ANALYSIS OF FLOW AND HEAT TRANSFER IN SMALL ARMS
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### Analysis of Flow and Heat Transfer in Small Arms

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**ABSTRACT**
A computer model of the pressure distribution and flow characteristics in small arms barrel has been developed. This model shows the sensitivity of the pressure history and launch velocity of surface concentration density of deterrent, flame front velocity, and projectile base heating.
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Abstract

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### NOMENCLATURE

<table>
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<tbody>
<tr>
<td>A</td>
<td>cross-sectional area of gun bore</td>
<td>ft²</td>
</tr>
<tr>
<td>a</td>
<td>speed of sound</td>
<td>ft/sec</td>
</tr>
<tr>
<td>cₚ</td>
<td>specific heat of propellant gas at constant pressure</td>
<td>BTU/lbm-°R</td>
</tr>
<tr>
<td>cᵥ</td>
<td>specific heat of propellant gas at constant volume</td>
<td>BTU/lbm-°R</td>
</tr>
<tr>
<td>D</td>
<td>diameter of gun bore</td>
<td>ft</td>
</tr>
<tr>
<td>D₀</td>
<td>initial diameter of propellant grain (weighted average)</td>
<td>ft</td>
</tr>
<tr>
<td>Fₚgas</td>
<td>energy per unit mass of gas</td>
<td>BTU/slug</td>
</tr>
<tr>
<td>Fₚd</td>
<td>drag force on propellent particles per unit volume</td>
<td>lbf/ft³</td>
</tr>
<tr>
<td>g</td>
<td>acceleration of gravity</td>
<td>ft/sec²</td>
</tr>
<tr>
<td>h</td>
<td>enthalpy</td>
<td>BTU/slug</td>
</tr>
<tr>
<td>J</td>
<td>mechanical equivalent of heat</td>
<td>ft-lbf/ BTU</td>
</tr>
<tr>
<td>L</td>
<td>distance burned normal to propellant grain surface</td>
<td>ft</td>
</tr>
<tr>
<td>mₚ</td>
<td>rate of mass of gas generated per unit length</td>
<td>slug/ft-sec</td>
</tr>
<tr>
<td>p</td>
<td>gas pressure</td>
<td>lbf/ft²</td>
</tr>
<tr>
<td>q</td>
<td>heat loss through barrel wall</td>
<td>BTU/ft²·sec</td>
</tr>
<tr>
<td>R</td>
<td>gas constant for propellant gas</td>
<td>ft-lbf/slug-°R</td>
</tr>
<tr>
<td>R̄</td>
<td>disk radius for an individual grain</td>
<td>ft</td>
</tr>
<tr>
<td>Rᵢ</td>
<td>one half of web thickness of individual grain</td>
<td>ft</td>
</tr>
<tr>
<td>r</td>
<td>linear burning rate of propellant</td>
<td>ft/sec</td>
</tr>
<tr>
<td>s</td>
<td>specific entropy</td>
<td>BTU/lbm-°R</td>
</tr>
<tr>
<td>Sₚ</td>
<td>surface area of an individual grain</td>
<td>ft²</td>
</tr>
<tr>
<td>Tₖ</td>
<td>gas temperature</td>
<td>°R</td>
</tr>
<tr>
<td>t</td>
<td>time</td>
<td>sec</td>
</tr>
<tr>
<td>uₚ</td>
<td>velocity of propellant particles</td>
<td>ft/sec</td>
</tr>
<tr>
<td>uₚ</td>
<td>velocity of gas</td>
<td>ft/sec</td>
</tr>
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</table>
$V_p$  volume of an individual grain, $ft^3$

$W$  powder potential, $cvT$, BTU/lbm

$W_r$  weight of projectile, lbm

$e$  particle volume fraction, dimensionless

$n$  number of particle grains per unit volume, $ft^{-3}$

$\rho_g$  gas density, slug/$ft^3$

$\rho_p$  propellant density, slug/$ft^3$

$\tau_w$  wall shearing stress, lbf/$ft^2$ or slug/$ft$-$sec^2$
Introduction

Investigations into propellant performance, barrel materials and weapon kinematics require accurate description of the weapon interior ballistics. The present study was undertaken to determine the effect of heat transfer on interior ballistics. In examining the interior ballistics of small arms, the effect of a number of parameters must be included. These parameters include chamber geometry, propellant geometry, propellant composition and burning rate.

The heat transfer rates in small arms are very high and are considered to be a major cause of barrel erosion. Accurate description of these heat transfer rates is necessary for analytical prediction of pressure and velocity histories in small arms.

The present study uses the method of characteristics to solve the governing equations. The model assumes the problem to be one dimensional and transient. Assuming a fixed propellant description, the effect of various heat transfer models has been examined. The performance standard is test data from reference 1 presented in figure 1.
Reference (2) presents a ballistic model developed from earlier work which was modified to include the effect of non-uniform propellant composition and non-simultaneous ignition. In this reference the propellant is represented by a composite of 9 different composition propellants. This variable propellant array allows the handling of different size grains and different deterrent concentration densities. Sample results are included in reference (2) which shows good agreement with test.

Reference (3) presents an analytical investigation of the flow in gun barrels with particular emphasis on the heat transfer. In addition to the transient nature of the flow, reference (3) considers the problem posed by the lack of a boundary layer at the base of the projectile. This condition occurs because in the projectile based region the gas velocity is equal to the projectile velocity. This condition means that the Reynolds analogy commonly used in heat transfer cannot be applied to this case. However, the reference does propose an expression for heat transfer coefficient in the projectile base region. The resulting convective coefficients are between 5 and 20 Btu/sec ft² °F. Comparisons between experiment and prediction presented in reference (3) indicate that even these high coefficients underestimate the actual results. However, the results of reference (3) represent the best available procedure for calculating the heating at the base of the projectile.

Reference (4) presented an analytical study of interior ballistics in which the governing equations were solved using a finite difference procedure. The geometric dimensions used in reference (4) did not correspond to standard small arms. In addition, the analysis was limited to stationary propellant and to propellant moving at the same velocity as the gas. In the area of heat transfer reference (4) also obtained very high heat transfer coefficients
\( \times 10 \text{Btu/sec ft}^2 \text{°F} \). It is worthy of note that for cases with and without heat transfer and wall friction the ballistic performance was not changed significantly. The present study has found that heat transfer particularly at the projectile base, is very significant.

Reference (5) presents an extensive study of the flame front movement through a porous bed propellant. The analysis is presented for a venting bomb type device. However, the results for flame propagation can be applied to the small arms case. Indeed, the functional relationship between flame velocity and pressure can be obtained from the data presented in reference (5). Reference (6) considers the same problem as reference (5) but applied to the small arms case. The results of reference (6) have been used in the present study to determine the progress of the flame front through the propellant bed. Reference (7) presents an iterative model for the interior ballistic problem in which parameters affecting the burning rate were changed until the calculated results agreed with test data. The caliber used in reference (7) is the same as the present study, (i.e. 5.56 MM); however, the experimental and analytical results are considerably different. This is most likely due to different charge weight and propellant type. In addition, a number of the loss factors considered in the present study are not included.

Reference (8) is the analytical basis for the present study. This reference presents the model which has been used and also outlines the calculation procedure for the method of characteristics. The nomenclature of reference (8) has been adopted in the present report. The present investigation has expended considerable effort in programming reference (8). Reference (8) describes the interior ballistic problem as a one dimensional transient flow problem. In the model the propellant is assumed to ignite instantaneously and burn at a rate proportional to the gas pressure. The geometry of the propellant is uniform and based upon reference (10).
Although heat transfer between the gas and propellant is neglected, drag between these two is considered. The transformation of the governing equations from the physical domain to the characteristic domain is described in detail in reference (9). The heat transfer to the wall and the wall friction are included in the model but the detailed descriptions are developed in the present study. In addition, the effect of chamberage and projectile resistive forces are also included in the model of reference (9).
Analysis

To determine the effect of heat transfer and to evaluate potential heat transfer relationships, an accurate model of the flow field must be developed. Considerable effort during the present study was devoted to developing a model which could reproduce test data. In addition, during the course of this model development it became apparent that the assumption of uniform ignition was not adequate and therefore a more realistic flame front model had to be developed. This section describes the flow field flame front analysis, and heat transfer models.

The flow is two phase, non-steady, including heat and mass addition. The basic model describing the problem has been developed in reference (8). Because of the geometry present, the problem is quasi-one dimensional. The resulting conservation equations form a set of quasi-linear partial differential equations of the first order and hyperbolic type. For this situation the method of characteristics provides the simplest means of solution. Reference (8) provides the details of the transformation of the governing equations to the characteristic form. In addition, reference (9) provides the theoretical foundation for the applications of the method of characteristic to the present problem.
Flow Field Governing Equations

The governing equations described in these sections are based upon the following assumptions:

1. Flow is quasi-one dimensional
2. The gas is assumed to be ideal
3. The specific heat and molecular weight are dependent upon the amount of deterred and non-deterred propellant burned
4. All propellants grains are initially of uniform surface area, volume, and radius. Weighted mean values based upon reference (10) and (11) are used.
5. Interactions between individual propellant particles and between particles and the boundary are negligible
6. The thermal motion of the propellant particles does not contribute to the pressure
7. The average gas temperature is defined by the heat content of the gas phase
8. The propellant particles are spheres for drag calculations
9. Heat dissipation due to viscosity is negligible
10. Radiation is negligible
11. Gravity is negligible
Overall Gas Continuity Equation
\[
\frac{\partial}{\partial t} (\rho_g A(1 - e)) + \frac{\partial}{\partial x} (\rho_g A U_g (1 - e)) = \dot{m}_g
\]  
(1)

where \(\dot{m}_g\) is the gas generated per unit length
\[
\dot{m}_g = e S_p A P / V_p
\]

Propellant Particle Continuity Equation
\[
\frac{D e}{D t} = - e \frac{\partial U_p}{\partial x} - \frac{\partial}{\partial x} \left( \frac{\rho_p A}{\rho_p} \right) - \dot{m}_g
\]  
(2)

Overall Gas Momentum Equation
\[
A \rho_g (1 - e) \frac{\partial U_g}{\partial t} + (1 - e) U_g \rho_g A \frac{\partial U_g}{\partial x} = - U_g \dot{m}_g - \tau_w \pi D - (1 - e) A \frac{\partial P}{\partial x} - F_d A
\]  
(3)

where \(\tau_w\) is shear stress at the wall and \(F_d\) is particle drag per unit volume

Propellant Particle Momentum Equation
\[
\frac{D U_p}{D t} = \frac{U_p \dot{m}_g}{\rho_p A e} + \frac{F_d}{\rho_p e}
\]  
(4)

Overall Gas Energy Equation
\[
\frac{\partial}{\partial t} \left( \left( 1 - e \right) \rho_g \left( U_g^2 + \frac{2}{3} c_v T_g \right) A \right) + \frac{\partial}{\partial x} \left( \left( 1 - e \right) \rho_g A U_g \left( \frac{2}{3} c_v T_g + \frac{P}{\rho_g} + \frac{U_g^2}{2} \right) \right) = - \frac{\partial}{\partial x} \frac{\partial}{\partial x} \frac{F_d A}{\rho} \left( U_g - U_p \right) + \frac{\partial}{\partial x} \left( W \dot{m}_g \right)
\]  
(5)

where \(W\) is the heat content of the propellant per pound
Overall Gas Entropy Equation

\[ \frac{DS}{Dt} = (C_{gas} + \frac{U_g F}{2}) - R_o) / (A(1-e) \rho_s T_g g) \]  \hspace{1cm} (6)

where,

\[ E_{gas} = -q \pi D - F_A \frac{U_g - U_o}{2} + g W \sin \gamma \]

\[ F = -U_g \sin \gamma + \frac{C_c \pi D}{2} + F_A \]

\[ R_o = \rho_s \frac{g c v T_g + U_g^2}{2} + \frac{PA}{2} - \frac{P \sin \gamma}{P_o} \]

In the preceding equations, the dependent gas variables are entropy, gas temperature, pressure, gas density and gas velocity.

