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This technical report has been reviewed and is approved for publication.

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<td>ABSTRACT</td>
<td>This report describes in detail the Vuilleumier (V-M) refrigeration cycle and various ways it has been applied to produce cryogenic temperatures. It starts with the most theoretical model of the Vuilleumier cycle and gradually adds complicating factors such as void volumes and undesirable heat losses until a real refrigerator is described. Included, are the factors and component characteristics that influence the refrigeration capacity, efficiency, and life of Vuilleumier refrigeration systems. The various ways different designers have mechanized this cycle in their quest for long life are discussed.</td>
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FOREWORD

This final report was prepared by Ronald White of the Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio. This report was written to satisfy one of the requirements for a Master of Mechanical Engineering degree at the University of Dayton. This report provides a basic description of the Vuilleumier cycle refrigerators and summarizes a number of Air Force and NASA development efforts.

This work was accomplished in the Environmental Control Branch (FEE), Vehicle Equipment Division (FE) under Project Number 2126 "Advanced Surveillance Technology"; Task Number 212603 "Cryo Cooler Technology." The time period covered by the effort was February 1975 to August 1975.

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SECTION I

THE VUILLEUMIER CYCLE

The Vuilleumier refrigeration cycle is not new, having been patented in 1918 by Rudolph Vuilleumier (Reference 1). However no known commercial use of the cycle was made until the mid 1960's. The cycle is difficult for laymen to comprehend since the primary power input is heat (at about 1200°F), while it is producing refrigeration at temperatures in the range of -445°F to -244°F. To aid in the understanding of this refrigerator, this report will start with a very theoretical model, and add complicating factors, one at a time, until a real refrigerator is described. It will also discuss the various ways different persons have mechanized this cycle.

The reason for the recent interest in this cycle is that a number of electronic devices for aircraft and spacecraft have been developed that required continuous cooling to cryogenic temperatures. In order to make these devices practical, long life reliable closed cycle refrigerators are needed. Closed cycle refrigerators require only power and a heat sink (such as a radiator), while open cycle systems require the replenishment of liquid, gaseous, or solid cryogens (with their attendant logistics problems). The Vuilleumier (V-M) cycle is especially attractive for aircraft and spacecraft cryogenic cooling requirements since it has the potential for long life, low wear, low noise, low vibration, low-to-moderate cost, low power consumption, and the ability to use heat directly as the input power. This latter feature (when heat is available) increases the total system efficiency since the losses encountered in converting heat...
to electricity and then converting the electricity to a refrigerator pressure change, are avoided. The degree to which the various advantages of this cycle can be realized is somewhat dependent on the details of how the cycle is mechanized and conversely, the objectives of the refrigerator designer. Therefore, design techniques that contribute to these advantages will be pointed out throughout this report.
SECTION II
THEORY OF OPERATION

The continuous production of refrigeration at low temperature requires the expenditure of a certain amount of energy. In most refrigeration cycles this is supplied as mechanical energy by an electric motor. In the Vuilleumier (V-M) cycle the energy is supplied in the form of heat. Externally, the V-M refrigerator is viewed as a machine which absorbs heat at both a high and low temperature and then rejects this heat at some intermediate temperature. The heat input at the hot end of the machine provides the energy required to produce the cooling effect at the cold end, while the heat from both the hot and cold ends is rejected at the ambient (intermediate) end.

The V-M refrigerator, in its simplest form, is composed of two cylinders with displacers, two thermal regenerators, three heat exchange areas, connecting passages, and a mechanism to drive the displacers in the proper sequence.

The displacer is a long loose fitting piston-like device whose function is to move (displace) gas from one end of the cylinder to the other (See Figure 1). The cylinder-displacer combination is arranged so that there is a gas passage connecting the two ends of the cylinder. With this connection, the effect of moving the displacer is to displace the gas from one end of the cylinder to the other. Note that the total volume of the gas remains constant (assuming an infinitely thin push rod). This is in contrast to a piston-cylinder combination that would change the gas volume. Since the only V-M that utilized a piston is the special-case self driven V-M, the term "displacer" will be used throughout this report as a reminder that the total gas volume of the system is not affected by the movement of the displacers. The pressure on each end of the displacer is
FIGURE I. DISPLACER AND CYLINDER
theoretically the same (in real refrigerators it differs by the pressure drop through the regenerator and connecting passages). Therefore, the energy needed to move the displacer is very low, especially when compared to a refrigerator using pistons. Since the two ends of the cylinder will be at significantly different temperatures the displacer is long with thin walls and packed with insulation to reduce the end-to-end heat conduction.

Next, a thermal regenerator is arranged to connect the two ends of the cylinder so that gas passing from one end of the cylinder to the other must pass through the regenerator. The thermal regenerator is an energy storage device whose heat capacity greatly exceeds that of the gas. In operation, gas passing through a regenerator is heated or cooled depending on flow direction. For example, gas flowing from the ambient temperature end to the high temperature end of a cylinder is heated by the hot regenerator. The energy added to the gas was stored in the regenerator packing, or matrix, by gas flow in the reverse direction during a previous part of the cycle. The regenerator matrix consists of a porous material with a high ratio of surface area to volume, a high heat capacity, and low heat conductivity in the direction in which the gas flows through it. Examples of regenerator matrix materials used in VM refrigerators are: Balls, stacked screens, and "wool" made of (depending on temperature) stainless steel, monel, bronze, copper, or lead. At each point in the regenerator there is a very high heat transfer coefficient between the matrix and the gas. The regenerator connects two regions of different temperatures, and therefore has a temperature gradient along its matrix. At one end,
the matrix has the temperature of one region and at the other end the temperature of the second region. Because of the presence of the regenerator, the gas can flow back and forth between the two volumes without serious loss of heat or cold. This thermal isolation (although not strictly true) tends to maintain each of the volumes at nearly constant, but different temperature levels. The energy storage capability of the regenerator is generally great enough that its temperature profile is only slightly altered by providing energy to the gas.

To better understand how these components function together to produce refrigeration, the power input section will be discussed first, followed by the refrigeration section, and then the total refrigerator.

The power input section, the first cylinder-displacer, is composed of the hot cylinder, hot displacer, hot regenerator, hot heat exhanger, and part of the ambient temperature heat exchanger. Theoretically if it were separated from the remainder of the refrigerator it would look like Figure 2. For this theoretical discussion, the regenerator is assumed to be 100% efficient with no significant gas void spaces. Therefore the gas in the system is distributed between the two (hot and ambient) active volumes. The term active volume is used to describe the volume swept by the displacer. When the displacer is in the top position the hot active volume is nearly zero and all of the gas is in the ambient active volume. The ambient volume is held at an ambient temperature (significantly below the hot temperature) by the ambient heat exchanger. Using the perfect gas law, the gas is at low temperature
FIGURE 2. POWER INPUT SECTION
(volume is constant) therefore it is at a low pressure. When the displacer is in the bottom position the ambient active volume is nearly zero and all of the gas is in the hot active volume. Since this volume is maintained at a high temperature (usually about 1200°F) by addition of heat through the hot end heat exchanger, the gas is hot and therefore at high pressure. Since there are no valves in the system, all of the gas throughout the system including any at the other end of the cylinder (or in any dead volumes to be discussed later) is at the same pressure. Therefore, it can be seen that by moving the hot displacer, the system pressure can be varied from a maximum to a minimum in a cyclic way and that heat is supplied to the hot end and rejected at the ambient end. The hot section acts as a thermal compressor (whose effect on pressure is similar to a piston in a dead ended cylinder). A similar thermal compressor was patented by V. Bush (Reference 2).

