CALCULATION OF INTERFERENCE FOR A POROUS WALL
WIND TUNNEL BY THE METHOD OF
BLOCK CYCLIC REDUCTION

PROPULSION WIND TUNNEL FACILITY
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This technical report has been reviewed and is approved for publication.

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The lift interference was calculated for a porous wall wind tunnel by a modified method of block cyclic reduction. This efficient, numerical method has indicated its accuracy by comparison with other available analytical and approximate solutions. A scheme is introduced to search for optimum wall configurations. The lift interference computed for an NACA 64-series finite airfoil in some optimum configurations has...
demonstrated the achievement of minimization of interference. The effect of test section length is also examined.
PREFACE

The work reported herein was conducted by the Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC), under Program Element 65807F. The results of the research presented were obtained by ARO, Inc. (a subsidiary of Sverdrup & Parcel and Associates, Inc.), contract operator of AEDC, AFSC, Arnold Air Force Station, Tennessee. The work was conducted under ARO Project Nos. PF422 and P32A-29A. The authors of this report were C. F. Lo and H. N. Glassman, ARO, Inc. The manuscript (ARO Control No. ARO-PWT-TR-75-63) was submitted for publication on May 21, 1975.
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1.0 INTRODUCTION

The minimization of tunnel wall interferences has become one of the major tasks after the introduction of ventilated transonic tunnels. A variable, but uniformly distributed, porosity wall was designed to reduce interferences at various Mach numbers, e.g., the Aerodynamic Wind Tunnel (4T) at AEDC. The recent requirement for an increase in the size of the testing model to achieve higher Reynolds number creates severe interference which prohibits obtaining useful data. In addition, the axial gradients of interference may cause interference on pitching moment for a long model. By introducing an axially distributed porosity in the walls of a slotted tunnel, the elimination of pitching moment and lift interferences was achieved in the experimental development of walls for V/STOL testing (Ref. 1). It is necessary to search for a theoretical optimum porosity distribution for the minimization of interference as the guideline for an experimental program.

The first theoretical approach to the problem has been carried out in Ref. 2 to reduce the interference in a two-dimensional perforated tunnel by a gaussian type distribution of porosity with an approximate method. Specifically, a system of integral equations was derived using Fourier transform and convolution theorems and then solved by the collocation method with a series form representing the unknown functions. The selection of a gaussian distribution is strictly based on the merits of mathematical simplicity. The reduction of interference is achieved (Ref. 2) by using a simple singularity to represent the test model. This has been extended to a finite chord airfoil to permit comparisons directly with experimental data (Ref. 3). However, the approximate method is limited to certain porosity distributions. The complete elimination of the magnitude and axial gradient of interference requires a nongaussian porosity distribution. To provide such a solution, a numerical method for computing the interference has been developed to search for an optimum configuration in the present study. The application of a modified method of Block Cyclic Reduction (Ref. 4) to the lift interference computation is presented. The scheme to search for an optimum configuration is discussed and extended to a finite airfoil. The lift interference is calculated for an NACA 64-series airfoil in an optimum configuration to demonstrate the achievement of minimization of interference. The effect of test section length is briefly examined.
2.0 GENERAL ANALYSIS

The lift interference in a two-dimensional porous transonic tunnel is formulated for tunnel walls with varying porosity distributions. The optimum porosity distribution may then be obtained by judicious selection for a given application.

2.1 FORMULATION OF MATHEMATICAL PROBLEM

The field equation of an inviscid, irrotational fluid for subsonic flow in terms of the perturbation velocity potential $\phi$ in $X$-$Y$ coordinates (Fig. 1) is

$$\beta^2 \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \quad (1)$$

For the boundary condition of the tunnel, the average mass flow is assumed proportional to the pressure drop across the porous wall as

$$R(x) \frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial y} = 0 \quad \text{at } Y = \pm h \quad (2)$$

where $R(x)$ is the empirical constant, or porosity parameter, of the porous wall and is a function of streamwise location.

---

**Figure 1. Boundary value problem for tunnel lift interference.**
Within the assumptions of linearized theory, the perturbation velocity potential may be divided into two parts as

$$ \phi = \phi + \phi_m $$

(3)

where $\phi$ is the interference potential caused by the presence of tunnel walls and $\phi_m$ is the disturbance potential induced by a model. The linearity of the field equation and boundary conditions in the normalized coordinates $x = X/\beta h$, $y = Y/h$ gives

$$ \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 $$

(4)

and

$$ \frac{R(x)}{\beta} \frac{\partial \phi}{\partial x} \pm \frac{\partial \phi}{\partial y} = - \left( \frac{R(x)}{\beta} \frac{\partial \phi_m}{\partial x} \pm \frac{\partial \phi_m}{\partial y} \right), \ y = \pm 1 $$

(5)

with the upstream and downstream conditions described as

$$ \phi(\pm \infty) = 0 $$

(6)

The formulation is completed with the set of Eqs. (4), (5), and (6). The finite difference method will be used to solve this system. An efficient numerical scheme is provided by the modification of Block Cyclic Reduction to yield a solution of the finite difference equations.

### 2.2 Finite Difference Equations

To develop the finite difference equations, it is assumed that the interference potential $\phi$ effectively becomes zero at a large finite distance from the model location. This distance will be denoted $x^*$. 

Consider the rectangular region

$$ \bar{R} = \{-x^* \leq x \leq x^* \} \cap \{-1 \leq y \leq 1 \} $$

Let $N$ be any positive integer and let $k$ be any nonnegative integer.
Define $M = 2^k$. Let the region $\bar{R}$ be overlaid with a rectangular net with spacings

$$\delta x_i = x_{i+1} - x_i$$

$$i = 0, \ldots, N-1$$

where the mesh points in the $x$ direction may be distributed as desired. It will be required that $x_0 = -x^*$ and $x_N = x^*$.

In the $y$ direction, $\delta y = \frac{1}{M}$ and $y_j = j \delta y \quad j = 0, \pm 1, \ldots, \pm M$.

For notational convenience, column vectors such as

$$\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix}$$

will be denoted as

$$\mathbf{u} = \text{col} \left( u_1, u_2, \ldots, u_N \right)$$

and any $N \times N$ tridiagonal matrix $K$ of the form

$$K = \begin{bmatrix} b_1 & c_1 \\ a_2 & b_2 & c_2 \\ \vdots & \vdots & \vdots \\ a_N & b_N \end{bmatrix}$$

will be denoted by $K = (a_i, b_i, c_i)_{N \times N}$.
Let the value of the solution of the finite difference equations at the point \((x_i, y_j)\) be denoted as \(\phi_{i,j}\) and let

\[
\phi_i = \text{col}(\phi_{1,i}, \phi_{2,i}, \ldots, \phi_{N-1,i})
\]  

\(l = 0, \pm 1, \ldots, \pm M.\)

