IN-LINE AND TRANSVERSE FORCES ON SMOOTH AND SAND-ROUGHENED CYLINDERS IN OSCILLATORY FLOW AT HIGH REYNOLDS NUMBERS

By

TURGUT SARPKAYA

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IN-LINE AND TRANSVERSE FORCES ON SMOOTH AND SAND-ROUGHENED CYLINDERS IN OSCILLATORY FLOW AT HIGH REYNOLDS NUMBERS

This report presents the results of an extensive experimental investigation of the in-line and transverse forces acting on smooth and sand-roughened circular cylinders placed in oscillatory flow at Reynolds numbers up to 1,500,000; Keulegan-Carpenter numbers up to 100; and relative sand-roughnesses from 1/800 to 1/50. The drag and inertia coefficients have been determined through the use of the Fourier analysis and the least-squares method. The transverse force (lift) has been analysed in terms of its maximum, semi peak-to-peak, and root-mean-square values. In addition, the frequency of vortex shedding and the Strouhal number have been determined.

The results have shown that (a) for smooth cylinders, all of the coefficients cited above are functions of the Reynolds and Keulegan-Carpenter numbers, particularly for Reynolds numbers larger than about 20,000; (b) for rough cylinders, the force coefficients also depend on the relative roughness k/D and differ significantly from those corresponding to the smooth cylinder; and that (c) the use of the 'frequency parameter' $D^2/vT$ and the roughness Reynolds number $Umk/v$ allow a new interpretation of the present as well as the previously obtained data.

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NOMENCLATURE

A  Amplitude of oscillations at the test section
A₀  Amplitude of oscillations at the free surface
Cₙ  Drag coefficient
Cₙₐ  Maximum lift coefficient
Cₙₐₐ  Inertia coefficient
D  Diameter of the test cylinder
F  Force
Fₙ  Drag force
F₁  Inertia force
f  Frequency of oscillations, l/T
fᵥ  Frequency of vortex shedding, (first harmonic)
g  Gravitational acceleration
H  Elevation, (see Fig. 1)
K  Keulegan-Carpenter number, UₘT/D
k  Roughness height
L  Length of the test cylinder
p  Pressure
Pₚ  Reynolds number, UₘD/ν
T  Period of oscillations in the tunnel
t  Time
U  Instantaneous velocity
Uₘ  Maximum velocity in a cycle
Uₘₖ/ν  Roughness Reynolds number
β  Frequency parameter, D²/νT
γ  Specific weight of water
θ  2πt/T
ν  Kinematic viscosity of water at a given temperature
ρ  Density of water
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This report is dedicated to Hans von Nikolausberg for his lifelong friendship.
INTRODUCTION

The design of structures for the marine environment requires the prediction of the forces generated by waves and currents. Much of the present knowledge has been obtained by means of model tests at Reynolds numbers generally two to three orders of magnitude smaller than prototype Reynolds numbers. These model tests have relied heavily on the so-called Morison formula for expressing the force as the sum of a drag and inertia force. The values of the drag and inertia coefficients to be used in the Morison equation became the subject of many experimental studies in the last twenty years. The correlation of these coefficients with the relative amplitude of the waves (or the Keulegan-Carpenter number) has been generally inconclusive. The complexity of the problem stems partly from the difficulty of accurately defining the kinematics of the flow field, partly from the difficulty of accounting properly for the effects of time-dependent separation and vortex shedding, and partly from the difficulty in extrapolating the laborator findings to various conditions of the marine environment where three-dimensional effects and reduced spanwise coherence play important roles. It thus became clear that much is to be gained by considering plane oscillatory flow about cylinders at high Reynolds numbers in order to isolate the influence of individual factors such as relative amplitude, Reynolds number, and the relative roughness on vortex shedding and resistance. It is with this realization that the present investigation was undertaken and the preliminary results obtained with smooth cylinders in a small U-shaped water tunnel operating at relatively low Reynolds numbers (2,500 to 25,000) have been previously reported [1].
The present paper deals with in-line and transverse forces acting on smooth and sand-roughened circular cylinders in harmonic flow at critical and transcritical Reynolds numbers.

APPARATUS AND PROCEDURE

Of the two possible methods of generating relative harmonic fluid motion about bluff bodies, namely, oscillating the fluid or the body, the former has been chosen. The relative merits and shortcomings of the two methods have been amply discussed [2] and will not be repeated here. Suffice it to note that large amplitude structural and free-surface oscillations commonly encountered in oscillating the body in a fluid otherwise at rest do not lead to reliable data. The advantages of the apparatus used herein for the purpose under consideration have already been demonstrated [1, 2] and will become further evident from the data to be presented.

The oscillating flow system consisted of a large U-shaped vertical water tunnel as shown in Fig. 1. The cross-section of the two vertical legs is 3 ft by 6 ft and that of the test section is 3 ft by 3 ft. The two corners of the tunnel were carefully streamlined to prevent flow separation. This design proved to be more than adequate for no separation was encountered, and also the desired frequency and amplitude of oscillation were achieved. The auxiliary components of the tunnel consisted of plumbing for hot and cold water, butterfly-valve system, and the air-supply system.

