ESCALATION PROVISIONS IN DoD PROCUREMENT:
A REVIEW OF THE PROBLEM AND A FRAMEWORK FOR ANALYSIS

Robert William Eberth
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Thesis Advisor: M. K. Block

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Block #20 Continued

After this review a framework was designed to facilitate the analysis of the relationship of escalation provisions and price level uncertainty. The specific model employed examined interacting objectives of the government and a firm in a sole-source contract negotiation scenario. A method was developed to approximate the increase in contract price required by the firm as compensation for accepting the risk of uncertain price levels. A criterion was established for the employment of the escalation provision in the modeled scenario.
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by

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ABSTRACT

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I. INTRODUCTION

A. BACKGROUND

1. The Impact of Inflation

By July of 1974, consumer prices, as measured by the Consumer Price Index, had risen to 147% of their 1967 level and had shown the sharpest annual rise, 11.1%, since 1947. At the same time that this inflation is reflected in greatly increased nominal dollar program costs for the Department of Defense (as for other producers and consumers), Congressional reaction to inflation has been, in part, to place tighter constraints on the budget of the DoD. Thus the final impact on DoD of the current inflation has been a constant-dollar decrease of 36% in weapons systems procurement from that of 20 years ago.

2. Treatment of Inflation in Private-Sector Contracts

In private sector procurement contracting, there are several methods of alleviating the effects of inflation, in particular, and uncertain price levels, in general:

(i) contingency pricing

With this method, the contractor projects his estimates of wage and material price changes over the term of the contract, and adds a contingency sum to the estimates to

1 These figures are from Department of Labor computations as reported in Ref. 1; p. 1, 4.

2 Department of Defense, [2; p. 7, 23].
protect himself from the possibility of having under-estimated the price changes.³

(ii) escalation provisions

A variety of pricing clauses fall under this heading, including

(a) graduated price schedules; the later the delivery, the higher the price.

(b) "escalators"; whether based on experienced cost increases or on the movements of price indices, these compensate the contractor for cost increases during contract performance.

(iii) price at time of delivery

Although an estimate of the future actual price may be given, the contractor is not bound to it; rather, he determines the price after all his costs are known, at the time of shipment or delivery.

3. Treatment of Inflation in DoD Contracts

In DoD procurement contracting, the existence of contingency factors is recognized, but at present only certain broad types of "escalators" are generally authorized for use in prime contracts. These, as defined by the Armed Services Procurement Regulation (ASPR) Committee, are:⁴

"(i) adjustment based on established prices - price adjustments are based on an increase or decrease from an agreed-upon level in published or established prices

³Logistics Management Institute, [3; p. 13].

⁴Department of Defense, [4; p. 5].
of either specific items or price levels of contract end items.

(ii) adjustment based on labor or material costs (Actual Costs Method) - price adjustments are based on an increase or decrease in specified costs of labor or material actually experienced by the contractor during performance of the contract.

(iii) adjustment based on labor or material costs (Cost Index Method) - price adjustments are based on an increase or decrease from specified labor or material cost standards or indices made applicable to the contract."

The problem of when to use escalation clauses (as opposed to contingency factors for uncertain price levels) is necessarily related to the problem of the "proper" structure of those clauses. In May of 1968, the Logistics Management Institute (LMI) published a study addressing some elements of both problems. The Institute's major recommendations were:

"(i) The use of escalation provisions is generally to be preferred to adding estimates of future price level changes in contract prices.

(ii) Indexes should encompass the widest possible industrial base compatible with the objectives of escalation provisions to avoid the possibility that contractors may influence the index and that escalation adjustments may contribute to spiraling price levels.

---

5 The terms "escalators" and "escalation clauses" are in common use to refer to compensating adjustments for uncertain prices; however, ASPR has adopted the term "economic price adjustment clauses" instead to avoid ambiguity since such clauses apply to both inflationary and deflationary price movements.

6 Logistics Management Institute, [3; p. 3].
(iii) Escalation provisions should not require audit or statement of actual costs as a condition for applying the escalation adjustment.

(iv) Escalation provisions should be included in all multi-year procurement contracts and in contracts containing priced options.

(v) Studies should be made to determine the appropriate labor and material indexes for ... major commodity areas where long-term contracts are employed."

These recommendations resulted primarily from LMI's conclusions that "constant dollar" pricing was the preferred method of paying for escalation, that the only "escalation" to be compensated for should be that due purely to general economic trends and outside the influence of the individual contractor, and that suitable indexes could be chosen or constructed to reflect such trends. Additionally, LMI recommended that there be no ceiling or floor to the amount of the adjustment provided by an escalation clause, that such adjustments should be made without regard to contract delivery date, that target costs and ceiling price should be reset to reflect escalation, and that an increment of profit should be included in any escalation adjustment. 7

In 1969, in comment on LMI's study, the Navy Secretariat generally disagreed with the recommendations of that study, stating that the consensus of opinion of senior Navy contracting personnel was that prospective (i.e., contingency factor) pricing was strongly preferred to more extensive use of escalation clauses. The principal reason

7 Logistics Management Institute, [3; p. 53-57].
given was that, "As generally recognized, prospective pricing, by its very nature, encourages management attention to performance within firm budgets and provides a significant inducement to proficiency in performance." The Navy did agree, however, to the possible need for wider use of escalation clauses, on a selective basis, where long-term contracts and highly unstable prices might make price projections too unreliable for use of prospective pricing. It was also agreed that, when escalation clauses were used, indexes with the widest possible bases should be chosen and that audits of actual costs should not be required. The remaining areas of disagreement appeared to stem from the Navy's desire to maintain maximal incentives on the contractor to exert strong managerial control on his costs and on meeting his contracted delivery data; in particular, the Navy responded negatively to LMI's contention that escalation should continue to be paid on delayed deliveries, and should have no ceiling.

