RESEARCH STUDY OF RELIABILITY
PREDICTION OF MECHANICAL AND
STRUCTURAL SYSTEMS
Volume II
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U.S. ARMY MATERIEL SYSTEMS ANALYSIS ACTIVITY
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This study was intended to assess the state-of-the-art of predicting the reliability of items that fail due to mechanical stresses, and to identify those areas in which development effort should be intensified. In order to achieve an orientation toward application rather than analytical theories, a "sample" approach has been chosen by AMSAA, the details of the sample being selected by consultation between AMSAA and the University of Delaware. The study revealed a dearth of design-oriented data on mechanical failures and a
virtual absence of analytical methods and expertise to use such data. Development of a data base is recommended, initially from design testing. This experience and its result will provide a basis of analytical expertise to use the data for reliability prediction.
Most of the analytical activity of the Army Materiel Systems Analysis Activity (AMSAA) is concerned with anticipating the future results of present or proposed actions or conditions. This projection of the expected effect of present or assumed causes is usually called prediction. The basis of prediction is recorded experience, adequately correlated with causes. Unfortunately most experience records are correlated with factors having little or no relation to true causes. But a more unfortunate aspect of prevalent prediction methods is that analysts are usually unaware of this problem. This report presents the results of a Research Study by the University of Delaware, under contract with AMSAA, to make a "sample" investigation of prediction methodology in order to discover the most significant shortcomings and determine the most effective means of correcting them. Reliability prediction was chosen as a familiar and active area for a sample, and mechanics as the field of scientific design most difficult to predict. The sample approach was chosen after a review of the literature found a dearth of application-oriented methods. Instead the approach has been to find a new or "unique" method of performing a predictive analysis and then fit it to the application.

The approach used here may at times have a superficial appearance of developing particular methods. But it is intended to be illustrative and has the following threefold purposes: (1) to avoid an orientation toward analytical theories, which has not developed sound or effective methods; (2) to enable analysts to appreciate the interrelation of their disciplines with the scientific disciplines in the field of application; and (3) to provide designers and analysts some idea of the interdependence of statistics and inductive inference. The specialists in each area are unlikely to understand fully all that is presented in the other area, a "difficulty" that will illustrate the need for effective communication between analysts and designers.

AMSA Special Publication No. 19, Vol 1, "R&M Prediction in Engineering Development," addresses the basic principles of statistical prediction, showing that the conclusions of this study are applicable in other areas. It also provides a perspective for the study in more detail than would have been appropriate in this report.

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US Army Materiel Systems Analysis Activity
Research Study of Reliability Prediction of Mechanical and Structural Systems

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## CONTENTS

1. INTRODUCTION ................................................. 1

2. CONCLUSIONS AND RECOMMENDATIONS ....................... 4
   2.1 Failure Mode Analysis .................................... 4
   2.2 Relation of Failure Modes to Design Disciplines .......... 4
   2.3 Reliability Analyses .................................... 5
   2.4 Data System ........................................... 6

3. STATE OF THE ART ........................................... 8
   3.1 Engineering Design .................................... 8
   3.2 Statistical Analyses .................................... 8
   3.3 Improved Reliability Prediction ......................... 9
   3.4 Relative to the Recommendations ....................... 10

4. VEHICLE MODEL ............................................. 13
   4.1 Subsystem Reliability ................................... 14
   4.2 System Reliability ..................................... 15
   4.3 Operational Conditions and Failure Modes ............... 17

5. SUSPENSION SUBSYSTEM ..................................... 20

6. ANNOTATED BIBLIOGRAPHY .................................. 26
1. INTRODUCTION

The objective of this study, briefly, is to assess the state-of-the-art of reliability prediction of items that fail due to mechanical stresses and to discover promising frontiers for the next stage of development. In particular, the areas of interest are those in which a relevant or adequate effort is not in progress.

Recent technology has emphasized both the development of complex systems and a sophisticated interest in predicting their characteristics. These systems, often consisting of large numbers of subsystems, can fail because of the failure of a single and perhaps inexpensive part or component. The prediction of failure as a chance consequence has evolved into the subject of Reliability, which has become a concern for those engaged in developing systems, from design through manufacturing and to ultimate use.

Statisticians have served the needs of engineers and scientists in developing new statistical techniques and methods particularly suited to reliability studies. The first step in the application of mathematics to a physical problem is the construction of a mathematical model which embodies the main observational and theoretical features of the phenomenon. When phenomena are chance dependent, the mathematical model has a probabilistic character. With a mathematical model of the phenomenon one uses appropriate mathematical techniques to study the reliability and other aspects of a system.

Reliability gained especial recognition in World War II. In June 1943 the Vacuum Tube Department Committee (VTDC) was formed to establish joint Army and Navy parts standards. Following the VTDC, an airlines group formed in 1946 aimed at the development of better electronic tubes. This was followed by parallel studies conducted by
Aeronautical Radio, Inc. and Cornell University, in which respectively, 45,000 and 100,000 defective tubes were examined. Between 1949 and 1953 Vitro Laboratories pursued similar studies on the failure of parts other than vacuum tubes, e.g. resistors, capacitors, transformers, relays, etc. In 1950 the Department of Defense established an ad hoc committee on reliability, which in 1952 became a permanent group called the Advisory Group on the Reliability of Electronic Equipment (AGREE). An AGREE report was published in 1957 which was shortly followed by a specification on the reliability of military electronic equipment. Since the mid-1950's much work has been done on reliability analysis. The title "Reliability Engineer" is common. Many bibliographies have been published [1,2]. One quickly recognizes that the early reliability analyses were mainly concerned with electronic equipment. However, since the mid-1950's the new concept has evolved in structural [3,4,5], and mechanical engineering [6].

In many ways the problems encountered in mechanical-structural reliability studies are stubborn and advances have not been rapid. The main thrust of these reliability prediction efforts is toward existing equipment. Failure data is marshalled and statistically based curve fitting techniques are used to predict the reliability of nominally identical existing equipment, i.e. in effect a reliability assessment. This type of work is both important and useful and, hopefully, will continue.

