Effects of Atmospheric Winds and Aerodynamic Lift on the Inclination of the Orbit of the S3-1 Satellite

Space Sciences Laboratory
The Ivan A. Getting Laboratories
The Aerospace Corporation
El Segundo, Calif. 90245

27 October 1976
Interim Report

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Prepared for
SPACE AND MISSILE SYSTEMS ORGANIZATION
AIR FORCE SYSTEMS COMMAND
Los Angeles Air Force Station
P.O. Box 92960, Worldway Postal Center
Los Angeles, Calif. 90009
This report was submitted by The Aerospace Corporation, El Segundo, CA 90245, under Contract F04701-76-C-0077 with the Space and Missile Systems Organization, Deputy for Advanced Space Programs, P.O. Box 92960, Worldway Postal Center, Los Angeles, CA 90009. It was reviewed and approved for The Aerospace Corporation by G. A. Paulikas, Director, Space Sciences Laboratory. Lieutenant Jean Bogert, SAMSO/YAPT, was the project officer.

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FOR THE COMMANDER

Jean Bogert
1 Lt, United States Air Force
Technology Plans Division
Deputy for Advanced Space Programs
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<th>2. GOVT ACCESSION NO.</th>
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<td>EFFECTS OF ATMOSPHERIC WINDS AND AERODYNAMIC LIFT ON THE INCLINATION OF THE ORBIT OF THE S3-1 SATELLITE.</td>
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<td>Barbara K. Ching, David R. Hickman, and Joe M. Straus</td>
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<td>Superrotation</td>
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<td>S3-1 Satellite</td>
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<td>Aerodynamic drag and lift effects on the inclination of the orbit of the S3-1 satellite have been used to infer atmospheric</td>
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<td>zonal wind speeds at an altitude of 175 km and to estimate the satellite surface thermal accommodation coefficient. The</td>
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<td>unusually high quality of the orbital and attitude data permitted the 7-month data span to be analyzed in 6 non-overlapping</td>
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<td>subintervals, thereby resulting in finer local-time and spatial resolution than is normally obtained in such studies. The</td>
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<td>aerodynamic lift force was found to be quite...</td>
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sensitive to the assumed value of the thermal accommodation coefficient, a quantity that is neither well known nor understood. The inclination data were best fit (in a least squares sense) when the accommodation coefficient was greater than 0.95. Best agreement with theoretical models in terms of wind speed and direction was obtained when the accommodation coefficient was taken to be nearly 1.0. In this case, the inferred wind was $\approx 400 \text{ m/sec}$ eastward at local times 19-21 hr, and $\approx 200 \text{ m/sec}$ westward at local times 7-9 hr. The study indicates that if accurate zonal wind velocities are to be derived from satellite orbital inclination changes, care must be taken to ensure that the shape and orientation of the satellite are such that the lift forces generated are sufficiently small that the resulting uncertainty due to inadequate knowledge of the accommodation coefficient does not cause significant error.
PREFACE

We would like to thank Dr. C. J. Rice for providing us with the S3-1 orbital data.
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INTRODUCTION

The analysis of long-term changes in the inclination of earth-orbiting satellites is a valuable tool for investigating the characteristics of upper-atmospheric winds. The majority of this work has been carried out by King-Hele and his colleagues (see King-Hele, 1972a for a review) and has revealed an average eastward rotation of the atmosphere in the altitude range 200-350 km. Little information regarding the north-south component of the winds has been gathered in this way because of the small effect of this wind component on long-term satellite inclination changes; however, recent studies (King-Hele, 1976) have been aimed at this aspect of the problem. Furthermore, the diurnal variations of the wind field have been treated in this way only very recently (Blum and Schuchardt, 1976), but this study applied only to exospheric heights. The primary obstacle in such a study arises from the fact that the local time at perigee, where the forces normal to the orbit plane due to winds are most effective, changes considerably on the time scale necessary for significant changes in inclination to occur; thus, the diurnal variations in winds are difficult to isolate. However, in principle, this technique is complementary to incoherent scatter radar (Evans, 1972; Amayenc, 1974) and Fabry-Perot interferometric (Hernandez and Roble, 1976) determinations of thermospheric winds.
None of the previous studies of satellite inclination changes has taken explicit account of possible effects of aerodynamic lift. This is primarily because no accurate attitude information has been generally available. Furthermore, the aerodynamic lift effects on spherical or near-spherical satellites should be negligible. However, Karr and Yen (1970) and Karr et al. (1975) have shown that the effects of aerodynamic lift on various orbital elements can be considerable, especially for a satellite which has large planar surfaces or a fixed attitude relative to its velocity vector.