The butterfly-valve system (mounted on top of one of the legs of the tunnel) consisted of four plates, each 18 inches wide and 36 inches long. All four valves were simultaneously driven by a simple rack and pinion.
system actuated by an air-driven piston and a three-way pneumatic valve. Initially, the butterfly valves were closed and air was introduced to that side of the tunnel, with an electrically-controlled ball valve, to create the desired differential water level between the two legs of the tunnel. Then the valves were opened with the help of the rack and pinion system and the three-way control valve. This action set the fluid in the tunnel in oscillatory motion with a natural period of $T = 5.500$ seconds. The elevation, acceleration, and all force traces were absolutely free from secondary oscillations so that no filters whatsoever were used between the outputs of the transducers and the recording equipment (see Fig. 2).

Throughout the investigation, the monitoring of the characteristics of the oscillations in the tunnel was of prime importance in view of the fact that most of the difficulties in the past in the determination of the drag, lift, and inertia coefficients resulted from the difficulty of generating a purely harmonic motion free from vibrations or from applying theoretically derived rather than measured values for velocities and accelerations.

Three transducers were used to generate three independent d.c. signals, each proportional to the instantaneous value of elevation, velocity, and acceleration. The first one consisted of a platinum wire stretched vertically in one leg of the tunnel. The response of the wire was perfectly linear within the range of oscillations encountered. The second method consisted of the measurement of the instantaneous acceleration by means of a differential-pressure transducer connected to two pressure taps placed horizontally 2 ft apart and 4 ft to one side of the test section. The instantaneous acceleration was then calculated from $\Delta p = \rho_s \frac{dU}{dt}$ where
Δp is the differential pressure, s the distance between the pressure taps, and dU/dt is the instantaneous acceleration of the fluid. The third method again consisted of the measurement of the differential pressure between two pressure taps placed symmetrically on the two vertical legs of the tunnel at an elevation H ft below the mean water level. The linear differential-pressure transducer yielded the instantaneous elevation and hence the amplitude of oscillation since, according to Bernoulli's equation

\[ A = 2A_0 = \frac{\Delta p/y}{\max} \left[ 1 - \frac{(2\pi/T)^2 H/g}{1} \right] \]

in which g and T are constant and H is kept constant.

All three methods gave nearly identical results and yielded the amplitude A, the maximum velocity \( U_m = 2\pi A/T \), or the maximum acceleration \( a_m = (2\pi/T)^2 A \) to an accuracy of about 2 percent relative to each other.

In addition to the methods cited above, the velocity at the test section was directly measured with a calibrated magnetic velocimeter. These comparisons, as well as the perfectly sinusoidal and noise-free character of all pressure and force traces, speak for the suitability of the unique test facility used in this study. The additional details of the apparatus and procedure are given in [2].

The in-line and transverse forces were measured with two identical, cantilever type, force transducers, one at each end of the cylinder. The gages had a capacity of 250 lbs and the deflection of the cantilever end was less than 0.008 inches. A special housing was built for each gage so that it can be mounted on the tunnel window and rotated to measure either the in-line or the transverse force alone. The test cylinders were placed...
in the test section by retracting the gages from their housing and then pushing them into the bearings mounted on each end of the cylinders. This allowed a gap of 1/32 inch between the cylinder and the tunnel wall. The natural frequency of the cylinder and force-transducer combination in water was about 20 times larger than the frequency of oscillation of the water column and about 10 times larger than the largest frequency of vortex shedding.

Seven circular cylinders with diameters ranging in size from 2 inches to 6.5 inches were used. The cylinders were turned on a lathe from aluminum pipes or plexiglass rods and polished to a mirror-shine surface. Same cylinders were also used as rough cylinders. For this purpose, sand was sieved to obtain the desired relative roughness and applied uniformly on the cylinder surface with an air-drying epoxy paint. After a series of tests with water at various temperatures, the cylinders were polished again and covered with sand of different size. This procedure was continued until the desired ranges of all the governing parameters were covered.

Previously [2], sand paper, sand, and polystyrene beads were used as roughness elements for a given cylinder in order to achieve the desired relative roughness in a given Reynolds number range. A detailed study of the effective roughness of each type of roughness element and the discussions with the manufacturer have shown that the effective roughness of the sand paper is larger than the height of the mean sand particles applied on it. Furthermore, the gluing of the sand paper on the cylinder invariably resulted in a 'joint' along the cylinder which might have generated larger disturbances and promoted earlier transition. The polystyrene beads, on the other hand, present an effective-roughness
height which is often smaller than their actual size [3]. In spite of these differences, however, the terminal values of the drag coefficients in the transcritical region remained practically the same for a given actual effective relative roughness whether the data were obtained with sand alone or with a combination of other roughness elements. Evidently, it will be most interesting and desirable to carry out similar experiments with cylinders roughened in the ocean environment. The results presented herein show that the testing of roughened cylinders with steady uniform flow is not sufficient for the purposes under consideration, namely the determination of the fluid loading on offshore structures.

FORCE COEFFICIENTS AND GOVERNING PARAMETERS

Data reduction for the forces in-line with the direction of oscillation is based on Morison equation [4] and three different analysis of the force records, namely, Fourier analysis, least squares, and a modified least squares method.

The in-line force which consists of the drag force $F_d$ and the inertia force $F_i$ is assumed to be given by [4]

$$F = F_d + F_i = 0.5C_dLDp|U|U + 0.25C_mL^2d\mu\mu dt$$  \hspace{1cm} (2)

in which $C_d$ and $C_m$ represent respectively the drag and inertia coefficients and $U$ the instantaneous velocity of the ambient flow. For an oscillating flow represented by $U = -U_m\cos\theta$, with $\theta = 2\pi t/T$, the Fourier averages of $C_d$ and $C_m$ are given by Keulegan and Carpenter as [5]
\[ C_d = -0.75 \int_0^{2\pi} (F_m \cos \omega \rho U_m^2 D) d\omega \] 

(3)

and

\[ C_m = \frac{2\pi}{2 \sin T} \int_0^{2\pi} (F_m \sin \omega \rho U_m^2 D) d\omega \] 

(4)

in which \( F_m \) represents the measured force.

