A USER'S MANUAL FOR THE REPSIL CODE

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Prepared for:
Army Materiel Command

October 1974

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A USER'S MANUAL FOR THE REP'IL CODE

REPSIL is a finite difference computer program which calculates the large deflection, transient motion of thin Kirchhoff shells. The program marches out the solution by cyclically solving an explicit, centered time difference formula for displacements. The formulation handles elastic-plastic behavior, including strain hardening and strain rate effects, and permits the modeling of a variety of initial shell configurations and impulsive/pressure loadings.
Item 20 (Continued) ABSTRACT

The manual gives instructions for correctly setting up problems and estimating machine time and storage requirements. Two illustrative problems are set up and the resulting solutions given. The numerical algorithm employed by REPSIL is outlined and instructions for programming additional initial geometries and loadings are given. The REPSIL plotting program, which produces isometric and cross-sectional displays and time histories of energies, deflections and strains, is also described. Listings of both the REPSIL program and the REPSIL plotting program are included.
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1. INTRODUCTION

REPSIL* is a FORTRAN IV computer program developed at the BRL to treat the large transient deformations of anelastic shells under blast loadings. The program uses the finite difference technique to solve the equations governing the motion of thin Kirchhoff shells of negligible rotational inertia. These equations are derived in a recent BRL report [1]**. That report, which also treats more general Kirchhoff shells, constitutes the theoretical documentation for REPSIL. The present report is a companion user's manual.

The equations on which REPSIL is based impose certain restrictions on the types of shells and deformations that can be treated. As already mentioned, only thin Kirchhoff shells of negligible rotational inertia can be treated. Moreover, as presently formulated, the program can only handle shells of uniform thickness having no cutouts, stiffeners, or bifurcations. On the other hand, the formulation does treat the geometric nonlinearities due to large displacements exactly.

Within the limits of the above geometric restrictions, REPSIL can accept a wide variety of initial shell geometries. This is made possible by the finite difference formulation being coded independent of any particular initial geometry. Hence, accommodating any initial geometry simply entails programming it into the initial geometry subroutine according to the instructions contained in Chapter 6. The current version of REPSIL comes with initial geometry subroutines for the rectangular plate, the cylindrical shell and the conical shell; Section 2.3 contains instructions for their use.

The program also can accept arbitrary distributions over the shell surface of initial impulse velocities and time varying pressures. The minor programming necessary to specify such distributions is described in Chapter 6. Presently, initial velocities at mesh points and over rectangular regions of the shell surface can be specified on input cards without need of programming, as detailed in Section 3.2.

REPSIL can simulate a number of boundary conditions along the four edges bounding the shell. Three edges can have any combination of clamped or hinged boundary conditions. The remaining edge and two of the previous edges can be edges constrained to move in symmetry planes of the problem. The boundary conditions are described in detail in Section 3.2, where the procedure for selecting them is given.

* Response of Elastic-Plastic Shells to Impulsive Loadings

** Numbers in brackets correspond to the list of references on page 126.
REPSIL can model a variety of anelastic material responses, although the present version is limited to isotropic materials. Within that limitation, the response can be either completely elastic or elastoplastic. In the elastic range the behavior is linear. The plastic response can be perfect or strain hardening and in either case it can be made strain rate dependent. Section 3.2 gives the procedure for eliciting these material properties.

A useful feature of the code is the automatic determination of an optimum, stable time increment. REPSIL uses an explicit finite difference approximation to the equations of motion; as is well known, such explicit schemes are subject to numerical instability unless the size of the time increment is limited to some maximum increment. The REPSIL program initially computes such a maximum increment based on criteria given in Section 2.2 and then uses this increment or the increment specified by the initial data (see Section 3.2), whichever is smaller, for the (constant) time increment used to generate the solution. Hence, a stable solution is guaranteed.

The program is provided with a damping option that permits the rapid attainment of final deformed configurations by numerically damping out the motion of the shell. Damping is mainly achieved through the artifice of automatically annihilating the kinetic energy whenever it reaches a local maximum. Details on this procedure are given in Section 2.4, while instructions for executing the option are in Section 3.2.

REPSIL comes equipped with a number of print options for outputting results of the calculations at regular intervals. Printed out are such local measures of the deformation as the displacements and surface strains, and such global measures as the kinetic energy, strain energy and plastic work. There is also a companion plotting program that at regular intervals draws isometric and cross-sectional views of the deforming shell and that graphs time histories of the printed measures of deformations just mentioned. The output options are described in Chapter 4 and illustrated by example problems in Chapter 5. Appendix D contains a description and a listing of the plotting program.

This report is organized as follows: Chapter 2 presents the assumptions and equations of the REPSIL formulation and outlines the computational algorithm used to solve the equations; Chapter 3 describes the input data, shows how to initiate and continue a problem and select the various options, and gives rules-of-thumb for estimating the memory and machine time requirements for a problem; Chapter 4 describes the various output data generated by REPSIL and the formats in which they are printed; Chapter 5 gives two example problems, describing how they are set up and the kind of output data generated; Chapter 6 gives instructions for programming arbitrary loadings and initial geometries. Chapter 3, 4 and 5 contain the necessary information to run the program as listed in Appendix E, with the associated loading and initial geometry subroutines. A user desiring to model other loadings or initial geometries...
should read Chapter 6 for programming instructions. Such a user would find a reading of Chapter 2 useful as background. On the other hand, the analyst or programmer who intends to make extensive changes in the formulation or numerics of the program would do well to thoroughly study Chapter 2 before proceeding to the study of Appendix E with the aid of Appendix C, the list of program variables.

2. DESCRIPTION OF PROGRAM

2.1 Synopsis of Program

REPSIL treats the transient deformation of shells by approximating the initial value problem with an explicit nonlinear finite difference scheme. The finite difference equations are solved at each time increment to update the deformation variables. This procedure is repeated cyclically in order to generate the time history of the deformation. This chapter presents the equations of the REPSIL formulation and describes the computational algorithm by which REPSIL solves them.

The formulation is based on equations that use a material or Lagrangian description: the dependent variables of the theory are functions of \( \eta^a \), the material coordinates of the middle surface of the shell; moreover variables defined outside the middle surface also depend upon \( \zeta \), the normal distance from the middle surface. All variables are functions of \( t \), the time.

Some comments on the physical significance of the material coordinates of the middle surface are in order. For simple initial geometries, the middle surface material coordinates often have significance as distances, arc lengths, or angles; however, this need not be the case, not even for simple geometries. It is better to regard the material coordinates abstractly as a pair of parameters defining the position of the middle surface in space. In other words, the material coordinates serve as parameters for the parametric representation of the middle surface in space. That the parameters \( \eta^a \) are also material coordinates simply means that as the image of the middle surface changes in space as a function of time, the pair of values \( \eta^a \) associated with a given material particle on the middle surface does not change, but remains fixed.

* Index notation is used, with Greek indices ranging over the integers 1 and 2; hence, \( \eta^a = \eta^1, \eta^2 \)

** The concept of material coordinates for a shell is defined more precisely in [1].

*** For example, see the cases of the flat plate, cylindrical shell and conical shell in Section 3.2.
The finite difference scheme is obtained by making the dependent variables of the theory discrete functions of \( n^1, n^2, \zeta, t \). Discretizing the dependence on \( \eta^a \) makes the dependent variables functions of the intersection points of the two dimensional rectangular mesh resulting from the division of the domain of \( \eta^a \) into increments (see Figure 3.1 and 6.1); more precisely, the variables become functions of the mesh number pair \((m,n)\), the integer pair corresponding to the mesh intersection whose coordinates \( \eta^a \) satisfy the relations

\[
n^1 = m \Delta n^1 + \eta_0^1, \quad n^2 = n \Delta n^2 + \eta_0^2, \tag{2.1}
\]

with \( \Delta n^a \) constant increments of the coordinates and \( \eta_0^a \) the coordinates of a conveniently chosen origin.

The dependence on \( \zeta \) is made discrete by conceptually dividing the shell into \( K \) layers of equal thickness. Within each layer those variables defined outside the middle surface are assumed constant with respect to \( \zeta \). Numbering each layer in sequence, \( k = 1, \ldots, K \), the variables become functions of the layer number \( k \), as well as functions of \((m,n)\). When needed, the value of \( \zeta \) in each layer is taken to be its mid-layer value:

\[
\zeta = \left( k - \frac{K+1}{2} \right) \frac{h}{K} ; \quad k = 1, \ldots, K, \tag{2.2}
\]

where \( h \) is the thickness of the shell.

The principal reason for making the dependence on \( \zeta \) discrete is in order to model the variations of the stresses through the thickness. For, while the functional form of the dependence of the strains on \( \zeta \) is prescribed by the Kirchhoff hypothesis *, no such simplification is possible for the stresses. Plasticity makes the \textit{a priori} determination of the dependence of the stresses on \( \zeta \) not feasible.

Strain hardening behavior is modelled using the "mechanical sublayer model" **: the stress tensor at a point is assumed to be a weighted sum of \( J \) "sublayer" stress tensors or, more simply, \( J \) "substress" tensors, with each sublayer obeying the same linear incremental stress-strain relation, but having different yield stresses. Using this model, there will be \( K \) stress tensor values at each mesh point \((m,n)\) and \( J \times K \) substress tensor values.

---

* Cf equation (2.15) of this report.

** This method of modelling strain hardening behavior has been developed and extensively used at the Aeroelastic and Structure Research Lab. of MIT. A detailed description of the method is given in [2; Sect. 5.4.2].
The dependence on $t$, the time, is made discrete by replacing time derivatives by equivalent finite difference operators. As already mentioned, an explicit finite difference scheme results. The scheme characterizes the change undergone by the dependent variables during a time interval $\Delta t$. A constant time interval is used, which is automatically determined by the program to assure numerical stability.

The basic function of the finite difference scheme is to advance the values of the following fundamental variables.

- $y^i(m,n)$, the Cartesian coordinates of the middle surface at mesh point $(m,n)$;
- $n^i(m,n)$, the Cartesian components of the unit normal to the middle surface at mesh point $(m,n)$;
- $\Delta u^i(m,n)$, the Cartesian components of the displacement increment undergone by the middle surface at mesh point $(m,n)$ during the time interval $\Delta t$;
- $\sigma^{ij}(m,n,k)$, the contravariant components of the stress tensor (or, in the case of strain hardening, the substress tensor) tangential to the middle surface at mesh point $(m,n)$ at layer (or sublayer) station $k$.

The dependence of the variables on $(m,n)$ or $(m,n,k)$ is indicated explicitly to emphasize that their values are stored as 2- and 3-dimensional arrays in the program. From these fundamental variables all the other dependent variables of the theory are determined.

For the sake of clarity, the description of the computational algorithm used by REPSIL is separated into four groups of calculations:

- Initialization calculations,
- Finite difference calculations,
- Energy calculations,
- Surface strains calculations.

Within each group the description is organized by units of calculations. Units can occur within the main program or as subroutines. In the latter case, the subroutine description, which follows the subroutine name, is indented. When a subroutine occurs within a subroutine, the description

*Latin indices indicate Cartesian components in Euclidean 3-space and range over the integers 1, 2, and 3.
of the embedded subroutine is further indented. All subroutine names are completely capitalized.

2.2 Initialization Calculations

The initialization calculations determine the program constants and the initial values of the dependent variables from the input data. The flow chart in Figure 2.1 summarizes the calculations, whose description follows.

START Figure 2.2 outlines this subroutine. The input data, including material properties, is read into storage and program constants are calculated. The minimum and maximum values of the mesh numbers are determined from the boundary conditions. Subroutine INGEOM is called within this subroutine.

INGEOM This subroutine generates the initial geometry of the shell. First, the dimensions characterizing the particular geometry being treated are read off the input cards. Next, the finite difference increments \( \Delta \eta \) are calculated. Finally, the subroutine computes the initial Cartesian coordinates \( y^i(m,n) \) of the middle surface and stores the resulting arrays. The user not finding a suitable geometry among those described in Section 3.2 (flat plate, cylinder or cone) is referred to Section 6.4 for instructions on programming initial geometries.

Returning to START, the program calculates the interpolation coefficients at locations where STRAIN * computes the surface strains and the mesh number of the mesh point bracketing these locations. Next, a stable time interval \( \Delta t \) is found from the criteria

\[
\Delta t_M = \frac{2}{c} \left[ \frac{1}{(\eta n)^2} + \frac{1}{(\eta^2)^2} \right]^{-\frac{1}{2}}, \quad c = \sqrt{\frac{E}{\rho (1-\nu^2)}} ;
\]

\[
\Delta t_B = \frac{1}{2hc} \left[ \frac{1}{12} \left( 1 - \frac{1}{k^2} \right) \left( \frac{1}{(\eta n)^4} + \frac{1}{(\eta^2)^4} + \frac{\nu + \nu}{8 (\eta^2 \eta n^2)^2} \right) \right]^{-\frac{1}{2}} .
\]

These criteria result from a von Neumann stability analysis of an elastic flat plate **: \( \Delta t_M \) is the maximum stable interval for membrane motion and \( \Delta t_B \) for bending motion. The program

---

* Subroutine STRAIN is described in Section 2.5.

** A report describing this analysis is being prepared.
Figure 2.1 Flow Chart for Initialization Calculations
Read input data

Calculate program constants

Determine boundary mesh numbers

INGEOM

Evaluate interpolation coefficients for STRAIN

Determine stable $\Delta t$

Calculate program constants

Return

Figure 2.2 Flow Chart for Subroutine START
selects for use in the finite difference calculations the
minimum of $\Delta t_M$, $\Delta t_B$ and the $\Delta t$ prescribed in the input data.

The subroutine ends with the calculation of the remaining
program constants, most of which require for their determination
the values of $\Delta n^B$ computed in INGEOM.

The main program next determines whether the problem being solved
is a new one, starting from an initial (stress-free) configuration at
time $t_0 = 0$ or a continuation of a problem whose solution up to some
time $t_0 > 0$ has already been calculated, called a restart problem or,
simply, a restart. If it is a restart, subroutine WRTAPE is called in
order to read data off the restart tape, after which subroutine PDATA
is called and the initialization calculations end (see Figure 2.1).

WRTAPE Depending on the instructions of the main program, this sub-
routine either reads off or writes on a tape, the restart
tape, the values of the fundamental variables $y^i$, $n^i$, $\Delta u^i$ and
$\sigma^B$, the surface strains and the energy variables at time step
prescribed by the initial data. This information is sufficient
to permit the program to continue the solution of the problem
from any of the prescribed time steps. During the initialization
portion of the program this subroutine reads this information
off the tape and during the finite difference portion it stores
the information.

