AN EXPERIMENTAL INVESTIGATION OF WING TRAILING VORTEX FORMATION

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**An Experimental Investigation of Wing Trailing Vortex Formation**

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**Abstract:**
The incompressible flowfield in the vicinity of a lifting rectangular finite wing is investigated experimentally to ascertain the nature and detailed characteristics of the formation and early development of a trailing vortex. The apparatus was designed and fabricated to facilitate a direct comparison of real flow data with existing theories and flow models. The mean velocity field was mapped directly using a linearized constant temperature hot-wire anemometry probe in conjunction with a precision wind visualization system.
Abstract (cont'd)

tunnel traversing mechanism. The associated vorticity field was inferred from these measurements through a spatial contour integration procedure.

The existence of several identifiable "flow regions" exhibiting similar characteristics was established and verified with both hot-wire and flow visualization data. Among the regions examined extensively are the three-dimensional wing boundary layer, the inboard potential flow region and the region of vortex shedding and roll-up adjacent to the wing tip.

Measurements of the three-dimensional boundary layer were employed to determine the accuracy of the measurement technique close to the solid surface boundary and to insure that the boundary layers over the wing remained thin and attached.

The inboard potential flow region was observed to exhibit the simplifying characteristic of a spanwise cross-flow velocity component which is independent of the surface normal coordinate. As a result, a possible ambiguity in the determination of the stream-wise vorticity component was removed. Graphs of the near-surface (bound) vorticity distribution are presented along with a map of "Prandtl bound vortex filaments." The associated bound circulation function could be expressed approximately as the product of two separable functions of the planform spatial coordinates. Measurements at the wing trailing edge are employed to show that the properly non-dimensionalized characteristic vorticity distribution in this region is independent of angle of attack.

Shed vorticity emanating from the lower aerodynamic surface near the wing tip was observed to roll-up adjacent to the tip and roll over onto the upper wing surface at a predictable chordwise location. The presence and behavior of this structure which possesses an identifiable "core" is seen as an explanation to the modified pressure distribution (lift increment) normally observed in this region.

A comparison of the flowfield in the near wake region with this data reveals that roll-up occurs much more violently near the solid surface than in the boundary free zone downstream of the wing. The normalized description of the roll-up process in the near wake region is observed to be nearly independent of both aspect ratio and angle-of-attack.
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NOMENCLATURE

A - Area, usually referring to that enclosed by a contour

A(x) - Function expressing chordwise variation of circulation

AR - Aspect ratio

B - A constant in the hot-wire response equation

B(y) - Function expressing spanwise variation of circulation

CD - Wing drag coefficient

CL - Wing lift coefficient

D - Drag, usually out of the wing

E - Output voltage of hot-wire bridge

Eo - Output voltage of hot-wire bridge at zero velocity

E - Average output voltage of hot-wire bridge (Note: overbar denotes "average"; usually with respect to time. If the averaged quantity appears with an adjacent variable in parentheses, the average is taken with respect to that variable, e.g., \( \bar{E}(x) \) is an average with respect to the x coordinate.)

Istag - Contribution to "A" type contour integrals by flow field in the vicinity of the leading edge stagnation line

Ivis - Contribution to "A" type contour integrals by flow field over that portion of the contour intercepting the boundary (or viscous) layer

K - A constant, sometimes subscripted if necessary

L - Lift, usually of the wing

M - Slope of the lift curve, \( C_L = Ma \)

M - A dimensionless parameter relating the ratio of crossflow velocity to mainflow velocity
P(x) - Integral of the pressure differential over the local span,
\int_0^b \Delta p(x,y) dy

Q - Dynamic pressure of the free stream, 1/2 \rho U_0^2

S - Surface area, usually of the wing

U - General symbol for velocity, subscripted to denote specific spatial components (U_x, U_y, U_z). Also used as velocity component along hot-wire probe direction

U_o - Free stream tunnel velocity

U|| - Component of velocity in mainflow direction parallel (locally) to the wing surface

U_\theta - Azimuthal component of velocity in cylindrical coordinate system

V - Velocity component perpendicular to hot-wire probe direction

W - Velocity component perpendicular to plane of X-probe

b - Wing semispan, geometric

b* - Effective wing semispan, with respect to the spacing of trailing vortices

c - Wing chord length, geometric; or symbol for "contour"

C_l - Wing section lift coefficient

e' - Fluctuating component of hot-wire bridge output voltage, E = \bar{E} + e', prime denotes a fluctuating quantity

h - A thickness, or thickness parameter--used as slab thickness or that of body plus displacement regions

l - Path distance along a contour, c.

l* - Distance from wing trailing edge to a point downstream where vortex has rolled up

\circ - Symbol meaning "of the order"
- Pressure
- Pressure on lower (aerodynamically) wing surface
- Pressure on upper (aerodynamically) wing surface
- Radial co-ordinate in cylindrical coordinate system; also radius of curvature
- Wing thickness, usually a function of x, t(x)
- Fluctuating component of velocity in x-direction, along probe direction of hot-wire probe
- Fluctuating component of velocity perpendicular to hot-wire probe direction
- Flow direction parallel to free stream
- Flow direction perpendicular to free stream extending spanwise along the wing
- Flow direction perpendicular to free stream extending normal to x-y plane; also the axial component in cylindrical coordinates
- Wing angle of attack; also hot-wire crossflow angle
- Circulation
- Circulation of a contour enveloping the wing plus boundary layer at the centerline spanwise location
- Circulation around a contour oriented in the xz-plane which surrounds all or part of the wing
- Circulation around a contour oriented in the yz-plane which surrounds all or part of the wing
- "The change in"--when used as a prefix to another symbol it implies the difference in two values of that quantity. In this paper used exclusively to reflect the difference between values on the upper and lower surfaces for the same chordwise and spanwise locations
- Boundary layer thickness, sometimes subscripted to denote the reference surface
- Vorticity
\[ \vec{\xi} = \xi_x \hat{i} + \xi_y \hat{j} + \xi_z \hat{k} \]

- Vorticity vector, \( \vec{\xi} = \xi_x \hat{i} + \xi_y \hat{j} + \xi_z \hat{k} \)

\( \Lambda \)
- Wing tip sweep angle

\( \rho \)
- Density of fluid, usually air

\( \sigma \)
- Sensitivity coefficient of hot-wire anemometer

\( \phi \)
- X-meter wire inclination angle

\( \theta \)
- Complement of X-meter wire inclination angle, \( \phi = \pi/2 - \theta \); also used as azimuthal coordinate when referring to a cylindrical coordinate system

\( \tau \)
- A reference time, or time constant
CHAPTER I

INTRODUCTION

This investigation is concerned with the formation and early development of a trailing vortex resulting from lift generation on a finite wing. The process is examined experimentally employing measurements of the velocity and vorticity fields adjacent to and immediately downstream of a rectangular wing immersed in a uniform stream. The experiments were conducted in a low speed wind tunnel and involve nominally unseparated flow conditions over the lifting face.

The significance of the problem can clearly understood by considering the entire flow field which results from the passage of fluid over a lifting surface (Figure 1). The disturbances resulting from the wing's presence lead to the appearance of a wake which persists for a long time period (far downstream of the wing) and which evolves through several identifiable stages.

The flow near the solid surface is classically termed the "boundary layer"--that region which is the source of all vorticity generated in the flow field. The behavior of the flow in this region together with
the neighboring "external" flow is directly related to the surface load distribution and is the primary arena of interest in this study. This region is more fully discussed below.

The sector immediately downstream of the wing, referred to as the "near wake" or rollup region, is usually strongly three dimensional. Vortex filaments shed into this area are free to influence each other which results in the ultimate development of two discrete vortices. This region is one of considerable importance in aircraft applications since a good portion of an aircraft structure (tail surface and controls) is usually immersed in it. The behavior of this flow can also influence the behavior of other aircraft flying in this region, e.g., formation flying, aerial refueling. The mechanics of rollup is the subject of a number of recent investigations most of which are numerical in nature. It is examined to a limited extent in the present work.

Further downstream the wake is observed to be composed of two separate vortical structures having opposite directions of rotation. Each of these highly stable forms is characterized by a cylindrically symmetric shape and composed of a viscous core and an outer flow which behaves like that induced by a potential line vortex. Frequently, an axial deficit or excess is
detected in the core region which prohibits its being analyzed as a two dimensional field. The behavior of this part of the flow field has drawn increased attention recently due to the hazard it poses to encountering aircraft. Its persistence in high density airport traffic areas has been the cause for much concern especially when generated by heavy "jumbo-jet" type aircraft.

A host of investigators have studied the motion of these vortices, including their stability, their interaction with various environmental conditions, and their decay. Prediction methods for their interaction with encountering aircraft have also been developed. They have been studied theoretically, numerically and experimentally in both laboratory experiments and in flight tests. In short, many aspects of the flow in the wake region are reasonably well documented.

This is not the case, however, with the flow in the immediate vicinity of the wing. Though the problem of lift generation was one of the first addressed by fluid mechanists at the beginning of this century, the primary objective was and continues to be the determination of airloads on the wing motivated primarily by structural and design considerations. The qualitative
nature of the flow was sketched by Lanchester,\textsuperscript{29} and visualized by Hoerner,\textsuperscript{30} but little has since been done to ascertain its detailed character in the immediate vicinity of the wing.

Early experimental efforts concentrated largely on the measurements of integral effects (e.g., lift coefficient) and the surface pressure distribution. Direct information regarding the flow field was obtained primarily through simple flow visualization techniques.

Analytical studies were confined to inviscid models since the pressure distribution was found to vary only slightly through the boundary layer over a large portion of the wing surface. In fact, the qualitative aspects of the flow field were usually prescribed a priori, reference Prandtl's lifting line theory. Recent numerical efforts\textsuperscript{31,32} which have attempted to predict the structure of the vorticity field are hampered by the necessity for user supplied "initial conditions" for the flow. An iteration scheme is usually employed in which the flow field is allowed to deform slightly which in turn leads to a new set of interaction conditions. The procedure is repeated until a "stable" situation is attained depending on the operating limits of the computer employed.

Another shortcoming of most finite wing analyses is the calculated behavior of the flow near the wing
tip. A mathematical singularity in the vorticity distribution corresponding to an infinite velocity is usually found there which is physically impossible. Difficulties encountered in further calculations are usually resolved via mathematical methods which remove or ignore the singularity.\textsuperscript{33}

The actual physical situation is related to the analytical model through the concept of bound vorticity. In potential theory this entity is usually ascribed to the "body" in contrast with free vorticity which remains with the fluid particles (or elements). In reality, the bound vorticity is actually that which is continually generated by shear in the wing boundary layer and which appears "attached" to the wing in the appropriate Eulerian reference frame. In two-dimensional flow or at the symmetry plane of a finite wing, the vorticity vector is perpendicular to the free stream direction.

Of equal significance is the apparent redirection of vorticity toward the free stream direction, i.e., the horseshoe vortex concept. The actual re-orientation of vorticity is caused by the spanwise variation of the pressure gradient over the surface. If one considers the effects on a fluid element which lies near the surface as shown in Figure 2, it is
observed to deform and rotate due to the influence of the crossflow velocity gradient as well as that in the mainflow direction. As it moves within the boundary layer it continues to be effected by this rotational tendency depending, of course, on the strength and character of the crossflow shear. Eventually the element enters the free field at some point of boundary layer separation (trailing edge, wing tip or other location where the layer separates) where it becomes part of a free vortex filament in the wake. The above description is not strictly accurate since the hypothetical element is, of course, distorted and diffused as it moves over even a small distance, but the discussion serves a pedagogical purpose in describing the origins of mainstream directed vorticity.

In addition, this simplistic view is complicated by the presence of free vorticity near the surface which is not accounted for in the "sheet" models employed to describe most wings. This vorticity, like that shed at the wing tip, can alter the predicted pressure gradient which can, in turn, reinfluence the flow geometry. Hence a simple cause-effect relationship can never be established.

These difficulties are discussed in the ensuing pages in an attempt to provide insight into the physical
mechanisms operating on this type of flow field. Hopefully, the experimental evidence uncovered in this investigation will serve to aid in describing flows over other wing configurations.
CHAPTER II

THE EXPERIMENT--APPARATUS AND PROCEDURE

II.1 The Experimental Facilities

Most experiments were conducted in the 2 foot x 2 foot cross-section low speed wind tunnel at the University of Colorado (see Figure 3). This open return, suction type tunnel is driven by a single stage axial flow impeller powered by a variable speed, direct current motor. The useful velocity range of this facility extended from 5 to 110 feet per second.

Air drawn into the settling chamber was initially filtered through a furnace filter type dust trap at the inlet. After passing through a hexagonal-honeycomb structure and several screens it was accelerated into the test section through a 16:1 area ratio contraction. The resulting free stream turbulence level \( \left( \frac{1}{\frac{1}{2}} \sqrt[1/2]{U_o} \right) \) was measured at 0.06\% for \( U_o = 60 \) feet per second. The air was then exhausted back into the room through a conical diffuser.

The 14 foot long test section is composed of two separate units. The first unit (nearest the inlet) is 4 feet in length and constructed of transite insulating material on all sides. A small quartz window
is located on one wall as a provision for optical heating of immersed bodies. The remaining 10 foot section is constructed of plywood with a formica finish on three sides. The fourth wall is a solid, 1/2 inch thick plexiglas sheet. This latter section was the one utilized during the course of this experiment primarily due to the need for optical viewing, which will be discussed below. Variation of the static pressure gradient was less than 1% over the entire test section length. Cross-sectional variation of mean velocity was less than 0.4% outside the boundary layer in the empty tunnel.

The tunnel environment was a large air-conditioned room nominally maintained at 70°F. Thermal equilibrium with the tunnel operating (60 ft. per sec.) was reached in approximately two hours. All data were obtained with the system in total thermal equilibrium.

