THE DEPTH OF THE SURFACE BOUNDARY LAYER

ARMY ELECTRONICS COMMAND
FORT MONMOUTH, NEW JERSEY

JUNE 1976
RESEARCH AND DEVELOPMENT TECHNICAL REPORT
ECOM-5596

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June 1976

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Politics Task No. 1TT62111AH71A102

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Fort Monmouth, New Jersey 07703

June 1976

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19. SUPPLEMENTARY NOTES

Surface boundary layer Richardson number
Micrometeorology Reynolds stress
Dynamic similarity theory

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The depth of the surface boundary layer of the atmosphere can be related to a characteristic dimension defined by Obukhov as the scaling length L. Utilizing the form of the diabatic wind and temperature profiles, the depth of the surface layer is demonstrated to be proportional to and a function of the slope of the wind profile and the Obukhov length. The methodology was verified utilizing experimental data extracted from the literature. Generally, the surface boundary layer depth approaches the depth of the planetary boundary layer in near adiabatic conditions near sunrise and sunset.
SUMMARY

In general, the approach presented to determine the depth of the surface boundary layer is simple and relatively straightforward. The depths of the surface boundary layer as indicated in Figure 1 are representative averages, while the depths shown in the remaining figures are relative owing to the smoothing techniques utilized. The binomial filtering method has a tendency to reduce peaks drastically. The results obtained appear to be reasonable and agree with the experimental determinations of Lumley and Panofsky.
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INTRODUCTION

Within the planetary boundary layer known as the friction zone of the atmosphere, a sublayer exists where the heat and momentum fluxes are considered to be invariant with height. This surface boundary layer is further characterized by a scalar Reynolds stress along the mean wind, the direction of which is also invariant with height since coriolis effects may be assumed to be insignificant. The depth of the surface boundary layer can vary from a few meters to perhaps a hundred or more meters, depending upon surface roughness, stability, and windspeed. In turn, these parameters and the thickness of the surface boundary layer control the maximum values of the eddy conductivity and eddy viscosity that occur in the planetary boundary layer.

A method for determining the thickness (or depth) of the surface boundary layer can be easily developed from the dynamic similarity theory of Obukhov [1], in terms of measurable and calculated micrometeorological parameters. The scheme is simple, readily implemented, and allows rapid determination of the depth of the surface boundary layer and exchange coefficient maxims for the planetary boundary layer.

DYNAMIC SIMILARITY IN THE SURFACE BOUNDARY LAYER

Obukhov [1] postulated that heat and momentum fluxes in the surface boundary layer were dynamically similar. Drawing on Richardson's [2] criterion and the Schmidt [3] exchange coefficient hypothesis, Obukhov reasoned that heat and momentum transfer processes in the surface boundary layer could be stated as

\[ H = \frac{K_H}{K_M} \phi (Ri) c_p \rho \frac{\partial}{\partial z} \left( \frac{\partial V}{\partial z} \right) \frac{\partial \theta}{\partial z} \]  

(1)

and

\[ \tau = -\phi (Ri) \rho \frac{\partial}{\partial z} \left( \frac{\partial V}{\partial z} \right)^2 \]  

(2)

where \( H \) is the vertical heat flux, \( c_p \) the specific heat of air at constant pressure, \( \phi \), the potential temperature, \( V \) the mean windspeed, \( K_H/K_M \) the ratio of the exchange coefficients for heat and momentum, \( \rho \) density, \( \tau \) the Reynolds stress, \( \phi (Ri) \) a monotonously decreasing universal function, \( \ell \) the Prandtl [4] mixing length, and \( z \) height. Parallelizing, and in conjunction with Equations (1) and (2), the conjugate laws for the wind and temperature profiles in differential form may be written as:

\[ \frac{\partial V}{\partial z} = \frac{u_w}{kz} \phi_M \]  

