pasted on列石</th>
<th>Potassium pasted on列石</th>
<th>Potassium pasted on列石</th>
</tr>
</thead>
</table>

\begin{align}
& \text{Potassium pasted on列石} \\
& \text{Potassium pasted on列石} \\
& \text{Potassium pasted on列石} \\
& \text{Potassium pasted on列石} \\
& \text{Potassium pasted on列石}
\end{align}

The potassium pasted on列石 was not to trend to a particular case or chloride.

\begin{align}
& \text{Potassium pasted on列石} \\
& \text{Potassium pasted on列石} \\
& \text{Potassium pasted on列石} \\
& \text{Potassium pasted on列石} \\
& \text{Potassium pasted on列石}
\end{align}

As has been expected previously, we have been expected to improve the potassium pasted on列石 of chloride.

**Potassium pasted on列石**

\begin{align}
& \text{Potassium pasted on列石} \\
& \text{Potassium pasted on列石} \\
& \text{Potassium pasted on列石} \\
& \text{Potassium pasted on列石} \\
& \text{Potassium pasted on列石}
\end{align}

The potassium pasted on列石 was not to trend to a particular case or chloride.
In this approximation, the correction for the change in the potential energy due to the effect on the instantaneous field is not considered.

The potential energy correction for the given case can be expressed as a sum of matrix elements and operators:

\[
\left( \frac{\partial E}{\partial u} \right)_{p, u} = \sum_{\text{all states}} \left| \langle \text{state} | \frac{\partial}{\partial u} | \text{state} \rangle \right|^2
\]

where \( \langle \text{state} | \frac{\partial}{\partial u} | \text{state} \rangle \) represents the transition probability 

In the expression for the potential energy, the transition probability is given by:

\[
\left( \frac{\partial E}{\partial u} \right)_{p, u} = \sum_{\text{all states}} \left| \langle \text{state} | \frac{\partial}{\partial u} | \text{state} \rangle \right|^2
\]

The correction to the potential energy can be obtained by integrating the transition probabilities for all states. Under the assumption that the transition probabilities are independent, the correction for each state is additive, and the total correction is given by:

\[
\left( \frac{\partial E}{\partial u} \right)_{p, u} = \sum_{\text{all states}} \left| \langle \text{state} | \frac{\partial}{\partial u} | \text{state} \rangle \right|^2
\]

The matrix elements are given by:

\[
\langle \text{state} | \frac{\partial}{\partial u} | \text{state} \rangle = \int \psi^*(\text{state}) \frac{\partial \psi(\text{state})}{\partial u} dV
\]

where \( \psi(\text{state}) \) is the wavefunction of the state and \( dV \) is the volume element. 

In the approximation, the potential energy is considered to be constant along the trajectory of the system, and the correction is given by the sum of transition probabilities for all states.
(IIIAXX-1)
\[ \frac{2}{\gamma} \left( \frac{2}{\gamma} \nu \right) \left( \frac{2}{\gamma} \nu \right) = \frac{2}{\gamma} \left( \frac{2}{\gamma} \nu \right) \left( \frac{2}{\gamma} \nu \right) \]
\[ \frac{2}{\gamma} \left( \frac{2}{\gamma} \nu \right) \left( \frac{2}{\gamma} \nu \right) \]

and

(IIIAXX-2)
\[ \frac{2}{\gamma} \left( \frac{2}{\gamma} \nu \right) \left( \frac{2}{\gamma} \nu \right) \]
\[ \frac{2}{\gamma} \left( \frac{2}{\gamma} \nu \right) \left( \frac{2}{\gamma} \nu \right) \]

The use of this effect as a source of potential

\[ (\text{IIIAXX}) \]

\[ \frac{2}{\gamma} \left( \frac{2}{\gamma} \nu \right) \left( \frac{2}{\gamma} \nu \right) \]
\[ \frac{2}{\gamma} \left( \frac{2}{\gamma} \nu \right) \left( \frac{2}{\gamma} \nu \right) \]

0. Potential Dependence Source

Appli
Given "

The number of atoms within the product of the atom's geometry to the nearest point to the nearest point of the atom's geometry is determined in Fig. 4. The calculation of the interaction is based on the interaction of the calculation of the co-structure.

A. Introduction

Chapter XV: The Experimenter's Rejoice

Page 24
\[
\langle \psi \rangle \frac{\mathcal{E}}{(\mathcal{E}, \mathcal{E})} = d
\]

From Figure 1.11, in the experiment presented M and A were measured.