The next step in the solution procedure is to apply the method of characteristics. In this approach the dependent variables become the P and Q Riemann variables where:

\[ P = \frac{2}{5-1} a + U_g \]

\[ Q = \frac{2}{5-1} a - U_g \]

Solutions are determined along these P and Q characteristics. The characteristics represent lines along which infinitesimal disturbances are propagated at the local speed of sound in the gas phase. Along each characteristic the following relationships between x, t are true:

\[ \frac{dx}{dt} = U_g + a \]  \hspace{1cm} (along P characteristic)

\[ \frac{dx}{dt} = U_g - a \]  \hspace{1cm} (along Q characteristic)

Equation (1) and (3) can be combined into two equations for P and Q along the respective characteristic using the ideal gas relations (for details see reference (3)):

\[ a^2 = \frac{2 R T_g}{5-1} \]

\[ \ln P = \frac{2}{5-1} \ln a - \frac{g T S}{R} \]

yielding equations (7) and (8).
\[
\frac{S + P}{\partial t} = -a \frac{U_g}{2} \frac{\partial \ln A}{\partial x} + \frac{a g J}{8} \frac{DS}{\partial t} + \frac{a g J}{8} \frac{S + S}{8} - \frac{T_{vD}}{A(1-e)R} - \frac{a D}{8} \frac{\partial \ln (1-e)}{\partial t}
\]

\[
-\frac{F_d}{p_g(1-e)} - \frac{U_g \ln g}{A(1-e) p_g} + \frac{a^3 \ln g}{A(1-e) P^g}
\]

\[
\frac{S - P}{\partial t} = -a \frac{U_g}{2} \frac{\partial \ln A}{\partial x} + \frac{(3-1) a g J}{8} \frac{DS}{\partial t}
\]

\[
+ \frac{a g J}{8} \frac{S - S}{8} + \frac{T_{vD}}{A(1-e) p_g} - \frac{a D}{8} \frac{\partial \ln (1-e)}{\partial t}
\]

\[
+ \frac{F_d}{p_g(1-e)} + \frac{U_g \ln g}{A(1-e) p_g} + \frac{a^3 \ln g}{A(1-e) P^g}
\]

where:

\[
\frac{S + S}{\partial t} = \frac{2}{\partial t} + \frac{(U_g \pm a) \frac{2}{\partial x}}{\partial t}
\]

Equation (6), (7) and (8) along with the appropriate ideal gas relations are the governing equations for the gas phase. Equations (2) and (4) describe the propellant particle phase.

**Initial and Boundary Conditions**

At \(t=0\), the first \(Q\) wave leaves the base of the projectile and travels toward the breech. During the time prior to the arrival of the \(Q\) wave at the breech, it is assumed that the gas in the chamber is at rest and that the temperature is constant at the flame temperature, \(T_0\). The pressure in the region between the projectile and breech will rise as the \(Q\) wave advances.
due to the burning of propellant. The slope of this first Q wave is given by equation (9).

\[ \frac{dx}{dt} = -a_0 = -\sqrt{8RT_0} \] (9)

The Q wave is divided into a specified number of equally spaced nodes. The nodes are the origin for the P wave. At each node the gas temperature and speed of sound are known, the other parameters are given by the following relations:

\[ t_{i+1} = t_i + \frac{(x_{i+1} - x_i)}{a_0} \]

\[ \frac{DS}{Dt} = (qW \dot{m}_q - \dot{m}_q (gC_v T_q + \frac{P}{\rho} ) - \frac{Pa}{3} \frac{DS}{Dt} )/ (A (1 - c) \rho_q T_q) \]

\[ \frac{De}{Dt} = -\frac{\dot{m}_q}{A c} \]

\[ S_{i+1} = S_i + \frac{DS}{Dt} (t_{i+1} - t_i) \]
Since the speed of sound is constant along this wave, Q is also constant. This procedure is different from that described in reference 8, but it yields the same results.

The spatial boundaries of the problem are the breech (x=0) and the base of the projectile. The breech is at a fixed condition with the gas and propellant velocity equal to zero. The location of the projectile base is determined by the intersection of a P wave and the equation of motion of the projectile. The two equations governing the position of the projectile are:

\[ \frac{d^2x}{dt^2} = \frac{q_{A_p}}{W_t} - \frac{q_{R_1}}{W_t} \]  

where \( R_1 \) is the engraving force and \( W_t \) is the weight of the projectile.

At the projectile base the gas velocity, propellant velocity and projectile velocity are all equal.
Numerical Procedure

The numerical solutions to the governing equation will be obtained using the finite difference procedure. In this approach, derivatives will be replaced by finite differences (explicit) and these will be used to generate the desired characteristic grid. Fig 1 shows the proposed grid system.

Figure 1: Schematic of Characteristic Grid Points

There are three types of grid points, interior points, \((a,b,c)\), breech points \((e,f,g)\) and projectile base points \((h, i, j)\). The governing equations and solution procedures are different for each of these types of grid points and therefore, each will be handled separately.
Interior Points

Figure 2 shows a typical interior unknown grid point (6). It is on a P wave from point (4) and a Q wave from point (3).

![Diagram of Points Used to Calculate Interim Grid Points](image)

Point (7) represents the location of a gas particle which intersects point (6) and the line connecting points (3) and (4). Point (5) is similar to point (7) except it represents a propellant particle. The slope of the P and Q characteristics are given by equations (12) and (13).

\[
\frac{x_6 - x_5}{t_6 - t_5} = U_{g3,6} + a_{3,6} \tag{12}
\]

\[
\frac{x_6 - x_3}{t_6 - t_3} = U_{g4,6} - a_{4,6} \tag{13}
\]

where

\[
U_{g3,6} = \frac{(U_{g3} + U_{g6})}{2}
\]

\[
a_{3,6} = \frac{(a_3 + a_6)}{2}
\]

\[
U_{g4,6} = \frac{(U_{g4} + U_{g6})}{2}
\]

\[
a_{4,6} = \frac{(a_3 + a_6)}{2}
\]

Solving for \(t_6\) and \(x_6\)

\[
t_6 = \frac{x_6 - x_3 + t_3 (U_{g3,6} + a_{3,6}) - t_4 (U_{g4,6} - a_{4,6})}{(U_{g3,6} + a_{3,6}) - (U_{g4,6} - a_{4,6})} \tag{14}
\]

\[
x_6 = x_3 + (U_{g3,6} + a_{3,6})(t_6 - t_3) \tag{15}
\]
Since the values at point 6 are initially unknown, an assumed value of a given parameter is used and then the procedure is repeated until the assumed value and calculated value agree to within a specified tolerance. The initial assumption is an arithmetic average of the values at points (3) and (4).

Knowing the location of point 6, it is possible using the procedure described in reference (8) pages 23 through 30 to generate the data required for the solution to proceed. For brevity, the details will be omitted; however, certain key points should be mentioned. The average burning rate between points (5) and (6) given by:

\[ r_{5,6} = B P_{5,6}^n \left( 1 + \frac{1}{k} \left( \frac{U_p - U_{p,6}}{U_p - U_{p,5}} \right) \right) \]

The distance burned normal to a particle surface at (6) is:

\[ \delta 6 = \delta 5 + r_{5,6} (t_6 - t_5) \]

Knowing \( \delta 6 \) and \( \delta 5 \) the average particle volume and surface area between (5) and (6) can be determined

\[ V_{p5,6} = \frac{(V_{p5}(\delta 5) + V_{p6}(\delta 6))}{2} \]

\[ S_{p5,6} = \frac{(S_{p5}(\delta 5) + S_{p6}(\delta 6))}{2} \]

The gas per unit length generated along the propellant path (5) and (6) is

\[ m_{g,5,6} = \left( \frac{\varepsilon_{p,6}}{V_{p5,6}} \right) S_{p5,6} r_{5,6} P_p A \]
The rate of change in propellant particle velocity between (5) and (6) is given by equation (16)

\[
\frac{Dv_p}{Dt} = \frac{U_{p5,4} m_{3,4}}{R_p A E_{5,4}} + \frac{F_{d3,4}}{R_p E_{5,4}}
\]  

(16)

The rate of change of propellant particle volume fraction is given by equation (17)

\[
\frac{De}{Dt} = -\frac{\varepsilon_{2,4} Dv_p}{U_{p8,4}} - \frac{2 e_{2,4} U_{p8,4} (A_4 - A_3)}{(A_4 + A_3) (x_4 - x_3)} - \frac{m_{2,4}}{A_{1p}}
\]  

(17)

The rate of entropy change for the gas can now be computed using equation (6). The change in P and Q along the appropriate characteristic is given by equations (18) and (19)

\[
P_6 = P_3 + \frac{e_{2,4} P_3}{\varepsilon_4} (t_6 - t_3)
\]  

(18)

\[
Q_6 = Q_4 + \frac{e_{2,4} Q_4}{\varepsilon_4} (t_6 - t_4)
\]  

(19)

where \(\frac{e_{2,4} P_3}{\varepsilon_4}\) and \(\frac{e_{2,4} Q_4}{\varepsilon_4}\) are given by the equations (20) and (21)

\[
\frac{e_{2,4} P_3}{\varepsilon_4} = -\frac{2 A_{2,4} U_{p8,4} (A_4 - A_3)}{(A_4 + A_3) (x_4 - x_3)} + \frac{(2 - 1) A_{2,4}}{R} \frac{J}{R} \frac{DS}{D\varepsilon_4} + \frac{A_{2,4} J}{(1 - e_{2,4}) R} \frac{[D_4] + A_{2,4} (Z_4 - \varepsilon_4)}{(x_4 - x_3)} - \frac{J_0 \Gamma D}{A (1 - e_{2,4}) R_{3,4}} + \frac{A_{2,4}}{(1 - e_{2,4})} \left( \frac{D_4}{\varepsilon_4} + \frac{e_{2,4} \varepsilon_4}{A P_{3,4}} \right) - \frac{1}{e_{2,4} (x_4 - x_3)} \left( F_3 + U_{p8,4} \varepsilon_4 \right)
\]  

(20)
Using the calculated values for \( P_6 \) and \( Q_6 \), new values for the gas velocity and speed of sound can be computed

\[
U_{g6} = \frac{(P_6 - Q_6)}{2} \tag{22}
\]

\[
a_6 = \frac{1}{4} (P_6 + Q_6) \tag{23}
\]

The entropy at point 6 can be calculated from equation (24)

\[
S_6 = S_7 + \frac{DS}{DT} (t_6 - t_7) \tag{24}
\]

The new values of \( U_{g6} \), \( a_6 \), and \( S_6 \) can be used to calculate the remaining thermodynamic properties based upon the equations on page 34 reference 8.

In addition \( U_{g6} \) and \( a_6 \) are available to obtain a new estimation of the location of point (6) (i.e., \( t_6, X_6 \)). The procedure described above is then repeated until the new values of \( a_6 \) and \( U_{g6} \) agrees with the previously calculated ones to within a specified tolerance.
Projectile Base Point

The solution at the projectile base is determined by the intersection of a P wave and the path of the projectile. Figure 3 presents a diagram of this intersection. The location and thermodynamic properties at points 1 and 2 are known. This section describes the procedure to find point 4. The slope of the P wave from 2 to 4 is given by equation (25)

\[
\frac{x_4 - x_2}{t_4 - t_2} = U_{q_{A_{2}}} + a_{A_{2}}
\]  

\hspace{1cm} (25)

In addition integration of the equation of motion of the projectile yields equation (26)

\[
x_4 = x_1 + U_{q_{1}}(t_4 - t_1) + \frac{1}{2} \left[ \frac{(P_1 + P_2)}{2} \frac{a A}{W_2} - \frac{P_1}{W_2} \right] (t_4 - t_1)^2
\]  

\hspace{1cm} (26)
Simultaneous solution of equations (25) and (26) yields equation (27)

\[ t_4 = \frac{-B - \sqrt{B^2 - 4AC}}{2A} \]  

(27)

where

\[ C = (x_1 - x_2) + (u_{34} + a_{34})t_2 - \left( \frac{(P_1 + P_2) \gamma A - R_{1g}}{2w_k} \right) t_2^2 \]

\[ - t_1 \left( u_{21} - t_1 \left( \frac{2A(P_2 + R_{1g})}{w_k^2} \right) \right) \]

\[ B = u_{31} - \left( \frac{(P_1 + P_2) \gamma A - R_{1g}}{2w_k} \right) t_1 - \left( u_{32} + a_{32} \right) \]

\[ A = \frac{1}{2} \left( \frac{(P_1 + P_2) \gamma A - R_{1g}}{w_k^2} \right) \]

Knowing \( t_4 \), \( X_4 \) can be obtained from equation (25). At the base of the projectile the projectile velocity equals the gas velocity (eq. (28)).

\[ U_{g4} = U_{g1} + \left( \frac{(P_1 + P_2) \gamma A - R_{1g}}{2w_k} \right) (t_4 - t_1) \]

(28)

Having established a first approximation for the location and the velocity at point 4, the change in P along the characteristic 2-4 can be determined. The details regarding this procedure are given in reference 8, pages 36 to 40. The rates of change of P (\( \Delta P / \Delta t \)) and P4 are given by equations (29) and (30)
Knowing $P_4$ and $U_{g4}$, a new velocity of sound, $a_4$, can be computed from equation (31)

$$a_4 = (P_4 - U_{g4})(\gamma - 1)/2$$

Using $a_4$ and $s_4$, revised values for the other thermodynamic variable can be computed. The procedure is then repeated until the values of $U_{g4}$ and $a_4$ for two consecutive calculations agree within a specified tolerance.
After the projectile leaves the barrel the procedure is the same as described on pages 49 and 50 of reference 8. It is assumed that the gas velocity is sonic at muzzle exit and that Q waves after the projectile leaves, originate from the muzzle exit. Under these conditions $X_{24}$ is fixed and the following relations exist:

$$t_4 = t_2 + \frac{(X_4 - X_2)}{(U_{q2} + a_4)}$$

$$a_4 = \frac{P_4 (\gamma - 1)}{(\gamma + 1)}$$

$$U_{q4} = a_4$$
Breech Points

The breech is located at X=0. Figure 4 diagrams the intersection of a Q wave with the breech.