PDATA This subroutine stores on a tape, the plotting tape, the data
required by the REPSIL plotting program. A description of these
data are given in Section 4.2. Appendix D describes and lists
the plotting program.

If, on the other hand, the program is trying a new problem, WRTAPE is
not called at all and PDATA is not called until later (see Figure 2.1).
Rather, for a new problem the program reads in the information
otherwise obtained by WRTAPE.

As discussed in Section 3.2, the program treats symmetric deforma-
tions about one or two planes. For such deformations, the shell
variables, in particular the middle surface coordinates $y^i$, are
symmetrically distributed about symmetry planes. The program imposes
this condition numerically by relating the coordinates one mesh spacing
outside the symmetry edge to the coordinates one mesh spacing inside
the edge. Typically, for the symmetry plane located in the $y^2$, $y^3$
coordinate plane and intersecting the middle surface along the curve
with mesh number $m$ (see Figures 3.6 - 3.8 where $m = 2$) these relations
are
\[ y^1(m-1,n) = -y^1(m+1,n) \]
\[ y^2(m-1,n) = y^2(m+1,n) \]
\[ y^3(m-1,n) = y^3(m+1,n) \]
\[ y^1(m,n) = 0, \quad (2.4) \]

for all allowable \( n \).

The initial time \( t_o \), kinetic energy \( T \), strain energy \( V \), damping work \( W \) and external work \( W \) are next set equal to zero. This is followed by the arrays for the displacement increments \( \Delta u^i \), the pressure \( P \), the tangential strain components \( \varepsilon_{\alpha\beta} \) and the tangential stress components all being cleared to zero. If pressure loads are acting on the shell, the initial pressure distribution is obtained from subroutine PRESS, whose description is postponed until Section 2.3. If no pressure loads are acting, the program skips PRESS and goes directly to DGEOM. DGEOM is described in Section 2.3, where it is shown that at this point in the program it is called in order to calculate the normal \( \hat{n}^i \), the augmented pressure \( P^* \), the time constant \( \Delta t^2/(a_o^2 \Gamma_o) \) and the initial values of the tensor \( a_{\alpha\beta} \) and \( b_{\alpha\beta} \). Next the program goes to STRAIN where the values of \( a_{\alpha\xi} \) and \( b_{\alpha\beta} \) are interpolated to give the metric tensor \( G_{\alpha\beta} \) on the bounding surfaces of the shell; the details of this calculation are given in Section 2.5.

The remainder of the initialization calculations are concerned with obtaining the initial values of the displacement increment arrays \( \Delta u^i(m,n) \) and the associated values of the kinetic energy \( T \) and external work \( W \). The initial displacement increments can arise in three ways: (1) from an initial impulse velocity distribution; (2) from an initial pressure distribution or (3) from a combination of both. Should they be due to an initial velocity distribution or combination, the program calls INVEL.

INVEL. This subroutine determines the magnitude of the initial velocity \( v \) at each mesh point and multiplies this by the normal \( n^i(m,n) \) to give the initial velocity distribution:

\[ v^i(m,n) = v \ n^i(m,n). \quad (2.5) \]

The initial velocity magnitudes can either be read off input data cards, as shown in Section 3.2, or the user can program an analytical expression for these magnitudes, as shown in Section 6.2.
The program then calculates the displacement increments due to the initial velocity distribution

\[ \Delta u^i(m,n) = v^i(m,n) \Delta t \quad (2.6) \]

and proceeds to subroutine BOUNDU. This subroutine, described in Section 2.3, adjusts the displacement increments one mesh space in from clamped edge boundaries so that clamped edge conditions are satisfied and also generates the displacement increment one mesh spacing outside symmetry boundaries from the values one mesh spacing inside, using equations very much like (2.3) for \( y^i \). Next the kinetic energy \( T \) associated with the initial displacement increment distribution is determined in subroutine KINET, described in Section 2.4. If there is no initial pressure distribution, the displacement increments \( \Delta u^i(m,n) \) and kinetic energy just determined are associated with the time interval from \( t_o = 0 \) to \( t_1 = \Delta t \) and the initialization calculations terminate with the calling of subroutine PDATA. On the other hand, if the shell is subjected to an initial pressure distribution, the displacement increments \( \Delta u^i(m,n) \) and kinetic energy \( T \) just determined are associated with the time interval from \( t_{-1} = -\Delta t \) to \( t_o = 0 * \) and the external work due to the pressure distribution acting on the displacement increments just determined is computed in subroutine PWORK, described in Section 2.4. The program then proceeds to calculate the displacement increments for the time interval \([t_o, t_1]\) using an equation which is nothing more than the equation of motion (2.35) with the stresses set equal to zero:

\[ \Delta u^i_+ = \Delta u^i - \frac{\Delta t^2}{r_o^2} \frac{p^i}{r_o} \quad (2.7) \]

with \( \Delta u^i_+ \) and \( \Delta u^i \) the displacement increments for the time intervals \([t_o, t_1]\) and \([t_{-1}, t_o]\), respectively. When there is no initial impulse velocity distribution, but only a pressure distribution the program goes from STRAIN to directly computing the displacement increments for the interval \([t_o, t_1]\) using (2.7), with, however, the displacement

* The reason that the displacement increments were previously associated with the time interval \([t_o, t_1]\) rather than the interval \([t_{-1}, t_o]\) is that with no pressure acting and the shell being stress free at the time \( t_o \), the displacement increments are the same for both intervals. On the other hand, a pressure distribution at time \( t_o \) will cause the displacement increments for \([t_o, t_1]\) to change from those for \([t_{-1}, t_o]\).
increments for \([t_{-1}, t_0]\) set equal to zero.

Again, the new \(\Delta u^i\) are adjusted in BOUNDU to satisfy clamped edge and symmetry edge conditions and the additional external work resulting for the initial pressure distribution acting on the displacement increments for the interval \([t_0, t_1]\) is computed in PWORK. Lastly, the kinetic energy during the time interval \([t_0, t_1]\) is calculated in KINET and the program proceed to PDATA where information is gathered for the plotting program. With this last operation the initialization calculation end and the program proceeds to the finite difference calculational loop.

### 2.3 Finite Difference Calculations

The finite difference calculations follow the initialization calculations and are repeated each time step. They solve the finite difference equations for the current values of the fundamental variables. In this way, the values of these variables are advanced each time step and the history of the deformation is generated.

Aside from minor simplifications in notation, the finite difference equations presented in this section are identical with those given in [1; Sect. 7.3]. They are written in compact form with only finite time derivatives being shown explicitly; finite differences with respect to material coordinates \(\eta^\alpha\) are symbolically represented by their corresponding partials. At interior points and along symmetry boundaries the program uses central difference operators, while along hinged and clamped boundaries it uses forward or backward difference operator, all operators being of order \(|\Delta \eta^\alpha|^2\) accuracy.

Figure 2.3 outlines the finite difference calculations. They comprise a sequence or, better still, a loop of calculations, which, as already mentioned, are repeated every time step. The description of this loop begins after \(k-1\) time steps have elapsed, at the generic time \(t_\ast = (k-1)\Delta t\). The values of the middle surface coordinates, the normal to the middle surface and the tangential stress or substress component are assumed known at this time and are denoted by appending subscripted minus signs: \(\gamma^i_, n^i_, \text{ and } \sigma^\alpha^\beta_\). Also the components of the displacement increment for the next time interval, from the time \(t_\ast = (k-1)\Delta t\) to the time \(t = k\Delta t\), are known and are denoted simply as \(\Delta u^i\). Using these values of the fundamental variables, the finite differences calculations generate succeeding values of the fundamental

* Appendix A summarizes the explicit forms of these operators.
Figure 2.3 Flow Chart for Finite Difference, Energy and Surface Strain Calculations
variable $y_i$, $n_i$, $\sigma_{\alpha\beta}$ and $\Delta u_i$ at the time $t = \Delta t$ as follows.

First the time is updated:

$$t = t + \Delta t,$$

(2.8)

and checked against a prescribed maximum, $t_{\text{max}}$. If the maximum is exceeded, then the solution has progresses to completion and calculations stop. Otherwise, the program checks as to whether there is a pressure loading acting on the shell at this time; if there is then the program calls subroutines PRESS, POSITN, DGEOM, STRAIN, MOTION, PDATA and DAMP in that order; if not, it skips PRESS and calls the remainder of the sequence.

PRESS This subroutine supplies the values of pressure at mesh intersection points for the given time and stores them in the array $P(m,n)$. The user is expected to supply this information either by programming some analytical expression for the pressure, as outlined in Section 6.3, or by generating a tape with this data in numerical form.

POSITN This subroutine simply calculates the coordinates of the middle surface at the current time $t$ using the formula

$$y_i = y_{i-} + \Delta u_i,$$

(2.9)

storing the values in the arrays $y_i(m,n)$.

DGEOM This subroutine is summarized in Figure 2.4. It begins by calling subroutine GRAD.

GRAD This subroutine, using appropriate finite difference operators, determines the first and second gradients of $y_i$ and $\Delta u_i$ with respect to the material coordinates $\eta^\alpha$:

$$y_i^\alpha = \frac{\partial y_i}{\partial \eta^\alpha}, \quad y_i^{\alpha\beta} = \frac{\partial^2 y_i}{\partial \eta^\alpha \partial \eta^\beta},$$

(2.10)

$$u_i^\alpha = \frac{\partial \Delta u_i}{\partial \eta^\alpha}, \quad u_i^{\alpha\beta} = \frac{\partial^2 \Delta u_i}{\partial \eta^\alpha \partial \eta^\beta}$$

After GRAD the program stores the preceding values of the components of the normal $n_i$ for later use in calculating strain increments. From the first and second gradient of $y_i$, the program determines the differential geometric quantities characterizing...
Figure 2.4 Flow Chart for Subroutine DGEOM
the current position of the middle surface: the covariant components of the metric $a_{\alpha \beta}$, the second fundamental tensor $b_{\alpha \beta}$

and the current normal $n^i$, and also the determinant $a$ of the metric as follows.

$$a_{\alpha \beta} = y^i_\alpha y^i_\beta, \quad a = a_{11} a_{22} - (a_{12})^2,$$  \hspace{1cm} (2.11)

$$n^i = e^{ijk} y^j_1 y^k_2 / a^i, \quad b_{\alpha \beta} = n^i y^i_{\alpha \beta},$$

where $e^{ijk}$ is the permutation symbol.** Next, the pressure distribution is modified for use in the equation of motion:

$$p^* = a^i p^i.$$ \hspace{1cm} (2.12)

The remainder of the subroutine's calculations depend on the time. Since initially the shell is assumed to be in a stress-free state, there is no need to calculate the stress field in DGEOM. Rather, the subroutine determines certain time constants and stores these for later use: $\Delta t^2 / (a_0^2 \Gamma_0)$ is calculated for use in the equations of motion and the expression for the kinetic energy and the initial values of $a_{\alpha \beta}$ and $b_{\alpha \beta}$ at prescribed mesh points are selected using subroutine ABINIT for use in STRAIN, as described in Section 2.5.

At all subsequent times, the subroutine calculates the stress field and other stress related quantities as follows. First, the contravariant components $a^i_{\alpha}$ of the metric, the Christoffel symbols $\Gamma^\gamma_{\alpha \beta}$, and the mixed components $b^\alpha_{\beta}$ of the second fundamental tensor are calculated:

$$a_{11} = a_{22} / a, \quad a_{12} = a_{22} / a, \quad a_{22} = a_{11} / a,$$ \hspace{1cm} (2.13)

$$\Gamma^\gamma_{\alpha \beta} = a^\gamma_i y^i_\delta y^i_{\alpha \beta}, \quad b^\alpha_{\beta} = a^\alpha_\delta b^\delta_{\beta}$$

* We introduce the summation convention: terms or products of terms having the same index appearing twice are to be summed over the range of the index. In the case of repeated Latin indices both will be superscript since their basis is Cartesian, while repeated Greek indices will always appear as paired superscript and subscript.

** That is, $e^{ijk} = 1$ for $i, j, k$ an even permutation of $1, 2, 3$; $-1$ for $i, j, k$ an odd permutation; $0$ for $i, j, k$ non-distinct.
Next the incremental changes in $a_{\alpha\beta}$ and $b_{\alpha\beta}$ due to the incremental displacement $\Delta u^i$ are determined:

$$\Delta a_{\alpha\beta} = \gamma_{\alpha\beta} u^i u^i + \gamma_{\alpha\beta} u^i u^i + \gamma_{\alpha\beta} u^i u^i. \quad (2.14)$$

These expressions are exact with no approximations based on the smallness of $\Delta u^i$ being used. The corresponding increments undergone by the tangential strain components $\varepsilon_{\alpha\beta}$ at the station $z(k)$ distant from the middle surface are then calculated using the equation

$$\Delta \varepsilon_{\alpha\beta} = \frac{1}{2} \Delta a_{\alpha\beta} - \frac{1}{2} \Delta b_{\alpha\beta} \quad (2.15)$$

This equation is based on the thin shell approximation *. Subroutine STRESS follows.

STRESS The principal function of this subroutine, outlined in Figure 2.5, is to calculate the current stress or substress component from their preceding values and the incremental change in the strain component. Calculations begin by evaluating the metric $g_{\alpha\beta}$ and inverse $g^{\alpha\beta}$ for the lamella distance from the middle surface:

$$g_{\alpha\beta} = a_{\alpha\beta} - 2 \xi b_{\alpha\beta}, \quad g = g_{11} g_{22} - (g_{12})^2,$$

$$g_{11} = g_{22}/g, \quad g_{12} = -g_{12}/g, \quad g_{22} = g_{11}/g. \quad (2.16)$$

With these terms evaluated, the mixed components of the incremental strain and preceding stress are immediately obtained:

$$\Delta \varepsilon^\alpha_{\beta} = g^{\alpha\delta} \Delta \varepsilon_{\delta\gamma} \quad (2.17)$$

Next, assuming that the incremental deformation is elastic, the increments in the stress components are calculated using the linear isotropic law:

*Cf [1; Sect. 7.1].
Figure 2.5 Flow Chart for Subroutine STRESS
\[
\Delta \sigma_{\beta}^\alpha = \frac{E}{1+\nu} (\Delta \varepsilon_{\beta}^\alpha + \frac{\nu}{1+\nu} \delta_{\beta}^\alpha \Delta \varepsilon_{\gamma}^\gamma). \tag{2.18}
\]

In the case of strain hardening, \(\Delta \sigma_{\beta}^\alpha\) are elastic increments in the substress components and are the same for each layer. These increments are added to the preceding values to give the components of a trial stress or substress

\[
\sigma_{\beta}^\alpha = \sigma_{\beta}^\alpha - \Delta \sigma_{\beta}^\alpha. \tag{2.19}
\]

The trial stress is tested against the yield function

\[
\phi(\sigma_{\beta}^\alpha) = \frac{3}{2} \sigma_{\beta0}^2 - \frac{1}{2} (\sigma_{\alpha}^\alpha)^2 - \sigma_0^2, \tag{2.20}
\]

where \(\sigma_0\) is the uniaxial yield stress or, in the case of strain hardening, the yield substress; for rate-sensitive behavior \(\sigma_0\) is assumed to depend on the second invariant of the strain rate deviator:

\[
\sigma_0 = \sigma_0(\text{static}) \left[1 + \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^{1/\eta}\right], \tag{2.21}
\]

with

\[
\dot{\varepsilon} = \frac{1}{\dot{\varepsilon}_0} \left[\frac{3}{2} \Delta \varepsilon_{\beta}^\alpha \Delta \varepsilon_{\alpha}^\beta - \frac{1}{2} (\Delta \varepsilon_{\gamma}^\gamma)^2\right]^{1/4}. \tag{2.22}
\]

If \(\phi(\sigma_{\beta}^\alpha) \leq 0\), then the trial stress is on or within the yield surface and its components define acceptable values for the current stress components \(\sigma_{\beta}^\alpha = \sigma_{\beta}^\alpha\). Hence, the stress increment is indeed elastic and the calculations for a plastic stress increment are skipped, the program proceeding directly to (2.29) below.