However, there is perhaps a beginning recognition [7] that the more fundamental problem is one of predicting the reliability of a mechanical system at the design stage - perhaps from blueprints of non-existent equipment. This report emphasizes this phase of mechanical-structural reliability. It indicates the state-of-the-art of applicable predictive techniques and the nature of the initial tasks in the development of structural-mechanical reliability models capable of predicting the reliability of aircraft and surface vehicles. For those areas for which expertise was available for the study the approximate
This report of the study is divided into six sections. The conclusions and recommendations are placed after the introduction rather than at the end as is usually done. This is to provide a context within which the significance of the next three sections may be more easily discerned. The crux of mechanical reliability prediction lies in its dependence on the interrelationships of several factors which do not have an obvious significance when considered separately. Section 3 deals with reliability in general and the state of art. Section 4 concerns a mathematical model of the vehicle system to determine the relation between the reliability of individual subsystems and the reliability of the entire vehicle. Section 5 describes one subsystem - The Suspension System - and illustrates the analytic approach by use of examples. Section 6 is an annotated bibliography.


2. CONCLUSIONS AND RECOMMENDATIONS

A greater potential for reliability improvement appears realizable at the design stage, i.e., reliability prediction. At that stage, perhaps from blueprints of non-existent equipment, there is better flexibility to implement improvements in quality, reliability, availability, and maintainability of equipment. The Army’s reliability needs are assumed to be not only tactical and specific, but strategic and general as well. In view of this a four fold program is recommended which, by its focus upon root matters, can bring mechanical reliability prediction capabilities closer to the electronic counterpart. The four parts of the recommended program are: first, analysis of failure modes; second, determination of the relationship between failure modes and design disciplines; third, probabilistic expressions intrinsically related to design disciplines for the reliabilities relative to those failure modes; fourth, a data system to support and monitor the program. These are outlined in turn.

2.1 Failure Mode Analysis.

For reliability prediction it is essential that the failure modes be analysed.

Failure mode analyses have three outcomes. First, failures that may be directly related to the design disciplines; so-called intrinsic modes. Second, statistical correlations may indicate as yet unexplained relation of failures to design; so-called empirical modes. Finally, there may not be sufficient information for a modal classification; so-called random failures.

2.2 Relation of Failure Modes to Design Disciplines.

A fundamental study of mechanical and structural systems can yield the relationships between strengths and stresses in deterministic and probabilistic terms. They are functions of design and environmental constants and corresponding failure modes. These relations may be
called the physics of failure or the "physics of design."

Elements (systems, subsystems, components, parts, etc.) and appropriate design disciplines and constants are identified (See listing in Section 4 and the final page of Section 5). Failure modes are identified by stresses exceeding strengths and related to design constants. This may, for example, reveal the effect of degradation or other changes in some components upon others, say the effect of a worn shock absorber upon, say, the axle housing.

It should be noted that much of the above information exists. Often the need is to properly marshall such information for the probabilistic analyses described next.

2.3 Reliability Analyses.

The above analyses, which relate failure modes to design disciplines yield stochastic models. These models are the bases for conducting appropriate reliability analyses. These, in turn, are the ingredients of a reliability prediction model.

Since World War II an extensive literature on probability theory has emerged. The development has often been theoretically motivated and application to mechanical reliability has not been completely effective. As a consequence, some techniques whose development is envisioned mainly relate to i) the approximation of probabilities from statistical and phenomenological information (e.g. Chebyshev inequality), ii) operational functions for the manipulation of stochastic models (e.g. convolution properties of characteristic functions and Mellin Transforms), iii) dependencies among random variables (e.g. two parts made from the same batch of material), iv) capabilities to aggregate lower-level reliabilities into higher-level ones, and v) representation of empirical statistical distributions and their manipulation (e.g. calculus techniques) with mathematical and computer tractability.
To illustrate these: For (i), the Chebyshev inequality, known for about a century, has only recently been extended to multi-dimensional cases. Other approximation techniques are also needed to more fully utilize available information in the estimation of probabilities when exact expressions are impractical or unnecessary. For (ii), operational techniques are well known for such simple random models as sums and products of independent random variables. More flexibility is needed to cope with the relations between parameters and failure modes in an effective way. For (iii), the interdependence of failure modes of materials, fabricating and design parameters, of stresses, of time dependent parameter changes, distinguishing between relevant and irrelevant dependencies, etc. requires a better developed ability for multivariate analyses. For (iv), an important part of the program deals with a) the levels of precision needed to achieve particular reliability predictions and b) means for relating precision levels to each other. Finally, for (v), combinations of simple exponential functions have desirable characteristics.

Development of techniques for the five areas of application listed above require that approximate data and/or models be available. Only for the last area, i.e. empirical distributions, are these in evidence, as represented by "wear-out" and repair distributions for use in reliability, availability, and dependability models. It is estimated that an effort of the same order of magnitude as that represented by this report would produce significant results. About one-tenth of this work may be defined in sufficient detail to constitute an initial task. Further work in this area as well as work in the other four may be defined with implementation of a data system (described below) and parallel effort in constructing models for which it is designed.

2.4 Data System.

A properly designed data system is vital to support failure
mode analysis, environmental and operational inputs, and reliability analysis. It also provides the common purposes of parameter estimation.

The collection of data, by qualified people, will help to identify prominent failure modes and significant underlying factors. Appropriate statistical relationships (e.g. a regression) are used empirically to identify possible causal relations and the character of appropriate environmental and operational conditions. Statistical techniques may be useful to probe for possible relationships between stress and stress distributions. Statistical analyses are relied upon to provide parameter information and prediction feedback.

Note that the data system considered here requires three characteristics; 1) it must rely on qualified people particularly in the learning phases (e.g. to enable secondary analyses for certain qualitative assessments), 2) it must play an integral role to the development of the first two phases in addition to providing customary parameter and curve fitting information, 3) it is intended, hopefully, as an instrument of feedback and control.
3. STATE OF THE ART

There is now a well developed reliability literature, and "state of the art" considerations are available as, for example, in the attached annotated bibliography. Since the Contract for which this report is prepared had as a primary purpose the guidance of future efforts, this section on "state of the art" seeks to expose a gap in the reliability field and to support the main recommendations of this study.