The manner in which the cold section, the second cylinder-displacer, of the V-M refrigerator uses the changing pressure to produce a cooling effect is similar to that of the cold section of the Stirling cycle refrigerator, since the method by which the cyclic pressure is produced (thermal compressor or mechanical compressor) has little effect on the cold section.

Referring to Figure 3, if the displacer is in the top position and pressure from the compressor is increasing, gas will flow into the ambient active volume (the cold active volume is zero) and increase in pressure. This compression causes the temperature of the ambient volume gas to increase, but this heat is rejected by the ambient heat
COLD HEAT EXCHANGER

COLD ACTIVE VOLUME

DISPLACER

CYLINDER

SEAL

AMBIENT ACTIVE VOLUME

GAS SUPPLIED FROM COMPRESSOR

AMBIENT HEAT EXCHANGER

VERY THIN PUSH ROD

FIGURE 3. COLD SECTION
exchanger so that at the end of the compression process the ambient active volume is filled with high pressure ambient temperature gas. The cold displacer is then moved to its lower position and the gas is displaced from the ambient end, through the cold regenerator to the cold end. As the gas passes through the cold regenerator, it deposits its heat in the regenerator matrix and emerges at the cold end as cold high pressure gas (any decrease in volume of the gas is made up by the compressor). After all the gas had been displaced to the cold active volume, the compressor displacer is moved so that the pressure decreases to the minimum. This decreases the pressure throughout the refrigerator including the cold section. The gas in the cold active volume undergoes an expansion (some of it goes back through the regenerator toward the compressor) becoming colder low pressure gas. This colder gas then has the ability to absorb heat from the device to be cooled (refrigeration). After the gas has absorbed the heat, the cold displacer is moved to the upper position, displacing the gas from the cold volume through the regenerator where it picks up the heat that was deposited on the previous half cycle and emerges at the ambient end where it will reject the heat picked up from the refrigeration load. After this displacement is completed, the compressor starts to increase the pressure and the cycle begins again.

From this description it can be seen that by combining two cylinder-displacer-regenerator assemblies and phasing their motion properly, useful refrigeration can be obtained.
To this point, nothing has been said about what sort of mechanism drives the displacer push rods. In the previous discussion a stop-start square wave motion was implied to simplify the discussion of the events in each section. In almost all of the real V-M refrigerators built to date, the displacers have been driven harmonically. Harmonic drives (such as a crankshaft with connecting rods) are easy to fabricate and avoid the problem of high acceleration at the end of the stroke. However, harmonic drives do complicate the discussion of the V-M cycle. A schematic of a harmonically (sinusoidal) driven V-M refrigerator is shown in Figure 4.

The cycle operates through the use of displacers moving the gas from one section to another without the requirement to compress the gas in a closed volume. Therefore, the pressures throughout the system are nearly equal at any moment. The seals shown in the schematic are to force all of the displaced gas through the regenerators. Since the pressure drop across the regenerators in real refrigerators is only a few pounds per square inch the loading on the seals and displacer drive bearings is minimal, which contributes significantly to the long life of V-M refrigerators. Only a small (few watt) timing motor is needed to drive the mechanism, since the forces are so small.

The equations to be presented in this report assume isothermal operation in the refrigerator expansion and compression volumes since such an assumption makes it possible to derive a set of thermodynamic equations that are relatively simple and that are fairly representative of the thermodynamics of the V-M refrigerator. Isothermal operation is not achieved in an actual refrigerator, but can be approached by careful design.
GAS-FILLED WORKING VOLUME (TYPICALLY HELIUM AT HIGH PRESSURE)

HEAT EXCHANGER ABSORBS HEAT FROM LOAD AT LOW TEMPERATURE

Vc

COOLING CYLINDER

COOLING DISPLACER

COLD DISPLACER SEAL

HEAT EXCHANGER REJECTS HEAT TO AMBIENT

HEAT EXCHANGER ABSORBS HEAT FROM HEAT SOURCE

VA

CRANKSHAFT

CRANKCASE

CRANK THROW

POWER CYLINDER

SOURCE OF HEAT AT HIGH TEMPERATURE (~1200 °F)

FIGURE 4. SCHEMATIC OF VUILLEUMIER REFRIGERATOR
As shown in Figure 4, the idealized V-M refrigerator has two cylinders fitted with displacers which separate the refrigerator into three volumes $V_H$, $V_A$, and $V_C$. The subscript $H$ is the hot temperature level, $A$ is the ambient temperature level, and $C$ is the cold temperature level. $V_A$ actually consists of the summation of the two active volumes at the ambient end of the two cylinders. The crankcase is fitted with filler blocks so that the volume in the crankcase region is only the ambient active volume. It is assumed that there are no pressure drops within the refrigerator in the idealized model. The thermal regenerators are assumed to be perfect; i.e., no temperature difference is required between gas flows in each direction in order to transfer heat, and therefore no heat flows through the regenerators over a complete cycle. The regenerators are assumed to have an infinite heat capacity and therefore the temperature of the regenerator is invariant with time. In the idealized refrigerator it is assumed that there are no dead volumes, that is, all the volume inside the refrigerator is active volume ($V_H$, $V_A$, and $V_C$). This means the gas volume in the regenerators, heat exchangers, around the sides of the displacers, around crankcase parts, at the ends of cylinders, etc. is assumed negligible (this will be modified later). The heat conduction axially through the regenerator matrix, and along the walls of the displacers and cylinders is assumed negligible.

Attempts have been made to describe the cycle using a temperature-entropy (T-S) diagram such as Figure 5.
It is apparent that the diagram describes more than one unit mass of gas and the diagram is somewhat artificial since a great many simultaneous operations occur. For an example, let us look at three masses of gas that cycle between the ambient end and the hot end. The mass of gas in the hot end nearest the hot regenerator entrance will be the last mass in and (if no mixing occurs) the first mass out. Its T-S diagram would look like Figure 6.

The second mass (between the first and last) would look like Figure 7.

The third mass of gas is the first one in the hot end and the last one out. It would have a T-S diagram that looks like Figure 8. It can be seen that in order to use the T-S diagram to describe this cycle a large number of individual T-S diagrams for both ends of the refrigerator would need to be summed (mixing could further complicate the analysis). However, the pressure-volume (P-V) diagram for each volume provides a clearer and more accurate picture of the operation than does the T-S diagram.