(3)
\[ K_M = ku_*z \vartheta_M^{-1} \]  \hfill (4)

\[ \frac{\partial \vartheta}{\partial z} = \frac{T^*}{z} \vartheta_H \]  \hfill (5)

\[ K_H = ku_*z \vartheta_H^{-1} \]  \hfill (6)

where \( u_* \) is a friction velocity, \( k \) Karman's constant, \( \vartheta_M = [\vartheta(Ri)]^{-\frac{1}{2}} \), \( \vartheta_H = [\vartheta(Ri)]^{-1} \), and \( T^* \) a scaling temperature defined as

\[ T^* = -\frac{1}{ku_*} \frac{H}{c_p \rho}. \]  \hfill (7)

The gradient Richardson number, \( Ri \), is generally written as

\[ Ri = \frac{g}{\Theta} \frac{\partial \Theta/\partial z}{(\partial \Theta/\partial z)^2} \]  \hfill (8)

where \( g \) is the gravitational acceleration. Operating upon Equation (8) with respect to \( \Theta(Ri) \), Obukhov found that Equation (8) could be stated as

\[ Ri = \frac{K_M}{K_H} \frac{kgzH}{u_*^3c_p \rho \Theta} [\vartheta(Ri)]^{\frac{1}{2}}. \]  \hfill (9)

If Equation (9) is differentiated with respect to \( z \), what is now known as the Obukhov scaling length is represented by

\[ L = -\frac{u_*^3c_p \rho \Theta}{kgH} \]  \hfill (10)

where \( L \) is a characteristic length representative of and proportional to the depth of the surface boundary layer. By definition it may also be stated that

\[ \frac{Z}{L} = \frac{\vartheta_M Ri K_H}{K_M} \]  \hfill (11)

which may also be stated as \( z/L = \vartheta_M R_f \) where \( R_f \) is the flux form of the Richardson number.
A universal form of the wind profile may now be developed from Equations (3) and (11). Writing Equation (3) as

$$\frac{\partial v}{\partial z} = \frac{u_*}{kz} \frac{z/L}{R_f},$$

(12)

Then adding and subtracting 1 to \(z/L \ R_f^{-1}\) and multiplying and dividing by \(z/L\),

$$\frac{\partial v}{\partial z} = \frac{u_*}{kz} \left[ 1 + \frac{z}{L} \frac{z/L - R_f}{R_f z/L} \right].$$

(13)

Defining \(z/L - R_f (R_f z/L)^{-1}\) as an arbitrary variable \(\beta\), then integrating Equation (13),

$$\Gamma = \frac{u_*}{k} \left[ \ln \frac{z}{z_o} + \bar{\beta} \frac{z}{L} \right]$$

(14)

where \(\bar{\beta}\) is the average \(\beta\) over the layer \(z_g = (z_1 z_2)\) and \(z_o\) is the roughness length. Since \(R_f = R_i K_H/K_M\), then

$$1 + \frac{z}{L} \frac{z/L - R_f}{R_i z/L} \frac{K_M}{K_H} = 1 + \bar{\beta} \frac{z}{L} = \frac{K_H}{K_M} (\bar{\beta} - 1)$$

and

$$V = \frac{u_*}{k} \left[ \ln \frac{z}{z_o} + \frac{K_H}{K_M} (\bar{\beta} - 1) \right]$$

(15)

which is valid for both stable and unstable thermally stratified flow in the surface boundary layer.

**THE DEPTH OF THE SURFACE BOUNDARY LAYER**

Lumley and Panofsky [5] suggest that the depth of the surface boundary layer may be empirically determined from

$$h = 20 \tau_o$$

(16)
where \( h \) is in meters and \( \tau_0 \) is the surface stress. It may be assumed that \( \tau_0 = \tau = \rho u_*^2 \), the Reynolds stress. Equation (16) is based upon the assumption that \( \tau \) varies about 20 percent from \( z_0 \) to \( h \).

