Stratospheric Turbulence and Vertical Effective Diffusion Coefficients

N. W. Rosenberg
E. M. Dewan

29 September 1975

Approved for public release; distribution unlimited.

AERONOMY LABORATORY PROJECT 6687
AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
HANSCOM AFB, MASSACHUSETTS 01731

AIR FORCE SYSTEMS COMMAND, USAF
We have obtained an estimate of residence times in the stratosphere in terms of an effective diffusion coefficient, $K_e$. Our approach is based on the hypotheses that (1) CAT (clear-air turbulence) is the major source of vertical transport, and that (2) almost all CAT is caused by the Kelvin-Helmholtz shear instability. According to the best current evidence, turbulent instability occurs whenever the Richardson number $(Ri)$ is less than approximately 0.25.

Our calculations used velocity data from a NASA report of 200 rocket smoke-trail wind profiles at 25 m resolution. Our analysis of stratospheric...
Shears as a function of altitude from this sample (30,000 data points) revealed that about 2% of the altitude consists of thin sporadic layers of high shear, separated by large regions of low shear. Richardson numbers and turbulence frequencies were computed from these shears on the basis of a standard temperature profile.

A model for vertical transport by such intermittently-occurring turbulent mixing zones, separated by regions of negligible mixing, was generated using the turbulence statistics. It leads to a value of $K_o$ which is approximately 0.3 m$^2$/s between 12-18 km, when data at 25 m resolution is used. When 100-m resolution is used, the diffusion estimates are slightly smaller. These results agree with other methods of measuring diffusivity (radioactive fallouts, CH$_4$ loss) and seem to indicate that CAT plays a prominent role in vertical transport in the stratosphere.
STRATOSPHERIC TURBULENCE AND VERTICAL EFFECTIVE DIFFUSION COEFFICIENTS

N.W. ROSENBERG AND E.M. DEWAN
Air Force Cambridge Research Laboratories
L.G. Hanscom Field
Bedford, Massachusetts

ABSTRACT: We have obtained an estimate of residence times in the stratosphere in terms of an effective diffusion coefficient, $K_e$. Our approach is based on the hypothesis that (1) CAT (clear-air turbulence) is the major source of vertical transport, and that (2) almost all CAT is caused by the Kelvin-Helmholtz shear instability. According to the best current evidence, turbulent instability occurs whenever the Richardson number ($R_i$) is less than approximately 0.25.

Our calculations used velocity data from a NASA report of 200 rocket smoke-trail wind profiles at 25 m resolution. Our analysis of stratospheric shears as a function of altitude from this sample (30,000 data points) revealed that about 2% of the altitude consists of thin sporadic layers of high shear, separated by large regions of low shear. Richardson numbers and turbulence frequencies were computed from these shears on the basis of a standard temperature profile.

A model for vertical transport by such intermittently-occurring turbulent mixing zones, separated by regions of negligible mixing, was generated using the turbulence statistics. It leads to a value of $K_e$ which is approximately 0.3 m$^2$/s between 12-18 km, when data at 25 m resolution is used. When 100-m resolution is used, the diffusion estimates are slightly smaller. These results agree with other methods of measuring diffusivity (radioactive fallout, $\text{Cl}_3^-$ loss) and seem to indicate that CAT plays a prominent role in vertical transport in the stratosphere.

INTRODUCTION

In this paper we report computations of the observed frequency distribution of the magnitudes of vertical shears of horizontal winds between 5 and 20 km altitude. We then derive the probability distribution of turbulent layers for various values of vertical thickness, on the basis of accepted relationships between shear and turbulence. An effective vertical-diffusion coefficient, $K_e$, is estimated from a simple model using this empirical turbulence probability distribution.

Specifically, we calculate the effect of clear-air turbulence (CAT) upon vertical transport, using a model consisting of a vertical column of thin, randomly-spaced mixing (turbulent) layers separated by thick non-mixing atmospheric layers. The turbulent layers correspond to the intermittent sporadic CAT “blini” described in the literature (Bretherton, 1969). We also assume that essentially all CAT is due to shear instability of the Kelvin-Helmholtz type, which can occur in stratified fluids. This assumption is shown to have wide experimental support in the current literature. Our results lead to an estimated diffusion coefficient of approximately 0.3 m$^2$/s in the stratosphere. This is roughly consistent with vertical diffusivities estimated from radioactive fallout, which have values ranging from 0.1 m$^2$/s to 1.0 m$^2$/s for tropical and polar stratospheres respectively (Junge, 1963).