Using the centered second difference approximation for \(\frac{\partial^2 \phi}{\partial x^2}\) and \(\frac{\partial^2 \phi}{\partial y^2}\) as given in Ref. 5 for variable steps, the finite difference approximation to Eq. (4) on \(\mathcal{R}\) becomes

\[
\begin{align*}
\frac{\phi_{i+1,j}}{\delta x_i (\delta x_i + \delta x_{i-1})} - \frac{\phi_{i,j}}{\delta x_i \delta x_{i-1}} + \frac{\phi_{i-1,j}}{\delta x_{i-1} (\delta x_i + \delta x_{i-1})} \\
+ \frac{\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}}{2\delta y^2} = 0
\end{align*}
\]

\(i = 1, \ldots, N-1\)

\(j = 0, \pm 1, \ldots, \pm (M-1)\)

Let

\[
a_i = 2\delta y^2 / [\delta x_i (\delta x_i + \delta x_{i-1})]
\]

\[
b_i = -2 \left[1 + \delta y^2 / (\delta x_i \delta x_{i-1})\right]
\]

\[
c_i = 2\delta y^2 / [\delta x_{i-1} (\delta x_i + \delta x_{i-1})]
\]

Since it is required that \(\phi(-x^*, y) = \phi(x^*, y) = 0\), there results

\[
\phi_{N,j} = \phi_0, j = 0, j = 0, \pm 1, \ldots, \pm M.\]

Then Eq. (8) may be written

\[
\phi_{j+1} + A\phi_j + \phi_{j-1} = 0
\]
where $A$ is the matrix

$$A = (c_i, b_i, a_i)^{N-1 \times N-1}$$  \quad (9)

The procedure along the boundaries $y = \pm 1$ is as follows:

Let

$$B^+_i = -\left(\frac{\partial \phi_m(x_i, \pm 1)}{\partial x} \pm T_i \frac{\partial \phi_m(x_i, \pm 1)}{\partial y}\right)$$

where $T_i = \beta/R(x_i) \quad i = 1, 2, \ldots, N-1$

On $y = +1$, the difference approximations given in Ref. 5 are again used to approximate Eq. (5) resulting in

$$P_i \phi_{i+1,M} + Q_i \phi_{i,M} + R_i \phi_{i-1,M}$$

$$+ T_i \frac{\phi_{i,M+1} - \phi_{i,M-1}}{2\delta y} = B^+_i$$

where

$$P_i = \frac{\delta x_i}{\delta x_i} \left[\delta x_i \left(\delta x_i + \delta x_{i-1}\right)\right]$$

$$Q_i = \frac{(\delta x_i - \delta x_{i-1})}{(\delta x_i \delta x_{i-1})}$$

and

$$R_i = -\frac{\delta x_i}{\delta x_i} \left[\delta x_{i-1} \left(\delta x_i + \delta x_{i-1}\right)\right]$$

Eq. (4) is also required to hold for $y = +1$ which gives

$$\phi_{i,M+1} + a_i \phi_{i+1,M} + b_i \phi_{i,M} + c_i \phi_{i-1,M} + d_i \phi_{i,M-1} = 0$$

Eliminating $\phi_{i,M+1}$ from these two equations yields

$$T \phi_{M-1} = E \phi_M + f^+$$  \quad (11)
where $E$ is the matrix

$$E = \begin{bmatrix} \frac{1}{2} (r_i^* - T_i c_i), & \frac{1}{2} (q_i^* - T_i b_i), & \frac{1}{2} (p_i^* - T_i a_i) \end{bmatrix} \quad N-1 \times N-1$$

with $q_i^* = 2\delta y_q_i^*$, $p_i^* = 2\delta y p_i^*$, $r_i^* = 2\delta y r_i^*$, $f_i^* = -\delta y b_i^*$

and where

$$T = \begin{bmatrix} 0, & T_i, & 0 \end{bmatrix} \quad N-1 \times N-1.$$

In a similar manner it can be shown that on the boundary $y = -1$,

$$T^{\phi}_{\gamma-M} = E^{\phi}_{\gamma-M} + f^-. \quad (12)$$

The set of finite difference equations (Eqs. (8), (11), and (12)) is readily solved for the determination of lift interference once the lift potential is established.

### 3.0 LIFT INTERFERENCE

The lift interference factor is defined by

$$\delta = \frac{C}{SC_L} \frac{1}{U} \frac{\partial \phi}{\partial y}$$

In particular, the factor along the centerline, $y = 0$, can be obtained by

$$\delta = \frac{C}{SC_L} \frac{1}{U} \frac{\phi_1 - \phi_{-1}}{2\delta y} \quad (13)$$

where $\phi_1 - \phi_{-1}$ can be computed by solving an $N-1$ system of equations using the Modified Method of Block Cyclic Reduction which is described in Appendixes A and B.
3.1 SMALL CHORD AIRFOIL

In the first step, a simple vortex is chosen to represent the lift model as

\[ \phi_m = \frac{-\Gamma}{2\pi} \tan^{-1} \frac{y}{x} \]  

(14)

A solution for the case of a wall with a uniform porosity distribution has been obtained to check with the known analytical solution case and is shown in Fig. 2. The second case, computed for an inverse gaussian distribution of \( R/\beta \), is compared with results obtained by the approximate method (Ref. 2) in Fig. 3. The agreement between the results using the proposed technique and previous solutions for the above cases indicates that the accuracy of the present numerical solution is satisfactory.

![Graph showing comparison of analytical and numerical methods](image)

Figure 2. Comparison of block cyclic reduction and analytic solutions for walls with uniform porosity distribution.

3.2 OPTIMUM POROSITY DISTRIBUTION

The ideal porosity distribution for a tunnel wall is defined as that which induces no lift interference anywhere in the test section. In the mathematical sense, the upwash interference, \( \partial \phi / \partial y \), vanishes everywhere; or the interference potential is a trivial solution of the system
of Eqs. (4), (5), and (6). This solution can be obtained by observation as the right-hand side of Eq. (5) becomes zero and substituting Eq. (4) then

\[
R(x)/\beta = \left( \frac{\partial \phi}{\partial y} / \frac{\partial \phi}{\partial x} \right) y = \pm 1
\]

\[= x\]  

However, the porosity parameter for the perforated wall \( R/\beta \) can only have a positive value because the mass flow is always from the high-pressure to the low-pressure side. Thus, a distribution of \( R(x)/\beta \) is selected and shown in Fig. 4 and Table 1 denoted by Configuration C as

\[
R(x)/\beta = \begin{cases} 
  x/\beta h & x > 0 \\
  0 & x \leq 0
\end{cases} \]

(16)

to evaluate the interference. The lift interference factor for Configuration C has been calculated and is shown in Fig. 5. The interference factors for three additional Configurations D, E, and F (Fig. 4 and Table 1)
Figure 4. Wall configuration with various porosity distributions.

Table 1. Wall Porosity Distribution, R/β for Various Configurations

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14
with a slight variation from Configuration C have been calculated and are presented in Fig. 5. Also plotted in Fig. 5 are results for walls with uniform (Configuration A) and inverse gaussian (Configuration B) porosity distributions. It seems that Configurations C and D give, overall, less interference.

