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TRAJECTORY GENERATION BY PIECEWISE SPLINE INTERPOLATION

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# Trajectory Generation by Piecewise Spline Interpolation

**Abstract**: A method of generating vehicle trajectories from sets of position coordinates, acceleration components, or velocity vector directions specified at discrete points in time is described. The method uses piecewise cubic spline polynomials fitted to the trajectory position coordinates, thus ensuring continuity of position, velocity, and acceleration along the trajectory. The mathematical basis of cubic spline interpolation is derived, and the description of a computer program for generating the trajectory interpolating polynomial coefficients is included in the report.

**Key Words**: Vehicle trajectories, Piecewise cubic spline, Polynomials, Cubic spline interpolation.
Block 20. Abstract (Concluded)

Calculation of vehicle Euler angles is also contained as an option in the program; these are expressed as interpolating polynomial coefficients in a manner similar to that used for the trajectory. The Euler angle calculation permits the inclusion of aerodynamic angles of attack for an air-supported vehicle under the assumption that all maneuvers use coordinated turns.
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I. INTRODUCTION

In determining the performance, by simulation methods, of a missile system designed to intercept moving targets, it is frequently necessary to generate the trajectory of the targets during the course of complicated maneuvers. Various possibilities exist for expressing the target trajectories without resorting to a full numerical solution of the target dynamic equations which would require complete knowledge of all the forces acting on the target.

Target trajectories can be expressed deterministically in terms of position, velocity, or acceleration components as functions of time with respect to a particular frame of reference. It is only necessary to use one of these formulations since the others are obtainable either by integration or differentiation. Whichever form is used, an interpolation process is usually required since typical maneuvers are not expressible in terms of simple algebraic functions but must, in general, be represented as arbitrary tabulated functions of time. It is important that the interpolation process maintain mathematical continuity of at least target position and velocity since discontinuities in either of these parameters could have deleterious effects in a simulation. A typical example is the effect on simulations containing recursive target tracking filters in a digital tracking signal processor. A further requirement of the trajectory generation procedure is that it be computationally efficient with respect to computer time usage. This is an aspect of particular importance in simulations based on Monte Carlo sampling since this type tends to consume large amounts of computer time and a relatively small time reduction in each sample run can produce significant overall savings.

This report describes a method of trajectory generation which is designed to satisfy the requirements outlined in the foregoing. It is based on fitting a series of cubic splines in a piecewise manner to a set of coordinates describing the position of the target body at discrete points in time. The use of cubic splines implies that a third order polynomial is fitted between the discrete time points, and that continuity of first and second derivatives is maintained at the breakpoints. In this way the technique ensures continuity of position, velocity, and acceleration. Calculation of the polynomial interpolation coefficients is performed offline to the simulation and then input and stored by the simulation program. At any point in the course of the simulation the target position, velocity, and acceleration are available by the evaluation of third, second, and first order polynomials respectively, one set for each trajectory component.

This report includes a description of a computer program which performs the offline calculation of interpolation coefficients with the added facility that the input target trajectory may be specified in
terms of position, acceleration, or flight path angles at discrete
time points. Target position data input is a particularly useful option
when test range measured positions in an actual trajectory are required
to be input; accelerations are required to be specified in a target-
fixed frame based on the target velocity vector direction.

In addition to the target trajectory in terms of the kinematic
parameters, it is often necessary to know the orientation of the
target. This is particularly the case when detailed radar models are
used which depend on target aspect angles relative to the radar
antenna. The computer program contains an option to calculate the
interpolation coefficients for three Euler angles in the same manner
as for the trajectory position. The Euler angle representation
includes the effect of target aerodynamic angle of attack, provided
the requisite input data concerning target lift curve and wing loading
are supplied.

II. TRAJECTORY REPRESENTATION

Output trajectories, expressed in terms of spline interpola-
tion coefficients as functions of time, are defined relative to the
Cartesian axes of an inertial frame which has its origin at a point
on the earth's surface. Input data to the trajectory calculation are
expressed optionally in one of three forms, two of which employ a
reference frame defined by the body velocity vector. The various
reference frames are described in the following paragraphs.

A. Input Reference Frame

This is an orthogonal, right-handed Cartesian frame
with the origin at an arbitrary reference point on the earth's surface.
The X and Y axes lie in the plane of the local horizontal, and the Z
axis is along the downward vertical. Input data to the trajectory
calculation which are referenced to an inertial frame use this frame.
Those input data included in this classification are the position
coordinates in option 1 and the initial position and velocity components
of options 2 and 3 (see Section IV for descriptions of these options).

B. Output Inertial Frame

The output inertial frame is also an orthogonal, right-
handed Cartesian system and is related to the input inertial frame,
in general, by a translation and a rotation. The translation represents
the displacement vector \( \mathbf{r}_0 \) of the output frame origin relative to the
input frame, and the rotation is expressed in terms of three Euler
angles \( \psi_0, \theta_0, \phi_0 \) through which the input frame rotates in order to
align itself with the output frame. The relationship of the two frames is shown in Figure 1, and the angular rotations in going from the input frame to the output frame are shown in Figure 2. In mathematical terms the transformation equation is

\[ \mathbf{r}_{\text{out}} = [T]_{01} (\mathbf{r}_{\text{in}} - \mathbf{r}_0) \]  

where \([T]_{01}\) is the matrix of direction cosines of the output frame relative to the input frame, and \(\mathbf{r}_{\text{in}}, \mathbf{r}_{\text{out}}\) are vectors expressed respectively relative to the input and output frames. Expansion of \([T]_{01}\) into its components gives

\[ T_{11} = \cos \psi \cos \theta \]  
\[ T_{21} = \cos \psi \sin \theta \sin \phi - \sin \psi \cos \phi \]  
\[ T_{31} = \cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi \]  
\[ T_{12} = \sin \psi \cos \theta \]  
\[ T_{22} = \cos \psi \cos \phi + \sin \psi \sin \theta \sin \phi \]  
\[ T_{32} = \sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi \]  
\[ T_{13} = -\sin \theta \]  
\[ T_{23} = \cos \theta \sin \phi \]  
\[ T_{33} = \cos \theta \cos \phi \]  

C. Vehicle Velocity Frame

The vehicle velocity frame is a noninertial Cartesian axis system in which the positive \(X_v\) axis is directed along the body velocity vector. The \(Y_v\) axis is directed horizontally to the right when looking from the origin along \(X_v\), and the \(Z_v\) axis is normal to the \(X_v - Y_v\) plane and thus lies in a vertical plane through the velocity vector.

In relation to the input inertial frame, the velocity frame is obtained by a rotation \(\psi_T\) about the input inertial frame \(Z\) axis, and a rotation \(\theta_T\) about the new \(Y\) axis position. Origin of the velocity frame lies at the CG of the body. Figure 3 illustrates the velocity frame and its relationship to the input inertial frame.
Figure 1. Input and output inertial frame.

Figure 2. Input-output frame angular rotations.
Figure 3. Orientation of velocity frame relative to input frame.

Trajectory specification options 2 and 3 express the body acceleration in this frame. Acceleration components determine the rotational rate components $\psi_T$ and $\theta_T$ which are integrated to give the orientation of the body velocity frame to the input inertial frame. Let the acceleration components in the velocity frame be $A_{xv}$, $A_{yv}$, $A_{zv}$, and let the vehicle velocity be $V$, then

\[ \dot{V} = A_{xv} \]  
\[ \dot{\psi}_T = \frac{A_{yv}}{V \cos \theta_T} \]
\[
\theta_T = -A \frac{z}{v},
\]

from which velocity components in the input inertial frame are

\[
V_{XI} = V \cos \theta_T \cos \psi_T
\]

\[
V_{YI} = V \cos \theta_T \sin \psi_T
\]

\[
V_{ZI} = -V \sin \theta_T
\]

These velocity components are integrated to yield the position of the body as a function of time from the given initial position and velocity. Initial position and velocity components are expressed relative to the input inertial frame. If the initial velocity components relative to the input frame are \(V_{XIC}, V_{YIC}, V_{ZIC}\), then

\[
\psi_{TIC} = \tan^{-1} \left( \frac{V_{YIC}}{V_{XIC}} \right)
\]

and

\[
\theta_{TIC} = \sin^{-1} \left( \frac{-V_{ZIC}}{\sqrt{V_{XIC}^2 + V_{YIC}^2 + V_{ZIC}^2}} \right)
\]

III. CUBIC SPLINE INTERPOLATION

This section contains a brief outline of the underlying theory of curve fitting by means of cubic splines. A more detailed analysis is given by Schultz.¹

The fitting of smooth curves to function values given at discrete intervals of an independent variable has received considerable attention in the past. A well-known procedure is that due to Lagrange in which an \( N^{th} \) order polynomial is fitted to \( N + 1 \) function values. However, it is also well-known that for values of \( N \) of moderate size and larger, the procedure can diverge and produce highly unsatisfactory results.\(^2\)

The modern approach to curve fitting has received much impetus from finite element techniques and has led to the use of piecewise polynomial fitting with constraints on the derivatives at the end points of each interval. A requirement for continuity of first derivatives at the interval end points yields a set of interpolating polynomials which are "local" in the sense that each polynomial depends only on the independent variable and function values at the end points of each interval. As such, it is an extension of the method of piecewise linear interpolation which is in common use. This technique, using cubic polynomials, is described as follows. It should be noted that although cubic polynomials are often used, the procedure can readily be generalized to higher order polynomials.