On the other hand, if \(\phi(\sigma_{\beta}^\alpha) > 0\), then the trial stress is outside the yield surface and, hence, unacceptable. In this case, the subroutine determines a correction to the trial stress due to plastic flow. First, the components of a corrector stress are determined:

\[
\frac{\Delta \sigma_{\beta}^\alpha}{\sigma_{\beta}^\alpha} = 3 (1-\nu) \sigma_{\beta}^\alpha \dot{\varepsilon} - (1-2\nu) \sigma_{\alpha}^\alpha \delta_{\beta}^\alpha \delta_{\gamma}^\gamma \quad \sigma_{\beta}^\alpha \tag{2.23}
\]
The corrector stress gives the direction in stress space in which to apply a correction to $\sigma_B^\alpha$ in order to bring the resultant stress components

$$\sigma_B^\alpha = \sigma_B^\alpha - \Delta \lambda \sigma_\beta^\alpha$$

(2.24)

back to the yield surface, so that

$$\phi(\sigma_B^\alpha - \Delta \lambda \sigma_\beta^\alpha) = 0.$$  

(2.25)

This gives a quadratic equation in $\Delta \lambda$

$$A \Delta \lambda^2 - 2B \Delta \lambda + C = 0,$$

(2.26)

where

$$A = \frac{D_\alpha \sigma_\beta^\alpha - 1}{3} \frac{D_\alpha \sigma_\beta^\alpha}{\sigma_\beta^\alpha}$$

$$B = \frac{D_\alpha \sigma_\beta^\alpha - 1}{3} \frac{D_\alpha \sigma_\beta^\alpha}{\sigma_\beta^\alpha}$$

(2.27)

$$C = \frac{D_\alpha \sigma_\beta^\alpha - 1}{3} \frac{D_\alpha \sigma_\beta^\alpha}{\sigma_\beta^\alpha} - \frac{2}{3} \sigma_\beta^\alpha$$

which the program solves for the smallest positive value of $\Delta \lambda$:

$$\Delta \lambda = B - (B^2 - AC)^{1/2}.$$  

(2.28)

With $\Delta \lambda$ known, current stresses or substress components are determined from (2.24) and are then put into contravariant form

$$\sigma^\alpha_\beta = g^{\alpha \delta} \sigma^\delta_\beta.$$  

(2.29)

* If $\Delta \lambda$ turns out to be negative or complex, then the subroutine uses a procedure described in [3; Sect. IV], which divides the elastic increment components $\Delta \sigma_B^{E\alpha}$ into $L$ elastic subincrements and applies a correction to each step such that $\Delta \lambda$ is always positive.
In the case of strain hardening, a weighted sum of the substress components of the layer is taken to obtain the layer stress components before being put in contravariant form. The subroutine finishes by computing the contribution to the strain energy of the current stress using (2.39) of Section 2.4 and returning to DGEOM.

Next, the components of strain at prescribed locations on the bounding surfaces of the shell are evaluated using (2.55) as explained in Section 2.5. Subroutines RESULT and SYMTRY are called next.

RESULT This subroutine numerically integrates the stress components (note: not the substress components) and their moments through the thickness to give components of the membrane and bending resultants:

\[ Q^{\alpha\beta} = a^1 \sum_k a^{\alpha\beta} (1 - \zeta b^\gamma) \Delta \zeta, \]

\[ M^{\alpha\beta} = a^2 \sum_k [\sigma^{\alpha\beta} (1 - \zeta b^\gamma) - \frac{\zeta}{2} (\sigma^{\alpha\delta} b^\delta + \sigma^{\beta\delta} b^\delta)] \Delta \zeta. \]  

From these components the subroutine determines the components of the stress resultant:

\[ N^{i\alpha} = Q^{\alpha\beta} y^i_\beta + \Gamma^i_{\beta\gamma} M^{\beta\gamma} n^i. \]  

SYMTRY This subroutine imposes the symmetry edge conditions on \( n^i, M^{\alpha\beta} \) and \( N^{i\alpha} \) relating the values of these variables one mesh spacing outside the symmetry edge to their values one mesh spacing inside. Typically, for the symmetry plane located in the \( y_2, y_3 \) coordinate plane and intersecting the middle surface along the \( m \) equals a constant curve these relations are:

\[ n^1(m-1,n) = -n^1(m+1,n), \]

\[ n^2(m-1,n) = n^2(m+1,n), \]  

\[ n^3(m-1,n) = n^3(m+1,n). \]  

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\[ M^{11}(m-1,n) = M^{11}(m+1,n) \]
\[ M^{12}(m-1,n) = -M^{12}(m+1,n) \]
\[ M^{22}(m-1,n) = M^{22}(m+1,n) \]
\[ N^{11}(m-1,n) = N^{11}(m+1,n) \]
\[ N^{12}(m-1,n) = -N^{12}(m+1,n) \]
\[ N^{21}(m-1,n) = -N^{21}(m+1,n) \]
\[ N^{31}(m-1,n) = -N^{31}(m+1,n) \]
\[ N^{12}(m-1,n) = -N^{12}(m+1,n) \]
\[ N^{22}(m-1,n) = N^{22}(m+1,n) \]
\[ N^{32}(m-1,n) = N^{32}(m+1,n) \]

for all admissible \( n \). These relations as well as relations for other symmetry plane locations used in the program are derived in [1; Sect. 6.4 and App. B].

Returning to DGEOM, the program determines the strain energy by summing the contribution of each mesh point and layer as described in Section 2.5. This ends DGEOM and calculations return to the main program.

Next, the strain components at selected points on the bounding surfaces are determined in subroutine STRAIN, as described in Section 2.5, after which subroutine MOTION is called.

**MOTION** This subroutine is summarized in Figure 2.6. Its principal function is to determine the values \( \Delta u^i_+ \) of the components of the displacement increments undergone by the middle surface in the time interval \( [t, t + \Delta t] \). First a check is performed to determine whether a pressure distribution is currently acting on the shell. If a pressure distribution is acting, then subroutine PWORK, which is described in Section 2.4, is called in order to determine the contribution to the external work of the pressure acting through the displacement increments \( \Delta u^i_+ \) of the time interval \( [t - \Delta t, t] \). With no pressure loads acting, the subroutine skips PWORK and proceeds directly to determine \( \Delta u^i_+ \) from the finite difference form of the equations of motion.
Figure 2.6 Flow Chart for Subroutine MOTION
As indicated earlier, the partials appearing in the right hand side of these equations represent finite difference derivatives. These equations do not coincide with those given in [1; Eq. (7.38)] due to the addition of the term \( D \frac{\Delta u^i_+ + \Delta u^i_-}{2 \Delta t} \), representing a linear viscous damping effect. However, this damping term only appears when the damping option is used otherwise, the term is set equal to zero. Next, subroutine BOUNDU is called.

BOUNDU This subroutine generates the additional values of \( \Delta u^i_+ \) need along symmetry edges and modifies the values of \( \Delta u^i_+ \) along clamped edges so that clamped edge conditions are satisfied. As already described in subroutine SYMTRY, the values of variables one mesh spacing outside a symmetry edge are related to their values one mesh spacing in. The conditions imposed on \( \Delta u^i_+ \) are similar to those imposed on \( y^i \) in (2.4): for the symmetry plane located in the \( y^2, y^3 \) coordinate plane intersecting the middle surface along the \( m \) equals a constant curve

\[
\Delta u^i_+ (m-1,n) = -\Delta u^i_+ (m+1,n) ,
\]

\[
\Delta u^i_+ (m-1,n) = \Delta u^i_+ (m+1,n) , \quad (2.36)
\]

\[
\Delta u^i_+ (m-1,n) = \Delta u^i_+ (m+1,n) ,
\]

\[
\Delta u^i_+ (m,n) = 0 .
\]

for all admissible \( n \). Along clamped edges the components of the normal \( n^i \) must remained fixed at their initial values. This condition is achieved by adjusting \( \Delta u^i_+ \) one mesh spacing in from the clamped edge so that its component in the direction of the edge normal is 1/4 the value of the corresponding component of \( \Delta u^i_+ \) two mesh spacings in from the edge. This adjustment guarantees
that the tangent to the middle surface at a clamped edge, which is computed using a one-sided finite derivative, will always lie in a fixed plane perpendicular to the original normal. Typically, for a clamped edge fixed in the \( y^3, y^1 \) coordinate plane intersecting the middle surface along a curve with fixed mesh number \( n \), the relations used are

\[
\Delta u^i_i (m,n+1) = \Delta u^i_i (m,n+1) - \Delta u^i_n (m,n) , \tag{2.37}
\]

where

\[
\Delta u_n = n^j (m,n) [\Delta u^j^i (m,n+1) - \frac{1}{4} \Delta u^j_j (m,n+2)] \tag{2.38}
\]

and \( \Delta u^i_i \) are the unadjusted components of the displacement increments one mesh spacing in from the clamped edge as obtained from the equations of motion.

The remainder of MOTION is concerned with energy calculations, the details of which are covered in Section 2.4. Briefly, the kinetic energy due to \( \Delta u^i_i \) is calculated and, if there is a pressure loading, the portion of the external work due \( \Delta u^i_i \) is also calculated. Then the total external work is computed and finally the energy dissipated by plasticity is determined.

Returning to the main program, see Figure 2.3, the next subroutine called is PDATA, which is described in Section 2.2. Following PDATA, subroutine DAMP is called in order to compute the energy removed by the damping, as described in Section 2.4. A check is next performed to determine if information for a restart should be collected by subroutine WRTAPE at this time step. If no restart information need be gathered, then the finite difference calculational loop is complete; otherwise, WRTAPE and PDATA are called, again completing the finite difference loop. This sequence of calculation has generated new values \( y^i_i, n^i_i, \sigma^a^i_i \) and \( \Delta u^i_i \) of the fundamental variables from the old values \( y^i_i, n^i_i, \sigma^a^i_i \) and \( \Delta u^i_i \); the solution has been advanced a time step.

2.4 Energy Calculations

The energy calculations and the finite difference calculations are performed concurrently. The energy calculations use the results of the finite difference calculations to determine the current values of the kinetic energy, the strain energy, the external (pressure) work and the plastic work (i.e. the energy dissipated by plasticity). When the damping option is used, they also determine the energy removed by damping, called
the damping work. The energy calculations have no influence on the finite difference calculations and, hence, on the solution, except when the damping option is employed. This section presents the equations and procedure used in the REPSIL energy calculations; a theoretical report justifying the use of these will be forthcoming shortly.

Because the energy calculations are embedded in the finite difference calculations, the flow chart, Figure 2.3, used in describing the latter calculation still pertains and will be referred to in the discussion that follows.

The first calculation performed is for the strain energy. Immediately after the current values of the mixed components of stress \( \sigma_{i}^{0} \) are computed in STRESS, Figure 2.5, the strain energy density \( \phi \) per unit material coordinate volume at mesh point \( (m,n) \) and layer \( k \) is calculated using:

\[
\phi = \frac{1}{2E} \left[ (\sigma_{1}^{1} + \sigma_{2}^{2})^2 - 2(1+\nu) (\sigma_{1}^{1} \sigma_{2}^{2} - \sigma_{1}^{0} \sigma_{2}^{0}) \right] \Delta t (2.39)
\]

and simultaneously summed over all mesh points and layers. The strain energy calculation is completed in DGEOM, where the last sum is multiplied by the finite difference volume element giving the strain energy of the shell:

\[
V = \sum_{m,n,k} \phi \Delta \eta_{1} \Delta \eta_{2} \Delta \zeta . \tag{2.40}
\]

The kinetic energy calculation is accomplished in subroutine KINET, which is, as shown in Figure 2.6, called by subroutine MOTION.

**KINET**

The kinetic energy density \( \psi \) per unit material coordinate area of the middle surface at the mesh point \( (m,n) \) is determined:

\[
\psi = \frac{1}{2} \Gamma_0 a_o \frac{\Delta u_i^1}{\Delta t} \frac{\Delta u_i^1}{\Delta x} . \tag{2.41}
\]

Summing \( \psi \) over all mesh numbers \( (m,n) \) and multiplying by the finite difference area element, the kinetic energy of the shell is obtained:

\[
T(t + \frac{1}{2} \Delta t) = \sum_{m,n} \psi \Delta \eta_{1} \Delta \eta_{2} \Delta \zeta . \tag{2.42}
\]

Notice, since the displacement increments \( \Delta u_i^1 \) are for the time interval \([t,t+\Delta t]\), the kinetic energy is properly centered at the time \( t + 1/2 \Delta t \), as indicated. The kinetic energy at the
time \( t \) is determined by averaging the values of the kinetic energy at times \( t + \frac{1}{2} \Delta t \) and \( t - \frac{1}{2} \Delta t \):

\[
T(t) = \frac{1}{2} \left[ T(t + \frac{1}{2} \Delta t) + T(t - \frac{1}{2} \Delta t) \right] . \tag{2.43}
\]

The external work calculation is done in two steps, accomplished by \texttt{MOTION} calling subroutine \texttt{PWORK} twice, see Figure 2.6.

\texttt{PWORK} This subroutine calculates the work \( w \) per unit material coordinate area of the middle surface at the mesh point \((m, n)\) due to the pressure \( P \) acting during half the displacement increment \( \Delta u_i \) using the equation

\[
\omega = -\frac{1}{2} \Delta u_i n_i , \tag{2.44}
\]

where the negative sign is a consequence of the pressure being oppositely directed to \( n_i \). The subroutine then sums \( \omega \) over the mesh points and returns to \texttt{MOTION}.