The position taken, with respect to the use of escalation clauses, by private industry was typified by a letter from the Council of Defense and Space Industry Associations (CDSIA) submitted to the ASPR Committee in June 1972. It stated, in part:

---

8 Office of the Secretary of the Navy, [5; p. 2-3].

9 Ibid., p. 3-7.

10 Council of Defense and Space Industry Associations, [6; p. 1-2].
The first and strongest conviction of these associations is that the inclusion in the ASPR of guidelines for Price Escalation Clauses would significantly improve the realism of price proposals. The importance of the need for this improvement cannot be over-emphasized. We believe that there is a general understanding that, without this improvement, the continuation of "cost-growth" on many large procurements will undeniably have the effect of stifling our defense efforts to the danger point. It is furthermore undeniable that the rate of inflation in recent years has been one of the principal contributing factors to this "cost-growth" situation; and it may continue to remain one.

"In summary, it appears that the bulk of informed opinion does favor the use of escalation provisions; provided that suitable indexes exist or can be developed."

The importance of "suitable indexes," as mentioned above, was underscored by a letter, forwarded to the Secretary of the Navy by a United States Senator, from a potential competitor for the shafting systems contract for the Navy's Patrol Frigate program. The letter presented some evidence that the Navy's chosen material price level index for the escalation clause of this long-term contract did not accurately reflect price level changes within the industry concerned and that, if a suitable index were not adopted, "[this company], and others interested in this shafting procurement, will thus be in a non-responsive position relative to the Patrol Frigate Program contract terms on price escalation..." The particular company did not indicate a desire to use an index structured from its own individual data; rather, it had constructed a "suitable" index from a set of indexes, published by the Bureau of Labor Statistics (BLS),

11 National Forge Company, [7; p. 2-3].
by "weighting" each index and summing. Such a method is used in some Navy shipbuilding escalation clauses, but at a higher level of aggregation.

Besides the problem of choosing the proper index, there is the problem of how to best use the index to compute escalation payments. LMI noted that sub-contract commitments were generally made, and priced, some time before the expenses were actually incurred, but that escalation was computed at the latter time, due to expenditure rather than commitment accounting methods.\textsuperscript{12} LMI concluded that, for major subcontracts, escalation should be computed at time of commitment, not actual expenditure.\textsuperscript{13} At present, this is not being done; rather, in long-term negotiated procurements, the expenditure rates and relative cost weights (contract profile and contract mix) of labor and material are set at the time of negotiations and are not changed to reflect actual commitments or expenditures. In competitive contracts, the contracting officer unilaterally specifies the above rates and weights, based on the average profiles of all companies to be solicited.\textsuperscript{14} In regard to this method for competitive contracts, the General Accounting Office (GAO) has expressed concern that,

\textsuperscript{12}Logistics Management Institute, [3; p. 44-46].

\textsuperscript{13}\textit{Ibid.}, p. 47.

\textsuperscript{14}Department of Defense, [4; p. 9-12].
"Because such advance provisions based on averages can be unrealistic to the offerors' plans for contract performance, each offeror can be expected to increase or decrease his offered price for the provisions which do not conform to his contract performance plans. These actions can result in estimates for inflation being a substantial factor in proposed prices."\(^\text{15}\)

Concern has also been expressed by the Naval Ship Systems Command, and others, that care must be taken to avoid "gamesmanship" with respect to the contracted expenditure rates and mix.\(^\text{16}\) Such "gamesmanship" could also result from the manner of computing escalation payments, as previously mentioned.

Finally, there still appears to be some conceptual disagreement among various DoD institutions as to the objectives of escalation provisions. The Naval Material Command has stated that the objectives of an optimum escalation system should be:\(^\text{17}\)

(i) The system should provide coverage adequate to substantially remove contingencies for economic unknowns while retaining adequate incentives for the contractor to maintain control of his unit labor and material costs.

(ii) The measure to which escalation is paid must be objectively determinable so that disputes are avoided.

(iii) The cost and effort associated with the functioning and administration of the system should be minimal.

The above objectives agree, generally, with those espoused by other Systems Commands, all of whom appear oriented

\(^{15}\) General Accounting Office, [8; p. 2].

\(^{16}\) Naval Ship Systems Command, [9; encl. 1, p. 1].

\(^{17}\) Naval Material Command, [10; p. 1-2].
toward the use of escalation provisions to minimize the cost of a given program. At the policy-setting levels, there are additional, and occasionally conflicting, objectives concerning the overall defense budget. In particular, the Assistant Secretary of the Navy for Installations and Logistics has stated that,\textsuperscript{18}

"The more we can do to continue to separate [inflation-caused] cost growth from those that the Defense Department has normally been charged, the better our image will be. ...These budget elements (procurement and escalation) should remain as separate as possible."

