BUCKLING, PREBUCKLING AND VIBRATION OF STIFFENED AND UNSTIFFENED SHELLS

Josef Singer, et al

Technion - Israel Institute of Technology

Prepared for:

Air Force Office of Scientific Research
European Office of Aerospace Research and Development

10 February 1975

DISTRIBUTED BY:

NTIS
National Technical Information Service
U. S. DEPARTMENT OF COMMERCE
BUCKLING, PREBUCKLING AND VIBRATION
OF STIFFENED AND UNSTIFFENED SHELLS

FINAL REPORT,

JOSEF SINGER AND MENACHEM BARUCH
Department of Aeronautical Engineering,
Technion - Israel Institute of Technology,
Haifa, Israel.

10 FEBRUARY 1975.

Approved for public release; distribution unlimited.

Prepared for Air Force Office of Scientific Research, United
States Air Force, 1400 Wilson Blvd. Arlington, VA 22209,

and

European Office of Aerospace Research and Development, London,
England.
BUCKLING, PREBUCKLING AND VIBRATION OF STIFFENED AND UNSTIFFENED SHELLS

This report complements the Interim Report (TAE 228), which summarized the major portion of the investigation. It discusses the stability and vibrations of stiffened and unstiffened shells under axial compression, as well as thermal buckling. The studies include theoretical and experimental work, with emphasis on elastic constraints, on correlation between vibrations and buckling and on the influence of initial imperfections on vibrations and stability. A new incremental theory for the nonlinear elastic-plastic behavior of structures is formulated.

PRICE SUBJECT TO CHANGE
SUMMARY

This report complements the Intarim Report (T.A.E. 228), which summarized the major portion of the investigation. It discusses the stability and vibrations of stiffened and unstiffened shells under axial compression, as well as thermal buckling. The studies include theoretical and experimental work, with emphasis on elastic constraints, on correlation between vibrations and buckling and on the influence of initial imperfections on vibrations and stability. A new incremental theory for the nonlinear elasto-plastic behavior of structures is formulated.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>SUMMARY</th>
<th>PAGE NO.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summary</td>
<td>1</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td>iii</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>1 - 2</td>
</tr>
<tr>
<td>SUMMARY OF INVESTIGATIONS IN PERIOD 1 JULY 1974 - 31 DECEMBER 1974</td>
<td>3</td>
</tr>
<tr>
<td>1. Vibrations and Buckling of Axially Compressed Stiffened Cylindrical Shells with Elastic Restraints</td>
<td>3</td>
</tr>
<tr>
<td>2. Elastic-Plastic Instability of Shells</td>
<td>3 - 4</td>
</tr>
<tr>
<td>3. Influence of Asymmetric Initial Geometrical Imperfections on the Vibrations of Cylindrical Shells</td>
<td>4 - 5</td>
</tr>
<tr>
<td>5. Thermal Buckling</td>
<td>5 - 6</td>
</tr>
<tr>
<td>6. Study of the Growth of Initial Imperfections of Axially Compressed Stiffened Cylindrical Shells</td>
<td>6 - 7</td>
</tr>
<tr>
<td>INVENTION STATEMENT</td>
<td>7</td>
</tr>
</tbody>
</table>
INTRODUCTION

This final report describes part of a continuing investigation of stability and vibrations of stiffened and unstiffened shells under different loads and load combinations as well as heating. In the investigation, new methods for non-destructive determination of buckling loads for elastic bars, plates and shells, as well as a new incremental theory for the non-linear elastic-plastic behaviour of structures, have been developed. An improved technique for measurement of imperfections has also been established and the influence of imperfections on vibrations had been studied. The work reported was performed at the Department of Aeronautical Engineering of the Technion, Israel Institute of Technology under Grant 72-2394, during the period 16 September 1972 to 31 December 1974.

The major portion of the work has been summarized in an Interim Report (covering the period 16 September 1972 to 30 June 1974), T.A.E. Report 228 "Buckling, Prebuckling and Vibration of Stiffened and Unstiffened Shells" by J. Singer and M. Baruch, August 1974. This Interim Report may be considered as the first part of the present Final Report. Hence only the work performed in the remaining period of the Grant, 1 July 1974 to 31 December 1974 is described here in detail.

During the period 16 September 1972 to 31 December 1974, 13 Scientific Reports were completed, 10 of them were detailed in the Interim Report (pp. 1-3 of T.A.E. 228) and in the remaining period (1 July 1974 – 31 December 1974) the following additional ones were completed:


During the same period 16 September 1972 to 31 December 1974, 14 papers some of which are condensed versions of the above cited T.A.E. Reports, were published or accepted for publication. 13 of them were detailed in the Interim Report (pp. 3-5), and one Synoptic has recently been accepted for publication in the AIAA Journal: "Buckling of Discretely Stringer-Stiffened Cylindrical Shells and Elastically Restrained Panels by J. Singer and R. Haftka.

One of those mentioned in the Interim Report as accepted has since been published: "Behaviour of An Incrementally Elastic Thick Walled Sphere under Internal and External Pressure", by D. Durban and M. Baruch, International Journal of Non-Linear Mechanics, Vol. 9, 1974, pp. 105-119.

