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ON THE DEFLECTION OF PROJECTILES DUE TO ROTATION OF THE EARTH

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An analysis of the effect of the Coriolis acceleration on the flight of projectiles fired from the surface of the Earth has been extended to include bodies fired or released from any altitude above the surface of the Earth. An equation for Coriolis deflection perpendicular to the line of fire and the general condition for zero deflection are derived. Directions of Coriolis deflections are tabulated for bodies released horizontally at altitude y_o.

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ON THE DEFLECTION OF PROJECTILES DUE TO ROTATION OF THE EARTH

by

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FOREWORD

This report, referred to as Reference 7 in NWL Technical Report TR-3061 published in April, 1974, is one of several written on the effect of Coriolis acceleration on the flight of a projectile. It contains a derivation of the expression for DOMEGA, the Coriolis deflection perpendicular to the line of fire.

This work was performed in the Aeroballistics Division of the Warfare Analysis Department under ORDTASK 551028090.

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ABSTRACT

An analysis of the effect of the Coriolis acceleration on the flight of projectiles fired from the surface of the Earth has been extended to include bodies fired or released from any altitude above the surface of the Earth. An equation for Coriolis deflection perpendicular to the line of fire and the general condition for zero deflection are derived. Directions of Coriolis deflections are tabulated for bodies released horizontally at altitude y_0.

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I. INTRODUCTION

An analysis was made[1] of the effect of the Coriolis acceleration on the flight of projectiles fired from the surface of the Earth and the condition for zero deflection was derived. It was assumed that the acceleration of gravity is constant; velocities due to Coriolis accelerations may be neglected in computation of Coriolis accelerations; air resistance is negligible; and the impact position is in the plane tangent to the Earth at the gun. The analysis has been extended, using the same assumptions, to include bodies fired or released from any altitude (y_o) above the surface of the Earth.

II. ANALYSIS AND RESULTS

Consider a projectile fired from altitude y_o above a point on the Earth at latitude θ in the northern hemisphere with initial velocity V_o having azimuth ϕ (measured clockwise from north) and elevation angle ϵ . The time of flight is given by

$$T = (V_o \sin \epsilon + (V_o^2 \sin^2 \epsilon + 2g'y_o)^{\frac{1}{2}})/g' \qquad (1)$$

where $g' = g - 2V_o\omega \cos \epsilon \sin \phi \cos \theta$. The second term in the expression for g' is the vertical component $(2V_E\omega \cos \theta)$ of the Coriolis acceleration. V_E is the magnitude of the horizontal east component of projectile velocity and ω is the angular rate of rotation of the Earth.

The horizontal Coriolis displacements are, in the east (x) direction[1],

$$x = V_o\omega T^2 \sin \theta \cos \epsilon \cos \phi$$

resulting from the horizontal north component $(\vec{V}_N)$ of projectile velocity; and, in the south (z) direction[1],

$$z = V_o\omega T^2 \sin \theta \cos \epsilon \sin \phi$$

resulting from the horizontal east component $(\vec{V}_E)$. In the west (-x) direction

$$d^2x/dt^2 = -2V_V\omega \sin (90 - \theta)$$

from which, using $V_V = V_o \sin \epsilon - g't$ and integrating from $t = 0$ to $t = T$,

$$x = -V_o\omega T^2 \sin \epsilon \cos \theta + \omega g'T^3(\cos \theta)/3.$$

V_V is the magnitude of the vertical component of projectile velocity.

The net horizontal displacement perpendicular to the line of fire is

$$D_T = V_o\omega T^2 (\sin \theta \cos \epsilon \cos^2\phi + \sin \theta \cos \epsilon \sin^2\phi - \sin \epsilon \cos \theta \cos \phi + \frac{g'T}{3V_o} \cos \theta \cos \phi)$$

$$= V_o\omega T^2(\sin \theta \cos \epsilon - \sin \epsilon \cos \theta \cos \phi + \frac{g'T}{3V_o} \cos \theta \cos \phi). \qquad (2)$$

For no deflection $\qquad D_T = 0$

and $$\tan \theta = (\tan \epsilon - \frac{g'T}{3V_o \cos \epsilon}) \cos \phi. \qquad (3)$$

This is the general condition for zero deflection.