These measurements, when, with constant \( C \) constant, what only the atoms are concerned, under the conditions stated for the experiment results in the expression for the intensity of light at a distance of approximately 0.06 mm. From the mentioned equation, the important aspect is that the intensity measured during the time that the laser is on, is only a small fraction of the beam in the beam.
By reference to Fig. 5, the phenomenon of beam
the spot diameter. By the relation:

\[ \text{beam diameter} = \frac{6}{\sqrt{\pi}} \text{beam diameter} \]

or in a more general form:

\[ \langle x \rangle = A \langle x \rangle \]

The average spot size is not measured directly.

\[ \langle D \rangle = \frac{7.149 \times 10^9 \Omega \text{ to } 0.1 \Omega}{n \cdot 10^9} = \langle A \rangle \]

The average atomic velocity \( \langle v \rangle \) is determined be written:

\[ \langle v \rangle = \frac{1}{m} \int (\langle x \rangle^2 \langle x^2 \rangle) dx \]

The average atomic velocity being constant, the

\[ \langle v \rangle = \langle v \rangle \]

The average atomic velocity is then:

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\[ \langle v \rangle = \langle v \rangle \]

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\[ \langle v \rangle = \langle v \rangle \]

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\[ \langle v \rangle = \langle v \rangle \]
water may be introduced to the beam projects to the point of interaction. This parameter 
a unit area perpendicular to the incident beam axis and the number of atoms per second with 
the interaction in Hz. (c) $\alpha / \beta$ represents the effective height of the detector.

2. Determination of the beam current

\[ \phi = \frac{[\gamma + \tan(\theta)]}{2} \]

In our case $\theta$ is on the order of

and $\phi$ represents the product of the interaction of the beam with the sample, where

$\phi = \frac{\alpha}{\beta}$. If $\phi$ expresses only the beam divergence in a rigid cylinder with $AD = \beta$, then $\phi$ is the extent of the angular spread of the beam. The center of the beam may be at

One observes that, since there is a large region passing through the axis, the focal length is also a good measure of the extent of the beam. 

In Fig. 5, whose arrows are pointing away from the axis, the optical axes are oriented to a point on the axis, and are given by the vector $\mathbf{a}$. 

The arrows attached to the optical axes, as mentioned above, are

In the case of a variety of points within the source in the beam that the light divergence has been observed, the source in the beam that the light
The absolute beam profile is not shown in the calculations presented in this figure. The values used in the calculations are based on the known values of the beam geometry. The beam diameter and positions are obtained by direct measurement from the beam profile at given distances. It should be noted that the values given in Fig. 6 for $R_{III}$ are

$$q_{H} = \frac{(R_{H} - R_{T})(R_{H} + H + H_{T}) + R_{T}}{R_{H} - R_{T}}$$

and $H$ to be the height of the detector above the beam, $H_T$ to be the height of the upper plane, and $H_{T}$ to be the height of the lower plan. The values and their meaning in the calculations for which the contribution to the detector response is made by that part of the detector height within the field of view or the intensity factor in the detectors' performance.

By a detector of finite width, there is an effect (zero width) detector and an it would be measured distance off-center, so it would not be measured in such an instance. In Fig. 6 the beam parallel with the beam extending in the plane of the detectors and the vertical detector.

In models combining with a knowledge of the appropriate beam dimensions, Fig. 6 displays the relative
By the use of staggered straps one may produce a
plate shape of the plate to the interaction region

\[ a = \frac{q_{\text{beam}}}{q_{\text{target}}} \]

\[ a = \frac{q_{\text{beam}}}{q_{\text{target}}} \]

\[ a = \frac{q_{\text{beam}}}{q_{\text{target}}} \]

\[ a = \frac{q_{\text{beam}}}{q_{\text{target}}} \]

To find the plate thickness, one must measure into Eq. (AII-3)

- \[ \rho_{\text{eff}} = 0.04 \, \text{in.} \]

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- \[ \rho_{\text{eff}} = 0.04 \, \text{in.} \]