The P-V or indicator diagrams for the three sections of a well designed V-M refrigerator \( P_1 \leq P_2 = P_{\text{max}} \) and \( P_3 \leq P_4 = P_{\text{min}} \) are shown in Figures 9 to 11. Note that the ambient volume is the sum of the ambient end volumes of the hot and cold cylinder.
Figure 5. Theoretical T-S Diagram

Figure 6. T-S Diagram for First Mass of Gas
Figure 7. T-S Diagram for Second Mass of Gas

Figure 8. T-S Diagram for Third Mass of Gas
FIGURE 9 COLD VOLUME INDICATOR DIAGRAM

FIGURE 10 HOT VOLUME INDICATOR DIAGRAM

FIGURE 11 AMBIENT VOLUME INDICATOR DIAGRAM
At each volume for a complete cycle, the first law reduced to:

\[ Q = (h_2 - h_1) + W \]

where \( Q \) and \( W \) represent energy per unit mass of gas flowing into or out of the active volume. The temperature of the gas crossing the boundaries does not vary with time (since ideal regenerators were assumed), so \( h_2 = h_1 \) and \( Q = W \) or:

\[ Q = W = \int P dV \]

The heat flow is equal to the work at that volume. There is, of course, no net work resulting from the idealized refrigerator because no pressure differences exist within the machine at any given time. Since the pressure at each end of each displacer is equal and by elementary geometry the change of volume at one end of a cylinder is exactly equal to the negative of the volume change at the other end:

\[ \int P_A dV_A = \int P_H dV_H + \int P_C dV_C \]

Since the displacer motion has been assumed harmonic, the pressure changes are assumed to occur isothermally, and if the phase separation angle is assumed to be \( 90^0 \), the volume at each of the ends can be described by the maximum active hot and cold volumes.

\[ V_H = \frac{1}{2} V_{HM} (1 - \cos \theta) \quad (1) \]
\[ V_C = \frac{1}{2} V_{CM} (1 - \sin \theta) \quad (2) \]
\[ V_A = \frac{1}{2} V_{HM} (1 + \cos \theta) + \frac{1}{2} V_{CM} (1 + \sin \theta) \quad (3) \]

Where:

- \( V_{HM} \) = maximum swept volume of the hot end
- \( V_{CM} \) = maximum swept volume of the cold end
If the gas is assumed to be ideal, 

\[ n = \frac{PV}{RT} \]

\[ n = \frac{P}{R} \left\{ \frac{V_{HM}}{2T_H} (1 - \cos \theta) + \frac{V_{CM}}{2T_C} (1 - \sin \theta) + \frac{V_{HM}}{2T_A} (1 + \cos \theta) + \frac{V_{CM}}{2T_A} (1 + \sin \theta) \right\} \]  \hspace{1cm} (4)

rearranging, the equation becomes:

\[ n = \frac{P}{R} \left\{ V_{HM} \left( \frac{1}{2T_H} + \frac{1}{2T_A} \right) + V_{CM} \left( \frac{1}{2T_C} + \frac{1}{2T_A} \right) + V_{HM} \left( \frac{1}{2T_A} - \frac{1}{2T_H} \right) \cos \theta + V_{CM} \left( \frac{1}{2T_A} - \frac{1}{2T_C} \right) \sin \theta \right\} \]  \hspace{1cm} (4)

multiplying numerator and denominator by \(2T_A\) the equation becomes:

\[ n = \frac{P}{2T_AR} \left\{ V_{HM} \left( \frac{T_A}{T_H} + 1 \right) + V_{CM} \left( \frac{T_A}{T_C} + 1 \right) + V_{HM} \left( 1 - \frac{T_A}{T_H} \right) \cos \theta + V_{CM} \left( 1 - \frac{T_A}{T_C} \right) \sin \theta \right\} \]  \hspace{1cm} (5)

This equation can be simplified by grouping the parameters, using the symbols \(a, b,\) and \(c\) defined by:

\[ a = V_{HM} \left( 1 + \frac{T_A}{T_H} \right) + V_{CM} \left( 1 + \frac{T_A}{T_C} \right) \]  \hspace{1cm} (6)

\[ b = V_{HM} \left( 1 - \frac{T_A}{T_H} \right) \]  \hspace{1cm} (7)

\[ c = V_{CM} \left( 1 - \frac{T_A}{T_C} \right) \]  \hspace{1cm} (8)

Where "a" reflects the charge of gas in the refrigerator, "b" reflects the effect of the movement of the hot end displacer on the total pressure, and "c" reflects the effect of the cold-end displacer on the pressure.

The pressure, \(P\), at any instant and at every point in the working medium is:

\[ P = \frac{2nRT_A}{a + b \cos \theta + c \sin \theta} \]  \hspace{1cm} (9)

To obtain the angle \(\theta\) at which \(P\) is maximum or minimum, differentiate the equation and set \(dP/d\theta = 0\). Then:

\[ \tan \theta = \frac{c}{b} \]
Using Equation 9 to calculate the area inside the P-V diagrams:

\[
Q_H = \int PdV_H = nRT_AV_{HM} \int_0^{2\pi} \frac{\sin \theta}{a + b \cos \theta + c \sin \theta} d\theta \\
Q_C = \int PdV_C = nRT_AV_{CM} \int_0^{2\pi} \frac{\cos \theta}{a + b \cos \theta + c \sin \theta} d\theta
\]

(10)

(11)

where: \(Q_H\) = heat input to the hot end

\[Q_C = \text{heat input to the cold end}\]

These equations can be integrated using formula 2.558-2. from Table of Integrals Series and Products by I. S. Gradshteyn and I. M. Ryshik. Note that \(a^2 > (b^2 + c^2)\) and that \(\int d\theta/(a + b \cos \theta + c \sin \theta)\) must be integrated from 0 to \(\pi\) and from \(\pi\) to \(2\pi\) with \(-\pi/2 \leq \arctan \Theta \leq \pi/2\). The result is:

\[
Q_H = \frac{2\pi \cdot a \cdot c R}{b^2 + c^2} V_{HM} T_A \left\{ 1 - \frac{a}{\sqrt{a^2 - b^2 - c^2}} \right\}
\]

\[
Q_C = -\frac{2\pi \cdot b \cdot c R}{b^2 + c^2} V_{CM} T_A \left\{ 1 - \frac{a}{\sqrt{a^2 - b^2 - c^2}} \right\}
\]

(12)

(13)

These are the equations for the heat input to the hot end and the heat absorbed by the cold end (per cycle) for an ideal V-M refrigerator.

The relationship between the maximum hot active volume and the maximum cold active volume is:

\[
\frac{V_{HM}}{V_{CM}} = \frac{T_H (T_A - T_C)}{T_C (T_H - T_A)}
\]

(14)

This equation is derived from the ideal work and the ideal refrigeration that takes place in the machine during a cycle, when the pressure at crank position 1 is assumed equal to the pressure at crank position 2 as indicated in Figure 4.
The coefficient of performance of this ideal refrigerator is:

\[
\text{COP} = \frac{Q_C}{Q_H} = \left\{ \frac{V_{CM}}{V_{HM}} \right\} \left\{ \frac{V_{CM}}{V_{HM}} \right\} = \left\{ \frac{V_{HM}}{V_{CM}} \frac{1 - \frac{T_A}{T_H}}{1 - \frac{T_A}{T_C}} \right\}
\]

which is the same as a Carnot engine, driving a Carnot refrigerator, therefore the figure of merit (FOM) is one.

The maximum pressure ratio is obtained by differentiating the pressure equation:

\[
P_{\text{max}} = \frac{a + \sqrt{b^2 + c^2}}{a - \sqrt{b^2 + c^2}}
\]

In order to better understand this ideal refrigerator an example was calculated and plotted in Figure 12. The assumptions were:

Temperature of hot end = 812^\circ K
Temperature of ambient end = 366^\circ K
Temperature of cold end = 77^\circ K
\(V_{HM}\) is 87.23% of the total volume
\(V_{CM}\) is 12.77% of the total volume
Phase angle is 90^\circ
\(\theta = 0\) at hot displacer top dead center

The volumes \(V_H, V_A,\) and \(V_C\) were plotted to show how they vary with crank position, \(\theta\). The percentage of the total number of molecules \(N_H, N_A, N_C\), in each of the volumes vs. crank position was also plotted, as was the resulting pressure (in arbitrary units).
Figure 12. Ideal Refrigerator Interactions
From Figure 12, it can be seen that although the maximum cold volume is only 12.77% of the total, at one point of the cycle 52% of the molecules of the system are in the cold end. Also the maximum percentage of molecules ever in the hot end is 55%. The maximum pressure occurs 45° before the maximum hot volume occurs and the minimum pressure occurs 45° before the minimum hot volume occurs. Noting the shape of the pressure curve in the region of 180° to 270°, it can be seen that the pressure curve is not quite sinusoidal.

