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ORBIT DETERMINATION IN THE PRESENCE OF UNMODELED ACCELERATIONS

B. D. Tapley

Texas University at Austin

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B. D. Tapley
Principal Investigator

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Department of Aerospace Engineering
and Engineering Mechanics
The University of Texas at Austin
Austin, Texas

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This report summarizes the results of three years work on Grant AFOSR 72-2233. Based on the results obtained during the course of study, it can be concluded that using observations from topocentric tracking stations, the Dynamic Model Compensation Method can be used to obtain an accurate approximation of the unmodeled aerodynamic forces acting on a space vehicle due to 1) Uncertainties in the atmospheric model and 2) Variations in the drag parameter. The accuracy of the estimate will be influenced by the accuracy and density of the observations and the tracking station distribution. The observation accuracy

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20. Abstract (Continued)

requirements for successful implementation require participating satellites; that is, accurate estimates of the density cannot be obtained with "skin-track" or passive observations. However, there are alternate formulations which hold potential for drag estimation using passive satellite observations.

ORBIT DETERMINATION IN THE PRESENCE OF UNMODELED ACCELERATIONS

Introduction

The determination and prediction of the orbit of a satellite in the near-earth environment is complicated by the fact that the satellite is influenced by the non-spherical effects of the Earth's gravitational field as well as the dissipative effects of the Earth's atmosphere. The effects of the atmosphere are difficult to determine because the atmospheric density and, hence, the drag undergo large unmodeled fluctuations. For many artificial satellites this fluctuation in the drag, rather than the observational error, is the main source of error in orbital predictions. The actual fluctuations in the drag can be correlated with the dynamic state of the Earth's atmosphere, i.e., the hourly, daily, and monthly fluctuations in the atmospheric density, as well as in the orientation of the spacecraft's attitude. This latter effect, i.e., changes in the spacecraft's attitude will directly influence both the drag coefficient and the projected cross-sectional area. Previous discussions of the effect of atmospheric drag on the determination and prediction of the orbits of near-earth satellites are given in References 1-18.

In the presence of unmodeled acceleration due to dynamic model error, both batch and sequential estimation algorithms can yield inaccurate results. Due to the "random nature" of fluctuations in the atmospheric drag, this is an important consideration in establishing the accuracy with which the orbit of a near-earth satellite can be determined. The nature of the errors, which occur when the batch processor is used, is discussed in Ref. (8); while a discussion of the errors which occur when the sequential processor is used are discussed in Refs. (9), (10), and (11). The correction for the effects of dynamic model error (or unmodeled accelerations) can be implemented, most easily, using the sequential estimation algorithm. The algorithms which account for noise in the state equations, i.e., the equations of motion, are derived by adding a term to the equation for propagating the covariance matrix associated with the state estimate. This additive term is chosen to account for the uncertainty in the mathematical model. This method of accounting for the effects of unmodeled accelerations suffers from two significant disadvatages:

1) The accuracy with which the estimate can be obtained depends on the value chosen for the additive state-noise covariance term. If the additive term is too large, undue emphases will be placed on the most recent observations. If the term is too small, the state estimate convariance can become too small and the estimate can diverge.

2) Such an estimation algorithm does not yield any direct information concerning the values of the unmodeled accelerations.

In Ref. (12), a method for estimating the state of a spacecraft in the presence of dynamic model error (unmodeled accelerations) is described. The proposed method, referred to as the Dynamic Model Compensation method, processes the data in a sequential fashion. The method has the advantage that in addition to obtaining a more precise estimate of the state of the spacecraft, the values of the unmodeled accelerations are determined also as a function of time. The method is statistically categorized as a first-order auto-regressive estimation procedure in which the unmodeled acceleration in each of the three components of the equations of motion is modeled as a first-order Gauss-Markov process.[13] The correlation times are treated as unknown parameters and are estimated also during the orbit determination procedure.

In Ref. (14), the method is applied to the problem of estimating the unmodeled accelerations acting on a lunar satellite. Range-rate data obtained during the Apollo 10 and 11 missions is processed and a significant improvement in the accuracy with which the orbit is determined over that obtained by the NASA Manned Spacecraft Center using a conventional batch processor is obtained. In some regions, the range-rate residual is reduced by a factor of approximately ten. The estimated values of the unmodeled accelerations are repeatable from revolution to revolution within a given mission and from mission to mission when the spacecraft covers the same ground track. In addition, the variations in the unmodeled acceleration show a high physical correlation with the reported location of lunar surface mascons[15]. In Refs. (16) and (17), the estimates of the unmodeled accelerations are shown to be an accurate representation of the acceleration due to the lunar surface mascons. These results yield a high confidence in the ability of the algorithm to estimate the unmodeled accelerations. In Ref. (18), the application of the procedure to the problem of estimating the random accelerations acting on the rotational motion of the earth is discussed.