The method of least squares consists of the minimization of the error between the measured and calculated forces. This procedure yields [6]

\[ C_{dls} = -\frac{8}{3\pi} \int_0^{2\pi} (F_m |\cos| \cos \rho D U_m^2) d\omega \] 

(5)

and \( C_{mls} = C_m \). Evidently, the Fourier analysis and the method of least squares yield identical \( C_m \) values and that \( C_d \) values differ only slightly. The details of the modified least-squares method may be found in [6] and will not be repeated here.

The transverse force has been expressed in terms of various coefficients. Some of these are: (a) the maximum lift coefficient defined by \( C_L = \frac{\text{maximum amplitude of the transverse force in a cycle}}{0.5 \rho D U_m^2} \); (b) the semi peak-to-peak value of the transverse force normalized as above; and (c) the normalized root-mean-square value of the transverse force. In addition, the frequency of the oscillations of the transverse force and the Strouhal number have been evaluated.

It is recognized that the coefficients cited above are not constant throughout the cycle and are either time-invariant averages or peak values at a particular moment in the cycle. A simple dimensional analysis of the flow under consideration shows that the time-dependent coefficients may be written as

\[ \frac{F}{0.5 \rho D U_m^2} = f(U_m T/D, U_m D/\nu, k/D, t/T) \] 

(6)
in which $F$ represents the in-line or the transverse force. Equation (6), combined with Eq. (2), assuming for now that the latter is indeed valid, yields

$$C_d = f_1(K, \text{Re}, k/D, t/T)$$  \hspace{1cm} (7) \\
$$C_m = f_2(K, \text{Re}, k/D, t/T)$$  \hspace{1cm} (8)

in which $K = U_m T/D$ and $\text{Re} = U_m D/\nu$, and $k/D$ represents the relative roughness.

There is no simple way to deal with Eqs. (7) and (8) even for the most manageable time-dependent flows. Another and perhaps the only other alternative is to eliminate time as an independent variable and consider suitable time-invariant averages as given by Eqs. (3), (4), and (5).

---

*The one-parameter characterization of the effect of roughness needs some justification. It is of course recognized that not only the relative size of the roughness elements but also their shape and distribution may be quite important. It is in fact partly for the difficulty of uniquely specifying the 'roughness' and partly for the differences in other test conditions that there are considerable differences between the steady-flow data reported by various workers [7-10], particularly in the drag crisis region. For example, the effective surface roughness may be larger or smaller than the nominal relative roughness based on the geometric size of the roughness element depending on the shape and arrangement of the roughness elements [3]. It is partly for this reason that it has been thought advisable to investigate afresh the effect of roughness on cylinders in harmonic flow using only sand of uniform size and packing rather than three different types of roughness elements, (see Fig. 3 for sample photographs of the sand-roughened surfaces).
Thus, one has

\[
[C_d, C_m, C_L, \ldots] = f_i(K, Re, k/D) \tag{9}
\]

It appears, for the purposes of Eq. (9), that the Reynolds number is not the most suitable parameter involving viscosity. The primary reasons for this are that the effect of viscosity is relatively small and that \(U_m\) appears in both \(K\) and \(Re\). Thus, replacing \(Re\) by \(Re/K = D^2/\nu T\) in Eq. (9), one has

\[
C_i[a \text{ coefficient}] = f_i(K, \beta, k/D) \tag{10}
\]

in which \(\beta = D^2/\nu T\) and shall be called the 'frequency parameter'.

From the standpoint of dimensional analysis, either the Reynolds number or \(\beta\) could be used as an independent variable. Evidently, \(\beta\) is constant for a series of experiments conducted with a cylinder of diameter \(D\) in water of uniform and constant temperature since \(T\) is kept constant in a U-shaped oscillating flow tunnel. Then the variation of a force coefficient with \(K\) may be plotted for constant values of \(\beta\). Subsequently, one can easily recover the Reynolds number from \(Re = \beta K\) and connect the points, on each \(\beta\) = constant curve, representing a given Reynolds number.

From the standpoint of laminar boundary layer theory, \(\beta\) represents the ratio of the rate of diffusion of vorticity through a distance \(\delta\) (the boundary-layer thickness) to the rate of diffusion through a distance \(D\). This ratio is also equal to \((D/\delta)^2\) and, when it is large, gradients of velocity in the direction of flow are small compared with the gradients normal to the boundary, a situation to which the boundary-layer theory is applicable [11]. It should be noted in passing that the added mass and drag coefficients for a cylinder or sphere undergoing harmonic oscillations...
without separation in a fluid otherwise at rest are determined uniquely in terms of $\beta$ [11].

Let us now re-examine a set of data previously obtained by others [5] partly to illustrate the use and significance of $\beta$ as one of the governing parameters and partly to take up the question of the effect of Reynolds number on the force coefficients.