The first time \texttt{PWORK} uses the values of the displacement increments for the time interval \([t - \Delta t, t]\), determining the contribution to the external work of the interval \([t - 1/2 \Delta t, t]\); the second time it uses the values for the increment \([t, t + \Delta t]\), determining the contributions of the interval \([t, t + 1/2 \Delta t]\). These contributions are added and the result multiplied by the finite difference area element to give the total external work during the time interval \([t - 1/2 \Delta t, t + 1/2 \Delta t]\):

\[
\Delta W(t) = \sum_{m,n} \left[ \omega(t - \frac{1}{4} \Delta t) + \omega(t + \frac{1}{4} \Delta t) \right] \Delta n^1 \Delta n^2 , \\
= -\sum_{m,n} \left[ \frac{\Delta u_i^1 + \Delta u_i^2}{2} n^i P^* \right] \Delta n^1 \Delta n^2 . \tag{2.45}
\]

This work increment, which is properly centered at time \( t \), is then averaged with the work increment at time \( t - \Delta t \) to give the work increment for the time interval \([t - \Delta t, t]\) centered at time \( t - 1/2 \Delta t \):

\[
\Delta W(t - 1/2 \Delta t) = \frac{1}{2} \left[ \Delta W(t - \Delta t) + \Delta W(t) \right] . \tag{2.46}
\]

This average work increment is added to the total external work up to the time \( t - \Delta t \) to give the external work done up to the current time \( t \):
\[ W(t) = W(t - \Delta t) + \Delta W(t - 1/2 \Delta t). \quad (2.47) \]

The reasons for doing the external work calculation in this rather elaborate way are made clear in the forthcoming theoretical report on the energy calculations.

If the damping option is not in effect, then the only means by which energy is dissipated is through plastic flow. This unavailable energy is measured by the plastic work \( W_p \), which is simply the difference of the total external work and the sum of the kinetic energy and strain energy:

\[ W_p = W - T - V. \quad (2.48) \]

The plastic work is computed in MOTION, just following the external work calculation, see Figure 2.6. With this calculation, the generation of current values of \( W, T \), and \( V \) is complete and the energy calculations end.

On the other hand if the damping option is in force, then there is an additional means of energy dissipation, measured by the damping work \( W_d \). In this case the plastic work \( W_p \) is computed from

\[ W_p = W - T - V - W_d. \quad (2.49) \]

The damping work \( W_d \) is computed in subroutine DAMP which is called after MOTION, in which the above calculation for \( W_p \) is performed. Hence, for a proper sequencing of calculations, DAMP must compute the damping work up to the next time step \( t + \Delta t \).

DAMP This subroutine, which is schematically summarized in Figure 2.7, controls the entire damping operations. As already mentioned in the introduction, these operations remove the kinetic energy of the system efficiently so that the shell approaches a static equilibrium configuration quickly. The kinetic energy is removed in two ways: first through viscous damping and second through the use of a kinetic energy annihilation (KEA) procedure. The KEA procedure is the principal means of energy removal, while the viscous damping mainly serves to smooth out disturbances in the solution caused by the abrupt nature of the KEA procedure. Conceptually, the KEA procedure involves "freezing" the position of the shell whenever the kinetic energy achieves a local maximum, so that the velocity and hence the kinetic energy vanish instantaneously, followed by an immediate "release" of the shell.

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Figure 2.7 Flow Chart for Subroutine DAMP
Since it is highly unlikely that the shell will be "frozen" in a static equilibrium configuration, it will resume its motion on being "released", but with the total energy reduced by the amount of kinetic energy present when the maximum was achieved. This procedure is repeated at each maximum of the kinetic energy until the energy removed at some maximum is a small enough fraction of the total energy removed, at which time the shell is considered sufficiently close to its static equilibrium configuration and computations terminate*. This procedure is motivated by two considerations: first, the removal of energy is accomplished efficiently, since it occurs when the kinetic energy is at a peak and, second, should the kinetic energy be at its absolute maximum, then the corresponding configuration would be a static equilibrium configuration.

The subroutine begins by determining if the damping procedure is in force by checking the time $t$ against the prescribed time $t_D$ at which this procedure is to begin. If $t_D$ is not exceeded the remainder of the subroutine is not used and calculations return to the main program. When $t_D$ is exceeded, first the pressure $P$ is set to zero. Second, the kinetic energy is checked as to whether it has just reached a local maximum by comparing its value $T_+$ at the time $t + 1/2 \Delta t$ to its value $T_-$ at $t - 1/2 \Delta t$.** If $T_+ > T_-$ then no maximum has been reached and the subroutine adds the increase in the damping work due to viscous damping over this interval, which is simply a linear function of $T_+$, to the damping work $W_D$ at the time $t$ to obtain the damping work $W_D$ at the time $t + \Delta t$:

$$W_D^+ = W_D + \frac{2 \Gamma_0 \Delta t}{\Gamma_0} T_+.$$  \hspace{1cm} (2.50)

On the other hand, if $T_+ < T_-$, a local maximum has been reached and the KEA procedure goes into effect. To maintain the energy balance, the kinetic energy removed at this time $t$ is added to the damping work:

$$W_D^+ = W_D + T.$$  \hspace{1cm} (2.51)

* This method for reducing kinetic energy appears to be in common use; see, for example, DAHL, BEELER and BOURQUIN [4] who use this method to obtain computer solutions of some solid state physics problems.

** Cf. the description of subroutine KINET, where the notation $T(t+1/2 \Delta t)$ and $T(t-1/2 \Delta t)$ was used for $T_+$ and $T_-$. 

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The sum of kinetic energy $T$ removed at this time and the kinetic energy $T^*$ removed by the previous KEA calculation is compared to the damping work $W_D^*$. If this sum is a small enough fraction of $W_D^*$, then the maximum time for the problem $t_{\text{max}}$ is set equal to the present time $t$, after which the main program causes the problem to terminate, see Figure 2.3. If the sum is not sufficiently small, then the displacement increments $\Delta u^i$ are set equal to zero, making the velocities at time $t + 1/2 \Delta t$ and, hence, the kinetic energy $T^*_+$ vanish. Since $\Delta u^i = 0$, the position of the shell and stress field at the time $t + \Delta t$ will remain the same as at the previous time $t$. Hence, these variables need not be recalculated and the subroutine can proceed to increase the time step and call MOTION in order to calculate the displacement increments $\Delta u^i_{++}$ for the interval $[t + \Delta t, t + 2 \Delta t]$ and the kinetic energy $T_{++}$ at the time $t + 3/2 \Delta t$. The kinetic energy $T_{++}$ is next compared with $T_-$. If $T_{++} \leq T_-$ then the subroutine finishes the damping operations by calling PDATA in order to collect some plotting information and returns to the main program. However, if $T_{++} > T_-$ then experience has shown that a numerical instability due to the KEA operations is likely to occur. To remedy this the subroutine calls subroutine ESTEP in order to compute a smaller stable time increment $\Delta t^*$. 

DESTEP This subroutine calculates a decreased time step $\Delta t^*$ that prevents the KEA operations from causing an instability:

$$\Delta t^* = \left( \frac{T_-}{T_{++}} \right)^{1/2} \Delta t. \quad (2.52)$$

Then the values of $\Delta u^i$ and $T_{++}$ are adjusted for the decreased time step by scaling:

$$\Delta u^i_{++} = \frac{2 \Gamma_0 + D\Delta t}{2 \Gamma_0 + D\Delta t^*} \left( \frac{\Delta t^*}{\Delta t} \right)^2 \Delta u^i_{++}, \quad (2.53)$$

$$T_{++} = \left( \frac{2 \Gamma_0 + D\Delta t}{2 \Gamma_0 + D\Delta t^*} \right)^2 \left( \frac{\Delta t^*}{\Delta t} \right)^2 T_{++}. \quad (2.54)$$

Returning from DESTEP back to DAMP, PDATA is called and the calculations return to the main program as before.
2.5 Surface Strain Calculations

The surface strain calculations, like the energy calculations, are embedded in the finite difference sequence of calculations. They do not influence the solution, but their results are a consequence of the solution and provide useful local measures of the validity and reliability of the solution. The elongational strains in prescribed directions at prescribed locations on the bounding surfaces of the shell are calculated. The equations used are derived in [1; App. D] and give the exact elongation per unit initial length, with no approximations based on the smallness of strain being invoked. The strains are intended to simulate the readings of strain gages bonded to the shell at these locations.

The strain calculations take place in subroutine STRAIN. However, before these calculations are performed, the interpolation coefficients and the mesh numbers bracketing the strain locations are calculated in START, see Figure 2.2, and subroutine ABINIT is called during the initial pass through DGEOM, see Figure 2.4.

ABINIT This subroutine uses the mesh numbers of the mesh points bracketing the strain locations, as determined in START, to select the initial values of $a_{\alpha\beta}$ and $b_{\alpha\beta}$ at the bracketing mesh points. These values are stored in arrays for later use in STRAIN.

The above calculations are performed initially and only once. The covariant components of strain $\varepsilon_{\alpha\beta}$ on the bounding surfaces of shell are computed in DGEOM each time step for every mesh point using the equation

$$\varepsilon_{\alpha\beta} = \varepsilon_{\alpha\beta}^- + \frac{1}{2} (\Delta a_{\alpha\beta} \pm h \Delta b_{\alpha\beta}), \quad (2.55)$$

where $\varepsilon_{\alpha\beta}^-$ are the covariant strain components at the previous time $(t - \Delta t)$, $h$ is the shell thickness and the $+$ or $-$ sign depends on whether the bounding surface is on the negative or positive side of the normal $n_i$, respectively. The covariant components of strain are used in STRAIN, immediately following DGEOM.

STRAIN This subroutine calculates the elongational strains in predetermined directions at predetermined locations on the surfaces of the shell. It also computes the components of the total displacement of a predetermined locations on the middle surface. These directions and locations are specified in the input data, see Section 3.2. At each strain location four elongational strains are found: two along the coordinate curves and two in directions specified in the input data by the angles $\theta$ made with the $n^i$ coordinate curve, see Figure 3.5. The first time
Figure 2.8 Flow Chart for Subroutine STRAIN
this subroutine is executed, see Figure 2.8, the interpolation coefficients calculated in START are used to linearly interpolate the initial values of $a_{\alpha\beta}$ and $b_{\alpha\beta}$ to obtain the corresponding values $A_{\alpha\beta}$ and $B_{\alpha\beta}$ at the strain locations. From these values, the components of the surface metric $G_{\alpha\beta}$ at these locations are calculated:

$$G_{\alpha\beta} = A_{\alpha\beta} + \beta B_{\alpha\beta},$$

(2.56)

where, as in (2.55), $+$ or $-$ depends on the surface lying in the negative or positive direction of $n_i$. Also, the direction coefficients $a$ and $\beta$ are calculated from the angle $\theta$ specifying the strain directions:

$$a = \sin \theta / \sqrt{1 - \delta^2}, \quad \beta = \cos \theta - a \delta$$

(2.57)

where $\delta$ is a function of $G_{\alpha\beta}$:

$$\delta = G_{12} / \sqrt{G_{11}G_{22}}$$

(2.58)

This calculation ends the initial pass through the subroutine. For all subsequent time steps, the subroutine uses a different computational loop to calculate the elongational strains and the components of total displacement, as shown in Figure 2.8. First, the values $\epsilon_{\alpha\beta}$ at the mesh points bracketing the strain locations are linearly interpolated to give the covariant components of strain $E_{\alpha\beta}$ at the strain locations. These components are combined with $G_{\alpha\beta}$ to give the intermediate strain components

$$\epsilon_1 = E_{11}/G_{11}, \quad \epsilon_2 = E_{22}/G_{22}, \quad \gamma = E_{12}/\sqrt{G_{11}G_{22}}.$$  

(2.59)

From these intermediate components, the subroutine determines the elongational strains along the $\eta^1$ and $\eta^2$ coordinate curves:

$$E_1 = \sqrt{1 + 2 \epsilon_1} - 1, \quad E_2 = \sqrt{1 + 2 \epsilon_1} - 1$$

(2.60)
and in the direction specified by $\theta$:

$$E_0 = \sqrt{1 + 2 \left( s^2 \varepsilon_1 + 2 \alpha \gamma + a^2 \varepsilon_2 \right) - 1} \quad (2.61)$$

The remainder of the subroutine involves determining the total displacement components $U^i$ at a predetermined location. The components of displacement increments $\Delta U^i$ at the mesh point surrounding the displacement location are linearly interpolated to give the components of displacement increment $\Delta U^i$ at this location. These components are added to the previous values of the components of the total displacement $U^i$ to give their current values.

$$U^i = U^i - \Delta U^i \quad (2.62)$$
3. DESCRIPTION OF INPUT

3.1 Input Cards

The data needed to run REPSIL are supplied on input cards and, in the case of certain pressure loadings, on a user-generated input tape. Instructions for generating the pressure input tape are given in Section 6.2. The input cards assign values to the FORTRAN variables listed in Table 3.1 in that order using the formats indicated.

Table 3.1 List of Input Cards

<table>
<thead>
<tr>
<th>CARD</th>
<th>VARIABLES</th>
<th>FORMAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>TITLE</td>
<td>10A8</td>
</tr>
<tr>
<td>2</td>
<td>MESH, NMESH, LAYER, YLDFAC</td>
<td>3IS,E12.6</td>
</tr>
<tr>
<td>3</td>
<td>MAXC, NCONT, NRITE, DELTAT</td>
<td>3IS,E12.6</td>
</tr>
<tr>
<td>4</td>
<td>IBEC1, IBEC2, IBEC3, IBEC4</td>
<td>4IS</td>
</tr>
<tr>
<td>5</td>
<td>LOAD, LPRESS, MDAMP, DAMPF, DFACT</td>
<td>3IS,2E12.6</td>
</tr>
<tr>
<td>6</td>
<td>E, FNU, SIGZ, RHO, THICKN, NSFL, ISR</td>
<td>5E12.6,2IS</td>
</tr>
<tr>
<td>7</td>
<td>(SSIG(J), SEPS(J), DSR(J), PSR(J), J=1, NSFL)</td>
<td>4E15.7</td>
</tr>
<tr>
<td>8</td>
<td>NPRINT, (JCHK(J), J=1,3)</td>
<td>4IS</td>
</tr>
<tr>
<td>9</td>
<td>NUMCY, (NCYCH(J), J=1, NUMCY)</td>
<td>16IS</td>
</tr>
<tr>
<td>10</td>
<td>NLPRIN, (JCyNLP(J), J=1,NLPRIN)</td>
<td>16IS</td>
</tr>
<tr>
<td>11</td>
<td>N3D (NC3DP(J), J=1, N3D)</td>
<td>16IS</td>
</tr>
<tr>
<td>12</td>
<td>ETADI, ETAD2, NSTRN</td>
<td>2E10.4,15</td>
</tr>
<tr>
<td>13</td>
<td>(ETAG1(I), ETAG2(I), ANGLE(I), ANGLB(I), NETAG(I), I=1, NSTRN)</td>
<td>4E10.4,15</td>
</tr>
<tr>
<td>14</td>
<td>LENGTH, WIDTH [for flat plate]</td>
<td>2E12.6</td>
</tr>
<tr>
<td></td>
<td>LENGTH, RADIUS, THETA [for cylindrical shell]</td>
<td>3E12.6</td>
</tr>
<tr>
<td></td>
<td>LENGTH, RADI, RADF, THETA, MASH [for conical shell]</td>
<td>4E12.6,15</td>
</tr>
<tr>
<td>15</td>
<td>MI, MF, NI, NF, VR, NV</td>
<td>4IS,E12.6,15</td>
</tr>
<tr>
<td>16</td>
<td>M, N, V</td>
<td>2IS,E12.6</td>
</tr>
</tbody>
</table>
The following rules should be obeyed in preparing the input cards.