![Diagram of Points Used to Calculate Breech Point](image)

The time $t_3$ is given by equation (32)

$$t_3 = -\frac{X_2}{Ug_2} - a_2, \ z + t_2$$  \hspace{1cm} (32)

In equation (32) the initial estimate if $a_3$ is made by assuming the same values as point (1). The change in $Q$ from 2 to 3 is then computed and a revised $a_3$ can be obtained from equation (33)

$$Q_3 = \frac{(z-1)}{z} Q_3$$  \hspace{1cm} (33)

The rate of change of propellant volume fraction and of gas entropy are given by equations (34) and (35)

$$\frac{De}{Dt} = -\frac{\frac{Eg}{A}}{\frac{Dm}{3x}}$$  \hspace{1cm} (34)

$$\frac{Df}{Dt} = \frac{Egas - Rs}{A(1 - \epsilon_{x})} \frac{Q_{1,3}}{T_{1,3}}$$  \hspace{1cm} (35)
where

\[
\begin{align*}
E_{\text{eq}} &= -g \tau D + g W \bar{m}_g \\
R_{\text{eq}} &= \bar{m}_g (q c_v T_{\text{eq}}) + P_{1,3} A \frac{\bar{D}_g}{\dot{t}} \\
&\quad + \frac{P_{1,3} \bar{m}_g}{\rho_{1,3} g}
\end{align*}
\]

The rate of change in \( Q \) along the characteristics from 2 to 3 is given by equation (36)

\[
\frac{\bar{D} \bar{q}_{1,3}}{\bar{t}} = \frac{a_{1,3} q_{1,3}}{\bar{t}} \frac{\bar{D}_g}{\dot{t}} + \frac{a_{1,3}}{(1 - e_{1,3})} \frac{\bar{D}_g}{\dot{t}}
\]

\[
+ \frac{a^3_{1,3} \bar{m}_g}{A (1 - e_{1,3})} P_{1,3} T
\]

(36)

The value of \( Q \) and \( S \) at point 3 is given by equation (37) and (38)

\[
\begin{align*}
Q_3 &= Q_1 + \frac{\bar{D} Q_{1,3}}{\bar{t}} (t_3 - t_1) \\
S_3 &= S_1 + \frac{\bar{D}_g}{\dot{t}} (t_3 - t_1)
\end{align*}
\]

(37)  (38)

Using equation (33) and (37), a new value of \( a_3 \) can be calculated. In addition, \( a_3 \) and \( S_3 \) can be used to calculate the remaining thermodynamic variables. The procedure is repeated until the value at \( a_3 \) from two successive calculations agrees within a specified tolerance.
The preceding procedure has been programmed and the details of the program are presented in Appendix A.

Flame Front Governing Equations

Initially it was assumed that there was instantaneous ignition of the entire propellant bed. This resulted in excessive breech pressure and low projectile base pressure. A better approximation is obtained by using a flame front with a finite velocity. Based upon the results from references (4) and (12), it was determined that the flame front velocity was proportional to the pressure behind the front. Using data from these references the following relationship was developed

\[ V_F = \alpha p^\beta \]  \hspace{1cm} (39)

where

\[ \alpha = 31.5 \text{ in/sec/(spi)} \]
\[ \beta = .48 \]

The effect of the flame front progressing through the propellant bed can then be determined by integrating from the breech to the projectile base. The pressure and temperature behind the flame front was assumed uniform. The gas velocity was also assumed to be zero. For a given time increment the distance the flame front moves is given by equation (40).

\[ dL = \int_{t_1}^{t_2} V_F \, dt \]  \hspace{1cm} (40)

In equation (40) the velocity is based upon the initial pressure (at \( t_1 \)) and an assumed final pressure (at \( t_2 \)). The procedure is repeated until the assumed final pressure and calculated final pressure agree to within 5%. The distance burned normal to the propellant surface in the same time period is given by equation (41)

\[ d \, \ell = (B \, p^\gamma) (t_2 - t_1) \]  \hspace{1cm} (41)
Using the results of reference (10) the new volume a propellant grain resulting from the surface recession given in equation (41) is

\[ L_{i,j} = L_{i,j-1} + \Delta x \]
\[ RL = R_i - L_{i,j} \]
\[ V_{i,j} = 2\pi (RL')(RL) + \pi^2 RL'^2 (RR + 4RL') \]  

(42)

where

\[ RR = R_o - R_i \]
\[ R_o = \text{equivalent mean radius of propellant grain} \]
\[ R_i = \text{one half web} \]

The total gas generated in the cell defined by \( dL \) from the time \( t = 0 \) to \( t = t_2 \) is given by equation (43).

\[ x_{mg} = dL \eta a (V_{oi} - V_{i,j}) P_f \]  

(43)

Equation (44) gives the volume fraction.

\[ \varepsilon_{i,j} = \frac{(\eta A dL - x_{mg}/P_f)}{(\eta A dL)} \]  

(44)

The pressure behind the flame front is obtained by applying a modified perfect gas relationship to the total gas generated from every cell.

\[ P = \frac{(X TC \cdot W + P_f A 2L)}{(A 2L - V_{oi} - X TC b)} \]  

(45)
where

\[ \text{sum of gas generated in each cell at time } t_2 \]
\[ \text{distance flame front has progressed from } t = 0 \text{ to } t_2 \]
\[ \text{total volume of solids behind flame front} \]
\[ \text{propellant covolume} \]

The pressure given by equation (45) applies to the entire region behind the flame front. This pressure is averaged with the pressure from the preceding time period to determine the flame front velocity. The procedure is repeated until the pressure in equation (45) for two successive passes agrees to within 5%.

The flame front calculation procedure has been programmed and the details of the program are presented in Appendix B.
Heat Transfer Models

Since the analytical model considers the problem in three distinct regions, the breech, the interior, and the projectile base, the heat transfer models will be considered in the same manner. The approach in defining the heat rates has been to use the best data either experimental or analytical.

Breech Region

In the breech region, the hot gases stagnate and as such, no boundary layer is formed. Although there may be recirculation flows in this region, analytical description would be extremely difficult. However, there is data available on heating in the projectile case region. Reference (12) developed experimental data on the heating rate at various locations in ammunition case region or a function of time. Figure 5 presents the results from reference 12. To obtain the heating at the breech, the data in figure 5 was extrapolated to \( x = 0 \) and then replotted in figure 6. The peak heating occurs at .25 milsec with a value of 3.7 Btu/ft\(^2\)sec. After reaching this peak value the breech heating decreases exponentially with time. Rather than trying to match the peak heating, the approach taken was to satisfy the long term heating by fitting the following equation to the data in figure 6.

\[
q = K_1 e^{K_2 (T_f - T_w)}
\]

Using 1 and 2 milsec as the matching times, the following values for \( K_1 \) and \( K_2 \) were obtained:

\[
K_1 = 3.24 \text{ Btu/ft}^2\text{sec} \\
K_2 = 587.78 \text{ sec}^{-1}
\]

The dotted line in figure 6 gives the profile of the fitted curve. To determine the effect of breech heating, the value of \( K_1 \) was varied from 0 to 3.24 on a series of computer runs. The results showed that breech heating had a negligible effect of projectile launch velocity and peak pressure.
HEAT TRANSFER DATA REF. 12

FIGURE 5

DISTANCE FROM BREECH - IN

HEAT TRANSFER - COEFF - BTU/HR-SEC-FT

TIME (MIN SEC)

- 0.25
- 0.50
- 0.75
- 1.00
EXTRAPOLATION OF HEAT TRANSFER DATA TO X = 0.0

DATA REF 12 EXTRAPOLATED TO BREECH

$\eta = 3.242 \times 10^{-11}$

FIGURE 6
Interior Region

The interior region refers to those locations not considered to be the breech or projectile base. In the interior region, the boundary layer along the wall is fairly well defined. For this reason the approach has been to use the standard Reynolds Analogy:

\[
\frac{n}{\rho V_{cp}} = \frac{f}{2}
\]

The selection of the friction factor, \( f \), is somewhat arbitrary but values between .001 and .006 have been used.

Projectile Base Region

Describing the heat transfer in this region is very difficult because the normal boundary layer approximations are not satisfied. In this region the gas velocity at the wall or base of the projectile equals the projectile velocity which is the highest velocity in the problem. In addition, the level of projectile heating (barrel heat transfer at projectile) has a profound effect upon the peak projectile base pressure and therefore, the launch velocity.

Initially, the approach was to simply use the Reynolds Analogy in this region with the gas velocity equal to the projectile velocity. However, this approach grossly underestimates the peak pressure at the base. Since the pressure increases with entropy decrease assuming constant gas temperature, this condition is valid as long as the propellant is burning. Entropy can be decreased by higher heat transfer rates. The approach taken therefore, was to arbitrarily increase the heat transfer coefficient. Factors of ten were necessary to significantly increase the base pressure. The requirement of a factor of this size seems unreasonable. In addition the peak heating rate occurs near the barrel exit which does not agree with test data.
The next approach was to use the heating rate based upon a shock tube end condition. This procedure was developed in reference (13). In this formulation the following equation was used:

\[ n = \left( \frac{\rho R_{e} c_{v}}{k} \right)^{\frac{1}{2}} T_{\text{gas}} (0.5 + \frac{1}{2} \left( \frac{T_{\text{proj}}}{T_{\text{gas}}} \right)^{2} - \frac{T_{\text{proj}}}{T_{\text{gas}}})^{\frac{1}{2}} \]

In this representation the shape of the heat transfer coefficient as a function of time is correct. However, the overall levels are too low. The constant, K, can vary by a factor of 2 (reference (13)) but this is still inadequate to predict the required levels of heating.

The most promising procedure is the one described in reference (3). In this reference the flow in the barrel is examined qualitatively and quantitatively. The reference notes that away from the projectile the Reynolds analogy could be used. However, in the projectile base region the momentum thickness goes to zero because the gas velocity and projectile velocity are equal. If a boundary layer analysis were attempted this would yield an infinite Nusselt number.

The approach taken in reference (3) was to use the usual boundary layer method away from the projectile and breech. These locations are handled using approximate solutions. The analysis develops the integral momentum equation and solves this equation introducing empirical data when needed. The projectile heat transfer is given by:

\[ Nu = \frac{4 a_{*} R_{e} P_{t} V_{0}}{R_{e} \cdot 0.6} \]
where:

\[ \text{Nu} = \text{Nusselt Number } \frac{h d}{k} \]
\[ \alpha = \text{shear law constant } = 0.332 \]
\[ Pr = \text{Prandtl Number} \]
\[ Re = \text{Reynolds Number based upon gas velocity and } L \]
\[ Re_{\theta} = \text{Reynolds Number based upon momentum thickness at transition} \]
\[ L = \text{Distance from projectile to breech} \]

The transition Reynolds \( Re_{\theta} \) taken as 200. The heat transfer prediction method of reference (3) was used in the computer model because it provided the right profile and heat transfer rates of the right magnitude.
Numerical Results

Using the analysis described in the preceding sections, an analytical model computer program was developed and number of cases were examined. The most important parameter affecting the pressure and velocity history is the surface concentration density (SCD) of deterrent. Figures 7 and 8 present the pressure and velocity histories for an SCD of 0.1 and 0.2 respectively. In these two cases it is assumed that the flame velocity through the propellant bed is infinite. The surface concentration density of deterrent refers to the distribution of deterrent through the propellant grain. For all cases using the same propellant, the total amount of deterrent is the same; however, for an SCD of 0.1 compared to 0.2 the deterrent is spread to a greater grain depth. This means that the effect of the deterrent is less over a greater depth. For instance the SCD = 0.1 deterred flame temperature is 4615 °R compared to 3331 °R for an SCD = 0.2. From figure 7 it can be seen that the launch velocity for an SCD = 0.1 is 2500 fps. Launch occurs 1.27 milsec after ignition. For SCD = 0.2, (figure 8) the launch velocity is 1700 fps at 1.69 milsec. For both these runs the heat transfer rates at the projectile base were defined by the results of reference (3). Comparing the results from figure 7 and 8 with figure 1, the experimental data, it can be seen that considerable differences exist. In figure 7 the launch velocity and peak projectile base pressure are closer to the experimental results than figure 8. However, the launch velocity is only 78% of the measured data and the peak projectile pressure is only 59% of the corresponding experimental results. In contrast the peak breech pressure from figure 7 is 183% of the results in figure 1. It is felt that these differences can be attributed to:

(1) projectile heat transfer

(2) flame front velocity
Based upon earlier heat transfer studies it was determined that projectile heat transfer had a strong influence on projectile velocity and projectile base pressure. For comparison the projectile heat rate for an SCD of 0.1 and 0.2 are presented versus travel in figure 9. The peak projectile heating is 14,500 Btu/ft² sec at approximately 7 inches of travel. This is approximately one half the predicted peak from reference (3). In addition, the peak occurs further downstream than anticipated by reference (3) and experimental experience.

The large variation in peak breech pressure was attributed to the high flame speed. To examine this effect, a finite flame program was written and run. The analytical basis for this program was described earlier. Table 1 presents some results from the finite flame velocity analysis.

<table>
<thead>
<tr>
<th>SCD</th>
<th>Max Pressure behind projectile-psia</th>
<th>Flame Travel Time - sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>414081</td>
<td>0.595x10⁻³</td>
</tr>
<tr>
<td>0.2</td>
<td>14,720</td>
<td>0.756x10⁻³</td>
</tr>
<tr>
<td>0.1*</td>
<td>16,972</td>
<td>0.731x10⁻³</td>
</tr>
</tbody>
</table>

*SCD=0.1 but 1/2 burning rate to simulate ignition delay

Figure 10 presents the volume fraction, ε, distribution versus distance for the same cases on Table 1.