Consider a set of \( N + 2 \) real numbers \( x_0 < x_1 < x_2 \ldots \ldots < x_N + 1 \) and an associated set of function values \( \{f_i, f'_i\} \) \( i = 0, 1, 2, \ldots, N + 1 \) where the prime superscript implies differentiation with respect to \( x \). Let \( p(x) \) be a cubic polynomial such that

\[
p(x_i) = f_i, \quad p(x_i + 1) = f'_i + 1,
\]

\[
dp(x_i)/dx = f'_i \quad \text{and} \quad dp(x_i + 1)/dx = f'_i + 1.
\]

Also let the interpolated value of \( f \) be \( F \); then \( F \) may be expressed as

\[
F = \sum_{i=0}^{N+1} \{f_i h_i(x) + f'_i h'_i(x)\}
\]

where \( h_i(x) \) and \( h'_i(x) \) are "basis" functions of \( x \) with the following properties:

\[
h_i(x_j) = \delta_{ij}, \quad 0 \leq i, j \leq N + 1
\]

\( h_i'(x_j) = 0 \) \hspace{2cm} (12)

\( \frac{d}{dx} h_i'(x_j) = \delta_{ij} \) \hspace{2cm} (13)

\( \delta_{ij} \) is the Kronecker delta function. For \( i = 1, 2, 3, \ldots, N \)

\[
h_i(x) = \begin{cases} 
\frac{(x - x_{i-1})^2}{(x_i - x_{i-1})^2} & \left\{ \begin{array}{l}
3 - \frac{2(x - x_{i-1})}{(x_i - x_{i-1})^2} \\
x_{i-1} \leq x \leq x_i 
\end{array} \right.
\end{cases}
\]

\( x \leq x_{i-1}, x \geq x_{i+1} \) \hspace{2cm} (14)

\[
h_i'(x) = \begin{cases} 
\frac{(x - x_i)(x - x_{i-1})^2}{(x_i - x_{i-1})^2} & \left\{ \begin{array}{l}
3 - \frac{2(x - x_i)}{(x_i - x_{i-1})^2} \\
x_{i-1} \leq x \leq x_i 
\end{array} \right.
\end{cases}
\]

\( x \leq x_{i-1}, x \geq x_{i+1} \) \hspace{2cm} (15)

For \( i = 0 \)

\[
h_0(x) = \begin{cases} 
\frac{(x - x_0)^2}{(x_1 - x_0)^2} & \left\{ \begin{array}{l}
2(x - x_0) \\
(x_1 - x_0)^2 - 3 \\
+ 1 \\
x_0 \leq x \leq x_1 
\end{array} \right.
\end{cases}
\]

\( x_1 \leq x \leq x_N + 1 \) \hspace{2cm} (16)
\[
h_0^i(x) = \begin{cases} 
\frac{(x - x_0)(x_1 - x)^2}{(x_1 - x_0)^2} & x_0 \leq x \leq x_1 \\
0 & x_1 \leq x \leq x_{N+1}
\end{cases}
\quad (17)
\]

and for \(i = N+1\)

\[
h_{N+1}^i(x) = \begin{cases} 
\frac{(x - x_N)^2}{(x_{N+1} - x_N)^2} \left(3 - \frac{2(x - x_N)}{x_{N+1} - x_N}\right) & x_N \leq x \leq x_{N+1} \\
0 & 0 \leq x \leq x_N
\end{cases}
\quad (18)
\]

Expanding Equation (10) for the case of \(x_i \leq x \leq x_i + 1\) gives

\[
F = f_1 + \frac{(f_{i+1} - f_i)(x - x_i)^2}{(x + 1 - x_i)^2} \left\{3 - \frac{2(x - x_i)}{x_{i+1} - x_i}\right\} \\
+ \frac{(x - x_i)}{(x + 1 - x_i)^2} \left\{f_{i+1}(x + 1 - x)^2 + f_i(x - x_i)(x - x_{i+1})\right\}.
\quad (20)
\]

If a piecewise independent variable is written \(\Delta x\), where \(\Delta x = x - x_i\), then a piecewise cubic polynomial may be defined for \(x_i \leq x \leq x_i + 1\) as
\[ p(\Delta x) = a_0 + a_1 \Delta x + a_2 \Delta x^2 + a_3 \Delta x^3, \] (21)

and the coefficients are obtained from Equation (20) as

\[ a_0 = f_1 \] (22)

\[ a_1 = f_1' \] (23)

\[ a_2 = \frac{3(f_1 + 1 - f_1)}{(x_1 + 1 - x_1)^2} - \frac{2f_1' + f_1 + 1}{x_1 + 1 - x_1} \] (24)

\[ a_3 = \frac{-2(f_1 + 1 - f_1)}{(x_1 + 1 - x_1)^3} + \frac{f_1' + f_1 + 1}{(x_1 + 1 - x_1)^2}. \] (25)

In many cases the derivatives \( f_1' \) of the function \( f \) are not available and it becomes necessary to approximate them from values of \( f_1 \).

A method of performing this approximation is to use local cubic Lagrange interpolation polynomials to fit a curve through groups of four points and obtain \( f_1' \) from these polynomials. The procedure is described as follows. Let the Lagrange polynomials be \( r_k(x) \) defined by

\[ r_k(x) = \sum_{i=0}^{3} \eta_{k,i}(x) f_{k+i} \] (for \( N \geq 2 \)) (26)

where

\[ \eta_{k,i}(x) = \frac{\prod_{j=0}^{3} (x - x_{k+j})}{\prod_{j=0, j \neq i}^{3} (x_{k+1} - x_{k+j})} \] (27)
which interpolates \( f_k + 1 \) for \( i = 0, 1, 2, 3 \). Derivatives of \( f \) may then be approximated as follows:

\[
 f'_i = \frac{df(x_i)}{dx} = \begin{cases} 
 \frac{d}{dx} \left( r_1(x_i) \right) & i = 0 \\
 \frac{d}{dx} \left( r_{i-1}(x_i) \right) & i = 1 \\
 \frac{1}{2} \left\{ \frac{d}{dx} \left( r_{i-2}(x_i) \right) + \frac{d}{dx} \left( r_{i-1}(x_i) \right) \right\} & 2 \leq i \leq N-1 \\
 \frac{d}{dx} \left( r_{i-2}(x_i) \right) & i = N \\
 \frac{d}{dx} \left( r_{i-3}(x_i) \right) & i = N+1 
\end{cases}
\] (28)

The term "spline interpolation" implies the fitting of polynomials in a piecewise manner as described but with the added constraint that second and higher derivatives of the function are given continuity at the interval end points, thus simulating the effect of forcing a thin, flexible spline to pass through the function points. The physical process of clamping a spline to a certain number of function values is one that is commonly used in naval architecture and ship design. For the purposes of this report, the spline functions considered will be restricted to cubic polynomials and only second derivatives will be equated at interval end points. The general case of \( k \)th order polynomials used is often referred to as B-spline fitting.

To illustrate the cubic spline interpolation process, consider again the set of function values \( f_i \) corresponding to a set of independent variable breakpoints \( x_i, i = 0, 1, 2, \ldots, N+1 \). Let \( p(x) \) be the cubic interpolating polynomial for the interval \( x_{i-1} \leq x \leq x_i \) and let \( q(x) \) be the cubic interpolating polynomial for the interval \( x_i \leq x \leq x_{i+1} \) with the following properties for \( 1 \leq i \leq N \):

\[
p(x_i) = q(x_i) = f_i
\] (29)
\[
\frac{d}{dx} p(x) = \frac{d}{dx} q(x) = f'
\]

Using Equation (20), the polynomials \( p(x) \) and \( q(x) \) may be expanded about the point \( x_i \) to give

\[
p(x) = p(x_i) + Dp(x_i)(x - x_i) + \left\{ \frac{3}{\Delta x_i - 1} [p(x_i + 1) - p(x_i)] \right\}
\]

\[
+ Dp(x_i + 1) + 2Dp(x_i) \frac{(x - x_i)^2}{\Delta x_i - 1}
\]

\[
+ \left\{ \frac{2}{\Delta x_i - 1} [p(x_i + 1) - p(x_i)] + Dp(x_i + 1) \right\}
\]

\[
+ Dp(x_i + 1) + 2Dp(x_i) \frac{(x - x_i)^3}{\Delta x_i - 1}
\]

\[
q(x) = q(x_i) + Dq(x_i)(x - x_i) + \left\{ \frac{3}{\Delta x_i} [q(x_i + 1) - q(x_i)] \right\}
\]

\[
+ Dq(x_i + 1) + 2Dq(x_i) \frac{(x - x_i)^2}{\Delta x_i}
\]

\[
+ \left\{ \frac{2}{\Delta x_i} [q(x_i + 1) - q(x_i)] + Dq(x_i + 1) \right\}
\]

\[
+ 2Dq(x_i) \frac{(x - x_i)^3}{\Delta x_i}
\]

where

\[
D^n = \frac{d^n}{dx^n}
\]

\[
\Delta x_i - 1 = x_j - x_i - 1
\]

\[
\Delta x_i = x_i + 1 - x_i
\]
The requirement for continuity of second derivatives at the point $x_i$ is

$$D^2 p(x_i) = D^2 q(x_i) \quad (33)$$

and by analogy with the Taylor Series expansion about $x_i$, then

$$D^2 p(x_i) = \frac{2}{\Delta x_i} \left\{ \frac{3}{\Delta x_i} [p(x_i - 1) - p(x_i)] 
+ Dp(x_i - 1) + 2Dp(x_i) \right\} \quad (34)$$

$$D^2 q(x_i) = \frac{2}{\Delta x_i} \left\{ \frac{3}{\Delta x_i} [q(x_i + 1) - q(x_i)] 
+ Dq(x_i - 1) + 2Dq(x_i) \right\} \quad (35)$$

which leads to the requirement

$$\Delta x_i f'_i - 1 + 2f'_i (\Delta x_i + \Delta x_i - 1) + \Delta x_i - 1 f'_i + 1
= 3 \left\{ \frac{\Delta x_i - 1}{\Delta x_i} (f'_i + 1 - f_i) 
+ \frac{\Delta x_i}{\Delta x_i - 1} (f'_i - f_i - 1) \right\} \quad 1 \leq i \leq N \quad (36)$$