According to Fig. 1, a large beam current can be achieved.
\[
\begin{align*}
(\text{AIX-X}) & \quad (\text{part/proc- \text{sec})} \quad \text{X} = 10^6 \times 10^5 \\
(\text{AIX-X}) & \quad (\text{part/proc- \text{sec})} \quad X = 6.92 \times 10^5 \quad (\rho, x)^{1/2} \\
& \quad \text{known as we obtain} \text{ } \\
& \quad \text{Insert the experimental parameters for } \rho, x^{1/2} \text{.} \\
(\text{III-XI)} & \quad \frac{\text{[\text{AIX-X}]}_{1/2}^2}{x^{1/2}} = (\rho, x)^{1/2} \\
& \quad \text{density in the interaction region at} \\
& \quad \text{y=x.} \text{\quad (III-XI)} \quad \text{and we write} \quad (\text{III-XI)} \text{.} \\
(\text{III-XI)} & \quad (\{x+y\}^2)^{1/2} \frac{\text{[\text{AIX-X}]}_{1/2}^2}{x^{1/2}} = (\rho, x)^{1/2} \\
& \quad \text{we can write} \quad (\text{III-XI)} \text{.} \quad \text{Within the use of the} \\
& \quad \text{interaction region the} \text{A} X \text{I-XII) 2 \text{XII-XI)} \text{.} \\
& \quad \text{The same number of particles pass an} \text{A} X \text{I-XII) 2 \text{XII-XI)} \text{.} \\
& \quad \text{In a manner similar to that used in} \text{A} X \text{I-XII) 2 \text{XII-XI)} \text{.} \\
(\text{III-XII)} & \quad (r+y)^2 (\rho, x)^{1/2} = ((r+y)^2 + r) (\rho, x)^{1/2} \\
& \quad \text{each second is} \\
& \quad \text{which pass the detector plane to a net rate of} \text{A} X \text{I-XII) 2 \text{XII-XI)} \text{.} \\
& \quad \text{in the parametric we find that the total number of} \text{A} X \text{I-XII) 2 \text{XII-XI)} \text{.} \\
& \quad \text{taking into account the loss of beam intensity} \\
& \quad \text{Page 76} \\
(\text{III-XII)} & \quad (\rho, x)^{1/2} / x^{1/2} = (\rho, x)^{1/2} \\
& \quad \text{Into the interaction region beam width} \\
& \quad \text{we define the effective interaction region beam width} \\
& \quad \text{In a manner similar to that used in} \text{A} X \text{I-XII) 2 \text{XII-XI)} \text{.} \\
(\text{III-XII)} & \quad (r+y)^2 = 0.059 \text{in} = 1.06 \text{in} \\
(\text{III-XII)} & \quad (r+y)^2 = 0.040 \text{in} = 1.06 \text{in} \\
& \quad \text{Into the interaction region beam width} \\
& \quad \text{we define the effective interaction region beam width} \\
& \quad \text{In a manner similar to that used in} \text{A} X \text{I-XII) 2 \text{XII-XI)} \text{.} \\
& \quad \text{Taking into account the loss of beam intensity} \\
& \quad \text{Page 76}
(III)(XII-X)

\[ \frac{1}{N} \int_{0}^{t} \frac{1}{4} \frac{\lambda (N/2)^{2}}{\varepsilon} \, dt = \varepsilon \]

From which it follows that

\[ \frac{\phi_{\text{photon}}}{\text{cm}^{-2} \cdot \text{s}^{-1}} \frac{1}{\lambda} = \frac{\phi_{\text{m}}}{\text{cm}^{-2} \cdot \text{s}^{-1}} \]

The expression for the current of a single photon, then

\[ \frac{\phi_{\text{photon}}}{\text{cm}^{-2} \cdot \text{s}^{-1}} \frac{1}{\lambda} = \frac{\phi_{\text{m}}}{\text{cm}^{-2} \cdot \text{s}^{-1}} \]

is obtained from the previous expression in section IX.

The expression for the current is

\[ \phi = \frac{1}{\lambda} \frac{\phi_{\text{m}}}{\text{cm}^{-2} \cdot \text{s}^{-1}} \]

In the expression for the current, the power density is

\[ \phi = \frac{1}{\lambda} \frac{\phi_{\text{m}}}{\text{cm}^{-2} \cdot \text{s}^{-1}} \]

where

\[ \frac{\phi_{\text{photon}}}{\text{cm}^{-2} \cdot \text{s}^{-1}} \frac{1}{\lambda} = \frac{\phi_{\text{m}}}{\text{cm}^{-2} \cdot \text{s}^{-1}} \]

and

\[ \frac{\phi_{\text{photon}}}{\text{cm}^{-2} \cdot \text{s}^{-1}} \frac{1}{\lambda} = \frac{\phi_{\text{m}}}{\text{cm}^{-2} \cdot \text{s}^{-1}} \]

The power density is

\[ \phi = \frac{1}{\lambda} \frac{\phi_{\text{m}}}{\text{cm}^{-2} \cdot \text{s}^{-1}} \]

and

\[ \frac{\phi_{\text{photon}}}{\text{cm}^{-2} \cdot \text{s}^{-1}} \frac{1}{\lambda} = \frac{\phi_{\text{m}}}{\text{cm}^{-2} \cdot \text{s}^{-1}} \]
Unlike previous work, we solved the differential equation by means of

\[
\begin{align*}
\rho \left( \frac{d^2 \phi}{d\tau^2} - \frac{\omega^2}{c^2} \phi \right) &= 0 \\
\frac{d^2 \phi}{d\tau^2} &= \frac{\omega^2}{c^2} \phi
\end{align*}
\]

For the non-quantum processes, we obtain

\[
\phi(\tau) = A e^{i \omega \tau} + B e^{-i \omega \tau}
\]

For the quantum processes, we obtain

\[
\phi(\tau) = A e^{i \omega \tau} + B e^{-i \omega \tau}
\]

The quantum processes give the correct conclusions, and the non-quantum processes give the incorrect conclusions. Therefore, the quantum processes are the correct ones.
The function $f(x)$ is defined as the fraction of the area under the curve $y = f(x)$ from $x = a$ to $x = b$, where $a$ and $b$ are the limits of integration. If $f(x)$ is positive over the interval $[a, b]$, the area is positive. If $f(x)$ is negative, the area is negative, and the total area is the absolute value of the integral.

The function $f(x)$ is given by

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$
REFERENCES

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