3.3 FINITE CHORD AIRFOIL CASE

For a finite chord airfoil with camber and incidence, a discrete distribution of vortices can be used as

$$\phi_m = -\frac{1}{2\pi} \sum \alpha_j(\zeta_j) \Delta \zeta \cdot \tan^{-1} \frac{v}{x - \zeta_j}$$  \hspace{1cm} (17)
The results for the NACA 64-series airfoil with a chord $C = 0.5/\beta h$ are presented in Fig. 6 and indicate that Configurations D and E exhibit the most satisfactory distribution of porosity to obtain the minimum interference factor.

![Figure 6. Lift interference on a finite chord airfoil in tunnels with various wall configurations.](image)

3.4 EFFECT OF TEST SECTION LENGTH

Most analytical approaches in wind tunnel theory have assumed the length of test section to be infinite for mathematical simplicity. The effect of test section length on the lift interference is of interest since the actual tunnel test section length is usually about two to three times the test section height. The versatility of the present approach can be applied to examine the effect of test section length. For the uniform porosity distribution case, the comparison of lift interference of a finite test section as $-2 \leq x/\beta h \leq 3$ (upstream and downstream regions using solid walls) with the infinite test section is shown in Fig. 7 and indicates the effect on the interference in the region $x/\beta h > 2$. It can be seen that the assumption of an infinite length test section for the calculation of interference in the neighborhood of the model appears reasonable.
4.0 CONCLUDING REMARKS

An efficient numerical scheme has been developed by a modification to the Block Cyclic Reduction Method for computing lift interference in a wind tunnel with an arbitrary distribution of wall porosity. A comparison with other available analytical and approximate solutions has demonstrated the accuracy of the present numerical method. The optimum porosity distribution to minimize interference is obtained by the variation of the ideal mathematical configuration which produces exact interference-free condition. The minimization of interference is also presented for a finite chord airfoil in the optimum wall configurations.

The optimum wall porosity configurations have been calculated for both a simplified ideal airfoil and a finite chord airfoil. The effect of test section length has been also studied.

REFERENCES


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APPENDIX A
METHOD OF BLOCK CYCLIC REDUCTION

Consider the problem of solving the finite difference analog to Laplace's equation with the boundary conditions

$$\phi(-x^*, y) = \phi(x^*, y) = 0 \quad -1 \leq y \leq 1$$  \hspace{1cm} (A-1)

$$\phi(x, \pm 1) = g^\pm(x) \quad -x^* \leq x \leq x^*$$

and where $g^\pm(x)$ are given functions with

$$g^\pm(x^*) = g^\pm(-x^*) = 0.$$  

It is well known that replacing Laplace's equation on the region $R$ by a centered second difference approximation and imposing the boundary conditions given in Eq. (A-1) yields the problem

$$D \phi = y$$  \hspace{1cm} (A-2)

where $D$ is the $(2M-1) (N-1) \times (2M-1) (N-1)$ real symmetric matrix which has the block tridiagonal form

$$D = [I, A, I]_{2M-1 \times 2M-1}$$

and $A$ is the matrix defined in Eq. (9).

The vector $\phi$ will be given in partitioned form as

$$\phi = \text{col} \left( \phi_{M-1}, \phi_{M-2}, \cdots, \phi_{1-M} \right).$$

Likewise, the vector $y$ is given by

$$y = \text{col} \left( -\phi_M, 0, \cdots, 0, -\phi_{-M} \right).$$

In their description of Block Cyclic Reduction, Buzbee, Golub, and Nielson (Ref. 6) first write Eq. (A-2) as
\[ A_{M-1}^j + \phi_{M-2} = Y_{M-1} \]  
\[ \phi_{j+1} + A \phi_j + \phi_{j-1} = 0 \quad j=0, \pm 1, \ldots, \pm (M-2) \]  
\[ \phi_{2-M} + A \phi_{1-M} = Y_{1-M} \cdot \]  

Then for \( j = l-1, l, l+1 \) where \( l = -M+2, \ldots, M-2 \), Eq. (A-3b) can be written

\[ \phi_{l+1} + A \phi_l + \phi_{l-1} = 0 \]
\[ \phi_{l+2} + A \phi_{l+1} + \phi_l \]
\[ \phi_{l} + A \phi_{l-2} + \phi_{l-2} = 0 \cdot \]

Then multiplying the middle equation by \(-A\) and adding the three equations yields

\[ \phi_{l+2} + (2I-A^2)\phi_l + \phi_{l-2} = 0 \]
for \( l = -M+2, \ldots, M-2 \).

Buneman, as described by Hockney (Ref. 7) proceeds by these steps and then reapplies the method. Thus with

\[ A_1 = 2I-A^2 \cdot \]
\[ \phi_{l+4} + A_1 \phi_{l+2} + \phi_l \]
\[ \phi_{l+2} + A_1 \phi_l + \phi_{l-2} \cdot \]
\[ \phi_{l} + A_1 \phi_{l-2} + \phi_{l-4} = 0 \]

where \( l = -M+4, \ldots, M-4 \)

and again, multiplying the middle equation by \(-A\), and adding yields

\[ \phi_{l+4} + (2I-A_1^2)\phi_l + \phi_{l-4} = 0 \cdot \]

Then repeating the process of cyclic reduction recursively, Buneman obtains for the \( i^{th} \) recursion
$\phi_{j+2i} + A_i \phi_j + \phi_{j-2i} = 0$

$A_i = 2I - A_i^{-1}$

$A_0 = A$

Hence, when $j = 0$ and $i = k$ there results

$\phi_M + A_k \phi_0 + \phi_{-M} = 0$

so that

$\phi_0 = -A_k^{-1} (\phi_M + \phi_{-M})$.

$\phi_M$ and $\phi_{-M}$ are known values from Eq. (A-1); hence, $\phi_0$ may be found by inverting an $(N-1) \times (N-1)$ matrix. Once $\phi_0$ is known, the method may be repeated on the regions

$R_U = \left\{ (x,y) - x^* \leq x \leq x^*, \quad 0 \leq y \leq 1 \right\}$

and $R_L = \left\{ (x,y) - x^* \leq x \leq x^*, \quad -1 \leq y \leq 0 \right\}$

solving for $\frac{\phi_M}{2}$ and $\frac{\phi_{-M}}{2}$.

These steps are repeated until all the vectors $\phi_l \ l = 0, \pm 1, \ldots, \pm (M-1)$ are found. Each step requires finding the solution to $N-1$ linear equations.
APPENDIX B
MODIFICATION OF BLOCK CYCLIC REDUCTION

The set of finite difference equations (Eqs. (8), (11), and (12)) was developed in Section 2.2 and is given by

\[ \phi_{j+1} + A \phi_j + \phi_{j-1} = 0 \]
\[ j = 0, \pm 1, \ldots, (M-1) \quad (B-1) \]

\[ T \phi_{M-1} = E \phi_M + \phi^+ \]
\[ (B-2a) \]

\[ T \phi_{1-M} = E \phi_{-M} + \phi^- \]
\[ (B-2b) \]

It will be shown that the vectors \( \phi_M \) and \( \phi_{-M} \) can be found as the solution to a system of \( 2(N-1) \) linear equations.