Equation (36) represents a set of $N$ linear equations in $f'_i$ which may be written in vector form as

$$[B_{ij}] f'_i = c \quad 1 \leq i, j \leq N \quad (37)$$

where the matrix $[B_{ij}]$ is given by
\[ B_{ij} = \begin{cases} 
2(\Delta x_i + \Delta_i - 1) & 1 \leq i = j \leq N \\
\Delta x_i & 1 \leq j = i - 1 \leq N - 1 \\
\Delta x_i - 1 & 2 \leq j = i + 1 \leq N \\
0 & \text{otherwise} 
\end{cases} \] (38)

and the vector \( \mathbf{c} \) is given by

\[ \mathbf{c}_i = \begin{cases} 
3 \left[ \frac{\Delta x_0}{\Delta x_0} \frac{\Delta f_1}{\Delta x_1} + \frac{\Delta x_1}{\Delta x_0} \frac{\Delta f_0}{\Delta x_0} \right] - \Delta x_i f_i' & i = 1 \\
3 \left[ \frac{\Delta x_i - 1}{\Delta x_i} \frac{\Delta f_{i+1}}{\Delta x_{i+1}} + \frac{\Delta x_{i+1}}{\Delta x_i - 1} \frac{\Delta f_i}{\Delta x_i - 1} \right] - \Delta x_{i+1} f_{i+1}' & 1 < i < N \\
3 \left[ \frac{\Delta x_{N-1}}{\Delta x_N} \frac{\Delta f_N}{\Delta x_N} + \frac{\Delta x_N}{\Delta x_{N-1}} \frac{\Delta f_{N-1}}{\Delta x_{N-1}} \right] - \Delta x_N f_N' & i = N 
\end{cases} \] (39)

where

\[ \Delta f_{i+1} = f_{i+1} - f_i \]

\[ \Delta f_i - 1 = f_i - f_{i-1} \]

As can be observed from definition Equation (38), the matrix \( B_{ij} \) is tridiagonal and it can be shown that the system of Equations (37) has a unique solution; in fact, Equations (37) can be readily solved by Gaussian elimination.

Interpolated function values are then obtained by substituting in Equation (10), yielding the interpolant \( F \) as

\[ F = \sum_{i=0}^{N} f_i h_i(x) + f_0' h_0'(x) + \sum_{i=1}^{N} f_{i+1}' h_i(x) + f_{N+1}' h_{N+1}'(x) \] (40)

\(^3\)Schultz, loc. cit.
where \( f_1' \) (\( i = 1, 2, \ldots, N \)) have been obtained from the solution of Equations (37). Note that \( F \) is no longer a "local" interpolating function since all \( f_1 \) are used in obtaining \( f_1' \). Piecewise polynomial interpolation coefficients are given by Equations (22) through (25).

It remains only to choose a method of calculating the extremity derivatives \( f_0' \) and \( f_{N+1}' \). These derivatives may be obtained from local cubic Lagrange polynomials at \( k = 0 \) and \( k = N - 2 \) as given in Equations (17) and (18), i.e.,

\[
\begin{align*}
r_0(x) &= \sum_{i=0}^{3} \eta_{0,i}(x) \cdot f_1 \\ r_{N-2}(x) &= \sum_{i=0}^{3} \eta_{N-2,i}(x) \cdot f_{N-2+i}
\end{align*}
\]

and the derivatives \( f_0' \) and \( f_{N+1}' \) are approximated as

\[
\begin{align*}
f_0' &= D r_0( x_0 ) \\ f_{N+1}' &= D r_{N-2}( x_{N+1} )
\end{align*}
\]

An example of cubic spline curve fitting is contained in Figure 4, in which function values are indicated at the appropriate breakpoints in \( x \), and the full line is that obtained by the spline interpolation process.

IV. TRAJECTORY GENERATION

The input data to the trajectory generation program may be specified in one of three ways, chosen at the option of the user. These options are described in the following paragraphs. All three options require that the origin translation and frame rotation of the output frame relative to the input frame be specified. For options 2 and 3, initial body position and velocity relative to the input frame must be included in the data.
Figure 4. Cubic spline interpolation example.
A. Option 1 - Position Input

For this case the input data consists of sets of values of body position coordinates relative to the input reference frame and an associated value of time. Data sets should cover the time span of the trajectory at intervals demanded by the rates of change of trajectory parameters.

B. Option 2 - Acceleration Input

Applied acceleration components in the body velocity frame are specified as arbitrary functions of time over the required time span of the trajectory. The acceleration functions are required to be specified in units of local gravitational acceleration, and the local gravity is related to sea level gravity (assuming a spherical earth) by

\[ g = g_0 \left( \frac{R_0}{R_0 + H} \right)^2 \]  

\[ H = \sqrt{X_I^2 + Y_I^2 + (R_0 - Z_I)^2 - R_0} \]  

where \( g_0 \) is mean sea-level gravity acceleration, \( R_0 \) is the mean radius of the earth, \( X_I, Y_I, Z_I \) are body coordinates relative to the input reference frame, and \( H \) is the body altitude above the earth's surface. Derivation of Equation (46) is readily apparent from Figure 5.

The body trajectory is obtained by integrating the kinematic equations to give position coordinates, as indicated by Equations (2) through (7). Acceleration functions are assumed to be continuous between the specified data points; linear interpolation in the functions is used to obtain intermediate acceleration values.

C. Option 3 - Flight Path Angle Input

For option 3 input data are specified by expressing flight path angles \( \psi_T \) and \( \theta_T \) as arbitrary functions of time, together with the longitudinal acceleration in the velocity frame as a function of time. The changes in flight path angles between consecutive pairs of time breakpoints are converted to equivalent accelerations and the body trajectory is obtained as in option 2. For this case, however, the acceleration is assumed to be constant between adjacent pairs of time breakpoints, and to change discontinuously at each breakpoint.
Figure 5. Vehicle altitude above earth's surface.

D. Trajectory Interpolation

Program output consists of sets of polynomial coefficients at discrete time points contained within the input data. The polynomial coefficients generate the vehicle position as a function of time in the following manner. Let $t$ be the current time and let the discrete time breakpoints be $t_0 < t_1 < t_2 \ldots < t_N$, and let $\Delta t = t - t_i$ for some $0 \leq i < N$ where $t_i < t < t_{i+1}$, then

$$X_0 = a_0 + a_1 \Delta t + a_2 \Delta t^2 + a_3 \Delta t^3 \quad (47)$$

where $X_0$ is the $X$ coordinate of vehicle position relative to the output reference frame and $a_0$, $a_1$, $a_2$, $a_3$ are the polynomial coefficients for the $X$ coordinate within the time interval $t_i$ to $t_{i+1}$. Coordinates
Y and Z are generated by similar sets of coefficients. Coefficients are calculated for time intervals from \( t_0 \) to \( t_N \); thus there are \( N \) sets of coefficients for the \( N + 1 \) breakpoints.

Velocity and acceleration components are generated by differentiation, i.e.,

\[
\begin{align*}
V_{x0} &= a_1 + 2a_2 \Delta t + 3a_3 \Delta t^2 \\
A_{x0} &= 2a_2 + 6a_3 \Delta t
\end{align*}
\] (48) (49)

From the above it can be seen that the interpolation process consists simply of determining \( i \) for a given value of \( t \) and evaluating Equations (47), (48), and (49) to obtain the vehicle trajectory. In simulation applications \( t \) is inevitably monotonically increasing, which makes the determination of \( i \) a simple matter and avoids extensive searching of the table of time breakpoints.

V. VEHICLE EULERIAN ANGLES

The Euler angles of the vehicle with respect to the output reference frame are expressed in the same form as the final output trajectory, i.e., as sets of spline interpolation coefficients at discrete intervals in time. An option in the calculation permits the inclusion of aerodynamic angles of attack under the assumption that the vehicle is air-supported by fixed wings and that all maneuvering turns are coordinated. That is, the body sideslip angle is always zero.

Euler angles of the vehicle with respect to the output reference frame are defined by three rotations required to align the output frame with a vehicle-fixed frame. The rotations are \( \psi_B \) about the \( Z_{out} \) axis, \( \theta_B \) about the resultant position of the \( Y_{out} \) axis, and \( \phi_B \) about the vehicle \( X \) axis.

The vehicle velocity frame relative to the output reference frame has angles \( \psi_{T0} \) and \( \theta_{T0} \) defined similarly to \( \psi_T \) and \( \theta_T \) for the input frame. However, \( \psi_{T0} \) and \( \theta_{T0} \) are calculated from velocity components relative to the output reference frame obtained by differentiating the cubic spline representation of the trajectory, as indicated in Section IV. If the vehicle velocity components relative to the output frame are \( V_{x0} \), \( V_{y0} \), \( V_{z0} \), then
The body acceleration, represented by the trajectory interpolation coefficients, is $A_0$ relative to the output frame. For the force balance of Figure 6, this must be transformed to the body velocity frame through the angles $\psi_{TO}$ and $\theta_{TO}$. The lateral acceleration terms in the velocity frame are thus

$$A_{yv0} = -A_{x0} \sin \psi_{TO} + A_{y0} \cos \psi_{TO}$$
Figure 6. Vehicle lateral force balance.

\[
A_{zv0} = A_{x0} \cos \psi_{T0} \sin \theta_{T0} + A_{y0} \sin \psi_{T0} \sin \theta_{T0} + A_{z0} \cos \theta_{T0}
\]

(57)

and the gravitational acceleration transformed to the body frame is, in the lateral directions

\[
g_{yv0} = -g_{x0} \sin \psi_{T0} + g_{y0} \cos \psi_{T0}
\]

(58)

\[
g_{zv0} = g_{x0} \cos \psi_{T0} \sin \theta_{T0} + g_{y0} \sin \psi_{T0} \sin \theta_{T0} + g_{z0} \cos \theta_{T0}
\]

(59)
where \( A_0 \), \( A_y \), \( A_z \) are components of \( A \) in the output frame. The vehicle lift force \( L \) must provide a component along \( Z_v \), which has a resultant acceleration of \( A_{zv0} \) and a horizontal component which has a resultant acceleration of \( A_{yv0} \) with \( L \) acting normal to the vehicle wingspan and velocity vector. Thus, the vehicle bank angle is given by

\[
\phi_{T0} = \tan^{-1}\left( \frac{A_{yv0} - g_{yv0}}{g_{zv0} - A_{zv0}} \right)
\]

and the lift force \( L \) is given by

\[
L = \frac{W}{g} \sqrt{(A_{yv0} - g_{yv0})^2 + (g_{zv0} - A_{zv0})^2},
\]

where \( W \) is the vehicle weight. Note that in Equation (60) if both numerator and denominator are zero, \( \phi_{T0} \) is undefined since this would be the case of the body falling freely under gravity. A further point to be carefully considered concerns the definition of the vehicle velocity frame. If \( \theta_0 \neq 0 \) or \( \phi_0 \neq 0 \), then the velocity frames obtained by rotating \( \psi_T, \theta_T \) from the input reference frame and \( \psi_{T0}, \theta_{T0} \) from the output reference frame will differ by an apparent roll angle, as can be deduced from Equation (58) in which the \( Y_v \) axis has a gravitational component acting along it. For this reason the acceleration components \( A_{yv0} \) and \( A_{zv0} \) are, in general, not equal respectively to \( A_y \) and \( A_z \) of Section II.