CLEAR-AIR TURBULENCE AND KELVIN-HELMHOLTZ BILLOWS

In order to clarify our model, it seems appropriate to briefly review the main background information concerning the Kelvin-Helmholtz (K-H) instability, as well as the evidence that CAT, at the altitude of interest, is almost always due directly to the K-H phenomenon.

The K-H Instability

Kelvin's original paper (Kelvin, 1910) on this phenomenon treated the influence of wind on water waves. Helmholtz was the first, however, to discuss the instability of surfaces separating
ROSENBERG AND DEWAN

fluids which have different velocities. Subsequent literature assigns their names to the more general instability which occurs when there are vertical shears of horizontal velocities across finitely thick layers of vertically stratified fluids. This instability frequently occurs in both the ocean and the atmosphere. Whenever a horizontal layer is buoyantly stable but has a sufficiently high velocity-shear across it, a small perturbation will result in a growing wave which eventually breaks and generates a patch of turbulence. Such breaking “gravity waves” are usually organized in clusters and result in horizontally wide but vertically thin turbulent layers. The criterion for instability is given in terms of Richardson number, Ri, which is defined as

\[ \text{Ri} = \frac{g(\partial \rho / \partial z) / [\rho (\partial u / \partial z)^2]} {\left(\frac{\partial T / \partial z}{\Gamma} + \frac{\Gamma}{(\partial u / \partial z)^2}\right)} \]  \hspace{1cm} (1)

The first form on the right is often used in oceanography, and the second form in atmospheric physics. \( g \) is the acceleration of gravity, \( \rho \) is the fluid density, \( z \) the vertical coordinate, \( u \) the horizontal velocity, \( T \) the temperature in °K, and \( \Gamma \) the adiabatic lapse rate (dry air assumed) (Monin and Yaglom, 1971).

The criterion for stability is that \( \text{Ri} \) be \( >0.25 \) everywhere in the flow (Taylor, 1931; Goldstein, 1931; Miles, 1961; Hazel, 1972; Turner, 1973). This is a general result which has been accepted (within certain restrictions). This criterion does not mean, however, that instability and turbulent breakdown occur whenever and wherever \( \text{Ri} < 0.25 \); it simply means that \( \text{Ri} < 0.25 \) is a necessary condition for turbulence to occur. The physical reason for the value 0.25 is that, for this value of \( \text{Ri} \) (Ludlam, 1967; Businger, 1969a), the available kinetic energy due to the velocity difference across the layer is equal to the work which must be done against buoyancy forces in order to exchange fluid parcels across the layer. Once the energy is available, a perturbation may result in a growing nonlinear oscillation of the layer, in the form of a wave. When turbulence has started (after the wave breaks), mixing can occur within the layer. Businger (1969) has shown that in this case, \( \text{Ri} = 1 \) represents equality between potential energy and available kinetic energy (thus explaining Richardson’s original \( \text{Ri} = 1 \) criterion for the instability threshold). Once a layer has become turbulent, then, one would expect it to continue to be “fed” energy until \( \text{Ri} \) has increased to 1. This occurs when the layer thickens enough to sufficiently lower the shear across it. After such a point, turbulence would be damped by the forces of stable buoyancy.

Internal K-H billows below the ocean surface have been investigated by oceanographers. One simple model for such observations has been given by Woods (1969). He assumed that \( \text{Ri} = 0.25 \) is also a sufficient condition for turbulence. Experimental evidence to date seems to indicate that a value of \( \text{Ri} \) around 0.25 is indeed a sufficient condition in the free ocean and atmosphere (away from boundaries and in the absence of obliquely-shearing oblique winds, etc. (see Hines, 1971). Woods also assumed that \( \text{Ri} = 1 \) was the “cutoff” condition for turbulence. He thus postulated a “hysteresis effect” that would give a stable layer until \( \text{Ri} < 0.25 \), and would then become turbulent until the layer thickened enough to make \( \text{Ri} = 1 \).