All of the above equations and discussion assumed an ideal refrigerator, now the various nonideal factors will be added to these equations until a real refrigerator is discussed.

There are a number of void volumes in a real refrigerator. These include clearances around the displacers to prevent scraping, clearance at the ends of the cylinders to prevent hitting the end and to allow for thermal expansion, gas flow spaces in the regenerators, heat exchangers, gas transfer passages, and clearances in and around the mechanism. The void volumes must undergo the pressure variations of the cycle but do not contribute to the cooling effect. The void volumes decrease the maximum pressure ratio produced and, therefore, the amount of cooling produced.

To illustrate the magnitude of some of these void volumes, the cold regenerator flow passage void volume is usually 1.5 to 3.5 times the active cold volume, while the displacer side clearance void volume is usually about 0.17 times the active cold volume. The effect of void volumes is related to the temperature of the void volume. The void volumes at the coldest temperatures have the greatest effect on the refrigerator performance while the void volumes at the hot end have the least effect on refrigerator performance. The effect of void volumes is more pronounced in miniature V-M refrigerators, due in part
to ordinary manufacturing tolerances. In miniature V-M refrigerators, decreasing
the void volume is both essential and expensive. Reduction of void volume requires
close tolerances and unusual shaped parts (especially in the crankcase). Void
volume reduction techniques used in current refrigerators even include the use
of epoxy to fill the screw slots in the internal screws.

The effect of void volume can be accounted for by adding terms to
Equation 4 of the form:

\[ V_{vl} \frac{T_{vl}}{T_{vl}} + V_{v2} \frac{T_{v2}}{T_{v2}} + \cdots + V_{vn} \frac{T_{vn}}{T_{vn}} \]

where: \( V_{vl} \) = a void volume

\( T_{vl} \) = temperature of that void volume

or

\[ \frac{V_{v}}{T_{v}} \]

where: \( V_{v} \) = sum of void volumes

\[ T_{v} = \frac{\int T_{v} dV_{v}}{\int dV_{v}} \]

The addition of void volumes will change Equation 6 to:

\[ a = V_{HM} \left\{ 1 + \frac{T_{A}}{T_{H}} \right\} + V_{CM} \left\{ 1 + \frac{T_{A}}{T_{C}} \right\} + 2 V_{v} \left\{ \frac{T_{A}}{T_{v}} \right\} \]  

(17)

The other Equations 9 thru 16 remain the same except the numerical value
of "a" has changed. Typical values of the reduced void volume ratio (reduced
void volume to cold end volume) for small V-M refrigerators built to date are
in the range of 1.5 to 3.7, where reduced void volume is defined as:

\[ V_{v,\text{reduced}} = V_{v,\text{actual}} \times \frac{T_{C}}{T_{v,\text{actual}}} \]  

(18)
Another correction needed for the basic V-M equations is the compressibility factor for the working fluid which is helium. Helium has been used as the working fluid in all V-M refrigerators built to date since it behaves as a nearly perfect gas and has the lowest temperature capability. The effect of compressibility of helium is slight until temperatures below 70^0K are reached. Adding the compressibility factor (Z) to Equation 4:

\[ n = \frac{P}{2R} \left\{ \frac{V_{HM}(1-\cos\theta)}{T_H Z_H} + \frac{V_{CM}(1-\sin\theta)}{T_C Z_C} + \frac{V_{HM}(1+\cos\theta)}{T_A Z_A} + \frac{V_{CM}(1+\sin\theta)}{T_A Z_A} + \frac{2V_v}{T_v Z_v} \right\} \]  

which changes Equations 6 or 17, 7, 8, and 14 to:

\[ a = \frac{V_{HM}}{Z_H} \left\{ \frac{Z_H}{Z_A} + \frac{T_A}{T_H} \right\} + \frac{V_{CM}}{Z_C} \left\{ \frac{Z_C}{Z_A} + \frac{T_A}{T_C} \right\} + \frac{2V_v T_A}{Z_v T_v} \]  

(20)

\[ b = \frac{V_{HM}}{Z_H} \left\{ \frac{Z_H}{Z_A} - \frac{T_A}{T_H} \right\} \]  

(21)

\[ c = \frac{V_{CM}}{Z_C} \left\{ \frac{Z_C}{Z_A} - \frac{T_A}{T_C} \right\} \]  

(22)

\[ V_{HM} = \frac{T_H Z_H}{T_H Z_H - T_A Z_A} \left\{ \frac{T_A Z_A - T_C Z_C}{T_C Z_C} \right\} \]  

(23)

\[ V_{CM} = \frac{T_H Z_H}{T_H Z_H - T_A Z_A} \left\{ \frac{T_C Z_C}{T_A Z_A - T_C Z_C} \right\} \]  

(24)

Equations 9 through 13 and 16 are still valid, however the values of "a", "b", and "c" within these equations have changed slightly.

To summarize the theoretical V-M section, if compressibility effects and void volumes are to be accounted for, the equations to use are 9 through 13, 16, and 18 through 24.
Equation 13 $Q_C$, is the gross refrigeration produced by the refrigerator. To obtain the net refrigeration produced by the refrigerator, all of the various cold end losses must be subtracted from the gross refrigeration. Equation 12, $Q_H$, is the P-V heat input to the hot cylinder. To obtain the actual heat (power) input needed, the various hot end losses must be added to the P-V input.
SECTION III
INHERENT THERMODYNAMIC AND HEAT TRANSFER LOSSES

Since it is impossible to build perfect regenerators and to eliminate all undesirable heat transfer processes in the refrigerator, these losses must be subtracted from the gross refrigeration to determine the net refrigeration available at the cold end. Similar losses must be added to the heat input to the gas to determine the required heat input to the hot end.

1. SHUTTLE LOSS

Shuttle loss is caused by the mismatch of thermal gradients between the displacer and the cylinder. The cold cylinder wall is at ambient temperature at one end and at cryogenic temperature at the other. It has a fixed length and a gradient from warm to cold that is approximately linear. The displacer is shorter than the cylinder wall by the length of the stroke, however it has the same temperature extremes; warm at one end and cold at the other. When the displacer is at one extreme of its travel the temperature gradients are somewhat mismatched. As it passes thru its stroke and reaches the other extreme the gradients are again mismatched, but now they are mismatched in the opposite direction so that the displacer picks up heat from the cylinder when it is at the warm end and it gives off heat to the cylinder when it is at the cold end of its stroke. Hence, there is a picking up of heat at the warm end, a shuttling of the displacer to the cold end where it drops off the energy to the cylinder. It is thus termed a shuttle loss.

If the motion of the displacer is approximately harmonic and if the thermal time lag of the cylinder and displacer materials is small compared
with the reciprocating time, the shuttle heat transfer can be computed from
the following equation (see Reference 3 for derivation):

\[ Q_{SH} = 0.186 \gamma^2 C \frac{kg}{S} \frac{(T_w - T_c)}{L_{cy}} \]  

(25)

where:
- \( Q_{SH} \): shuttle heat loss
- \( \gamma \): stroke
- \( C \): wetted perimeter = \( \pi \) x diameter
- \( S \): radial clearance between displacer and cylinder
- \( T_w \): temperature of warm end of cylinder
- \( T_c \): temperature of cool end of cylinder
- \( L_{cy} \): length of cylinder
- \( kg \): thermal conductivity of the gas (helium)

The terms "warm" and "cool" are used since this equation is used to compute
the shuttle losses of both the hot and the cold ends of V-M refrigerators.