The purpose of the present paper is to evaluate the effects of upper-atmospheric winds and aerodynamic lift on the inclination of the S3-1 satellite. As will become clear, the facts that the orbit was nearly sun-synchronous and that the orientation of the satellite was almost fixed and was closely monitored allow a detailed study of diurnal variations of upper-atmospheric winds and the effects of aerodynamic lift on the inferred values of wind speeds, and an estimate of the thermal accommodation coefficient.
THE SATELLITE AND ITS ORBIT

S3-1, a research satellite developed under the U.S. Air Force Space Test Program, was placed into near sun-synchronous orbit in late 1974. The payload consisted of a number of instruments designed to measure thermospheric density and composition. On November 4, 1974, characteristics of the orbit were as follows:

- Apogee height: 3767 km
- Perigee height: 159 km
- Eccentricity, $e$: 0.217
- Inclination, $i$: 96.98 deg
- Right ascension of ascending node, $\Omega$: 29.04 deg
- Argument of perigee, $\omega$: 125.43 deg
- Period, $T$: 126.1 min.

Figure 1 shows the evolution of $i$, $\omega$ and $T$ during the period November 4, 1974 to May 26, 1975. Since the orbit is nearly polar, the latitude of perigee $\lambda$ can be determined by inspection of $\omega$ in Fig. 1 ($\sin \lambda = \sin \omega \sin i$). The local time of perigee occurred approximately in the meridian plane containing the local times 8 and 20 hr. (Further information regarding the local time of perigee is given in Table 2 below.)

The orbit of the S3-1 satellite was especially suitable for drag analysis because of the unusually high quality and completeness of orbit and attitude parameters. Tracking was performed by the USAF S-band Space-Ground Link System (SGLS), which utilizes an up-link active tracking
Figure 1. Satellite inclination $i$, argument of perigee $\omega$ and orbital period $T$ as a function of time.
signal having a pseudo-random noise (PRN) ranging code which is echoed back to the remote tracking station by a phase-locked transponder. Round-trip timing of the signals along with Doppler shift measurements allowed range and range-rate to be derived. These quantities along with the observed satellite azimuth and elevation were used to compute orbital elements. On-board earth horizon sensor, sun angle sensor, and three-axis magnetometers allowed determination of the satellite attitude. Magnetic torque coils enabled the spin axis to be maintained nearly normal to the orbit plane. The satellite was contacted about six times per day in order to ensure adequate tracking information. Orbital elements and spin axis orientation were computed at least once per day throughout the satellite lifetime. The inclination data has a root-mean-square (rms) error on the order of $10^{-3}$ deg. Two hundred thirteen sets of orbital elements and spin axis orientation parameters (right ascension and declination) were used in the present study.

In addition to the orbital elements and attitude parameters, the satellite shape is of importance in considering aerodynamic effects. Photographs of the actual satellite and access to an accurate full sized mock-up ensured adequate knowledge of shape and dimensions for this study.
CALCULATION OF AERODYNAMIC FORCES

The aerodynamic forces exerted on a body traveling through a gas in the free molecular flow regime depend on the details of the interaction of the impinging gas particles with the surface of the body. A two-parameter model (Karr and Yen, 1970; Karr et al., 1975) has been used to describe this interaction. The forces acting on a flat plate of area S are illustrated in Fig. 2. As viewed from the plate, gas particles with average velocity \( v_\infty \) approach from a direction making an angle \( \chi \) with the surface normal. The velocity and angular distribution of the reflected particles are characterized by two parameters, \( \alpha \) and \( P \), respectively. The velocity of the reflected particles is

\[
v_r = v_\infty \sqrt{1 - \alpha},
\]

so that \( \alpha \) resembles the usual thermal accommodation coefficient, \( \alpha_T \), which appears in the relationship

\[
v_r^2 = v_\infty^2 (1 - \alpha_T) + v_s^2 \alpha_T.
\]

The velocity \( v_s \), which characterizes the surface temperature, \( T_s \), of the plate, is given in terms of \( k \), Boltzmann's constant, and \( m \), the molecular weight of the gas molecules, by \( v_s^2 = 3kT_s/2m \). The average angle of reflection is
Figure 2. Geometry for lift and drag forces due to gas molecules impinging on a flat plate.
\[
\chi_r = \chi (1 - P), \tag{3}
\]

and the angular distribution of the reflected particles ranges from pure specular at \( P = 0 \) to diffuse at \( P = 1 \).