Figure 1 (based on his report) shows the sequence of events he observed by means of dye tracers. The initially stable layer goes into oscillation. A wave builds up until nonlinear effects predominate, causing a characteristic “roll-up.” Finally there is turbulent breakdown and layer spread until \( \text{Ri} = 1 \). He assumed that the density (\( \rho \)) and velocity (\( V \)) gradients were zero outside the layer, but that \( \rho \) and \( V \) at the top and bottom layer surfaces remained constant. Under these assumptions, the transition from \( \text{Ri} = 0.25 \) to \( \text{Ri} = 1 \) causes the layer to thicken by a factor of 4. At \( \text{Ri} = 1 \) the turbulence subsides and the layer becomes stable.

The original (and now famous) paper which first described this “roll-up” effect was by Rosenhead (1931); it showed a numerically-generated billow effect. This effect has also been beautifully reproduced in the laboratory by Thorpe (1973), who generated K-H billows by inducing a shear between two initially stable liquids of different densities. His measurements indicated that the actual “cutoff” for turbulence is more like 0.4 ± 0.1 rather than the overly simple theoretical value mentioned above, so \( \text{Ri} = 0.4 \) is known in the literature as the Thorpe number (Garrett and Munk, 1972).
ROSENBERG AND DEWAN

Events were preceded by a period of time in which \( R_i \) was around 0.25. The events themselves lasted approximately 500 sec. on the average, and the condition \( R_i < 0.25 \) was often maintained for over one-half hour prior to the K-H events in the figures he gave. This and other evidence reviewed by Dutton (1971, 1973) make a convincing case that CAT is most likely due to the K-H phenomenon. Thorpe himself refers (1973) to underwater K-H as “underwater CAT.”

\[
\begin{align*}
R_i > 1/4 \\
R_i < 1/4
\end{align*}
\]

(ROSENHEAD)

\[
\begin{align*}
| \frac{\Delta V_x}{\Delta z} | + | \frac{\Delta V_y}{\Delta z} | \leq 0.014 \text{ s}^{-1}
\end{align*}
\]

Figure 2. Schematic of a K-H billow event in the atmosphere as detected by means of radar. (After Browning and Watkins, 1970.)

WIND-PROFILE DATA AND THE STATISTICAL SPATIAL STRUCTURE OF CAT

The experimental data base for our calculation was a series of reports containing 200 vertical profiles of horizontal winds measured from smoke trails by Miller, Henry, and Rowe* (1959-1962, 1965, 1968). They gave wind velocity vectors as a function of height at 25-m intervals with a precision of 0.1 m/s. Figure 3 shows a typical smoke trail. This montage shows the progressive distortion due to shears. In all there were 90,000 data points: 50,000 below 12 km, 30,000 between 12 and 16 km, and 10,000 above 16 km. From these, shears were calculated as \( | (\Delta V_x/\Delta z)^2 + (\Delta V_y/\Delta z)^2 |^{1/2} \), where \( V_x \) and \( V_y \) represent the horizontal wind components.

Figure 4 shows the cumulative frequency distribution for component shears measured at 25-m spacing, where the ordinate is given in units of standard deviations. The linear portions of these curves (the central 90%) correspond to a Gaussian distribution with a standard deviation of 0.014 \( \text{s}^{-1} \) in both components at both altitudes. However, high shears occur more

---

* All profiles were obtained in beautiful weather, and this selection aspect of the method of data collection must be kept in mind.
predict. When the non-Gaussian portions of these curves were plotted with a log-normal scale for
the ordinate, they became linear. According to
Gibson, Stegen, and Williams (1970), who
referred to predictions of Kolmogoroff,
Obukhoff, and Yaglom, there is now good
evidence that the probability distributions of
velocity derivatives are log-normal, provided that
these velocities are part of an inertial-range
turbulent velocity field. This raises a very inter-
esting question: Why should the high shears
which would bring about turbulence have the
log-normal statistics which would be expected
from the turbulent process itself? One might
speculate that the high shears (of horizontal and
presumably laminar winds) have their origin in a
much larger scale of turbulence. We will not
discuss this phenomenon further in this paper,
but simply note that it seems to be of interest.

The critical shear was computed from
$R_i = 0.25$, but since temperature measurements
made simultaneously with the wind data were
not available, we used U.S. Standard Atmosphere
mean temperature gradients of $-6^\circ$/km in the
troposphere and $0^\circ$/km in the stratosphere, with
a $-9.8^\circ$/km adiabatic lapse rate. This led to a
frequently than a Gaussian distribution would

critical shear, $S_c$, of $0.025\ \text{s}^{-1}$ in the 5-12 km

region (troposphere), while for the 12 km - 19

km region (stratosphere), $S_c$ was $0.045\ \text{s}^{-1}$. The

latter higher value, of course, reflects the higher

stability of the stratosphere. Figure 4 shows that

many of the above-threshold shears are in the

log-normal portion of the curves, especially for

the stratosphere.