The data given by Keulegan and Carpenter [5] may be represented by 12 different values of $\beta$. The drag and inertia coefficients are plotted in Figs. 4 and 5 and connected with straightline segments. Evidently, the identification of the individual data points in terms of the cylinder diameter, as was done by Keulegan and Carpenter [5] and also by Sarpkaya [1], irrespective of the $\beta$ values gives the impression of a scatter in the data and invites one to draw a mean drag curve through all data points. Such a temptation is further increased by the fact that the data for each $\beta$ span over only a small range of $K$ values. Evidently, the drawing of such a mean curve eliminates the dependence of $C_d$ and/or $C_m$ on $\beta$ and hence on Re.

Also shown in Figs. 4 and 5 are points representing four selected Reynolds numbers. The corresponding $K$ values for each Re and $\beta$ were calculated from $K = Re/\beta$. The points corresponding to the selected Reynolds numbers are reproduced in Figs. 6 and 7. These figures show, within the range of Re and $K$ values encountered in Keulegan-Carpenter data, that (a) $C_d$ depends on both $K$ and Re and decreases with increasing Re for a given $K$; and that (b) $C_m$ depends on both $K$ and Re for $K$ larger than approximately 15 and decreases with increasing Re. A similar analysis of Sarpkaya's data [1] also shows that $C_d$ and $C_m$ depend on both $K$ and Re and that $C_m$ increases with increasing Re. Notwithstanding this difference in the variation of $C_m$ between the two
sets of data, Figs. 6 and 7 put to rest the long standing controversy regarding the dependence or lack of dependence of $C_d$ and $C_m$ on Re and show the importance of $\beta$ as one of the governing parameters in interpreting the data, in interpolating the $K$ values for a given Re, and in providing guide lines for further experiments as far as the ranges of $K$ and $\beta$ are concerned.

RESULTS AND DISCUSSIONS

Drag and Inertia Coefficients for Smooth Cylinders

Only the representative data will be presented herein for sake of brevity. The tabulated as well as plotted data for all force coefficients for smooth circular cylinders are given in [2].

Figures 8 and 9 show $C_d$ versus $K$ and $C_m$ versus $K$ for five representative values of $\beta$. Evidently, there is very little scatter in the data even though the figures represent the results of four independent runs. Mean lines drawn through the data shown in Figs. 8 and 9 are presented in Figs. 10 and 11 together with the constant Reynolds number lines obtained through the use of $K = \frac{Re}{\beta}$. Evidently, there is a remarkable correlation between the force coefficients, Reynolds number, and the Keulegan-Carpenter number. The smoothness of the constant Re lines is another indication of the consistency of the data from one cylinder to another.

*Additional data obtained since the writing of Ref. [2] are included herein in the figures showing $C_d$ and $C_m$ as a function of the Reynolds number for constant values of $K$ and/or $k/D$. 

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Figures 10 and 11 show that $C_d$ and $C_m$ do not vary appreciably with $Re$ for $Re$ smaller than about 20,000 and help to explain the conclusions previously reached by Keulegan and Carpenter [5] and Sarpkaya [1].

The entire data, obtained since the inception of the investigation about five years ago, are shown as a function of $Re$ for constant values of $K$ in Figs. 12 and 13. These figures clearly show that $C_d$ decreases with increasing $Re$ to a value of about 0.5 (depending on $K$) and then gradually rises to a constant transcritical value within the range of Reynolds numbers encountered. The inertia coefficient $C_m$ increases with increasing $Re$, reaches a maximum, and then gradually approaches a value of about 1.75. It will be recalled that the Keulegan-Carpenter data indicated an opposite trend. It is believed that the Keulegan-Carpenter data for $C_m$ are not quite reliable for $K > 15$. This is also evident from the observation that the data corresponding to $\beta = 141$ appear to be out of place (see Fig. 5) relative to those corresponding to $\beta = 97$ and $\beta = 217$. Suffice it to say that the results presented in Figs. 12 and 13 shed new light on the variations of the drag and inertia coefficients and partly explain the reasons for the large scatter encountered in the plots of $C_d$ versus $Re$ and $C_m$ versus $Re$ as compiled by Wiegel [12].

**Drag and Inertia Coefficients for Sand-Roughened Cylinders**

In view of the fact that each coefficient depends on at least three independent parameters ($Re$, $K$, and $k/D$), it is not possible to show on two-dimensional plots the variation of either $C_d$ or $C_m$ for all values of $Re$, $K$, and $k/D$. However, this difficulty is alleviated by the fact that the variation of a given force coefficient for a given $Re$ and $k/D$ is not.
very strong from one K to another. Thus it has been decided to choose five representative K values, namely K = 20, 30, 40, 60, and 100, to present the variation of \( C_d \) and \( C_m \) with Re for five representative k/D values.

Figures 14 through 18 show \( C_d \) and \( C_m \) for five values of K as a function of the Reynolds number. Each curve on each plot corresponds to a particular relative roughness. Also shown on each figure is the corresponding drag or inertia coefficient for the smooth cylinder at the corresponding K value.

The k/D = constant curves on each \( C_d \) plot are quite similar to those found for steady flow about rough cylinders [7-10]. For a given relative roughness, the drag coefficient does not significantly differ from its smooth cylinder value at very low Reynolds numbers. As the Reynolds number increases, \( C_d \) for the rough cylinder decreases rapidly, goes through the region of 'drag crisis' at a Reynolds number considerably lower than that for the smooth cylinder and then rises sharply to a nearly constant transcritical value. The larger the relative roughness the larger is the magnitude of the minimum \( C_d \) and the smaller is the Reynolds number at which that minimum occurs. However, there appears to be a minimum Reynolds number below which the results for rough cylinders do not significantly differ from those corresponding to smooth cylinders. In other words, the Reynolds number must be sufficiently high for the roughness to play a role on the drag and flow characteristics of the cylinder.