In reply to the memorandum containing the above statement, the Assistant Secretary of the Navy for Financial Management stated that,\textsuperscript{19}

"OSD does attempt to separate and identify escalation to the Hill whenever possible, but the policy of doing so does not dominate their overall review and presentation of the budget. [There are instances] where their concern for the availability of prior year assets and the makeup of the budget overrode any concern they may have for identifying and funding escalation separately."

The ASPR Committee appears most concerned with the "pure" economics of the government-industry relationship. Their objectives with respect to escalation provisions was summed up in one sentence,\textsuperscript{20}

"[This is] the crux of the entire escalation concept; i.e., to reimburse the contractor for economic fluctuations beyond his control."

\textsuperscript{18}Assistant Secretary of the Navy (Installations and Logistics), [11; p. 1].

\textsuperscript{19}Assistant Secretary of the Navy (Financial Management), [12; p. 2].

\textsuperscript{20}Armed Services Procurement Regulations Subcommittee, [13; p. 5].
4. **Summary of the Situation**

The entire question of escalation clauses appears unresolved, even to the extent that the proper objectives of escalation provisions have not been agreed upon among the agencies which direct their use and those which must apply the provisions. The "ideal" structure of escalation clauses and of any indexes used therein remains subject to controversy. Finally, there is no general agreement as to the proper method for computing and paying escalation adjustments.

**B. PURPOSE OF THE STUDY**

The areas of controversy with regard to escalation provisions appeared both too wide and too interrelated to be able to concentrate on any one (e.g.; structure of indexes, when to use some form of an escalation clause, etc.) without first obtaining a more thorough understanding of how escalation provisions affect a firm's attitude toward inflation, or uncertain price levels, and how this attitude is reflected in its pricing of a contract. To this end, it was felt necessary to "start fresh"; to develop an analytical framework which would both permit such an understanding of the theoretic nature of escalation provisions and provide the basis for further, more detailed investigation of the effects of employing differing types of escalation provisions.
II. THE DEVELOPMENT OF THE ANALYSIS

The analysis was designed to examine the contractual objectives of both the government and the firm, the requirements to jointly meet these objectives (in a negotiation, as compared to a competitive bid, situation), and the changes in such requirements due to successive "uncertainties." In particular, it was desired to isolate and examine changes due to uncertain price levels.

The notation and initial formulation drew heavily on the works of Sandmo [Ref. 14] and Baron [Ref. 15], combining theoretic elements of each to best meet the purpose of the study.

A. THE MODEL OF THE FIRM

The firm was assumed to be competitive in its private-sector production operations. Since it was desired to examine the firm's behavior under uncertainty, it was also assumed that the firm was a maximizer of expected utility of future wealth and that it was risk averse. Finally, the model was static and thus (to eventually introduce uncertain contract costs) it was assumed that contract costs were paid by the firm upon completion of the contract, at which time the government paid the firm in accordance with the original contract terms.²¹

²¹The alternative, that the firm paid all costs at the time the contract was let, proved unworkable in a simple static model.
The problem faced by the firm at the time of contract negotiation was two-fold:

(i) allocation of initial wealth between a "riskless" asset and production in the private sector, and

(ii) the decision as to accepting or rejecting the preferred government contract.

Notation:

\[ W_0 = \text{initial wealth of the firm} \]

\[ W_t = \text{wealth of the firm at the end of the term of the contract (whether or not it was actually accepted)} \]

\[ M = \text{amount invested in the "riskless" asset, which gave rate of return (1+r) over the term of the contract} \]

\[ S(x) = \text{cost of producing output quantity x in the private sector} \]

\[ R = \text{market-clearing price of a unit of the firm's private-sector output} \]

\[ C = \text{cost of performing the work specified by the contract} \]

\[ C_t = \text{contract target cost} \]

\[ \alpha = \text{contract profit rate where } \alpha C_t \text{ was "target profit"; } \alpha > 0 \]

\[ \beta = \text{incentive profit rate where } \beta(C_t - C) \text{ was paid to the firm if } C_t > C \text{ and by the firm to the government if } C_t < C; 0 \leq \beta \leq 1. \]

The firm's initial allocation possibilities were given by:

\[ W_0 = M + S(x). \quad (1) \]

Its future wealth was then determined by:

\[ W_t = M(1+r) + Rx + \delta[\alpha C_t + \beta(C_t - C)] \quad (2) \]
where

\[ \delta = \begin{cases} 
1 & \text{if contract were accepted} \\
0 & \text{if contract were rejected} 
\end{cases} \]

Solving (1) for M and substituting into (2) gave (after rearrangement of terms):

\[ W_t = W_0(1+r) + [Rx - (1+r)S(x)] + \delta[(\alpha+\beta)C_t-\beta C]. \quad (3) \]

If \( \beta = 1 \), the contract was of the Fixed Cost-Fixed Fee type; if \( \beta = 0 \), it was of the Cost Plus Fixed Fee Type.