2. PUMPING LOSS

The pumping loss is due to the fixed clearance volume that must exist
between the displacer and the cylinder so that the displacer can move
without rubbing. This volume is bounded on one end (usually the ambient
end) by a displacer seal and is open on the other end. Because this volume
around the displacer allowed for running clearance is fixed, the mass of
gas in this volume at any one time is proportional to the (cycling) pressure.
At the minimum pressure point there is a minimum mass of gas in this volume.
As the pressure increases toward the maximum pressure point, gas flows into
this volume proportional to the increasing pressure. Therefore, using the
cold cylinder as an example, cold gas flows into this volume and down into
the warm areas where it picks up energy. Then as the gas pressure falls from
the maximum back to the minimum pressure, some warm gas flows out of this volume into the cold regions. This causes a loss of refrigeration called the pumping loss. This loss occurs on both the cold end and the hot end of V-M refrigerators when displacer seals are used to force gas flow thru the regenerators. The following equation for pumping loss was derived in Reference 3:

\[
Q_{pu} = \frac{2(\pi Dc)^{0.6} L_{cy} (P_{max} - P_{min})^{1.6} N^{1.6} C_p^{1.6} (T_w - T_c)^{2.6}}{1.5^Z R^{1.6} k g^{0.6} \left(\frac{T_w + T_c}{2}\right)^{1.6}}
\]

(26)

where: 
- \(Q_{pu}\) = pumping loss
- \(Dc\) = diameter of cylinder
- \(L_{cy}\) = length of cylinder
- \(P_{max}\) = maximum pressure
- \(P_{min}\) = minimum pressure
- \(N\) = cycle speed
- \(C_p\) = specific heat of gas
- \(T_w\) = temperature at warm end
- \(T_c\) = temperature at cool end
- \(S\) = radial clearance between displacer and cylinder
- \(Z\) = compressibility factor for gas
- \(R\) = gas constant
- \(kg\) = conductivity of gas

By examining Equations (25) and (26) it can be seen that there is an optimum gap (clearance) for best performance (lowest losses). If the gap is too small the shuttle loss will be high and if the gap is too large the pumping loss will be high. Hence, for a fixed diameter and stroke, there is an optimum gap for least losses and there is much to be gained by maintaining the gap...
accurately. To accomplish this the materials of the cylinder and displacer are selected so that their coefficients of thermal expansion are matched.

3. HEAT TRANSFER THROUGH DISPLACER

This loss is the heat transfer thru the displacer due to the difference in the temperatures of the ends of the displacer. It is expressed as:

$$Q_D = \frac{k_D A_D (T_w - T_c)}{L_D}$$  \hspace{1cm} (27)

or if the displacer is hollow:

$$Q_D = \frac{\pi k_D (D_{OD}^2 - D_{ID}^2) (T_w - T_c)}{4L_D}$$  \hspace{1cm} (28)

where:
- $Q_D =$ conduction heat loss thru displacer
- $k_D =$ conductivity of displacer material
- $A_D =$ area of end of displacer
- $L_D =$ length of displacer
- $T_w =$ temperature of warm end
- $T_c =$ temperature of cool end
- $D_{OD} =$ outside diameter of displacer
- $D_{ID} =$ inside diameter of displacer

4. HEAT TRANSFER THROUGH CYLINDER WALL

This loss is the heat transfer due to the difference in the temperatures of the ends of the cylinder. If the inside and outside diameters of the cylinder are known Equation 28 with the appropriate conductivity and dimensions can be used. If the refrigerator is being designed and the thickness of the cylinder wall is not known the following equations can be used:

$$Q_{CY} = \frac{\pi k_{CY} P_{max} D_{CY}^2 (T_w - T_c)}{2\sigma_{CY} \left(1 - \frac{P_{max}}{2\sigma}\right)}$$  \hspace{1cm} (29)
where: $Q_{CY} = \text{conduction heat loss thru cylinder}$

$k_{cy} = \text{conductivity of cylinder material}$

$P_{max} = \text{maximum pressure}$

$D_{cy} = \text{inside diameter of cylinder}$

$L_{cy} = \text{length of cylinder}$

$\sigma = \text{allowable stress}$

5. HEAT GENERATED BY FRICTION BETWEEN DISPLACER AND CYLINDER

This loss is due to friction between the cylinder wall and the displacer riders and seals.

$$Q_F = \int F_f \, dY$$

where: $F_f = \text{friction force in direction of travel}$

$Y = \text{stroke}$

It is rather difficult to evaluate this loss since much must be known about the seal forces, rider loads, and the locations at which this loss is generated.

6. REGENERATOR LOSSES

The regenerators of the V-M refrigerator must be very efficient if the refrigerator is to be a practical device for producing refrigeration at temperatures below 100°K. In fact, the performance of the machine as a whole depends directly on the efficiency of the regenerators. The regenerator losses can be divided into two types:

a. Heat Load Due to Friction in the Regenerator

This refrigeration loss is caused by aerodynamic heating when the working medium flows thru the regenerator. Its derivation is discussed in Reference 3.

$$Q_{RF} = \frac{\Delta P_{\text{Reg}} (P_{max} + P_{min}) V_{CMN}}{\rho j Z R T_c}$$

(30)
where:

\[ \Delta \text{reg} = \frac{(P_{\text{max}} + P_{\text{min}})^2 V_{CM}^2 N^2 f_{LR}}{2 Z^2 R^2 T_c^2 g_c A_{\text{reg}}^2 D_e \rho} \]  

or:

\[ Q_{RF} = \frac{(P_{\text{max}} + P_{\text{min}})^3 V_{CM}^3 N^3 f_{LR}}{2 Z^3 R^3 T_c^3 g_c A_{\text{reg}}^3 \rho^2 D_e J} \]

where: \( Q_{RF} \) = loss due to aerodynamic friction in the regenerator

- \( P_{\text{max}} \) = maximum system pressure
- \( P_{\text{min}} \) = minimum system pressure
- \( V_{CM} \) = maximum displaced cold volume (if a hot regenerator is being calculated this would be maximum displaced hot volume)
- \( T_c \) = temperature of the displaced volume (hot or cold)
- \( N \) = cycle speed
- \( f \) = friction factor
- \( L_R \) = length of regenerator
- \( Z \) = compressibility factor for working fluid
- \( R \) = gas constant for working fluid
- \( A_{\text{reg}} \) = crossectional area of regenerator
- \( \rho \) = density of working fluid
- \( D_e \) = equivalent hydraulic diameter
- \( g_c \) = gravitational conversion factor
- \( J \) = conversion factor

b. Heat Load Due to Limiting Value of Film Coefficient in Regenerator

The limiting value of the film coefficient in the regenerator prevents sufficient cooling of the working fluid as it flows from the hot to the cold end of the regenerator. Reference 3 discussed in general the derivation of the following equations that evaluate the amount of energy that remains in the working fluid because of this limiting value of film coefficient.
\[
Q_{RH} = (1 - N_R)C_p(T_w - T_c) \frac{(P_{max} + P_{min}) V_{CM} N}{2RT_cZ}
\]

where:
\[
N_R = \frac{h'h''L_R^2}{h'h''L_R^2 + h''L_RC_pG \frac{r_h}{r_h} + h'L_RC_pG \frac{r_h}{r_h}}
\]

where: \(Q_{RH}\) = loss due to limiting value of film coefficient in regenerator

\(N_R\) = regenerator efficiency

\(C_p\) = specific heat of working fluid

\((T_w - T_c)\) = temperature difference across regenerator

\(V_{CM}\) = maximum displaced cold volume (if a hot regenerator is being calculated this would be the maximum displaced hot volume)

\(T_C\) = temperature of the displaced volume (hot or cold)

\(N\) = cycle speed

\(R\) = gas constant of working fluid

\(Z\) = compressibility factor

\(h'\) = heat transfer coefficient - flow out

\(h''\) = heat transfer coefficient - flow in

\(L_R\) = length of regenerator

\(G\) = mass velocity (mass flow rate per unit crosssectional area)

\(r_h\) = hydraulic radius

The value of \(N_R\) for cold regenerators is usually in the range from 0.995 to almost 1.0 and for hot regenerators is in the range of 0.9 to 1.0.