The figures for the drag coefficient also exhibit a few other interesting features. First, even a relative roughness as small as 1/800 can give rise to transcritical drag coefficients which are considerably higher than those for the smooth cylinder. Secondly, the asymptotic values of the drag coefficient for roughened cylinders within the range of Reynolds numbers
encountered, can reach values which are considerably higher than those obtained with steady flows over cylinders of similar roughness ratio. In other words, it is not safe to assume that the transcritical drag coefficient in harmonic flows will be identical to those found in steady flows and will not exceed a value of about unity. On the basis of the present results it may be said that such a conjecture is not accurate even for $K$ values as large as 100 (corresponding to a wave height-to-diameter ratio of about 30). In steady flow about a cylinder, roughness precipitates the occurrence of 'drag crisis' and gives rise to a minimum drag coefficient which is larger than that obtained with a smooth cylinder [7-10]. This is partly because of the transition to turbulence of the free shear layers at relatively lower Reynolds numbers (due to disturbances brought about by the roughness elements) and partly because of the retardation of the boundary-layer flow by roughness (higher skin friction) and, hence, earlier separation. In harmonic flow about a cylinder, roughness appears to play an even more complex role because of the time-dependence of the boundary layer and the position of the separation points. In particular, the magnitude of the transverse force (to be discussed subsequently) strongly suggests that the combined effect of uniformly-distributed roughness and time dependence (even in the drag dominated region of $K$ values) is to increase the strength of vortices and the spanwise coherence relative to that in steady flow at the same $Re$ about the same cylinder.

The Reynolds number at which the drag crisis occurs gives rise to an 'inertia crisis'. In other words, for a given relative roughness, $C_m$ rises rapidly to a maximum at a Reynolds number which corresponds to that at which $C_d$ drops to a minimum. At relatively higher Reynolds numbers, $C_m$ decreases
somewhat and then attains nearly constant values which are lower than those corresponding to the smooth cylinders. It is also apparent from the inertia coefficient curves that the smaller the relative roughness the larger is the maximum inertia coefficient. For relatively smaller roughnesses such as \( k/D = 1/800 \), the terminal value of \( C_m \) is nearly equal to that of a smooth cylinder. The behavior of \( C_m \) is not entirely unexpected. It has long been noted [5] that whenever there is a rise in the drag coefficient, there also is a decrease in the inertia coefficient.

Before closing the discussion of the drag and inertia coefficients, it is necessary to point out the remarkably consistent behavior of the data points, particularly for \( C_d \). Perhaps it would not have been too surprising had the data been obtained for one relative roughness through the use of only one cylinder. In the present investigation, the use of several cylinders and several temperatures for a given cylinder always provided data for nearly identical \( k/D \), \( Re \), and \( K \) values. For instance, the \( C_d \) and \( C_m \) values obtained at a given \( K \), \( Re \), and relative roughness \( k/D \), using a 5 inch cylinder at a low temperature corresponds to the \( C_d \) and \( C_m \) values using a 4-in. cylinder at a high temperature. Remembering the fact that not only the actual size of the cylinders but also the size of the sand grains differed in order to obtain the same \( k/D \), and the fact that the experiments were carried out at different temperatures and times, one fully realizes that the correlation of the data and the relatively small scatter are indeed quite remarkable. This is due not only to the repeatability of the tests but also due to the vibration-free operation of the entire tunnel system.

The correlation length along the cylinders was not directly measured.
However, one series of experiments was conducted with a 2.18-diameters (12 inches) long, centrally located, section of a 5.5-inch cylinder which 'floated' on the ends of the force transducers with small gaps (0.002 in.) between the section and the rest of the rigidly supported 12-inch long sections. The floating and dummy sections were carefully polished and tested as a smooth cylinder. Then both sections were coated with sand for a relative roughness of $k/D = 1/100$. The comparison of the lift, drag, and inertia coefficients obtained with the short section with those obtained with the longer section spanning the entire test section has shown that the two sets of coefficients are nearly identical for both smooth and roughened cylinders. The differences were in the order of experimental errors and, if anything, the drag coefficients obtained with the short section were about 3% larger than those obtained with the 3-foot section. Evidently, the force-cancelling effects of phase shifts which may have been brought about by three-dimensional effects were either insignificant or non-existent. Thus, it is concluded that both the three-dimensionality effects and the boundary-layer effects played very little or no role in the present investigation.

A comparison of the results shown in Figs. 14 through 18 with the previously reported [2, 13] preliminary results for roughened cylinders for $K = 50$ alone indicates that the type of roughness may affect the variation of the drag coefficient with Reynolds number, particularly in the drag-crisis region. Previously, sand paper, sand, and polystyrene beads were used [2] as roughness elements for a given cylinder in order to achieve the desired relative roughness in a given Reynolds number range. In spite of the differences in the 'effective roughness' of the three types of roughness elements, however, the terminal values of the drag coefficients
in the transcritical region remained practically the same for a given actual effective relative roughness whether the data were obtained with sand alone or with a combination of other roughness elements. Evidently, it will be most interesting and desirable to carry out similar experiments with cylinders roughened in the ocean environment.