With \( E \) representing the expectation operator, the firm's problem was then formulated as:

\[
\max_{x,\delta} \left\{ U[W_0(1+r) + Rx - (1+r)S(x) + \delta((\alpha+\beta)C_t-\beta C)] \right\} \quad (4)
\]

subject to \( x \geq 0 \)

\[ \delta \in \{0,1\}. \]

The firm's preference ordering over wealth, represented by \( U(W) \), was required to be monotonic increasing (for all positive utility levels) and unique up to a positive linear transformation. This, and the added assumption of risk aversion, required that

\[
\begin{align*}
U'(W) &> 0 \\
U''(W) &< 0
\end{align*}
\quad (5)
\]

for all \( W \), \( U(W) > 0 \)

\[
\begin{align*}
U'(W) &> 0 \\
U''(W) &< 0
\end{align*}
\quad (6)
\]

B. THE MODEL OF THE GOVERNMENT

The government was assumed to be an expected cost minimizer with the constraint that the contract must be awarded to the
particular firm described in the preceding section. The contract parameters \( \alpha \) and \( \beta \) were assumed to be pre-determined. The government then had to negotiate the contract target cost.

Using the previous notation, the government's problem was formulated as:

\[
\min_{C_t} E[\delta (\alpha + \beta) C_t - \beta C] \\
\text{subject to } \delta = 1.
\]  

(7)

C. THE CONTRACT NEGOTIATIONS ANALYSIS

1. The Negotiations Model

This model represented the interaction, through negotiation, of the previous models of the firm and the government by defining a problem whose solution satisfied the objectives of each party to the negotiations. This was formulated as:

\[
\min_{C_t} \\
\text{subject to:} \\
\max_{x} E[U[W_0 (1+r) + R_x - (1+r)S(x) + (\alpha + \beta)C_t - \beta C]} \\
= \max_{x} E[U[W_0 (1+r) + R_x - (1+r)S(x)]]
\]  

(8)

The \( C_t \) which satisfied (8) was then the minimum target cost such that the firm was indifferent between accepting and rejecting the contract.\(^{22}\)

\(^{22}\)It was recognized that this condition did not guarantee acceptance, since the firm was indifferent to the contract. Such indifference, however, was assumed to result in acceptance.
2. The Certainty Solution

Under complete certainty conditions (i.e., R, S(x), and C, as well as the other parameters, known with certainty), (8) simplified to:

\[
\min C_t \\
\text{subject to:}
\]

\[
\max_x \{Rx - (1+r)S(x) + (\alpha+\beta)C_t - \beta C\}
\]

\[
= \max_x \{Rx - (1+r)S(x)\} .
\] (9)

For a unique and internal optimal output to exist in the firm's private sector production, it was necessary to assume increasing marginal costs of the factors of production. With this assumption made, the first and second-order conditions for private-sector optimality were:

\[
R - (1+r)S'(x) = 0 \quad (10)
\]

and

\[
-(1+r)S''(x) < 0, \text{ respectively.} \quad (11)
\]

With complete certainty, equations (10) and (11) held for both sides of the equality constraint of (9) and therefore a unique x optimized both. This implied that, for the equality to hold, the solution to (9) was:

\[
C_t = \frac{\beta C}{\alpha+\beta} .
\]

3. Risk Aversion as Applied to the Negotiations Model

a. Risk Aversion Functions

The Arrow-Pratt functions of absolute and relative risk aversion allowed further specification of the firm's attitude toward risk. These were defined as:
Absolute Risk Aversion; \( F_A(W) \equiv - \frac{U''(W)}{U'(W)} \) \hspace{2cm} (13)

Relative Risk Aversion; \( F_R(W) \equiv - \frac{U''(W)}{U'(W)} W \) \hspace{2cm} (14)

The meaning of these functions was characterized by Pratt as:\(^{23}\)

"If the amount invested in the risky asset increases (decreases) with wealth, the investor has decreasing (increasing) absolute risk aversion. If the fraction of wealth invested in the risky asset increases (decreases) with wealth, the investor has decreasing (increasing) relative risk aversion."

This analysis made the assumption of decreasing absolute risk aversion; however, no assumption was made with respect to relative risk aversion since none appeared capable of adding any qualitative value to the analysis or facilitated the interpretation of its results.

b. The Risk Aversion Increment

With \( U \) as previously defined, and under the assumption of risk aversion, it was assured (when any uncertainty existed) that:

\[
E[U(W_t)] < U[E(W_t)], \quad W_t = f(W_t). \hspace{2cm} (15)
\]

The certainty equivalent, \( D \), was defined such that:\(^{24}\)

\[
U(D) = E[U(W_t)]. \hspace{2cm} (16)
\]

\( D \), then was the dollar amount such that the firm was indifferent to accepting \( D \) with certainty or taking the risky action with expected outcome \( E(W_t) \).

---

\(^{23}\) This characterization was quoted from: [16; p. 107].

\(^{24}\) Berhold, M., [17; p. 46].
The risk aversion increment, also a dollar amount, was then defined by:\textsuperscript{25}

\[ I(W_t) \equiv E[W_t] - D. \] \hspace{1cm} (17)

From the definition of \( D \) and equation (5), assuming the condition on it held, it was assured that \( I(W_t) \) was positive.