The aerodynamic force on the plate may be resolved into a drag component, \( F_D \), parallel to \( v_\infty \), and a lift component, \( F_L \), perpendicular to \( v_\infty \) and in the plane of \( v_\infty \) and the normal to the surface, given by

\[
F_D = \frac{1}{2} \rho v_\infty^2 S \left\{ 2 \cos \chi - 2 \sqrt{1 - \alpha} \cos \chi \cos \left[ \pi - (2 - P)\chi \right] \right\}, \tag{4}
\]

\[
F_L = \frac{1}{2} \rho v_\infty^2 S \left\{ 2 \sqrt{1 - \alpha} \cos \chi \sin \left[ \pi - (2 - P)\chi \right] \right\}, \tag{5}
\]

where \( \rho \) is the mass density of the gas. The quantities in the braces of Equations (4) and (5) are the so-called drag \( (C_D) \) and lift \( (C_L) \) coefficients, respectively.

The S3-1 satellite, schematically illustrated in Fig. 3 is approximated as an assembly of eight flat plates. In orbit it was maintained with its spin axis nominally perpendicular to the orbit plane with a spin rate of about 5 rpm. Equations (4) and (5) may be used to calculate the lift and drag forces averaged over one spin rotation of the satellite.

Let \( \hat{x}, \hat{y}, \hat{z} \) be mutually orthogonal unit vectors defining a rectangular coordinate system. The spin axis is chosen to lie along the \( z \) axis. Consider a pair of identical flat plates, each of area \( S \), symmetrically placed on opposite sides of the \( z \) axis. Let the outward unit normal vectors for the plates, \( \hat{n}_1 \) and \( \hat{n}_2 \), be given by
Figure 3. S3-1 satellite shape.
\[ \hat{\mathbf{L}}_1 = \cos \xi \sin \eta \hat{x} + \sin \xi \sin \eta \hat{y} + \cos \eta \hat{z}, \]  
(6)

\[ \hat{\mathbf{L}}_2 = -\cos \xi \sin \eta \hat{x} - \sin \xi \sin \eta \hat{y} + \cos \eta \hat{z}. \]  
(7)

where \( \xi \) and \( \eta \) are the azimuthal and polar angles of \( \hat{\mathbf{n}}_1 \), respectively.

For \( \xi = 0 \) the geometry is as illustrated in Fig. 4.

Consider a stream of gas incident on the plates coming from the direction defined by azimuthal and polar angles \( \varphi_v \) and \( \theta_v \). The unit vector in the direction of \( \varphi_v \) and \( \theta_v \) is given by

\[ \hat{\mathbf{c}} = \sin \theta_v \cos \varphi_v \hat{x} + \sin \theta_v \cos \varphi_v \hat{y} + \cos \theta_v \hat{z}. \]  
(8)

Thus the velocity of the gas molecules is \(- | \mathbf{v} | \hat{\mathbf{c}} \). The magnitudes of the drag and lift forces on plate 1 are then given by Equations (4) and (5) with \( \cos \chi = \mathbf{c} \cdot \hat{\mathbf{L}}_1 \). If \( \cos \chi < 0 \), the gas stream is incident on the back side of the plate, in which case both drag and lift forces will be considered to be zero. (The plate is shadowed by other surfaces of the satellite.)

Unit vectors in the drag and lift directions are \( \hat{\mathbf{F}}_D = -\hat{\mathbf{c}} \) and \( \hat{\mathbf{F}}_L = (\hat{\mathbf{n}}_1 \times \hat{\mathbf{c}}) \times \hat{\mathbf{c}} \).

Denoting either drag or lift vectors by \( \hat{\mathbf{F}}(\xi) \), a spin averaged force acting on the pair of plates is given by

\[ <\hat{\mathbf{F}}_D> = \frac{1}{\pi} \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\xi}{\xi_o + \frac{\pi}{2}} \hat{\mathbf{F}}(\xi) \, d\xi, \]  
(9)

where \( \xi_o = \varphi_v \) is the spin angle for which \( \hat{\mathbf{n}}_1 \) is closest to \( \hat{\mathbf{c}} \).
Figure 4. Geometry for calculation of spin averaged lift and drag forces on a symmetrically placed pair of flat plates (shown for $\xi = 0$).
Equation (9) was used to calculate spin-averaged drag and lift forces for the S3-1 satellite. The asymmetric shape due to the inclined "roof" causes a non-zero average lift force. The ratio of lift to drag forces for various values of $\alpha$ and $P$ is shown in Fig. 5. It will be shown later that only values of $\alpha$ near 1 appear to be reasonable. The dependence on $P$ of the lift to drag force ratio for large values of $\alpha$ is seen to be weak.

In addition, other investigations of satellite drag phenomena have given results compatible with diffuse rather than specular reflection (Imbro, et al., 1975; Beletskii, 1970). Hereafter we will therefore consider only the case $P = 1$ (diffuse reflection).