**In-Line Force Coefficients and the Roughness Reynolds Number**

The data given in Figs. 18a and 18b are replotted in Fig. 19 as a function of the roughness Reynolds number defined by $U_m k/v$ for all values of $k/D$. Similar plots may be prepared for other values of $K$ through the use of Figs. 14 through 17.

It is rather remarkable that $C_d$ and $C_m$ become practically independent of $k/D$ for $U_m k/v$ larger than about 300. In other words, for sufficiently large values of the roughness Reynolds number, the drag and inertia coefficients for a roughened cylinder in a given harmonic flow are determined by the height of the excrescences rather than by the diameter of the cylinder. The importance and the consequences of this result are self evident for supercritical Reynolds number simulation for flow over circular cylinders. A detailed discussion of this and other pertinent concepts for steady flow over roughened cylinders is presented by Szechenyi [10] and will not be repeated here.

**Transverse Force and Vortex Shedding for Smooth and Rough Cylinders**

The data for the maximum lift coefficient for smooth cylinders are summarized in Figs. 20 and 21. The original data in plotted and tabulated form are presented in [2]. Evidently, the lift coefficient depends on Re for Re larger than about 20,000 and rapidly decreases to about 0.2 for
larger values of Re and K. It is also evident that the lift force is a major portion of the total force acting on the cylinder and cannot be neglected in the design of structures.

The frequency of the alternating transverse force is shown in Fig. 22 in terms of \( f_r = f_v/f \) as a function of K and Re. It is apparent that \( f_r \) is not constant and increases with increasing K and Re. Furthermore, a quick calculation through the use of Fig. 22 shows that the Strouhal number given by \( St = f_v D/U_m = f_r/K \) is not constant at 0.2, as in steady flow, and depends on both Re and K.

Several additional facts are of special importance in connection with the variation of the lift coefficient and the frequency ratio. Firstly, the lift curve begins at K = 5. There is an occasional vortex shedding for K values between 4 and 5. The minimum value of K at which lift or asymmetry in the vortices develops is, by the very nature of vortices, extremely sensitive to the experimental conditions. Evidently, what is of special importance is not the highest value of K at which the symmetry of the vortices can be maintained with extreme care but the lowest value of K below which asymmetry cannot be initiated in spite of the magnitude of the external disturbances. A careful analysis of all the lift traces has shown that there is a 90% chance that the asymmetry will occur at K = 5. At K = 4, there is only a 5% chance that the asymmetry will appear for very short periods of time. Secondly, each \( f_r = N \) line does not represent an absolute line of demarcation between the frequencies \( N-1 \) and \( N+1 \). Occasionally, a frequency of \( N+1 \) will occur on the \( N-1 \) side of the \( N \) line, and vice versa. Thirdly, the frequency of vortex shedding is not a pure multiple of the frequency of flow oscillation. Evidently, \( f_r \), as an integer, is a measure of the number.
of vortices actually shed during a cycle. However, all of the vortices are not fully developed or completely shed. Thus, the fractional part of \( f_r \) indicates an incomplete vortex shedding. This phenomenon is particularly common for \( f_r \) values in the neighborhood of 3 and also for large values of \( K \) and \( Re \) where the oscillations of the transverse force become quite irregular. The significance of the foregoing relative to the in-line force will be taken up separately.

The maximum lift coefficient for the roughened cylinders are presented in Fig. 23 as a function of \( K \) for various values of \( \beta \) and one particular value of \( k/D \). Additional details may be found in [14]. Evidently, \( C_L \) does not vary appreciably with either \( \beta \) or \( Re \). The data presented in [14] for other values of \( k/D \) show that \( C_L \) does not vary with \( k/D \) also within the range of the parameters encountered. If there is some variation with these parameters (\( Re \) and \( \beta \)), it is certainly masked by the scatter of the data resulting from the random nature of the lift force. In fact, it is not too uncommon to obtain a variation of 20-25\% for a given value of \( K \). This fact is of importance in discussing the effect of the Reynolds number on the lift coefficient. Also shown in Fig. 23 is the lift coefficient for smooth cylinders for \( \beta \) in the range of 1000 to 2000. It is rather surprising that the smooth cylinder data at relatively low values of \( \beta \) form more or less the upper limit of the rough cylinder data. In other words, the lift coefficient for rough cylinders does not depend on \( Re \) and becomes almost identical to that for smooth cylinders at very low Reynolds numbers. The consequences of this observation for model testing purposes are rather obvious. The behavior of the lift coefficient for rough cylinders is in conformity with the fact that the transcritical drag coefficient for rough cylinders (steady or harmonic flow) nearly returns to its subcritical value.
As noted earlier, the alternating nature of the transverse force is as important as its magnitude. It is for this reason that the frequency of the lift force has been determined for rough cylinders also. A close examination of the frequency ratios shows that $f_r/K$ remains essentially constant at a value of about 0.22 [14]. To be sure, there are variations from one cylinder to another and from a given combination of Re and $K$ to another. Nevertheless, the Strouhal number is fairly constant for all roughnesses, relative amplitudes, and Reynolds numbers larger than about 20,000. This fact is of special importance in determining the in-line and transverse vibrational response of the elements of a structure to wave-induced forces. One must, however, keep in mind the fact that the spanwise coherence along a vertical cylinder in the ocean environment may be reduced by the variation of the velocity vector with time and depth and that the lift coefficients presented herein represent the maximum possible values of the transverse force.