At this point, a change of notation will be made for convenience.

Let

\[ V_{\ell+M} = \phi_{\ell} \quad \ell = -M, \ldots, M \quad (B-3) \]

Applying Eq. (B-3) to Eq. (A-4) results in

\[ V_{j+2+M} + A \phi_{j+M} + V_{j-2+M} = 0 \]
\[ (B-4) \]

Theorem 1

\[ V_{\ell+1} = F_n \phi_{\ell} + G_n \phi_{\ell+2n} \]
\[ (B-5) \]

where

\[ n = 1, 2, \ldots, K+1 \]

and \( \ell \) is such that

\[ \ell \geq 0 \text{ and } 2^n + \ell \leq 2M \]
where

\[ F_n = F_{n-1} - G_{n-1} A_{n-1}^{-1} \quad n = 2, \ldots, K+1 \]  \hspace{4cm} (B-6)

\[ G_n = - G_{n-1} A_{n-1}^{-1} \quad n = 2, \ldots, K+1 \]  \hspace{4cm} (B-7)

and where

\[ F_1 = - A_0^{-1}, \quad G_1 = - A_0^{-1}. \]

**Proof**

In Eq. (B-4) let \( j = 1 - m + \ell \) and \( i = 0 \).

Then

\[ V_{2+\ell} + A_0 V_{1+\ell} + V_\ell = 0 \]

hence

\[ V_{\ell+1} = - A_0^{-1} (V_\ell + V_{\ell+2}) \]

\[ = F_1 V_\ell + G_1 V_{\ell+2} \]

so the theorem holds for \( n = 1 \). These steps would complete the proof for \( K = 0 \) so now assume \( K > 0 \) and suppose Eq. (B-5) holds for \( n = L, L = 1, \ldots, K \).

Then

\[ V_{\ell+1} = F_L V_\ell + G_L V_{\ell+2L} \]  \hspace{4cm} (B-8)

where \( \ell \) is such that

\[ \ell \geq 0 \quad \text{and} \quad 2^L + \ell < M. \]

In Eq. (B-4) then, let \( j = 2^L - 2K + \ell \) and let \( i = L \).

Then

\[ V_{2^L+1+\ell} + A_L V_{2^L+\ell} + V_\ell = 0 \]
or

\[ V_{2L+\ell} = -A_{L}^{-1}(V_{\ell} + V_{2L+1+\ell}) \]  \hspace{1cm} (B-9)

Substituting Eq. (B-9) into the inductive hypothesis Eq. (B-8) gives

\[
V_{\ell+1} = F_{L} V_{\ell} + G_{L} \left[ -A_{L}^{-1}(V_{\ell} + V_{2L+1+\ell}) \right]
\]

\[
= \left( F_{L} - G_{L} A_{L}^{-1} \right) V_{\ell} - G_{L} A_{L}^{-1} V_{2L+1+\ell}
\]

\[
= F_{L+1} V_{\ell} + G_{L+1} V_{2L+1+\ell}
\]

and hence the proof is complete.

Then with \( n = K+1 \) and \( \ell = 0 \). Eq. (B-5) becomes

\[
V_{1} = F_{K+1} V_{0} + G_{K+1} V_{2K+1}
\]  \hspace{1cm} (B-10)

\[
= F_{K+1} V_{0} + G_{K+1} V_{2M} .
\]

In a similar manner, it may be shown that

\[
V_{2M-1} = F_{K+1} V_{2M} + G_{K+1} V_{0} .
\]  \hspace{1cm} (B-11)

Applying Eq. (B-3) to Eqs. (B-10) and (B-11) gives

\[
\phi_{1-M} = F_{K+1} \phi_{-M} + G_{K+1} \phi_{M}
\]  \hspace{1cm} (B-12)

\[
\phi_{M-1} = F_{K+1} \phi_{M} + G_{K+1} \phi_{-M} .
\]  \hspace{1cm} (B-13)

Substituting Eqs. (B-10) and (B-11) into Eqs. (B-2a) and (B-2b) respectively, yields

\[
T \left[ F_{K+1} \phi_{M} + G_{K+1} \phi_{-M} \right] = E_{M} + \phi^{+}
\]  \hspace{1cm} (B-14)

\[
T \left[ F_{K+1} \phi_{-M} + G_{K+1} \phi_{M} \right] = E_{M} + \phi^{-}
\]  \hspace{1cm} (B-15)
which may be written in Block Matrix form as

\[
\begin{bmatrix}
    T_F K+1 - E & G_{K+1} \\
    G_{K+1} & T_F K+1 - E
\end{bmatrix}
\begin{bmatrix}
    \phi^+_M \\
    \phi^-_M
\end{bmatrix}
= \begin{bmatrix}
    f^+ \\
    f^-
\end{bmatrix}. \tag{B-16}
\]

It will be noted that Eq. (B-16) is a linear system of 2(N-1) equations which can be solved for $\phi^+_M$ and $\phi^-_M$. Once these vectors are known, the problem becomes one of the Dirchlet type which can be solved by the methods of Appendix A.

In Theorem I, let $n = K$ and $\ell = M$; then Eq. (B-5) becomes

\[
V_{M+1} = F_K V^M + G_K V^{2M}. \tag{B-17}
\]

Also, writing Eq. (B-4) with $j = 0$, $i = K$ and again noting that $2^K = M$ results in

\[
V^{2M} + A_K V^M + V_0 = 0
\]
or

\[
V^M = -A_K^{-1}(V_0 + V^{2M}). \tag{B-18}
\]

Substituting Eq. (B-18) into Eq. (B-17) gives

\[
V_{M+1} = F_K \left[-A_K^{-1}(V_0 + V^{2M})\right] + G_K V^{2M}
\]
\[
= (G_K - F_K A_K^{-1})V^{2M} - F_K A_K^{-1} V_0.
\]

Letting

\[
S = G_K - F_K A_K^{-1} \text{ and } W = -F_K A_K^{-1}
\]
gives

\[
V_{M+1} = S V^{2M} + W V_0
\]
and then by use of Eq. (B-3)

\[
\phi_1 = S\phi^+_M + W\phi^-_M.
\]
In a similar manner it is found that

\[ \phi_{-1} = W\phi_M + S\phi_{-M}. \]

Then

\[ \phi_1 - \phi_{-1} = (S-W) (\phi_M - \phi_{-M}). \]  \hspace{1cm} (B-19)

Subtracting Eq. (B-15) from Eq. (B-14) results in

\[ (T_{FK_{+1}} - T_{GK_{+1}} - E) (\phi_M - \phi_{-M}) = (f^+ - f^-) \]

so that

\[ \phi_1 - \phi_{-1} = (S-W) (T_{FK_{+1}} - T_{GK_{+1}} - E)^{-1} (f^+ - f^-) \]  \hspace{1cm} (B-20)

hence, \( \phi_1 - \phi_{-1} \) can be computed by solving an N-1 system of equations and the interference factor in Eq. (13) is obtained.
APPENDIX C
METHOD OF EVALUATION

The evaluation of the lift interference by use of Eq. (B-20) is greatly hindered by the number of operations required to evaluate the recursion matrices \( F_{K+1} \) and \( G_{K+1} \). However, these computations may be greatly simplified by a simple application of induction and it is shown by use of Eqs. (B-6) and (B-7) that

\[
G_K = (-1)^K (A_{K-1} \ldots A_1 A_0)^{-1} \quad (C-1)
\]

and

\[
F_K = \sum_{\ell=1}^K G_\ell
\]

so that

\[
F_{K+1} - G_{K+1} = \sum_{\ell=1}^{K+1} G_\ell - G_{K+1} = F_K.
\]

Since \( S - W = G_K \), Eq. (B-20) becomes

\[
\phi_1 - \phi_{-1} = G_K (T F_K - E)^{-1} (\tilde{f}^+ - \tilde{f}^-).
\]

Now consider the matrix \( A \) given by Eq. (9).