These equations for the regenerator losses are only approximate since they are based on average mass flow rates rather than instantaneous mass flow rates. Another way to compute the performance of the V-M refrigerator is partially outlined in Reference 4. It involves cutting the V-M refrigerator into a large number of control volumes and determining the
mass flow rate into and out of each of these control volumes as a function of crank position. The losses, especially the regenerator and pumping losses, can be calculated and their effect on the pressure can be iterated with the mass flow equations so that a more accurate description of the refrigeration at every crank position is obtained. This can be done for an existing refrigerator design with the aid of a computer but is extremely difficult to do when optimizing a new design since the control volumes themselves are being changed during the optimization process. Therefore, the equations presented above or similar equations are usually used for optimizing new designs.

7. NET REFRIGERATION

To obtain the net refrigeration, the cold end losses for each stage are summed and subtracted from the gross refrigeration for that stage. In multistage refrigerators some of the losses from colder stages appear as increased refrigeration at warmer refrigeration stages. These include conduction losses, regeneration heat loss, shuttle loss, and pumping loss. These losses from a colder stage are called interstage heat flow and should be added to the gross refrigeration of the warmer stage. Other cold end losses that were not discussed here but might be applicable depending on refrigerator design are regenerator conduction, insulation, seal leakage losses, and heat leaks down instrumentation leads.
8. TOTAL HEATER POWER INPUT

To calculate the heater power required at the hot end, the hot end losses are summed and added to the heat input to the gas. Losses not discussed above that should be included depending on the actual design are the hot end insulation loss, conduction losses down heater and instrumentation leads, and heat leaks thru the insulation inside the hollow hot displacer.
SECTION IV
V-M COOLER VARIATIONS

1. MULTISTAGE V-M COOLERS

In many applications there is a need to produce useful refrigeration at more than one temperature level at the same time. An example of this is cooling an electronic device to a very low temperature while cooling a dewar heat shield surrounding the device to an intermediate temperature to intercept the dewar heat leaks. This saves considerable power since the majority of the heat is removed at the higher temperature for much less input power (and refrigerator size) than if all the heat had been removed at the lower temperature.

Two mechanisms have been used to add additional cold stages to V-M refrigerators. The parallel cylinder system (Figure 13) adds another smaller diameter cold cylinder parallel to the first cold cylinder. In this configuration the displacer seals are at the ambient temperature region of the displacers (an advantage) but fabrication of the cylinder assembly is more difficult. Keeping the two cylinders straight and parallel from brazing temperature down through cryogenic temperature is difficult and expensive.

The other configuration is the series cold cylinder configuration (Figure 14). This configuration is easier to fabricate but requires a displacer seal at the base of the second stage displacer that seals properly at the first stage (cryogenic) temperature.

Adding additional cold stages affects the previously developed equations in the following ways. Additional cold stage factors
SECOND STAGE HEAT EXCHANGER
SECOND STAGE REGENERATOR
SECOND STAGE (COLDEST) ACTIVE VOLUME
FIRST STAGE ACTIVE VOLUME
FIRST STAGE HEAT EXCHANGER
FIRST STAGE REGENERATOR
THERMAL SHORTING BLOCK
AMBIENT TEMPERATURE SEALS
AMBIENT HEAT EXCHANGER
CONNECTING ROD

Figure 13. Parallel Cold Displacers
Figure 14. Series Cold Displacers
(second term) are added to Equation 20. Additional terms are added to Equation 22 and additional equations similar to Equation 13 are written to describe the gross refrigeration of the additional stages. The ratio between the cold active volumes of a two stage refrigerator is obtained by taking the ratio of the gross refrigeration equations (similar to Equation 13). The result is:

\[ \frac{Q_{C_1}}{Q_{C_2}} = \frac{V_{CM_1}}{V_{CM_2}} \]  

(35)

2. PHASE ANGLE

Phase angles other than 90° were investigated by E. B. Quale and T. T. Rule (Reference 5) and by B. Leo (Reference 3). Phase angles other than 90° complicate the equations (1 through 13) considerably and make the fabrication of parts more difficult. The optimum phase angle is a function of the active swept volumes of the hot and cold cylinders and of the thermal boundary conditions. The investigations showed that for refrigerators with heat rejection temperatures near room temperature, the optimum phase angle would be in the range from 90° to 102° depending on cold end temperature (the lower the temperature the greater the angle). Multistage refrigerators further complicate these relationships and reduce the range of optimum phase angles. Since the gross refrigeration is changing very slowly with respect to phase angle near the optimum, most manufacturers are using a 90° phase angle.
3. SIMILAR CYCLES

There are other heat powered refrigeration cycles similar to the V-M cycle. One by Bush (Reference 6) is quite similar to the V-M except that the two ambient volumes (one at the ambient end of the cold cylinder and one at the ambient end of the hot cylinder) are separated by a thermal regenerator and reject heat to heat sinks at different temperatures. Another heat powered refrigerator was patented by Taconis (Reference 7). It differs from the V-M in the timing of the movements (three instead of four motions) of the displacers. Another heat powered refrigerator was patented by Hogan (References 8 and 9). It produced cooling in the 10° to 20°K range, while the hot end absorbed heat at room temperature and the heat rejection was at 77°K (the heat was rejected to liquid nitrogen). A patent by Cowans (Reference 10) describes a modification to the V-M refrigerator that allows it to drive its own displacers and produce useful shaft power. This is done by increasing the crosssectional area of either the hot displacer connecting rod or both connecting rods, so that with the addition of connecting rod seals, and by lowering the crankcase pressure below the minimum pressure in the V-M cycle, a net force can be created to drive the refrigerator. This has the advantage that the small timing motor used on most V-M refrigerators is not needed (but something must give it a shove to get it started). However this adds the life limiting problem of dynamic connecting rod seals that must be able to seal against the full cycle pressure (several hundred pounds per square inch). This type of sealing problem is
avoided by most V-M refrigerators since in a "pure" V-M cycle the only dynamic seals in the system are the displacer seals. Displacer seals usually experience very small pressure differences of 5 to 15 psi which contributes to their very long life.
SECTION V

V-M REFRIGERATOR MECHANIZATION

The V-M refrigerator is a very compact high performance refrigerator that can produce refrigeration at cryogenic temperatures for long periods of time without maintenance. It can be powered by electrical heating, direct solar energy, exothermic chemical reactions, a gas burner, and even nuclear energy or isotopes. The noise level of the V-M refrigerator is low because of the very small gas pressure difference between the faces of the displacers. This coupled with the low speed of the refrigerator results in low bearing loadings, considerably less wear, and long life. The low speeds and low loads present opportunities to use contamination control techniques, unavailable to highly loaded machines, that significantly improve the time between servicing.