ON THE APPLICABILITY OF MORISON'S EQUATION AND THE PRESENT DATA

Since its inception, questions have been raised regarding the applicability of Morison's equation to time-dependent flows in general and to wavy flows in particular. It has been known that the equation predicts quite accurately the in-line force for both very small values of $K$ ($K$ smaller than about 10) and for large values of $K$ ($K$ larger than about 20). For intermediate values of $K$, differences have been observed between the measured and calculated values. These differences have been attributed either to the imprecise measurement of the kinematics of the flow or to the shortcomings of the equation. It is now realized that not only these two factors (namely the heuristic nature of the equation and the difficulty of measuring the local
velocities and accelerations) but also the three-dimensional nature of the wavy flows and decreased spanwise coherence must be partly responsible for the differences between the measured and calculated forces. In fact, it would have been extremely difficult to draw meaningful conclusions regarding the applicability of Morison's equation through the use of the field data. It is only through the use of carefully conducted two-dimensional harmonic flow experiments that one can ascertain the degree of applicability of Morison's equation.

Figure 24 shows the calculated and measured forces normalized by $0.5DL\mu u_m^2$ together with the normalized velocity and the difference between the measured and calculated forces for a relatively large value of $k/D$. It is evident that there is often a remarkable correspondence between the measured and calculated forces particularly for $K$ values larger than about 20. This is also true for $K$ smaller than about 10. In the disturbance-sensitive region of vortex formation and onset of asymmetry, the growth and shedding of single or alternating vortices have profound effects not only on the measured in-line force but also on the force coefficients calculated. The fractional shedding of vortices and the vortex-induced oscillations in the in-line force give rise to an asymmetry in the magnitude of the in-line force. In other words, the in-line force can no longer be represented by an odd harmonic function. Thus, it is clear that part of the reason for the larger differences between the measured and calculated forces even in perfectly two-dimensional harmonic flows is due to the use of the force coefficients which are derived by assuming the in-line force to be given by an odd harmonic function. In the range of $K$ values from about 10 to 20, particularly for low values of $Re$, this assumption is not quite correct as evidenced by the present experiments, (see also Ref. [2]).
In considering the relevance of the coefficients presented herein and of the equation devised by Morison to wave induced loads on offshore structures, it is of course important to take into account the differences between uniform two-dimensional harmonic motion and the wave motion where the velocity vector both rotates with time at a point and decays in magnitude with depth. The spanwise variations of the flow in general lead to reduced spanwise coherence. It is safe to assume that both the three-dimensionality of the flow and the reduction of the correlation length along the cylinder, in an ocean environment, tend to increase the base pressure and thus give rise to transcritical drag coefficients which are smaller than those obtained with purely two-dimensional flows. The drag coefficients presented herein obviously represent their maximum possible values since they have resulted from a uniform, two-dimensional flow where the instantaneous wake of the cylinder has the highest possible degree of spanwise coherence. The similarity between the reduced drag coefficient due to reduced spanwise coherence in wavy flows and the drag coefficient in steady flows (both for roughened cylinders) is rather fortuitous and does not imply the equality of the two drag coefficients in the drag-dominated region of the K values. It is rather unfortunate that even the experiments with wavy flows cannot be expected to isolate the effect of reduced spanwise coherence since such experiments surely bring in other factors whose influence is combined in a complex way with that of the reduced correlation. Thus, the value of the results presented herein lies in the fact that the designer now knows the maximum possible value of the coefficients under consideration, if not the values which might be more appropriate to the conditions under which the structure must survive and function.
CONCLUSIONS

The results presented herein warrant the following conclusions:

(a) For smooth cylinders, the drag, lift, and the inertia coefficients depend on both the Reynolds and Keulegan and Carpenter numbers; (b) For roughened cylinders, the drag and inertia coefficients depend on Re, K, and k/D; (c) The drag coefficient undergoes a 'drag crisis' and rises to a nearly constant value (depending on k/D) within the range of Reynolds and Keulegan-Carpenter numbers tested; (d) The asymptotic values of the drag coefficient for K < 100 are larger than the transcritical drag coefficients for roughened cylinders in steady flow; (e) The results show that neither the acceleration modulus $R_1 = 2\pi/K$ nor the ratio of the maximum inertial force to maximum drag force $R_2 = (\pi^2/K)(C_m/C_d$) is small enough (for the K, Re, and k/D values tested) for the harmonic flow to behave like a pseudo-steady flow over a roughened cylinder, (e.g. for K = 100, Re = 1,000,000, and k/D = 1/100, one has $C_d = 1.55$, $C_m = 1.6$, and $R_1 = 0.063$, $R_2 = 0.10); (f) The importance of the role played by roughness in harmonic flow is further evidenced by the fact that in the same range of K and Re values, the transcritical $C_d$ values for steady and harmonic flows become nearly identical even though the force ratio $R_2$ may be larger than that for the rough cylinder case, (e.g. for K = 100, Re = 1,000,000, one has $C_d = 0.65$, $C_m = 1.75$, and $R_1 = 0.063$, and $R_2 = 0.27$); (g) The physical mechanism responsible for the surprisingly large effect of roughness on resistance in flows with relatively small accelerations and decelerations needs careful study; (g) The inertia coefficient also undergoes an 'inertia crisis' at Re values corresponding to the 'drag crisis' at which $C_m$ reaches a maximum value and then asymptotically decreases; (h) The drag and inertia coefficients become independent of k/D for roughness Reynolds numbers larger than about 300; (i) The transverse
force is a significant fraction of the total resistance at all Reynolds numbers for both smooth and rough cylinders and must be taken into consideration in the design of structures; (j) The Strouhal number for smooth cylinders depends on both $K$ and $Re$. For rough cylinders, it remains nearly constant for all Reynolds numbers at about 0.22; (k) The results presented herein and the conclusions arrived at are applicable only to cylinders in harmonic flow with zero mean velocity within the range of the $Re$, $K$, and $k/D$ values encountered in the tests. The force coefficients for wavy flows (with or without a mean velocity superimposed on them) may differ somewhat from those presented herein partly due to the reduced coherence along the length of the cylinder, partly due to the three-dimensionality of the flow, and partly due to the nonlinear interaction between the current and the waves. It is thus hoped that the data and the discussions presented herein will draw serious attention to the effect of roughness, three dimensionality, and spanwise coherence in wavy flows and will accentuate the need for additional laboratory and full scale experiments.