- Omit card 7 if either NSFL = 0 or NSFL = 1 and ISR = 0; otherwise, the number of card 7's must match NSFL.
- The number of card 13's must match NSTRAN > 1.
- Only one card 14 is used, with the data matching the particular subroutine INGEOM used.
- Omit cards 15 and 16 if LOAD = 1 or NCONT > 0.
- Omit card 16 if NV = 0; otherwise, the number of card 16's must equal NV.

The input cards can be grouped according to the type of data they supply:

- Cards 2,3: Data controlling the finite difference and numerical analysis.
- Card 4: Parameters for selecting boundary conditions.
- Card 5: Data controlling the type of loading and the damping option.
- Card 6,7: Material properties.
- Card 8,9,10,11: Printing and plotting control numbers.
- Cards 12,13: Data specifying locations where displacement components and surface strains are to be calculated.
- Card 14: Dimensions of shell.
- Card 15,16: Data characterizing the initial impulse velocity.

### 3.2 Description of Input Variables

The input variables are described below in the order in which they appear on the input cards, as listed in Table 3.1. The dimensions of a variable are indicated by capital letters in square brackets following the short underlined description of the variable, with F representing force, L length, and T time. The program is written to accept any consistent set of dimension units. For example, the mass density in the pound-inch-second system of units for a material weighing 1 pound per cubic inch would be \( \frac{1}{386} \frac{1b-sec^2}{in^4} \).
Card 1 TITLE Title to identify run. Not to exceed 80 alphanumeric characters.

Card 2 MESH Number of mesh intervals in the $\eta^1$ direction.

NMESH Number of mesh intervals in the $\eta^2$ direction.

Figures 3.6 - 3.8 show the orientation of the mesh relative to the $\eta^1$ and $\eta^2$ directions for the initial geometries presently programmed in REPSIL. The choice of MESH and NMESH should only be based on the portion of the shell to be actually analyzed; additional intervals due to exterior mesh points along symmetry boundaries should be disregarded. MESH and NMESH are limited by the maximum number of M and N mesh elements permitted by DIMENSION and COMMON statements (see Section 3.3):

\[
\text{MESH} \leq M_{\text{max}} - \begin{cases} 
2 & \text{IBCE3} = 1,3 \\
3 & \text{IBCE3} = 2 
\end{cases}
\]

\[
\text{NMESH} \leq N_{\text{max}} - \begin{cases} 
1 & \text{IBCE2} = 1,3 \\
2 & \text{IBCE2} = 2 
\end{cases}
\]

LAYER Number of layers into which the shell thickness is divided. The shell is divided into layers, within which the stress is assumed constant, in order to facilitate the modelling through the thickness of the stress profile resulting from plasticity. Hence, the greater the number of layers used, the more accurately is the stress profile presumably modelled, but at the expense of longer computation times and greater memory requirements. LAYER = 4 has been found to be a good compromise giving reasonably accurate deflections.

YLDFAC Parameter controlling the "thickness" of ellipsoidal annuli surrounding yield surface in stress space. The ellipsoidal annuli divided the excursions of the stress increment outside yield surface into subincrements making the calculation of the stress on the yield surface more accurate, see Appendix B. Accuracy increases with value of YLDFAC, but at the expense of increased computation times, with YLDFAC = 1 a good compromise. In order not to use this option set YLDFAC = 0.

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Card 3  MAXC  Time step at which it is desired to terminate the problem.

NCONT  Time step at which it is desired to begin the problem. For new problems NCONT = 0 and for restart problems NCONT = time step from which solution is continued. Notice that always NCONT < MAXC.

NRITE  Time steps elapsed between the gathering of restart data. Restart information written on the restart tape (tape #1) every NRITE time steps. If it is planned to restart a problem, NRITE ≤ MAXC; if restart information is not desired, make NRITE > MAXC and omit tape.

DELTAT  Finite difference time increment [T]. Using equations (2.3), the program calculates time increments that assure numerical stability in the membrane and bending modes of vibration and then chooses the minimum of these and the input DELTAT to use in the finite difference calculations. If DELTAT = 0.0, the program chooses the minimum stable time increment.

Card 4  IBEC1  Number prescribing boundary conditions along the edges of the shell. In Figure 3.1 the edges are innumerated relative to the (M,N) grid and the admissible values of the boundary control numbers at each edge are listed.

IBEC2  Clamped edge condition (1). Coordinates of middle surface \( y^i \) and components of normal \( n^i \) are fixed at their initial values along this edge.

IBEC3  Symmetry edge condition (2). Edge lies in a symmetry plane about which the shell and the loads are symmetrically distributed. Edge 1 is always a symmetry edge located in the \( y^1 = 0 \) symmetry plane, see Figures 3.6 - 3.8. The symmetry plane for edge 2 is \( y^2 = \text{LENGTH} \), see Figure 3.6 and 3.7. The symmetry plane for edge 3 is the same as that for edge 1, namely \( y^1 = 0 \), and hence is applicable to shell intersecting this plane twice, such as cylinders and cones, see Figure 3.7 and 3.8. Care should be taken that the symmetry edge condition be compatible with the particular shell geometry treated, e.g. although IBCE3 = 2 is admissible, it is certainly not appropriate to the flat plate (Figure 3.6) or the cylindrical panel with \( \Theta = \pi \) (Figure 3.7).
EDGE 1: IBCE1 = 2 (SYMMETRY)
EDGE 2: IBCE2 = 1 (CLAMPED), 2 (SYMMETRY) OR 3 (HINGED)
EDGE 3: IBCE3 = 1 (CLAMPED), 2 (SYMMETRY) OR 3 (HINGED)
EDGE 4: IBCE4 = 1 (CLAMPED) OR 3 (HINGED)

Figure 3.1 Admissible Boundary Conditions
Hinged edge conditions (3). Coordinates of middle surface $y^i$ are fixed at their initial values along this edge, but normal $n^i$ is free to rotate about edge.

Card 5 LOAD

Number controlling mode of loading

-1, initial impulse velocity and pressure-time loading,
0, initial impulse velocity,
1, pressure-time loading.

Initial velocity distribution, other than those representable by cards 15 and 16, and pressure-time histories must be supplied by the user in appropriate form through subroutines INVEL and PRESS; instructions for doing this are in Sections 6.2 and 6.3, respectively.

LPRESS

Distribution of pressures over shell after time steps LPRESS fixed at the LPRESS distribution. If LOAD = 0, set LPRESS = 0. If user does not desire to fix pressure distribution, make LPRESS > MAXC.

MDAMP

Time step at which damping starts. Numerical damping is used to rapidly slow down the motion of the shell in order to obtain a final equilibrium configuration. MDAMP is selected after most of the plastic dissipation is over. This entails a preliminary run in order to estimate from the energy balance when plastic deformation is virtually finished; the damping run is continued from the time step closest to MDAMP as a restart problem (see Section 3.4). If damping is not desired set MDAMP > MAXC, otherwise values for DAMPF and DFACT must be supplied below.

DAMPF

Viscous damping coefficient used in smoothing solution during damping [FT/L$^3$]. Should not be too large in order to avoid overdamping and consequently prolonging the time to reach a final configuration.

DFACT

Parameter controlling termination of problem during damping. If the ratio of the sum of the energies removed in two consecutive kinetic energy annihilations to the damping work is less than DFACT, the problem terminates (see Figure 2.7). The smaller DFACT is made, the less the residual kinetic energy at termination, but at the expense of longer machine times.
Card 6  E    Young's modulus \([F/L^2]\).

FNU    Poisson's ratio.

SIGZ    Yield stress \([F/L^2]\). For perfectly plastic behavior SIGZ is the maximum stress \(\sigma_0\) on the uniaxial loading curve, Figure 3.2; for strain hardening behavior SIGZ is the stress \(\sigma_1\) at the first change in slope in the polygonal approximation to the loading curve, Figure 3.3.

RHO    Initial mass density per unit volume \([FT^2/L^4]\).

THICKN    Thickness of shell \([L]\).

NSFL    Number of changes in slope in the polygonal approximations to the uniaxial loading curve (equal to the number of stress sublayers):

  = 0, no plasticity \(\equiv\) elastic behavior,
  = 1, elastoplastic with no strain hardening,
  > 1, elastoplastic with strain hardening.

ISR    Strain rate sensitivity control

  = 0, plasticity is strain rate independent,
  = 1, plasticity is strain rate dependent.

Card 7* SSIG(J), SEPS(J)    Stress \([F/L^2]\) and strain \([L/L]\) at points of slope change of the polygonal approximation to the uniaxial loading curve, Figure 3.3, where \(J = 1, NSFL\). The program automatically makes these data compatible with those on Card 6 by setting SSIG(1) = SIGZ and SEPS(1) = SIGZ/E. For the strain rate sensitive case, take these data from the static loading curve.

DSR(J), PSR(J)    Empirical constants used to model strain rate sensitive behavior, \(d\) and \(p\) of equation (2.21). Pair of constants must be specified for each slope change (i.e. each stress sublayer) on the polygonal approximation to the loading curve, \(J = 1, NSFL\). On the stress-strain diagrams, Figure 3.4, the straight line portions of the

* Omit card 7 if either NSFL = 0 or NSFL = 1 and ISR = 0.
Figure 3.2 Uniaxial Loading Curve for the Elastic/Perfectly-Plastic Constitutive Model

Figure 3.3 Uniaxial Loading Curve for the Strain Hardening Constitutive Model and Polygonal Approximation to Loading Curve
a. Strain rate parameters the same at each point of slope change.

b. Strain rate parameters differ at each point of slope change.

Figure 3.4 Polygonal Approximations to the Uniaxial Loading Curve at Various Constant Strain Rate Levels for Strain Rate Sensitive Materials
constant strain rate curves are parallel to the corresponding portions of the static loading curve. The strain \( \varepsilon_j \) at each point of slope change is magnified from the corresponding static strain \( \varepsilon_j^* \) by the rate sensitivity factor:

\[
\varepsilon_j = \varepsilon_j^* \left[ 1 + \left( \frac{\dot{e}}{\eta j} \right) \right],
\]

where \( \dot{e} \) is the strain rate, see (2.22). If the same values of DSR(J) and PSR(J) are used for all J, the stress \( \sigma_j \) at each point of slope change is also magnified from the corresponding static stress \( \sigma_j^* \):

\[
\sigma_j = \sigma_j^* \left[ 1 + \left( \frac{\dot{e}}{\eta j} \right) \right],
\]

so that

\[
\sigma_j = \left( \frac{\sigma_j^*}{\varepsilon_j^*} \right) \varepsilon_j,
\]

as illustrated in Figure 3.4a.

Card 8 NPRINT Number of elapsed time steps between surface strain prints. This print is described in Section 4.1.5. The remainder of the strain print data is specified on Card 13. If this print is not desired, set NPRINT > MAXC.

JCHK(J) Numbers controlling the printing of output data:

JCHK(1), components of displacement increments,
JCHK(2), coordinates of middle surface and the pressure,
JCHK(3), components of surface normal;

\[
JCHK(J) = \begin{cases} 
0, & \text{data not printed,} \\
1, & \text{data printed.} 
\end{cases}
\]

Card 9 NUMCY Number of time steps for which JCHK(J) controlled data and energy balance data are to be printed. Sections 4.1.2 and 4.1.3 describe these prints.
NCYCH(J)  Time steps at which JCHK(J) controlled data and energy balance data are printed. If these prints are not desired, set NUMCY = 1 andNCYCH(1) > MAXC.

Card 10 NLPRIN  Number of time steps for which the LMAT (M,N,K) array is printed. See Section 4.1.4 for a description of this print.

JCYNLP(J)  Time steps at which the LMAT (M,N,K) array is printed. If this print is not desired, set NLPRIN = 1 and JCYNLP(1) > MAXC.

Card 11 N3D  Number of time steps for which isometric and cross-sectional plots are drawn. A description of the plotting capabilities of REPSIL is given in Section 4.2.

NC3DP(J)  Time steps at which isometric and cross-sectional plots are drawn. If plots are not desired, set N3D=1 and NC3DP(1) > MAXC.

Card 12 ETADI,ETAD2  Material coordinates of location at which the components of displacements are calculated and plotted [dimensions correspond to those for ETAG1(I), ETAG2(I) below].

NSTRN  Number of locations at which surface strains are calculated and plotted.

Card 13 ETAG1(I), ETAG2(I)  Material coordinates of locations at which surface strains are calculated and plotted, Figure 3.5. Dimensions depend on subroutine INGEOM:

- Flat plate, distance along width and length, Figure 3.6,
- Cylinder, angle in degrees from the symmetry plane and distance along axis, Figures 3.7.
- Cone, angle in degrees from the symmetry plane and arclength along the cone generator, Figure 3.8.

ANGLE(I), ANGLB(I)  Angles measured in degrees from the direction counterclockwise about the normal of the directions in which surface strains are calculated, Figure 3.5, where 0 < 6 < 180°.

NETAG(I)  Number selecting the bounding surface on which surface strain calculations are performed, Figure 3.5:

= 0, surface on positive side of normal,
= 1, surface on negative side of normal.
Figure 3.5 Material Coordinates $\eta^a$ Locating Surface Strains Position and Orientation of Strain Direction Relative to the $\eta^1$ Direction
Card 14

Dimensions of shell specified on this card. Data specifying dimensions must be compatible with the particular subroutine INGEOM used. The next three figures show the orientations relative to the Cartesian coordinate axes $y$ and the associated compatible boundary conditions for the three INGEOM subroutines presently programmed in REPSIL.