If it is assumed that the lift curve for the vehicle can be approximated by the form given in Figure 7 and is an odd function about \( \alpha = 0 \), the angle of attack can be approximated by first calculating the vehicle lift coefficient as

\[
C_L = \frac{W}{S} \frac{\left( A_{yv0} - g_{yv0} \right)^2 + \left( g_{zv0} - A_{zv0} \right)^2}{\frac{1}{2} \rho v^2}
\]

(62)
Figure 7. Idealized lift curve slope.

where $W/S$ is the vehicle wing loading, $\rho$ is the ambient air density, and $V$ is the vehicle total speed. The angle of attack is then

$$\alpha = \begin{cases} \frac{C_L}{\left(\frac{dC_L}{d\alpha}\right)} & \alpha \leq \alpha_1 \\ C_L - \alpha_1 \left(\frac{dC_L}{d\alpha}\right)_1 & \alpha \geq \alpha_1 \end{cases}$$

$\rho$ is a function of vehicle altitude $H$, where $H$ is given by Equation (46) in which $X_1$, $Y_1$, $Z_1$ are referenced to the input frame. Thus, for position coordinates referenced to the output frame, $H$ must be determined by a transformation of the trajectory coordinates to the input reference frame. Lift curve parameters $\alpha_1$, $(dC_L/d\alpha)_1$, and $(dC_L/d\alpha)_2$
are illustrated in Figure 7. Note that $\alpha$ as defined by Equation (63) is always positive. A set of rotations from the output reference frame to the vehicle fixed frame is $\psi'_0$, $\theta'_0$, $\phi'_0$, $\alpha'_0$ where

$$
\alpha'_0 = \begin{cases} 
\alpha & \text{if } (g_{zv0} - A_{zv0}) \geq 0 \\
-\alpha & \text{if } (g_{zv0} - A_{zv0}) < 0
\end{cases}
$$

These rotations may be expressed as a set of matrix products to form a transformation matrix of direction cosines which rotates the output reference frame to the vehicle fixed frame as follows:

$$
[T]_{V0} = [t_\alpha][t_\psi][t_\theta][t_\phi]
$$

where

$$
[t_\alpha] = \begin{bmatrix}
\cos \alpha_{T0} & 0 & -\sin \alpha_{T0} \\
0 & 1 & 0 \\
\sin \alpha_{T0} & 0 & \cos \alpha_{T0}
\end{bmatrix}
$$

$$
[t_\psi] = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \phi_{T0} & \sin \phi_{T0} \\
0 & -\sin \phi_{T0} & \cos \phi_{T0}
\end{bmatrix}
$$

$$
[t_\theta] = \begin{bmatrix}
\cos \theta_{T0} & 0 & -\sin \theta_{T0} \\
0 & 1 & 0 \\
\sin \theta_{T0} & 0 & \cos \theta_{T0}
\end{bmatrix}
$$

$$
[t_\phi] = \begin{bmatrix}
\cos \phi_{T0} & \sin \phi_{T0} & 0 \\
-\sin \phi_{T0} & \cos \phi_{T0} & 0 \\
0 & 0 & 1
\end{bmatrix}
$$
The transformation matrix \([T]_{O1} \) is equivalent to a matrix of direction cosines formed from the Euler angles \(\psi_B, \theta_B, \phi_B\). The direction cosine matrix contains elements of the same form as that given for the expansion of \([T]_{OI} \) (Paragraph II.B) with \(\psi_B, \theta_B, \phi_B\) substituted for \(\psi_0, \theta_0, \phi_0\). By inspection of the terms in the expansion of \([T]_{OI} \), the Euler angles are given by

\[
\psi_B = \tan^{-1} \left( \frac{t_{12}}{t_{11}} \right)
\]

(70)

\[
\theta_B = \sin^{-1} \left( -t_{13} \right)
\]

(71)

\[
\phi_B = \tan^{-1} \left( \frac{t_{23}}{t_{33}} \right)
\]

(72)

where \(t_{12}, t_{13}, t_{23}, t_{33}\) are elements of the matrix \([T]_{O1} \). Evaluating these terms from Equations (66) through (69), the vehicle Euler angles are

\[
\psi_B = \tan^{-1} \left( \frac{\cos \theta_{TO} \sin \psi_{TO} \cos \alpha_{TO} - \sin \theta_{TO} \cos \psi_{TO} \cos \phi_{TO} \sin \alpha_{TO}}{\cos \theta_{TO} \cos \psi_{TO} \cos \alpha_{TO} - \sin \psi_{TO} \sin \phi_{TO} \sin \alpha_{TO}} \right)
\]

(73)

\[
\theta_B = \sin^{-1} \left( \sin \theta_{TO} \cos \alpha_{TO} + \cos \theta_{TO} \cos \phi_{TO} \sin \alpha_{TO} \right)
\]

(74)

\[
\phi_B = \tan^{-1} \left( \frac{\cos \theta_{TO} \sin \phi_{TO}}{\cos \theta_{TO} \cos \phi_{TO} \cos \alpha_{TO} - \sin \theta_{TO} \sin \alpha_{TO}} \right)
\]

(75)

It should be noted that if the vehicle angle of attack is negligible, then Euler angles are easily deducible from the trajectory interpolation coefficients via the first and second derivatives of the interpolating polynomial and Equations (50), (51), and (60). In this case the calculation of Euler angle interpolation coefficients is unnecessary.

A further consideration in the calculation of polynomial interpolation coefficients for the Euler angles concerns the range of values taken by the Euler angles themselves. Equations (73) and (75) normally yield principal values of the angles, i.e., in the range ±π.
Thus, when the angles pass from a value close to $\pi$ to one close to $-\pi$ the spline fitting process will assume that the angle has passed through zero giving rise to incorrect results. To counteract this effect, the range of values taken by the angle must be extended by converting the principal values to 0 to $2\pi$ and permitting the angle to go beyond this range by adding or subtracting $2\pi$ when the angle crosses from the fourth to the first quadrant or from first to the fourth quadrant.

This calculation is described as follows:

$$
\psi_{BE} = \begin{cases} 
\psi_B + 2n\pi & \psi_B \geq 0 \\
(2\pi + \psi_B) + 2n\pi & \psi_B < 0 
\end{cases} 
$$

where $\psi_{BE}$ is the extended Euler angle and $n$ is determined by maintaining a record of the first/fourth quadrant crossings as $\psi_B$ varies with time. To illustrate this, let the index of discrete time points at which trajectory data are given be $i$ which takes values of $i = 1, 2, ..., k$ for $k$ breakpoints, then

$$
n = 0 \text{ for } i = 1 
$$

$$
\Delta\psi_{BE_i} = \psi_{BE_i} - \psi_{BE_{i-1}} \quad i = 2, 3, ..., k 
$$

$$
n = n + 1 \text{ when } \Delta\psi_{BE_i} < -\pi 
$$

$$
n = n - 1 \text{ when } \Delta\psi_{BE_i} > \pi 
$$

This process is applied to the calculation of $\psi_B$ since this Euler angle is most likely to require extension beyond principal values. It should be recognized that the Euler angle calculation contains limitations on the range of $\theta_B$ to $\pm\pi/2$ and $\phi_B$ to $\pm\pi$, and that $\phi_B$ is calculated by assuming that the vehicle makes coordinated turns.

VI. COMPUTER PROGRAM

A computer program has been written to generate the sets of spline interpolation coefficients which represent a vehicle trajectory and the corresponding Euler angles. The program is intended for use in an offline mode where the trajectory coefficients are punched on cards or written to some other peripheral device for use in a missile-target intercept simulation which requires a moving target.
All three options described in Section IV are included in the program. The user selects a desired option via the program's input data. The program is written in FORTRAN for a CDC 6600 series machine. A listing is contained in Appendix A.

A. Program Composition

The program consists of the main program which performs the input/output operations, calculates accelerations and angle of attack, performs coordinate frame transformation, and controls the overall operation and the following subroutines:

1) AVELIN - Calculates cubic spline interpolation coefficients for three functions of one independent variable.

2) RK4 - Performs Runge-Kutta fourth order integration of velocity frame rotational rates, longitudinal acceleration, and inertial frame velocities.

3) ATTAK - Interpolates in a trajectory using coefficients produced by AVELIN. First and second derivatives are also calculated plus angles $\psi_T^0$ and $\theta_T^0$ for the case of trajectory $X, Y, Z$ coordinates.

4) ATMOS - Calculates atmospheric density as a function of altitude.

5) GRVALT - Calculates local gravitational acceleration and vehicle altitude above the earth's surface as a function of position.

B. Input Data

Each trajectory generated requires one set of input data cards which contains the following elements, punched according to the given formats.

1. Title Cards. These are punched free field and are intended to contain a descriptive heading for the trajectory. The number of title cards is unrestricted; the last card must contain ENDT in columns 1 to 4 and blanks in columns 5 to 10.