The writers would like to take the opportunity to thank the authors whose names appear on the Scientific Reports and publications for their invaluable contributions to the work performed. They would also like to acknowledge the continuous assistance and encouragement given by the Air Force Office of Scientific Research and its European Office of Aerospace Research.
SUMMARY OF INVESTIGATIONS IN PERIOD 1 JULY 1974 - 31 DECEMBER 1974

1. Vibrations and Buckling of Axially Compressed Stiffened Cylindrical Shells with Elastic Restraints

The studies summarized in Sections A.9 and B.2 of the Interim Report (T.A.E. 228) have been continued. Additional computations have been carried out with the theory described there, that takes into account elastic restraints at the boundaries, in order to study in detail the influence of these restraints on the vibrations and buckling of stiffened cylindrical shells. The theoretical predictions have again been compared with experimental results obtained in recent tests. The method of experimental definition of equivalent elastically restrained boundary conditions from vibration tests, and the resulting reductions in the scatter of ratio of experimental to predicted buckling loads, have been reconfirmed. Details are given in T.A.E. Report 208.

2. Elastic-Plastic Instability of Shells

The basic theory which has earlier been applied to particular cases (Section A.6 of Interim Report) is now formulated.

The proposed incremental elasto-plastic theory is based on the assumption that the incremental strain tensor may be caused only by an incremental change of the stress tensor in which the rigid body motion has been excluded. The displacement can be very large. For small displacements the theory degenerates to the classical elasto-plastic theory.
The tensors used in the theory are treated as full and invariant quantities. It seems that this approach is an inherent requirement of the proposed theory.

Details are given in T.A.E. Report 193.

3. Influence of Asymmetric Initial Geometrical Imperfections on the Vibrations of Cylindrical Shells.

Since the influence of initial geometrical imperfections both axisymmetric and asymmetric, on the buckling of cylindrical shells is known to be significant, their influence on vibrations has been studied. First a dynamical extension of Koiter's well known 1963 study for axisymmetric imperfections was carried out (see Section A.12 of Interim Report and T.A.E. Report 182), and the influence of axisymmetric initial imperfections on the vibrations of axially loaded cylindrical shells was found to be large even at zero load.

The study has now been extended to the influence of non-axisymmetric initial imperfections on vibrations and buckling of isotropic cylindrical shells. The results show that the influence strongly depends on the mode of the imperfection and the mode of vibration, as well as on their relation. The corresponding parameters determine whether the frequency is raised or lowered by the imperfections. Asymmetric imperfections also affect the buckling of the shells, and apparently can not only lower the buckling load, but also raise it and even change the behavior to promote large bending deflections without buckling. These are only preliminary
results which will be extended to more practical imperfection and vibration modes. Details are given in T.A.E. Report 212.


Study of the indirect curve fitting technique developed (see Section B.4 of Interim Report) for prediction of buckling loads from experimentally observed frequency variation with load has continued. Preliminary results of this technique applied to additional shells tested are encouraging. Further study of this technique is, however, required, including correlation with many tests, to provide a basis for a reliable nondestructive test method.

5. Thermal Buckling

The theoretical study of heat transfer in a cylindrical shell heated along a generator of finite width and the non-linear analysis for thermal buckling of cylindrical shells have been continued. Preliminary results have been obtained, but further calculations are required to verify the derivations and the computer program.

The present phase of the experimental study is aimed at the interaction of torsion and heating. An extension of the test rig to permit tests on buckling of cylindrical shells due to combination of torque, axial load and heating along two generators has been designed and built. An early test indicated some torsional
slippage at high loads and resulted in improvements in the torque loading device. Two additional preliminary tests have been carried without any slippage, but the torque loading still introduced some lateral load. The torque mounting has been redesigned to ensure pure torque loading. This change will be incorporated before further tests are performed.


Development of the semi-automatic device for measurement of initial imperfections and the growth of these imperfections during axial compression has been continued. The test apparatus is now operational.

The rig employs a spiral scanning path in order to obtain a continuous record of the imperfections. The pick-up position is transmitted from two potentiometers, one indicating the vertical position and one the circumferential position. The signal transmitted in parallel to the magnetic tape of the Beckman Acquisition System, and as an input to an X-Y recorder on which the scanning is monitored.

The pick-up is a contacting type inductive displacement transducer. Other types of pick-ups, a non-contacting inductance transducer and a non-contacting capacitance transducer, were also tried, but the contacting type used was found to be very satisfactory. The output of the pick-up is monitored on a digital voltmeter,
whose readings are recorded on the magnetic tape of the Beckman System together with the position recordings. In parallel the output is added to the Y of the X-Y recorder, for visual indication of the imperfections during the test. The data reduction is performed on the IBM 370-185 computer of the Technion computing center.

A special load cell, consisting of a short shell, measures the circumferential load distribution during the test. A method of data acquisition by automatic sampling and feeding to the Beckman Data Acquisition system has been developed. A computer program which analyses the experimental data has been prepared. Some preliminary tests have been carried out.

INVENTION STATEMENT

No inventions were conceived or made during the period 16 September 1972 - 31 December 1974.