Using these definitions, the following maximization statements were equivalent:

\[
\begin{align*}
\max E[U(W_t)] & \iff \max U(D) \quad \text{(from definition of \( D \))} \\
& \iff \max D \quad \text{(from definition of \( D \) and \( U \))} \\
& \iff \max [E(W_t) - I(W_t)] \quad \text{(from definition of \( I(W_t) \)).} \\
\end{align*}
\]

The solution constraint of the negotiations model was then restated in the format of (18) as:

\[
\max \{E[W_o(1+r) + Rx - (1+r)S(x) + (\alpha+\beta)C_t - \beta C] - I(W_t^C)\} \nonumber \]

\[
= \max \{E[W_o(1+r) + Rx - (1+r)S(x)] - I(W_t^R)\} \quad (19)
\]

where the superscripts of \( I(W_t^C) \) and \( I(W_t^R) \) were used to denote the risk aversion increments with and without the contract, respectively.

c. The Risk Aversion Increment as a Function of the Mean and Variance of Future Wealth

With the assumption that the probability distribution over future wealth (with or without the contract)
could be approximated by its quadratic expansion, the risk aversion increment was re-expressed as:

$$I = I(\mu_W, V_W)$$

(21)

where

$$\mu_W = E(W)$$

(22)

and

$$V_W = E(W^2) - [E(W)]^2 \text{ (the variance of } W)$$

(23)

The total differential of $I$ was then:

$$dI = \frac{\partial I}{\partial \mu_W} d\mu_W + \frac{\partial I}{\partial V_W} dV_W.$$  

(24)

It was noted that the signs of the partial derivatives in (24) were determined by the assumptions of risk aversion and decreasing absolute risk aversion as:

$$\text{Risk Aversion } \Rightarrow \frac{\partial I}{\partial V_W} > 0$$  

(25)

$$\text{Decreasing Absolute Risk Aversion } \Rightarrow \frac{\partial I}{\partial \mu_W} < 0.$$

(26)

4. The General First-Order Optimality Condition with Uncertainties

Assuming statistical independence between uncertain contract costs and an uncertain market-clearing price, then:

$$\mu_W = E(W) = W_0(1+r) + \mu_R x - (1+r)S(x) + (\alpha+\beta)C_t \cdot \beta \mu_C$$

(27)

(28)

(29)

(where $\mu_R$ and $\mu_C$ were $E(R)$ and $E(C)$, respectively), and the variance of future wealth was:

$$V_W = x^2 \sigma_R^2 + \beta^2 \sigma_C^2$$

(28)
(where \( \sigma^2_R \) and \( \sigma^2_C \) were the variances of \( R \) and \( C \), respectively, and \( \sigma_R \) and \( \sigma_C \) were the standard deviations of each).

Then, with \( I \) as postulated above, the first-order optimality condition for the left-hand side of the equality constraint of (19) was:

\[
\left[ \mu_R - (1+r)S'(x) \right] dx + (\alpha+\beta) dC_t - \beta d\mu_c
- \frac{\partial I(W^C_t)}{\partial \mu_{W_t}} \left\{ \left[ \mu_R - (1+r)S'(x) \right] dx + (\alpha+\beta) dC_t - \beta d\mu_c \right\} dx

- \frac{\partial I(W^C_t)}{\partial V_{W_t}} \left\{ 2x\sigma^2_R dx + 2\beta^2\sigma_C d\sigma_c \right\} = 0 . \quad (29)
\]

Rearranging terms, (29) becomes:

\[
\left\{ \left[ \mu_R - (1+r)S'(x) \right] \left[ 1 - \frac{\partial I(W^C_t)}{\partial \mu_{W_t}} \right] - 2x\sigma^2_R \frac{\partial I(W^C_t)}{\partial V_{W_t}} \right\} dx

+ \left\{ (\alpha+\beta) \left[ 1 - \frac{\partial I(W^C_t)}{\partial \mu_{W_t}} \right] \right\} dC_t - \left\{ \beta \left[ 1 - \frac{\partial I(W^C_t)}{\partial \mu_{W_t}} \right] \right\} d\mu_c

- \left\{ 2\beta^2\sigma_c \frac{\partial I(W^C_t)}{\partial V_{W_t}} \right\} d\sigma_c = 0 . \quad (30)
\]

For the right-hand side of the equality constraint, the first-order optimality condition became, after rearranging terms:

\[
\left\{ \left[ \mu_R - (1+r)S'(x) \right] \left[ 1 - \frac{\partial I(W^C_t)}{\partial \mu_{W_t}} \right] - 2x\sigma^2_R \frac{\partial I(W^C_t)}{\partial V_{W_t}} \right\} = 0 . \quad (31)
\]
The next assumption made dealt with qualifying the definition of minimum contract target price. In particular, it was assumed that the government had, as a second objective, that the private-sector output of the firm not change as a result of taking the contract. This assumption changed the formulation of the contract negotiations problem to:

$$\min C_t$$

subject to:

$$E[W_0(1+r) + Rx^* - (1+r)S(x^*) + (\alpha + \beta)C_t - \beta C] - I(W^C_t) = E[W_0(1+r) + Rx^* - (1+r)S(x^*)] - I(W^n_t)$$

(32)

and

$$[\mu_R - (1+r)S'(x^*)][1 - \frac{\partial I(W^n_t)}{\partial \mu_{W_t}}] - 2x^*\sigma^2 \frac{\partial I(W^n_t)}{\partial V_{W_t}} = 0.$$  

The second constraint of (32) implied that each side of the first constraint must be optimized, with respect to \(x\), at private sector output \(x^*\).