3.1 Engineering Design

Engineering designs are generally based upon experimental and theoretical knowledge largely rooted in the physical sciences and traditional engineering subjects. In brief, they can be termed "cause and effect" designs. More recently, computer aided designs have vastly increased the designers alternatives. Selections can be made from among several feasible designs rather than a single one or two. In addition, concerted efforts are being made to achieve optimal or near optimal designs. It is also apparent that a fuller utilization of the existing software for computer aided design requires an improved precision in the design input. This often means that the potential for sharply improved designs depends upon better developed considerations of randomness in the material and environment incident to the design.

3.2 Statistical Analyses

Statistical analyses are widely used in reliability studies and they have an important role. However statistical analyses tend to contribute more to the assessment of existing designs than to the prediction of proposed designs. Also, statistical analyses are not always related to the underlying physical principles but rather to observed operational effects. For example, while regression and correlation techniques provide important clues to inadequate performance,
the inadequate roots in the physics of the design limit their usefulness for prediction purposes.

3.3 Improved Reliability Prediction

Having briefly cited some pertinent aspects of engineering design and of statistical analyses, it is useful to suggest what one seeks in an advance of the reliability prediction art.

There appear to be three broad characteristics of improved reliability prediction as recommended in the Report. First, elements of the design function (physical sciences and engineering) and probabilistic and statistical analyses (characterization of randomness) are utilized simultaneously. Second, reliability improvements more readily enter the design in the planning stages and diminish the need for corrective action in later stages. Third, the predictive approach as used in the Report emphasizes the phenomena and the random model to describe it. Thus, it seeks to aggregate a properly detailed consideration of elements.

In suggesting this predictive approach, it may be useful to attempt to assess why the proposed course was not pursued earlier. There are a few plausible reasons. First, the subject has simply reached an appropriate stage in its maturity. The reliability analysis of electronic equipment is easier to achieve and consequently it preceded structural-mechanical reliability prediction. Second, the post World War II specialist tendency and the attendant separation of disciplines is now being eroded. Third, not only have structural-mechanical engineers only recently seriously studied probability, but also there has not been an appropriate development of adequate applied probability techniques. While, of course, there is an ample probability literature, in the main it has derived from the needs and interests of mathematicians; not from the needs and interests of engineers. Finally, there appears to be an inadequate data base to permit the identification of needed analytic techniques and of the relation of failure modes to design.
3.4 Relative To The Recommendation

This section seeks to relate the "state of the art" somewhat more closely to the Conclusions and Recommendations (see Section 2.) of this Report. A simply supported beam is used to provide a specific example to illustrate ideas. It is a fundamental structural-mechanical system known to engineering sophomores. Yet, important aspects of its analyses related to randomness are hardly known. Specifically, a series of random loads $F_1, F_2, F_3$ act at respective random distances $X_1, X_2, X_3$ from the left end of the weightless beam of length $L$. The bending moment at a point $y$, $M(y)$, is given by

$$M(y) = \sum_{i=1}^{3} F_i \left\{ \frac{y(L-X_i)}{L} \mathcal{H}(X_i-Y) + \frac{L-y}{L} X_i \mathcal{H}(Y-X_i) \right\}$$

where $\mathcal{H}(X_i-Y)$ is a step function which is unity for a non-negative argument $(X_i-Y)$ and zero otherwise.

**Failure Mode Analysis.** MIL Handbook 217A provides examples of well done failure mode analyses. The long range objective of reliability prediction studies is to achieve an analogue in the structural-mechanical area. Of course, a good basis for failure mode analysis already exists. The need is for a reassessment with a predictive end in view (e.g., the physical failure mode may be different from the way it is packaged). Thus, if the simply supported beam is
serially packaged with other components, and if the beam has a moment failure, current practice is apt to simply record that the "package" has failed. A predictive reliability analysis seeks the more specific information that the moment failure occurred in the beam.

Relation of Failure Modes to Design Disciplines. A vast literature exists (e.g., current design handbooks). Some research may prove desirable (e.g. statistical design aspects are not well documented) but in the main it appears as if a marshalling of existing results may be adequate.

Quantitative Analyses. A serious impediment to the development of predictive reliability studies is the lack of appropriate quantitative techniques. As mentioned, there is a vast probabilistic literature, but it has not usually been developed for engineering needs. The additional techniques envisioned are of three broad types: First, decision rules are needed to evaluate the required levels of precision to describe the random phenomena at hand. For example, the probability distribution for the bending moment may not actually be needed and the mean and variance may be adequate for the design purpose at hand (c.f. pp 305, 316 of [8]). Second, techniques are needed for evaluating complex interrelationships among subsystems. For example, to separate fatigue from wear effects; to identify elements made from the same batch of material; to identify failure effects of one system upon another, or to identify the effect of wear of one component, such as a shock absorber, upon others. In brief, techniques are needed to assess the dependencies among random variables. Third, operational techniques possessing analytic convenience and powers of generality are needed to deal with the relationships between subsystems and design failure modes. For example, only recently have general expressions for the probability function of the bending moment for the simply supported beam been available. Even expressions for fundamental random vector operations appeared only recently [9].
Data Analysis. While analyses are often very well developed a comparable effort is needed in the collection and assessment of data. For example, assessment of environmental loads on the simply supported beam are needed to establish their probability distributions. These needs are elaborated in Section 2.

4. VEHICLE MODEL

Reliability is the probability that a system performs according to specifications and for a given time. A reliability analysis for a vehicle depends upon reducing the system into convenient units for study. One subdivision of a vehicle into subsystems is:

1. Engine System
2. Transmission System
3. Steering System
4. Electrical System - Ignition and Lights
5. Brake System
6. Suspension System
7. Body System
8. Instruments
9. Vision

Each subsystem can be further subdivided into smaller parts and components. An engine subsystem, e.g., can be divided into such components as:

1. Fuel System
2. Cooling System
3. Engine Body
4. Lubrication System

Further, each of these components is another system, whose detail can be complex. For example, a fuel system can be divided into the parts:

1. Fuel Tank
2. Fuel Pump Assembly
3. Carburetor
4. Filter
5. Piping
Finally, each of these parts consists of a number of elements. The carburetor, for example, is composed of the elements:

1. Casing
2. Jet Nozzles
3. Float
4. Flaper
5. Fixing Screws
6. Gaskets

A detailed reliability model properly aggregates the reliabilities of lower-level elements into higher ones.