The pressure and velocity history using a finite flame velocity is given in figure 11 for SCD=0.2. The launch velocity is increased by 400 fps and the projectile base pressure is increased slightly. The breech pressure is reduced significantly because the finite flame velocity analysis sets the volume fraction at the breech equal to zero. Figure 12 presents the same results for an SCD=0.1. The launch velocity in figure 12 is 3800 fps. The peak projectile base pressure is 64,500 psia. This represents a substantial change over the results in figure 7. The flat response of the
FIGURE 7

PRESSURE & VELOCITY

TRAVEL

BREACH PRESSURE

PROJECTILE VELOCITY

PROJECTILE BASE PRESSURE

S.C.D. = 0.1

FIGURE 7
Figure 8

Pressure & Velocity vs. Travel

S.G.D. = 0.2

Breech Pressure

 Projectile Velocity

 Projectile Base Pressure

Travel - Inches
Pressure & Velocity vs Travel

S.C.D. = 0.2
Finite Flame Speed

Figure III

[Graph showing pressure, velocity, and travel]
breech pressure curve is again attributed to the zero volume fraction at
the breech.

To eliminate the flat response at the breech when using the finite
flame criteria, it was decided to set the volume fraction at breech
equal to the first station in the flame front analysis. This change
increases the breech pressure significantly. For an SCD of 0.1, the
results are not reasonable. To moderate this effect, the one half burn-
ing rate was used in the flame front analysis (see Table 1). With the
breech volume fraction equal to zero, the peak breech pressure was
approximately 17,000 psia. The peak projectile bore pressure and launch
velocity were 57,500 psia and 3640 fps respectively. Figure 13 presents
the complete pressure and velocity history for the non zero breech volume
fraction. In this figure it can be seen that the peak breech pressure is
130,000 psia. This is approximately twice the experimental results. For
this case the starting breech volume fraction was 0.574. A lower value
would reduce this pressure.

Regarding the choice of heat transfer model, a number of runs were
made using the usual Reynolds analogy. The results from these runs yielded
launch velocities and pressure at the projectile well below test data.
Some parametric runs were made to determine the necessary increase in heat
transfer required to yield agreement with test. These results indicated
a factor of 10 or 20 which does not seem reasonable. In addition the
general shape of the profile predicted by the Reynolds analogy is in-
correct. The Reynolds heating peaks with peak velocity. This means that
peak heating would occur near launch. For this reason, the decision was
made to use the heat transfer model described in reference (3) exclusively.
Conclusions and Recommendations

Computer models have been used to describe the flow field in small arms and also to describe the propagation of flame front through a porous propellant bed. The model determines the pressure, velocity, propellant volume fraction, and temperature versus time and position.

Numerical results have shown the dependence of projectile velocity and barrel pressure history upon deterrent concentration density and flame velocity. It has also been determined that the heat transfer to the base of the projectile also has a significant effect upon the launch condition.

Unfortunately, it was not possible to simulate the experimental data presented in figure 1. However, with some additional effort, the author feels that the results of figure 1 can be duplicated. Toward this goal, the following recommendations are made:

(1) Vary breech volume fraction to simulate breech pressure.
(2) Parametric study of projectile base heat transfer to obtain a steeper pressure profile.
(3) Parametric study of projectile resistive force.
(4) Development of analytical equations for flow at the base of the projectile.
References

(1) Goldstein, S., "Study of the Pressure Distribution Behind the M193 Projectile When Fixed in the M16 Rifle Barrel" Report R-2066, Frankford Arsenal, Jan., 1973


(6) Kitchens Jr., Clarence W., "Flame Spreading in Small Arms Ball Propellant", BRL 1604, August, 1972. (AD750567)


(10) Goldstein, S., "A Simplified Model for Predicting the Burning Rate and Thermochemical Properties of Deterred Rolled Ball Propellant", TN-1184, Frankford Arsenal


APPENDIX A

Description and listing of
Flow Model Computer Program
**COMPUTER PROGRAM**

The computer program implementing the numerical procedures described in preceding sections composed of a main program and 10 subprograms. The main program controls the flow of the calculation. The subprogram performs certain specific functions during the calculation. The name and function of each subprogram is given below:

<table>
<thead>
<tr>
<th>SUBPROGRAM</th>
<th>FUNCTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIME</td>
<td>calculates the surface area, Sp, and volume, Vp, of propellant grain as a function of distance normal to the surface.</td>
</tr>
<tr>
<td>START</td>
<td>generates the pressure, entropy time history along the first Q wave from the starting point to the breech. The required number of points is specified in the input and program divides the distance from projectile base to breech into equal increments.</td>
</tr>
<tr>
<td>INTER</td>
<td>calculates the interior points using value at (3) and (4) (figure 2) as known</td>
</tr>
<tr>
<td>BREACH</td>
<td>calculates the breech points</td>
</tr>
<tr>
<td>BASEP</td>
<td>calculates the projectile base points</td>
</tr>
<tr>
<td>OUTPUT</td>
<td>points the required output information in the specified format</td>
</tr>
<tr>
<td>DIAGO</td>
<td>a diagnostic program which prints certain information if the calculations in INTER, BREACH, and BASEP do not converge within 10 tries</td>
</tr>
<tr>
<td>RFORCE</td>
<td>calculates the engraving force as a function of X based upon tabular input data</td>
</tr>
<tr>
<td>PROP</td>
<td>calculates the specific heat ratio, gas constant and specific heat at constant volume as a function of the amount of deterred and non-deterred propellant burned. The program is called only after the deterred layer is burned through</td>
</tr>
</tbody>
</table>
This is a subroutine which calculates the tube cross section area as a function of $X$ based on tabulated inputs.

The main program maintains control of the overall program and stores the calculated values of various parameters (i.e., gas velocity, propellant velocity, pressure, etc.) This data is stored in a double subscripted $(i,j)$ array. When $i$ equals $j$, the points correspond to the base of the projectile. Along a given $Q$ wave the $j$ subscript is constant. For a fixed value of $j$, the number of $i$ values which determines the grid size is an input parameter. Figure 5 presents an abbreviated flow diagram for the main program.
FIGURE 5: Flow Diagram for Computer Program
DESCRIPTION OF ARRAYS AND VARIABLES

X(I,J) X, location of grid points
TIME(I,J) t, time of grid points
VEL(I,J) gas velocity at grid points
SS(I,J) speed of sound at grid points
PRESS(I,J) pressure at grid points
TEMP(I,J) gas temperature at grid points
dENWS(I,J) gas density at grid points
VELP(I,J) propellant velocity at grid points
ENTR(I,J) entropy of gas at grid points
VOLFRA(I,J) volume fraction at grid points
A(I,J) distance burned normal to propellant
grain surface at grid points

The following are used in the main program and subroutine INTER (see Figure 2)

X3, X4, X6 spatial location of points 3, 4 and 6
T3, T4, T6 time location of points 3, 4 and 6
UG3, UG4, UG6 gas velocity at points 3, 4, and 6
A3, A4, A6 speed of sound at points 3, 4 and 6
PSI3, PSI4, PSI6 gas pressure at points 3, 4 and 6
AREA3, AREA4, AREA6 tube cross sectional area at points 3, 4 and 6
TG3, TG4, TG6 gas temperature at points 3, 4 and 6
RH03, RH04, RH06 gas density at points 3, 4 and 6
UP3, UP4, UP6
S3, S4, S6
EPL3, EPL4, EPL6
L3, L4, L5, L6

propellant velocity at points 3, 4 and 6
entropy at points 3, 4 and 6
propellant volume fraction at points 3, 4 and 6
distance burned normal to propellant grain at points 3, 4, 5 and 6

The following are used in the main program and subroutines BREACH and START (see figure 4)
X12
T11, T12, T13
UG11, UG12, UG13
A11, A12, A13
PSI1, PSI2, PSI3
TG11, TG12, TG13
RH011, RH012, RH013
S11, S12, S13
EPL11, EPL12, EPL13
AREA11, AREA12, AREA13
L11, L13
UP12

spatial location of point 2
time location of point 1, 2 and 3
gas velocity at points 1, 2 and 3
speed of sound at points 1, 2 and 3
gas pressure at points 1, 2 and 3
gas temperature at points 1, 2 and 3
gas density at points 1, 2 and 3
entropy at points 1, 2 and 3
propellant volume fraction at points 1, 2 and 3
tube cross sectional area at points 1, 2 and 3
distance burned normal to propellant grain at points 1 and 3
propellant velocity at point 2

The following are used in the main program and subroutine BASEP (see figure 5)
X21, X22, X24
T21, T22, T24

spatial location of points 1, 2 and 4
time location of points 1, 2 and 4
UG21, UG22, UG24
A21, A22, A24
PS121, PS122, PS124
TG21, TG22, TG24
RH21, RH22, RH24
S21, S22, S24
FPL21, EPL22, EPL24
AREA21, AREA22, AREA24
L21, L24

gas velocity at points 1, 2 and 4
speed of sound at points 1, 2 and 4
gas pressure at points 1, 2 and 4
gas temperature at points 1, 2 and 4
gas density at points 1, 2 and 4
entropy at points 1, 2 and 4
propellant volume fraction at points 1, 2 and 4
tube cross sectional area at points 1, 2 and 4
distance burned normal to the propellant surface at points 1 and 4
the value of the I index for the Q wave
SUBROUTINES:

Name: INTER

Purpose: To calculate the characteristics and thermodynamic properties at any interior point in the mesh

Input: X3, X4, T3, T4, UG3, UG4, A3, A4, PSI3, PSI4, TG3, TG1, RH03, RH04, UP3, UP4, S3, S4, LPL3, LPL4, AREA3, AREA4, L3, L4

Output: X6, T6, UG6, A6, PSI6, TG6, TH06, UP6, S6, EPL6, AREA6

Method: An iterative scheme is used until convergence to within a given tolerance is obtained. Normally, the convergence check is made on the calculated speed of sound and gas velocity. However, for gas velocities below 300 ft/sec only the speed of sound is checked. This procedure was necessary because below 300 ft/sec the accuracy requirements on P and Q for calculating gas velocity become excessive. At 300 ft/sec, a one percent error in the speed of sound can be translated into approximate a 10 percent in the gas velocity. At higher gas velocities, the percentage tolerance on gas velocity and speed of sound are the same.

Name: BRAC

Purpose: To calculate the characteristics and thermodynamic properties at the location X=0

Input: X12, T11, T12, UG11, UG12, A11, A12, PSI11, PSI12, TG11, TG12, RH011, RH012, S11, S12, LPL11, LPL12, AREA11, AREA12, L11, UP12
Output: T13, UG13, A13, PSI13, TG13, RHO13, S13, EPL13, L13
Method: An iterative scheme is used until convergence to within a given tolerance is obtained.
Name: BASi3P
Purpose: To calculate the characteristics and thermodynamic properties at the base of the projectile and to locate the base of the projectile.
Input: X21, X22, T21, T22, UG21, UG22, A21, A22, PSI21, PSI22, TG22, RHO21, RHO22, S21, S22, EPL21, EPL22, AREA21, AREA22, L21, NSTART
Output: X24, T24, UG24, PS124, TG24, WH1024, S24, EPL24, AREA24, L24

Method: An iterative scheme is used until convergence to within a given tolerance is obtained.

Name: DIME (A, B, SURA, SURB, VOLA, VOLB)

Purpose: To calculate the average propellant grain surface area and volume.

Input: A, B corresponding to the distance burned normal to the surface (L) at two points.

Output: SURA, SURB surface of grain at respective points.

VOLA, VOLB volume of grain at respective points.

Method: The following equations are used:

Volume = \( \frac{1}{2} \pi (R - R_i)^2 (R_e - L) + \frac{4}{3} \pi \left( R_e - L \right)^3 \left( \frac{R_e}{R_i} + \frac{2}{3} \left( R_e - L \right) \right) \)

Surface area = \( 2 \pi \left( \frac{R_i}{R_e} \right) \left( \frac{R_i - R_e}{R_e} \right)^2 + \frac{2^3}{3 \pi} \left( R_e - L \right)^3 \)

Name: OUTPUT

Purpose: To print the calculated results.

Input: Calculated velocities and thermodynamic properties at all of the grid points.

Output: Printed results including values of P and Q at each grid point.

Method: Format of printed results specified in subroutine.