To date there have been a number of V-M refrigerators built for a variety of applications by a number of different designers from several companies. The design philosophy for these refrigerators has varied widely due to constraints imposed by the applications and preferences of the designers.

To date there have been three major philosophies on how to mechanize the V-M cycle to produce long life.

The first mechanization concept (Figure 15) is a refrigeration configuration with the hot and cold cylinders at 90° (sometimes 180°) to each other. The displacers are driven by a simple crank mechanism that is relatively easy to balance if the displacers are the same weight. The crankcase volume is part of the ambient active volume and odd shaped bits of metal are used to reduce the void volume in the crankcase. Dynamic seals are used at the ambient end of both displacers to force the gas to flow thru the regenerators. One of the
more successful seal configurations has been the "C" crosssection glass loaded teflon lip seals. These seals have a special spring inside the "C" to expand the seal lip and provide continuous sealing under a wide variety of temperature and wear conditions. These seals are lightly loaded since they need to seal against only the pressure drop across the regenerator (about 5 to 15 psi). The displacers are guided within the cylinders by rider rings at each end of the displacer. The rider rings act as solid lubricated linear bearings. Common materials used are filled teflons in the cold and ambient regions and fluoride eutectic lubricated composites or carbon in the hot region. The only forces on the displacers in this design are caused by the product of the displacer area and the regenerator pressure drop (a few psi), therefore the bearing loads are very small and only a small motor is needed to drive the mechanism. These low loads coupled with the low speed contributes to the long life of this concept. The bearings usually used are solid lubricant film transfer ball bearings. Journal bearings have also been used, especially in the wrist pins. To prevent contamination of the working fluid that would freeze out in the cold end of the refrigerator, only solid lubricants are used (and the bearings loads derated). Motor windings are kept outside the working fluid space so that the contaminates trapped in the windings and outgassing of the wire insulation will not contaminate the working fluid. This has been accomplished by using an AC induction motor. A matching inverter is used to convert to the proper AC frequency. The motor rotor (solid metal) is inside the working space and is separated from the stator (with its winding) by a thin nonmagnetic pressure shell containing the working fluid. The pressure ratio attained in
this type of refrigerator (maximum cycle pressure to minimum cycle pressure) has been in the range of 1.3 to 1.7. This concept has a finite life since riders, seals, and bearings are wearing components. To date, the most critical life limiting component is the hot displacer rider ring. This mechanization concept results in a long life, relatively compact, rugged refrigerator that is easy to apply, and needs no special handling. One, two, and three stage refrigerators have been built using this concept (References 4 and 11) and a number of these refrigerators have successfully completed environmental and flight qualification testing.

The second long life concept (Figure 16) avoids the seal wear problems by using a combination of labyrinth and clearance seals to provide displacer sealing. Since these seals allow a certain amount of leakage to occur, the refrigerator must be somewhat larger to overcome the effects of this leakage. To minimize the leakage past these displacer seals the pressure drop through the regenerators and therefore across the seals is kept low by increasing the flow area of the regenerators. This requires better gas flow distributors at the ends of the regenerators and low pressure drop heat exchangers. These add to the dead volume of the refrigerator. The effect of adding these dead volumes is to reduce the pressure ratio (maximum cycle pressure to minimum cycle pressure) to about 1.15 which means the refrigerator is larger and less compact than the first concept. The bearings used in this concept are hard-on-hard materials lubricated with MoS$_2$. This has been used for both the linear and rotary bearings. These bearings show excellent promise of long life but have not yet been
tested to destruction in a refrigerator. In this concept there are no organic compounds (not even seals and riders) to contaminate the working fluid. This concept requires parts machined to closer tolerances especially in the bearing, seal, and heat exchanger regions than the refrigerators of concept one. The motor used in this concept is similar to the motor used in the first concept. To date only three versions of this concept are known to exist. All of them are single stage refrigerators (References 12 through 18).

The third long life design concept (Figure 17) is a radical departure from the first two. To attain long life in this concept the entire crankcase mechanism is oil lubricated and the displacer rods are supported on hydrostatic capillary-compensated oil bearings. To prevent the lubricating oil from contaminating the working fluid, a rolling sock diaphragm seal is used on the connecting rods to separate the helium working fluid from the oil filled crankcase. These roll seals are special polyurethane material with a "U" shaped cross section that rolls rather than stretches as the displacer rod moves (Figure 18). These seals are capable of very long life if properly supported by an oil cushion that limits the pressure difference seen by the seal to about 4 atmospheres. The oil cushion pressure must be maintained at all times at the correct value since a pressure reversal across the roll sock seal would cause a failure and too large a pressure difference across the roll sock would cause it to stretch and produce an early failure. Therefore a separate oil pump is used to supply the oil for the oil cushion and a regulator is used to sense the working fluid (helium) pressure and maintain the oil cushion at a fixed differential (about 4 atmospheres) below the working fluid pressure. The
Figure 17. Oil Lubricated V-M Crankcase
Figure 18. Roll Sock Seal Installed
refrigerator includes a safety system to dump part of the helium so that the maximum pressure difference across the rolling diaphragms will not be greater than 4 atmospheres in case the oil cushion pump stops. It should be noted that the oil cushion must be maintained at all times when the working fluid is pressurized above 6 atmospheres, whether the refrigerator is running or not.

Since the diaphragm material with the best life is somewhat permeable to helium a helium refill system was added to the refrigerator to replace the helium lost by permeation and by activation of the safety system. The refill system is composed of a high pressure helium bottle, a pressure regulator valve for filling the refrigerator and a shutoff valve to prevent emptying the refill system when the safety system is activated. Since the refrigerator is designed to operate in a zero gravity environment, it uses a crankcase completely filled with oil so that the oil pump intakes will see only oil. To have a completely oil filled crankcase, a bellows system was necessary to compensate for the change in crankcase volume caused by the movement of the displacer rods. Also, since helium diffuses into the crankcase through the rolling sock seals, a system to remove the helium from the crankcase oil was included. A crankcase heater was included to maintain the crankcase oil within the proper temperature (viscosity) range. The crankcase mechanism is a unique rhombic drive which can be balanced to reduce vibration to an extremely low level. The bearings in the rhombic mechanism are oil lubricated for very long life. These hydrodynamic and squeeze film journal bearings are supplied oil by two oil pumps directly connected to the refrigerator motors.
The refrigerator uses two counter rotating brushless DC motors to drive the rhombic mechanism. The motors are geared together for proper timing and to provide redundancy in case one motor fails. The rhombic drive provides a straight pushpull on the displacer rods. The cold displacer seal is a long close fitting clearance seal that in theory does not touch the cylinder or touches very lightly. The hot displacer is supported by the combination of a hydrostatic bearing in the crankcase and a five-inch long dry lubricated rider-seal on the ambient end of the displacer (a hot rider ring is not needed). This refrigerator has been built in a three stage configuration. The oil lubricated mechanism (with only one dry lubricated rider) potentially offers very long life, however the large part count of the supporting items may detract from this long life potential. Although parts of this system have been tested in other refrigerators, a complete system has not been life tested. This life test is scheduled to begin in the near future.
SECTION VI
V-M ACCESSORIES AND COMPONENTS