Study Objectives

During the course of this investigation, the algorithm for estimating the state of a dynamical system in the presence of unmodeled accelerations, i.e., the Dynamic Model Compensation (DMC) algorithm, has been applied to the problem of estimating the motion of a near-earth satellite influenced by the non-central components of the earth's gravitational field and the effects of atmospheric drag. Since one of the major limiting factors in the 14th Aerospace Force's orbit determination and prediction capability is the effects of the drag fluctuations due to the unmodeled changes in the atmospheric density, a basic objective of the investigation was to improve the precison of the orbit determination and prediction capabilities of the 14th Aerospace Force. The initial objective of the study was the evaluation of the capability of the DMC method proposed in Ref. (12) to improve the precision with which the state of an orbiting object influenced by unmodeled atmospheric drag can be determined. A secondary objective was to improve the prediction capability by analyzing the time history of the random fluctuations in the atmospheric drag over a given orbit to obtain smoothed coefficients for describing the secular effects of the drag. Additional possible applications of the method include the development of an improved capability for terminal impact prediction, the improvement of system calibration methods and finally the possibility of determining indirectly the attitude behavior or orbiting objects. Since the atmospheric drag force will depend on both the projected cross-sectional area and the drag coefficient and since these functions will depend on the attitude of the spacecraft, a sufficiently fine delineation of the random accelerations acting on an orbiting objective will yield information which can be correlated directly with the attitude of the spacecraft.

As a related matter, the computational characteristics of batch and sequential estimation algorithms were studied with regard to computing time, computer storage requirements, estimation accuracy, and convergence characteristics to determine the efficiency with which each algorithm performs the real-time orbit determination functions.

Results

The results obtained during the course of these studies are described in Refs. (19) through (45). In Ref. (34) and (45), results are presented which are pertinent to the problem of estimating the atmospheric drag on satellites in circular and small eccentricity orbits. In Ref. (45), three different procedures were developed. The proposed methods were tested by numerically simulating the orbit determination problem with the simulated true atmosphere defined to be the analytic Jacchi-Roberts atmosphere[21] and the filter atmosphere approximated locally with an exponential atmosphere. The results obtained in this investigation indicate that the simulated world atmospheric density and ballistic coefficient for the satellite could be accurately estimated with either of the two empirically adaptive processes. In Ref. (34), it was shown that under favorable tracking conditions, errors in the atmospheric density can be estimated to one part in 10^{-15}g/cm. The dynamic model compensation representation used in Ref. (34) can be described as follows. If the atmospheric density, ρ, and the drag parameter, B_c, are assumed to vary as follows:

$$\dot{\rho} = d[\rho_o \exp[-\beta(r-r_o)]/dt + u_1 \tag{1}$$

$$\ddot{B}_c = \omega^2[B_c - B_o] + u_3 \tag{2}$$

$$\dot{\beta} = u_2, \; \dot{B}_o = u_4, \; \dot{\omega} = u_5 \tag{3}$$

where r_o is a reference radius, ρ_o is the density at radius r_o, B_o is the mean drag parameter, ε, ω, and ϕ are constants and the random noise components u_i, $(i = 0, 1, \ldots, 5)$, are each assumed to satisfy the conditions $E[u_i] = 0$ and $E[u_i(t)u_j(\tau)] = q_i(t)\delta(t-\tau)\delta_{ij}$ where $\delta(t-\tau)$ is the Dirac Delta function and δ_{ij} is the Kronecker Delta, then the differential equations governing the model compensated motion can be expressed as follows:

$$\begin{aligned}
&\dot{\underline{r}} = \underline{v} \\
&\dot{\underline{v}} = \underline{a}_m(r,v,t) + X_7 X_9 |v_a| \underline{v}_a \\
&\dot{X}_7 = X_8 \\
&\dot{X}_8 = -X_{10}^2(X_7 - X_{11}) + u_1 \\
&\dot{X}_9 = \{\tfrac{d}{dt} \{X_9 (t_k) \exp [-X_{12} (r-r_i)]\} + u_2 \\
&\dot{X}_{10} = u_3 \\
&\dot{X}_{11} = u_4 \\
&\dot{X}_{12} = u_5
\end{aligned} \tag{4}$$

where $\underline{r}$ is the position vector, $\underline{v}$ is the velocity vector and $\underline{v}_a$ is the velocity of the satellite with respect to atmosphere.