If $y_o = 0$, $T = 2V_o(\sin \epsilon)/g'$, and Eq. (3) reduces to

$$\tan \epsilon = 3(\tan \theta)/\cos \phi. \tag{4}$$

Conclusions for this case are given in the previous analysis.[1]

If $\epsilon = 0$, $T = (2y_o/g')^{\frac{1}{2}}$, and we have from Eq. (3)

$$V_o = -(2g'y_o)^{\frac{1}{2}} (\cos \phi)/(3 \tan \theta). \tag{5}$$

Therefore, neglecting air resistance, the initial velocity for zero deflection of a body projected horizontally at azimuth ϕ from altitude y_o above the surface of the Earth at latitude θ varies directly as the square root of the altitude.

III. CONCLUSIONS

The following conclusions are drawn from a study of Eqs. (3) and (5) and Figure 1 for a body projected horizontally or dropped vertically downward from a point above the surface of the Earth:

(1) If the body is released horizontally in a northward direction ($270^o < \phi < 90^o$) in the northern hemisphere (Figure 1), the deflection is to the right for all velocities. If it is released horizontally in a southward direction ($90^o < \phi < 270^o$) in the northern hemisphere, the deflection is to the right or left depending on whether the horizontal velocity is greater or less, respectively, than that given by Eq. (5) for zero deflection.

Horizontal velocities required for zero deflection are given in Table I for different latitudes for azimuth 180 degrees. These were calculated using $g = 32$ ft/sec^2, the approximate average value for altitudes between zero and 100,000 feet. To obtain values for other azimuth angles, multiply the table values by the absolute value of $\cos \phi$.

(2) If the body is released horizontally in a southward direction in the southern hemisphere (Figure 1), the deflection is to the left for all velocities. If it is released horizontally in a northward direction in the southern hemisphere, the deflection is to the left or right depending on whether the horizontal velocity is greater or less than that given by Eq. (5) for zero deflection.

(3) A body projected or released directly east ($\phi = 90^o$) or west ($\phi = 270^o$) from a point above the surface of the Earth will deflect to the right in the northern hemisphere and to the left in the southern hemisphere; if released directly east or west horizontally from a point directly above the equator, the deflection is zero.

(4) A body projected horizontally from a point directly above the north pole will deflect to the right; a body projected horizontally from a point directly above the south pole will deflect to the left.

(5) A body dropped vertically down from a point above the surface of the Earth, except at the poles, will deflect toward east; if dropped vertically down from a point above either pole, the deflection is zero.

A summary of Coriolis deflections of bodies released horizontally or vertically downward at altitude y_o above the surface of the Earth is given in Table II.