4.1 Subsystem Reliability

A schematic of a subsystem appears in Fig. 4.1.

Survival of circuit AB depends upon the functioning of all of the "X" elements but only on some combination of the "Y" elements. This analysis of the "Y" element redundancy can take many forms. However, the specific form is not pertinent to the immediate task. As an illustration consider the following: Combinations of Y's can be replaced by equivalent Z's so that the reliability of subsystem [AB] is the probability of the event (all "X"'s and at least one "Z" survive).
Assuming independence of component parts:

\[ R_s(t) = \left[ \prod_{i=1}^{n} R_i(t) \right] \left[ 1 - \prod_{j=1}^{m} P_j(t) \right] \]

where

- \( R_s(t) \) = Reliability of the subsystem.
- \( R_i(t) \) = Reliability of \( i^{th} \) series component (X).
- \( P_j(t) \) = Probability of failure of \( j^{th} \) parallel components (Z).

In general, if there are \( q \) sets of parallel components, equation (4.1) can be generalized as

\[ R_s(t) = \left[ \prod_{i=1}^{n} R_i(t) \right] \left[ \prod_{k=1}^{q} \left( 1 - \prod_{j=1}^{m} P_{jk}(t) \right) \right] \]

where \( P_{jk}(t) \) is the probability of failure of \( j^{th} \) parallel component in the \( k^{th} \) set.

Consider the engine subsystem as an example of these formulas in the evaluation of vehicle reliability. The \( X_i \) parts are, for example, cylinders, casings, crank shaft, flywheel, camshafts, valves, etc. and \( Y_i \) parts might be machine screws to secure the crankcase to the engine body or the bolts of cylinder head, etc. Knowing the reliability of each component, the reliability of the subsystem can easily be evaluated. Practical element combinations involve the study of parallel elements \( Y_i \) and the assessment of equivalent \( Z_i \).

4.2 System Reliability

Schematically, a reliability model of a vehicle system might appear as in Figure 4.2.
Environmental loads such as terrain excitation, wind load, drag force, passenger or cargo load, explosion force, friction forces etc., are represented by $P_j$, $j = 1 \ldots A$. A function, $C_{ij}$, transfers the $j^{th}$ environmental loads to the subsystem. This transfer function is usually deterministic e.g. the spring constants in the suspension subsystem in the next section. However, it may be random as for example, shock absorbers which exhibit statistical behavior as a consequence of their deterioration in time. Moreover the transfer functions may have interdependencies. If we assume independence as a first approximation, then the total load experienced by the $i^{th}$ subsystem due to environmental loads is $\sum_j C_{ij} P_j$. Dependencies may be accounted for by terms, e.g. $C_{ijk} P_j P_k$ for the joint effect of the $j^{th}$ and $k^{th}$ loads on the $i^{th}$ part.

An important aspect of the reliability prediction research to be recommended here focuses upon exposing such design relationships. This contrasts with the reliability assessment approach which largely ignores design relationships. Using the electrical case as an analogy, the predictive approach depends upon an analysis of the electric circuit. System reliabilities are determined from subsystem reliabilities in much the same manner.
4.3 Operational Conditions and Failure Modes

Operational conditions give rise to the loads experienced by the vehicle components. The design relates the loads (stresses) to the strengths of these components for appropriate failure modes. These design relationships are used in reliability prediction. The designer generally has much less control over operational conditions than over strengths of components. Therefore, a data system is required to provide adequate inputs to the vehicle model and, ultimately, to the reliability prediction. Operational loads acting on vehicle components include passengers and cargo, terrain excitation, wind load, friction, and impact. These loads contribute to the failure modes. One failure mode classification, called "SCWFT", appears in Table 4.1. Failure modes are identified from appropriate data. Therefore, it is essential that the data system provide adequate failure mode identification, thus providing design clues.

Component reliabilities are related to failure modes. For example, consider a component that can fail from fatigue, corrosion, wear or shock. Assume known probability distributions. Let

$N_t =$ number of loading cycles to time $t$

$C_t =$ corrosion stress

$W_t =$ wear stress

$I_t =$ shock stress

$[S] =$ Event that the component survives to time $t$.

An event relationship for survival to time $t$ is written as the union:

$[S_t] = [N_t < N_o] \cup [C_t < C_o] \cup [W_t < W_o] \cup [I_t < I_o]$

The zero subscript indicates failure limits. This event relation expresses the fact that a component survives if none of the
## TABLE 4.1

### DEFINITION OF FAILURE MECHANISMS

<table>
<thead>
<tr>
<th>Code</th>
<th>Failure Mechanism</th>
<th>Producing Load</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>Creep or Stress Rupture</td>
<td>Constant or static mechanical (brief or extended duration)</td>
<td>Phenomena leading to failure under constant load in a period of time.</td>
</tr>
<tr>
<td>C</td>
<td>Corrosion</td>
<td>Chemical or chemical and mechanical</td>
<td>Deterioration of a metal by chemical or electrochemical reaction.</td>
</tr>
<tr>
<td>W</td>
<td>Wear</td>
<td>Various mechanical</td>
<td>Removal of material from a solid surface as a result of mechanical action.</td>
</tr>
<tr>
<td>I</td>
<td>Impact</td>
<td>Constant, moving, mechanical</td>
<td>Phenomena leading to failure as a result of the sudden application of a moving load.</td>
</tr>
<tr>
<td>F</td>
<td>Fatigue</td>
<td>Repeated or fluctuating, mechanical</td>
<td>Phenomena leading to failure under repeated or fluctuating stress less than the tensile strength.</td>
</tr>
<tr>
<td>T</td>
<td>Thermal</td>
<td>Heat (brief or extended duration)</td>
<td>Deterioration of material by melting, vaporization, decomposition and welding as a result of high temperatures.</td>
</tr>
</tbody>
</table>
failure modes is active prior to time \( t \). It follows that the reliability,
\[ P_s(t) = P \{ [N_t < N_o] \cup [C_t < C_o] \cup [W_t < W_o] \cup [I_t < I_o] \} \]

Earlier remarks apply here for the continued analysis of this relationship, and aggregation of similar relationships for other components into higher-level reliabilities.
5. SUSPENSION SUBSYSTEM

In a study of the analyses required to establish interrelationships among stresses, design strengths, and failure modes, only one present need was evident for development of relatively complex probabilistic analysis. This involves the effect of damper or shock absorbers on stresses imposed on vehicle components, and is illustrated in this section as part of deriving vehicle stress inputs through the suspension subsystem. The analysis also provides an example of the interaction between the disciplines of mechanics and probability theory that is indispensable to a true reliability prediction of new designs and configurations. The last page lists some typical elements or components, with characteristic functions, failure modes and related design disciplines.