Following the demonstration of the applicability of the representation shown above to the problem of estimating the atmospheric density and time varying drag coefficient, for satellites in circular and small eccentricity orbits the application of the method to the problem of estimating t ie drag acceleration on a highly eccentric orbit using actual observed data for comparison was initiated. The data obtained by tracking the Atmospheric Explorer-C (AE-C) satellite is particularily appropriate for such a study since this satellite is instrumented with a set of minature electostatic accelerometers (MESA) from which the estimates of the total drag force can be obtained. In the proposed study, The U ersity of Texas Orbit Processor Incorporating Statstical Analysis (UTOPIA) was to be used to perfrom the computational requirements of the study. During the final stages of study, simulated observations were used to evaluate the effects of the tracking network and the observation quality on the accuracy of estimates of the parameters in the model atmosphere representation. To develop an adaptive model atmosphere which could be used for long term prediction, the exponential model used in Eq. (1) was replaced with the Modified Harris-Priester Model.[21] In this model, the density is expressed as follows:

$$\rho(h) = \rho_o(h)\ (1 + P_1 t)\ (1 + P_2 \operatorname{Cos} P_3 \tfrac{\psi}{2})$$

$\rho(h)$ is the atmospheric density at altitude h above the reference ellipsoid,

$\rho_o(h)$ is determined from the Harris-Priester table by exponential interpolation,

P_1, P_2, P_3 are adjustable model parameters

t is the integration time and

ψ is the angle between satellite position vector and the vector toward the apex of the maximum diurnal density bulge.

Using this algorithm, observation data obtained from the AE-C was to be processed in order to determine the applicability of the alternate algorithm using actual tracking observations.

In addition to comparing the Analytic-Jacchia and the Modified Harris-Priester atmospheres, the study, also, compared the Russian model atmosphere proposed in Ref. 47. The conclusions of this study were that the Russian model atmosphere requires the least computational effort. The Analytic Jacchia was the most accurate. An additional conclusion from the comparison was that the modified Harris-Priester atmosphere variation could be adjusted to have the same frequency and phase angle as the Jacchia atmosphere with an associated computing time which was only slightly greater than the Russian atmosphere. As a consequence, it was concluded that the Modified Harris-Priester model holds the greatest promise for developing an adaptive atmospheric model.

Finally, a significant effort has been made to determine the most efficient method for propagating the state error covariance matrix. In the investigation, both quadratic and square root propagation algorithms have been evaluated in an attempt to define the operational advantages and disadvantages of each method. The methods evaluated include:

1) numerical integration of the matrix Racitti equation,
$\dot{\overline{P}} = \overline{A}P + \overline{PA}^T + Q$

2) the algorithm $\overline{P} = \phi P \phi^T + \tilde{Q}$ where $\dot{\phi} = A\phi,, \phi(t_k, t_k) = I$ and

3) a new algorithm for propagating the square root of the covariance

matrix in lower triangular form.

Conclusion

Based on the results obtained during the course of study, it can be concluded that using observations from topocentric tracking stations, the Dynamic Model Compensation Method can be used to obtain an accurate approximation of the unmodeled aerodynamic forces acting on a space vehicle due to 1) Uncertainties in the atmospheric model and 2) Variations in the drag parameter. The accuracy of the estimate will be influenced by the accuracy and density of the observations and the tracking station distribution. The observation accuracy requirements for successful implementation require participating satellites; that is, accurate estimates of the density can not be obtained with "skin-track" or passive observations. However, there are alternate formulations which hold potential for drag estimation using passive satellite observations.

Recommendations for Further Study

Topics which can be recommended as candidates for further study include the following.

1) Utilization of the DMC algorithm to process actual tracking data from the Atmospheric Explorer-C (AE-C) satellite would be of interest. The AE-C satellite has a capability for directly measuring the atmospheric drag using very precise accelerometers, and, consequently, the drag estimated with the DMC-algorithm can be compared with the drag measured by the accelerometer to provide verification of both approaches.

2) Further study should be made to determine the relative accuracy and computing speed associated with integrating the state transition matrix and the covariance matrix differential equations to obtain the propagated values for the covariance matrix. In this study, particular attention should be given to the efficiences of complete numerical solution to the problem versus approximate analytic determinations of the state noise covariance matrix contribution. In particular, the question of which procedure is best suited to the short arc fit and which procedure is best suited to the joint short arc multi-revolution orbit prediction fit should be investigated.

3) Further studies comparing the characteristics of the batch and the sequential estimation algorithms should be made. In particular, the following specific topics should be considered:

- A comparison of the batch and sequential estimation convergence characteristics with regard to the data accuracies and data type, i.e., both skin track data and active satellite data should be considered.

- Mean Motion - B-Coefficient Study - An attempt to determine the minimum data span required to effectively separate contributions of drag and gravity should be made. The influence of data-type, tracking station location and orbit geometry as well as satellite altitude should be investigated in this study.

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