LENGTH
Length of plate along symmetry boundary [L], Figure 3.6. Length of cylinder axis [L], Figure 3.7. Length of cone along axis [L], Figure 3.8.

WIDTH
Width of plate up to symmetry boundary [L], Figure 3.6.

RADIUS
Radius of cylinder [L], Figure 3.7.

RADI, RADF
Small and large radii of cone [L], Figure 3.8.

THETA
Angle subtended by cylindrical or conical panel measured from the symmetry plane [Degrees], Figures 3.7 and 3.8.

MASH
Number controlling mesh proportions for cone:

= 0, equal mesh intervals along meridian, Figure 6.2a,

= 1, constant mesh proportions, Figure 6.2b.

For the details of this option see Section 6.4.

Card 15*

This card gives the data on the uniform initial impulse velocity field and gives the number of points with nonuniform impulse velocity, see Figure 3.9. As indicated in the figure, velocities are directed in the opposite sense to the normal.

MI, MF
Minimum and maximum values of mesh number $M$ for point receiving uniform initial impulse velocity $V_R$.

$2 \leq MI < MF \leq MESH + 2$.

NI, NF
Minimum and maximum values of mesh number $N$ for point receiving uniform initial impulse velocity $V_R$.

$1 \leq NI < NF \leq NMESH + 1$.

* Cards 15 and 16 omitted if $LOAD = 1$ or $NCONT > 0$. 
Boundary conditions compatible with flat plate geometry (see Figure 3.1 for key):

\[
\begin{align*}
\text{IBCE}_1 &= 2 \\
\text{IBCE}_2 &= 1, 2 \text{ or } 3 \\
\text{IBCE}_3 &= 1 \text{ or } 3 \\
\text{IBCE}_4 &= 1 \text{ or } 3 
\end{align*}
\]

Figure 3.6 Flat Plate Geometry
Boundary conditions compatible with cylindrical geometry (see Figure 3.1 for key):

- **IBCE1** = 2
- **IBCE2** = 1, 2 or 3
- **IBCE3** = 1 or 3 when $0 < \sigma < 180^\circ$ (as shown)
- **IBCE4** = 1 or 3

Figure 3.7 Cylindrical Shell Geometry
Boundary conditions compatible with conical geometry (see Figure 3.1 for key):

- IBCE3 = 2
- IBCE2 = 1 or 3*
- IBCE3 = 1 or 3 when $0 < \theta < 180^\circ$
- IBCE3 = 1, 2 or 3 when $\theta = 180^\circ$ (as shown)
- IBCE4 = 1 or 3

*IBCE2 = 2 would result in discontinuous slope across boundary.

Figure 3.8 Conical Shell Geometry
Figure 3.9 Uniform Initial Impulse Velocity Distributed over a Rectangular Region Defined by the Limits $M_{L} \leq M \leq M_{F}$ and $N_{I} \leq N \leq N_{F}$
VR Uniform initial impulse velocity received by mesh points \((M, N)\) in the intervals \(MI \leq M \leq MF \) & \(NI \leq N \leq NF\). \((L/T)\).

NV Number of mesh point receiving nonzero initial impulse velocity other than uniform velocity VR.

Card 16* Each card 16 gives data for each mesh point receiving nonuniform initial impulse velocity; hence total number of card 16's must equal NV.

M, N Mesh numbers of point receiving initial impulse velocity \(V\) other than uniform velocity VR.

\(V\) Initial impulse velocity at mesh point \(\text{(M, N)}\) \((L/T)\).

3.3 Array Size, Memory Requirements and Computation Times

Often, the size of arrays must be adjusted in order for the program to accommodate a new problem. Such adjustments entail changing DIMENSION and COMMON statements to assure that array size equals or exceeds the maximum values of array indices required by the problem.

The maximum values of array indices are easily determined from the input data, as shall now be shown. First, denote the maximum values of the indices \(M, N, K, J\) and \(KJ\) (see Appendix C.1 for their definitions) by appending the subscript "max" to each. Then \(M_{\text{max}}\) and \(N_{\text{max}}\) depend on the number of mesh intervals in the \(\eta^1\) and \(\eta^2\) directions (specified on input card 2) and the boundary conditions along edges 3 and 2 respectively (card 4):

\[
M_{\text{max}} = \begin{cases} 
M + 2 ; & \text{IBCE3} = 1, 3 \\
M + 3 ; & \text{IBCE3} = 2 
\end{cases} \quad (3.1)
\]

\[
N_{\text{max}} = \begin{cases} 
N + 1 ; & \text{IBCE2} = 1, 3 \\
N + 2 ; & \text{IBCE2} = 2 
\end{cases} \quad (3.2)
\]

Moreover, \(K_{\text{max}}\) equals the number of layers into which the shell is divided (card 2):

\[
K_{\text{max}} = \text{LAYER} \quad , \quad (3.3)
\]

* Cards 15 and 16 omitted if \(\text{JAD} = 1\) or \(\text{NCONT} > 0\).
\( J_{\text{max}} \) equals the number of breaks in the polygonal approximation to the loading curve (Card 6):

\[
J_{\text{max}} = \text{NSFL},
\]

(3.4)

and \( K_{\text{Jmax}} \) is the product of \( K_{\text{max}} \) and \( J_{\text{max}} \):

\[
K_{\text{Jmax}} = \text{LAYER} \times \text{NSFL}.
\]

(3.5)

The arrays affected by changes in the maximum values of these indices are listed in Appendices C.2 and C.4. The maximum values of the remaining array indices usually need no adjusting because they are sufficiently large for most problems.

The storage of arrays constitutes the major portion of the memory requirements of REPSIL, with the rest of the memory requirements used to store the remainder of the program. The memory needed to store the remainder is more or less fixed and depends on the compiler that the computer uses; for example, the BRL ESC computer at the BRL uses between 11,000 and 12,000 words for this purpose. The memory required to store the arrays depends on their size, specified by prescribing the maximum values of array indices, and hence may change with the problem being solved.

It is useful to have an estimate of the memory required by a problem, to see if the computer can accommodate it. For an estimate it is sufficient to consider only the large arrays — the two and three dimensional arrays. A count of these arrays, listed in Appendices C.2 and C.4, shows that there are 26 two dimensional \((M,N)\) arrays, 3 three dimensional \((M,N,K)\) arrays and 1 three dimensional \((M,N,K)\) array. Hence, a problem whose maximum index values are \( M_{\text{max}}, N_{\text{max}}, K_{\text{max}} \) and \( K_{\text{Jmax}} \) will use

\[
S_A = M_{\text{max}} \times N_{\text{max}} \times (26 + K_{\text{max}} \times (3 \times J_{\text{max}} + 1))
\]

(3.6)

words of memory to store these arrays. An estimate on the amount of memory a given computer uses to store the remainder of the program is most easily obtained as the difference between the total memory used on a given problem and \( S_A \) for the given problem. If this difference is \( S_R \), then for any new problem a good estimate of the total number of words of memory needed is

\[
S_T = S_R + S_A = S_R + M_{\text{max}} \times N_{\text{max}} \times (26 + K_{\text{max}} \times (3 \times J_{\text{max}} + 1)).
\]

(3.7)
The maximum values $M_{\text{max}}$, $N_{\text{max}}$, $K_{\text{max}}$, and $J_{\text{max}}$ can also be used to estimate the time a given problem will take on a computer. This follows from the fact that the computer takes approximately the same time to solve the finite difference scheme for a given substress at a given layer and mesh point; hence, the total computation time is roughly proportional to the product of the number of substresses $J_{\text{max}}$, of layers $K_{\text{max}}$, of mesh points $M_{\text{max}} \times N_{\text{max}}$, and the total number of time steps $N_{t}$:

$$N_{t} = \text{MAXC} - \text{NCONT} \quad (3.8)$$

Therefore, once the time $T^{*}$ taken by the computer to solve a problem with the values $J_{\text{max}}^{*}$, $K_{\text{max}}^{*}$, $M_{\text{max}}^{*}$, $N_{\text{max}}^{*}$, and $N_{t}^{*}$ is established, then an estimate of the time $T$ for any other problem with the values $J_{\text{max}}$, $K_{\text{max}}$, $M_{\text{max}}$, $N_{\text{max}}$, and $N_{t}$ is given by the ratio

$$\frac{T}{T^{*}} = \frac{M_{\text{max}} \times N_{\text{max}} \times J_{\text{max}} \times K_{\text{max}} \times N_{t}}{M_{\text{max}}^{*} \times N_{\text{max}}^{*} \times J_{\text{max}}^{*} \times K_{\text{max}}^{*} \times N_{t}^{*}} \quad (3.9)$$

This relation only gives approximate times because it neglects such factors as the operations at boundary point differing from those at interior points, the different number of iterations for the plastic stress at different mesh points, the compile times not being proportional. However, the relation serves as a useful rule-of-thumb, giving overestimates as the number of mesh intervals increases.

3.4 Continuation of Problem (Restart)

Every $\text{NRITE}$ number of time steps information is written on the restart tape about the current state of the solution. As discussed in Chapter 2, this information is sufficient for the program to continue the solution from any of these prescribed time steps; such a continuation is called a restart. The restart of a problem entails certain necessary, advisable and permissible changes in the input data, as follow.

Necessary changes are changes without which the restart problem cannot be solved. These changes are confined to Card 3. The initial time step $\text{NCONT}$ must be set equal to some multiple of $\text{NRITE}$ at which time step there is written information on the restart tape; this is usually chosen as the last time step on the tape. Also, the last time step $\text{MAXC}$ must be changed to a value greater than $\text{NCONT}$.

Advisable changes are changes that either reduce the amount of input data or assure meaningful output data. First, Cards 15 and 16 may be omitted. Second, the time steps specified on Cards 9, 10, and 11 at which output data is collected should be changed so as to fall within the new interval between $\text{NCONT}$ and $\text{MAXC}$.
Permissible changes are changes that affect the collecting of restart and output data or the functioning of the damping operations. On Card 3, NWRITE may be changed so that restart information is collected at a different interval. Also, on Card 8 the interval between the times at which surface strains are printed may be changed by altering NPRINT and a different selection of arrays may be printed by altering JCHK(J). On Card 5, DAMPF and DFACT, which control damping operations, may be freely changed. Also, if LPRESS and MDAMP were greater than the new value of NCONT, they may be changed.

Finally, a few words of caution. If it is planned to continue a problem, provide a tape to collect the restart information; otherwise, omit the restart tape. Any changes in the input data that affect the size of arrays should never be made. Lastly, note that there is no restriction on repeatedly continuing the solution of a problem.

4. DESCRIPTION OF OUTPUT

REPSIL outputs the results of calculations in two forms: printed output and plotted output. This chapter describes the various output options available and how these are controlled through the input data. Reference will be made to tables and figures in Chapter 5 as samples of printed and plotted output.

4.1 Printed Output

4.1.1 Input Data. The printed output begins with a title page restating the input data and giving the critical Δt resulting from the REPSIL stability check, see Tables 5.3 and 5.15. If LOAD = 0 or -1, the uniform initial impulse velocity specified on Cards 15 and 16 will be printed out next, Table 5.16. There follows a print of the initial values of the Cartesian coordinates Yi and pressure P at all mesh intersections (M,N). Table 5.4 and 5.16 give samples of these arrays at select values of M. The user need not request any of the above prints; they are automatically produced by REPSIL for any initial run. For a restart run only the title page is printed.

4.1.2 Displacement Increment, Cartesian Coordinate, Pressure and Surface Normal Arrays. By setting the input variables JCHK(I) = 1 for I = 1,..,3 (see Section 3.2, Card 8), the values of the displacement increments Ui, Cartesian coordinates Yi and pressures P, and surface normal NIK at every intersection (M,N) are printed at each NCYCH(J) time step (Card 9). Samples of these arrays at select values of M appear in Tables 5.5 - 5.7, 5.9 - 5.11 and 5.17 - 5.22. Notice that the displacement increments are for the time increment just preceding time step NCYCH(J).
4.1.3 Energy Balance. The program prints the values of the kinetic energy $\text{CINET}$, strain energy $\text{STREN}$, plastic work $\text{PLAST}$ and external work $\text{TNRG}$ for the entire shell every $\text{NCYCH(J)}$ time steps (Section 3.2, Card 9), as shown in Tables 5.13 and 5.23.

4.1.4 Stress Subincrement Array. The stress subincrement array $\text{LMAT(M,N,K)}$ is printed every $\text{JCYNLP(J)}$ time steps (Section 3.2, Card 10), see Tables 5.8, 5.12, 5.18, 5.20 and 5.22. The value of $\text{LMAT}$ at the location $(M,N,K)$ is an approximate measure of the amount of plasticity occurring there during the given time interval: if $\text{LMAT} = 0$ the stress increment lies within the yield surface and, hence, is elastic; otherwise $\text{LMAT}$ equal the number of stress annuli outside the yield surface traversed by the stress increment, see Appendix B for a detailed description.

4.1.5 Surface Strains. The surface strains $\text{EPSSI(I)}$, $\text{EPSS2(I)}$, $\text{EPSANB(I)}$ and $\text{EPSANG(I)}$ at locations specified on input Card 13 are printed every $\text{NPRINT}$ elapsed time steps (Card 8). These strains, shown in Tables 5.13 and 5.23, simulate the reading of strain gages aligned at the angles indicated there relative to the $\eta^1$ direction, see also Figure 3.5.

4.1.6 Error Messages. An inability to satisfactorily calculate a plastic stress increment at a location $(M,N,K)$ results in an error print, wherein the values of the quantities involved in computing the stress at this location are printed and the calculations terminate. This print occurs for two reasons: either the lead coefficient $\text{AA}$ in the quadratic equation for $\text{TAMBDA}$ is negative or the values of $\text{TAMBDA}$ continue to remain complex even after the use of 100 stress subdivision. The reasons that both these results are unacceptable and do not permit the continuation of the solution are given in [3; Sect III].

4.2 Plotted Output

The plotted output is generated by a separate plotting program described in Appendix D. This program works from a tape generated by REPSIL on which plotting data is stored. The program employs the Cal Comp Standard Plotting Package SCOOP. The plots shown in this report are generated by the Cal Comp Model 790 Plotter.