Name: DICO (L, MIG, AA)

Purpose: A diagnostic tool to print results calculated prior to failure to converge event.
Input: LA indication of which subroutine failed
LA-1 INTER
LA-2 BASEP
LA-3 BREACH

Output: BUG last calculated gas velocity
         AA last calculated speed of sound

Method: Format used is specified in the subroutine

Name: RFORCE(A,B)

Purpose: Calculates the engraving force or a function of distance
         from initial projectile location

Input: A, location of the base of the projectile

Output: B, resistance force in lbf

Method: Resistance force tabulated or function of distance based upon
         data from Report R-2066, "Study of the Pressure Distribution
         Behind the M193 Projectile When Fired in the M16 Rifle Barrel"

Name: PROP(A)

Purpose: Calculates the CV, GAMMA, and RGAS as a function of the
         percentage deterred and non-deterred propellant burned.
         Subroutine is used only when distance burned normal to the
         surface is greater than the deterred thickness of 0.00084 ft

Input: A distance burned normal to the propellant surface at a given
        mesh point

Output: CV, GAMMA, RGAS

Method: Calculation assumes the gas generated to be ideal and that
         properties are based upon mass fraction of components present
Name: START

Purpose: Calculates the thermodynamic properties along the first Q wave at specified intervals

Input: Same as BREACH but here they refer to data along first Q wave

Output: Same as BREACH but here they refer to data along first Q wave

Method: The subroutine is called a specified number of time (i.e., an input parameter). Each time it calculates the thermodynamic properties along the first Q wave. The results are equally spaced between the projectile initial location and the breech. The gas velocity and heat transfer along this wave are zero
SUBFNC: TAREA(ALOC)

Name: TAREA(ALOC)

Purpose: Calculates the tube cross sectional area as a function of
distance from the breech

Input: ALOC location of the grid point

Output: TAREA tube cross sectional in $\text{ft}^2$

Method: Interpolates data from table which is supplied
THIS PROGRAM CALCULATES THE PRESSURE, TEMPERATURE AND VELOCITY TIME HISTORIES BASED UPON A GIVEN PRCELLENT CHARACTERISTIC

C THIS IS THE MAIN PROGRAM

REAL L5, L6, L3, L4, L21, L24, L11, L13
COMMON/COM1/X3, X4, X5, T3, T4, T6, UG3, UG4, UG6, A3, A4, A6, PSI3, PSI4, PSI6, STG3, TGO, TGO, TGO, RHO3, RHO4, RHC6, UP3, UP4, UP6, S3, S4, S6, EPL3, EPL4, EPL6, ARE
A3, AREA4, AREA6, L3, L4, L5, L6
COMMON/CM2/X12, T11, T12, T13, UG11, UG12, UG13, A11, A12, A13, PSI11, PSI12
COMMON/CM3/X21, X22, X24, T21, T22, T24, UG21, UG22, UG24, A21, A22, A24, PSI21, PSI22, PSI24, TGO, TGO, TGO, RHO21, RHO22, RHO24, S21, S22, S24, EPL21, EPL22, EPL4, AREA21, AREA22, AREA24, L21, L24, NSTART
COMMON/SAME/RGAS, PSI1, GAMMA, DZERC, PIE, BEE, VOLFRA, CV, BASE, GZERO, WT, TOL, AN
COMMON/STAR/KSTART
COMMON/OUT/MAK, MAXI
COMMON/EXIT/XLIM, TMUZE
COMMON/BHEAT/FACT, TWALL, FRICT
COMMON/PARM/IN1, IN2, IN3, PAR1A, PAR1B, PAR2A, PAR2B, PAR3A, PAR3B, PAR4, PAR5
COMMON/PCWDER/CV1, CV2, RGAS1, RGAS2, GAM1, GAM2, DLAY
NAMELIST/IN/PAR, PARIN1, PARIN2, PARIN3, PARIN4, PARIN5, PARIN6, PARIN7, PARIN8
IVER(5, INPOT) READ(5, INPOT) WRITE(6, PROPEL)
NAMELIST/IN/BASE, WT, TCL, XLIM, PSI1, DZERO, XFZERO, XZERO, FRAC1, ALL1

NAMELIST/PRCPEL/PP1, PP2, CV1, CV2, BEE1, BEE2, RGAS1, RGAS2, DLAY, GAM1, GAZ

999 READ(5, INPOT)
READ(5, PAR)
WRITE(6, PROPEL)
WRITE(6, INPOT)
CONTINUE
IF(I GT 0) GC TO 20
C CALCULATION OF FIRST TWO POINTS
X(1, 1) = ZERO
X(2, 1) = 0.0
TIME(1, 1) = 0.0
SS(1, 1) = 5QRT(GAMMA*RGAS*ZER0)
VOLFRA(1, 1) = FRAC1
VEL(1, 1) = 0.0
VELP(1, 1) = 0.0
PRESS(1, 1) = PSI1
TEMP(1, 1) = ZERO
DENS(1, 1) = PSI1/RGAS/2075
ENTR(1, 1) = (RGAS/778 * CV1) * ALOG(TZERO/TATM) - RGAS/778 * ALOG(PSI1/2075)
ENTRY(1,1)=ENTRY(1,1) / *778./GAS/GAMMA
AL(1,1)=ALL1
IF(AL11 GT CLAY) BEE=BEE1
IF(AL11 GT CLAY) PODPOT=FP2
IF(AL11 GT CLAY) CALL PROP(ALL1)
KSTART=J
DO 3 I=1,KSTART
K=I+1
X12=X(I,1)
T12=TIME(I,1)
EPL12=VOLFRA(I,1)
A12=SS(I,1)
UG12=VEL(I,1)
UP12=VELP(I,1)
PS112=PRESS(I,1)
TG12=TEMP(I,1)
RHO12=DENS(I,1)
S12=ENTR(I,1)
L11=AL(I,1)
CALL START
PRPSI=PSI13/144.
WRITE(6,5002)A13,L13,PRPSI,TG13,RHO13,S13,EPL13,T13
X(K1)=X12
TIME(K1)=T13
VOLFRA(K1)=EPL13
SS(K1)=A13
VEL(K1)=UG13
VELP(K1)=UG13
PRESS(K1)=PS113
TEMP(K1)=TG13
DENS(K1)=RHO13
ENTR(K1)=S13
AL(K1)=L13
3 CONTINUE
NCHECK=0
JLAST=1
5 DO 50 I=2,80
IF(NCHECK GT 0) LAST=JLAST+1
NCHECK=0
JQ=JLAST+1
DO 49 JJ=JQ,1
J=JJ
IF(JJ.EQ.1) GO TO 55
X4=X(I-1,J)
X3=X(I,J-1)
T4=TIME(I-1,J)
T3=TIME(I,J-1)
UG4=VEL(I-1,J)
UG3=VEL(I,J-1)
A4=SS(I-1,J)
A3=SS(I,J-1)
PS14=PRESS(I-1,J)
PS13=PRESS(I,J-1)
TG4=TEMP(I-1,J)
TG3=TEMP(I,J-1)
RHC4=DENS(I-1,J)
RHC3=DENS(I,J-1)
UP4=VELP(I-1,J)
UP3=VELP(I,J-1)
S4=ENTR(I-1,J)
S3=ENTR(I,J-1)
EPL4=VCLFRA(I-1,J)
EPL3 = VOLFRA(I, J-1)
L4 = AL(I-1, J)
L3 = AL(I, J-1)
TOOL = (L3+L4)/2.
IF (TCOL.GT. CLAY) GO TO 10
GO TO 14
10 BEE = BEE2
PODPOT = PP2
CALL PROP(TCOL)
14 AREA3 = TAREA(X3)
AREA4 = TAREA(X4)
WRITE(6, 5003) X3, X4, T3, T4, UG3, UG4, A3, A4, PSI3, PSI4, TG3, TG4, RHO3, RHO4
$UP3, UP4, EPL3, EPL4, L3, L4, AREA3, AREA4
CALL INTER
RGAS = RGAS1
BEE = BEE1
GAMMA = GAM1
CV = CV1
PODPCT = PP1
PRPSI = PSI6/144.
WRITE(6, 5002) A6, L6, PRPSI, TG6, RHO6, S6, EPL6, T6, X6, UG6, UP6
5002 FORMAT (1X, 11E11.3)
X(I, J) = X6
TIME(I, J) = T6
VEL(I, J) = UG6
PRESS(I, J) = PSI6
TEMP(I, J) = TG6
DENS(I, J) = RHO6
VELP(I, J) = UP6
ENTR(I, J) = S6
VOLFRA(I, J) = EPL6
SS(I, J) = A6
AL(I, J) = L6
IF (X3 .LE. 0.0) GO TO 40
IF (T6 .GE. TMUZE) GO TO 48
GO TO 49
55 L = JJ-1
NSTART = I-1
K = I
X21 = X(NSTART, L)
X22 = X(K, L)
T21 = TIME(NSTART, L)
T22 = TIME(K, L)
UG21 = VEL(NSTART, L)
UG22 = VEL(K, L)
A21 = SS(NSTART, L)
A22 = SS(K, L)
PSI21 = PRESS(NSTART, L)
PSI22 = PRESS(K, L)
TG21 = TEMP(NSTART, L)
TG22 = TEMP(K, L)
RHC21 = DENS(NSTART, L)
RHC22 = DENS(K, L)
S21 = ENTR(NSTART, L)
S22 = ENTR(K, L)
EPL21 = VOLFRA(NSTART, L)
EPL22 = VOLFRA(K, L)
L21 = AL(NSTART, L)
L22 = AL(K, L)
IF (L21 .GT. DLAY) BEE = BEE2
IF (L21 .GT. DLAY) PODPCT = PP2
IF (L21 .GT. DLAY) CALL PROP(L21)
AREA21 = TAREA(X21)
AREA22 = TAREA(X22)
CALL BASEP
NSTART=1
POPDOT=PP1
CV=CV1
GAMMA=GAM1
BEE=BE1
RGAS=RGAS1
PRPSI=PSI24/144.
WRITE(6,5001)A24,L24,PRPSI,TG24,RH024,S24,EPL24,T24,X24,UG24
5001 FORMAT(1X,10E11.3)
X(NSTART,J)=X24
TIME(NSTART,J)=T24
VEL(NSTART,J)=UG24
VELP(NSTART,J)=UG24
SS(NSTART,J)=A24
PRESS(NSTART,J)=PSI24
TEMP(NSTART,J)=TG24
DENS(NSTART,J)=RH024
ENTR(NSTART,J)=S24
VOLFRA(NSTART,J)=EPL24
AL(NSTART,J)=L24
GO TO 49
40 NCHECK=1
II=1
XI2=X(II,J)
T11=13
T12=TIME(II,J)
UG11=0.0
UG12=VEL(II,J)
X1=0.0
A11=A13
A12=SS(II,J)
PSI11=PSI13
PSI12=PRESS(II,J)
TG11=TG13
TG12=TEMP(II,J)
RHC11=RHC13
RH012=DENS(II,J)
S11=S13
S12=ENTR(II,J)
EPL11=EPL13
EPL12=VCLFRA(II,J)
UP12=VELP(II,J)
L11=L13
IF(L11,G,Delay) BEE=BE2
IF(L11,G,Delay) POPDOT=PP2
IF(L11,G,Delay) CALL PROP(L11)
AREA11=TAREA(X11)
AREA12=TAREA(X12)
CALL BREACH
RGAS=RGAS1
BEE=BE1
GAMMA=GAM1
CV=CV1
POCPCT=PP1
PRPSI=PSI13/144.
WRITE(6,5002)A13,L13,PRPSI,TG13,RH013,S13,EPL13,T13,X13
L=I+1
K=J
AL(L,K)=L13
TIME(L,K)=T13
SS(L,K)=A13
PRESS(L,K)=PSI13
TEMP(L,K)=T13
DENS(L,K)=RHO13
VEL(L,K)=0,0
VELP(L,K)=0,0
ENTR(L,K)=S13
X(L,K)=0,0
VOLFRA(L,K)=EPL13
MAXJ=JJ
IF(T13.GE.TMUZE) GO TO 48
49 CONTINUE
48 MAXI=I
IF(T13.GE.TMUZE) GO TO 56
50 CONTINUE
56 CALL OUTPUT
MAXI=MAXI+1
DO 51 I=MAXJ,MAXI
WRITE(7,5003)X(I,MAXJ),TIME(I,MAXJ),VOLFRA(I,MAXJ),SS(I,MAXJ),VEL(I,MAXJ),VELP(I,MAXJ),PRESS(I,MAXJ),DENS(I,MAXJ),ENTR(I,MAXJ),AL(I,MAXJ),TEMP(I,MAXJ)
5003 FORMAT(3E20.5)
51 CONTINUE
GO TO 999
20 NSTART=1
KSTART=J
NLAST=1
NCHECK=0
KK=J+1
DO 52 I=1,KK
READ(5,5003)X(I,1),TIME(I,1),VOLFRA(I,1),SS(I,1),VEL(I,1),VELP(I,1)
PRESS(I,1),DENS(I,1),ENTR(I,1),AL(I,1),TEMP(I,1)
WRITE(6,5003)X(I,1),TIME(I,1),VOLFRA(I,1),SS(I,1),VEL(I,1),VELP(I,1)
PRESS(I,1),DENS(I,1),ENTR(I,1),AL(I,1),TEMP(I,1)
52 CONTINUE
T13=TIME(KK,1)
A13=SS(KK,1)
PSI13=PRESS(KK,1)
T13=TEMP(KK,1)
RHO13=DENS(KK,1)
S13=ENTR(KK,1)
EPL13=VOLFRA(KK,1)
L13=AL(KK,1)
GO TO 5
END
SUBROUTINE BASEP
C SUBROUTINE BASEP
REAL L5,L6,L7,L8,L9,L10,L11,L12
C THIS SUBROUTINE CALCULATES POINTS AT THE BASE OF THE PROJECTILE
COMMON/XLIM,TMUZE,FRIC
COMMON/SAME/RHCP,PSI1,GAMMA,DZERC,PIE,BEE,POTV,CV,BASE,ALPHA,WT,
>TO,
COMMON/EXIT/FRIC
COMMON/CH/FACT,TWALL,FRIC
COMMON/PARM1,PARN1,PARN2,PARN3,PARN81,PARN82,PARN83,PARNBR1,PARNBR
COMMON/RHCP,REC,DELLT,3,1085,0.9,0.000005/
A1=SQRT(GAMMA*RHCP*530.)
XTRAV=(X21-0.125)*12.
WRITE(6,5000)POTV,CV,BEE,L21,XTRAV
TRY=1
NSTART=NSTART+1
5 PSI124=PSI121
EPL24=EPL21
A24=A21
P22=2.*A22/(GAMMA-1.)*L22
RHC24=RHC21
UG24=UG21
TG24=TG21
ABASE=BASE*(PS124+PS121)
10 SLOPEP=(UG22*A22+UG24*A24)/2.
CALL RFORCE(X21,R1)
C=X21-X22+SLOPEP*T22
WORK=(ABASE/2.-R1)*GZERO*T21/WT
C=C-WORK*T21/2.-T21*(UG21-WORK)
B=UG21-WORK-SLOPEP
A=(ABASE/2.-R1)*GZERO/WT/2.
5000 FORMAT(1X,********,//,1X,5E20.5)
T24=(-B-SQRT(B*B-4.*A*C))/2./A
X24=X22+SLOPEP*(T24-T22)
IF(X24.GE.XLIM) GO TO 75
11 AREA24=AREA(X24)
AREA=(AREA24+AREA21)/2.
UGMN=(UG21+UG24)/2.
PSIMN=(PS124+PS121)/2.
BR=BEER*((PSIMN/144.)/**AN)/12.
DPDX=0.0
DL=BR*(T24-T21)
L24=L21+DL
IF(L24.GT.0.000625)L24=0.000625
CALL DIME(L21,L24,SURF21,SURF24,VCL21,VOL24)
IF(VOL21.LE.0.000)SURF21=0.00
IF(VOL24.LE.0.000)SURF24=0.00
DBAR=DZERO-L24-L21
IF(DBAR.LE.0.000625)DBAR=0.00
DBAR=DBAR**3
FOR24=-PIE*DBAR*DPDX/6.
EPLMN=(EPL21+EPL24)/2.
VOLMN=(VOL21+VOL24)/2.
SURFMN=(SURF21+SURF24)/2.
IF(VOLMN.LE.0.000) GO TO 12
ETAMN=EPLMN/VOLMN
GO TO 13
12 ETAMN=0.00
13 CONTINUE
AMGMN=ETAMN*SURFMN*BR*RHCP*AREA
DPDX=(UG24-UG21)/(X24-X21)
DEPLCT=EPL21*(1.0-EPL21)*RHO24-(RHO24-RHCP)/(RHCP-T21)-(EPL21+(1.0-EPL21)*RHO2
#1/RHCP)*AMGMN*(T24-T21)/RHC21/AREA
DEPLCT=DEPLCT/(T24-T21)
14 CONTINUE
FD24=EPLMN*FOR24/VOLMN
RHMN=(RHO24+RHO21)/2.
TIMN=(TG24+TG21)/2.
DIA=SQR1T(4.*AREA/PIE)
AMN=(A22+A24)/2.
TAU=TIMN*(1.0+C1)*GAMMA-1.)*UGMN/AMN/AMN
QHEAT=0.000298*(C1*.333)+RHMN*UGMN
QHEAT=QHEAT1*(GAMMA-TWALL)
EGAS=QHEAT*PIE*DIA+AMM*PODPOT*GZERO
TAU0=0.0809*RHMN*UGMN*UGMN
F1= UGMN*AMMN-TAU0*PIE*PIE+FD24*AREA
F1=F1/778.
RS1=AMMN*(CV*TIMN*UGMN/UGMN/1556.0)*AREA*DEPLCT
AMN=RHMN**PSIMN/778.
DSDT=AMMN*(PODPOT*GZERO-(CV+RAS/778.0)*TIMN)+(TAU0*UGMN/778.0)
SUBROUTINE RFORCE(A,B)
DIMENSION XX(20),RF(20)