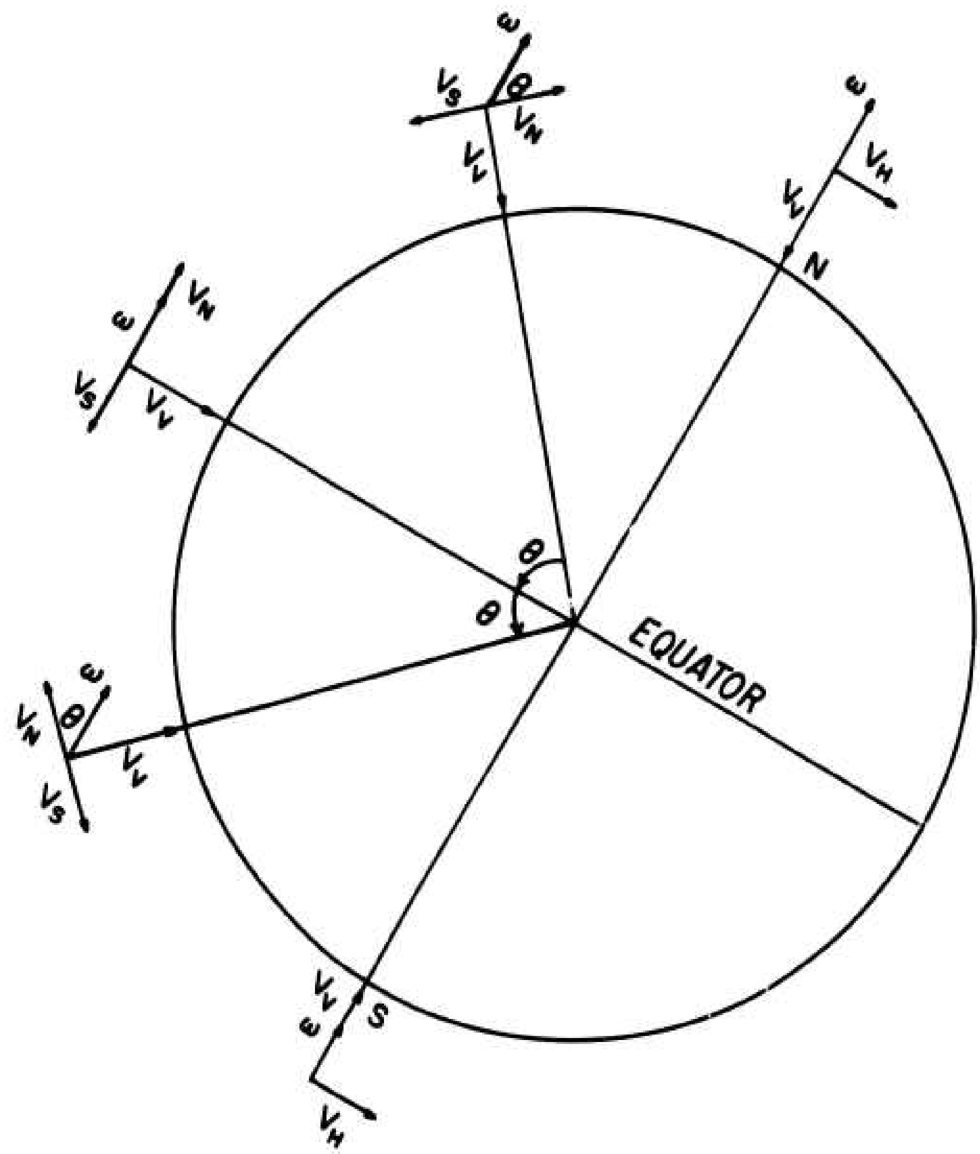

FIGURE 1

EARTH AND BODY VELOCITY COMPONENTS FOR HORIZONTAL, BODY PROJECTION

TABLE I

Horizontal Velocities (ft/sec) Required for Zero Coriolis Deflection ($\emptyset$ = 180°)*

Altitude (ft)	Latitude (deg) 15	30	45	60	75
0	0	0	0	0	0
500	223	103	60	34	16
1000	315	146	84	49	23
1500	385	179	103	60	28
2000	445	207	119	69	32
2500	498	231	133	77	36
3000	545	253	146	84	39
4000	629	292	169	97	45
5000	704	327	189	109	51
6000	771	358	207	119	55
7000	833	386	223	129	60
8000	890	413	239	138	64
9000	944	438	253	146	68
10000	995	462	267	154	71
20000	1408	653	377	216	101
30000	1724	800	462	267	124
40000	1991	924	533	308	143
50000	2226	1033	596	344	160
60000	2438	1131	653	377	175
70000	2633	1222	706	407	189

*To obtain values for other azimuth angles, multiply the table values by the absolute value of cos $\emptyset$.

TABLE II

Summary of Coriolis Deflections

Direction of Projection[b]	Direction of Deflection[a]			
	In Northern Hemisphere	In Southern Hemisphere	At the Equator	At the Poles
East	Right	Left	No Deflection	------------
West	Right	Left	No Deflection	------------
North	Right	L if $V_o > V_c$ R if $V_o \leq V_c$	Right	Left at South Pole
South	R if $V_o > V_c$ L if $V_o < V_c$	Left	Left	Right at North Pole
Vertically Down	East	East	East	No Deflection
North from Latitude θ, Azimuth Ø, Altitude y_o	Right	L if $V_o \geq V_c$ R if $V_o \leq V_c$	----	------------
South from Latitude θ, Azimuth Ø, Altitude y_o	R if $V_o \geq V_c$ L if $V_o \leq V_c$	Left	----	------------

a Deflection is zero if $V_o = -(2g'y_o)^{\frac{1}{2}} (\cos \emptyset)/(3 \tan \theta) = V_c$

b $\epsilon = 0$

Note: L, Left; R, Right

REFERENCES

1. Burns, G. Preston, "Deflection of Projectiles due to Rotation of the Earth," American Journal of Physics, November 1971, p. 1329.