A precision traversing mechanism was constructed to provide accurate positioning of the measuring probes (see Figure 4 for a photograph of this mechanism). The center body could be moved either vertically or horizontally with a drive provided by miniature d.c. motors. Movement was accomplished through a screw drive for vertical movement and a rack-and-pinion system for horizontal movement. Geared potentiometers were utilized to monitor distances in both these directions. The
resultant system spatial resolution was ±0.005 inches in horizontal movement and ±0.002 inches in vertical movement. Axial movement of the entire traversing system was accomplished manually by adjustment of a connecting cable which runs the length of the usable test section.

A Gaertner optical cathotometer was used to establish reference distances between the model and the probes in various configurations. Utilizing this device, probes could be positioned to within .005 inches of the model surface without establishing physical contact. In addition, the instrument was used to monitor the calibration of the vertical positioning system of the traversing mechanism.

Flow visualization photographs were obtained in the 2 foot x 3 foot cross-section subsonic wind tunnel at the USAF Academy. This closed circuit facility, powered by a motor-generator driven single stage fan has a useful test speed range of 50 to 400 feet per second. A Sage Action, Inc., soap bubble generator was located in the stilling chamber just upstream of the test section. Helium filled, neutrally buoyant soap bubbles were created and injected into the free stream at approximately the free stream velocity. These bubbles were then convected into the test section and utilized
as the medium for observing flow field behavior. The plexiglass test section side walls provided a window for photographic purposes.

II.2. The Test Wing

The selection of a suitable lifting shape for the experiment was predicated on both aerodynamic considerations and facility limitations. A simple geometry having a thin cross-section seemed desirable in order to simplify the theoretical aspects of the problem. An airfoil shape which reasonably satisfied the conditions of thin airfoil theory while yielding minimal wake and boundary layer displacement characteristics was required to physically emulate the simple, potential flow theoretical models. Since the detailed flow field about the wing was to be investigated, considerations of the position resolution capabilities of the apparatus and the minimum effective probe size demanded a reasonable wing chord. The relatively small tunnel cross-section, however, imposed a severe restriction on the span parameter thus limiting the maximum effective aspect ratio. (Initial measurements behind a wall-mounted rectangular wing, NACA 2412 cross-section, with half-aspect ratio 3 indicated that almost all vorticity production occurred within a 1 chord length distance of the tip in the spanwise direction.)
On these bases a rectangular, untwisted wing having an NACA 64009 laminar flow airfoil cross-section was constructed from solid wood with a sharp, aluminum trailing edge. The surface was finely polished in an attempt to preserve the laminar flow characteristics. Maximum surface ripple was less than 0.02 inches over the entire span of the model. The surface roughness was minimal except for two small irregularities on the inboard portion of the leading edge. To provide an accurate geometric reference for the measurements, dark black spanwise lines were drawn over the white finish on both upper and lower surfaces and extended around the exposed tip side at 5% chord increments from the leading to the trailing edge. A final coat of plastic sealer was applied to insure a minimal effect of line ink "thickness" on the uniform surface roughness.

With an 8 inch chord and 12 inch semispan (half aspect-ratio of 1.5), the wing was attached to the tunnel wall with a large endplate and protruded to the center of the test section. The angle-of-attack was varied to within ±1/4 degree by a rotation mechanism outside the tunnel.

II.3. Measurement of Fluid Velocities

A constant temperature hot-wire anemometer was employed for the measurement of both mean and turbulent
fluid velocities. Turbulence properties were measured with an anemometer based on a design by Freymuth\textsuperscript{34} which is detailed in Figure 5. Utilizing only three active elements in the sensing circuit, this instrument exhibits low noise characteristics (0.02 microvolts/°C RMS measured at the input). An anemometer designed by Bank\textsuperscript{35} utilizing an operational amplifier as the error sensing element was employed for the majority of measurements of the mean flow field (see Figure 6). Though somewhat noisier than the transistor version, this circuit's d.c. stability over long time periods (less than 2 mV per hour at constant speed equilibrium) rendered it extremely advantageous for the measurement of the potential flow field.

The frequency response of these instruments was obtained with a standard square wave response test. The response was never below 17 kHz (±3 db) for the hot-wires employed in this experiment.

King\textsuperscript{36} investigated the heat transfer relation for heated cylinders in a uniform stream. The application of his results to hot-wire anemometry gives rise to a functional relation between the wire output voltage and the free stream flow velocity--

\[ E^2 - E_0^2 = B \, U^n \]  

\text{(II-1)}
where B is a constant depending on the fluid and wire properties.

For the speed range employed in this experiment the exponent "n" was found to be very close to 0.5 in value (Figure 7). As a result, an analog circuit utilizing Motorola MC1594 multipliers in a squaring mode was employed to linearize the anemometer output. The basic schematic is shown in Figure 8. Squaring element number 2 multiplies the anemometer output by itself giving $E^2$. The constant $B^2_0$ is subtracted electronically by lowering the tunnel speed to zero and adjusting potentiometer, P1. The linearized output is then obtained by squaring $(E^2 - B^2_0)$ with element number 4. Amplifiers 1 and 3 are employed for gain control making it useful over a wide velocity range. The linearizer frequency response was measured flat (±3 db) to 29 kHz.

The hot-wire probes utilized silver coated Pt. - 10% Rh. Wollaston wire which was soft-soldered between the ends of the needle supports. A weak nitric acid solution was used in electro-chemically etching the silver jacket thus exposing the effective sensing element. The wire core was 0.0001 inches in diameter, and wires were etched to a length/diameter ratio of approximately 300.
Since X-meters were the primary probe types employed, adder and subtractor circuits were constructed utilizing Fairchild Type 741 operational amplifiers. An X-meter circuit constructed by McMichael was utilized for filtering, averaging and integrating the turbulent (a.c.) component of the hot-wire signals. The circuitry for these elements is detailed in Figures 9 and 10.

Turbulence intensities were also measured with a Bruel and Kjaer Random Noise Voltmeter (Type 2417). Variable time constants from 0.3 seconds to 100 seconds were available through this instrument with a d.c. output proportional to the RMS value of the input signal. The instrument bandwidth was 2-20,000 Hz.

Data was recorded from digital voltmeters and a Moseley type 2D-2A X-Y plotter. A Tektronix type 502A oscilloscope was used for constant signal monitoring.

The nature of the investigation demanded that the velocity field be resolved into components. A tri-axial 3-wire system was considered but discarded due to its limited spatial resolution, complicated analog circuitry, and complex calibration procedures. Since an unseparated, surface bounded flow was to be studied, an X-meter type probe was constructed having a variable
head angle (see Figure 11). Alignment of the probe so that the plane of the "X" was locally parallel to the wing surface effectively provided the total velocity vector in most measurement regions. Several standard fixed angle X-meters were also employed for other flow field measurements. The wire angle was maintained nominally at 45° inclined to the tunnel free stream. For a two-dimensional flow, or a locally two-dimensional situation, the X-meter equations are given below. Refer to Figure 12 for a graphical illustration of the variables:

**Mean Flow**

\[
\tan \alpha = \frac{E_1 - E_2}{E_1 + E_2} \cdot \frac{1}{\tan \phi} \tag{II-2}
\]

\[
U = \frac{\sigma (E_1 + E_2)}{2 \cos \phi} \tag{II-3}
\]

\[
V = \frac{\sigma (E_2 - E_1)}{2 \sin \phi} \tag{II-4}
\]

**Turbulence Properties**

\[
\bar{u}^2 = \frac{\sigma^2(e_1 + e_2)^2}{4 \cos^2 \phi} \tag{II-5}
\]

\[
\bar{v}^2 = \frac{\sigma^2(e_2 - e_1)^2}{4 \sin^2 \phi} \tag{II-6}
\]

\[
\bar{uv} = \frac{\sigma^2 (\bar{e}_2 - \bar{e}_1)}{4 \sin \phi \cos \phi} \tag{II-7}
\]
where the angles between the free stream and inclined wires, \( \phi_i \), are equal and the two anemometer system sensitivities have been adjusted to the same value, \( \sigma \).

The effective sensing region of the X-probes was a rectangular box of dimension 0.02 inches x 0.02 inches x 0.005 inches (volume = \( 2.0 \times 10^{-6} \) inches).

II.4. **Probe Calibration**

Rather than rely on derived relationships between the velocity components and linearized anemometer voltages, a direct calibration of probe properties was performed in the tunnel free stream. Employing the proportionality relationships (equation (II-3), (II-4)) for the X-probe, the instrument was first aligned so that its symmetry axis coincided with the free stream direction and the tunnel speed varied to give the linearized output for "U," a typical value shown in Figure 13. The probe was then rotated through known angles in the free stream at constant speed. A typical calibration curve is shown in Figure 14. This curve is found to be independent of the free stream velocity. The validity of this static calibration technique was verified by Singh\(^{38}\) who compared it with a dynamic calibration procedure. Agreement was found to be within 1%. Figure 14 shows that the crossflow component, \( \tilde{V} \), is linear up to a value of 0.5 \( \tilde{U} \) allowing for the
measurement of large crossflow components. Periodic probe calibrations revealed that virtually constant characteristics were maintained over the life of the probe.

Since the wire angle with respect to the free stream is never exact (the design angle), two calibrations were made for each probe. First, the probe angle was found for which the inclined wires made equal angles with the undisturbed free stream. This reference angle was then used as the basis of the first calibration. Secondly, the probe was aligned so that its geometric axis coincided with that of the free stream velocity vector. A second calibration utilizing this "zero angle" was obtained. The effect of this "forced calibration" is to shift the curve to one side or another and create an asymmetric characteristic. All measurements taken for this experiment, however, were well inside the linear bounds for all the probes used.

II.5. Measurement Procedure

Measurements of the velocity field were taken in both the wake region downstream of the wing and in close proximity to the surface. For data obtained in the former region either a geometric or an aerodynamic (e.g., vortex core center) reference could be utilized. Measurements near the wing surface were based
on a geometric reference, usually the wingtip, established with the traverse positioning system in conjunction with the optical cathotometer.

In all cases, the probe was first positioned in the undisturbed free stream with the wing at the zero lift angle. The linearizers were then adjusted to give the desirable output. The probe was repositioned at a point convenient to establishing the required reference. For situations involving measurements close to the wing, the distance between the probe and the surface was established at the position nearest contact by comparing the cathotometer readings for the surface line reference and sensing element locations. Traverses were then made in horizontal or vertical directions either parallel to or away from the wing. Graphs relating velocity to position were obtained with the X-Y recorder. In places where the turbulence level was significant, the data were taken pointwise by time averaging the signal.

In addition to direct measurements of the velocity field, information regarding the vorticity content of various spatial regions was essential. Three techniques were employed to obtain the rotational properties of the flow field:

1) In the far wake region, the isolated trailing vortex becomes axisymmetric, its circulation being given by--
A probe traversal through the vortex center with the X-plane perpendicular to the traverse direction allows for the measurement of $u_0(r)$. Hence, $\Gamma$ can be calculated using equation (II-8). If the vortex is completely rolled up, a constant value $\Gamma_0$ will be reached. Symmetry was never completely achieved in this experiment due to the limitation on maximum downstream measurement location (x/c = 7.5 chord lengths downstream of the trailing edge). Singh indicates complete rollup (i.e., cylindrical symmetry of the crossflow velocity field) for his higher aspect ratio wing for x/c > 10.

2) In regions of concentrated vorticity, a mechanical probe similar to a "paddle wheel" was utilized to obtain the local value of the vorticity. The device is described by Grow. Difficulties in using this device in a quantitative manner stem from its rather large effective volume (i.e., it must average the vorticity over a relatively large region in space) and its inherent mechanical friction which limited its use in weakly rotational regions. It provided, however, a qualitative description of the centers of concentrated vorticity (e.g., vortex cores) and their location around and downstream of the wing.
3) A third technique employs the definition of circulation as follows:

\[ \Gamma_c = \oint \mathbf{U} \cdot d\mathbf{I} = \oint_{\Gamma} \mathbf{\xi} \cdot d\mathbf{A} \]  

Contour integral paths were traced out in physical space by the probe traversing mechanism. The restriction to independent horizontal or vertical movement restricted these curves to being rectangles in the Trefftz plane--sometimes referred to as the "crossflow plane" when the contour envelops the wing. By orienting the probe such that the X-plane is parallel to the traversal direction, the quantities appearing in equations (II-4) and (II-9) were evaluated. The restrictions and limitations of this technique will be discussed below.

In all cases, the probes were returned to the zero reference location upon completion of the measurement to examine the electronic drift of the instrumentation. In situations where this characteristic exceeded one third of 1% based on full-scale values, the data were discarded and remeasured.

II.6. Restrictions and Limitations of the Measurements

The techniques described in the foregoing sections are not, in general, valid for all flow situations. Certain restrictions on the nature of the
flow field are required to insure compatibility with the experimental apparatus.

The measurements taken near the wing surface rely heavily on the assumptions of locally parallel and unseparated flow near the surface. The first assumption is invalid in the vicinity of the wing tip due to the surface discontinuity that exists there and to the presence of large rotational gradients which result from the rollup of accumulated tip-shed vorticity. As a result, direct velocity field measurements in this region are limited and are presented only when they can be corrected for these effects (e.g., Qualitative nature of the streamlines).

Though not a problem in this experiment, the effect of separated flow can induce large errors into the experimental results. Large reverse flow velocities with respect to the probe body are not readily detected due to the presence of the interfering probe afterbody. Measurements upstream of the separation point suffer errors due to the non-linear probe characteristics which are significant for low values of $\bar{U}$ due to resulting large values of $\bar{V}/\bar{U}$ (see Figure 14).

An additional difficulty encountered in measurements near a solid boundary with a hot-wire anemometer is that of probe-wall heat transfer. This phenomenon was investigated by Wills for a laminar boundary
layer with an extension to turbulent boundary layers and by Repik and Ponomareva for a turbulent boundary layer. The closest point of approach with the probes used in this experiment was 0.05 inches from the wing surface, giving a ratio of probe-wall distance to wire diameter of 500. Wills showed that the required velocity correction at this distance is negligible (<1%). Data from the other reference yield a similar result.