DATA XX/0.,0.3333,0.4167,0.4991,0.6660,0.1000,0.3333,0.
>1.0000,0.1333,0.6660,0.1250,12.
DATA RF/0.,.5200,5250,5200,4200,2550,2150,1950,1830,17

END

FORMAT(1X,5E20.5) T24=T22+(X24-X22)/SLOPEP
TMUE=T24
GO TO 11

END

SUBROUTINE RFORCE(A,B)
DIMENSION XX(20),RF(20)

DATA XX/0.,0.3333,0.4167,0.4991,0.6660,0.1000,0.3333,0.
>1.0000,0.1333,0.6660,0.1250,12.
DATA RF/0.,.5200,5250,5200,4200,2550,2150,1950,1830,17

END

FORMAT(1X,5E20.5) T24=T22+(X24-X22)/SLOPEP
TMUE=T24
GO TO 11

END

SUBROUTINE RFORCE(A,B)
DIMENSION XX(20),RF(20)

DATA XX/0.,0.3333,0.4167,0.4991,0.6660,0.1000,0.3333,0.
>1.0000,0.1333,0.6660,0.1250,12.
DATA RF/0.,.5200,5250,5200,4200,2550,2150,1950,1830,17

END

FORMAT(1X,5E20.5) T24=T22+(X24-X22)/SLOPEP
TMUE=T24
GO TO 11

END
SUBROUTINE BREACH

C THIS SUBROUTINE CALCULATES THE POINTS LOCATED ON THE BREECH

REAL L5,L6,L3,L4,L21,L24,L11,L13
COMMON/COM2/X12,T11,T12,T13,UG11,UG12,UG13,A11,A12,A13,PSI11,PSI12
*,PSI13,TG11,TG12,TG13,RHC11,RH012,RH013,S11,S12,S13,EPL11,EPL12,EP
*13,AREA11,AREA12,AREA13,L11,L13,LP12
COMMON/SAME/RGAS,PSI1,GAMMA,DZERC,PIE,BEE,PODPOT,CV,BASE,GZERO,WT
>TO,AN
COMMON/EXIT/XLIM,TMUZE
COMMON/PARM/PAR11,PAR12,PAR13,PARBA1,PARBA2,PARBA3,PARBR1,PARBR
$2,PARBR3
A1=SQRT(GAMMA*RGAS*530.)
ITRY=1
CONST=-0.58778E+03
TWALL=530.
UG13=0.0
A1=A11
RHOP=3.1085
AREA=AREA11
Q12=A12/(GAMMA-1.)
PSI13=PSI11
EPL13=EPL11

30 SLOPEQ=(A13+UG12-A12)/2.
T13=X12/SLOPEQ+T12
IF(T13.GE.TMUZE) T13=TMUZE
PSIMN=(PSI13+PSI11)/2.
BR13=BEE*((PSIMN/144.)*AN)/12
DL13=BR13*(T13-T11)
L13=L11+DL13
IF(L13.GT.0.000625)L13=0.000625
CALL DIME(L11,L13,SURF11,SURF13,VCL11,VOL13)
IF(VOL11.LE.0.00)SURF11=C.00
IF(VOL13.LE.0.00)SURF13=C.00
EPLMN=(EPL13+EPL11)/2.
VOLMN=VCL13+VOL11/2.
IF(VOLMN.LE.0.00) GO TO 31
ETAMN=EPLMN/VOLMN
GO TO 32

31 ETAMN=0.00
32 CONTINUE
AMGMN=ETAMN*BR13*RHOP*AREA
AMGMN=(AMGMN*(SURF13+SURF11)/2.*
DEPLOT=-AMGMN/AREA/RHOP-EPLMN*UP12/X12

300 FORMAT(1X,5E20.5)
TG13=A13*A13/GAMMA/RGAS
TG1N=(TG13+TG11)/2.
RHC13=PSI13/RGAS/TG13
RHC1N=(RH013+RH011)/2.
DIA=SQRT(4.*AREA/PIE)
FILM=3.24*EXP(CONST*T13)*DIA/2./X12
QHEAT=FILM*(TG13-TWALL)
QHEAT=QHEAT*PARBR1
EGAS=QHEAT*PIE*DIA*PODPOT*AMGMN*GZERO
RS1=AMGMN*(CV*TG1N+PSIPN/778.)/RHOPN+PSIMN*AREA*DEPLOT/778.
WORK=AREA*(1.-EPLMN)*R*OMN*TG1N
OSD1=AMGMN*(PODPOT*GZERO-(CV/RGAS/778.)*TG1N) -QHEAT*PIE*DIA -
150 CONTINUE

DELQDT = ((A11 + A13) / 2.0) * (778.0 * DSDT / RGAS + DEPLDT / (1.0 - EPLMN))
DELQDT = DELQDT + ((A11 / 2.0 + A13 / 2.0) * 3.0) * AMGN/AREA/PSIMN/(1.0 - EPLMN) / GAMMA

Q13 = Q12 + DELQDT * (T13 - T12)
P13 = P12 + DELQDT * (T13 - T12)

R13 = R12 + DELQDT * (T13 - T12)

IF (T13 .GT. PARBR2) GO TO 200

DELQDTu4(ALIeA13)/2.0)*(778.*OSDTiAtGAS4DEPLDT/(l.-EPLMN))

OELQOT-OELQCT* ( (All/2..A13/2.)**3)*AN4GMN/AREA/PSIMN/(1.-EPL4N)/GAMMA

Q13a = Q12 + DELQDT*(T13-T12)
P13 = P12 + DELQDT*(T13-T12)


R13 = R12 + DELQDT*(T13-T12)

EPL13 = EPLL1 + DELQDT*(T13-T11)

IF (EPL13 .LE. 0.00) EPL13 = 0.0

CHECK = ABS((ANEW13 / A13) / A13)

IF (T13 .GE. T1'UZE) RETURN

IF (CHECK .GT. TOL) GO TO 180

RETURN

180 ITRY = ITRY + 1

IF (ITRY .LT. 11) GO TO 30

LOC = 3

CALL DIAGO(LUCG, UG13, A13)

RETURN

200 WRITE(6,300)111, 113, VJL11, VG113, SURF11, SURF13, EPLMN, ETAMN, AMGN/DE

#PLDT, QHEAT, EGAS, DSDT, AREA

GO TO 150

END

C THIS IS A DIAGANOSTIC SUBROUTINE

REAL L5, L6, L3, L4, L21, L24, L11, L13

COMMON/X(99,99), TIME(99,99), VEL(99,99), SS(99,99), PRESS(99,99), TE


COMMON/COM1/X3, X4, X6, T3, T4, T6, UG3, UG4, UG6, A3, A4, A6, PSI3, PSI4, PSI6, $T3, T4, T6, RH03, RH04, RHC6, UP3, UP4, UP6, S3, S4, S6, EPL3, EPL4, EPL6, ARE

> A3, AREA4, AREA6, L3, L4, L5, L6

COMMON/CCM2/X12, T11, T12, T13, UG11, UG12, UG13, A11, A12, A13, PSI11, PSI12, $, PSI13, TGI1, TGI2, TGI3, RHC11, RHC12, RHC13, S11, S12, S13, EPL11, EPL12, EPL

$13, AREA11, AREA12, AREA13, L11, L13, UP12

COMMON/CCM3/X21, X22, X24, T21, T22, T24, UG21, UG22, UG24, A21, A22, A24, PSI

$21, PSI22, PSI24, T21, T22, T24, RH21, RH22, RH24, S21, S22, S24, EPL21, $EPL22, EPL24, AREA21, AREA22, AREA24, L21, L24, NSTART

COMMON/SAME/RGAS, PSI11, GAMMA, ZEREC, PIE, BEE, PDPOT, CV, BASE, GZERO, WT, $>TOL, AN

COMMON/OUT/MAXJ, MAXI

GO TO (10,11,12), LA

10 WRITE(6,100)

100 FORMAT(1H1, 'EXCEEDED ITERATION LIMIT IN SUBROUTINE INTER')

GO TC 15

11 WRITE(6,101)

101 FORMAT(1H1, 'EXCEEDED I:TERATION LIMIT IN SUBROUTINE BASE')

GO TC 15

12 WRITE(6,102)

102 FORMAT(1H1, 'EXCEEDED ITERATION LIMIT IN SUBROUTINE BREACH')

15 WRITE(6,110) BUG, AA

110 FORMAT(1H10, 'LAST CALCULATED GAS VELOCITY', E12.3, '//2x', 'LAST CALCULATED SPEED OF SOUND', E12.3)

C SUBROUTINE OUTPLT
C THIS SUBROUTINE PRINTS RESULTS
SUBROUTINE OUTPLT
MP(99,99),DENS(99,99),VELP(99,99),ENTR(99,99),VOLFRA(99,99),AL(99,
?99)
COMMON/SAME/RGAS,PSI1,GAMMA,DZERC,PIE,BEE,PODPOT,CV,BASE,GZERO,WT,
>TOL,AN
COMMON/OUT/MAXJ,MAXI
WRITE(6,100)
100 FORMAT('**** SOLUTION TO GAS FLOW IN TUBE *******')
WRITE(6,104)
104 FORMAT('FLOW PARAMETERS',/3X,'I',3X,'J',3X,'POSITION',8X,'TIM
>E',9X,'VELOCITY',3X,'SPEED OF SOUND',3X,'PRESS',3X,'TEMP',3X,'DENSI
TY',3X,'FRACTION')
*2.3,2X,E12.3,2X,F6.3)
C SUBROUTINE START