The nominal orientation of the spin axis perpendicular to the orbit plane was maintained to within about 5 degrees. To test for sensitivity to departures from nominal orientation, the ratio of spin averaged lift to drag force was calculated for a range of pitch angles on either side of the nominal. The results, shown in Fig. 6, indicate a strong dependence on actual spin axis orientation, a pitch angle of only 4 deg being sufficient to reduce the lift force to zero. For the calculations described below we have therefore used the flight-determined spin axis location rather than assuming nominal orientation. Only the availability of frequently determined, high quality attitude and ephemeris data has allowed reliable interpretation of variations of orbital elements for this satellite with its large lift to drag force ratio.
Figure 5. Variation of spin averaged lift to drag force ratio with $P$ for selected values of $\alpha$. 

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Figure 6. Variation of spin averaged lift to drag force ratio with pitch angle for selected values of $\alpha$. 
EFFECTS OF WINDS AND LIFT ON ORBITAL INCLINATION

To determine atmospheric winds from changes in orbital inclination, it was first necessary to correct the data for other known effects. Specifically, lunisolar and geopotential zonal harmonic perturbations were removed; solar radiation pressure was found to be negligible, and no corrections were made for it.

In principle, aerodynamic lift effects could have been removed from the inclination data in the same manner that the other perturbations were handled. It was found, however, that the magnitude of the lift effect was very sensitive to the value adopted for the accommodation coefficient, which is not well known or understood. It was decided, therefore, that the wind analysis needed to be conducted for a range of reasonable values of the accommodation coefficient. The derivation of the lift force was presented in the previous section. Here, the lift force is assumed to be known, and its effect on the inclination will be discussed.

A particular problem that existed in the present study was that the orbital eccentricity in the early portion of the data span exceeded a value of 0.20. This precluded the straightforward application of King-Hele's well-known analytic expressions (for small eccentricities) that relate changes in orbital inclination to atmospheric winds (e.g. King-Hele, 1966). Similarly, in deriving the inclination change due to aerodynamic lift, the "small eccentricity" approximation was avoided.
The problem of "large eccentricities" has recently been treated analytically by King-Hele and Walker (1976), but as their work was not available to us at the time of our analysis, we shall proceed to describe our approach, which was numerical rather than analytic.

The theory relating orbital inclination changes to atmospheric winds and other forces normal to the orbit has been presented in detail by King-Hele (e.g. 1964, 1966); therefore no attempt will be made here to reproduce his thorough work. It shall be assumed that the reader is familiar with King-Hele's work, and only major points of relevance to the present study will be pursued.

Changes in the orbital inclination arise only from forces normal to the orbit plane. In an orbital period $T$, the average change $\Delta i$ resulting from a force per unit mass $f$ that is normal to the orbit may be written (King-Hele, 1964)

$$\Delta i = \int_0^T \frac{rf \cos u}{\mu a (1 - e^2)^{5/2}} \, dt,$$

where $r$ is the geocentric distance of the satellite, $\mu$ is the gravitational constant of the earth, $a$ is the semi-major axis, and $e$ is the eccentricity. The angle $u$ is the argument of latitude, equal to the sum of the argument of perigee $\omega$ and the true anomaly $\beta$. The forces normal to the orbit that are of concern are those due to aerodynamic drag (i.e. atmospheric winds) and aerodynamic lift.
Drag. Using the eccentric anomaly $E$ instead of time $t$ as the independent variable, King-Hele (1966, equation 35) has shown that the change in inclination per orbital revolution arising from the drag force normal to the orbit is

$$
\Delta i_D = \frac{\delta p_p}{2} \int_0^{2\pi} r^2 \exp(z \cos E - z) \left[ \frac{r(1 + e \cos E)}{v \mu(1 - e^2)} \right]^{1/2} \times
$$

$$
\left[ \frac{\omega \sin i \cos^2 (w + \theta) - \frac{\delta}{2} \cos i \cos (w + \theta)}{1 - \sin^2 i \sin^2 (w + \theta)} \right]^{1/2} dE
$$

where $\rho_p$ is the density at perigee, and $z = ae/H$, $H$ being the density scale height at perigee. (Because of the high eccentricity of the orbit, constant scale height and spherical atmosphere assumptions are utilized.) The quantities $\omega$ and $\delta$ are, respectively, the zonal and meridional angular velocities of the atmosphere, measured positive eastward and northward. The function $\gamma$ is nearly constant and may be taken to be

$$
\sqrt{\gamma} \equiv 1 - \frac{r_p \omega \cos i}{v_p},
$$

where $r_p$ is the perigee distance and $v_p$ the velocity at perigee. The quantity $\delta$ in Equation (11) is a function of the physical characteristics of the satellite:

$$
\delta = \gamma S_D C_D /m_s,
$$

-25-
where $C_D$ is the drag coefficient appropriate for a reference area $S_D$ of the satellite, and $m_s$ is the mass of the satellite.