4.2.1 Isometric and Cross-sectional Plots. REPSIL stores on tape the Cartesian coordinate array $Y_1$ at the initial time step and at subsequent time steps as specified by the values of $\text{NC3DP(J)}$ on Card 11. From these data the plotting program generates two types of plots at each of these time step: an isometric drawing of the distorted image of the finite difference mesh passing through the middle surface; and a pair of cross-sectional drawing through the $Y_1 = 0$ symmetry plane and a plane normal to the $Y_2$ axis, as specified in the input to the plotting program. The scale of the drawing and a factor to magnify the displacements from the initial position must also be specified as input. These plots are illustrated by Figures 5.3 and 5.8. The zeroth time step plots are
automatically generated without the user requesting them. Notice that these plots print the input data: the mesh number \( N \) of the crosssection, the scale and the magnification factor.

4.2.2 Energy, Displacement and Surface Strain Histories. At every time step REPSIL also stores on tape three groups of data: the kinetic energy \( C_{\text{INET}} \), strain energy \( S_{\text{TREN}} \), plastic work \( P_{\text{LAST}} \), external work \( T_{\text{NRG}} \) and, when the damping procedure is used, the damping work \( T_{\text{DAMP}} \); the components of the displacement at the location specified on Card 12; and the elongational surface strains in the \( \eta^1 \) and \( \eta^2 \) coordinate direction at the location specified on Card 13.

The plotting program, using the first group of data, plots a time history of the balance of energies during deformation, as illustrated in Figures 5.4 and 5.9; notice that graph b of Figure 5.9 is an enlargement of the last 400 microseconds of graph a to bring out the details of the energy balance during damping operations. In Figures 5.4 and 5.9a the top line represents the external work, with that in Figure 5.4 being due to a pressure loading and in Figure 5.9a due to an initial impulse velocity; in Figure 5.9b the external work line falls off the graph. In all three graphs, the bottom line is the kinetic energy and the line second from the bottom is the total energy of the shell. Hence, the difference between the bottom line and the second from bottom line represents the strain energy and the difference between the top line and the second from bottom line represents the energy dissipated.

When damping operations are not used, as in Figure 5.4 and the first 405 microseconds in Figure 5.9, the energy dissipated is solely due to the plastic work. However, at the inception of the damping procedure (405 microseconds in Figure 5.9b) a third line appears dividing the energy dissipated into two parts: the plastic work represented by the difference between the top line and this new line and the damping work represented by the remaining difference.*

The second group of data is used by the plotting program to plot a time history of the components of displacement at a given location, as illustrated in Figures 5.5 and 5.10. The location as specified in the REPSIL input is printed with the plot.

* In principle, the plastic work and the damping work are monotone increasing function of time and when the external work is constant the total energy is monotone decreasing. That this is only approximately true of Figure 5.9b is a consequence of the numerical inaccuracy of the finite difference solution. For the same reason, in purely elastic problems it is found that the total energy oscillates about the external work rather than coinciding. However, an excessive rise of the total energy over the external work is usually an indication that something is going wrong with the solution, e.g. too large a time increment leading to a numerical instability.
The third group is used to plot time histories of the elongational strains on the surface of the shell in the $\eta^1$ and $\eta^2$ coordinate directions (see Figures 3.6 - 3.8 for the coordinate directions of the various geometries employe, as well as Figure 3.5) at prescribed locations. These plots are illustrated by Figures 5.6 and 5.11, with the solid line being the strain in the $\eta^1$ direction and the dash line the strain in the $\eta^2$ direction. The locations as specified in REPSIL, including which bounding surface, are reproduced on the plots. A maximum of 6 locations can be plotted at present.

5. EXAMPLE PROBLEMS

This chapter demonstrates the use of REPSIL to solve two typical shell problems. Correct preparation of input data, including proper implementation of various options available in REPSIL is illustrated. Portions of the printed and plotted output are displayed. These serve as checks on the proper functioning of the code, especially useful to users adapting REPSIL to their computers. For reasons of economy, no attempt is made to discuss the accuracy or significance of results.

5.1 Example 1: Pressure Loaded Flat Plate

The first problem involves finding the deformation history of a simple-supported, rectangular, steel plate subjected to loads resulting from the detonation of an explosive charge. Figure 5.1 shows the dimensions and orientation of the plate and charge. This example illustrates the use of the following REPSIL options.

* Flat plate initial geometry
* Analytically specified pressure loading
* Hinged edge and symmetry edge boundaries
* Strain hardening - strain rate dependent material behavior
* Problem restart or continuation

The material properties of the steel in the elastic range are:

- Young's modulus $E = 30 \times 10^6$ psi
- Poisson's ratio $\nu = 0.3$
- Mass density $\rho = 7.3235 \times 10^4 \frac{lb - sec^2}{in^4}$

In the plastic range the steel is assumed to strain harden in a strain rate dependent manner. This behavior is approximated by using a 3 substress model with the following values of the parameters.
Figure 5.1 Geometry for Example Problem 1
\[ \sigma_1 = 78,000 \text{ psi} \quad \epsilon_1 = 0.026 \text{ in/in} \quad d_1 = 40 \text{ in/in-sec} \quad P_1=5 \]
\[ \sigma_2 = 120,000 \text{ psi} \quad \epsilon_2 = 0.0082 \text{ in/in} \quad d_2 = 400 \text{ in/in-sec} \quad P_2=3 \]
\[ \sigma_3 = 180,000 \text{ psi} \quad \epsilon_3 = 0.0482 \text{ in/in} \quad d_3 = 4000 \text{ in/in-sec} \quad P_3=1 \]

Figure 5.2 shows the resulting polygonal approximations to the uniaxial loading curves at four strain rate levels.

The pressure loading is simulated by programming into subroutine PRESS the pressure relation

\[ P(m,n) = \begin{cases} 
0; & \text{for } t < t_a \\
\frac{225 \times 24465.5}{225 + r^2(m,n)} e^{-13000(t-t_a)}; & \text{for } t \geq t_a,
\end{cases} \]

where \( r(m,n) \) is the distance from the center of the plate to the mesh intersection \((m,n)\) and

\[ t_a = \sqrt{\frac{225 + r^2(m,n)}{144000}} \]

is the arrival time of the shock front at \((m,n)\).

Taking advantage of the two fold symmetry of the problem, only the lower right quarter of the plate, as shown in Figure 5.1, is treated. Consequently, edges 1 and 2 are symmetry boundaries and edges 3 and 4 hinged boundaries (compare with Figures 3.1 and 3.6). Also, only the half width and half length of the plate are prescribed as input dimensions. The problem uses a 20 x 32 square mesh and 4 layers through the thickness. A time increment of 4 microseconds is prescribed, a figure well below the critical time increment predicted by the REPSIL stability criteria (see Table 5.3).

The problem is solved in three successive runs. The initial run is set for a maximum of 400 time steps, giving a solution for the first 1600 microseconds. Table 5.1 gives the input for this run in the same order as outlined in Table 3.1. The first restart or continuation run is prescribed for the next 400 time step. Only the input data on Cards 3, 9, 10 and 11 are changed, as shown in Table 5.2. The second restart run continues the solution 389 time steps further and requires changes in Card 3, 9, 10 and 11 as before.
Figure 5.2 Polygonal Approximations to Uniaxial Loading Curves at Four Levels of Strain Rate
Table 5.1 Input Data Cards for Initial Run of Flat Plate Problem

<table>
<thead>
<tr>
<th>Card</th>
<th>Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>EXAMPLE 1 FLAT PLATE WITH PRESSURE LOADING</td>
</tr>
<tr>
<td>2</td>
<td>20 32 4 .100000E 01</td>
</tr>
<tr>
<td>3</td>
<td>400 0 100 .400000E-05</td>
</tr>
<tr>
<td>4</td>
<td>2 2 3 3</td>
</tr>
<tr>
<td>5</td>
<td>1 1500 1500 0.0 0.0</td>
</tr>
<tr>
<td>6</td>
<td>.300000E 08 .300000E 00 .780000E 05 .732350E-03 .100000E 00 3 1</td>
</tr>
<tr>
<td>7a</td>
<td>78000.0 .0026 40.0 5.0</td>
</tr>
<tr>
<td>7b</td>
<td>120000.0 .0082 400.0 3.0</td>
</tr>
<tr>
<td>7c</td>
<td>180000.0 .0482 4000.0 1.0</td>
</tr>
<tr>
<td>8</td>
<td>100 1 1 1</td>
</tr>
<tr>
<td>9</td>
<td>4 100 200 300 400</td>
</tr>
<tr>
<td>10</td>
<td>4 100 200 300 400</td>
</tr>
<tr>
<td>11</td>
<td>4 100 200 300 400</td>
</tr>
<tr>
<td>12</td>
<td>0.0 32.0 6</td>
</tr>
<tr>
<td>13a</td>
<td>0.0 32.0 45.0 22.5 0</td>
</tr>
<tr>
<td>13b</td>
<td>0.0 32.0 45.0 22.5 1</td>
</tr>
<tr>
<td>13c</td>
<td>0.0 0.0 45.0 22.5 0</td>
</tr>
<tr>
<td>13d</td>
<td>0.0 0.0 45.0 22.5 1</td>
</tr>
<tr>
<td>13e</td>
<td>20.0 32.0 45.0 22.5 0</td>
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<tr>
<td>13f</td>
<td>20.0 32.0 45.0 22.5 1</td>
</tr>
<tr>
<td>14</td>
<td>32.0 20.0</td>
</tr>
</tbody>
</table>

Table 5.2 Input Data Cards Changed for the First Restart Run

<table>
<thead>
<tr>
<th>Card</th>
<th>Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>800 400 100 .400000E-05</td>
</tr>
<tr>
<td>9</td>
<td>4 500 600 700 800</td>
</tr>
<tr>
<td>10</td>
<td>4 500 600 700 800</td>
</tr>
<tr>
<td>11</td>
<td>4 500 600 700 800</td>
</tr>
</tbody>
</table>
Sample listings of the printed output for the initial run are given in Tables 5.3 - 5.13. The program prints the $U_i(M,N)$, $Y_i(M,N)$, $SN_i(M,N)$ arrays over the entire range of $(M,N)$, of which Tables 5.4 - 5.7 and 5.9 - 5.11 give the values for $M = 2, 10$ and $1 \leq N \leq 33$ (i.e. the values along $y^1 = 0$ and $y^1 = 8$, see Figure 5.1). Also the program prints the $LMAT(M,N,K)$ array at every station through the thickness, of which Tables 5.6 and 5.12 give the array values at $K = 3, 4$. Table 5.13 shows the surface strains and energy balance prints.

Figures 5.3 - 5.6 present examples of the plotted output as obtained on the Cal Comp Model 780 Plotter using the plotting program described in Appendix E. Notice that the plots with time as the abscissa, Figures 5.4 - 5.6, are for the initial run plus the two successive restarts, an automatic feature of the plotting program.
### Table 5.3
First Page of Printed Output Summarizing the Input Data and Results of Stable Time Increment Check

**EXAMPLE 1** | **POSSIBLE LOAD**
---|---
20 meshes in the axial direction (D310 = 0.100000E01) | 30 meshes in the radial direction (D310 = 0.100000E01)

**BENDING TIME INCREMENT**: 0.614304E-09
**MEMBRANE TIME INCREMENT**: 0.664304E-09
**INPUT TIME INCREMENT**: 0.400000E-09

**TIME INCREMENT USED BY ANPC**: 0.400000E-09

**YOUNG’S MODULUS**: 0.300000E04
**POISSON’S RATIO**: 0.300000E00
**MASS DENSITY**: 0.733333E03
**WORLD STRESS**: 0.760000E00
**THICKNESS**: 0.100000E01

**START AT TIME STEP**: 0
**FINAL TIME STEP**: 400
**SURFACE STRAIN**: EVERY 100 TIME STEP
**RESTART WRITE**: EVERY 100 TIME STEP

**LAYER**: 4
**LAYER**: 1
**STRAIN**: 0
**STRAIN**: 0

**LAYER**: 4
**LAYER**: 1
**STRAIN**: 0
**STRAIN**: 0

**BOUNDARY CONDITIONS**: 1/2/3 = CLAMPED/SYMMETRY/HINGED
EDGE1 = 2
EDGE2 = 3
EDGE3 = 3
EDGE4 = 3

**PRINT OPTION CONTROL CARD**: 0/1 = NO PRINT/PRINT
1 = DISPLACEMENT INCREMENTS
1 = CARTESIAN COORDINATES, PRESSURE
1 = SURFACE NORMAL VECTOR COMPONENTS

**PRINT INFORMATION AT THE FOLLOWING TIME STEPS**
100 200 300 400

**PRINT L MATRIX (LMAT) AT THE FOLLOWING TIME STEPS**
100 200 300 400

**3-D PLOTS FOR THE FOLLOWING TIME STEPS**
100 200 300 400

**CONSTITUTIVE RELATION**: ELASTOPLASTIC-WORK HARDENING-STRAIN RATE DEPENDENT
**STRESS-STRAIN APPROXIMATION WAS**: 3 LAYERS

**STRESS-STRAIN AND STRAIN RATE PARAMETERS**

<table>
<thead>
<tr>
<th>J</th>
<th>0.600000E04</th>
<th>0.600000E04</th>
<th>1/0.600000E04</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.600000E04</td>
<td>2.000000E00</td>
<td>2.000000E00</td>
</tr>
<tr>
<td>2</td>
<td>1.200000E00</td>
<td>6.000000E00</td>
<td>4.000000E00</td>
</tr>
<tr>
<td>3</td>
<td>1.800000E00</td>
<td>4.000000E00</td>
<td>4.000000E00</td>
</tr>
</tbody>
</table>

**START DAMPING AFTER TIME STEP**: 1500
**TIME**: 0.600000E-02
**DAMPF**: 0.000000E00
**NFAC**: 0.000000E00

---

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Table 5.4 Sample of Initial Values of Cartesian Coordinates and Pressure Arrays for \( M = 2 \) and 10

<table>
<thead>
<tr>
<th>M</th>
<th>( x_m )</th>
<th>( y_m )</th>
<th>( z_m )</th>
<th>( x_m )</th>
<th>( y_m )</th>
<th>( z_m )</th>
<th>( p_{n,m} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>0.0000000000000000</td>
<td>0.0000000000000000</td>
<td>0.0000000000000000</td>
<td>0.0000000000000000</td>
<td>0.0000000000000000</td>
<td>0.0000000000000000</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>0.0000000000000000</td>
<td>0.0000000000000000</td>
<td>0.0000000000000000</td>
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<td>0.0000000000000000</td>
<td>0.0000000000000000</td>
<td>0.0000000000000000</td>
<td>0.0000000000000000</td>
<td>0.0000000000000000</td>
<td>0.0000000000000000</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>0.0000000000000000</td>
<td>0.0000000000000000</td>
<td>0.0000000000000000</td>
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<tr>
<td>6</td>
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<td>0.0000000000000000</td>
<td>0.0000000000000000</td>
<td>0.0000000000000000</td>
<td>0.0000000000000000</td>
<td>0.0000000000000000</td>
<td>0.0000000000000000</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>0.0000000000000000</td>
<td>0.0000000000000000</td>
<td>0.0000000000000000</td>
<td>0.0000000000000000</td>
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<td>0.0000000000000000</td>
<td>0.0000000000000000</td>
<td>0.0000000000000000</td>
<td>0.0000000000000000</td>
<td>0.0000000000000000</td>
<td>0.0000000000000000</td>
</tr>
<tr>
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<td>8</td>
<td>0.0000000000000000</td>
<td>0.0000000000000000</td>
<td>0.0000000000000000</td>
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<td>0.0000000000000000</td>
<td>0.0000000000000000</td>
<td>0.0000000000000000</td>
<td>0.0000000000000000</td>
</tr>
</tbody>
</table>