2. Trajectory, Option and Breakpoint Numbers. This card contains three integers punched in 315 format. The first integer is a trajectory identification number and must be an integer in the range 1 to 5; the second number selects the input data option (Paragraph VI.A.5) and must be an integer in the range 1 to 3. The third integer is the number of points in time (breakpoints) at which input data are provided; the range of this number is 3 to 100.
3. **Euler Angle Indicator and Angle of Attack Data.**

This card contains up to eight entries punched in floating point format 8F10.0. The first entry is an indicator to determine whether vehicle Euler angle interpolation coefficients are required to be output — a zero value indicates that Euler angles are not required, in which case the next six parameters on this card are unused and may be set equal to zero; i.e., only \( \xi_0 \) is required. The remaining seven parameters have the following meaning:

\[
\frac{dC_L}{da} = \text{Lift curve slope (rad)} \\
\alpha_1 = \text{Expressed in degrees} \\
\frac{dC_L}{da} = \text{Lift curve slope (rad)} \\
C = \text{Wing loading in appropriate units (Paragraph VI.D.)} \\
\rho_{SL} = \text{Sea-level air density in appropriate units} \\
H_{NORM} = \text{Number of feet per unit of length in which the trajectory is expressed} \\
\xi_0 = \text{Sea-level gravitational acceleration in units consistent with the trajectory data.}
\]

4. **Output Reference Frame Transformation Data.** This card contains six quantities read in 6F10.0 format and represent the translation and rotation of the output reference frame. The data are \( \xi_0 \) as components \( X_0, Y_0, Z_0 \) in the same units as the trajectory and \( \psi_0, \theta_0, \phi_0 \) in degrees.

5. **Trajectory Input Data.** All data are read with a format of 8F10.0. Card contents depend on the option number in Paragraph VI.A.2 as follows:

(a) Option 1 — Each card contains two sets of data (except the last card which may contain only one set) containing values of time and associated values of \( X, Y, Z \) trajectory coordinates relative to the input reference frame. Time is required to be expressed in units of seconds and trajectory coordinates in units which are selectable by the user (Paragraph VI.D).
(b) Option 2 — The first card must contain the three position coordinates and three velocity components of the vehicle relative to the input frame, at the first time breakpoint contained in the subsequent data.

Subsequent cards contain two sets of data, as in option 1, but each set consists of time in seconds and acceleration in units of g along the vehicle velocity reference axes.

(c) Option 3 — Data for this option is similar to option 2. The first card contains an identical set of parameters, and subsequent cards contain sets of four items consisting of time, acceleration along the velocity frame X axis and angles $\psi_T$ and $\theta_T$ of the velocity frame relative to the input frame. Angles are required to be expressed in degrees.

Note that in all three options, the number of sets of data consisting of time and three associated parameters is given by the third integer of the first data card after the title. Example sets of input data cards are given in Appendix B.

C. Output of Results

Results are output by the program in two forms. Trajectory results, in which the piecewise interpolation coefficients and associated times are included, are output to logical unit 7 (TAPE7 in CDC 6600 SCOPE) preceded by the title read from the input data. This part of the output contains the following records, all in 80 column card image format:

1) Title records identical to the input title terminated by a record containing ENDT in positions 1 to 4 and blanks in positions 5 to 10.

2) A record containing the trajectory number in the fifth character position and the number of sets of interpolation coefficients in the ninth and tenth character positions. This latter number is always one less than the number of time breakpoints contained in the input data.

3) Interpolation coefficients and their associated time breakpoint in the order $a_0$, $a_1$, $a_2$, $a_3$, $b_0$, $b_1$, $b_2$, $b_3$, $c_0$, $c_1$, $c_2$, $c_3$, $t$ where $a_i$, $b_i$, $c_i$, $1 = 1, 2, 3$ are respectively X, Y, Z coordinate coefficients. Each record containing these data has the trajectory number and the record sequence number contained in the first five positions of the record in the format 12, 13. The remainder of the record contains interpolation data in the format 7E15.7 output as a string starting with the first time breakpoint set.
4) For the case where Euler angle interpolation coefficients are required, these results are output in identical format to 3) in the foregoing. Sequence numbers are reset to commence with 1 for this set.

The second form of the output consists of printed results which includes the input data suitably annotated, the interpolation coefficients, and the vehicle trajectory at 0.25-sec intervals over the input time span. The printed trajectory is obtained by use of the piecewise interpolation coefficients. In addition, if the Euler angle option has been selected, the Euler angle and angle of attack input data (generated within the program), interpolation coefficients and interpolated Euler angles at 0.25-sec intervals are printed. Additionally, the resultant lateral accelerations (along velocity frame Y and Z axes) in terms of g are printed with the interpolated trajectory position and velocity data.

Sets of example printed results are given in Appendix B.

D. Dimensions and Units

The program is designed to permit the user to select the physical dimensions and units of the final output trajectory data. For this purpose the input data must be consistent within the desired units system. Units must be chosen for the following input data:

1) Option 1 position data.
2) Options 2 and 3 gravitational acceleration and initial position and velocity.
3) For the angle of attack option: Vehicle wing loading, sea level air density, altitude normalizing parameter $H_{\text{NORM}}$.
4) Output frame origin shift relative to input frame.

Table 1 contains the units of each of the above for the British and SI systems.

E. Error Messages

The following error message may occur followed by program termination:

TRAJECTORY NUMBER OUT OF RANGE XXX OR TOO MANY SEGMENTS XXX

where XXX are respectively the trajectory number and number of breakpoints read from the input data. This message appears when the trajectory number is greater than 5 or less than 1, or when the number of breakpoints exceeds 100.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>British</th>
<th>SI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle position</td>
<td>ft</td>
<td>m</td>
</tr>
<tr>
<td>Gravitational acceleration</td>
<td>32.17 ft/sec²</td>
<td>9.807 m/sec²</td>
</tr>
<tr>
<td>Velocity</td>
<td>ft/sec</td>
<td>m/sec</td>
</tr>
<tr>
<td>Vehicle wing loading</td>
<td>1 lb/ft²</td>
<td>kg (weight)/m²</td>
</tr>
<tr>
<td>Sea-level air density</td>
<td>0.002378 slugs/ft³</td>
<td>0.1244 kg (mass)/m³</td>
</tr>
<tr>
<td>Altitude normalizer H NORM</td>
<td>1.0 ft/ft</td>
<td>3.280843 ft/m</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>-----------------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>$A_0$</td>
<td>Acceleration vector relative to the output reference frame</td>
<td></td>
</tr>
<tr>
<td>$A_{x0}, A_{y0}, A_{z0}$</td>
<td>Components of $A_0$ along the output reference frame axes directions</td>
<td></td>
</tr>
<tr>
<td>$A_{xv}, A_{yv}, A_{zv}$</td>
<td>Components of vehicle acceleration along the vehicle velocity frame axes directions (velocity frame from the input frame)</td>
<td></td>
</tr>
<tr>
<td>$A_{xv0}, A_{yv0}, A_{zv0}$</td>
<td>Components of vehicle acceleration along the vehicle velocity frame axes directions (velocity frame from the output frame)</td>
<td></td>
</tr>
<tr>
<td>$a_0, a_1, a_2, a_3$</td>
<td>Coefficients of the piecewise cubic polynomials</td>
<td></td>
</tr>
<tr>
<td>$[B_{ij}]$</td>
<td>Tridiagonal matrix of independent variable breakpoint intervals for calculation of cubic spline coefficients [Equation (37)]</td>
<td></td>
</tr>
<tr>
<td>$c$</td>
<td>Vector function of independent variable intervals and dependent function values required for calculation of cubic spline coefficients [Equations (37) and (39)]</td>
<td></td>
</tr>
<tr>
<td>$C_L$</td>
<td>Vehicle lift coefficient</td>
<td></td>
</tr>
<tr>
<td>$D$</td>
<td>Operator representing $d/dx$</td>
<td></td>
</tr>
<tr>
<td>$F$</td>
<td>Interpolated function representing $f$ the true function</td>
<td></td>
</tr>
<tr>
<td>$f_i$</td>
<td>Function values, to which the piecewise splines are fitted, at the $i$th independent variable breakpoint</td>
<td></td>
</tr>
<tr>
<td>$f_i'$</td>
<td>$Df_i$</td>
<td></td>
</tr>
<tr>
<td>$g$</td>
<td>Local gravitational acceleration (magnitude)</td>
<td></td>
</tr>
<tr>
<td>$g_0$</td>
<td>Sea-level gravitational acceleration (magnitude)</td>
<td></td>
</tr>
<tr>
<td>$g_0$</td>
<td>Gravitational acceleration vector in the output reference frame</td>
<td></td>
</tr>
<tr>
<td>$g_{x0}, g_{y0}, g_{z0}$</td>
<td>Components of gravitational acceleration along the output frame axes directions</td>
<td></td>
</tr>
</tbody>
</table>
$g_{yv0}, g_{zv0}$ Components of gravitational acceleration along the velocity frame axes directions (velocity frame from the output frame)

$H$ Vehicle altitude above the earth's surface

$H_{\text{NORM}}$ Units parameter: number of feet per linear unit of the trajectory data

$h_i(x), h'_i(x)$ Cubic polynomial piecewise interpolation basis functions

$i, j, k$ Integer indices

$L$ Vehicle aerodynamic lift

$N$ The number of independent variable breakpoints over the interpolated range is $N + 2$

$n$ Integer index

$p(x)$ Piecewise cubic polynomial

$q(x)$ Piecewise cubic polynomial

$R_0$ Radius of the earth

$r_{\text{in}}$ Position vector relative to the input reference frame

$r_k(x)$ Piecewise cubic polynomial

$r_{0}$ Translation of the output frame origin relative to the input frame

$r_{\text{out}}$ Position vector relative to the output reference frame

$S$ Vehicle wing area

$[T]_{0i}$ Direction cosine matrix of the output reference frame relative to the input reference frame

$[T]_{v0}$ Matrix of direction cosines between the output reference frame and the vehicle velocity frame