1. HOT END TEMPERATURE CONTROLLER

One important accessory required by electrically heated V-M refrigerators is the hot end temperature controller. By examining the V-M theoretical equations, it can be seen that the higher the temperature of the hot end, the higher the efficiency of the refrigerator. V-M refrigerators are usually designed to operate at the highest temperature possible, consistent with metallurgical limits. The most popular hot end material is Inconel 718. The strength of this material falls off quite rapidly above 1250°F, so V-M refrigerators are usually designed to operate at about 1200°F. However, there are several problems with trying to operate at 1200°F. A change of input voltage to the heater can change the heater power and the hot end temperature. Aircraft power supply voltages can vary as much as ±1/6 of the mean voltage. In addition, the ambient heat rejection temperature aboard an aircraft can vary as much as 200°F, which will affect the power requirement and therefore the hot end temperature. Cold end load changes also have an effect on the power required and the hot end temperature. Refrigerator malfunctions such as loss of working fluid or a stalled motor prevent the working fluid from absorbing sufficient heat and causes the hot end to overheat. The heater is usually sized to supply the correct power at the minimum voltage and maximum ambient temperature. To prevent hot end overheating problems, a hot end temperature controller is used.
Proportional controllers are usually used since maintaining the heater at a nearly uniform temperature reduces the heater stresses and improves the life of this component. Frequently a simple ON-OFF controller, set at a higher temperature, backs up the primary controller as an additional safety measure. A variety of controller concepts have been used. One of the most popular is pulse width modulation, due to its high efficiency. However this type controller requires considerable filtering and shielding to prevent the electromagnetic interference it creates from affecting nearby equipment. Other concepts include linear proportional control, zero voltage (AC) switching, slow ON-OFF switching, and mechanical devices such as curie point switches and vapor bulb thermometers. On large V-M refrigerators the controller problem is reduced by calculating both the minimum power and the maximum power required. The minimum power is then supplied by a large heater with a simple ON-OFF controller for malfunction protection only. The difference between the minimum and maximum power is supplied by a smaller heater with a proportional controller and necessary shielding. This arrangement is more efficient and reduces the size and weight of the controller and electronic filters.

2. HEATERS

Two types of electric hot end heaters have been used in V-M refrigerators. The furnace type is a ribbon of heater wire wrapped on a ceramic mandrel and held in place with cement. The ceramic furnace surrounds the hot end of the hot cylinder and transfers heat by radiation. The Calrod type sheathed heater has been the most popular.
Heat is transferred either by radiation (Reference 12) or by brazing the heater sheath directly to the hot cylinder. Since the watt density of the heaters required by most V-M designs is very high (for this type of heater) the heater must be properly heat sunk to the hot cylinder or burnouts will occur. Care also must be taken to be sure that the active (heat producing) portion of the heater terminates while still thermally connected to the hot cylinder. The larger diameter low resistance lead-in wire that runs between the heater wire and the terminal (inside the sheath) must be of a material that does not embrittle or corrode when exposed to insulation or atmospheric contaminants. Single ended straight wire heaters have caused numerous failures and have been largely abandoned in favor of two ended helically wound single wire heaters. Straight wire heaters are available in smaller sheath diameters but must use smaller diameter heater wire since the total length of the heater wire is less. This, coupled with the possible nonuniform reduction of heater wire diameter during the swaging of the heater sheath and higher stresses during heater cycling have contributed to numerous straight wire heater failures. The helical single wire heaters have larger heater sheaths (less convenient for the refrigerator designer) but have a larger diameter longer heater wire that does not change crosssection (the helix angle changes) during the swaging of the heater sheath. The heater wire is closer to the sheath (less temperature drop) and is less sensitive to thermal cycling. These heaters are much more reliable than straight wire heaters. In critical applications additional redundant heaters are added to avoid
scrapping an expensive hot cylinder assembly due to a burned out heater.

3. MOTORS

As mentioned earlier, AC induction motors with the rotor inside the helium space and the stator with its windings outside the helium space have been successfully used in a large number of applications. These motors are either two or three phase and are purchased with a matching inverter. Total efficiency for the motor and inverter is about 25%. In small V-M refrigerators, the motor power is a small fraction of the total power, therefore cleanliness and reliability are more important than efficiency. In a few applications an inverter could not be used (due to space or ambient temperature problems) and since life was less critical a DC brush type motor was used. Special brush materials were used along with special commutator coatings. The motor windings were potted to reduce the generation of contaminants. Very little data has been gathered on this motor, so its limitations are still unknown. A brushless DC motor is being used on one refrigerator. The motor efficiency is expected to be at least 55%. It will be in the oil filled crankcase of the refrigerator and should present no contamination problem.

4. REGENERATORS

The cold regenerators have been previously discussed and are discussed in considerable detail in all of the references. The term internal regenerator is used for regenerators inside the displacer and the term external regenerator is used for regenerators attached to or
a part of the cylinder. Cold regenerator matrix materials are usually screens of 100 mesh to 500 mesh in copper alloys or stainless steel and balls of monel or lead in sizes down to 0.002 inch. Lead balls are usually used for temperatures below 50°K since lead is one of the few materials with appreciable specific heat at these temperatures.

The hot regenerators are described as internal or external also, however the forms of this matrix are more varied. Stacked screens, balls, tubes, and the annulus have been used for the hot regenerator matrix. The internal annular regenerator is composed of the walls of the displacer and the cylinder. In this configuration the gas flows between the displacer and the cylinder. A displacer seal is not used. This eliminates one wearing part (the seal) and eliminates the pumping loss, but makes the radial location of the displacer within the cylinder very critical, which in turn makes hot rider ring wear extremely critical. As an example, if the regenerator is designed with a 0.007 inch radial gap between the displacer and cylinder, and if rider wear allows the displacer to be out of concentricity by 0.002 inch, 15% of the gas flow is on the narrow side of the regenerator while 85% of the flow is on the wide side. This causes the regenerator loss to be doubled (Reference 3). Since lubrication and wear of the hot rider is a serious problem the internal annular regenerator is rarely used anymore. The external annular regenerator is composed of the walls of the cylinder and one or more linear sleeves. The linear sleeves must be very thin and concentric with the cylinder. A displacer seal riding
on the sleeve assures gas flow through the regenerator. Concentricity is a problem with this regenerator also.

The tubular regenerator is composed of small diameter thin wall tubes constrained (usually in a single layer) between the cylinder (or displacer) and a cylindrical liner with the tube axis in the direction of the cylinder axis. The gas flows in the axial direction either through the tubes or in the triangular shaped spaces between the tubes and the liner (or cylinder). This type of hot regenerator is easy to fabricate and is being used in several V-M refrigerators. The screen and ball regenerators are similar to the cold regenerators except matrix elements are larger and of materials such as monel and stainless steel.

5. HEAT REJECTION

Heat rejection has been accomplished in several ways. These include rejection to forced ambient air (Reference 4), rejection to a pumped liquid which in turn rejects the heat to air or a radiator (Reference 11) and rejection by heat pipes (References 12 through 18).
SECTION VII

CONCLUSIONS

This report has described in detail the theoretical V-M refrigerator and the ways it has been applied to produce cryogenic temperatures. Complicating factors such as void volumes and undesirable heat losses were added to the refrigerator description until a real refrigerator was described. Included in this study were the factors and component characteristics which influence the refrigeration capacity, efficiency, and life of Vuilleumier refrigeration systems. The various ways different designers have mechanized this cycle in the quest for long life were discussed. V-M refrigeration technology is now to the point where extended life tests can be run to determine the life of these different V-M refrigerator concepts.
REFERENCES