REAL L5, L6, L3, L4, L21, L24, L11, L13
COMMON/COM2/X12, T11, T12, T13, UG11, UG12, UG13, A11, A12, A13, PSI11, PSI12, 
# PSI13, TG11, TG12, TG13, RHO11, RHO12, RHO13, S11, S12, S13, EPL11, EPL12, EP 
S13, AREA11, AREA12, AREA13, L11, L13, UP12
COMMON/SAME/STAR, PSI1, GAPMA, DZERCV, PIE, BEE, PODLOT, CV, BASE, GZERCV, WT, 
# TOL, AN
COMMON/STAR/KSTART
AK = KSTART
ITRY = 1
UG13 = 0.0
A13 = A12
DELX = -0.125/AK
PSI13 = PSI12
RHOP = 3.1085
X13 = X12 + DELX
AREA12 = TAREA(X12)
AREA13 = TAREA(X13)
AREA = (AREA13 + AREA12)/2.
Q12 = 2.0*A12/(GAMMA - 1.)
EPL13 = EPL12

30 SLOPEQ = -(A13 + A12)/2.
PSIMN = (PSI12 + PSI13)/2.
T13 = T12 + (X13 - X12)/SLOPEQ
BR13 = BEE**((PSIMN/144.)**AN)/12.
DL13 = BR13*T13
L13 = L11 + DL13
CALL DIME(L11, L13, SURF12, SURF13, VCL12, VOL13)
EPLMN = (EPL13 + EPL12)/2.
VOLMN = (VOL13 + VOL12)/2.
ETAMN = EPLMN/VOLMN
AMGMN = ETAMN*BR13*RHOP*AREA*(SURF13 + SURF12)/2.
DEPLDT = AMGMN/AREA/RHCF
TG13 = A13*A13/GAMMA/RGAS
TG13 = (TG13 + TG12)/2.
RHO13 = PSI13/RGAS/TG13
RHO13 = (RHO13 + RHO12)/2.
EGAS = PODLOT*AMGMN/GZERCV
RS1 = AMGMN*(CVCV*TMMN + PSI1N/778.)*RHCN + PSIMN*AREA*DEPLDT/778.
WORK = AREA*(1.-EPLMN)*RHO1N*TGMN
DSDT = (EGAS - RS1)/WORK
DELDQT = ((A12 + A13)/2.)*(778.*DSDT/RGAS*DEPLDT/(1.-EPLMN))
DELDQT = DELDQT + (((A12 + A13)/2.)**3)*AMGMN/AREA/PSIMN/(1.-EPLMN)/GAMM 
#A
Q13 = Q12 - DELCOT*(T13 - T12)
P13 = Q13
ANEW13 = (GAMMA - 1.)*Q13/2.
TG13 = ANEW13*ANEW13/GAMMA/RGAS
S13 = S12 + CSOT*778.*((T13 - T12)/RGAS/GAMMA
PSI13 = PSI12 + ((ANEW13/A12)**(Z.GAMMA/(GAMMA - 1.))))
PSI13 = PSI13*EXP(-GAMMA*(S13 - S12))
RHO13 = PSI13/RGAS/TG13
EPL13 = EPL12 + DEPLDT*(T13 - T12)
CHECK = ABS((ANEW13 - A13)/A13)
A13 = ANEW13
IF(CHECK > GT.TOL) GO TO 180
X12 = X13
RETURN

180 ITRY = ITRY + 1
IF(ITRY > 11) GO TO 30
LOC = 3
CALL DIAGO(LCC, UG13, A13)
FUNCTION TAREA(ALOC)
C THIS IS A SUB PROGRAM TO CALCULATE THE BORE CROSS SECTIONAL AREA
DIMENSION BCRE(11),POS(11)
DATA BORE/0.024900,0.02491,0.02771,0.02746,0.02514,0.01864,0.018184,0.01824,0.01824,0.01824/
DATA POS/-10.0,0.0,0.02797,0.1032,0.1078,0.1135,0.1300,1.542,3.0,5.0,15.0/
DO 10 I=1,11
IF(POS(I)*LE.ALOC) GO TO 5
IF(POS(I)*GT.ALOC) GO TO 7
GO TO 10
5 TAREA=3.141593*BORE(I)*BCRE(I)*0.25
GO TO 15
7 AM=(BORE(I-1)-BORE(I))/(POS(I-1)-POS(I))
B=BORE(I)
DD=AM*(ALCC-POS(I))
TAREA=3.141593*DD*DD/4.
GO TO 15
10 CONTINUE
15 RETURN
END
SUBROUTINE CIME(ABqSUPASURBVOLAVOLB)
C THIS IS A SUBROUTINE TO CALCULATE PROPELLANT SURFACE AREA AND VOLUME
DATA PI/R3.141593,0.0009590.0006249/
WORK=R-AB
WORK1=RBAR-AB
VOLA2.*PIE*WORK1*WORK1*BORE*BORE*(WCRK1+2.*WCRK/PIE)
SURA2.*PIE*(PIE*WORK*WCRK1+2.*WCRK/PIE)*WORK1*WORK1)
WORK=R-B
VOLB2.*PIE*WORK1*WORK1*BORE*BORE*(WCRK1+2.*WCRK/PIE)
SURB2.*PIE*(PIE*WORK*WCRK1+2.*WCRK/PIE)*WORK1*WORK1)
RETURN
END
SUBROUTINE INTER
C THIS SUBROUTINE CALCULATES INTERIOR NETWORK POINTS
REAL L5,L6,L3,L4,L21,L24,L11,L13
COMMON/CCM/X3,X4,X6,T3,T4,T6,UG3,UG4,UG6,A3,A4,A6,PSI3,PSI4,PSI6,$T1,TC4,TC6,TH3,TH4,THC6,UP3,UP4,UP6,S3,S4,S6,EPL3,EPL4,EPL6,ARE
A3,A44,A64,L4,L5,L6
COMMON/SAME/RGB,PSI5,GAIP,AZIM,PIE,BEE,POCPOT,CV,BASE,GZERO,HT,
TOL,AN
COMMON/EXIT/XLIP,TMUZE
COMMON/BHEAT/FACT,TWALL,FRICT
COMMON/PAIR/PAIR,PARIN2,PARIN3,PARBA1,PARBA2,PARBA3,PARBA4,PARB
$2,PARBR3
DATA AMU*RC,RHOP/1.0445E-06,0.9,3.1085/
FUG=1.0
*1=SQR(GAMMA*RGAS*530.)
1TRY=1
UG6=(UG3+UG4)/2.
A6=(A3+A4)/2.
TG6=(TG3+TG4)/2.
RHO6=(TH3+TH4)/2.
PSI6=(PSI3+PSI4)/2.
CALCULATION OF THE LOCATION OF THE NEW POINT ON X-T DIAGRAM

30 SLOPEP = (UG3 + A3 + UG6 + A6) / 2.
SLOPEQ = (UG4 - A4 + UG6 - A6) / 2.
T6 = (X4 - X3 + T3 * SLOPEP - T4 * SLOPEQ) / (SLOPEP - SLOPEQ)
IF (T6 HE.TMLE) GO TO 25C
X6 = X3 + SLOPEP * (T6 - T3)

31 AREA6 = AREA3 + AREA4 / 2.
IF (T4 EQ. T3) GO TO 32
T7 = X6 - X3 + T3 * WORK / (T4 - T3) - T6 * UG6
T7 = T7 / (WORK / (T4 - T3) - UG6)
GO TO 33

32 T7 = T3
33 CONTINUE
X7 = X6 - UG6 * (T6 - T7)
IF (T4 EQ. T3) GO TO 34
T5 = X6 - X3 + T3 * WORK / (T4 - T3) - T6 * UG6
T5 = T5 / (WORK / (T4 - T3) - UG6)
GO TO 35

34 T5 = T3
35 CONTINUE
X5 = X6 - UP6 * (T6 - T5)
IF (X5 LT 0) X5 = X3
WORK1 = (X7 - X3) / WORK
UG7 = UG3 + (UG4 - UG3) * WORK1
A7 = A3 + (A4 - A3) * WORK1
S7 = S3 + (S4 - S3) * WORK1
TG7 = A7 / (GAMMA * RGAS)
WORK2 = 2 * GAMMA / (GAMMA - 1)
PS17 = PS13 + (A7/A3) * WORK2
PS17 = PS17 * EXP (-GAMMA * (S7 - S3))
RH7 = PS17 / RGAS / TG7
WORK1 = (X5 - X3) / WORK
UG5 = UG3 + (UG4 - UG3) * WORK1
UP5 = UP3 + (UP4 - UP3) * WORK1
A5 = A3 + (A4 - A3) * WORK1
S5 = S3 + (S4 - S3) * WORK1
TG5 = A5 / (GAMMA * RGAS)
PS15 = PS13 + (A5/A3) * WORK2
PS15 = PS15 * EXP (-GAMMA * (S5 - S3))
RH5 = PS15 / RGAS / TG5
EPL5 = EPL3 + (EPL4 - EPL3) * WORK1
IF (EPL5 LE. G.00) EPL5 = 0.
L5 = L3 + (L4 - L3) * WORK1
UMN = (UG6 + UG7) / 2.
UGMN = UMN
AMN = (A6 + A7) / 2.
TGMN = (TG6 + TG7) / 2.
RHMN = (RH6 + RH7) / 2.
PSMN = (PS16 + PS17) / 2.
UP56 = (UP5 + UP6) / 2.
RELV5 = ABS(UG5 - UP5)
RELV6 = ABS(UG6 - UP6)
RELV56 = (RELV5 + RELV6) / 2.
PEE56 = (PS15 + PS16) / 2.
BR56 = (BR56 + BR56) / 2.
BR56 = (PEE56 * 144) * 1/12.
IF (RELV56 G 500) BR56 = BR56 * (1 + G.0004 * (RELV56 - 500))
DL56=BR56*(T6-T5)
L6=L5+DL56
IF(L6.GT.0.0L0625)L5=0.0L0625
IF(L6.LE.0.0L0625)L6=0.0L0625
CALL DIME(L5,L6,SURF5,SURF6,VOL5,VOL6)
IF(VOL5.LE.0.000)VOL5=0.00
IF(VOL6.LE.0.000)VOL6=0.00
EPL56=(EPL6+EPL5)/2.
DBAR=DZERO-L5-L6
IF(DBAR.LT.0.000652)DBAR=0.00
REY56=RELV56*DBAR/AMU
REY56=REY56*(RH05+RH06)/2.
IF(REY56.LE.0.000)GO TO 98
C DRAG=28./(REY56**((0.85)))+0.48
GO TO 99
98 C DRAG=0.0
GO TO 99
C CALCULATION OF VELOCITY AND PRESSURE GRADIENTS
99 IF(T3.GT.T4) GO TO 100
   T8=T4
   X8=(UG3+A3)*((T8-T3)+X3)
   PS18=PSI3+((PS16-PSI3)*((X8-X3)/(X6-X3))
   UP8=UP3+(UP6-UP3)*((X8-X3)/(X6-X3))
   DPDX=0.00
   DUPDX=(UP4-UP8)/((X4-X8))
   GO TO 125
100 T8=T3
   X8=(UG4-A4)*((T8-T4)+X4)
   PS18=PSI4+((PS16-PSI4)*((X8-X4)/(X4-X6))
   UP8=UP4+(UP6-UP4)*((X8-X4)/(X4-X6))
   DPDX=0.00
   DUPDX=(UP8-UP3)/((X8-X3))
125 CONTINUE
C DRAG FORCE ON PROPELLANT PARTICLE
FOR56=CDRA5+*(RHO5+RHO6)*RELV56*RELV56
FOR56=FOR56*DBAR*DBAR/16.
FOR56=FOR56*DPDX*(DBAR**3)/6.
FOR56=FOR56*PIE
IF(VOL5.LE.0.000 AND VOL6.LE.0.000)GO TO 1125
ETA56=2.*EPL56/(VOL5+VCL6)
GO TO 1126
1125 ETA56=0.00
1126 CONTINUE
FD56=ETA56*FOR56
C MASS OF GAS GENERATED GOING FROM 5 TC 6
AM56=ETA56*BR56*RHOP*AREA
AM56=AM56*AMG56*(SURF5+SURF6)/2.
AM56=AMG56*FUG
C CHANGE IN PARTICLE VELOCITY AND MASS FRACTION
IF(EPL56.LE.0.000) GO TO 126
DUPDT =UP56*AMG56/RHOP/AREA/EPL56
DUPDT=DUPDT*FD56/RHOP/EPL56
GO TO 127
126 DUPDT=0.00
127 CONTINUE
DEPLDT=-EPL56*DUPDT/UP56
DEPLDT=DEPLDT-AMG56/RHCP/AREA
DIA=SQT((4.*AREA)/PIE)
FILM=FRICT*GAMMA*RGAS*RHCMN*UGMN/2./(GAMMA-1.)/778.
TA=UGMN*(1.+REC*(GAMMA-1.))*(UGMN/UGMN/2./AMN/AMN1)
CHEAT=FACT*(TA-TWALL)
QHEAT=QHEAT*FACT
EGAS=QHEAT*PIE*DIA-FD56*AREA*RELV56/778.
C CALCULATE THE CHANGE IN P AND C

UG36 = (UG3 + UG6) / 2.
RHO36 = (RHO3 + RHO6) / 2.
PSI36 = (PSI3 + PSI6) / 2.
AREA36 = (AREA3 + AREA6) / 2.
A36 = (A3 + A6) / 2.
EPL36 = (EPL3 + EPL6) / 2.