The vibrations of a vehicle moving on rough terrain is influenced primarily by the vehicle and tire masses, the spring constants of the suspension system, damping of the shock absorbers and the irregular terrain. A simple vehicle-terrain model which reflects these primary characteristics appears in Figure 5.1.

---

Figure 5.1. Vehicle - Terrain Model

---
The coordinates \( x_1 \) and \( x_2 \) define the rest position of the tire and vehicle measured from a reference elevation, \( x_0 \). A vehicle moving with speed \( V \), travels a distance \( Vt \) in a time \( t \) when the terrain elevation is \( x_0 \) \((Vt)\). For a fixed speed \( x_0 \) depends on time \( t \). The equations of motion for this system are

\[
\begin{align*}
    m_2 \ddot{x}_2 + c(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) &= 0 \\
    m_1 \ddot{x}_1 - c(\dot{x}_2 - \dot{x}_1) - k_2(x_2 - x_1) + k_1x_1 + k_1x_0 &= 0
\end{align*}
\]

Let \( \zeta = \frac{c}{2/k_2 m_2} \),  \( \omega_1^2 = \frac{k_1}{m_1} \),  \( \omega_2^2 = \frac{k_2}{m_2} \),  \( \mu = \frac{m_2}{m_1} \)

Then the equations

\[
\begin{align*}
    \dot{x}_1 + 2\zeta\omega_2(\dot{x}_2 - \dot{x}_1) + \omega_2^2(x_2 - x_1) &= 0 \\
    \dot{x}_1 - 2\mu\zeta\omega_2(\dot{x}_2 - \dot{x}_1) - \mu\omega_2^2(x_2 - x_1) + \omega_1^2x_1 + \omega_1^2x_0 &= 0
\end{align*}
\]

The complex frequency response functions are

\[
\begin{align*}
    H_{x_1}(\omega) &= \frac{-\omega_1^2 \omega_2^2 - 2i\zeta\omega_1^2 \omega_2 + \omega_1^2 \omega_2^2}{\Delta} \\
    H_{x_2}(\omega) &= \frac{2i\zeta\omega_2 \omega_1 \omega_2 + \omega_1^2 \omega_2^2}{\Delta}
\end{align*}
\]

where

\[
\Delta = \begin{vmatrix}
    -\omega_1^2 + 2i\zeta\omega_2 \omega + \omega_2^2 & -(2i\zeta\omega_2 \omega + \omega_1^2) \\
    -(2i\zeta\omega_2 \omega + \mu\omega_2^2) & -\omega_1^2 + 2i\mu\zeta\omega_2 \omega + \mu\omega_2^2 + \omega_1^2
\end{vmatrix}
\]
The stresses in the suspension system and tire measured by the relative displacements \( y_1 = x_1 - x_0 \) and \( y_2 - x_1 \) and their complex frequency response functions are

\[
\begin{align*}
H_{y_2}(\omega) &= \frac{\omega^2}{\Delta}, \\
H_{y_1}(\omega) &= \frac{-\omega^4 + 2i\omega \omega_2^3 (1+u) + \omega_2^2 (1+u) \omega^2}{\Delta}
\end{align*}
\]

Random terrain is customarily assumed to be stationary, ergodic and Gaussian with zero mean. The response of a linear system subjected to such a terrain excitation proves also to be Gaussian. The normal probability density function can be constructed if response statistics are known. For the relative displacement of the tire, the power spectrum is

\[
S_{y_2}(\omega) = \left[ \frac{\omega_1^2 \omega_2^2}{\Delta} \frac{\alpha}{\beta + i\omega} \right]^2
\]

Therefore the mean square displacement is

\[
E[y_2^2] = \int_{-\infty}^{\infty} \frac{\omega_1 \omega_2^2}{\Delta (\beta + i\omega)} \, d\omega
\]

This integral can be evaluated by contour integration. Finally the joint probability density of the relative displacement and velocity is

\[
f_{y_2, \dot{y}_2}(y_2, \dot{y}_2) = \frac{1}{2\pi \sigma_{y_2} \sigma_{\dot{y}_2}} \exp\left[ -\frac{y_2^2}{2\sigma_{y_2}^2} - \frac{\dot{y}_2^2}{2\sigma_{\dot{y}_2}^2} \right]
\]
An underlying assumption in the response transfer function for the terrain excitation input is that the damping factor \( c \) for the shock absorbers is a constant. Actually, it is a random variable and can be described by a time dependent probability distribution to account for deterioration. To consider this random variable with the random terrain excitation escalates the complexity of the probabilistic relationships. For single degree of freedom system (next figure), the equation of motion is

\[
\ddot{y} + c \dot{y} + ky = -m\dot{x}_0
\]

where \( y = x_1 - x_0 \)

Let \( \omega^2 = \frac{k}{m} \) and \( \zeta = \frac{c}{2\sqrt{km}} \)

Then

\[
\ddot{y} + 2\zeta \omega \dot{y} + \omega^2 y = -x_0(t)
\]

---

The complex frequency response \( H(\omega) \) is

\[
H(\omega) = \frac{-1}{\omega^2 - \zeta^2 \omega^2 + 2\zeta \omega}
\]
Now if the damping factor \( c \) is described by a probability density function \( f(c) \), then the probability function for \( \zeta \) is

\[
g(\zeta) = |J| f(c^{-1}) = 2\sqrt{km} f(2\sqrt{km} \zeta)
\]

where \( J \) is the Jacobian of the transformation and since

\[
\zeta = \frac{c}{2\sqrt{km}}
\]