The approach used by King-Hele (1966) to evaluate the integral in Equation (11) involved expansions of the functions in powers of $e$, thereby limiting the resulting expression for $\Delta i$ to small values of $e$. While this approximation is adequate for the great majority of satellites, the error would be too large for this particular satellite, and thus numerical integration was employed in this study.

As pointed out by King-Hele, it is useful to eliminate the explicit dependence of $\Delta i$ (Equation 11) on the perigee density $\rho_p$ by use of the change in orbital period per revolution, $\Delta T$. It may be shown (King-Hele, 1964) that

$$\Delta T = -6\pi^2 \sqrt{a/\mu} \delta a^2 \rho_p e^{-z} B(z)$$

where $B(z) = I_0 + 2eI_1 + \frac{3}{4} e^2 (I_o + I_2) + \frac{e^3}{4} (3I_1 + I_3)$.

The $I_n$'s in the expression above are the modified Bessel functions of the first kind of argument $z = ae/m$. It should be noted that Equation (12) was also derived by expansion in powers of $e$, but that sufficient higher order terms were retained to permit its use without serious error for eccentricities $e \sim 0.2$. 

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Substitution of Equation (12) into (11) leads to

\[
\Delta i_D = \frac{\Delta T_D}{6\pi B(z)} \int_0^{2\pi} \frac{r}{a} \left( \frac{1 + e \cos E}{1 - e^2} \right)^{1/2} \exp(z \cos E) \times \\
\left[ \Lambda \sin \lambda \cos^2 (u + \theta) - \varepsilon \cos \lambda \cos(u + \theta) \right] \left( 1 - \sin^2 \lambda \sin^2 (u + \theta) \right)^{1/2} \right] dE, \quad (13)
\]

where \( \Delta T_D \) is the change in orbital period per revolution expressed as a fraction of a day, and \( \Lambda \) and \( \varepsilon \) are related to the zonal and meridional wind components by

\[
\Lambda = \frac{w}{w_e}, \\
\varepsilon = \frac{\phi}{w_e},
\]

where \( w_e = 2\pi \) radians per day. It is to be noted that the ratio \( r/a \) and the true anomaly \( \theta \) are functions of \( E \):

\[
r/a = 1 - e \cos E, \\
\sin \theta = \frac{a/r}{\sqrt{1-e^2}} \sin E, \\
\cos \theta = \frac{a}{r} (\cos E - e).
\]

**Lift.** Because of attitude control, the spin-averaged lift force on this satellite was at all times within a few degrees of the normal to the orbit. The magnitude of the lift force, like that of the drag force, is proportional to the square of the satellite velocity relative to the ambient air.
However, little error results if the magnitude is taken to be proportional to the square of the inertial velocity. The force per unit mass normal to the orbit produced by lift may then be written

\[ f_L = \frac{1}{2} \rho v^2 \left( \frac{S_L C_L}{m_s} \right), \quad (14) \]

where \( \rho \) is the atmospheric density, \( v \) is the inertial velocity of the satellite, and \( C_L \) is the coefficient of lift appropriate for an effective satellite area \( S_L \).

By use of Equations (10) and (14) and the relationship

\[ v = \mu (1 + e \cos E) / r, \]

as well as other relationships between the orbital parameters, it may be shown that the change in inclination per revolution due to lift is given by

\[ \Delta i_L = - \pi \rho \beta a(S_L C_L / m_s) e^{-2 A(z)} (1-e^2)^{-1/2} \cos \omega, \quad (15) \]

where

\[ A(z) = (1-e^2) I_1 - \frac{e}{2} I_0 + \frac{e}{2} I_2 \]

and, as before, the \( I_n \) are the modified Bessel functions of the first kind of argument \( z \).
Using Equations (10), (12) and (14) and letting $T_D$ be the orbital period in units of a day one obtains

$$\Delta i_L = \frac{\Delta T_D \cos w}{3T_D \sqrt{1-e^2}} \frac{S_L C_L}{S_D C_D} A(z) \frac{A(z)}{B(z)} \frac{A(z)}{B(z)} \frac{A(z)}{B(z)} \frac{A(z)}{B(z)} \frac{A(z)}{B(z)}$$

(16)

In Equation (16), deviation of the satellite spin vector from the nominal position, which would affect the lift force as described in the previous section, is taken into account by the term $\left(S_L C_L\right)$ which has been assumed constant over an orbital period, but varies on a longer timescale.