This in turn implied that, in equation (30):

$$[\mu_R - (1+r)S'(x^*)][1 - \frac{\partial I(W^n_t)}{\partial \mu_{W_t}}] - 2x^*\sigma^2 \frac{\partial I(W^n_t)}{\partial V_{W_t}} = 0.$$  

(33)

The second constraint of (32) and equation (33) were then rewritten as:

$$[\mu_R - (1+r)S'(x^*)] - 2x^*\sigma^2 \frac{\partial I(W^n_t)}{\partial V_{W_t}} \left\{ \frac{\partial I(W^n_t)}{\partial \mu_{W_t}} / \left(1 - \frac{\partial I(W^n_t)}{\partial \mu_{W_t}}\right) \right\} = 0$$  

(34)

and

$$[\mu_R - (1+r)S'(x^*)] - 2x^*\sigma^2 \frac{\partial I(W^n_t)}{\partial V_{W_t}} \left\{ \frac{\partial I(W^n_t)}{\partial \mu_{W_t}} / \left(1 - \frac{\partial I(W^n_t)}{\partial \mu_{W_t}}\right) \right\} = 0$$  

(35)
respectively, implying that (for \( \sigma_R^2 > 0 \)):

\[
\left\{ \frac{\partial I(W^R_t)}{\partial V_{W_t}} \right\} \left( 1 - \frac{\partial I(W^R_t)}{\partial \mu_{W_t}} \right) = \left\{ \frac{\partial I(W^C_t)}{\partial V_{W_t}} \right\} \left( 1 - \frac{\partial I(W^C_t)}{\partial \mu_{W_t}} \right)
\]

(36)

It was noted that (36) was not a restriction on the utility function of the firm; rather, it was the result of the added assumption concerning the objectives of the government.

With reference to equation (30), since the coefficient of \( dx \) was shown to be zero (by equation (33)), the first-order optimality condition became, after rearranging terms:

\[
(a+\beta)dC_t = \beta \mu_C + 2\beta^2 \sigma_C \left\{ \frac{\partial I(W^C_t)}{\partial V_{W_t}} \right\} \left( 1 - \frac{\partial I(W^C_t)}{\partial \mu_{W_t}} \right) d\sigma_C
\]

(37)

Applying equation (35), this was rewritten as:

\[
(a+\beta)dC_t = \beta \mu_C + 2\beta^2 \sigma_C \left[ \frac{\mu_R - (1+r)S'(x^*)}{2x^* \sigma_R^2} \right] d\sigma_C
\]

(38)

5. Examination of the Effect of Adding Individual Uncertainties

a. Market-Clearing Price Uncertainty

With \( R \) (of mean \( \mu_R \) and variance \( \sigma_R^2 \)) as the only uncertainty, then using equation (38) required considering \( C \) as deterministic (and equal to \( \mu_C \)) so that \( \sigma_C^2 \) was identically equal to zero and invariant. Equation (38) then simplified to:

\[
(a+\beta)dC_t = \beta \mu_C
\]

(39)
In this case, the solution was the same as for the complete certainty case as previously presented.  \[ C_t = \frac{\beta C}{\alpha + \beta}. \] (40)

b. Contract Cost Uncertainty Added

With \( C \) also a random variable, having mean \( \mu_C \) and variance \( \sigma_C^2 \), equation (38) applied directly, and is not restated here.

It was noted that, from (38), the requisite change in \( C_t \) to maintain optimality at the same private sector output level consisted of two distinct parts, one a result of a "pure" increment in the mean of contract costs, the other a result of a "pure" increment in the standard deviation of those costs. That the first was positive was obvious; for the second, it was only necessary to note that \( \text{[} \mu_R - (1+r)S'(x^*) \text{]} \) was the expected marginal profit (in the private sector) to assure that it was also positive.

6. Approximating the Effect of Uncertain Price Levels

The purpose of this section was to investigate the impact on \( C_t \) of changes in the price levels of the factors making up contract cost. To this end, the effect of uncertain price levels in the private sector was not explicitly examined, but rather it was assumed that any effects were compensatory, within the private sector, so that neither \( x^* \),

\[ 26 \text{Equation (40) was the definite integral:} \]
\[ C_t = \int_0^C \frac{\beta \, d\mu_C}{\alpha + \beta}; \text{ with } C \text{ known.} \]
nor the value of \([\mu_R - (1+r)S'(x^*)]\) of equation (38), would have changed.