The response transfer function \( H_y(\omega) \) is a function of the damping ratio \( \zeta \) i.e.

\[
H_y(\omega) = \phi(\zeta)
\]

Let \( \psi(H(\omega)) \) be the single valued inverse function of \( \zeta \). Then the probability density function for frequency response is

\[
g(H(\omega)) = |J| g(\psi(H))
\]

However, for the two degrees of freedom model of a vehicle considered earlier the frequency response is a complex function of two random variables. Improvements in reliability prediction require further investigations to evaluate correlation between these random variables. For example, accelerometers that are installed on an axle near a tire and above the corresponding shock absorber will yield data for correlation studies.
<table>
<thead>
<tr>
<th>ELEMENTS</th>
<th>FUNCTION</th>
<th>FAILURE MODES</th>
<th>DISCIPLINES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaft</td>
<td>Transmit Power</td>
<td>Flexure, Shear, Excessive Deflection</td>
<td>Shaft Design</td>
</tr>
<tr>
<td></td>
<td>Support Wheel</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stabilizer</td>
<td>Prevent Sway</td>
<td>Flexure, Shear</td>
<td>Shaft Design</td>
</tr>
<tr>
<td>Spring</td>
<td>Smooth Riding</td>
<td>Shear, Fatigue</td>
<td>Spring Design</td>
</tr>
<tr>
<td>Damper</td>
<td>Absorbing Shock</td>
<td>Leakage, Rupture</td>
<td>Fluid Mechanics</td>
</tr>
<tr>
<td>Brake</td>
<td>Stopping</td>
<td>Wear, Leakage</td>
<td>Bearing</td>
</tr>
<tr>
<td>Bearing</td>
<td>Reduce Friction</td>
<td>Fatigue, Abrasion, Corrosion</td>
<td>Bearing</td>
</tr>
<tr>
<td>Tire</td>
<td>Ride</td>
<td>Wear, Rupture</td>
<td>Strength, Surface Wear</td>
</tr>
<tr>
<td>Tire Rim</td>
<td>Support</td>
<td>Rupture</td>
<td>Structural Design</td>
</tr>
<tr>
<td>Screws &amp; Nuts</td>
<td>Fasteners</td>
<td>Shear</td>
<td>Structural Design</td>
</tr>
</tbody>
</table>
6. ANNOTATED BIBLIOGRAPHY

(1) AIAA Journal (TL 5Q1 A162)

"Optimum Structural Design with Failure Probability Constraints" by F. Moses and D. E. Kinser, Vol. 5, No. 6, p. 1152, June 1967. Techniques for finding the minimum weight design of a statically loaded structure and multiple load conditions with random characteristics are considered. A unique feature is the consideration of the statistical dependence between failure modes.

"Frequency Response Functions from Cross Correlation-Bartlett Weighting Function" by J. Soovere and B. L. Clarkson, Vol. 5, No. 3, March 1967, p. 601. The cross correlation between force and response can be used to obtain the frequency response function of a lightly damped system. This paper illustrates the method of using Bartlett triangular weighting function on the determination of frequency response function from correlation function of force and response.

"Exceedances of Structural Interaction Boundaries for Random Excitation" by J. D. Houbelt, Vol. 6, No. 11, p. 2175, Nov 1968. Numbers of passage of a given interaction boundary is studied. In structural design, one often has to consider the combined stresses such as axial and shear forces or bending and sheer stresses. Therefore the information about the exceedances of the combined stresses is of importance.

"Minimum Weight Analyses Based on Structural Reliability" by P. N. Murthy and G. Subramonian, Vol. 6, No. 10, p. 2037, Oct 1968. Minimum weight design is achieved under assumption that weights of components and their probabilities of failure are directly proportional for a structure whose weight is minimum with the restraint of a constant over all probability of failure.

(2) Int. Journal of Mechanical Science

"Continuous Structure Excited by Correlated Random Forces" by
Response of a continuous structure excited by distributed forces varying randomly with time is studied. Correlations on $x$, $x'$ and $T$ are considered under assumption of stationarity and ergodicity.

(3) International Jour. Non-linear Mechanics, QA427.A11157

"A Numerical Study of Non-linear Vibrations Induced by Correlated Random Loads" by D. J. Belz, Vol. 1, p. 139, 1966. A physically realizable stationary Gaussian, random load is simulated digitally and employed as the forcing function in the equation of motion of a damped elastic beam. The power-residue method for generating pseudo-random numbers is employed to construct a random function whose statistical properties correspond closely to a subsonic air jet. A random load is applied at the mid span of a non-linear beam. The response statistics and probability are obtained by a numerical method. A Ph.D Thesis on Digital Simulation of Random Vibrations was submitted to the University of Florida in 1964.

(4) International Jour. of Solids and Structures TA349158


(5) Journal, Acoustics Soc. America QC221A27

"System Reliability Prediction in Impulsive-Noise Environments, Based on Wear-Dependent Failure Rates, Using Response 'Peak' Statistics" by T. I. Smits and R. F. Lambert, Vol. 43, No. 6, p. 1344, 1968. An approach to system reliability under wearout and overload failure is outlined, where conventional models are used to define wear-dependent failure rates in terms of response "peak" statistics. Weibull distribution are found quite useful in approximating the various response peak statistics.
for certain impulsive-noise in random vibration.

"Behavior of Linear Systems with Random Parametric Excitation" by A. H. Gray, Jr., Vol. 37, No. 2, p. 235, Feb. 1965. The behavior of linear systems with parameters that vary as "physical" white noise is developed. The Fokker-Planck equation is used to develop moment equations.

"First-Occurrence Time of High-Level Crossings in a Continuous Random Process" by J. R. Rice and F. P. Beer, Vol. 39, No. 2, p. 323, 1966. Approximate forms of the first-occurrence and first-recurrence time densities are found by considering the successive crossings to form a renewal process. A simple exponential distribution is found to give an approximate representation of the limiting case when crossings of the level are statistically rare events. The method is used to evaluate survival probability for randomly excited mechanical systems subjected to failure upon occurrence of a sufficiently high load.