A major problem with the use of an X-meter in three dimensional flow fields is the error due to an out-of-plane velocity component. This error is calculated to be dependent on the inplane crossflow velocity component, the out-of-plane component, the wire inclination angle and the local convective or mainflow speed. For small values of the out-of-plane component, \( W \), the linearized relative error to \( \bar{V} \) is computed as--

\[
\text{Error} = \frac{1}{2} \left( \frac{\bar{W}}{\bar{U}} \right) \frac{1}{\sin^2 \theta - \cos^2 \theta \left( \frac{\bar{V}}{\bar{U}} \right)^2}
\]

For the standard 45 degree inclination employed in these experiments, this expression reduces further to--

\[
\text{Error} \approx \frac{(\bar{W}/\bar{U})^2}{1 - (\bar{V}/\bar{U})^2}
\]

It is apparent that the boundaries of the contours employed to determine circulation can greatly affect
measurement accuracy if they pass through a region where large out-of-plane component exists, e.g., regions of concentrated vorticity. These regions may, of course, be enclosed by the contour. All experimental results have been corrected for the out-of-planeness error.
CHAPTER III

VELOCITY FIELD SURROUNDING THE
LIFTING WING

III.1. Optimum Test Configuration

In addition to obtaining the parametric variation of certain integral wing properties, a primary objective of this study was to ascertain the detailed structure of the flow field surrounding the lift generator for some representative orientation. The selection of an optimum operating condition was based largely on two considerations:

1) High lift was desired to accentuate the crossflow velocities and therefore improve the accuracy of lift related flow field measurements and,

2) A minimal boundary layer displacement region was sought in an attempt to simulate the "flat plate" or idealized thin wing geometry. This condition is usually optimized through drag minimization. The angle of attack employed was therefore that corresponding to a maximum lift to drag ratio.

Rather than relying on available high Reynolds number test data, the gross wing properties were estimated through preliminary measurements taken in the
wake. The lift coefficient was calculated by determining the circulation about the isolated free tip vortex downstream of the wing and applying the approximate relation from two-dimensional potential airfoil theory:

\[ C_L = \frac{2\Gamma_0}{U_0 c} \]  

(CIII-1)

Circulation measurements were made at 7.5 chord lengths downstream of the trailing edge using the method outlined by Singh,\textsuperscript{38} Chapter II. The resultant lift curve is given in Figure 15.

The drag was approximated by evaluating the axial velocity defect in the wake. Through the measurement of this distribution across the wake (z-direction), the drag coefficient per unit span is given approximately as:

\[ C_D = \frac{2\int_0^{\infty} (U_o - \bar{U}_x) \, dz}{U_0 c} \]  

(CIII-2)

The drag variation with angle of attack is shown in Figure 16. This data is based on an average of four spanwise locations. A comparison of data obtained at several chordwise locations indicated an error of less than 5%.

The resultant expression for the approximate lift-to-drag ratio is then--
The measured characteristic curve is given in Figure 17. This result served as the basis for the selection of a primary test angle of attack of 4 degrees. This low value allows one to apply the small angle approximations \( \sin \alpha \approx \alpha \), \( \cos \alpha \approx 1 \) in relating the wing-fixed (or "body-fixed") and wind axis coordinate systems. The main flow axis here approximately coincides with the airfoil symmetry axis.

The technique used for obtaining the lift characteristic suffers from several deficiencies in this experimental situation. (These discrepancies will be discussed at length in Chapter V.) The method was, however, sufficiently accurate to reveal the shape of the \( L/D \) characteristic and provide an adequate choice in the selection of the test orientation.

A free stream tunnel velocity of 60 feet per second was selected as the maximum useable speed for which probe vibration effects were negligible. The corresponding Reynolds number based on wing chord was 247,000.

III.2. Flow Regions in the Crossflow Plane

An examination of the behavior of the flow field in any crossflow (Trefi) plane intersecting the wing
surface reveals a general structure which is qualitatively independent of the chordwise location. A sketch detailing this structure is given in Figure 18. The flow is subdivided into various sections which exhibit similar properties. The relative physical sizes of these regions will vary with chord location and other aerodynamic parameters. For example, the thickness of the inboard boundary layer (Region 2 and inboard portions of Region 3) will vary with chordwise location due to the pressure gradient ($\partial p/\partial x$) which is, in turn, a function of wing orientation. The wavy lines which depict the interface between regions are not necessarily rigidly defined but can be a function of wing construction uniformity and subject to some interpretation as, for example, is the thickness of a turbulent boundary layer. The advantage of this subdivision concept is that it facilitates the description and analysis within any given region. While this particular structure has resulted from an investigation of a specific rectangular wing shape, the result can be generalized to a broader class of planforms, aspect ratios and angles-of-attack providing the flow is unseparated in the inboard regions. A detailed explanation of this structure follows in the ensuing sections.

The simplest region from an analytical standpoint is that of two-dimensional ("airfoil") potential
flow inboard of the wing tip and outside the boundary layers on both surfaces (Region 1). It can be located about most constant chord, unswept wings of high aspect ratio but more frequently is realized only in the wing symmetry plane as is the case in this experiment. An adjacent two-dimensional boundary layer (Region 2) can also be present. This airfoil-like flow field coincides with the flattening of the spanwise lift distribution near the wing centerline.

Observations nearer the tip indicate that a spanwise velocity component is introduced resulting from the spanwise pressure gradient. This component becomes increasingly more significant as the tip is approached.

III.3. Characteristics of the Three-Dimensional Boundary Layer

The three-dimensional wing boundary layer (Region 3 in Figure 18) is a complex structure whose velocity field is a function of all spatial variables. It is this region that is solely responsible for the generation of vorticity in the main flow direction.

Measurements of the mean velocity components and turbulent intensities were made at various locations adjacent to the wing to establish the extent of the displacement region. The results of several representative measurements are graphically depicted in
Figures 19-27. In these Figures the coordinate reference for the surface normal direction \((z/c = 0)\) is the local surface location. The data contained in these illustrations refer to upper surface conditions only. The lower wing surface exhibited an extremely thin boundary layer (less than 0.03 inches) preventing any detailed measurements in its interior.

Figure 19 shows the variation of mainflow velocity \((\bar{U}_x)\) profiles with spanwise location. Figure 23 shows the variation of the same parameter with chordwise position. The local upper surface boundary layer thickness is seen to be a strong function of chordwise location but almost independent of spanwise position except in the immediate vicinity of the tip. The weak spanwise variation is probably due to irregularities in wing thickness and surface roughness related to construction tolerances. An examination of the spanwise variation of turbulence intensity (Figure 27) indicates that this may be the case.

The layer thickness varied from immeasurably thin (less than .03 inches, \(z/c = .0083\)) upstream of the midwing point to over 0.30 inches near the trailing edge. Measurements of \(U_z/U_x\) near the surface verify that the measured value of the downwash coincides well with the calculated value of the local surface slope. A
representative example of these data are given in Figure 28 at a chordwise location near the trailing edge. This provides some evidence as to the integrity of the mainflow measurements near the surface and therefore, the absence of separation. The absence of separation was confirmed by measurements over the angle of attack range of 0-8 degrees. Further discussion of this evidence will be found in Chapter V.

The boundary layer characteristics are observed to be almost independent of span location over a large portion of the inboard region, as noted in Figures 22 and 27. The tip region, however, is characterized by gross spatial non-uniformity in all variables. The crossflow profiles detailed in Figures 20, 21 and 24 dramatically illustrate this variation. These characteristics will be discussed fully in the next section.

The presence of a highly frequency selective waveform in the fluctuating velocity trace was observed at the outer edges of the boundary layer over the inboard region near the aft portion of the wing (x/c>0.70). These "Tollmien-Schlicting" waves are associated with the process of transition to a fully turbulent flow.

*These waves were first suggested by Rayleigh, examined extensively analytically by Tollmien and Schlicting and verified experimentally by Schubauer and Skramstad for the case of flow over a flat plate.
Experimental evidence of this phenomenon is displayed in the correlation function, Figure 26. These waves were observed through the entire angle of attack range of unseparated conditions (0-8 degrees), though the amplitude of the waveform was greatest at the lower angles.

As indicated above, the lower wing surface exhibited a much thinner displacement region than upper surface. Only in the immediate vicinity of the cusped trailing edge did a suddenly high turbulence level and shallow mainflow velocity gradient indicate possible local separation (or, incipient transition). The characteristics of the outer flow did not seem to be affected by this situation.

A diagram depicting the growth of the displacement region over the inboard portion of the wing is given in Figure 25. This scale illustration shows the chordwise variation of boundary layer parameters for $\alpha = 4^\circ$.

III.4. **Inboard Potential Flow Region**

A region of considerable interest is that of three-dimensional potential flow just outside the boundary layer inboard of the tip (Region 4 in Figure 18). It is this sector that will be involved in the computation of wing bound vorticity in Chapter IV.
Continuous spanwise traverses were made at various distances from the upper and lower surfaces to provide a complete quantitative description of this portion of the flow field. Both the mainflow (x) and crossflow (y) components were measured. Data were obtained in 5\% chord intervals from the leading edge to the 30\% chord point and in 10\% chord increments from there aft to the trailing edge. Figures 20, 21, 24 and 29-35 contain a series of various crossflow velocity profiles adjacent to both wing surfaces.

Examination of the crossflow variation reveals that it is apparently not dependent on the normal (z) coordinate in this region. This important simplification is more readily understood by examining the equation for the x-vorticity component:

\[
\xi_x = \frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z} \tag{III-4}
\]

The left side of this expression is zero since only potential flow is considered. Since, for the untwisted, non-porous wing with a thin uniform boundary layer the streamlines approximately follow the surface contour, then

\[
\frac{\partial u_z}{\partial y} = 0
\]

and the above equation reduces to--

\[
\frac{\partial u_y}{\partial z} = 0, u_y = u_y(x, y) \text{ only} \tag{III-5}
\]
This equation is verified by the existing experimental evidence. The approximation breaks down several inches away from the surface due to the influence of other elements of the flow field. This simplification does, however, allow a wide margin for error in the measurement location employed for calculating the bound vorticity, as will be noted in the next chapter.

The spanwise crossflow velocity profiles, Figures 29-35, represent a complete survey of the velocity field in the potential region adjacent to the inboard upper and lower surfaces of the wing. Each data point has resulted from a spatial average (in the z-direction) in velocity for the region for which equation (III-5) is valid. Error bars have been added for those locations where the range of values exceeds that represented by the symbols.

One disturbing feature that stands out in these data is the reversal of crossflow near the trailing edge on the upper surface and the leading edge on the lower surface. Though it occurs well inboard of the tip and is small in magnitude, this reversed secondary flow seems to oppose the expected spanwise pressure gradient. Two explanations for the existence of this phenomenon seem possible:
(a) Local separation within the boundary layer is affecting the outer flow in the cross-flow component, and
(b) The tunnel wall boundary layer is imposing some effect on the outer flow near the wing-wall attachment point.

Explanation (a) does not seem reasonable in light of the results of downwash and mainflow measurements in this region (see, for example, Figure 28). Additional measurements (Chapter V) which relate the crossflow distribution with angle-of-attack also invalidate this explanation. The effect of the tunnel wall boundary layer, however, is significant and may well be responsible for this situation. Significant turbulence (well above the free stream level) was detected as far as two inches distant from the wall surface. This is not surprising when it is realized that the test wing is mounted some 5 feet downstream of the test section inlet. The net effect of this "thick" tunnel boundary layer is to reduce the effective aspect ratio of the wing. This is done, however, in a rather non-uniform way as will be seen in illustrations presented later.

III.5. Flow Field Near the Wing Tip

The flow immediately adjacent to the wing tip is characterized by the presence of steep velocity
gradients over a small spatial region in addition to an irregular geometric structure. Unlike its regular behavior in the inboard region, the crossflow near the tip is observed to be highly dependent on all spatial coordinates as can be seen in Figure 36. This area, represented by Region 5 of Figure 18, is one characterized by concentrated vorticity which has accumulated due to shedding upstream in the tip region. The helical flow pattern so commonly associated with the far-field isolated vortices is observed to develop well upstream of the trailing edge as can be seen in the streak photograph in Figure 37. Further examination of qualitative data shows this "lump" of vorticity to adhere to both the sides and upper surface of the tip region.

The actual identity of this region is doubtlessly related to the wing tip shape in addition to other wing parameters. The measurements described here concern only a flat tip whose plane is perpendicular to the y-coordinate. Since the tip is usually the source of a singularity in theoretical or numerical analyses, the physical characteristics of this portion of the flow field are significant.

The approximate shape of this rollup region in various Trefftz-plane stations along the wing has been reconstructed using detailed X-wire data. Horizontal
and vertical traverses at finely spaced intervals were used to construct crossflow plane velocity vectors, local maxima and minima, and crossflow stagnation points, lines and regions. These reconstructions are shown in Figures 38-43.

The swirling secondary flow is seen to originate close to the leading edge (first observed at x/c = 0.05) and progress aft along the side of the wing. The small scale and radius of curvature are evident from the sketches. Its growth is restricted to the side portion of the tip until the 60% chord point where it apparently begins to "roll over" onto the upper surface. This repositioning movement is probably caused by the general influence of the pressure gradient around the tip. By the time the structure has reached the vicinity of the trailing edge it is positioned almost entirely above the upper surface. The complex structure of the secondary flow at various intermediate chordwise locations is caused by the local wing tip geometry.

Figure 44 illustrates more effectively the regions where "reversed" crossflow exists, i.e., regions where the local crossflow direction opposes that of the nominal outer flow pressure gradient. A line representing locations of zero upwash (i.e., z-velocity
component) is observed to lie almost parallel with the free stream extending rearward from the leading edge stagnation point. Measurements of its location are shown more accurately in Figure 45 which is a pictogram showing the locations of significant upwash parameters adjacent to the side of the tip. As noted in these Figures, this "stagnation line" intersects the upper surface location at the approximate point of rollover.