The random variable, \(\gamma\), having mean \(\mu_\gamma\) and variance \(\sigma^2_\gamma\), was defined as the "inflation factor" (with negative values of \(\gamma\) representing "deflation") such that the contract cost became:

\[
C_\gamma = \gamma C.
\]

It was further assumed that \(\gamma\) and \(C\) were statistically independent, so that:

\[
E(C_\gamma) = E(\gamma)E(C) = \mu_\gamma \mu_C
\]

and

\[
V(C_\gamma) = V(\gamma)V(C) + [E(\gamma)]^2V(C) + [E(C)]^2V(\gamma)
\]

\[
= \sigma^2_\gamma \sigma_C^2 + \mu_\gamma^2 \sigma_C^2 + \mu_C^2 \sigma_\gamma^2.
\]

Equation (38) was then rewritten in the form of an approximation as:

\[
\Delta C_t = \frac{\beta}{\alpha + \beta} \Delta \mu_C + \frac{2\beta \sigma_C^2}{\alpha + \beta} \left\{\frac{\mu_R - (1+r)S'(x^*)}{2x^* \sigma_R^2}\right\} \Delta \sigma_C
\]

where \(\Delta\) represented "incremental change in...”\(^{27}\)

In this case, the incremental changes were the result of adding the uncertain \(\gamma\) to the problem. In particular

\[
\Delta \mu_C = \mu_\gamma \mu_C - \mu_C = \mu_C (\mu_\gamma - 1)
\]

\(^{27}\)It was recognized that such an approximation was only valid in a small neighborhood of the optimal solution values; however, it was considered to be sufficient for the purpose.
\[ \Delta \sigma_c = (\sigma_Y^2 \sigma_c^2 + \mu_Y^2 \sigma_c^2 + \mu_c^2 \sigma_Y^2)^{1/2} - \sigma_c \]

\[ = [\sigma_c^2 (\sigma_Y^2 + \mu_Y^2) + \mu_c^2 \sigma_Y^2]^{1/2} - \sigma_c . \]  

(45)

The approximation equation, (43), used in conjunction with (44) and/or (45) could then be used to approximate the change in \( C_t \) caused by uncertain price levels. The expanded form of the approximation equation was:

\[ \Delta C_t = \frac{\beta}{\alpha + \beta} \left[ \mu_c (\mu_Y - 1) \right] + \frac{2 \beta^2 \sigma_c}{\alpha + \beta} \left\{ \frac{\mu_R - (1+r)S'(x^*)}{2x^* \sigma_R^2} \right\} \]

\[ \cdot \left\{ [\sigma_c^2 (\sigma_Y^2 + \mu_Y^2) + \mu_c^2 \sigma_Y^2]^{1/2} - \sigma_c \right\} . \]

(46)

7. The Alternative of the "Economic Price Adjustment (EPA)" Clause

It was noted that \( C_t \), as developed, consisted of both a charge, to the government, for reimbursement of expected target cost and an additional charge, the "contingency price," due to the uncertainty of that contract cost. Although in practice the government has been willing to absorb such contingency costs in return for less cost uncertainty accruing to itself, it has indicated an unwillingness to do so with regard to the increase in contingency costs resulting from price level uncertainty (specifically, uncertain inflation).

One alternative to paying such increases in contingency prices is the "economic price adjustment (EPA)" clause in government procurement contracts. In concept, such a clause protects the contractor from inflation by directly
reimbursing him for increased costs due purely to inflationary trends within the economy, while maintaining cost reduction incentives on the contractor.

No "typical" EPA clause, such as those authorized by the Armed Services Procurement Regulations, was applied to the negotiations model developed in this study; rather, it was assumed that the EPA clause available for application by the government would exactly compensate the firm for any cost increases due purely to inflation. It was also assumed that the employment of the EPA clause would result in a known administrative cost, $P$, which accrued solely to the government.

Under the above assumptions, and the previous development of the model, the firm was indifferent between:

(i) taking the risk inherent to the price level uncertainty and requiring the compensatory change in $C_t$ (as approximated by equation (46)), and

(ii) foregoing any such compensatory change in $C_t$ and accepting the EPA clause.

The government was not necessarily indifferent, however, due to the administrative cost, $P$, and the assumption that it was an expected cost minimizer. The government's problem, with respect to the EPA clause, was

$$\min_\theta \theta P + (1-\theta)\Delta C_t$$

where

$$\theta = \begin{cases} 1 \text{ if EPA clause were adopted} \\ 0 \text{ if EPA clause were rejected} \end{cases}$$

(47)
and $\Delta C_t$ represented the firm's requisite change in the target cost due to uncertain price levels (as previously developed and approximated by equation (46)).
III. REVIEW OF THE ASSUMPTIONS WITH IMPLICATIONS FOR FURTHER STUDY

A. SCENARIO ASSUMPTIONS

1. Sole-Source Negotiated Contract

The assumption that the contract was negotiated with a single firm, rather than competitively bid, was made to enable examination of the effect on target price due purely to varying price levels, without the added uncertainty of the award of the contract.

The assumed sole-source negotiation case is the actual case for many major government procurement contracts, but most are either competitive or awarded using a combination of the two methods.\(^2^8\) In this regard, the problem of risk-aversion incentives causing a firm to bid low (in comparison to risk-indifferent behavior) on a competitive contract has been addressed by Baron [Ref. 15], but in a form that did not appear readily applicable to determination of contingency price changes.

2. Firm Competitive in the Private Sector

This assumption initially appeared to be a virtual necessity to develop the approximation for contract price. It was decided, however, that if a suitable proxy (e.g., "expected profits from all investments other than the contract") were defined, then the assumption as used could be discarded.

\(^2^8\) Belden, D. L. and Cammack, E. G., [18; p. 102-114].
It was believed that the principal value of the "competitive" assumption would lie in its application in an investigation of the firm's incentives to maintain control over inflationary trends within its own organization after receiving a contract with an EPA clause. On an intuitive level, if the firm desired to remain competitive in the private sector, and if each individual factor of production were perfectly substitutable between the private and public sector operations of the firm, then the above incentives would not be affected by the firm's taking a public-sector contract with an EPA clause.