"Response of a Linear System to Nonstationary Shot Noise" by T. C. Chen., Vol. 41, No. 4, p. 822, May 1967. Limitation of Lin, Wang, and Uhlenbeck solutions to the Fokker-Planck equation is discussed. The response of more general class of linear system to nonstationary Gaussian shot noise is studied.

"Nonstationary Excitation and Response in Linear Systems Treated as Sequences of Random Pulses" by Y. K. Lin, Vol. 38, p. 453, Dec. 1964. The response (Probability Density) of a linear system to nonstationary excitation is found with superposition of random pulses technique. More general shape of excitation pulses and non-Poisson arrival rate are studied.

"Derivation and Application of the Fokker-Planck Equation to a Discrete Non-linear Dynamic System Subjected to White Random Excitation" by Thomas K. Caughey, Vol. 35, No. 11, p. 1683, Nov. 1963. The Fokker-Planck equation is derived. Stationary responses for a class of problems in which the non-linearities only involve displacements are found by solving the Fokker-Planck equation.


"Numerical Prediction of Response-Peak Statistics of a Linear System Excited by Impulsive Noise" by G. L. Hedin and R. F. Lambert, Vol. 43, No. 6, 1968. Methods of predicting response peak statistics (crests, positive maxima, and rises) for a linear system excited by Poisson impulsive noise are developed. Results are compared to the empirical method on a bimodal system.

"Random Vibration of the Connected Structures" by R. H. Lyon and E. Eichler, Vol. 36, p. 1344, No. 7, 1964. The sharing energy by connected randomly vibrating structure is studied. When large complex structures are randomly excited, parts of it receive the excitation directly and the others receive the energy through mechanical attachments. This paper deals with the method of analysis under such conditions.


"Mean-Square Response of Single Mechanical Systems to Non-stationary Random Excitation" by R. L. Barnoski and J. R. Maurer, Vol. 36, Series E, No. 2, p. 221, June 1969. This paper concerns the mean-square response of a linear oscillator to amplitude modulated random noise. Both the unit step and rectangular step functions are used for the amplitude modulation, and both white noise and noise with an exponentially decaying harmonic correlation function are considered.

"On the Dynamic Response of a Beam to a Random Moving Load" J. K. Knowles, Vol. 35, Series E, No. 1, p. 1, Mar. 1968. The problem considered is that of an infinitely long beam subjected to a moving load whose position is a random function of time. Statistics for the deflection and moments of the beam are obtained.

"Power Balance Method for Nonlinear Random Vibrations" by D. Karnopp, Vol. 34, Series E, No. 1, p. 213, March 1967. A power balance method is used to study vibratory systems forced by white noise. For certain nonlinear systems it provides means for estimating average amplitude of vibration and average rate of accumulation of plastic restraint.

"Some First Passage Problems in Random Vibration by S. H. Crandall, K. L. Chandiramani and R. G. Cook, Vol. 33, Series E, No. 3, p. 532, Sept. 1966. First passage problems for a linear oscillator excited by white noise are described by two types of initial conditions and three types of barrier configurations. The quantitative behavior of the probability of first passage time is discussed. The asymptotic behavior of the probability for long mean times to failure is discussed.

"Probability Distribution of Bilinear System Response to Impulse Excitation" by S. F. Felszeghy and W. T. Thomson, Vol. 33, Series E, No. 2, p. 384, June 1966. An oscillator with a bilinear spring is excited by a rectangular impulse of constant value, but whose amplitude has a probability distribution which is Gaussian. The peak response of the system is determined, and its probability distribution is plotted as a function of its peak value.
"Application of Nonstationary Shot Noise in the Study of System Response to a Class of Nonstationary Excitation" by Y. K. Lin, Vol. 30, pp. 555-558, Dec. 1963. The response of a linear system to the excitation of a general class may be generated by passing a nonstationary shot noise through a linear filter with proper parameters. Finally, the response of a linear system to a general class of random excitation is discussed.

"The analysis of Dynamics Stresses in Aircraft Structure During Landing as Nonstationary Random Processes" by Y. C. Fund, Vol. 22, p. 449, Dec. 1955. The landing-gear impact load is regarded as a nonstationary random process. Mean response and derivation and higher statistical moments are calculated for the aircraft structure. Life of the aircraft is predicted.

(7) Journal of Applied Physics

"On the Theory of Random Noise. Phenomenological Models I, II" by D. Middleton, Vol. 22, pp. 1143, 1153, 1326, Sept. 1951. Phenomenological models are considered from which statistical description of varieties of electronic noise (shot, thermal, etc.) can be constructed. The general probability density \( W_1(x_1, t_1, \ldots, x_s t_s) \) is constructed and moments and spectral density are also found.


"Probability of Structural Failure under Random Loading by M. Shinozuka, Vol. 95 EM5, p. 147, Oct. 1964. Bounds are given for the probability that a random process crosses barriers in a given time interval. A linear oscillator under non-stationary Gaussian non-white random input is investigated. The approach may be applicable to aircraft subject to infrequent severe atmospheric turbulence.

a class of nonstationary random processes by passing a Gaussian white noise through a system introducing desirable nonstationarity at some phase of simulation.


(9) Jour. of Math. Physics

"Experimental Confirmation of the Applicability of the Fokker-Planck Equation to a Nonlinear Oscillator" by J. B. Morton and S. Corrsin, Vol. 10, No. 2, p. 361, Feb. 1969. A traditional derivation of the Fokker-Planck equation is examined with emphasis on assumptions, especially sufficient inequalities between excitation and response characteristics times in a "real" system, i.e. a system with finite correlation time of the excitation. Comparison with experiment for an electronic "cubic spring" oscillator gives good agreement in both second and fourth moments of the response. Steady state response is compared with the analog computer solution.

"Random Walk on Lattices II Calculation of First Passage Times with Application to Exciton Trapping on Photo Synthetic Units by E. W. Montroll, Vol. 10, 4, p. 753, April 1969.


"Structural Fatigue under Non-stationary Random Loading" by J. B. Roberts, Vol. 9, p. 393, 1966. Reliability of structures to fatigue failure when subjected to non-stationary random process. An analytic expression is deduced for the expected number of crossing per second with positive slope at any amplitude level for a normal nonstationary random process and the result is combined with the cumulative damage hypothesis to give an
estimate of mean damage in a given time for a structure with a normally distributed stress process.