It would seem useful to be able to ascertain whether vorticity shed from the lower wing surface could roll-over onto the upper surface. To estimate this possibility consider a hypothetical slab, thickness h, as shown in Figure 46. Assume the pressure to be uniform adjacent to both surfaces having values $P_u$ on the upper surface and $P_l$ on the lower surface. A comparison of the characteristic times associated with the lateral pressure gradient and the main convective flow should provide an appropriate parameter for the estimate. That is, consider--

$$N \equiv \frac{T_{lateral}}{T_{convective}}$$  \hspace{1cm} (III-6)

where

$T_{lateral}$--time required for a fluid "packet" to traverse laterally from the lower to the upper surface (see Figure 46)
Tconvective -- time associated with flow in the main-stream direction.

The latter quantity can be estimated immediately as--

\[ T_{\text{convective}} = \frac{x}{U_0} \]  

(III-7)

To calculate the former term one requires that the flow proceed around the "tip" in a regular fashion. In fact, assume that it follows a semi-circular trajectory as viewed in the Trefftz plane (or in the absence of any main convective stream). With the radial pressure gradient balancing centripetal acceleration to allow this situation to occur, the momentum equation in the \( \theta \)-direction can be written--

\[ \frac{\partial p}{\partial r} = \rho \frac{U_0^2}{r} \]  

(III-8)

The average value of the lateral flow velocity over the assumed trajectory can be expressed as--

\[ \bar{U}_\theta = \frac{\pi r}{T_{\text{lateral}}} = \frac{\pi h}{2T_{\text{lateral}}} \]  

(III-9)

The radial pressure gradient can be estimated--

\[ \frac{\partial p}{\partial r} \approx \frac{\Delta p}{h} = \frac{p_1 - p_\infty}{h} \]  

(III-10)

Then--

\[ T_{\text{lateral}} \approx \pi h \sqrt{\frac{\rho}{2\Delta p}} \]  

(III-11)

and, further--

\[ N = \frac{\pi h U_0}{x} \sqrt{\frac{\rho}{2\Delta p}} \]  

(III-12)
The pressure differential can be expressed in terms of a lift coefficient as follows:

\[ \Delta p = \frac{L}{S} = \frac{1}{2} \rho \frac{U_0^2}{C_L} \quad \text{(III-13)} \]

Hence,

\[ N = \frac{n}{x} \sqrt{\frac{1}{C_L}} \quad \text{(III-14)} \]

The value for \( x \) must be selected and in this case should correspond to the length of the slab, \( c \). If the resulting value of the parameter is much greater than unity rollover will not occur onto the upper surface. If the value is less than one it will occur upstream of the "trailing edge." If the value is near unity it should occur in the vicinity of the trailing edge.

The case of an actual wing is more complex, however, and is complicated by a spatially variable surface pressure distribution and an airfoil shape quite different from that of the hypothetical slab. The geometrical wing orientation also becomes an important factor. Nonetheless, if known experimental parameters are substituted into the above equation for the experimental configuration (data at \( \alpha = 4^\circ \), \( h/x = \) maximum thickness ratio) it is found that

\[ N = 0.57, \]
a value that might be expected in view of the experimental observations.

To account for airfoil geometry one would have to determine the appropriate values for \( h, x \) and \( \Delta p \).

As might be expected the flow surrounding this region of concentrated vorticity is dominated by its presence (induced velocity via Biot-Savart Law). The streamlines in the outer region exhibit a much more well-organized spiral character approaching a cylindrical geometry as viewed in the crossflow plane (Region 6 of Figure 18).

The area denoted by Region 7 in that Figure is one dominated equally by the concentrated tip vortex structure and the influence of the inboard surface boundary.

III.6. Streamline Pattern

Employing the crossflow and mainflow data described above, the streamline pattern adjacent to both the upper and lower wing surfaces was reconstructed. These diagrams, assembled from planform velocity vectors are shown in Figures 47 and 48.

Both diagrams appear to indicate shear in certain locations. These are most obvious near the tip on both surfaces and near the trailing edge on the upper surface. No actual streamlines are drawn in the
immediate vicinity of the tip in Figure 47 due to the swirling non-planar flow pattern which exists there.

In places where the streamlines actually enter the displacement region the apparent "slippage" actually represents shear, or vorticity production. In regions where the streamlines lie wholly outside the boundary layer, however, the apparent shearing is accompanied by a variation of mainflow and downwash in the normal coordinate direction yielding an irrotational gradient.

In contrast to the violent variations observed in the tip region, the inboard flow exhibits very small values for the crossflow angle. Streak photographs taken of this flow sector (Figure 49) indicate almost undetectably small angles. The sensitive quantitative nature of the hot-wire anemometer does, however, detect the distribution accurately even at these small angles. Measurements at one inch in from the tip ($y/b = 0.083$) reveal a maximum crossflow angle of only 6.4 degrees (at $x/c = 0.80$), with the value dropping below one degree at $y/b = 0.41$ for the same chordwise station.
CHAPTER IV

VORTICITY FIELD SURROUNDING

THE LIFTING WING

IV.1. The Concept of Bound Vorticity

The subject of bound vorticity was discussed briefly in the introductory chapter (pp. 5-6). The viability of this concept is not as unphysical as it might first appear. If one regards the vorticity in the wing boundary layer as being attached or "bound" to the surface, the direct physical connection is apparent. The points at which the bound vorticity and free vorticity from the trailing sheet meet must then be a region of vortex shedding. In lifting line theory this occurs everywhere immediately downstream of and perpendicular to the lifting line itself. In actual flows the shedding will occur at any point where the boundary layer separates, e.g., the trailing edge, the wing tip, and any location on the wing where a separation line might exist.

The replacement of a complex wing having a thick airfoil cross-section with a lifting line or thin vortex sheet is found to yield integral results (overall wing characteristics) in good agreement with
experiment. Only recently have complex numerical techniques provided suitable results with regard to the flow field details with discrepancies, however, still existing depending on the mathematical formulation employed. In this chapter, the detailed vorticity field around the lifting wing will be examined to ascertain the significance of various portions of the flow field.

IV.2. Measurements of Spatial Vorticity Content

Chapter II (pp. 20-24) alluded to the use of a hot-wire anemometry probe to measure circulation via the definition of circulation. A more complete discussion of this technique is found in Reference 41. While this reference discusses measurements in unbounded flows, the technique can be employed in the vicinity of solid boundaries as well.

Consider a contour, \( c \), in close proximity to the cross-section of a lifting wing as shown in Figure 50. Furthermore, let the \( y \)-direction segments of the contour lie completely outside the upper and lower surface boundary layers. The circulation about region \( \Lambda \) can be expressed--
\[ \Gamma_A = \oint \mathbf{\hat{u}} \cdot d\mathbf{l} = \int_{-h/2}^{h/2} \left[ U_z(y, z') - U_z(y, z') \right] dz' \\
+ \int_{y}^{y+\Delta y} [U_y(y', -h/2) - U_y(y', h/2)] dy' \quad (IV-1) \]

From Stoke's theorem this integral is also related to the vorticity as follows:

\[ \Gamma_A = \iint_A \mathbf{\hat{\xi}} \cdot d\mathbf{A} = \int_{-h/2}^{h/2} \int_{y}^{y+\Delta y} \xi_x dy' dz' \quad (IV-2) \]

By equating these expressions and taking the limit as \( \Delta y \) approaches zero, the following expression is obtained--

\[ \int_{-h/2}^{h/2} \xi_x dz' = \Delta U_y(y) \quad (IV-3) \]

where

\[ \Delta U_y(y) = U_y(y, -h/2) - U_y(y, h/2) \]

Expressed in terms of an average in the z-direction--

\[ \bar{\xi}_x(y; n) = \frac{1}{h} \Delta U_y(y) \quad (IV-4) \]

Hence, a direct relationship between the boundary layer (or "bound") vorticity and the surface crossflow differential has been established. If the wing and boundary layer thicknesses are negligibly thin (\( h \to 0 \), a vortex sheet), the crossflow differential defines the strength of the sheet. A more complex derivation can be found in Karamcheti, p. 530-531. The reasons for the
selection of a thin wing with a small displacement region are thus apparent. A graph of $\Delta U_y$ as a function will then give the shape of the vorticity distribution when $\Delta z = h$ is kept constant along the span.

The utilization of this technique to measure vorticity close to the surface should then give accurate results over the inboard portions of the wing. In this region the streamlines closely follow the surface curvature and the downwash distribution, $W$, is found to be independent of the $y$-coordinate. The outer flow which exhibits a crossflow component independent of $z$ (see Chapter III) provides a suitable range of adjustment for the value of $h$. The applicability of this analysis is, so far as the vorticity distribution is concerned, predicated on the assumption that the wing plus boundary layer regions exhibit a sheet-like structure of vorticity with finite thickness, $h = c(t + \delta u + \delta l)$.

As noted in Chapter III and in later sections, the flow in the vicinity of the tip does not behave in this manner and in fact precludes the applicability of the traditional concept of a vortex sheet. The steep swirling gradients in velocity greatly enlarge the error in X-wire data due to out-of-planeness of the flow. A more extensive examination of the flow field in this region is therefore required.
If one considers a contour extending from $y = -\infty$ to some value of $y$ inboard of the tip and to the left of the wing centerline (as shown in Figure 51, contour B), the circulation about this closed curve can be expressed through an extension of equation (IV-1) as follows:

$$\Gamma(y) = \Gamma_{\text{tip}} + \int_{y_0}^{y} \Delta U_y(c, y') \, dy' \quad \text{(IV-5)}$$

then

$$\frac{\partial \Gamma(y)}{\partial y} = \Delta U_y(c, y) \quad \text{(IV-6)}$$

where $c$ denotes the chordwise location of the contour, in this case at the trailing edge. These equations are applicable over the inboard region of the wing surface. $\Gamma_{\text{tip}}$ represents the overall tip circulation in that region where high rotational gradients are present.

An additional relationship can be established by considering both contours in Figure 51. The one labelled "A" is taken to completely encompass all of the bound vorticity in a direction parallel to the local section. The resultant circulation about this contour will be labeled $\Gamma_A$. Similarly, contour "B" as explained above possesses circulation $\Gamma_B$. Conservation of vorticity requires that all of the bound vortex lines passing through the capping surface of "A" also pass through "B" if it extends far enough in the
outboard spanwise direction (-y). Separation is assumed to occur only at the wing trailing edge or adjacent to the side of the wing tip. Then--

\[ \Gamma_A = \Gamma_B \]  \hspace{1cm} (IV-7)

If it is further assumed that the wing is uniformly thin and at a small angle of attack, the following approximation can be written--

\[ \Gamma_A(y) \cong \int_0^C \Delta U_x(x', y)dx' + I_{stag}(y) + I_{vis}(y) \]  \hspace{1cm} (IV-8)

where \( I_{stag} \) represents the contribution to the overall contour integral in the vicinity of the leading edge (stagnation line) and \( I_{vis} \) relates the contribution due to the viscous wake near the trailing edge. The validity of this expression as it relates to lift will be considered in a later section. Comparing equations (IV-5), (IV-7) and (IV-8), the following is noted--

\[ \Gamma'_{tip} + \int_{Y_0}^Y \Delta U_y(c, y')dy' = \int_0^C \Delta U_x(x', y)dx' \]

\[ + I_{stag}(y) + I_{vis}(y) \]  \hspace{1cm} (IV-9)

This establishes a relationship between the crossflow differential and mainflow differential velocity fields. The equivalence of the \( \Gamma_A \) (at wing centerline) and \( \Gamma_B \) (for the entire semi-span) contours was first experimentally verified by Page and Simmons\(^{48} \) in 1925.
IV.3. **Velocity, Vorticity, Circulation and Pressure Fields**

A relationship between the crossflow velocity differential, "bound" vorticity, and circulation was established in the preceding section. These quantities are, in turn, related to the pressure field and, hence, the section and overall lift coefficients.

If the Kutta-Joukowskani condition is imposed on the velocity at the sharp trailing edge, the lift per unit span is known to be \( \rho U_0 \Gamma \), where \( \Gamma \) is the circulation around a circuit encompassing the wing chord and boundary layer and parallel to the free stream. The selection of this contour in an actual viscous flow is sensitive to a number of parameters, most importantly the selection of the region of intersection of the contour with the wake. Thwaites,\(^4^9\) pp. 192-194, indicates that the error in the selection of this location is negligible providing the wing lift-drag ratio is large. This connection between contour integrals in fully potential flow and actual viscous flow was actually first discussed by G. I. Taylor in a note to Reference 50.

If this is the case, the section lift coefficient can be computed as:

\[
  c_L(y) = \frac{2 \Gamma_A(y)}{U_0 c} 
\]  

(IV-10)
The above is essentially the same expression employed for estimating the lift of the entire wing as outlined in Chapter III. In that case \( \Gamma(y) \) was replaced by \( \Gamma_0 \). The wing lift coefficient, however, is not given by the above expression but is obviously dependent on the nature of the \( \Gamma \)-distribution. Figure 52 depicts more accurately the actual physical situation. As bound vorticity is shed at the trailing edge the section lift coefficient decreases toward the wing tip.

The resulting value for the parameter can then be expressed as—

\[
C_L = \bar{C}_1 = \frac{1}{b} \int_{0}^{b} c_1 \, dy
\]  

then—

\[
C_L = \frac{2}{U_0 c} \cdot \frac{1}{b} \int_{0}^{b} \Gamma_A(y) \, dy = \frac{2}{U_0 c} \int \Gamma_A(y)
\]  

The error involved in assuming a flat-rectangular distribution and say, an elliptic lift distribution is \((1 - \pi/4)\) or about \(21\%\). The error involved in the selection of the optimum lift-drag ratio angle of attack using the incorrect lift estimate is discussed in the Appendix.