Define the matrix

\[
D = \begin{bmatrix} 0, \ d_i, \ 0 \end{bmatrix}_{N-1 \times N-1}
\]

where \( d_1 = 1 \) and \( d_{j+1} = a_j \ d_j / C_{j+1}, \ j = 1, \ldots, N-2. \quad (C-2) \)

Define \( \hat{A} = D^{1/2} \ A \ D^{-1/2} = \begin{bmatrix} \hat{c}_i, \ \hat{b}_i, \ \hat{a}_i \end{bmatrix}_{N-1 \times N-1} \)

where \( \hat{b}_i = b_i \quad i = 1, \ldots, N-1 \)

and \( \hat{c}_i = \hat{a}_{i-1} = \sqrt{c_i \ a_{i-1}} \quad i = 2, \ldots, N-1. \)
Then $\hat{A}$ is a real symmetric matrix for which many well known computer programs can be used to compute the eigenvalues and eigenvectors $\lambda_j$ and $x_j \cdot j = 1, \ldots, N$.

From Eq. (C-3), $\hat{A}$ and $A$ are seen to be similar matrices hence $\lambda_j$ and $D^{-1/2}x_j$ form an eigenvalue, eigenvector pair for $A$.

Let $X$ be the unitary matrix whose columns are the set of orthonormal eigenvectors of $\hat{A}$.

Then $X^{-1} \hat{A} X = \Lambda$

where $\Lambda = \begin{bmatrix} 0, & \lambda_1, & 0 \end{bmatrix}_{N-1 \times N-1}$.

Hence $Q^{-1} A Q = \Lambda$ where $Q = D^{-1/2}X$.

Now suppose there exists a matrix $Q_k$ which diagonalizes $A_k$ so that $Q_k^{-1} A_k Q_k = \Lambda_k$.

Then since $A_{k+1} = 2I - A_k^2$,

$$ Q_k^{-1} A_{k+1} Q_k = Q_k^{-1} 2I Q_k - Q_k^{-1} A_k Q_k $$

$$ = 2I - Q_k^{-1} A_k Q_k Q_k^{-1} A_k Q_k $$

$$ = 2I - \Lambda_k^2. $$

Hence, $Q_k$ diagonalizes both $Q_k$ and $Q_{k+1}$. Then dropping the subscript on $Q$ gives

$$ Q^{-1} A_k Q = \Lambda_k $$

so that from Eq. (C-1)

$$ G_k = (-1)^k (A_{k-1} \ldots A_1 A_o)^{-1} $$

$$ = (-1)^k (Q \Lambda_{k-1} Q^{-1} Q \Lambda_{k-2} Q^{-1} \ldots Q \Lambda_o Q^{-1}) $$

$$ = (-1)^k (Q \Lambda_{k-1} \ldots \Lambda_1 \Lambda_o Q^{-1}) $$
But \( Q^{-1} = (D^{-1/2}X)^{-1} = X^{-1} D^{1/2} \)

and since \( X \) is a unitary matrix,

\[
Q^{-1} = X^T D^{\frac{1}{2}}. \quad \text{So finally,}
\]

\[
G_k = (-1)^k (D^{-\frac{1}{2}} X \lambda_{k-1} \ldots \lambda_1 \lambda_0 X^T D^{\frac{1}{2}}). 
\]

Hence the work of computing \( G_k \) is simplified since most matrices involved are diagonal matrices.

It will be noted that \( G_k \) and \( F_k \) are functions only of the mesh and hence the interference may be computed via Eq. (C-2) for many different porosity distributions \( T \) without having to recompute these matrices.
APPENDIX D
COMPUTER PROGRAM

Program Description

MAIN

The main program is for control purposes. SETUP should be called immediately. TFIX is called whenever a new porosity distribution is desired. EVALU8 is called to compute the lift interference. In the sample listing, the interference is computed for the four porosity distributions given in Table 1.

B

Function B is a user-supplied routine and is used to compute

\[ B_i^\pm = - \frac{R(x_i)}{\beta} \frac{\partial \phi_m(x_i, \pm 1)}{\partial x} \pm \frac{\partial \phi_m(x_i, \pm 1)}{\partial y}. \]

This equation is similar to the one following Eq. (9). It need be rewritten only when the model velocity potential is changed.

CHOLES

Subroutine CHOLES solves the linear system given by Eq. (C-2) by the method of matrix factorization.

DFFIX

Subroutine DFFIX computes the vector \( \xi^+ - \xi^- \).

EVALU8

This subroutine evaluates the finite difference coefficients, constructs the matrix \((TF_k - E) G_k^{-1}\), and calls subroutine CHOLES to solve Eq. (C-2).

FFIX

This subroutine constructs the matrices \( F_k \) and \( G_k \) by the methods described in Appendix C.
HDIAG

Subroutine HDIAG computes the eigenvectors and eigenvalues of the matrix $\hat{A}$ defined by Eq. (C-3).

MESH

This user-supplied subroutine is used to fill the X and DX arrays. Note that a value is assigned to X(0), namely X(0) = -X*.

MULT and MULT2

These routines perform the FORTRAN matrix replacements $B = AB$ and $A = AB$, respectively.

SETUP

This is a user-supplied routine used to initialize all program constants.

TFIX

TFIX loads the vector $T(x_i)$ by calling TFUNC.