Using the above values for $S_c$, we obtained

the cumulative frequency distribution, $P_l(L)$, for

finding turbulent layers of thickness L or greater.

Note that only 2% of shears at 25 m resolution

exceed threshold in Figure 4. $P_l(L)$ is related to

the probability $P(L)$ of finding a layer having a

thickness between $L$ and $L + dL$ by

$$P_l(L) = 1 - \int_0^L P(L') dL'$$

(2)

We can therefore derive $P(L)$ from our data by

calculating the difference between $P_l(L)$ for

neighboring values of $L$,

$$\frac{dP_l(L)}{dL} = P(L)$$

(3)

Figure 5 shows a plot of $P_l(L)$ vs. L. We used

the empirical results of Miller et al. (1965)

directly in our calculations. An extrapolation was

made for $L = 0$ by a least-squares fit of the form

$P_l(L) \propto \sqrt{L}$ on available points.

![Figure 5. Cumulative frequency distributions ($P_l(L)$) for various lengths L of unstable layers.](image)

THE VERTICAL-STACK DIFFUSION

MODEL

A one-dimensional model relating effective

vertical diffusivity $K_e$ to $P_l(L)$ is derived below.

In other words, the vertical structure of CAT

layers is determined from wind-shear data, using

the criterion $Ri < 0.25$, and this structure is then

used to determine the vertical transport by

means of the model.

Assumptions

First we shall assume that there is no vertical

transport between turbulent layers: in other

words, all such transport is assumed to take place

within CAT mixing layers. Second, we shall

assume that the horizontal rearrangements of the

layers and material being transported will have

no effect on the vertical transport. This type of

assumption is not unusual in oceanography and

allows the use of a simple one-dimensional model

(Garrett and Munk, 1972).

Model

The fundamental definition of the coefficient

diffusivity provides the basis for the

derivation of our model. Figure 6 shows a

horizontal slab through which material (or heat,

in the general case) diffuses vertically. Suppose

that the concentration $C$ of the material is held

to a constant at the top of the slab by means of

an infinite reservoir, and assume that the down-

ward diffusion is steady-state. Assume that all

material reaching the bottom of the slab drops

into an infinite sink at zero concentration. The

profile of $C$ would then be a straight line as

indicated.

The definition of the coefficient of diffusivity, $K$, is

$$K = \frac{(dn/dt)/(dc/dz)}{dc/dz}$$

(4)

where $dn/dt$ is the number of moles of material

flowing out through a unit of surface area at the

bottom of the slab per unit of time, and $dc/dz$ is

the constant gradient of the concentration with

respect to the altitude, $z$. When vertical motion is

not due to molecular transport effects, but is

instead due to turbulence of some sort, this

motion can be expressed in terms of effective
diffusivity, $K_e$.

To simulate stratospheric vertical motion,

we imagine a series of thin horizontal mixing
Since we assume that the random nature of our turbulent layers will ensure that vertical transport will be, on the average, diffusive in nature, we can assume that $K_e$ also satisfies the diffusion equation:

$$K_e \frac{\partial^2 C}{\partial t^2} = \frac{\partial C}{\partial t}$$

(6)

From this we can calculate the residence time for a layer of pollution. Assuming an initial Gaussian distribution, Eq. (6) leads to

$$C(z,t) = \frac{1}{\sqrt{4\pi t}} \exp \left( -\frac{z^2}{4 K_e t} \right)$$

(7)

(Korn and Korn, 1968). The residence time $t_R$, i.e., the time needed for the Gaussian radius (one-dimensional) to reach a distance $(z) = H$, is therefore

$$t_R = \frac{H}{2K_e}$$

(8)

In order to calculate $K_e$ from Eq. (5) and $t_R$ from Eq. (8) we need to have an estimate of $\Delta t$, and this will be discussed in the next section.

**GROWTH AND DECAY TIME, $\Delta t$, FOR A K-H BILLOW EVENT**

The $\Delta t$ in our model does not correspond to the duration of turbulence, but rather to the time needed for a K-H event to develop after $Ri$ has descended below 0.25. Thus $\Delta t$, or the time between profiles in the model, corresponds to the interval between $Ri < 0.25$ and the time when turbulent breakdown makes $Ri > 0.25$.