3. All Costs of the Contract Paid at Completion of the Contract

As mentioned, this assumption was necessary due to the static nature of the model. An alternative approach would be to formulate a dynamic model utilizing sequential-decision theory, including (among others) considerations of possible early termination, work delays and opportunity costs associated with the contract. Such a model would almost certainly be "more realistic", but would also be far more complex.

B. ASSUMPTIONS REGARDING THE BEHAVIOR OF THE FIRM AND THE GOVERNMENT

1. The Risk-Averse Nature of the Firm

The joint assumptions that the firm maximized expected utility and that it was risk averse are amply justified in the literature, as is the additional assumption of decreasing
absolute risk aversion. There did not seem to be any reason for (or positive value in) deviating from these assumptions.

The definition of decreasing absolute risk aversion implied that $\frac{\partial I}{\partial \mu_W}$ was negative, and considerations of boundedness of the utility function led to implications with regard to the signs of the second partials (specifically, that $\frac{\partial^2 I}{\partial \mu_W^2} > 0$ and $\frac{\partial^2 I}{\partial V_W^2} < 0$) which were not actually used; however, it was not found possible (within the time constraint of the study) to interpret, analytically, the concept of relative risk aversion in terms of the risk aversion increment. It was believed that doing so would have permitted a qualitative interpretation of the term,

$$\left[ \frac{\partial I}{\partial \mu_W} / (1 - \frac{\partial I}{\partial \mu_W}) \right]$$

and thus obviate the need for qualifying the objectives of the government.

2. The Risk-Indifferent Nature of the Government

The assumption that the government was an expected cost minimizer was equivalent to the assumption of risk indifference. This particular assumption, although possibly reflecting the avowed government objectives with respect to any given contract, could well be challenged. In this respect, it appeared that a goal-programming approach to the government's behavior could prove interesting.

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29 These assumptions are well discussed in Arrow [Ref. 19], Sandmo [Ref. 14], and Baron [Ref. 15], among others.
The definition of the "second objective" of the government had the same effect as the assumption that the term 
\[ \left( \frac{\partial I}{\partial V_{W_t}} / (1 - \frac{\partial I}{\partial \mu_{W_t}}) \right) \]
was constant would have had. For this reason, it is a "strong" assumption, and perhaps the most fragile part of the analysis. As mentioned, further study of relative risk aversion in terms of the risk aversion increment could result in discarding this assumption.

C. THE ECONOMIC PRICE ADJUSTMENT CLAUSE

The EPA clause was assumed to fully compensate for varying price levels since it was not possible, in this initial formulation, to directly address the problem of maintenance of control over inflationary trends, which would be required otherwise.

The assumption that the EPA clause had an associated administration cost accruing solely to the government was considered realistic since, with the clause "fully compensatory" the government would need to closely monitor the firm's costs. Any increase in costs to the firm (e.g.; restructuring its accounting system) were implicitly assumed to be directly passed on to the government as a certain increase in contract costs would have been, except that, in this case, the certain increase only occurred if the EPA clause were imposed and thus could be included in the administrative cost of the clause, \( P \).
IV. RESULTS AND CONCLUSIONS

A. RESULTS OF THE NEGOTIATIONS MODEL ANALYSIS

1. The Incremental Target Cost Approximation

The incremental target cost approximation, equation (43), was noted to be valid only in a small neighborhood about the solution, but was considered sufficient for the purpose; this was due to using the approximation only to define a "break-even" relationship between the dollar increase in price, due to inflation, and the cost of administering an EPA clause, which was implicitly assumed "small" compared to the total contract cost.

2. Application of the EPA Clause

The analysis led to the conclusion that, under the assumptions made, the EPA clause should be applied when the "inflation contingency", as evaluated by the approximation developed, exceeds the administrative cost of that clause.

3. Effect on Inflation and Cost-Reduction Incentives

The formulation of the model did not permit a direct analysis of the concurrent problem regarding the effect of using the EPA clause, as structured, on incentives to maintain control over inflationary trends within the firm. The analysis did not, however, disclose any effect of the EPA clause (or lack of it) on the cost-reduction incentives as represented by the incentive profit rate, $\beta$, leading to the conclusion that, if cost-reduction incentives were effective
without the EPA clause, they would be no less effective with
the clause. 30

B. THE FRAMEWORK OF THE ANALYSIS

The particular contingency price increment approximation
developed was concluded to be too restricted, by the as­
sumption used, to be directly applicable to actual contract
negotiations. Any such restrictions were not considered
binding on the framework of the analysis, however; it was
concluded that further study within the same framework would
allow many of the restrictive assumptions to be relaxed, so
that the results could be of direct use in procurement
contracting.

Finally, the methodology developed in this study appeared
to have one major advantage over others encountered in the
literature; it did not require postulating and parameterizing
a specific utility function (for the firm) as a prerequisite
to its use in quantifying the behavior of the firm under un­
certainty conditions.

30 For a specific analysis of incentive sharing ratios
(profit incentive rates), see Berholdt [Ref. 17].
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