Finally, in more concise form, the sum of drag (i.e. atmospheric wind) and lift effects on the orbital inclination is expressed by

$$\Delta i = \pi\left(\alpha\right) + A_G + \varepsilon G_m,$$

(17)

where $\pi\left(\alpha\right)$ is the aerodynamic lift contribution, given by $\Delta i_L / \Delta T_D$ (Equation 16), whose dependence on $\alpha$, the thermal "accommodation coefficient," arises from the ratio $\left(S_L C_L / S_D C_D\right)$. The ratio of lift to drag forces, discussed in the previous section, yields $\left(S_L C_L / S_D C_D\right)$. The second and third terms on the right hand side of Equation (17) represent the zonal and meridional wind contributions, respectively; expressions for $G_z$ and $G_m$ are readily derived from Equation (13).
RESULTS

Equation (17) relates the rate of change of inclination to the orbital parameters, the wind speeds (Λ and ε), the thermal accommodation coefficient α and the satellite geometry and orientation. Since all of these quantities except Λ, ε and α are known, values of Λ and ε may be determined which "best" fit the data corrected for luni-solar and geopotential perturbations for a given value of α. This best fit was obtained by integrating Equation (17) and evaluating Λ and ε and the initial value of i in such a way that the data were fit in a least-squares sense. A total of 213 data points were analyzed in this manner, and the results of this procedure are described in this section. As shown by King-Hele, the inferred atmospheric wind speeds apply to an altitude 3/4 of a scale height above the perigee altitude of \( \sim 160 \text{km} \).

In this analysis, the data set was treated in two ways. First, least-squares fits were carried out for the entire data set for six values of ε: 1.0, 0.99, 0.97, 0.95, 0.85 and 0.70. (Note that the effect of aerodynamic lift vanishes for \( \alpha = 1.0 \).) In all cases, it was found that the value of ε was poorly determined, and variations in the value of ε had little effect on the value of Λ deduced. This is primarily due to the fact that i is near 90 deg so that the effect of ε on i is small (cf. Equation 13). Therefore, the fits were made with ε = 0, and no further discussion of ε is given here. Figure 7 shows the least-squares fit to the data for \( \alpha = 1.0 \), for which the best fit value of Λ is 1.31 (solid curve) and for \( \alpha = 0.99 \), for which the best fit value of Λ is 1.22 (dashed curve). Both calculated curves reproduce the general evolution of the inclination with time,
Figure 7. Least squares fits to entire set of inclination data (corrected for lunisolar and geopotential perturbations) for $\alpha = 1.0$ (-----) and $\alpha = 0.99$ (---).
the greatest rate of change of inclination occurring during periods in which the satellite perigee was near the earth's equator \((w = 0, 180 \text{ deg})\) and near the end of the satellite's lifetime, when the orbital eccentricity decreased rapidly. However, quantitative comparison indicates that the calculated best fits depart substantially from the data (i.e., differences on the order of 0.01 - 0.03 deg).

Table 1 gives the calculated values of \(A\) for the six values of \(\alpha\) noted above, along with the rms error of the fit. Several features are evident. The inferred value of \(A\) is fairly sensitive to changes in \(\alpha\), especially for values of \(\alpha\) very near 1. In fact, the value of \(A\) inferred from this analysis indicates an average eastward wind relative to the rotating earth if \(\alpha > 0.85\) and a westward wind otherwise. However, inspection of the value of the rms error as a function of \(\alpha\) discloses that the rms error minimizes in the range \(0.95 < \alpha < 0.99\), so that values of \(\alpha\) in this range are more acceptable than others. Orbital considerations, coupled with theoretical treatments of thermospheric winds (Blum and Harris, 1975; Straus et al., 1975a, b), also tend to favor the value of \(A\) associated with the larger values of \(\alpha\). As noted earlier, the local time of satellite perigee lies nearly in the 8 hr local time - 20 hr local time plane (see Table 2). During the lifetime of the satellite, perigee occurred on the evening side of the earth approximately 67% of the time, so that the overall value of \(i\) is weighted twice as strongly by its value at \(\sim 20\) hr local time, where the theoretical studies cited above indicate a strong eastward wind, than by that
Table 1. Calculated values of $\Lambda$ for the entire data set for several assumed values of $\alpha$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\Lambda$</th>
<th>RMS error (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>1.31</td>
<td>$1.24 \times 10^{-2}$</td>
</tr>
<tr>
<td>0.99</td>
<td>1.22</td>
<td>$7.87 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.97</td>
<td>1.16</td>
<td>$6.59 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.95</td>
<td>1.12</td>
<td>$7.00 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.85</td>
<td>1.00</td>
<td>$1.19 \times 10^{-2}$</td>
</tr>
<tr>
<td>0.71</td>
<td>0.91</td>
<td>$1.74 \times 10^{-2}$</td>
</tr>
</tbody>
</table>
Table 2. Characteristics of the six data segments