We shall estimate $\Delta t$ directly from some observations by Browning (1971) of 17 K-H events. He made measurements simultaneously by radar and balloon soundings. Figure 2, taken from Browning and Watkins (1970), shows that in a typical event, the build-up and breakdown can take place in a period of approximately 1000 seconds. In Browning (1971), information on $\Delta t$ times was available for 6 of the 17 K-H events. The time elapsed between $Ri < 0.25$ and the end of a billow event ($Ri > 0.25$) varied between approximately 1000 seconds and 5000 seconds, averaging about 3000 seconds, and the average duration of the 17 billow events themselves is
approximately 500 seconds. (The exceptional case of a 4-hour billow event was omitted in the calculation of this average.) We chose a time of 3,000 sec for the 5-12 km altitude region which was studied by Browning.

The value of \( \Delta t \) for the stratosphere would differ from the above values. From Rosenhead (1971) it can be seen that \( \Delta t \) is proportional to \( \lambda/U \), where \( \lambda \) is the most unstable wavelength \( \approx 7h \); and where \( h \) is the layer thickness (Turner, 1973), and where \( U \) is half the difference between the velocities on each side of the layer. Thus

\[
\Delta t \propto 7h(du/dz)^{-1} (h/2)^{-1} \propto du/dz^{-1}
\]

where we take \( du/dz \) to be \( S_u \). Since the stratospheric \( S_u \) is approximately twice the size of the tropospheric \( S_u \), \( \Delta t \) in the stratosphere would be 1,500 sec, if we accept 3,000 seconds as the tropospheric \( \Delta t \).

The turbulence should start to decay when \( R_i \) exceeds 0.4, the Thorpe number mentioned above. As we have seen from the data of Browning, the duration of a billow is about 500 sec on the average, which presumably is the time needed to raise the \( R_i \) above the threshold once turbulence has commenced.

RESULTS

Figure 7 summarizes our main findings. The relation

\[
P(L) = 1 - \int_0^L P(L')dL'
\]

was explained earlier, and illustrated in Figure 5. We have also explained how the effective diffusion coefficient, \( K_e \), can be obtained from

\[
K_e = \frac{1}{2\Delta t} \int_0^L P(L')(L')^2dL'
\]

Figure 7 shows \( K_e \) as a function of the upper limit of integration \( L \). Among the 30,000 atmospheric shears, 60 were found with a thickness greater than 200 m \( (P_f(L) = 0.002) \), but none with a thickness greater than 300 m \( (P_f(L) = 0) \). A comparison of Figures 5 and 7 shows how \( P_f(L) \) affects the shape of \( K_e(L) \).

Figure 7 indicates that \( K_e = 0.068 \text{ m}^2/\text{s} \) for data at 25 m resolution. We were also interested in the effect of data resolution on \( K_e \). A four-point moving average was used to smooth component velocity profiles to simulate 100-m resolution. Figure 7 shows that this results in no significant change in the final value of \( K_e \), although the dependences of \( P_f \) and \( K_e \) on \( L \) have been markedly altered (\( K_e \approx 0.054 \)).

Next we consider the effect of vertical spreading of turbulence upon our estimate of \( K_e \). As was discussed earlier, once turbulence has been initiated in regions of high shear, it spreads vertically until the mean shear decreases so much that the increasing Richardson number reaches the "extinction" value of about 0.4.

Figure 8 shows "original" 25-m and 100-m resolution shear profiles. It also shows what happens if we allow turbulence spreading to bring supercritical shears down to their critical values (\( R_i = 0.25 \)).

Figure 9 demonstrates how the spreading was computed. The left side shows a jagged profile of shear vs. altitude, and the right side a hodograph (showing the velocity profile as seen from above). When a supercritical shear is encountered at an altitude \( z \) (e.g., altitude 16.50 km in Figure 9, between point 7 and 8 in the hodograph), a search is made to find the maximum height separation, centered at \( z \), which is still supercritical (\( R_i < 0.25 \)). It is assumed that the profile will take on a constant shear between those two altitudes, with the excess energy going into turbulence. Thus we joined heights 16.30 km and 16.65 km with a constant shear, and the
Figure 8. Effects of resolution and layer spreading of supercritical shear layers. The curve on the right side of each box shows the profile after spreading. These profiles consist of a superposition of 8 trails.

hodograph between these points with a straight-line vector.