The expression for the lift coefficient can then be written—
\[ C_L = \frac{2}{U_0 b} \left\{ \int_{0}^{b} \int_{0}^{c} \Delta U_x (x', y') \, dx' \, dy' + \int_{0}^{b} I_{stag} (y') \, dy' \right\} \]

+ \left\{ \int_{0}^{b} I_{vis} (y') \, dy' \right\} \quad (IV-13)

The corresponding expression substituting \( \Gamma_B \) as given by equation (IV-9) is not properly posed since the distribution of vorticity near the tip is expressed as a single integral term, \( \Gamma_{tip} \).

A general expression involving the "B"-type contours can, however, be developed by considering a more general form of these contours. Consider the two general circuits shown in Figure 53. These contours do not intersect at the trailing edge but at some point \((x, y)\). Again, they are assumed to encompass the entire displacement region. Again, the net vorticity passing through capping surface "A" is equivalent to that of "B". This is true at least for the case where the lift distribution is decreasing monotonically toward the tip (see Figure 52). Then, employing previous results--

\[ \Gamma (x, y) = \int_{0}^{x} \Delta U_x (x', y) \, dx' + I_{stag} (x, y) + I_{AB} (x, y) \]

\[ = \Gamma_{tip} (x, y_o) + \int_{y_o}^{y} \Delta U_y (x, y') \, dy' + I_{AB} (x, y) \quad (IV-14) \]

which holds for points over the inboard region on the wing. The term, \( I_{AB} \), represents a contribution to the...
contour integral from the downwash at point \((x,y)\)
where the contours intersect.

The lift coefficient can be established by
considering that group of "B"-contours which extend
all the way to the wing centerline, \(y = b\). Realizing
that equation (IV-12) merely expresses the lift coef-
ficient as an average value of the circulation over
the wing surface, an equivalent expression can be
written--

\[
C_L = \frac{2}{U_0 b} \int_0^c \Gamma_B(x')dx' \tag{IV-15}
\]

Then--

\[
C_L = \frac{2}{U_0 b} \left\{ \int_0^c \int_{y_o}^y \Delta U(y,x',y')dy' \, dx' + \int_0^c \Gamma_{\text{tip}}(x',y_o) \, dx' + \int_0^c \Gamma_{AB}(x',b) \, dx' \right\} \tag{IV-16}
\]

Comparing equations (IV-12) and (IV-15), one finds--

\[
C_L = \frac{2}{U_0 b} \, \Gamma_B(x) = \frac{2}{U_0 c} \, \Gamma_A(y) \tag{IV-17}
\]

where the averages refer to the spatial coordinates.

Since the lift is simply the surface integral
of the pressure distribution, it is found that--

\[
\int_0^c \Gamma_B(x') \, dx' = \int_0^b \Gamma_A(y') \, dy' = \frac{1}{\rho U_0} \int_0^c \int_0^b \Delta p(x',y') \, dy' \, dx' \tag{IV-18}
\]
If two contours a distance $\Delta y$ apart are considered (see Figure 54), the net lift on the enclosed strip can be computed--

\[
\Delta L = \int_y^{y+\Delta y} \rho U_o \Gamma_A (y') dy'
\]  

(IV-19)

and--

\[
\Delta L = \int_y^{y+\Delta y} c \int_0^c \Delta p(x',y') dx'dy'
\]  

(IV-20)

One can then deduce--

\[
\Gamma_A (y) = \frac{1}{\rho U_o} \int_0^c \Delta p(x',y) dx' = \Gamma_B (c,y)
\]  

(IV-21)

Similarly, it is also found that--

\[
\Gamma_B (x) = \frac{1}{\rho U_o} \int_0^b \Delta p(x,y') dy'
\]  

(IV-22)

The above relationships (with the exception of equation (IV-21)) are not strictly valid due to the breakdown of the concept of the sheet near the tip. They do, however, provide a good approximation which relates the velocity (circulation) and pressure fields in an integral fashion.

Lastly the crossflow at the trailing edge over the inboard region can be related to the pressure field through equations (IV-6) and (IV-21)--

\[
\Delta U_y (c,y) = \frac{1}{\rho U_o} \frac{\partial}{\partial y} \int_0^c \Delta p(x',y) dx'
\]  

(IV-23)
IV.4. Bound Vorticity in the Inboard Region

The surface crossflow differential distribution representing the x-component of bound vorticity over the inboard portion of the wing is presented graphically in Figures 55-57. Spanwise profiles are shown for 10% chord intervals along the surface.

Values of the crossflow velocity component have been rendered non-dimensional through division by the free stream velocity. The resulting parameter can be shown to be independent of the free stream value at the trailing edge, i.e.,

\[
\frac{\Delta U_y(c,y)}{U_0} = \frac{c}{2} \frac{dc}{dy},
\]

This result follows from the equations discussed above.

The parameter, \(h\), from equation (IV-4) has been adjusted to correspond with the maximum observed value--

\[
h = (t + \delta_u + \delta_l)_{\text{max}}
\]  

The experimentally determined value was found to be 0.72 inches or \(h/c = 0.09\).

Examination of the Figures shows that the net value of the crossflow reverses polarity well inboard of the tip, but not at the tunnel wall as one would hope. As indicated in the preceding chapter this is probably the effect of the tunnel wall boundary layer on the outer flow. More explicitly, it is the result of the
interaction of the tunnel wall boundary layer with the wall-bound tip vortex. Had the wall boundary layer been negligibly thin one might have expected this vortex to be "suppressed" with respect to the external flow. In this case, however, the tunnel wall boundary layer thickness is appreciable (nearly two inches) so that the complex interaction may well influence the outer flow.

A more comprehensive picture of this portion of the flow field can be obtained from Figure 58 which is a planform map of the crossflow differential distribution. It can be seen that the actual effect of the crossflow direction reversal is to reduce the effective aspect ratio, though in a highly irregular manner.

Experimentally determined values of the differential can be compared with theory through the use of the circulation function and equation (IV-6) which is restated below in more general form--

\[ \Delta U_y(x,y) = \frac{\Gamma}{2\pi} \mathcal{r}(x,y) \]  \hfill (IV-26)

In the case of lifting line theory, the elliptic lift distribution gives rise to a trailing edge value of the crossflow differential which is--

\[ \Delta U_y(c,y)_{\text{elliptic}} = \frac{\Gamma_0 (b-y)}{b\sqrt{2by-y^2}} \]  \hfill (IV-27)
If the Schrenk approximation is employed, the corresponding prediction for a rectangular wing becomes—

\[ \Delta U_y(c,y)_{\text{Schrenk}} = \frac{\Gamma_0(b-y)}{2b \sqrt{2by-y^2}} \]  

exactly half the above value. Other values can also be predicted provided that the circulation function is known.

These "predicted" values are compared with measured data at the trailing edge in Figure 59. Two predictions are shown for the Schrenk approximation. The solid line represents the value for a wing having a twelve inch semi-span which is expected on the basis of the experimental wing geometry. The other curve is determined for a wing of 8.5 inch semi-span which corresponds to the location where the crossflow differential reverses at the trailing edge. The elliptic prediction (dashed line) is determined only for the latter case.

Actual values are shown to lie between those predicted by the Schrenk and elliptic distributions, erring by as much as 40%. That the experimental data generate a curve closely resembling that of the predictors (in shape) is shown in Figure 60. Here the crossflow is non-dimensionalized through division by the value at one-inch inboard of the tip \((y/b = .083)\).
Hence the approximate curve shape is correctly established.

The use of the crossflow difference as a basis of comparison provides a more sensitive indicator as to the detailed nature of the bound vorticity than the more commonly employed circulation (or lift) distribution.

An additional question to be addressed by these results concerns the nature of the circulation function and the possibilities of a spatially separable solution. That is, can one express--

$$\Gamma(x,y) = A(x) B(y) \quad (IV-29)$$

Since many analyses deal with the span load distribution exclusively, one might ask whether it is reasonable to decouple the chordwise dependence in achieving such a solution. If this is, in fact, the case, then--

$$\frac{\Delta U_y(x,y)}{\Delta U_y(c,y)} = \frac{A(x)}{A(c)} \quad (IV-30)$$

which is a function of $x$-only.

Experimentally determined values of this function are shown in Figure 61 for several different spanwise locations. The parameter appears to be much more dependent on "$x" than span though its $y$-dependence is apparently not negligible on the basis of these results.

Equation (IV-29) might be more appropriately written as--
IV.5. Vorticity Content in the Tip Region

The presence of large rotational gradients and the complex distribution of concentrated vorticity in the tip region prohibit the application of the simple measurement technique described above. A more comprehensive description of spatial vorticity content was therefore obtained through the use of rectangular contours in the Trefftz plane near the wing tip.

A series of these contours forming a rectangular mesh was constructed about the tip and measurements were made of the crossflow velocity component tangent to the local path direction. Integration of the velocity field about the respective paths then yielded the circulation.

Figures 62-68 show the results of these measurements for the test condition. Data were obtained in chordwise increments of 10% from the leading edge to the trailing edge. The overall contour size was determined by the amount of spanwise distance influenced by the "rolled-up" portion of vorticity as well as the spatial extent of that vorticity. The distribution and sizes of the measurement cells were based on an attempt to minimize the effects of an out-of-plane flow component on the "X"-wire probe. The streamline
reconstruction technique and results discussed in Chapter III were used in making this determination.

A comparison of Figures 62-68 and corresponding Figures 38-43 indicates that the regions of secondary swirl adjacent to the surface do, in fact, exhibit high concentrations of vorticity. Additional traverses in this area indicated that the outer edges of the swirl zones roughly correspond to the boundary of the vorticity bearing regions as one would expect. (A similar correlation was established for the inboard regions of the wing with the additional correspondence between the "vorticity layer" and the boundary layer turbulence envelope.)

The production of vorticity (x-component) is observed to begin almost at the leading edge and predominantly on the lower wing surface. The concentration of vorticity there exceeds that adjacent to the opposing surface from the leading edge downstream to the 40% chord point. From there aft the opposite ratio is observed. Figure 69 depicts this ratio as a function of chordwise location for spanwise regions extending from the tip to one half and one inch inboard, respectively. The ratio, $\Gamma_{\text{upper}}/\Gamma_{\text{lower}}$, is observed to be independent of the spanwise location forward of the 60% chord location. The deviation of the two curves
downstream of this point is probably due to the "roll-over" of free vorticity onto the upper surface and its interaction with the surface boundary layer. This interaction is discussed more extensively in a later section.

The shedding of vorticity into the region opposite to the side of the tip is also observed to occur close to the leading edge. The significance of the vorticity content in this area is shown in Figure 70 which depicts the percentage of circulation associated with this region compared to the overall wing value as a function of the chordwise location. The amount is observed to increase steadily from the leading edge to the 40% chord point and thereafter remain constant to the trailing edge. This observation does not imply a sudden discontinuance of the shedding process but rather that a kind of equilibrium has been established. That is, the vorticity which is shed into the side region is equal in amount to that rolling over onto the upper surface and out of this region.

A graphic illustration of the comparative amount of shed vorticity is given in Figure 71 which shows the ratio of circulation in the tip region to the local chordwise value for the entire wing \( \left( \frac{\Gamma_B(x,0)}{\Gamma_B(x,b)} \right) \) again as a function of chordwise location. The resulting relation reflects a comparison of shed vorticity to that
being produced over the surface. The curve is seen to be relatively flat, decreasing only slightly toward the trailing edge. This indicates that the amount of vorticity shed into the side region nearly keeps pace with the local production.

IV.6. Effects of Tip-Shed Vorticity

Vorticity shed from the lower wing surface into the region outboard of the tip has been observed to roll up as it moves downstream and spill over onto the upper surface. The movement of this aggregate of vorticity is expected due to the influence of the nominal pressure field near the tip. While it lies adjacent to the tip this vorticity augments the upper and lower surface flows through a "Biot-Savart" kind of influence. In the region where it has rolled over onto the upper surface, however, a more direct viscous interaction with the conventional boundary layer is responsible for local deviations in the expected wing properties.

Early experimentalists discovered that the section lift coefficient near the tip of a wing could deviate markedly from the theoretically predicted value depending on the planform shape of the tip and the wing's orientation. The case of a rectangular wing is illustrated in Robinson, et al., 51 p. 460, Figure
15 for various aspect ratios. This graph is reproduced here as Figure 72 and shows a significant increment in the lift near the wing tip. Pearson\textsuperscript{52} was the first to formulate an empirical correction to theory to account for this deviation. When the results of these early pressure measurements are compared with the flow "picture" established earlier, a possible explanation for the discrepancy surfaces. The high velocity rotating flow above the upper surface causes a reduction in the local pressure coefficient which, in turn, gives rise to an incremental increase in the value of the section lift. This hypothesis is supported by measurements of section pitching moment which predict a more negative value than expected in this region and a corresponding chordwise shift in the center of pressure toward the trailing edge.\textsuperscript{52}

Actually, this result may be viewed as something of a limiting case of the "delta" wing which experiences leading edge separation. In that case, the resulting vortex passes over a major portion of the upper surface and augments the pressure field throughout (see Thwaites,\textsuperscript{49} pp. 511-517). The comparison would be much closer except for the fact that the tip thickness distribution controls the rollover location in conjunction with the wing orientation.
Unlike the case of the delta wing with separated flow, the vorticity which rolls over at low angles of attack is not physically displaced from the upper surface. Delta wing leading edge separation occurs predominantly at high angles of attack and the resulting vortices influence the surface conditions through their induced field. In this case, however, the close proximity of the "free" vorticity and that of the boundary layer results in a complex interaction which is not easily formulated due to the non-neglectable influence of viscosity. It is this interaction which actually prohibits the application of the concept of vortex sheet when modeling this region.