TFUNC

This is a user-supplied routine used to evaluate the function.

$$TFUNC (x) = R(x)/\beta$$

It should be noted here that in the program the array $T$ contains values of $R/\beta$ and not $\beta/R$ as given following Eq. (9).
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MAIN

IMPLICIT REAL*8(A-H,O-Z)
COMMON T(49),VEC(49),DUM(49,50)
COMMON/XX/XD,X(50)
COMMON/I1/N,M,N1,M1,K
COMMON/CTRL/IC
CALL SETUP
DO 50 ICC=1,4
IC=5-ICC
CALL TFIX
CALL EVALU8
DO 39 l=1,N1
39 WRITE(6,10)X(l),T(l),DUM(l),N
10 FORMAT(SX,I3,5X,F10.3,5X,8ETA/R0=,D16.8,X,
**DELTAS=,D16.8)
50 CONTINUE
STOP
END

B

DOUBLE PRECISION FUNCTION B(J)
IMPLICIT REAL*8(A-H,O-Z)
COMMON/I1/DXO,DX(49),DUM(5),XINF,P12
COMMON/XX/XD,EX(50)
COMMON T(49)
COMMON/BROWN/GAM
I=IABS(J)
X=EX(I)
Y=I/J
B=(-Y*T(I)+Y*X)/(X*X+1.DO)*GAM/P12
RETURN
END
SUBROUTINE CHOLES(A, N, NV, ID1, ID2, MATSYM)
REAL*8 A(ID1, ID2), SUM, TEMP
M = N + NV
NARD = N + 1
IF(A(I, I).NE.0.0) GO TO 47
DO 37 J = 2, N
IF(A(J, I).EQ.0.0) GO TO 37
IFLIP = J
GO TO 27
37 CONTINUE
GOTO 54321
27 DO 57 K = 1, M
TEMP = A(IFLIP, K)
A(IFLIP, K) = A(I, K)
AL(I, K) = TEMP
57 CONTINUE
47 T = 2*M
A(I, J) = A(I, J) / A(I, I)
2 CONTINUE
DO 6 I = 2, N
DO 7 J = 2, M
IF(MATSYM .EQ. 0) GO TO 49
IF(I - J).EQ. 69, 6B, 67
49 IF(J.GT.I) GO TO 69
68 K = J - 1
SUM = 0.0
DO 3 IR = 1, K
SUM = SUM + A(I, IR) * A(IR, J)
3 CONTINUE
A(I, J) = A(I, J) - SUM
GO TO 7
69 K = 1 - 1
SUM = 0.0
DO 4 IR = 1, K
SUM = SUM + A(I, IR) * A(IR, J)
4 CONTINUE
IF(A(I, I).EQ. 0.0) GO TO 54321
A(I, J) = (A(I, J) - SUM) / A(I, I)
GO TO 7
67 A(I, J) = A(I, J) * A(J, J)
7 CONTINUE
6 CONTINUE
DO 52 NPROB = NARD, M
DO 52 K = 2, N
I = N + 1 - K
SUM = 0.0
LL = I + 1
DO 51 IR = LL, N
SUM = SUM + A(I, IR) * A(IR, NPROB)
51 CONTINUE
A(I, NPROB) = A(I, NPROB) - SUM
52 CONTINUE
GO TO 12345
54321 N = -1
12345 RETURN
END
SUBROUTINE DFFIX
IMPLICIT REAL*8(A-H,O-Z)
COMMON T(49),VEC(49)
COMMON/I1/N1,N1,N1
COMMON/R1/DXO,DX(49),DY
DO 1 I=1,N1
1 VEC(I)=-DY*(B(I)-B(-1))
RETURN
END

SUBROUTINE EVALU8
IMPLICIT REAL*8(A-H,O-Z)
COMMON T(49),VEC(49),DUM(49,50),FK(49,49),APROD(49,49)
COMMON/R1/DXO,DX(49),DY,BETA,RO,W,CS,XINF,P12,Y,DY2
COMMON/I1/N1,M1,M1,K1,IDL,IDL2,IZ,N2
COMMON/ABC/A(49),B(49),C(49)
100 CALL DFFIX
DO 1 J=1,N1
1 DUM(J,1)=FK(I,J)
Q1=DY2*(DX(1)-DX(I))/D(1)+D(1)/DX(I)
R1=DY2*(DX(I-1)-DX(I))/D(1)+D(1)/DX(I)
DUM(1,1)=DUM(1,1)-50*(T(1)*Q1-B(1))
DUM(1,2)=DUM(1,2)-50*(T(1)*P1-A(1))
DO 65 I=2,N2
Q1=DY2*(DX(I)-DX(I-1))/D(1)+D(1)/DX(I-1)
R1=DY2*(DX(I-1)-DX(I))/D(1)+D(1)/DX(I-1)
DUM(I,1)=DUM(I,1)-50*(T(I)*Q1-B(I))
DUM(I,1)=DUM(I,1)-50*(T(I)*Q1-B(I))
65 DUM(I,I+1)=DUM(I,I+1)-50*(T(I)*P1-A(1))
R1=DY2*(DX(N1)-DX(N2))/D(1)+D(1)/DX(N2)
Q1=DY2*(DX(N1)-DX(N2))/D(1)+D(1)/DX(N2)
DUM(N1,N2)=DUM(N1,N2)-50*(T(N1)*R1-C(N1))
DUM(N1,N1)=DUM(N1,N1)-50*(T(N1)*Q1-B(N1))
CALL MULT2(DUM,APROD)
DO 20 I=1,N1
20 DUM(I,N)=VEC(I)
DO 20 I=1,N1
DO 20 I=1,N1
4 DUM(I,N)=DUM(I,N)/(2.0*DY)
RETURN
END
SUBROUTINE FFIX
IMPLICIT REAL*8(A-H,O-Z)
COMMON T(49),VEC(49),TMP(49,50),FK(49,49),APROD(49,49)
COMMON/R1/DX0,DX(49),DY,DUM(5),PI2
DIMENSION D(49),DG(49),DF(49)
COMMON/I1/N,M,N1,M1,K
COMMON/ABC/A(49),B(49),C(49)
DY2=2.DO*DY*DY
DO 20 I=1,N1
A(I)=DY2/(DX(I)*(DX(I)+DX(I-1)))
B(I)=-2.DO-DY2/(DX(I)*DX(I-1))
20 C(I)=DY2/(DX(I-1)*(DX(I)+DX(I-1)))
   D(I)=1.DO
N2=N1-1
DO 30 I=1,N2
30 D(I+1)=A(I)*D(I)/C(I+1)
   DO 60 I=1,N1
   DO 60 J=1,N1
60 FK(I,J)=0.DO
   FK(I,1)=B(I)
   DO 40 I=2,N1
   FK(I,I)=B(I)
   FK(I,I-1)=DSQRT(C(I)*A(I-1))
40 FK(I-1,I)=FK(I,I-1)
   IEGN=0
   CALL HDIAG(FK,N1,IEGN,TMP,NRN)
   DO 50 I=1,N1
   VEC(I)=1.DO/DSQRT(D(I))
50 D(I)=FK(I,I)
   DO 1 D(I)=0.DO
   1 DG(I)=1.DO
   DO 2 IK=1,K
   DO 2 I=1,N1
   DG(I)=D(I)*DG(I)
   DF(I)=(-1)**IK/DG(I)+DF(I)
2 D(I)=2.DO-D(I)**2
   EE=(-1)**K
   DO 10 I=1,N1
10 DG(I)=EE*DG(I)
   DO 501 J=1,N1
   DO 501 I=1,N1
   APROD(I,J)=TMP(I,J)*DG(J)
501 FK(I,J)=TMP(I,J)*DF(J)
   DO 502 I=1,N1
   DO 502 J=1,N1
   APROD(I,J)=VEC(I)*APROD(I,J)
FFIX

502 FK(I,J)=VEC(I)*FK(I,J)
   DO 503 I=2,N1
   IM1=I-1
   DO 503 J=1,IM1
   TEMP=TMP(I,J)
   TMP(I,J)=TMP(J,I)
503 TMP(J,I)=TEMP
   CALL MUL2(APROD,TMP)
   CALL MUL2(FK,TMP)
   DO 504 J=1,N1
   TOM=1.DO/VEC(J)
   DO 504 I=1,N1
   APROD(I,J)=APROD(I,J)*TOM
504 FK(I,J)=FK(I,J)*TOM
RETURN
END
SUBROUTINE HDIAG (H,N,IENGEN,U,NR)

PROGRAHED BY F. J. CARBATO AND N. MERWIN OF THE MIT
COMPUTATION CENTER.