A more precise argument can be developed from Equation (3). If Equation (3) is written in finite difference form for the unstable regime as

\[
\frac{\Delta V}{h \Delta \ln z} = \frac{u_*}{k} \frac{\Theta M}{Ri} L^{-1}
\]

where \( z = h = L \, Ri \), then

\[
h = L \frac{k}{u_*} \frac{Ri}{\Theta M} \frac{\Delta V}{\Delta \ln z}
\]

If \( \Delta V \) the wind gradient at the geometric mean height \( h \) is assumed to occur over the layer \( \Delta \ln z = \ln e = 1 \), then \( \Delta V^{-1} = Ri^{-1} \Theta M^{-1} \) and

\[
h = mL
\]

where \( m = \left| ku_*^{-1} \right| \) the slope of the normalized diabatic profile.

In thermally stratified stable flow, Equation (3) is written as

\[
h = L \frac{k}{u_*} \frac{Ri}{\Theta M} \frac{\Delta V}{\Delta \ln z}
\]

owing to the assumption that all fluxes are a result of mechanical turbulence only. This leads to \( K_H = K_M \) and \( z/L = Ri \Theta M \). If \( \Delta \ln z \) is again unity, then \( Ri \Delta V = \beta^{-1} \) and

\[
h = mL \beta^{-1} = mL \frac{\Theta M}{15}
\]

since for stable flow, Hansen [6] has shown that

\[
\frac{z}{L} = Ri + 15 \, Ri^2,
\]

\[
\Theta M = 1 + 15 \, Ri,
\]

and

\[
\beta = 15 \, \Theta M^{-1}.
\]
If \( h/L = \text{Re} \phi_M \), then from Equation (21)

\[
\text{Re}(h) = \frac{m}{15}
\]  

(25)

and

\[
h = L \left[ \text{Re}(h) + 15 \text{Re}^2(h) \right]
\]  

(26)

where the subscript denotes the height of the parameter evaluation.

**DISCUSSION**

Equations (19) and (21), the primary formulae for calculating the depth of the surface boundary layer, were evaluated by using experimental data extracted from studies reported on by Lettau and Davidson [7], Barad [8], Swinbank [9] and Izumi [10]. Generally, these data were observed in relatively stationary conditions and mostly over terrain that possessed a high degree of homogeneity. Of the 493 profiles available, only 36 were unusable because of the extremely low windspeeds reported.

Gradient Richardson numbers, scaling lengths, profile slopes, Reynolds stresses, and the dimensionless shears were calculated for each profile. Surface boundary layer depths were determined for both the stable and unstable flow regimes. These data were then averaged as a geometric progression as a function of \( L \) and are shown in Figure 1. The curve representing \( \text{h} \), calculated by using Equation (16), is also given. The agreement between Equation (16) and \( h \) as determined from Equations (19) and (21) is considered to be good. Included for definitive purposes are the smoothed average windspeeds observed over the entire stability range. The divergence of the two curves over the range \(-55 < L < 222\) meters is attributed to the fact that the profiles observed in this stability range were observed near sunrise and sunset under light wind conditions and indifferent thermal stratification. The dashed portions of the curves through neutral conditions are suggested shapes owing to a lack of data near \( L = \infty \).

Surface boundary layer depth with respect to windspeed at 2 m above the surface is shown in Figure 2. Both \( \text{V} \) and \( h \) were smoothed with a five-point binomial filter over 24 hours to obtain this representation. It is apparent that surface boundary layer depth is somewhat of a function of windspeed and, in turn, the Reynolds stresses.

Figures 3a and 3b present the surface boundary layer depth as a function of \( L \) and \( L^{-1} \), respectively. Again, these data were smoothed by using the binomial filter over 24 hours. Figures 3a and 3b show that in near neutral conditions, the depth of the surface boundary layer approaches the depth of the planetary boundary layer. If this is so, then the limit of the Obukhov [1] length will not be infinity but the overall depth of the planetary boundary layer in adiabatic flow conditions.
REFERENCES


Figure 1. Surface boundary layer depth as a function of the scaling length L.
Figure 3a. Surface boundary layer depth as a function of time and the scaling length L.
Figure 3b. Surface boundary layer depth as a function of time and the inverse of the scaling length.
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