While many investigators have sought to eliminate the "tip singularity" through mathematical manipulation and, in some cases, judicious choice of the tip shape, few have attempted to face the problem on physical grounds and grapple with its restrictions. Recently, numerical analysts have succeeded in verifying that rollup does, in fact, occur adjacent to the tip. Their results, however, do not illustrate the "rollover" phenomenon since the wing is modeled by an infinitesimally thin sheet of vorticity consisting of a small and finite number of bound filaments. In all cases, the application of the sheet concept is extended
over the entire wing surface including the tip itself. In these methods, regions of shedding are determined a priori by the investigator. The results in situations which simulate actual flows where rollover can be observed must be viewed with some skepticism.

A primary question, then, concerns the prediction of shedding in the tip region and the likelihood of rollover given a particular tip shape, specifically the planform characteristics. If one considers an "almost"* rectangular wing having possible tip shapes shown in Figure 73, qualitative statements can be made regarding the nature of the flow in that region. Figure 73a illustrates a possible situation where, under certain conditions, shedding will occur over the entire extent of the tip but rollover will not occur due to the inability of the pressure gradient to force the shed filaments over onto the upper surface. In Figure 73b, shedding itself might not exist at the tip due to the inability of the spanwise pressure gradient to create a "y"-velocity component large enough. (At high angles of attack, however, separation may occur over the tip.) The former case has been numerically verified by Rehbach\textsuperscript{54} for a slightly swept configuration.

---

\*The angle, $\Lambda$, is kept small enough so as not to effect the nominal rectangular wing load characteristics.
If the model of the rectangular slab employed in Chapter III (Figure 46) is recalled, the velocity around the semicircular secondary flow was found to be--

$$U_\theta = \left( \frac{\Delta p}{2\rho} \right)^{1/2}$$  \hspace{1cm} (IV-32)

This implies that the crossflow velocities at the tip on the upper and lower surfaces are given as--

$$U_y(h/2) = -U_y(-h/2) = \sqrt{\frac{\Delta p}{2\rho}}.$$  \hspace{1cm} (IV-33)

Now, one considers the case of a hypothetical, thin slab-wing at negligibly low angle of attack and having spatially uniform upper and lower surface pressures, $P_u$ and $P_\ell$, respectively. Then, since

$$L = \int \Delta p \, dS = \Delta p \cdot S = (P_\ell - P_u) \cdot S$$  \hspace{1cm} (IV-34)

it is readily observed that

$$U_y(h/2) = -U_y(-h/2) = \frac{U_\theta}{2} \sqrt{C_L}$$  \hspace{1cm} (IV-35)

which is independent of the slab thickness and overall surface area. For shedding to occur (case "b"), the angle, $\Lambda$, must not exceed the following value--

$$\Lambda \geq \tan^{-1} \frac{U_y}{U_\theta} = \tan^{-1} \frac{\sqrt{C_L}}{2}$$  \hspace{1cm} (IV-36)

Since only small angles have been considered, this becomes--

$$\Lambda \geq \frac{\sqrt{C_L}}{2}$$  \hspace{1cm} (IV-37)
The opposite situation is true for rollover (case "a"). The tip offset angle must be greater than the critical value (Equation IV-37) for rollover to be prevented. An additional restriction which surfaces is that of low to moderate values of angle of attack so that the required angle, \( \Lambda \), does not get too large invalidating the original assumption.

If the restriction on a "uniform" pressure field is relaxed, then

\[
\Lambda_{\text{critical}} = \tan^{-1} \frac{1}{U_o} \sqrt{\frac{\Delta p(x,t)}{2\rho}}
\]  

(IV-38)

If only case "a" is considered (prediction of shedding) and rollover is neglected, the pressure field at the tip can be written as a function of the centerline pressure. Then--

\[
\Lambda_{\text{critical}} = \tan^{-1} \frac{1}{U_o} \sqrt{\frac{K(b) \Delta p(x,b)}{2\rho}}
\]  

(IV-39)

or

\[
\Lambda_{\text{critical}} = \tan^{-1} \sqrt{\frac{K(b) \Delta p(x,b)}{4 Q_O}}
\]  

(IV-40)

Lastly, the case of an actual airfoil profile is employed as shown in Figure 74. In this case, it is readily shown that--

\[
U_y(-h/2) = - \sqrt{\frac{h(x)}{2\rho} \frac{\Delta p(x,t)}{t(x)}}
\]  

(IV-41)
where h is the local secondary flow (swirl) diameter as given in Chapter III. Then,

$$\Lambda_{\text{critical}} = \tan^{-1} \sqrt{\frac{h(x)}{t(x)}} \cdot \frac{K(b) \cdot \Delta p(x,b)}{4 \cdot Q_0}$$  (IV-42)

for the case where rollover does not influence the pressure field.

One can then qualitatively see that, for a more complex planform like the semi-circular tip shown in Figure 75, both shedding and rollover can occur to varying degrees at various locations along the side of the tip, depending on the airfoil shape and wing orientation.

IV.7. Overall Wing Vorticity Distribution

The quantitative descriptions presented in the preceding sections can be combined to yield the total distribution of vorticity surrounding the wing. Figure 76 shows the resulting circulation profiles at various chordwise locations compiled from these data. These profiles were generated with equation (IV-5).

All vorticity adjacent to the wing is included in this "running sum" description including the rolled-up region discussed above. Care must, therefore, be exercised in attempting to relate these data to the surface pressure differential in the tip zone. The
absence of detail in this region (in the Figure) is intentional due to the ineffectiveness of the experimental technique to correctly predict the distribution there.

Figure 77 is a replotted of the same data but referenced to the local chordwise maximum, \( \Gamma_B(x,b)_{\text{max}} \). The distinctive similarity in these profiles indicates a corresponding similarity in the various chordwise values of the spanwise pressure gradient. The deviation near the leading edge reflects a corresponding non-uniformity in the pressure differential function but is also due in part to measurement error. Large curvature and radial streamline gradients in this region made it virtually impossible to eliminate flow "out-of-planeness" errors.

Figure 70, referred to in an earlier section, illustrates the comparative influence of various spanwise regions on overall vorticity content. Two sets of data points are employed to display overall wing vorticity development in the chordwise direction, both reflecting different values of the "reference" circulation. In one case the trailing edge value, \( \Gamma_0 \), was employed while in the other the maximum observed value (which occurred at \( x/c = 0.80 \)) was used.

It is of interest that over half of the vorticity directed along the x-axis lies within one inch
(y/b = .083) of the wing. A significant portion also appears as shed vorticity adjacent to the tip. No attempt has been made to fit a curve through these points due to the complex and unknown nature of the chordwise lift variation, \( P(x) = \int_0^b \delta p(x,y)\,dy \). For comparison, however, the distribution predicted for a two-dimensional flat plate is also shown. A derivation of this distribution may be found in Thwaites, p. 299.

The vorticity content of the various spanwise segments exhibits a different form if referenced to a "local" maximum. Figure 78 illustrates this fact with several representative graphs of \( \Gamma_{B_2}(x,y)/\Gamma_{B_2}(c,y) \) versus chordwise location for several values of span. The vorticity content in various regions is observed to be nearly constant along the chord. This might indicate that the comparable growth or production of vorticity does not vary significantly with chordwise position at least for the inboard reaches of the wing.

The above data can be presented in a more comprehensive way and one more familiar to proponents of the various lifting wing theories. One considers a situation involving two contours, \( A_1 \) and \( B_2 \), to the left of the wing centerline (as shown in Figure 79) which terminate at points \((x_1, y_1)\) and \((x_2, y_2)\).
respectively. Assuming that both contours yield the same value of the circulation, then the net number of equal strength vortex filaments passing through $A_1$ must also pass through $B_2$. Since $\Gamma_{A_1}$ and $\Gamma_{B_2}$ are equivalent, then

$$\Gamma_{B_1} = \Gamma_{B_2} \quad \text{(IV-43)}$$

and the net numbers of filaments intersecting the capping surfaces of $B_1$ and $B_2$ are equal. By connecting surface points exhibiting equal values of $\Gamma_B(x,y)$, families of curves are obtained which represent lines of constant vorticity, or the traditional "bound vortex lines." It must be emphasized that this description is only valid where the sheet concept can be applied, i.e., interior to the interactive flow region near the tip.

Figure 80 is a planform map of the bound vortex lines developed for the wing at four degrees angle of attack. The interpretation of the contours is given in non-dimensional form for more universal applicability as will be discussed in the following chapter. An apparent discrepancy indicated by the map concerns the inboard movement of some of the vortex lines near the trailing edge. This tendency, though not governed by the external flow while the vorticity is "bound," seems to oppose the motion at the trailing edge which indicates that the
net crossflow, \( U_{\text{upper}} + U_{\text{lower}} \), is outward (-y direction). The most probable explanation for this difficulty is simply error in the detailed velocity measurements. Since the measurement of the circulation profiles is cumulative, i.e., \( \Gamma_B(x, y_o) \) is dependent on the flow in the entire region \( 0 < y < y_o \), any small errors in the velocity measurements are carried over and magnified through the computation of the circulation. If, in fact, these lines actually do tend toward the wing center line, possible explanations would have to include the lower surface separation at the trailing edge and the influence of the tunnel wall boundary layer as discussed above.

Another question which arises in light of the above results is that of an equivalent scheme employing the "A"-type contours alluded to earlier. Figure 81 depicts such a contour enveloping a wing section.

Measurements were made along the surface employing these contours (see equation (IV-8)) in an attempt to reconcile the two methods of measurement. The paths were selected so that all bound vorticity was completely enveloped and the vertical path segment near the trailing edge \( (x/c = 1.05) \) was perpendicular to the free stream. Selection of this wake portion of the contour was based on considerations discussed in
Thwaites,\textsuperscript{49} pp. 192-194, and reviewed briefly in Section IV.2.

Measurements of the locally parallel component of the mainflow were made in the inboard region with a representation sample of the results shown in Figure 82. Graphs of the resulting differential function have the form shown in Figure 83. Visual inspection of these curves reveals that a large fraction of the vorticity resides within one inch of the wing tip.

The largest source of error accompanying these measurements was that in the velocity data near the leading edge. The presence of the stagnation point and uncertainties in flow direction created difficulties in obtaining reliable results in this region.

The chordwise variation in vorticity content is obtained through the application of equation (IV-14). The contribution, $I_{AB}(x,y)$, was not measured except aft of the trailing edge so all profiles must be viewed only in a comparative light. Figure 84 illustrates this data when nondimensionalized by local spanwise maximum values. Again, the predicted two-dimensional flat plate distribution is presented for comparison. Except for the curve representing $y = 0.5$ inches ($y/b = 0.042$), the similarity in curve shape is remarkable. This evidence strongly supports the contentions regarding the
similarity in the pressure differential function and vorticity production made above. Specifically, the observations of Figures 70 and 84 indicate that—

\[
\frac{\Gamma_B(x,y)}{\Gamma_B(x,b)} = F(y) \quad \text{only (IV-44)}
\]

and

\[
\frac{\Gamma_A(x,y)}{\Gamma_A(c,y)} = G(x) \quad \text{only (IV-45)}
\]

Expanding the upper relation gives

\[
\Gamma_{\text{tip}}(x) + \int_{y_0}^{y} \Delta U_y(x,y')dy' = F(y) \left\{ \Gamma_{\text{tip}}(x) + \int_{y_0}^{b} \Delta U_y(x,y')dy' \right\} \quad \text{(IV-46)}
\]

Differentiation with respect to \( y \) indicates—

\[
\Delta U_y(x,y) = F'(y) \int_{y_0}^{b} \Delta U_y(x,y')dy' \quad \text{(IV-47)}
\]

The remaining integral is a function of \( x \) only, so that—

\[
\Delta U_y(x,y) = F'(y) R(x) \quad \text{(IV-48)}
\]

Similar treatment of the second relationship reveals—

\[
\Delta U_x(x,y) = G'(x) H(y) \quad \text{(IV-49)}
\]

In both cases, the flow velocity differentials are separable functions of the spatial coordinates. It must be emphasized that these relationships are valid only where curve similarity is observed, e.g.,
inboard of the tip and downstream of the leading edge.

These observations provide further support for the relationship posed by equation (IV-29) over a large portion of the surface.

A comparison of the measurements applied to the two types of integrals is shown in Figure 85. The functions $\Gamma_A(x,b)$ and $\Gamma_B(c,y)$ are shown to be in close agreement. Also depicted are the distributions obtained by DeYoung and Harper---modified lifting line theory, the Schrenk approximation and an elliptical lift distribution. Several data points obtained in early measurements of the surface pressure differential are also shown.

The pressure data and circulation measurements are seen to agree in all but the immediate vicinity of the tip, as expected. Only the Schrenk approximation appears to adequately approximate the correct behavior of the vorticity distribution. Again, however, large errors persist in the tip zone.
CHAPTER V

FLOWFIELD DOWNSTREAM OF THE WING

V.1. Measurements in the Near Wake

In addition to the investigation of the flow field surrounding the wing, measurements were made in the "near wake" region to determine its physical properties.

Velocity field data were obtained to a maximum of 5 chord lengths downstream of the trailing edge, specifically, at downstream locations of \( x/c = 1.0 \) (trailing edge), 2.0, 4.0 and 6.0. The wing angle of attack was also varied to values of 4, 6 and 8 degrees, all representing conditions of largely unseparated flow over the wing surface. The total wind tunnel test section length was responsible for limiting the maximum downstream measurement location.

The contour integration techniques discussed in preceding chapters were employed to obtain the spatial vorticity distribution in the wake region. Trefftz plane contours like that of Figure 86 were employed for the surveys. The \( z \)-coordinate levels (horizontal path elements) were selected to entirely encompass the identifiable turbulent portions of the
wake, as well as the central vortex core. Hence, all wing generated vorticity was contained within the contour bounds.

Measurements of the wake shape (turbulent edges and mainflow profiles) were also obtained in an effort to identify the physical shape of the vorticity package.

NACA and other experimental data were employed for purposes of comparison, where possible.