(11) *Journ. of Sound and Vibration* (England), QC221J68

"Random Vibration Techniques Applied to Motor Vehicle Structures" by C. J. Chisholm, Vol. 4, No. 2, p. 129, 1966. A vehicle moving on a road is treated as a vibratory system and the road surface a randomly varying excitation source. The power spectrum technique and the frequency response function are used to study the road surface and the vehicle spectrum which may be characterized by a few parameters. These response parameters together with those of the road surface and the vehicle permit comparisons of different vehicle systems. Experiments are carried out by Motor Industry Research Association in England using scale models on test pavements.

"The Behavior of Aircraft on Rudimentary Airstrips" by H.P.Y. Hitch, Vol. 3, No. 3, p. 445, 1966. Analysis of aircraft behavior on rudimentary airstrips has been carried out. It was found that low pressure types appear to be necessary and that in general, a simple r.m.s. measure of runway roughness is insufficient for prediction of loads and undercarriage requirements.

"Deduction from the Spectra of Vehicle Response Due to Road Profile Excitation" by J. D. Robson, Vol. 7, No. 2, p. 156, 1968. Under certain circumstances response of a vehicle may be considered as a function of a single displacement imposed by the roadway. Spectra of roadway and of vehicle response of various speeds are then simply related. It is shown here that in such a case the establishment of response spectra at two known vehicle velocities makes possible the prediction of response spectra at other speeds, and also of the form of the road profile spectrum.

"On the Response of Single and Multidegree of Freedom Systems
to Non-Stationary Random Excitation" by J. K. Hammond, Vol. 7, No. 3, p. 393, 1968. Two current theoretical spectral methods of dealing with non-stationary processes are presented. A non-stationary evolutionary spectral function proposed by Priestley is constructed and response of a mechanical system to such excitations is investigated. A non-stationary spectral function may be constructed from a single record on the basis of an assumption of an ergodic nature, i.e., that an average or averages over one record are representative of averages taken over hosts of records at a fixed time instant.

"On the Two-Sided Time-Dependent Barrier Problem" by M. Shinozuka and M. Shinozuka and J. T. P. Yao, Vol. 6, No. 1, p. 98, 1967. Bounds are obtained for the probability that a separable random process X(t) will not be confined within the time-dependent barriers -\( \lambda_1(t) \) and \( \lambda_2(t) \) in the interval \([0,T]\) where \( \lambda_1 \) and \( \lambda_2 \) are positive deterministic functions of time. The process \( X(t) \) is not required to be stationary, Gaussian, nor purely random.

"An Approach to the First-Passage Problem in Random Vibration" by J. B. Roberts, Vol. 8, No. 2, p. 301, 1968. A series solution to the first-passage problem in random vibration is derived which is valid for any type of response process and for both single and double-sided barriers. The special case of a normal response process is discussed in some detail. The theory is applied to the case of a lightly damped oscillator excited by white noise.

(12) Jour. of Appl. Math. and Mechanics
(Russia QA801P74)


(13) Journal of the Structural Division
ASCE

"Safety, Reliability and Structural Design" by A. M. Freudenthal, Vol. 87, No. ST2, Mar. 1961. Also a closing by the author, Vol. 88, No. ST3, June 1962. A rational approach is presented to estimate the probability of failure of statically loaded structures where both the strength and the load are treated as random variables.

"Bounds on the Reliability of Structural Systems", by C. A. Cornell, Vol. 93, No. ST1, p. 171, Feb. 1967. A modified approach is presented for the estimation of the reliability of statically loaded structures. The modification is aimed at including the realistic conditions of functional and probabilistic dependence among resistances (in time and in location in a structure) and among forces on different members of a structure.

"Safety Factors and Probability in Structural Designs" by A. H. S. Ang and M. Amin, Vol. 95, No. ST7, p. 1389, July 1969. A formulation of structural safety is proposed, and its associated basis for the design of safe structures is described. The proposed formulation combines certain aspects of the conventional method of design with an extended reliability theory. Its practical significance is the feasibility of developing workable design procedures based on the reliability concept, but retaining the simplicity of the conventional method.

the reliability to date are summarized. Numerical examples are also presented.

"Probability of Plastic Collapse Failure" by J. L. Jorgenson and J. E. Goldberg. The probability of plastic collapse failure for a single-story frame of random strength under random load is calculated by analyzing the failure mechanisms for the structure.

(14) NASA Reports

"Study of Taxing Problems Associated with Runway Roughness" by B. Milwitzky, Memo 2-21-59L, 1959. This paper briefly summarizes available statistical data on airplane taxi operations, examines the profiles and power spectra of four selected runways and taxiways covering a wide range of surface roughness, considers the loads resulting from taxiing on such runways over a range of speeds and, by synthesis of the aforementioned results, proposes new criteria for runway and taxiway smoothness which are applicable to new construction and may also be used as a guide for determining when repairs are necessary.

"Measurements and Power Spectra of Runway Roughness at Airports in Countries of the North Atlantic Treaty Organization" by W. E. Thompson, Tech. Note 4303, July 1958. Measurements of runway roughness obtained by a profile-survey method are presented. The results are presented as elevation profiles of the runways surveyed and in the form of power spectra.

(15) Books and Misc.


"The Variance in Palmgren-Miner Damage Due to Random Vibration"

"Second Order Properties of Nonlinear System Drivers by Random Noise" by L. Wolaver. ARL65-61, Applied Mat'l. Res. Lab. Wright-Patterson AFB, Ohio, April 1965. Four different methods, Fokker-Planck, linearization, perturbation, and numerical scheme, of solving non-linear system under white noise excitation are studied. Experimental verifications of the solution is also presented.


"Development of a Reliability Prediction Procedure for Shipboard Mechanical Equipments - Phase I", ARINC Research Publication No. 933-01-2-1079, Nov., 1970. This report recognizes the need for "scientific insight, judgement, and experience in selecting the best predictor equation from the many mathematical possibilities that may appear to be equally good." ARINC Research Corp., Annapolis, Md. 21401.