THIS SUBROUTINE COMPUTES THE EIGENVALUES AND EIGENVECTORS
OF A REAL SYMMETRIC MATRIX, H, OF ORDER N (WHERE N MUST BE LESS
THAN 51), AND PLACES THE EIGENVALUES IN THE DIAGONAL ELEMENTS OF
THE MATRIX H, AND PLACES THE EIGENVECTORS (NORMALIZED ) IN THE
COLUMNS OF THE MATRIX U. IENGEN IS SET AS 1 IF ONLY EIGENVALUES
ARE DESIRED, AND IS SET TO 0 WHEN VECTORS ARE REQUIRED. NR CON-
TAINS THE NUMBER OF ROTATIONS DONE.

H, N, IENGEN, U, AND NR OF THE ARGUMENT LIST ARE DUMMY VARIABLES
AND MAY BE NAMED DIFFERENTLY IN THE CALLING OF THE SUBROUTINE.

SUBROUTINE PLACES COMPUTER IN THE FLOATING TRAP MODE
THE SUBROUTINE OPERATES ONLY ON THE ELEMENTS OF H THAT ARE TO THE
RIGHT OF THE MAIN DIAGONAL. Thus, ONLY A TRIANGULAR
SECTION NEED BE STORED IN THE ARRAY H.

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION H(49,49),U(49,49),X(49),IQ(49)
2 FORMAT(14H MAX OFF DIAU=,F14.7,3HRNR=,I3)
2001 FORMAT(1X,BE15.8)
2002 FORMAT(18H ORTHOGONAL MATRIX)
2003 FORMAT(15H ROTATED MATRIX)
SQRT(X)=DSQRT(X)
ABS(X)=DABS(X)

IF(IENGEN,NE,0) GO TO 15
10 DO 14 I=1,N
   DO 14 J=1,N
   IF(I-J,NE,0) GO TO 12
   U(I,J)=0.0
   GO TO 14
12 U(I,J)=0.0
14 CONTINUE
15 NR = 0
   IF(N-1,LE,0) GO TO 1000
C SCAN FOR LARGEST OFF-DIAGONAL ELEMENT IN EACH ROW
C X(I,I) CONTAINS LARGEST ELEMENT IN ITH ROW
C IQ(I) HOLDS SECOND SUBSCRIPT DEFINING POSITION OF ELEMENT
17 NM1=N-1
   DO 30 I=1,NM1
      X(I) = 0.0
      IPII=I+1
      DO 30 J=IPII,N
         IF(X(I,J)-ABS(H(I,J)).LT.0.0) GO TO 30
      20 X(I,J)=ABS(H(I,J))
         IQ(I)=J
      30 CONTINUE
C SET INDICATOR FOR SHUT-OFF. RAP=2**-27 NR=NO. OF ROTATIONS
RAP=7.450580596D-9
HDTEST=1.0038
C FIND MAXIMUM OF X(I) S FOR PIVOT ELEMENT AND
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HDIAG

C TEST FOR END OF PROBLEM

DO 70 I=1,NMI
   IF(MMAX .LE. 0) GO TO 60
   IF(MMAX .GT. 0) GO TO 70

60 XMAX=X(I)
   IPV=I
   JPV=IQ(I)

70 CONTINUE

C IS MAX. ELEMENT EQUAL TO ZERO, IF LESS THAN HDTEST, REVISE HDTEST

80 IF(HTEST .LE. 0) GO TO 90
85 IF(HMAX .GT. HDTEST) GO TO 148

90 HDIMIN = ABS(H(I,I))
   DO 110 I=2,N
      IF(HDIHIN .LE. 0.0) GO TO 100
   110 CONTINUE

C HDTEST HDIMIN=RAP

C RETURN IF MAX. X(I,J) LESS THAN [2**27]ABS(H(I,I,J)-MIN)

100 IF(HTEST .LE. 0) GO TO 1002

C COMPUTE TANGENT, SINE AND COSINE, H(I,I,J)

150 TAN=SIGN(2.0,H(IPV,IPV)-H(IPV,JPIV))/ABS(H(IPV,IPV)+H(IPV,JPIV))
   SINE=TANG*COSINE
   HDI=H(IPV,IPV)

152 HTEMP = H(IPV,IPV)
   H(IPV,IPV) = H(JPIV,JPIV)
   H(JPIV,JPIV) = HTEMP

C RECOMPUTE SINE AND COS

HTEMP = SIGN(1.0,-SINE) * C SINE
   COSINE = ABS(SINE)

153 CONTINUE

C INSPECT THE I(J) BETWEEN I+1 AND N-1 TO DETERMINE

C WHETHER A NEW MAXIMUM VALUE SHOULD BE COMPUTED SINCE

C THE PRESENT MAXIMUM IS IN THE I OR J ROW.

DO 350 I=1,NMI
   IF(I-IPV.EQ.0) GO TO 350
   IF(I-IPV.LT.0) GO TO 210

200 IF(I-JPIV.EQ.0) GO TO 350
210 IF(I-Q(I)-IPV.EQ.0) GO TO 240
230 IF(I-Q(I)-JPIV.NE.0) GO TO 350
240 K = IQ(I)

250 HTEMP = H(I,K)
   H(I,K) = 0.0

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HDIAG

IPL1 = I+1
X(I) = 0.0

C SEARCH IN DEPLETED ROW FOR NEW MAXIMUM
DO 320 J = IPL1,N
IF( X(I) - ABS( H(I,J) ) ) GT 0.0 ) GO TO 320
300 X(I) = ABS( H(I,J) )
IQ(I) = J
320 CONTINUE