TRY = A36 * A36 * 778. * (S6 - S3) / (X6 - X3) / GAMMA / RGAS
DELPDT = A36 * UG36 * (AREA3 - AREA6) / (X3 - X6) / AREA
DELPCT = DELPDT + A36 * 778. * DSDT / RGAS
WORK = TAUW * PIE * DIA / AREA / (1. - EPL36) / RHO36
WORK = WCRC + FC56 / RHO36 / (1. - EPL36)
DELPCT = DELPCT - WORK
DELPDT = DELPDT + A36 * DEPLCT / (1. - EPL36) + ((A36**3) / PSI36 / GAMMA - UG36 / RH0

>36) * AMG56 / AREA / (1. - EPL36)

DELPCT = DELPCT - TRY
PSI46 = (PSI4 + PSI6) / 2.
EPL46 = (EPL4 + EPL6) / 2.
A46 = (A4 + A6) / 2.
UG46 = (UG4 + UG6) / 2.
RHO46 = (RHO4 + RHO6) / 2.
AREA46 = (AREA4 + AREA6) / 2.
TRY = A46 * A46 * 778. * (S6 - S4) / (X6 - X4) / GAMMA / RGAS
DELCQT = A46 * UG46 * (AREA4 - AREA6) / (X4 - X6) / AREA
DELQDT = DELQDT + A46 * 778. * DS/D / RGAS
WORK = TAUW * PIE * DIA / AREA / (1. - EPL46) / RHO46
DELQCT = DELQCT + WORK + A46 * DEPCT / (1. - EPL46)
DELQDT = DELQDT + ((A46**3) / PSI46 / GAMMA + UG46 / RHO46) * AMG56 / AREA / (1. - EPL

#46)

DELQDT = DELQCT - TRY
P6 = P3 + DELPDT * (T6 - T3)
Q6 = Q4 + DELQDT * (T6 - T4)
ANEW6 = (GAMMA - 1.) * (P6 + Q6) / 4.
UGNEW6 = (P6 - C6) / 2.
S6 = S7 + DS/D * (T6 - T7) * 778. / RGAS / GAMMA
EPL6 = EPL5 + DEPLDT * (T6 - T5)
IF (EPL6 * LE. 0.00) EPL6 = 0.0
UPN6 = UP5 + DUPDT * (T6 - T5)
IF (EPL6 * LE. 0.00) UPN6 = LGNEW6
TG6 = ANEW6 / ANEW / GAMMA / RGAS
PSI6 = PSI7 + (ANEW6 / A) * ** WCRC2
PSI6 = PSI6 * EXP (- GAMMA * (S6 - S7))
RHO6 = PSI6 / RGAS / TG6
IF ((T6 .GE. TMLZE)) GO TO 260

5003 FORMAT (1X, 5E11.3)
CHECK = ABS((ANEW6 - A6) / A6)
IF (CHECK.GT. TOLL) GO TO 180
CHECK = ABS((LGNEW6 - UG6) / UG6)
IF (UG6 .LT. 3.00) CHECK = 0.0001
TOLL = 10.0 * TCL
IF (CHECK.GT. TOLL) GO TO 180
180  ITRY = ITRY + 1
   IF( ITRY .LT. 21) GO TO 20
   CALL DIAGO(LOC, UGNEW6, ANEW6)
200  UG6 = (UGNEW6 + UG6) / 2.
   A6 = (ANEW6 + A6) / 2.
   UP6 = (UPN6 + UP6) / 2.
   GO TO 30
201  WRITE(6, 5004) VOL5, VOL6, SLRF5, SURF6, ETA56, AMG56, EPL56, DUPDT, DEPLDT
   WRITE(6, 5004) L5, L6, X5, X6, BR56
   WRITE(6, 5004) P6, Q6, A6, LG6, UP6
   WRITE(6, 5004) FD56, TAUW, EGAS, RS1, F1, UGMN, FOR56, UP3, UP4, UP5, DUPDX, UP
   #56, UG5, DBAR, DVOL, UPN6
5004  FORMAT(1X, 5E20.5)
   GO TO 128
250  T6 = TMUZE
   X6 = X3 + SLCPES*(T6-T3)
   AQ = (UG3+UOLC4-A3-AOLD4)/2.
   AP = (UOLD4+UG4+A4+AOLD4)/2.
   TI = (X6-AQ*T6-AP*T4)/(AQ-AP)
   XI = X4 + AP*(TI-T4)
   UG4 = (UG4-UOLD4)*(XI-X4)/(XCLD4-X4)*UG4
   A4 = (A4-AOLD4)*(XI-X4)/(XOLD4-X4)*A4
   TG4 = A4*A4/GAMMA/EGAS
   PSI4 = (PSI4-PSIQL4)*(XI-X4)/(XOLD4-X4)+PSI4
   RHO4 = PSI4/EGAS/TG4
   UP4 = (UP4-UPCLD4)*(XI-X4)/(XCLD4-X4)+UP4
   S4 = (S4-SOLD4)*(XI-X4)/(XCLC4-X4)+S4
   X4 = XI
   T4 = TI
   GO TO 31
260  UG6 = UGNEW6
   A6 = ANEW6
   UP6 = UPN6
   RETURN
END
APPENDIX B

Listing of Flame Front Program
C PROGRAM MCASSEY
C PROGRAM CALCULATES THE PROPAGATION OF A FLAMEFRONT THRU A
C POROUS PROPELLANT BED
C F=PROPELLANT IMPETUS (FT-LBS/LB)
C G= CHARGE WT (LBS)
C RHOP= PROPELLANT DENSITY (LBS/IN**3)
C ALFA=FLAMEFRONT VELOCITY PRESSURE COEFFICIENT (IN/SEC-PSI**BETA)
C BEYA=FLAMEFRONT VELOCITY PRESSURE EXPONENT (DIMEN)
C B=PROPELLANT BURNING RATE COEFFICIENT (IN/(SEC-PSI**EN))
C E=PROPELLANT BURNING RATE EXPONENT (DIMEN)
C R=EQV. MEAN RADIUS OF PROP. GRAIN (IN)
C R=1/2 WER 'IN'
C XLO=LENGTH OF COMPACTION REGION (IN)
C ETA=PROPELLANT CO/VOLUME (IN**3/LB)
C SUBSCRIPT J DENOTES THE NUMBER OF TIME INTERVALS (PT) DURING WHICH
C THE FLAMEFRONT PENETRATES THE PROPELLANT, J=1 CORRESPONDE TO IGNITIO
C DATA RHOP, G, RI, XLO, ETA/0.0567, 0.0115, 0.0075, 0.23, 32.2/
C READ(I,5000)F, ALFA, ETA, B, EN, FACT
C REAC(I,5000)RGAS, GAMMA, CV, TZERO
C 5000 FORMAT(2E20.5)
C WRITE(6,5000)F, GALFA, ETA, B, EN, FACT
C WRITE(6,5000)RGAS, GAMMA, CV, TZERO
C VEL=0.0
C AZERO=SORT(GAMMA*RGAS* TZERO)
C C=4.014E-03
C ICOUNT=0
C DO 10 J=1,30
C F(I,J)=0.0
C 10 XL(I,J)=0.0
C PIE=3.1415
C T=0.0001
C XTG=0.0
C RR=R-RR
C VOI=2.*PIE*R*R*RI+PIE*RI*RI*(RR+1.333*RI/PIE)
C XN=THE NUMBER OF GRAINS PER UNIT VOLUME
C A=CROSS SECTIONAL AREA OF CARTRIDGE
C D=DIAMETER OF CARTRIDGE CASF .335
C A=PIE*0.335*0.335/4.
C VC=VOLUME OCCUPIED BY COMPRESSED PROPELLANT
C VG=A*(1.5-XL0)
C XN=C/RH*P*VOI*VC
C WRITE(6,1234)
C P(I)=2750.
C N=1000 J=2,29
C P(I,J)=P(I-1)
C DT=N/.75E-04
C T=T+DT
C 30 ICOUNT=ICOUNT+1
C SUBSCRIPT J DENOTES THE NUMBER OF TIME INTERVAL(DT) DURING WHICH
C THE FLAMEFRONT PENETRATES THE PROPELLANT BED(IGNIT TIME=0.00015)
C C CALCULATE THE DISTANCE (D(I,J)) THE FLAMEFRONT HAS TRAVELED
C PHAR(I,J)=PIJ-1)*P(I,J).
C V(I,J)=ALFA*PHAR(I,J)**BETA
C DL(I,J)=V(I,J)*DT
C XL(I)=XL(I)+DL(I,J)
C CL(I)=XL(I)
IF(ZL(J) .GE. 1.5) ZL(J) = 1.5
IF(ZL(J) .GE. 1.5) DL(J) = ZL(J) - ZL(J-1)
IF(ZL(J) .GE. 1.5) T = T - DT
IF(ZL(J) .GE. 1.5) DT = DL(J)/VF(J)
IF(ZL(J) .GE. 1.5) T = T + DT

C CALCULATE THE DISTANCE(XL(I,J)) BURNED NORMAL TO PROP. SURFACE(IN)
CELL I DURING TIME DT, I=1 IS INITIAL COMPACTION REGION (NO PROP.)

PXTG = 0.0
VOLS = 0.0
DO 500 I = 2, J
DYL = PBAR(I,J)*EN*DT
DYL = DYL*FACT
XL(I,J) = XL(I,J-1) + DYL
RL(I,J) = I - XL(I,J)
V(I,J) = 2.*PIE*(RR**2*RL(I,J)) + PIE*PIE*RL(I,J)**2*(RR + 1.333/PIE*RL(I,J))

C XMG(I,J) IS THE AMOUNT OF GAS GENERATED FROM CELL I DURING TIME T

XMG(I,J) = DL(I)*XN*A*(VOL-V(I,J))*RHOP
XTG = THE TOTAL GAS GENERATED IN TIME T

XMG = XMG(I,J)
E(I,J) = (XN*A*DL(I)*VOL-XMG(I,J))/RHOP/A/DEL(I)
VOLS = VOLS + A*DEL(I)*E(I,J)
PXTG = PXTG + DXMG

500 CONTINUE

XNUM = 12.*PXTG + P(I)*A*ZL(I)
DENOM = A*ZL(I) - VOLS - PXTG * ETA
PPBAR = XNUM/DENOM

IMN = J
IF(ABS((PPBAR-P(J))/PPBAR) .GE. 0.05) GO TO 955

800 IF(ICOUNT .GT. 25) GO TO 1001
P(J) = PPBAR
GO TO 801

55 CONTINUE

XTG = PXTG
P(J) = PPBAR
ICOUNT = 0

DYL = 900 I = 2, J
WRITE(6,2468)T,P(J),ZL(J),XTG,E(I,J),XL(I,J),I,J
TIME = T

900 CONTINUE

IF(ZL(J) .GE. 1.5) GO TO 1001
K = J
CL(J) = ZL(J)

1234 FORMAT(1X,I19,9X,'TIME',15X,'HP(J)',15X,'ZL(J)',15X,'XTG',15X,'E(I,J)

2468 FORMAT(6(4X,ES15.6),2I3)

1000 CONTINUE

DO 1002 J = 2, N
DO 1002 I = 2, J
WRITE(6,2468)XL(I,J),DLV(I,J),XMG(I,J),RL(I,J),DL(I),I,J

1002 CONTINUE

GO TO 1005

1005 CONTINUE

J = IMN
P(J) = P(J) + 144.
CS = SP(I)/RGAS/TCERO
TH = (RGAS/778.*CV)*ALOG(TZERO/530.) - RGAS/778.*ALOG(P(J)/2075.)
ENTR = ENTR * 778.*RGAS/GAMMA
GO 1004 I = 1, IMN
JST = ZL(I)/12.
XL(I,J) = XL(I,J)/12.
TIM=T+(0.125-DIST)/AZERO
WRITE(7,4445)I,J
WRITE(7,4444)DIST,TIM,E(I,J),AZERO,VEL,VEL,P(I),DENS,ENTR,XL(I,J)
$ TZERO
WRITE(6,4444)DIST,TIM,E(I,J),AZERO,VEL,VEL,P(I),DENS,ENTR,XL(I,J)
$ TZERO
WRITE(7,4445)I,J
CONTINUE
4445 FORMAT(5X,*********,5X,5X,5X,*********,5X)
4444 FORMAT(3E20.5)
1005 CONTINUE
GO TO 2
END

/GO
0.32060E 06 0.31500E 02
0.48000E 00 0.26305E 02
0.70000E 00 0.10000E 01
0.25643E 04 0.12506E 01
0.11470E 02 0.46150E 04