$t$ Time

$[t_\alpha], [t_\phi], [t_\theta], [t_\psi]$ Component transformation matrices which form $[T]_{v0}$
$t_i$  
Discrete breakpoints in time at which trajectory data are given

$V$  
Magnitude of vehicle velocity

$V_{x1}, V_{y1}, V_{z1}$  
Velocity components along the input reference frame axes directions

$V_{xIC}, V_{yIC}, V_{zIC}$  
Initial condition velocity components in the input reference frame

$V_{x0}, V_{y0}, V_{z0}$  
Velocity components along the output reference frame axes directions

$W$  
Vehicle weight

$X_{in}, Y_{in}, Z_{in}$  
Input reference frame axes

$X_1, Y_1, Z_1$  
Coordinates relative to the input reference frame

$X_0, Y_0, Z_0$  
Components of $E_0$ relative to the input frame

$X_{out}, Y_{out}, Z_{out}$  
Output reference frame axes

$X_v, Y_v, Z_v$  
Velocity frame axes

$X', Y', Z'$  
Intermediate axes between the input and output reference frames

$x$  
Independent variable of the exact and interpolated functions $f$ and $F$

$x_i$  
Discrete values of $x$ representing breakpoints for the interpolation process

$\alpha$  
Aerodynamic angle of attack (magnitude)

$\alpha_i$  
Angle of attack at which idealized lift curve slope changes

$\alpha_{i0}$  
Aerodynamic angle of attack with appropriate sign

$\Delta f_i$  
Difference between successive pairs of $f_i$

$\Delta x_i$  
Independent variable breakpoint intervals

$\Delta x$  
Piecewise independent variable, $x - x_i$
$\Delta \psi_B$  
Increment in vehicle Euler angle $\psi_B$ between the $i$th and $(i - 1)$th breakpoints

$\Delta t$  
Piecewise time variable for trajectory interpolation

$\delta_{ij}$  
Kronecker delta function (0 when $i \neq j$, 1 when $i = j$)

$\eta_{k,i}(\chi)$  
Lagrange interpolating polynomial basis functions

$\psi_B', \theta_B', \phi_B$  
Vehicle Euler angles relative to the output reference frame

$\psi_0', \theta_0', \phi_0$  
Euler angles of the output reference frame relative to the input reference frame

$\psi_T', \theta_T$  
Euler angles the velocity frame relative to the input reference frame

$\psi_{TO}', \theta_{TO}', \phi_{TO}$  
Euler angles of the velocity frame relative to the output reference frame (but with $\phi_{TO}$ defined by the vehicle bank angle)

$\psi_{TIC}', \theta_{TIC}$  
Initial values of $\psi_T$ and $\theta_T$

$\psi_{BE}$  
Definition of $\psi_B$ extended beyond principal values to allow first/fourth quadrant crossings to be continuous

$\rho$  
Ambient atmospheric density

$\rho_{SL}$  
Sea-level atmospheric density
Appendix A. COMPUTER PROGRAM LISTING
THE PROGRAM GENERATES TRAJECTORIES IN THE FORM OF PIECEWISE SPLINE INTERPOLATION COEFFICIENTS AT GIVEN BREAKPOINTS IN TIME.

TRAJECTORIES ARE OUTPUT IN THE FORM OF SETS OF CUBIC SPLINE INTERPOLATION COEFFICIENTS REPRESENTING X, Y, Z COMPONENTS OF A TRAJECTORY TOGETHER WITH THE ASSOCIATED TIME. THERE ARE 13 TERMS FOR EACH TRAJECTORY POINT.

TARGET INPUT TRAJECTORIES MAY BE SPECIFIED IN TERMS OF X, Y, AND Z COMPONENTS OR AS ACCELERATION COMPONENTS - LONGITUDINAL, NORMAL AND RADIAL - AS FUNCTIONS OF TIME, FOR THE VARIOUS INPUT OPTIONS SEE THE PROGRAM DOCUMENTATION.

PIECEWISE SPLINE REPRESENTATION OF EULER ANGLES IS AVAILABLE AS AN OPTION IN WHICH CASE ANGLES OF ATTACK MAY BE INCLUDED. EULER ANGLES ARE CALCULATED ASSUMING COORDINATED TURNS.

THE OUTPUT FRAME MAY BE TRANSFORMED BY TRANSLATION AND ROTATION, RELATIVE TO THE INPUT DATA REFERENCE FRAME.

OUTPUT OF THE INTERPOLATION COEFFICIENTS IS TO UNIT 7 IN CARD IMAGE FORMAT (12,13,5,13,8) WHERE THE 5 INTEGERS ON EACH CARD ARE: TRAJECTORY NUMBER (1 TO 5) AND A SEQUENCE NUMBER. COEFFICIENTS IN THE ORDER AC, A1, A2, ..., AT AT EACH POINT.


*** INTERPOLATION STATEMENT FUNCTION

FUNCTION ACC, TINT, IJ = ACC(IJ) + TINT*(ACC(IJ+1) - ACC(IJ))

*** READ AND OUTPUT TRAJECTORY TITLE.

PAGE = 1
WRITE (6,97) TITLE
PAGE = PAGE + 1
LINES = 1
NPRINT = 1
10 READ (3,90) LABEL
IF (LINES.GT.10) GO TO 100
WRITE (3,90) LABEL
LINES = LINES + 1
WRITE (7,90) LABEL
GO TO 10

END
DO 5 I=1,99
   ITraj(I,1) = -1.56
C *** READ TRAJECTORY NUMBER AND TRAJECTORY GENERATOR OPTION AND
C *** NUMBER OF TRAJECTORY POINTS (UP TO A MAXIMUM OF 21)
C
   READ (8,191) Itraj,Opt,Nseg
   WRITE (6,990) Itraj, Opt, Nseg
   Lines = Lines+3
   IF (Itraj.LE.5 .AND. Itraj.GT.0 .AND. Nseg.LE.100) GO TO 21
   WRITE (6,971) Itraj, Opt, Nseg
   STC
C *** READ ANGLE OF ATTACK DATA
   READ (8,990) Alpha1,Slope1,Alpha2,Slope2,Pos,Ang,Norm,Grad
   IF (Ang.NE.0) WRITE (6,950) Slope1, Alpha1, Slope2, Pos, Ang, Norm
   IF (Ang.NE.0) Lines = Lines+4
   Alpha1 = Alpha1/Rad
C *** READ FRAME TRANSFORMATION DATA FOR TRANSFORMING TO THE OUTPUT
C *** TRAJECTORY FRAME. TRANSFORMATION PARAMETERS ARE TRANSLATION AND
C *** ROTATION COMPONENTS, THE LATTER IN DEGREES, THE FORMER IN
C *** CONSISTENTLY APPROPRIATE UNITS.
C
   READ (8,340) Xyz0, Psig, Theta, Phi
   WRITE (6,945) Xyz0, Psig, Theta, Phi
   WRITE (6,995)
   Lines = Lines+7
   GO TO (30,52,50) Opt
C *** TRAJECTORY INPUT AS A SERIES OF X, Y, Z COORDINATES
C
   Ncards = Nseg/2
   IF (Ncards.NE.0) Lines = Lines+1
   Go To 41,Ncards
   J = 2*(I-1)+1
   READ (8,340) Time(J), XYZ(1,J), XYZ(2,J), XYZ(3,J),
   1       Time(J+1), XYZ(1,J+1), XYZ(2,J+1), XYZ(3,J+1)
   WRITE (8,340) Time(J), XYZ(1,J), XYZ(2,J), XYZ(3,J),
   1       Time(J+1), XYZ(1,J+1), XYZ(2,J+1), XYZ(3,J+1)
   Lines = Lines+2
   CONTINUE
   GO TO 177
C *** OPTION 2: LONGITUDINAL, NORMAL AND HORIZONTAL ACCELERATIONS (IN
C *** G-S) ARE INPUT FOR EACH OF NSEG FLIGHT SEGMENTS. THE ACCELERATIONS
C *** ARE DEFINED IN A TARGET FRAME WHOSE AX-AXIS IS THE TARGET VELOCITY
C *** AND WHICH DOES NOT ROLL WITH THE TARGET. ACCELERATIONS ARE
C *** INTEGRATED IN AN INERTIAL FRAME TO YIELD THE TRAJECTORY OF
C *** POSITION DATA TO WHICH THE CUBIC SPLINE IS FITTED.
C *** FIRST READ TARGET INITIAL POSITION AND VELOCITY IN THE INERTIAL
C *** FRAME
C
   READ (8,340) Rto, Vto
C *** NEXT THE SETS OF ACCELERATION COMPONENTS AND ASSOCIATED TIMES
C
NCARDS = NSEG/2
IF (NCARDS GT 2 .AND. NSEG) NCARDS = NCARDS+1
DO FC = 1, NCARDS
J = 2*(I-1)+1
READ (1, 94) TIME(J), VOL(J), VDR(J), VDN(J),
1 TIML(J+1), VOL(J+1), VDR(J+1), VDN(J+1)
WRITE (8, 93) TIME(J), VOL(J), VDR(J), VDN(J),
1 TIML(J+1), VOL(J+1), VDR(J+1), VDN(J+1)
LINES = LINES+2
CONTINUE
C *** CONTINUE IF IOPT.EQ.3GO TO 90
C *** CALCULATE TARGET INITIAL CONDITIONS
C
TEMP1 = VT0(1)*VT0(1)+VTG(2)*VTG(2)
VT = SQRT(TEMP1)*VT0(1)
PS(T) = ATAN2(VTG(2), VTG(1))
THE = ATAN2(-VT3, SQRT(TEMP1))
CONTINUE
XYZ(1,1) = PT0(1)
XYZ(2,1) = RT0(2)
XYZ(3,1) = RT0(3)
C *** DIVIDE EACH TIME SEGMENT INTO 10 STEPS FOR INTEGRATION AND
C *** CALCULATE TARGET POSITION BY RK-4 NUMERICAL INTEGRATION.
C
JUP = NSEG-1
DO AC = 1, JUP
DELT = TIME(J+1)-TIME(J)
UT = DELT/10.
T = TIME(J)
CALL G=VALT(GG,RT3,HNORM,ALT)
VOLNG = G*VOL(T)
VDRNG = G*VDR(T)
VRADL = G*VRAD(T)
DO 7 AC = 1,10
IF (IOPT.EQ.3) GO TO 70
TINT = (T-TIME(J-1))/DELT
VOLNG = G*FIN(VOL,TINT,1)
VRADL = G*FIN(VDR,TINT,1)
VDRNG = G*FIN(VDN,TINT,1)
7 CALL FX4(VOLNG,VRADL,VRANG,T,DT)
XYZ(1,J+1) = RT0(1)
XYZ(2,J+1) = RT0(2)
XYZ(3,J+1) = RT0(3)
GO TO 17;
C *** OPTION 3: TRAJECTORY DATA ARE SPECIFIED AS LONGITUDINAL ACCEL-
C *** ERATION, TARGET FRAME PSIT AND TARGET FRAME THET AS ARBITRARY
C *** FUNCTIONS OF TIME, PSIT AND THET FUNCTIONS ARE CONVERTED TO
C *** EQUIVALENT ACCELERATIONS WHICH ARE INTEGRATED AS FOR OPTION 2.
C
PSIT = VJR(1)/RT0
THET = VJN(1)/RT0
VTI = VT3(1)
PROGRAM TRAJECTRY OPT=1