V.2. Trailing Edge Vorticity Distribution

Crossflow velocity differential measurements were made at the trailing edge \((x/c = 1.0)\) at the three test angles-of-attack. These data were obtained outside of the surface displacement regions but in the quasi-two-dimensional inboard region discussed in preceding chapters. Additional data were obtained which verify the distinctive properties of this region for all values of \(\alpha\) employed.

The inboard vorticity distribution is shown in Figure 87. Again, non-dimensionalization of the crossflow parameter is accomplished with the undisturbed mainflow velocity. The curve shapes show a marked similarity indicating similar local flow characteristics.

This similarity is even more graphically illustrated via the circulation function, as shown in Figure
88. Shown as distributions with respect to spanwise locations, these curves offer the additional feature of including the contribution of the tip increment to the overall vorticity sum. The data strongly support the contention that the wing flow field characteristics are independent of the angle-of-attack, at least over the inboard region. The unseparated nature of the surface flows is probably the key to this relationship.

Care must be exercised in inferring the similarity of the resulting pressure distributions, however, especially in the vicinity of the wingtip. Though an extensive survey of the surface flow was not made at all values of \( \alpha \), reasoning presented in earlier chapters indicates that vortex "spillover" onto the upper surface should occur farther upstream with increasing angles of attack. This would imply a higher value for the tip lift increment (due to interaction with the surface boundary layer) and a probable variation in the section center of pressure.

The inboard flow field, being independent of \( \alpha \), should then be expected to exhibit the appropriately non-dimensionalized characteristics of the test case discussed in preceding chapters. Therefore, the flow "picture" presented in Figure 80 is valid for other values of \( \alpha \) for which the inboard flow is unseparated.
Only the interactive tip zone characteristics should be expected to exhibit local variations.

V.3. Wing Lift Characteristics

Values of the overall wing lift coefficient were computed from the data described above and compared with each other as well as other theoretical and experimental data. The calculation employed equation (IV-16) and circulation data obtained aft of the trailing edge which will be discussed below.

Since a direct comparison with theory and other data was desired, the raw experimental data had to be corrected for various wind tunnel effects. Theoretical and empirical corrections for solid blockage (of both wing and traversing mechanism) and lift effect were utilized. A complete discussion of these and other wind tunnel interference effects can be found in Pankhurst and Holder,57 pp. 328-364. The effect of solid blockage was found to be negligible at the measurement conditions due largely to the small projected cross-section at these low angles of attack. The correction for lift effect, however, was more significant resulting in an effective increase in angle of attack of approximately 0.5 degrees.

The various curves of wing lift coefficient versus angle-of-attack are displayed in Figure 89.
Linear curve fits were employed for the experimental data due to the unseparated nature of the flow in this case.

Data are presented which illustrate the error incurred from the use of the "1/r" assumption for the vortex behavior in the near field (see section III.1). These measurements, corrected for the approximate load distribution, predict a lower value for $C_L$ than any of the theoretical schemes or the contour integral data.

Lifting surface theory, here due to Multhopp, is observed to predict the lift curve slope with more accuracy than the lifting line theory as would be expected.

Numerical results obtained by Rom, et al., using the iterative numerical technique discussed in Chapter I come even closer to the experimental data. These results combine more "details" of the flow field behavior than previous investigators have employed.

An empirical relationship attributed to Wood seems to provide the closest agreement in this case. Wood's relation is based largely on an average of a large array of empirical and experimental results.

V.4. Wing Wake Vorticity Distribution

The wing wake, whose detailed shape is discussed in a later section, can be envisioned as consisting of
a sheet-like structure extending beyond the trailing edge across the entire span and a core of concentrated vorticity in the outboard region. The formation of this core has been discussed in the preceding chapters.

Of interest regarding the rollup process is the rate of transfer of vorticity from the inboard wake to the core region, as well as the core's evolution. The former relationship was investigated for the near wake. The spanwise distribution of "x"-vorticity is displayed via the circulation function, with integration from $y = -\infty$ toward the wing centerline.

Variation of the circulation profiles downstream for fixed values of $\alpha$ is illustrated in Figures 90-92. The reference values of circulation, $I^*_o$, obtained for the various bounding contours is shown in Figure 93. Agreement is within experimental expectations. Vorticity generated outboard of the tip (at the trailing edge) is observed to move toward the "Betz" value of the spanwise location. A significant outboard movement of the "sheet" vorticity was not observed, however. Figure 94 shows more clearly the chordwise variation of circulation for several spanwise locations. An increase in this function with the "x"-coordinate would indicate vorticity transfer toward the vorticity centroid. No definitive statement about
this trend can be made over the region of investigation. Only at the highest angle of attack (8 degrees) does the expected behavior occur.

Some comparative data obtained by Fage and Simmons using a similar measurement scheme for an aspect ratio 6 rectangular wing is displayed in Figure 90. Their observations appear to fit within the experimental scatter of the present measurements. This result indicates that the rollup process is only weakly dependent on aspect ratio for the entire sheet. This conclusion is consistent with the results of Grow who achieved the same qualitative variation for higher angles of attack.

Figures 95 and 96 show the variation of x-vorticity content with angle-of-attack for fixed downstream locations. The correlation between outboard vorticity transfer and increasing angle-of-attack appears to be stronger than with downstream location.

V.5. Shape of the Near Wake

Since the "bound" vorticity adjacent to the wing surface is located within the boundary layer (unseparated displacement region), it is not unreasonable to expect that the shed vorticity downstream should be confined to the resulting wake. Except for the properties of the vortex core this region is
identifiable by the usual characteristics of a high turbulence level and a perceptible velocity defect in the mainflow direction. Both of these properties were studied to identify the wake location corresponding to the vorticity measurements.

Figures 97-99 illustrate the location of the wake edges for the various values of $\alpha$. The data represent spatial locations where the turbulence indicator, $W/U$, dropped below the value 0.002, which is three times the free stream value. Additional data indicating the locations of the maximum $x$-velocity ($U$) deficit are superimposed on these Figures.

It is observed that the wake at the wing center line descends more rapidly than at any other location. A marked curling of the inboard wake is also apparent. The vortex core, on the other hand, appears to extend downstream in nearly the free stream direction, positioned slightly above the projected $z$-location of the trailing edge.

These observations are in qualitative agreement with the results of both Spreiter and Sachs$^6$ and Chigier and Corsiglia$^{60}$ who experimentally investigated the wake shape in this region.

The wake is observed to be thickest in the inboard region with a marked "necking down" toward the
central core. Tight curling near the core is observed at \( x/c = 2.0 \) and to a lesser extent at other down-stream locations. Diffusion at the wake edges in this region tends to blur the expected spiral behavior.

Care must be exercised when employing these results in a comparison with other experiments (actual flow or numerical) due to the effects of wind tunnel wall interference. Hackett and Evans\(^2\) investigated the influence of the proximity of the wall boundaries and their geometry and noted that "Wall proximity causes tip vortices to sweep downward less and floor proximity has an obvious effect on the penetration of the center of the vortex sheet." Their experimental evidence substantiates this claim.

If one assumes that all non-core located vorticity lies along a line representing the wake center then the preceding description of the vorticity variation (section V.4) along with the geometry of this line sufficiently described the wake rollup. As indicated above, however, the wake exhibits a finite thickness which, in this case, cannot be presumed to be insignificant. The \( z \)-variation of the vorticity may, in fact, cause some modification in the wake evolution and rate of rollup. This variation was not examined in the present study.
V.6. **Rate of Rollup; Discussion**

The results of the above sections can be combined to give a more graphic picture of the rollup.

The planform location of vortex filaments can be constructed in a manner identical to that described in Chapter IV for the bound filaments. Had this been accomplished one would note only a slight spanwise displacement of the filaments over the region of interest at least for small angles. This indicates, perhaps, a qualitatively good reason for Prandtl's placement of the free vorticity elements in the lifting line theory.

One glaring conclusion is inescapable from the above result. The wake rollup process is not nearly as marked as one would expect based on the rapid evolution of the core adjacent to the wing surface. Though the near wake shape distorts and some transfer of vorticity into the core takes place, it is near the lifting surface that the most marked evolutive changes take place probably due to its ability to sustain shear.

A number of investigators have attempted to predict the roll-up distance (or time) employing various bases. Kaden's early work led to the belief that rollup had occurred fully at almost the wing itself.
His basis for the rollup time was the coincidence of the vortex core with the "Betz" centroid. His analysis, however, was applied to an open-sided infinite vortex sheet. A slightly modified analysis by Moore using a spanwise finite sheet predicted 99% rollup within 3.5 chord lengths of the trailing edge using the same criterion.

Spreiter and Sachs\textsuperscript{6} slightly improved analysis employing the bound vortex and a semi-infinite sheet yielded a value of 4.45 chord lengths for the present test case. Again the criterion was the same.

In all of the above cases, agreement between the estimates and the current experiment is poor. Perhaps the data is in error, but it is more probable that what is at fault is the rollup criterion. It has been shown that a vortex "center" does, in fact, coexist with a sheetlike structure of vorticity extending to the wing centerline.

A more realistic criterion would seem to be the requirement of \(1/r\) behavior of the velocity field, though this location is difficult to predict.

Another interesting parameter is the nondimensional distance between the trailing edge and some location, \(l^*\), where an infinitesimally weak (no vorticity) fluid element initially at the wing centerline
has revolved around the central core one revolution. A simplified analysis employing a fully rolled up potential vortex extended rearward from the tip at the trailing edge gives rise to an upper bound in this case. It can be shown that--

\[ l^* = \frac{4\pi^2 b^2 U}{C^0_o} \]  \hspace{1cm} (V-1)

where \( b^* \) is the effective semi-span. For an elliptically loaded wing this can be written--

\[ l^* = \frac{2\pi^3 b^*^2}{C_L c^2} \]  \hspace{1cm} (V-2)

In the present case, for \( \alpha = 4 \) degrees, the approximate value of this parameter is \( l^* = 612 \), i.e., 612 chord lengths downstream of the trailing edge.
CHAPTER VI

SUMMARY AND CONCLUSIONS

The following list summarizes the major results of this investigation and its primary conclusions:

1. The experimental technique employed for the measurement of the circulation and vorticity distributions appeared to provide satisfactory data for the flow conditions utilized.

2. The flow field surrounding the wing could be separated into various spatial regions each exhibiting unique properties. This separation facilitated the local simplification of the flow field from the more complex general three-dimensional situation.

3. The surface boundary layers were found to contain all of the wing generated vorticity except in the immediate proximity of the tip. This verification lends credence to the application of the vortex sheet as a replacement model for the actual wing if one envisions the surface boundary layer and wing thicknesses approaching zero.

4. The flow field near the wingtip is observed to exhibit a high degree of rollup immediately adjacent
to the wing surface similar to that expected in the wake. The resulting crossflow plane secondary flow is observed immediately downstream of the leading edge in the region opposite the side of the tip. Vortex shedding from the lower wing surface is responsible for this occurrence.

5. This rolled-up "free" vorticity is seen to roll over onto the upper surface at a predictable chordwise location for the simple geometric tip shape employed.

6. The behavior of the shed vorticity at the tip can modify the expected surface pressure distribution and resultantly invalidate the applicability of the vortex sheet concept due to direct viscous interaction with the conventional wing boundary layer.

7. Investigation of the inboard bound vorticity distribution revealed that the bound circulation function could be expressed approximately as the product of two separable functions of the spatial coordinates, x and y.

8. A rolled up vortex with an identifiable core was observed to emanate from the upper surface at the trailing edge. This region was found to contain over 60% of the total wing generated vorticity for the configuration tested.
9. The trailing edge vorticity distribution was found to be independent of the wing angle of attack for the unseparated conditions studied. There is good justification to assume that the entire surface bound vorticity field exhibits similar characteristics.

10. The wing lift coefficient is found to vary nearly linearly with angle-of-attack as expected. Corrected experimental results are found to be in good agreement with direct force measurements and various theoretical and numerical predictions.

11. A comparison of near surface data and measurements in the near wake region indicate that rollup occurs much more violently near the solid surface than in the boundary-free zone downstream of the wing.

12. The normalized description of the rollup process in the near wake region is observed to be nearly independent of both aspect ratio and angle-of-attack.

13. The isolated vortex core is observed to move downstream in nearly the free stream direction while the inboard wake is displaced downward (opposite the lift direction). The largest displacement is observed to occur near the wing centerline. A distinctive curling motion is apparent as the wake evolves.

14. The thickest portion of the wake (z-direction) occurs downstream of the wing centerline
with a marked necking down observed near the rolled up core. A complete examination of the roll up characteristics should include variation in the vorticity distribution in this direction.

15. Various estimates of the roll up rate reviewed in this study are found to err significantly, predicting a fully rolled up geometry far upstream of its actual occurrence. The "x"-oriented vorticity in the inboard wake region is observed to be ingested into the core at a slow rate in the near wake region.
BIBLIOGRAPHY


56. Schrenk, O. "A Simple Approximation Method for Obtaining the Spanwise Lift Distribution," NACA Technical Memorandum No. 948, August 19...