<table>
<thead>
<tr>
<th>SEGMENT</th>
<th>HEMISPHERE</th>
<th>LST</th>
<th>NO. POINTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Northern</td>
<td>21-20</td>
<td>60</td>
</tr>
<tr>
<td>2</td>
<td>Southern</td>
<td>20-21</td>
<td>38</td>
</tr>
<tr>
<td>3</td>
<td>Southern</td>
<td>8</td>
<td>37</td>
</tr>
<tr>
<td>4</td>
<td>Northern</td>
<td>7</td>
<td>32</td>
</tr>
<tr>
<td>5</td>
<td>Northern</td>
<td>19</td>
<td>31</td>
</tr>
<tr>
<td>6</td>
<td>Southern</td>
<td>19</td>
<td>15</td>
</tr>
</tbody>
</table>
at ~8 hr local time, where westward winds are expected. Thus, the calculated value of $\Lambda \sim 1.10 - 1.20$ is understandable, and does not necessarily indicate an average eastward motion of the atmosphere.

The excellent quality of the orbital data allows more detailed analysis of the changes in inclination than that described above. In order to investigate diurnal variations of $\Lambda$, and to further elucidate the effects of different assumed values of $\alpha$ on the inferred values of $\Lambda$, the data set has been treated in a second manner, as follows. As shown in Table 2 and in Fig. 8, the data set was divided into 6 segments in each of which the perigee latitude was restricted to lie in one hemisphere and also to occur on one side of the earth. For example, Segment 1 contains data points for which perigee occurred in the northern hemisphere on the evening (~22 hr local time) side of the earth, whereas Segment 2 covers the southern hemisphere on the evening side. This division thus allows us to distinguish variations of $\Lambda$ with local time as well as season.

The least-squares fit procedure was carried out for each of the six segments of data for four values of $\alpha$: 1.0, 0.99, 0.95 and 0.85. An example of the accuracy of the fit is given in Fig. 8, which displays every second data point as an "x", the calculated fit for $\alpha = 0.99$ as a piecewise continuous solid line, and the residual for each data point. The rms error for the entire data set in this case is $1.33 \times 10^{-3}$ deg, a factor of almost 6 better than that obtained when the entire data set was treated at once. This overall agreement is considerably better than that which
Figure 8. Least squares fits to the six segments of inclination data for $\alpha = 0.99$. Also shown is the residual of the fit at each data point.
has been attained in this type of analysis in the past.

The calculated values of $\Lambda$ for each of the six segments are given in Table 3. In each segment, a substantial variation with $\alpha$ of the inferred value of $\Lambda$ is evident, and, for a fixed value of $\alpha$, the computed value of $\Lambda$ varies from segment to segment. There is no a priori way to pick the correct value of $\alpha$. Examination of the residuals of the fit shows no strong tendency towards a particular value of $\alpha$, although the fits with $\alpha > 0.95$ are consistently better than those for $\alpha = 0.85$; however, in no case were the residuals for $\alpha = 1.0$ less than those for $\alpha = 0.99$. On the other hand, if a single value of $\alpha$ applies for the entire lifetime, the values of $\Lambda$ inferred for $\alpha \geq 0.99$ are preferable on the basis of comparison with the theoretical studies cited above. These studies predict an eastward ($\Lambda > 1$) motion during the evening (Segments 1, 2, 5 and 6) and a westward ($\Lambda < 1$) motion during the morning (Segments 3 and 4); only the results for values of $\alpha$ very near 1.0 show this behavior.
Table 3. Calculated values of \( \Delta \) for the six data segments for several assumed values of \( \alpha \)

<table>
<thead>
<tr>
<th>SEGMENT</th>
<th>( \alpha = 1.00 )</th>
<th>( \alpha = 0.99 )</th>
<th>( \alpha = 0.95 )</th>
<th>( \alpha = 0.85 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.82</td>
<td>1.71</td>
<td>1.60</td>
<td>1.47</td>
</tr>
<tr>
<td>2</td>
<td>1.92</td>
<td>1.61</td>
<td>1.29</td>
<td>0.91</td>
</tr>
<tr>
<td>3</td>
<td>0.61</td>
<td>1.03</td>
<td>1.48</td>
<td>1.98</td>
</tr>
<tr>
<td>4</td>
<td>0.38</td>
<td>0.47</td>
<td>0.57</td>
<td>0.67</td>
</tr>
<tr>
<td>5</td>
<td>1.80</td>
<td>1.41</td>
<td>0.99</td>
<td>0.52</td>
</tr>
<tr>
<td>6</td>
<td>1.79</td>
<td>1.41</td>
<td>1.00</td>
<td>0.55</td>
</tr>
</tbody>
</table>
DISCUSSION AND SUMMARY

The present paper has dealt with the effects of upper atmospheric winds and aerodynamic lift on the changes in inclination of the orbit of the S3-l satellite. Aerodynamic lift effects have not been considered in previous studies to determine atmospheric winds from changes in inclination, and only the high quality of orbit and attitude data for this satellite has permitted it in this case.