MSG = NSEG-1
CALL C+VALY(GO,RT6,HNOR,4,G,ALT)
DO 11: I=1,NSEG
DI = TIME(I)+TIME(I)
VI = VTOT*0.5*(VOL(I)+VOL(I+1))
TH = 0.5*(VON(1)+VON(I+1))/RTO
QA = VT0/(DII0+RT0)
7F = 0.5*(VON(I+1)+VON(I))
QA = 0.5*COS(THET)+*(VOR(I+1)-VOR(I))
VOR(I) = QA
VOR(I1) = -ZT
11. CONTINUE
VONINSEG) = 0.
VONINS.G) = 0.
C
C *** CALCULATE TARGET INITIAL CONDITIONS
C
VTI = VI(I)
VT(I) = VT*COS(THET)*COS(PT6)
VT(I) = VTI*SIN(THET)
VT(I) = VTI*COI,(THET)*SIN(PS6)
GO TO 65
C
C *** TRANSFORM TARGET POSITION COORDINATES TO THE OUTPUT FRAME.
C
17. CSI = COS(PS6/RTO)
SSI = SIN(PS6/RTO)
CTH = COS(THET/RTO)
STH = SIN(THET/RTO)
SFI = COS(PRH/RTO)
SFI = SIN(PRH/RTO)
TM(I,1) = CSI*CTH
TM(I,2) = CSI*CTH
TM(I,3) = CSI*CTH
TM(I,4) = CSI*CTH
TM(I,5) = CSI*CTH
TM(I,6) = CSI*CTH
GO 19 I=1,NSEG
GO 16 J = 1,3
10. RT(J,J) = XZ(J,1)-XZ(J,1)
GO 19 J=1,3
19. XYZ(J,1) = TM(J)*RT(J,1)+TM(J,2)*RT(J,2)+TM(J,3)*RT(J,3)
C
C *** NOW CALCULATE CUBIC SPLINE COEFFICIENTS
C
20. CALL AVELIN(TIME,XYZ,NSEG,3,MLAM,4M,0,0,0,4,4,4,4,4
C
C *** PRINT INTERPOLATION COEFFICIENTS
C
MSG = NSEG-1
WRITE (6,831)
WRITE (6,822) TRAJ(13,1),TRAJ(13,2)
DO 20 I=1,NSEG
WRITE (6,823) TRAJ(I,1),TRAJ(I,2)
43
SUBROUTINE TRJEC (74/74, GPT=1, FTN 4.2+74355, 34/14/7)

LIENS = LINES+1
IF (LIENS.LT.65) GO TO 29
WRITE (6,977) NPAGE
NPAGI = NPAGE+1
LIENS = 3
29 CONTINUE
LIENS = LINES+1
C *** OUTPUT THE INTERPOLATION COEFFICIENTS AND TIME TO UNIT 7 IN A
C STRING STARTING AT TRAJ(1,1) PLUS TRAJECTORY AND SEQUENCE
C NUMBER 3 ON EACH CARD.
C
MSG = (INSEG-1)*13
I = 1
ISEO = 1
21 J = I+1
IF (J.GT.NSEG) WRITE (6,951) ITRAJ, ISEQ, (XTRAJ(K), K=I,J)
I = J+1
ISEO = ISEQ+1
IF (J.LT.NSEG) GO TO 21
C *** GENERATE TARGET POSITION AND RATE TERMS AT 1/4 SECOND INTERVALS
C
TYPE = 0,
DT = .25
JUF = IFIX(TIME(NSEG)/DT)+1
WRITE (6,963) ITRAJ
951 IF (INPASS.EQ.0) WRITE (6,971)
IF (INPASS.EQ.0) WRITE (6,975)
LIENS = LINES+1
GO TO 22
I = 1, JUP
CALL ATTAK(TRAJ)
CALL GIVALT(GO, RT, MNORM, G, ALT)
CSI = COS(ANGL(1))
SSI = SIN(ANGL(1))
CTH = COS(ANGL(2))
STH = SIN(ANGL(2))
YACC = -ACC(1)*SSI+ACC(2)*CSI
ZACC = ACC(1)*CSI*STH+ACC(2)*SSI*STH+ACC(3)*CTH
YACC = SQRT(YACC*YACC+ZACC*ZACC)+G
PSIT = ANGL(1)*RTO
THET = ANGL(2)*RTO
IF (INPASS.EQ.0) WRITE (6,981) TIME, RT, VT, PSIT, THET, YACC
IF (INPASS.EQ.0) GO TO 22
DO 212 J=1, J
212 RT(J) = RT(J)*RTO
VT(J) = VT(J)*RTO
WRITE (6,995) TIME, RT, VT
206 CONTINUE
LIENS = LINES+1
IF (LIENS.LT.60) GO TO 215
WRITE (6,875) NPAGE
NPAGE = NPAGE+1
LIENS = 3
215 CONTINUE
TYPE = TIME+DT
CONTINUE
WRITE (6,199)
LINES = LINES+1
IF (LINES.LT.58) GO TO 225
WRITE (6,197)
LINES = 3
225 CONTINUE

C *** CALCULATE TARGET EULER ANGLES INCLUDING ANGLES OF ATTACK.
C *** TARGET LIFT COEFFICIENTS ARE CALCULATED FROM LIFT CURVE SLOPES
C *** AIR WING LOADING.
C
IF (AIMD.EQ.0) GO TO 4
WRITE (6,266)
LINES = LINES+6
CALL F_SET (TRAJ)
NREW = 0
C0 23 I=1,NSEG
TPM = TIME(I)
CALL ATT (TRAJ)
CALL G=ALT (GT , MNORM , G , ALT)
CALL ATMOS (ALT , RHOSL , RHO)
QA = *RHO* (VT(I) + VT(I+1) + VT(I+2) + VT(I+3) + VT(I+4))
C
C *** TRANSFORM GRAVITY TO OUTPUT FRAME
C
GX = u*TM(1,3)
GY = v*TM(2,3)
GZ = w*TM(3,3)
C
C *** TRANSFORM ACCELERATION AND OUTPUT FRAME GRAVITY TO VELOCITY FRAME
C
CSI = COS(ANGL(1))
SSI = SIN(ANGL(1))
CTH = COS(ANGL(2))
STH = SIN(ANGL(2))
YACC = (G XO - ACC(1)) * SSI + (ACC(2) - GYO) * CSI
ZACC = (G XO - ACC(1)) * CSI * STH + (GYO - ACC(2)) * SSI * STH + (GZO - ACC(3)) * GTH
IF (AMSL(ACC), LT, 1, E=30) GO TO 235
ANGL(3) = ATAN (YACC/ ZACC)
GO TO 236
C
235 ANGL(3) = 0
C
236 CONTINUE
ZF = WOS*SQRT(YACC*YACC + ZACC*ZACC)/G
ALFT = ZF/QA/SLOPE1
IF (ALFT .LE. ALFA1) GO TO 240
ALFT = (ZF/QA-ALFA1*SLOPE1)/SLOPE2+ALFA1
240 IF (ZACC .LE. 0.0) ALFT = -ALFT
CFI = COS(ANGL(3))
SFI = SIN(ANGL(3))
CAL = COS(ALFT)
SAL = SIN(ALFT)
XYZ(1, I) = ATAN2 (GTH*SSI*CAL-STH*CSI*CFI*SAL), (GTH*CSI*CAL-
1
SSI*CFI*SAL)
XYZ(2, I) = ATAN2 (GTH*SSI*CAL-STH*CSI*CFI*SAL)
XYZ(3, I) = ATAN2 (GTH*CFI, (GTH*CFI*CAL-STH*SAL))
SI = XYZ(1, 1)
PROGRAM TRAJEC 74/74  OPT=1