H(I,K) = HTEMP
350 CONTINUE

X(IPIV) = 0.0
X(JPIV) = 0.0

C CHANGE THE OTHER ELEMENTS OF H
DO 530 I = 1,N
IF( I - IPIV .EQ. 0 ) GO TO 530
IF( I - IPIV .GT. 0 ) GO TO 420
370 HTEMP = H(I,IPIV)
H(I,IPIV) = COSINE*HTEMP + SINE*H(I,JPIV)
IF( X(I) - ABS( H(I,IPIV) ) GE 0.0 ) GO TO 390
380 X(I) = ABS( H(I,IPIV) )
IQ(I) = IPIV
390 H(I,JPIV) = -SINE*HTEMP + COSINE*H(I,IPIV)
IF( X(I) - ABS( H(I,JPIV) ) GE 0.0 ) GO TO 530
400 X(I) = ABS( H(I,JPIV) )
IQ(I) = JPIV
420 IF( I - JPIV .EQ. 0 ) GO TO 530
IF( I - JPIV .GT. 0 ) GO TO 480
430 HTEMP = H(JPIV,I)
H(JPIV,I) = COSINE*HTEMP + SINE*H(I,JPIV)
IF( X(JPIV) - ABS( H(JPIV,I) ) GE 0.0 ) GO TO 450
440 X(JPIV) = ABS( H(JPIV,I) )
IQ(JPIV) = I
450 H(JPIV,I) = -SINE*HTEMP + COSINE*H(JPIV,I)
IF( X(JPIV) - ABS( H(JPIV,I) ) GE 0.0 ) GO TO 530
IF( X(JPIV) - ABS( H(JPIV,I) ) LT 0.0 ) GO TO 400
480 HTEMP = H(JPIV,I)
H(JPIV,I) = COSINE*HTEMP + SINE*H(JPIV,I)
IF( X(JPIV) - ABS( H(JPIV,I) ) GE 0.0 ) GO TO 500
490 X(JPIV) = ABS( H(JPIV,I) )
IQ(JPIV) = I
500 H(JPIV,I) = -SINE*HTEMP + COSINE*H(JPIV,I)
IF( X(JPIV) - ABS( H(JPIV,I) ) GE 0.0 ) GO TO 530
510 X(JPIV) = ABS( H(JPIV,I) )
IQ(JPIV) = I
530 CONTINUE

C TEST FOR COMPUTATION OF EIGENVECTORS
IF( IGEN .NE. 0 ) GO TO 40
540 DO 550 I = 1,N
HTEMP = U(I,IPIV)
U(I,IPIV) = COSINE*HTEMP + SINE*U(I,JPIV)
550 U(I,JPIV) = -SINE*HTEMP + COSINE*U(I,JPIV)
GO TO 40
592 CONTINUE
1000 RETURN

END
MESH

SUBROUTINE MESH
IMPLICIT REAL*8(A-H,O-Z)
COMMON/R1/DX0,DX(49),DY,BETA,RO,C,CS,XINF
COMMON/I,I/N,N,N1
COMMON/X/XD,X(50)
IZ=0
X(IZ)=-XINF
X(N)=XINF
DO 1 I=1,5
  1 X(I)=-XINF+I*3.00
  DO 2 I=6,44
    2 X(I)=-5.00+(I-5)*.2500
  DO 3 I=45,49
    3 X(I)=5.00+(I-45)*3.00
  DO 4 I=IZ,N1
    4 DX(I)=X(I+1)-X(I)
RETURN
END

MULT

SUBROUTINE MULT(A,B)
IMPLICIT REAL*8(A-H,O-Z)
COMMON/I,I/N,N,N1
DIMENSION A(49,1),B(49,1),TEMP(49)
DO 1 J=1,N1
  1 SUM=0.00
DO 2 K=1,N1
  2 SUM=SUM+A(J,K)*B(K,J)
2 TEMP(I)=SUM
  DO 1 I=1,N1
    1 B(I,J)=TEMP(I)
RETURN
END
MULT2

SUBROUTINE MULT2(A, B)
IMPLICIT REAL*8(A-H,O-Z)
COMMON/I1/N, M, N1
DIMENSION A(49,1), B(49,1), TEMP(49)
DO 1 I=1, N1
DO 2 J=1, N1
SUM=0.D0
DO 3 K=1, N1
3 SUM=SUM+A(I, K)*B(K, J)
2 TEMP(J)=SUM
DO 1 J=1, N1
1 A(I, J)=TEMP(J)
RETURN
END

SETUP

SUBROUTINE SETUP
IMPLICIT REAL*8(A-H,O-Z)
COMMON/R1/DX0, DX(49), DY, BETA, RO, C, CS, XINF, PI2, Y, DY2
COMMON/I1/N, M, N1, M1, K, ID1, ID2, IZ, N2
COMMON/BROWN/GAM
C
C THIS SUBROUTINE IS FOR INITIALIZING PROGRAM CONSTANTS
C
Y=1.0D0
XINF=20.0D0
N=50
M=16
K=3
BETA=4.0D0
RO=1.0D0
GAM=1.0D0
ID1=49
ID2=50
IZ=0
N1=N-1
N2=N-2
DY=2.0D0*Y/M
DY2=2.0D0*DY
PI=3.1415 9265 3589 7932 D0
PI2=2.0D0*PI
CALL MESH
CALL FFIX
RETURN
END
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TFIX

SUBROUTINE TFIX
IMPLICIT REAL*8(A-H,O-Z)
COMMON T(49)
COMMON/I/NI,M,N1
COMMON/R/DX0,DX(49),JY,BLTA,RO,C,CS,XINF
COMMON/XX/XX,XX(50)
DO 2 I=1,N1
2 T(I)=TFUNC(I,X(I))
RETURN
END

TFUNC

DOUBLE PRECISION FUNCTION TFUNC(I,X)
IMPLICIT REAL*8(A-H,O-Z)
COMMON/CTRL/IC
TFUNC=DMAX1(X,0.DO)
GOTO(1,2,3,4,IC)
1 RETURN
2 IF(I.EQ.26)TFUNC=0.DO
IF(I.EQ.27)TFUNC=.25DO
IF(I.EQ.28)TFUNC=.5D0
RETURN
3 IF(I.EQ.26.OR.I.EQ.27)TFUNC=0.DO
IF(I.EQ.28)TFUNC=.25DO
IF(I.EQ.29)TFUNC=.5D0
IF(I.EQ.30)TFUNC=.85DO
IF(I.EQ.31)TFUNC=1.2D0
IF(I.EQ.32)TFUNC=1.55D0
RETURN
4 IF(I.EQ.26.OR.I.EQ.27.OR.I.EQ.28)TFUNC=0.DO
IF(I.EQ.29)TFUNC=.20D0
IF(I.EQ.30)TFUNC=.45DO
IF(I.EQ.31)TFUNC=.7D0
IF(I.EQ.32)TFUNC=.975D0
IF(I.EQ.33)TFUNC=1.3D0
IF(I.EQ.34)TFUNC=1.6D0
IF(I.EQ.35)TFUNC=1.95D0
IF(I.EQ.36)TFUNC=2.375D0
IF(I.EQ.37)TFUNC=2.85D0
IF(I.EQ.38)TFUNC=3.25D0
RETURN
END
NOMENCLATURE

C  Airfoil chord length

$C_L$  Lift coefficient

h  Semiheight of tunnel

R(x)  Porosity parameter

S  Airfoil surface area

U  Free-stream velocity

x, y  Normalized Cartesian coordinates

X, Y  Cartesian coordinates (Fig. 1)

T  $\beta/R(x)$

$\beta$  Compressibility parameter

$\gamma$  Vortex strength

$\delta$  Lift interference factor, Eq. (13)

$\delta x, \delta y$  Finite spacing in x and y direction

$\Phi$  Perturbation velocity potential

$\phi$  Interference velocity potential

$\phi_m$  Model velocity potential