The argument is occasionally advanced that aerodynamic lift effects may be ignored for satellites which do not have an active orientation mechanism because such satellites undergo random tumbling motion. It should be noted, however, that in the absence of disturbances such as aerodynamic torque, a spinning satellite will maintain its angular momentum, and thus its spin axis will be inertially fixed. For asymmetric satellites (such as S3-1) the spin averaged lift force may be non-zero. Great care should therefore be exercised before aerodynamic lift effects are ignored for asymmetric satellite shapes, even for those which are tumbling "randomly."

As discussed in the previous section, the lift force, and hence the inferred values of the zonal wind speed and direction, are quite sensitive to the assumed value of the thermal accommodation coefficient. Analysis of the zonal winds derived using different assumptions of the value of \( \alpha \) indicates that best agreement with current theoretical thermospheric models is attained when nearly complete thermal accommodation is assumed (i.e., \( \alpha \approx 1.0 \)). There have been a few studies dealing with the value of \( \alpha \) appropriate for an orbiting satellite. Imbro et al. (1975),...
using spin rate decay data from Ariel 2 (perigee altitude ~300 km), estimated an accommodation coefficient near 0.9 and discussed the possible increase of $\alpha$ with decreasing altitude (associated with the increase in atmospheric concentration of atomic oxygen). In fact, Beletskii (1970) has analyzed spin rate data from the paddle wheel satellite Proton 2 (perigee altitude ~190 km) and concluded that $\alpha > 0.99$. Beletskii's results and the discussion of Imbro et al. thus support the assumption of large values of $\alpha$, which seem most reasonable on the basis of the effects of aerodynamic lift on inferred atmospheric wind. These values of $\alpha$ were derived assuming models which did not include possible surface chemistry effects. Atomic oxygen, a major constituent in the thermosphere, is highly reactive, and it is not unlikely that a satellite surface may become covered with a chemisorbed layer of atomic oxygen. An incoming atomic oxygen atom might then collide with the surface, recombine with a surface adatom, and be re-emitted as a molecule. Present knowledge of surface chemistry does not allow treatment of such effects with confidence.

Assuming that values of $\alpha \approx 1$ are indeed correct, the present study provides some information regarding the diurnal variations of the zonal wind speed and direction at ~175 km altitude. For local times in the range 19-21 hr the calculated wind is eastward and has a magnitude of ~400 m/sec. For local times of 7-8 hr, the calculated wind is westward and on the order of 200 m/sec. These directions are in agreement with the theoretical studies cited earlier, and the wind speeds are in general agreement with, or somewhat larger than, those predicted theoretically. The wind speeds also are similar to the results of other observations at comparable altitudes (Ching, 1971; King-Hele, 1972b; Forbes, 1975).
No seasonal variations in wind speed are evident in the present results with $\alpha \approx 1$. The possible dependence of wind speed on the level of magnetic activity, noted by Forbes (1975), could not be investigated, since the average value of $A_p$ for each of the 6 data segments was about the same.

Since the value of $\alpha$ is not well known, and since the inferred wind speeds and directions are so sensitive to small deviations of $\alpha$ from unity, the above-mentioned wind speeds can be regarded as being only approximate. Although it is gratifying that "acceptable" wind properties have been found to be consistent with "acceptable" accommodation coefficients, it is nevertheless apparent that accurate atmospheric wind speeds cannot be obtained from this particular satellite because of the uncertainty in the accommodation coefficient. The effects of lift and its dependence on the accommodation coefficient will, of course, depend on the individual characteristics of a satellite's shape and orientation. The almost wing-like shape of S3-1 undoubtedly is capable of generating lift forces larger than would more symmetrically shaped satellites. If accurate zonal wind velocities are to be derived from measurements of satellite orbital inclination changes, care must be taken to ensure that the shape and orientation of the satellite are such that the lift forces generated are sufficiently small that the resulting uncertainty due to inadequate knowledge of the accommodation coefficient does not cause significant error.
REFERENCES


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