Figures 5 and 7 show the effects of spreading on $P_f(L)$ and $K_c(L)$ at 25 m resolution. The $K_c$ estimate has been raised to 0.21 m$^2$/s because of the thickened layers. A decrease in resolution to 100 m is seen to decrease estimated $K_c$ to 0.15 m$^2$/s. From this we would expect that, if the resolution were improved beyond 25 m, one might find a larger value for $K_c$. Trails
with such higher resolution (10 m) are currently being analyzed in our laboratory. If the spreading were allowed to continue until $Ri = 0.4$, $K_e$ would be increased. The spread which would account for a change in $Ri$ from 0.25 to 0.4 is

$$Ri = -g \left( \frac{\theta'}{\bar{\theta}} \right) \frac{1}{(u')^2} = -\frac{g \theta' h^2}{\bar{\theta}(\Delta u)^2}$$  \hspace{1cm} (11)

where $\bar{\theta}$ is the average potential temperature in a layer, $\theta'$ is the potential-temperature gradient, $h$ is the layer thickness, and $\Delta u$ is the difference in horizontal velocity across the layer. $\theta'$ and $\Delta u$ can be presumed to remain approximately constant as the layer expands; thus $Ri$ is proportional to $h^2$. From this we see that $h^2$ would grow by a factor of $(0.4/0.25) = 1.6$. From Eq. (10) we see that an increase of all values of $L$ (or $h$) in this way amounts to multiplying $K_e$ by a factor of 1.6. A spread 25-m profile would then result in

$$K_e = 0.21 \times 1.6 \approx 0.3 \text{ m}^2/\text{s}.$$

Figure 9. Shear profile and velocity hodograph before and after spreading.
In order to see whether the extremely high shears (in excess of 2 \( S_2 \)) were an important factor for the value of \( K_* \), we edited out these high shears (amounting to 0.1% of the sample) and repeated the calculations. We obtained essentially identical results, and so ignored the very high shears thereafter.

Using Eq. (8) with \( H = 10 \) km (corresponding to the growth of a Gaussian radius located at 20 km down to the tropopause at 10 km), we obtain a residence time \( t_R = 3 \) years for \( K_* = 0.3 \) \( \text{m}^2/\text{s} \). Is this an overestimate or an underestimate? It is difficult to answer this question without further information on the reliability of our estimate of \( \Delta t \).

CONCLUDING REMARKS

We used a statistical analysis of 200 wind profiles in conjunction with a vertical-stack diffusion model to calculate the effective vertical diffusivities to be expected from CAT in the stratosphere. (It was also necessary to use the radar and balloon soundings of Browning for these calculations.) Our results indicate that it is likely that \( K_* \) in the stratosphere is in the 0.3 \( \text{m}^2/\text{s} \) range if the spreading of turbulent layers is taken into account. Our results seem to agree with measurements of radioactive fallout (see Junge, 1963) as well as measurements of \( \text{CH}_4 \) loss (see Wofsy and McElroy, 1973). The results are also consistent with the findings of Lilly, Waco, and Adelfang (1973), who derived vertical diffusivities from turbulence spectra observed by aircraft-borne instrumentation, and with the studies discussed by Justus (1973) for the altitude range of interest.

In this way we see that one need not resort to such mechanisms as stratospheric penetration by thunderstorms, "dumping" by global circulation to the poles, aerosol "precipitation", etc. in order to explain observed stratospheric residence times. In other words, it now appears that CAT plays the same prominent role in vertical transport in the stratosphere that "underwater CAT" plays in the World Ocean.

The next steps in this research should involve (a) a careful study of the high shears and possible instrumental effects, (b) better estimates of \( \Delta t \), and (c) analysis of higher-resolution velocity profiles.

ACKNOWLEDGMENTS

We thank Prof. Mark Beran of the University of Pennsylvania and Tel Aviv University, Israel for assistance in the early part of this work. We also thank S. Zimmerman and E. Good for stimulating discussions.

REFERENCES


Businger, J.A. (1969b), "Note on the critical Richardson number(s)," Quart. J. Roy. Met. Soc. 95, 653-654.


**DISCUSSION**

REITER: You've obtained diffusion coefficients on a relatively small scale. If you include synoptic disturbances, the residence times in the lower stratosphere become shorter.

ROSENBERG: Yes, this is only the vertical diffusivity; if other processes contribute, each will have to be weighted accordingly.