REFERENCES


Fig. 1a. Schematic drawing of the U-shaped vertical water tunnel.
Fig. 2 Force and Acceleration Traces
Fig. 3 Scanning Electron Microscope Photographs of Sand-Roughened Surface
Fig. 4 $C_d$ versus $K$ for various values of $\beta$ for the data provided by Keulegan and Carpenter [5]

Fig. 5 $C_m$ versus $K$ for various values of $\beta$ for the data provided by Keulegan and Carpenter [5]
Fig. 6 $C_d$ versus $K$ for Constant Values of $Re$ for the Data Provided by Keulegan and Carpenter [5]

Fig. 7 $C_m$ versus $K$ for Constant Values of $Re$ for the Data Provided by Keulegan and Carpenter [5]
Fig. 8 $C_d$ versus $K$ for various values of $\beta$, (present data).

Fig. 9 $C_m$ versus $K$ for various values of $\beta$, (present data).
Fig. 10 $C_d$ versus $K$ for constant values of $Re$ and $\beta$
(present data)

Fig. 11 $C_m$ versus $K$ for constant values of $Re$ and $\beta$
(present data)
Fig. 12 $C_d$ versus $Re$ for constant values of $K$, (Present data).
Fig. 14a $C_d$ versus Re for various values of $k/D$, ($K=20$).

Fig. 14b $C_m$ versus Re for various values of $k/D$, ($K=20$).
Fig. 15a  $C_d$ versus $Re$ for various values of $k/D$, $(K = 30)$.  

Fig. 15b  $C_m$ versus $Re$ for various values of $k/D$, $(K = 30)$.  

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Fig. 16a  $C_d$ versus Re for various values of $k/D$, ($K = 40$).

Fig. 16b  $C_m$ versus Re for various values of $k/D$, ($K = 40$).
Fig. 17a  $C_d$ versus Re for various values of $k/D$, ($K = 60$).

Fig. 17b  $C_m$ versus Re for various values of $k/D$, ($K = 60$).
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Fig. 18b $C_m$ versus $Re$ for various values of $k/D$, ($K = 100$).
Fig. 19 Drag and inertia coefficients variation of the roughness Reynolds number for various values of $\delta$. --- for $K = 100$. 
Fig. 20 $C_L$ versus $K$ for various values of $Re$ and $\beta$

(smooth cylinders)
Fig. 21 Maximum lift coefficient versus Reynolds number for various values of $K$
Fig. 22  $f_r$ as a function of $Re$ and $K$
(sMOOTH CYLINDER DATA)
Fig. 23a. Combined plot of $C_L(\text{max})$ data for $k/D = 1/200$ for various values of $\beta$. 

- Dotted line: All rough-cylinder data for $k/D = 1/200$
- Solid line: Smooth cylinder data for $\beta = 1000$ to 2000.
Fig. 23b  $C_L$(max) versus $K$ for various values of $k/D$ and $\beta$. 
Fig. 24. Comparison of measured and calculated forces.

\[ F = \text{Re} \times \frac{1}{d} \times \frac{1}{100} \]

\[ F = \text{Measured} \]

\[ F = \text{Calculated} \]

\[ F = \text{Measured} - F(\text{Calculated}) \]
Fig. 24b Comparison of measured and calculated forces

\[ K = 61.88 \]
\[ Re = 241,798 \]
\[ k/D = 1/100 \]
Fig. 24c Comparison of measured and calculated forces.

\[ F(\text{measured}) \]

\[ K = 56.58 \]
\[ Re = 221,088 \]
\[ k/D = 1/100 \]
Fig. 24d. Comparison of measured and calculated forces

K = 37.80
Re = 147,719
K/D = 1/100
Fig. 24e  Comparison of measured and calculated forces

K = 25.43
Re = 99,350
k/D = 1/100
Fig. 24f Comparison of measured and calculated forces

$K = 23.43$
$RE = 91,514$
$k/D = 1/100$
Fig. 24g. Comparison of measured and calculated forces

\[ K = 18.70 \]
\[ Re = 73,090 \]
\[ k/D = 1/100 \]
Fig. 24h  Comparison of measured and calculated forces

$K = 16.26$
$Re = 63,528$
$k/D = 1/100$
Fig. 241 Comparison of measured and calculated forces

K = 11.97
Re = 46,783
k/D = 1/100
Fig. 24j  Comparison of measured and calculated forces

\[ K = 7.85 \]
\[ Re = 30,670 \]
\[ k/D = 1/100 \]
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