EFFECT OF LIFT CURVE SLOPE ON THE MAXIMUM LIFT-DRAG ORIENTATION

The condition for maximum lift-drag ratio is simply--

\[ \frac{d}{d\alpha} (L/D) = 0 \]

For wings of moderate to high aspect ratio, the conventional relations are

\[ C_L = M\alpha \]
\[ C_D = K_1 + K_2 C_L^2 \]
\[ = K_1 + K_2 M^2 \alpha^2 \]

Then

\[ \alpha_{\text{max}} L/D = \left\{ \frac{K_1}{K_2 M^2} \right\}^{1/2} \]
\[ = \frac{1}{M} \sqrt{\frac{K_1}{K_2}} \]

A small error, \( \Delta M \), in the measurement of the lift curve slope results in an error in \( \alpha_{\text{max}} L/D \) of--

\[ \Delta \alpha_{\text{max}} L/D = \frac{\Delta M}{M} \]
Fig. 1. Stages of Evolution of a Trailing Vortex Pair.
Fig. 2. Behavior of Fluid Particles Adjacent to the Upper and Lower Wing Surfaces.
FIG 3 EXPERIMENTAL APPARATUS IN WIND TUNNEL FACILITY
FIG. 4 TEST SECTION TRAVERSING MECHANISM WITH HOT-WIRE PROBE
FIG. 5 CONSTANT TEMPERATURE HOT-WIRE ANEMOMETER AND SQUARE WAVE GENERATOR
FIG. 6 I.C. HOT-WIRE ANEMOMETER
FIG. 7 HOT-WIRE ANEMOMETER VELOCITY RESPONSE CHARACTERISTIC
FIG. 8. LINEARIZER CIRCUIT

INPUT

MC 1594

709

27k

50k

+15V 0 -15V

20k

47k

30k 4.7k

2.7k

E.LIN.
(C) LOW-PASS FILTER
0-20µf
46.4k

(F) RECTIFIER
15k
15k
15k
20k

(B) HIGH-PASS FILTER
15-150k
14.99k
10µf

(E) AMPLIFIER
5k
70µf
470k
470k

15k

(D) SUBTRACTOR (a - b)
15k
30k
30k
30k

8.2k
8.2k
8.2k
8.2k

(A) ADDER \((\frac{a + b}{2})\)
15k
30k
30k
30k

FIG. 10 DETAIL, X-METER CIRCUIT COMPONENTS
FIG. 12 X-METER GEOMETRY
VORTICES

WIND TUNNELS
HOT WIRE ANEMOMETERS
ANEMOMETERS WAKE

FLOW VISUALIZATION
INCOMPRESSIBLE FLOW
LIFTING SURFACES MODELS WIND TUNNEL MODELS

FLOW PROBES

THREE DIMENSIONAL WINGS
BOUNDARY LAYER
VORTEX SHEDDING FLOW FIELDS
OPTICAL EQUIPMENT

PRANDL BOUND VORTEX FILAMENTS STREAM LINES
GAERTNER OPTICAL CATHODOTOMETERS
Fig. 13. Linearized Hot-Wire Response Characteristic.

\[ \frac{E_1 + E_2}{2} \] (volts)

\[ U_0 \] (ft/sec)
\[ \frac{2(E_1 - E_2)}{(E_1 + E_2)} \]

\[ \frac{V}{U_0} \]

FIG. 14 TRANSVERSE VELOCITY CALIBRATION
X-METER, TYPICAL

DATA AT 60 FT/SEC
Fig. 15. Variation of Lift Coefficient with Angle of Attack (NACA 64009)
Fig. 16. Variation of Drag Coefficient with Angle of Attack (NACA 64009)
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Fig. 18. Flow Regions in the Crossflow Plane
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Fig. 20. Crossflow Velocity in the Boundary Layer, $x/c = 0.95$
Fig. 21. Crossflow Velocity in the Boundary Layer, \( x/c = 0.95 \)
Fig. 22. Intensity of Mainflow Turbulent Component in the Boundary Layer, $x/c = 0.95$
Fig. 23. Mainflow Velocity in the Boundary Layer, $y/b = .375$
Fig. 24. Crossflow Velocity in the Boundary Layer, $y/b = 0.375$
Boundary layer thickness less than 0.03" - lower surface

- Based on displacement thickness
- Distance from surface where local turbulence level is twice free stream value

Fig. 25. Chordwise Distribution of Boundary Layer Thickness, Inboard Spanwise Locations
Fig. 27. Spanwise Variation of Turbulence Level in Boundary Layer, \( x/c = 0.75, \ z/c = .01, \) Upper Surface
Surface Incidence Angle = 7.78°

\[ \frac{\gamma}{b} \]
- ○ - 0.083
- □ - 0.250
- △ - 0.500
- ▲ - 0.750

z/c

Fig. 28. Downwash Angle Variation with Distance from Upper Surface, x/c = 0.98
Fig. 29. Crossflow Velocity Variation in the Inboard Region, Upper Surface
Fig. 30. Crossflow Velocity Variation in the Inboard Region, Upper Surface
Fig. 31. Crossflow Velocity Variation in the Inboard Region, Upper Surface
Fig. 32. Crossflow Velocity Variation in the Inboard Region, Lower Surface
Fig. 33. Crossflow Velocity Variation in the Inboard Region, Lower Surface
Fig. 34. Crossflow Velocity Variation in the Inboard Region, Lower Surface
Fig. 35. Crossflow Velocity Variation in the Inboard Region, Lower Surface
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Fig. 38. Sketch of Streamlines in the Quasi-Trefftz Plane, $x/c = .99$, $\alpha = 4^\circ$
Wing Tip Region
Fig. 39. Sketch of Streamlines in the Quasi-Trefftz Plane, \( x/c = 0.80 \), \( \alpha = 4^\circ \)
Wing Tip Region
Fig. 40. Sketch of Streamlines in the Quasi-Trefftz Plane, x/c = .60, α = 4°
Wing Tip Region
Fig. 41. Sketch of Streamlines in the Quasi-Trefftz Plane, $x/c = 0.40$, $\alpha = 4^\circ$
Wing Tip Region
Fig. 42. Sketch of Streamlines in the Quasi-Trefftz Plane, $x/c = 0.20$, $\alpha = 4^\circ$
Wing Tip Region
Fig. 43. Sketch of Streamlines in the Quasi-Trefftz Plane, $x/c = 0.05$, $\alpha = 4^\circ$
Wing Tip Region
Fig. 44. Regions of "Reverse Crossflow", Wing Tip Region, $\alpha = 4^\circ$, $U_0 = 60 \text{ '/sec}$
○ - Upwash stagnation line
× - Lateral (z) location of maximum upwash
△ - Lateral (z) location of eddy center, v = w = 0

Fig. 45. Flow Characteristics Adjacent to the Flat Rectangular Wing Tip, α = 4°, U₀ = 60 °/sec
View of cross-section in the Trefftz plane

radius, $r$

$P_u$

$P_x$

Fig. 46. Idealized Rectangular "Slab" in Uniform Flow
Fig. 47. Streamlines Over Upper Surface; \( \alpha = 4^\circ \),
(0.1 inches \( z/c = 0.12 \) nominal)
Fig. 48. Streamlines Over Lower Surface, $\alpha = 4^\circ$
FIG. 49 FLOW OVER THE INBOARD REGION, UPPER SURFACE,
HELIUM BUBBLE TECHNIQUE, $\alpha = 4^\circ$,
$U_0 = 60$ ft/sec
Flow

Fig. 50. Use of Contour Integration
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Fig. 51. "A" (xz-plane) and "B" (yz-plane) Type Contour Integrals Meeting at the Trailing Edge
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Fig. 54. Chordwise Oriented Strip of Width, $\Delta y$
Fig. 55. Crossflow Velocity Differential as a Function of Spanwise Position
Fig. 56. Crossflow Velocity Differential as a Function of Spanwise Position
Fig. 57. Crossflow Velocity Differential as a Function of Spanwise Position
Fig. 58. Crossflow Velocity Differential ($\Delta U$)
Distribution Over the Planform Surface,
($U_0 = 60$ ft/sec, $\alpha = 4$ degrees)
Fig. 59. Comparison of Predicted and Measured Values of Crossflow Differential Velocity at the Trailing Edge
Fig. 60. Comparison of Predicted and Measured Values of Dimensionless Crossflow Differential Velocity at the Trailing Edge
Fig. 61. Uniformity of Dimensionless Circulation Function
Note: Data in rectangular regions refers to percentage of vorticity contained in that region. Upper number is referenced to circulation of overall tip contour. Lower number (in parentheses) is referenced to maximum wing circulation, \( \Gamma_0 = 5.781 \text{ ft}^2/\text{sec} \).

Fig. 62. Contour Integrals in the Tip Region, Trefftz Plane, \( x/c = 0.99 \)
### Flow

<table>
<thead>
<tr>
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<th>-</th>
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<td></td>
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<tr>
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Note: Data in rectangular regions refers to percentage of vorticity contained in that region. Upper number is referenced to circulation of overall tip contour. Lower number (in parentheses) is referenced to maximum wing circulation, \( \Gamma_0 = 5.781 \text{ ft}^2/\text{sec} \).

Fig. 63. Contour Integral in the Tip Region, Trefftz Plane, \( x/c = 0.90 \).
Flow

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<th>x/c = 0.70</th>
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<td>-</td>
<td>-0.3 (-0.1)</td>
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$\Gamma_{\text{tip}} = 3.484 \text{ ft}^2/\text{sec}$

(a) $x/c = 0.70$

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<thead>
<tr>
<th>x/c = 0.80</th>
<th>1.0 (0.6)</th>
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<td>12.5 (7.5)</td>
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</table>

$\Gamma_{\text{tip}} = 3.460 \text{ ft}^2/\text{sec}$

(b) $x/c = 0.80$

Fig. 64. Contour Integrals in the Tip Region, Trefftz Plane
Flow

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<td>0.2 (0.1)</td>
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</table>

\[ \Gamma_{\text{tip}} = 3.14 \text{ ft}^2/\text{sec} \]

(a) \( x/c = 0.50 \)

\[ \Gamma_{\text{tip}} = 3.44 \text{ ft}^2/\text{sec} \]

(b) \( x/c = 0.60 \)

---

Fig. 65. Contour Integrals in the Tip Region, Trefftz Plane
<table>
<thead>
<tr>
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<th>0.3</th>
</tr>
</thead>
<tbody>
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\( \Gamma_{\text{tip}} = 2.705 \text{ ft}^2/\text{sec} \)

(a) \( x/c = 0.30 \)

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\( \Gamma_{\text{tip}} = 2.97 \text{ ft}^2/\text{sec} \)

(b) \( x/c = 0.40 \)

---

**Fig. 66.** Contour Integrals in the Tip Region, Trefftz Plane
Fig. 67. Contour Integrals in the Tip Region, Trefftz Plane
Fig. 68. Contour Integral in the Tip Region, Trefftz Plane, x/c = 0.01

Γ_{tip} = 0.807 ft²/sec
Fig. 69. Ratio of Circulation of Flow on the Upper Surface to Corresponding Region on the Lower Surface
Fig. 70. Chordwise Variation of Wing Circulation

Legend
- $\Gamma(x,y = 10''), \Gamma_{\text{ref}} = \Gamma_0$
- $\Gamma(x,y = 10''), \Gamma_{\text{ref}} = \Gamma_{\text{max}}(x=c)$
- $\Gamma(x,y = 1''), \Gamma_{\text{ref}} = \Gamma_0$
- $\Gamma(x,y = 0), \Gamma_{\text{ref}} = \Gamma_0$
- Predicted Flat Plate Distribution
Fig. 71. Circulation of Fluid Outboard of the Wing Tip as a Function of Chord
Fig. 72 Span Load Distribution for a Rectangular Wing (Reference 50)
Fig. 73. Sweepback at the Wing Tip

(a) Negative Sweepback

(b) Positive Sweepback
Fig. 74. Airfoil and Flow "Thickness" at the Wing Tip
Fig. 75. Behavior of Vorticity Field Near a Rounded Wing Tip
Fig. 76. Spanwise Circulation Profiles as a Function of Chord, $\Gamma_0 = 5.781 \text{ ft}^2/\text{sec}$
Fig. 77. Spanwise Circulation Profiles (Referenced to Local Maximum Chordwise Circulation)
Fig. 78. Chordwise Variation of Circulation Function (Referenced to Maximum Chordwise Value)
Fig. 79. Contour Integrals and the Bound Vorticity Distribution
Fig. 80. Bound vortex elements over the ring surface
($\alpha = 4^\circ, u_o = 60 \text{ ft/sec}$)

Legend

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Fig. 81. Contour Description, "A" type Contours, (to Scale)
Fig. 82. Chordwise (x) Component of Velocity
Adjacent to the Wing Surface,
Inboard Region, y/b = 0.25
Fig. 83. Mainflow Differential Velocity as a Function of Chordwise Location
Fig. 84. Chordwise Variation of Circulation Function as Obtained by "A" - type Contours (referenced to local spanwise maximum value)
Fig. 85. Spanwise Variation of the Wing Circulation Function, $\Gamma(c, y)$
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Fig. 88. Circulation Function at the Trailing Edge
Fig. 89. Variation of Lift Coefficient with Angle of Attack - Theory and Experiment
Fig. 90. Non-dimensional Circulation Function at Various Downstream Locations, $\alpha = 4$ degrees.
Fig. 91. Non-dimensional Circulation Function at Various Downstream Locations, $\alpha = 6$ degrees
Fig. 92. Non-dimensional Circulation Function at Various Downstream Locations, $\alpha = 8$ degrees
Fig. 93. Variation of Circulation with Angle of Attack for Various Downstream Locations (Contour Integrals)

\( \Gamma (\text{ft}^2/\text{sec}) \)

\( x/c \)
- \( \circ \) - 1.00 (trailing edge)
- \( \triangle \) - 2.00
- \( \square \) - 4.00
- \( \diamond \) - 6.00

\( \alpha \) (degrees)
Fig. 94. Variation of Circulation Function Downstream of the Wing
Fig. 95. Non-dimensional Circulation Function at Various Angles-of-Attack, $x/c = 2.00$
Fig. 96. Non-dimensional Circulation Function at Various Angles-of-Attack, x/c = 4.00
Upper Surface - 1b Trailing Max. Edgex

\[ x/c \]

- ○ - 2, Upper Limit
- △ - 2, Lower Limit
- □ - 4, Upper Limit
- ◊ - 4, Lower Limit
- △ - 6, Upper Limit
- □ - 6, Lower Limit

Darkened symbols indicate core center locations

Side View

Fig. 97. Variation of Wake Shape in the Near Field, \( \alpha = 4 \) degrees
Fig. 98. Variation of Wake Shape in the Near Field, 
$\alpha = 6$ degrees
Fig. 99. Variation of Wake Shape in the Near Field, $\alpha = 8$ degrees
ACKNOWLEDGEMENTS

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