99 FORMAT (21X 7F12.2)  EMC
C ROUTINE AV.LIN.

C This subroutine calculates spline interpolation coefficients to fit the X, Y, Z coordinates transmitted in array XYZ.
C The number of sets of dependent variables is given by NY and the number of breakpoints in the independent variable is N.
C
C DIMENSION XYZ(3,22), TRAJ(13,2), H(I), XLM(1),
C MU(I), PI(I), Q(I), XY(I,Y,1), AZ(I,Y,1),
C A3(I,Y,1), TIME(I)
C
C *** CALCULATE INTERVAL IN THE INDEPENDENT VARIABLE.
C DI
C DO 1 K=2,N
C H(K) = TIME(K) - TIME(K-1)
C
C *** CALCULATE CONDITIONS AT THE EXTREMITIES.
C DO 1 = 2
J = 1
K = 2
I = 1
X0 = TIME(I-1)
X1 = TIME(I)
X2 = TIME(I+1)
X11 = X1*X1
X22 = X2*X2
H0 = H(I)
H1 = X2-X1
H2 = H(I+1)
C = 1./H0/H1/H2
D = 2.*TIME(J)
LO 2: IY=1,NY
Y0 = XYZ(IY,I-1)
Y1 = XYZ(IY,I)
Y2 = XYZ(IY,I+1)
S1 = (Y1-Y0)*Y2-(Y2-Y0)*X11+(Y2-Y1)*X01
S2 = H.5*Y2-H1*Y1-H2*Y0
GO TO 10
3 = I-1
J = K
K = 2
GO TO 10
C *** CALCULATE LAMDA, MU, P AND Q
C DI
C Q(I) = -0.5
INO = K-1
IF K < 5 THEN
XM(K) = X*(K+1)/((K+1)*X(K+1))
XKH(K) = 1.-XLM(K)
FI(K) = 1./((XLM(K))*Q(K-1)+2.)
GO TO 10
C *** CALCULATE C, U AND M
C
ROUTINE AVELIN 74/74 OPT=1

DO 70 IY=1,NY
   A2(IY,1) = XM(IY,1)
   IND = N-1
   DO 66 K=2,IND
      GA = S Lam(K) * (XYZ(IY,K1) - XYZ(IY,K-1)) / H(K1) * XM(U,K) * 
      (XYZ(IY,K+1) - XYZ(IY,K)) / H(K+1))
      A2(IY,K) = (GA * Lam(K) * A2(IY,K-1)) * P(K)
      K = K-1
      66 CONTINUE

C *** CALCULATION OF POLYNOMIAL COEFFICIENTS

DO 66 J=1,IND
   U1 = M(J+1)
   U12 = 1.0 / U1 / U1
   U13 = U12 / U1
   DO 66 K=1,NY
      Y0 = XYZ(IY,J+1) - XYZ(IY,J)
      X0 = XM(IY,J+1) + XM(IY,J)
       X1 = X0 * XM(IY,J)
       A2(IY,J) = U12 * Y0 / U1 * X1
       66 CONTINUE

C *** STORAGE OF COEFFICIENTS IN ARRAY TRAJ

IND = N*NY+1
DO 10 J=1,N
   J = J+1
   TRAJ(J,K-1) = XYZ(I,J,K-1)
   TRAJ(J-1,K-1) = XM(I,J,K-1)
   TRAJ(J+2,K-1) = A2(I,J,K-1)
   10 CONTINUE

RETURN
END
SUBROUTINE RK4(VOL, VCR, VON, TIME, DT)
C- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C THIS SUBROUTINE INTEGRATES THE TARGET ACCELERATION AND VELOCITY
C TO GIVE DISPLACEMENT IN AN INERTIAL FRAME. ACCELERATION COMPONENTS
C IN THE TARGET FRAME ARE CONVERTED TO THE TARGET VELOCITY AND
C TARGET FRAME EULER ANGLES PSI AND THET.
C- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
COMMON /KTGT/ TST(6), VT(3)
DIMENSION A(6), S(6), T(6), TACC(6)
DO 4 I=1,4
TACC(I) = VOL
CTHET = COS(TST(I))
IF (CTHET.EQ.0.) GO TO 5
TACC(2) = VOR/CTHET/TST(I)
3 TACC(3) = -VOR/TST(I)
STHET = SIN(TST(I))
CPSI = COS(TST(2))
SPSI = SIN(TST(2))
VT(1) = CTHET*CPSI*TST(I)
VT(2) = CTHET*SPSI*TST(I)
VT(3) = STHET*TST(I)
TACC(4) = VT(1)
TACC(5) = VT(2)
TACC(6) = VT(3)
GO TO 10
4 DO 10 I=1,6
VT(I) = TST(I)*DT
10 TST(I) = TST(I)+.5*S(I)
GO TO 40
5 DO 25 I=1,6
A(I) = VT(I)*TACC(I)
S(I) = S(I)+2.*A(I)
25 TST(I) = T(I)+.5*A(I)
GO TO 40
6 DO 55 I=1,6
A(I) = VT(I)*TACC(I)
S(I) = S(I)+2.*A(I)
55 TST(I) = T(I)*A(I)
GO TO 40
7 DO 35 I=1,6
35 TST(I) = T(I)+.5*(S(I)+DT*TACC(I))/6.
8 CONTINUE
TIME = TIME+DT
RETURN
END
SUBROUTINE ATTAK(TRAJ)

; This subroutine determines target position, velocity and acceleration as a function of time by cubic spline interpolation.

; Pre-calculated interpolation coefficients are stored in array TRAJ.

DIMENSION C(12), TRAJ(13,23)
COMMON /TGT/ TDISPL, Y(9), ACC(3)
DATA KSEG/1/

10 IF (T1-TRAJ(13,KSEG)) .EQ. -1.E6 KSEG = 1
11 IF (T1.LE.TRAJ(13,KSEG+1)) GO TO 20
15 IF (T1.LT.TRAJ(13,KSEG+1)) GO TO 30
20 KSEG = KSEG+1
30 IF (KSEG .LT. 20) GO TO 10
40 KSEG = KSEG-1
50 GO TO 10

T1 = TDISPL
Y1 = T1*C(3)*T1*C(4)
Y2 = T1*C(7)*T1*C(8)
Y3 = T1*C(11)+T1*C(12)
Y4 = T1*(C(11)*T1+C(12))
Y5 = C(1)*T1+C(2)*T2
Y6 = C(5)*T1+C(6)*T3
Y7 = C(9)*T1+C(10)*T4
Y8 = C(13)*T2+T2*T1*C(4)
Y9 = C(17)+T3+T3*T1*C(8)
Y10 = C(11)+T4+T4*T1*C(12)
Y11 = ATAN2(Y5,Y4)
Y12 = ATAN2(-Y6,SQRT(Y4)*Y4+Y3*Y5))
Y13 = 0.
ACC(1) = 2.*C(3)+6.*C(4)*T1
ACC(2) = 2.*C(7)+6.*C(8)*T1
ACC(3) = 2.*C(11)+6.*C(12)*T1
RETURN
END
SUBROUTINE ATMOS(RHOL,RHOSL)
C-THIS SUBROUTINE CALCULATES ATMOSPHERIC DENSITY AS A FUNCTION
C-OF ALTITUDE. DENSITY TABLE IS NORMALIZED BY ITS SEA LEVEL VALUE
C-SO THAT DIMENSIONS OF RHO ARE THOSE OF INPUT PARAMETER RHOSL.
C-THE ALTITUDE TABLE IS NORMALIZED INTO UNITS OF 1000 FEET. IF TARGET
C-ALTITUDE IS EXPRESSED IN METERS THE PARAMETER HNORM IN THE MAIN
C-PROGRAM MUST BE 3.28083.
C-%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
C-DIMENSION
C-ALTTB(32),RHTB(32)
C-DATA
C-RHTB/1., .97138, .94276, .91498, .88005, .84806, .81061, .76625, .76221, .75806, .74866, .73980, .73715, .73671, .73062, .73025, .72902, .72822, .72612, .72417, .72167, .71871, .71446, .71020, .70569, .70083, .69546, .69066, .68531, .67932, .67273, .66542, .65789, .64907, .63992, .62941, .61754, .60432./
C-HL = 3.98E13
C-NR = 3.28083
C-CONTINUE
C-I = 32
C-CONTINUE
C=END

SUBROUTINE GRVALT(RO,RH,GN,ALT)
C-THIS ROUTINE CALCULATES TARGET ALTITUDE ABOVE THE EARTH'S SURFACE
C-AND THE LOCAL GRAVITATIONAL ACCELERATION ASSUMING A SPHERICAL
C-EARTH OF RADIUS 20.065 FEET.
C-%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
C-DIMENSION
C-R(3),G(3)
C-DATA
C-RO/R20.065/2E6/
C-RR = (R(0)-R(3))*NN*2
C-GPR = GPR(R(1)+R(2))**NN*NN
C-ALT = SQRT(GPR+RSQ)-RO
C-G = G/NN/RR/RSQ
C-END
Appendix B. EXAMPLE TRAJECTORY RESULTS

This appendix contains a set of trajectory results corresponding to each of the three input options. In each case the Euler angle option has been selected. The first two trajectories are expressed in SI units and the third is calculated in British units.
### Trajectory Generation by Piecewise Spline Interpolation

**Trajectory No. = 1**

<table>
<thead>
<tr>
<th>Euler Angle Interpolation Selected</th>
<th>Output Frame Transformation</th>
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<tbody>
<tr>
<td>ALFA1</td>
<td>ZC</td>
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<tr>
<td>COCL/DA2</td>
<td>PSI (DEG)</td>
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<td>.750</td>
<td>THET (DEG)</td>
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<tr>
<td>.45</td>
<td>PSI (DEG)</td>
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#### Input Tables

<table>
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<th>TIME</th>
<th>X COMP</th>
<th>Y COMP</th>
<th>Z COMP</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>-6320.466</td>
<td>4.000</td>
<td>-1387.838</td>
</tr>
<tr>
<td>2.00</td>
<td>-5989.400</td>
<td>0.000</td>
<td>-1803.900</td>
</tr>
<tr>
<td>6.00</td>
<td>-5609.300</td>
<td>0.000</td>
<td>-2243.430</td>
</tr>
<tr>
<td>6.00</td>
<td>-5226.300</td>
<td>1.000</td>
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**Note:**

- The table above represents the trajectory data with coordinates and transformations.
- The spline interpolation coefficients are used to calculate intermediate points along the trajectory.
- The input tables list the time and corresponding X, Y, and Z components for each point.
- The spline coefficients are used to ensure smooth transitions between points.
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MUANEVERING TARGET TRAJECTORY.

OPTION 3. TRAJECTORY SPECIFIED BY FLIGHT PATH ANGULAR POSITION.

HORIZONTAL SPIRAL MANEUVER.

DIMENSIONS IN FEET AND FT/SEC

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