Note: The table continues with similar entries for \( M = 2 \) and \( M = 10 \).
Table 5.5 Sample of Displacement Increments Array During Time Increment 199 to 200 for N = 2 and 10

<table>
<thead>
<tr>
<th>N</th>
<th>U1(N, M)</th>
<th>U2(N, M)</th>
<th>U3(N, M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.0000000000000000E+00</td>
<td>0.0000000000000000E+00</td>
<td>0.0000000000000000E+00</td>
</tr>
<tr>
<td>3</td>
<td>0.0000000000000000E+00</td>
<td>-0.1381206393553796E-04</td>
<td>-0.616488329489617E-03</td>
</tr>
<tr>
<td>4</td>
<td>0.0000000000000000E+00</td>
<td>-0.1211680903176595E-04</td>
<td>-0.472004486989014E-04</td>
</tr>
<tr>
<td>5</td>
<td>0.0000000000000000E+00</td>
<td>-0.1050622978399257E-04</td>
<td>-0.177364250488556E-04</td>
</tr>
<tr>
<td>6</td>
<td>0.0000000000000000E+00</td>
<td>-0.971956587315917E-05</td>
<td>-0.250338305088261E-04</td>
</tr>
<tr>
<td>7</td>
<td>0.0000000000000000E+00</td>
<td>-0.892905659080736E-05</td>
<td>-0.45043119825864E-04</td>
</tr>
<tr>
<td>8</td>
<td>0.0000000000000000E+00</td>
<td>-0.816496582373936E-05</td>
<td>-0.214669682373936E-04</td>
</tr>
<tr>
<td>9</td>
<td>0.0000000000000000E+00</td>
<td>-0.7432077477079695E-05</td>
<td>-0.522072850204365E-04</td>
</tr>
<tr>
<td>10</td>
<td>0.0000000000000000E+00</td>
<td>-0.671041027326904E-05</td>
<td>-0.410174519626505E-04</td>
</tr>
<tr>
<td>11</td>
<td>0.0000000000000000E+00</td>
<td>-0.600000000000000E+00</td>
<td>-0.347658254107284E-04</td>
</tr>
<tr>
<td>12</td>
<td>0.0000000000000000E+00</td>
<td>-0.532036594292829E-05</td>
<td>-0.278682554107284E-04</td>
</tr>
<tr>
<td>13</td>
<td>0.0000000000000000E+00</td>
<td>-0.465698064838043E-05</td>
<td>-0.195381635225723E-04</td>
</tr>
<tr>
<td>14</td>
<td>0.0000000000000000E+00</td>
<td>-0.401854139986883E-05</td>
<td>-0.122959410136581E-04</td>
</tr>
<tr>
<td>15</td>
<td>0.0000000000000000E+00</td>
<td>-0.340465246234613E-05</td>
<td>-0.7635772677079695E-05</td>
</tr>
<tr>
<td>16</td>
<td>0.0000000000000000E+00</td>
<td>-0.287204505784250E-05</td>
<td>-0.77050430563238E-05</td>
</tr>
<tr>
<td>17</td>
<td>0.0000000000000000E+00</td>
<td>-0.234561451396983E-05</td>
<td>-0.723181635225723E-06</td>
</tr>
<tr>
<td>18</td>
<td>0.0000000000000000E+00</td>
<td>-0.182095941013658E-05</td>
<td>-0.69770543677719E-06</td>
</tr>
<tr>
<td>19</td>
<td>0.0000000000000000E+00</td>
<td>-0.1374065279720577E-05</td>
<td>-0.653876825541072E-06</td>
</tr>
<tr>
<td>20</td>
<td>0.0000000000000000E+00</td>
<td>-0.92474004662928E-06</td>
<td>-0.625077572649037E-06</td>
</tr>
<tr>
<td>21</td>
<td>0.0000000000000000E+00</td>
<td>-0.501980707367616E-06</td>
<td>-0.57698369028279E-06</td>
</tr>
<tr>
<td>22</td>
<td>0.0000000000000000E+00</td>
<td>-0.183472097984609E-06</td>
<td>-0.410174519626505E-04</td>
</tr>
<tr>
<td>23</td>
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<td>-0.119983869082797E-06</td>
<td>-0.347658254107284E-04</td>
</tr>
<tr>
<td>24</td>
<td>0.0000000000000000E+00</td>
<td>-0.868567007269784E-06</td>
<td>-0.278682554107284E-04</td>
</tr>
<tr>
<td>25</td>
<td>0.0000000000000000E+00</td>
<td>-0.576140139698383E-06</td>
<td>-0.214669682373936E-04</td>
</tr>
<tr>
<td>26</td>
<td>0.0000000000000000E+00</td>
<td>-0.379157894741586E-06</td>
<td>-0.150464525506708E-04</td>
</tr>
<tr>
<td>27</td>
<td>0.0000000000000000E+00</td>
<td>-0.24307592209293E-06</td>
<td>-0.802417135961302E-04</td>
</tr>
<tr>
<td>28</td>
<td>0.0000000000000000E+00</td>
<td>-0.130998032067519E-06</td>
<td>-0.546082470974037E-04</td>
</tr>
<tr>
<td>29</td>
<td>0.0000000000000000E+00</td>
<td>-0.801423748114468E-06</td>
<td>-0.265912946306651E-04</td>
</tr>
<tr>
<td>30</td>
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<td>-0.135953726447153E-04</td>
</tr>
<tr>
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<td>-0.719157637457296E-04</td>
</tr>
<tr>
<td>32</td>
<td>0.0000000000000000E+00</td>
<td>-0.777141027325423E-06</td>
<td>-0.365319360124109E-04</td>
</tr>
<tr>
<td>33</td>
<td>0.0000000000000000E+00</td>
<td>-0.316000000000000E+00</td>
<td>-0.130500000000000E+00</td>
</tr>
</tbody>
</table>

**Displacement Increments between T.S. 199 and 200**

**Notes:**
- The table provides a sample of displacement increments during time increment 199 to 200 for two and ten data points.
- The increments are calculated using specific mathematical formulas for each data point.\n- Each row represents a data point with corresponding increments in displacement for three different positions (U1, U2, U3).
- The increments are depicted in scientific notation for easier readability.

---

**Source:**
- The data is sourced from a structural analysis software or computational method used for engineering or scientific simulations.
- The increments are crucial for understanding the dynamic response of a system under applied forces or external influences.

---

**Further Reading:**
- Detailed mathematical formulations for calculating these increments are typically found in advanced mechanics, structural analysis, or computational fluid dynamics textbooks.
- Understanding these increments is vital for engineers in designing structures that can withstand various environmental and operational stresses.
### Table 5.6 Sample of Cartesian Coordinates and Pressure Arrays at Time Step 200 for M = 2 and 10

<table>
<thead>
<tr>
<th>TIME STEP 200</th>
<th>TIME step 0</th>
<th>CARTESIAN COORDINATES</th>
<th>PRESSURE (P/M.N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>T1(Tm, 200)</td>
<td>T2(Tm, 200)</td>
<td>T3(Tm, 200)</td>
</tr>
<tr>
<td>5</td>
<td>1.0</td>
<td>0.000000000000000E 00</td>
<td>0.000000000000000E 00</td>
</tr>
<tr>
<td>10</td>
<td>2.0</td>
<td>0.000000000000000E 00</td>
<td>0.000000000000000E 00</td>
</tr>
<tr>
<td>15</td>
<td>3.0</td>
<td>0.000000000000000E 00</td>
<td>0.000000000000000E 00</td>
</tr>
<tr>
<td>20</td>
<td>4.0</td>
<td>0.000000000000000E 00</td>
<td>0.000000000000000E 00</td>
</tr>
<tr>
<td>25</td>
<td>5.0</td>
<td>0.000000000000000E 00</td>
<td>0.000000000000000E 00</td>
</tr>
<tr>
<td>30</td>
<td>6.0</td>
<td>0.000000000000000E 00</td>
<td>0.000000000000000E 00</td>
</tr>
<tr>
<td>35</td>
<td>7.0</td>
<td>0.000000000000000E 00</td>
<td>0.000000000000000E 00</td>
</tr>
<tr>
<td>40</td>
<td>8.0</td>
<td>0.000000000000000E 00</td>
<td>0.000000000000000E 00</td>
</tr>
<tr>
<td>45</td>
<td>9.0</td>
<td>0.000000000000000E 00</td>
<td>0.000000000000000E 00</td>
</tr>
<tr>
<td>50</td>
<td>10.0</td>
<td>0.000000000000000E 00</td>
<td>0.000000000000000E 00</td>
</tr>
<tr>
<td>55</td>
<td>11.0</td>
<td>0.000000000000000E 00</td>
<td>0.000000000000000E 00</td>
</tr>
<tr>
<td>60</td>
<td>12.0</td>
<td>0.000000000000000E 00</td>
<td>0.000000000000000E 00</td>
</tr>
<tr>
<td>65</td>
<td>13.0</td>
<td>0.000000000000000E 00</td>
<td>0.000000000000000E 00</td>
</tr>
<tr>
<td>70</td>
<td>14.0</td>
<td>0.000000000000000E 00</td>
<td>0.000000000000000E 00</td>
</tr>
<tr>
<td>75</td>
<td>15.0</td>
<td>0.000000000000000E 00</td>
<td>0.000000000000000E 00</td>
</tr>
<tr>
<td>80</td>
<td>16.0</td>
<td>0.000000000000000E 00</td>
<td>0.000000000000000E 00</td>
</tr>
<tr>
<td>85</td>
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<td>0.000000000000000E 00</td>
</tr>
<tr>
<td>90</td>
<td>18.0</td>
<td>0.000000000000000E 00</td>
<td>0.000000000000000E 00</td>
</tr>
</tbody>
</table>

Note: The table continues with similar entries for different time steps and M values.
### Table 5.7 Sample of Surface Normal Components Array at Time Step 200 for M = 2 and 10

<table>
<thead>
<tr>
<th>TIME STEP</th>
<th>M</th>
<th>20</th>
<th>200</th>
<th>2000</th>
<th>20000</th>
<th>200000</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21</td>
<td>0.390000000000000000</td>
<td>0.579487560865093000</td>
<td>0.998317927214582600</td>
<td>0.998317927214582600</td>
<td>0.998317927214582600</td>
</tr>
<tr>
<td>2</td>
<td>21</td>
<td>0.500000000000000000</td>
<td>0.998317927214582600</td>
<td>0.998317927214582600</td>
<td>0.998317927214582600</td>
<td>0.998317927214582600</td>
</tr>
<tr>
<td>3</td>
<td>21</td>
<td>0.600000000000000000</td>
<td>0.998317927214582600</td>
<td>0.998317927214582600</td>
<td>0.998317927214582600</td>
<td>0.998317927214582600</td>
</tr>
<tr>
<td>4</td>
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<td>0.998317927214582600</td>
<td>0.998317927214582600</td>
<td>0.998317927214582600</td>
</tr>
<tr>
<td>5</td>
<td>21</td>
<td>0.800000000000000000</td>
<td>0.998317927214582600</td>
<td>0.998317927214582600</td>
<td>0.998317927214582600</td>
<td>0.998317927214582600</td>
</tr>
<tr>
<td>6</td>
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<td>0.900000000000000000</td>
<td>0.998317927214582600</td>
<td>0.998317927214582600</td>
<td>0.998317927214582600</td>
<td>0.998317927214582600</td>
</tr>
<tr>
<td>7</td>
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</tr>
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</table>

**M = 10**

<table>
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<th>200</th>
<th>2000</th>
<th>20000</th>
<th>200000</th>
</tr>
</thead>
<tbody>
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<td>0.000000000000000000</td>
<td>0.000000000000000000</td>
<td>0.000000000000000000</td>
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<tr>
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<td>0.000000000000000000</td>
<td>0.000000000000000000</td>
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</table>
Table 5.8 Sample of Stress Subincrement Array at Time Step 200 for Layers $K = 3$ and 4

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<th>8</th>
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Table 5.9 Sample of Displacement Increments Array During Time Increment 399 to 400 for M = 2 and 10
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<th>TEMP (K)</th>
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Table 5.10 Sample of Cartesian Coordinates and Pressure Arrays at Time Step 400 for M = 2 and 10
Table 5.11 Sample of Surface Normal Components Array at Time Step 400 for M = 2 and 10

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<th>SURFACE NORMAL VECTORS COMPONENTS</th>
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Sample of Surface Normal Components Array at Time Step 400 for M = 2 and 10.
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Table 5.12 Sample of Stress Subincrement Array at Time Step 400 for Layers K = 3 and 4

LMAT(m,N, 3)
Table 5.13 Samples of Energy Balance and of Surface Strains Printed Output at Time Steps 200 and 400

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<tr>
<th>TIME STEP 200</th>
<th>TIME= 0.80000000E-03</th>
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<td>SURFACE STRAINS</td>
<td>STRAIN GAGE READING</td>
</tr>
<tr>
<td>Eta1 Eta2 M N FACE</td>
<td>ANGLE 0</td>
</tr>
<tr>
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TOTAL ENERGY= 0.1257156E 07

TAPE 1 WRITTEN, N CYCLE= 200 TIME= 0.80000000E-03

<table>
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<th>TIME= 0.16000000E-02</th>
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TOTAL ENERGY= 0.12571482E 07

TAPE 1 WRITTEN, N CYCLE= 400 TIME= 0.16000000E-02

KINETIC= 0.8468797E 06 ELASTIC= 0.3499661E 06 PLASTIC= 0.630989CE 05
Figure 5.5 Isometric and Cross-sectional Cal Comp Plots of the Deformed Middle Surface at Time Steps 200 and 400 with Displacements Magnified by Factor of Three
Figure 5.4 Cal Comp Plot of the History of the Energy Balance, Showing in Ascending Order the Kinetic Energy, Total Energy and External Work.
Figure 5.1: Cai Corp Plot of the History of the Deflection at the Center of the Plate.
Figure 5.6 Cal Comp Plots of the History of the Surface Elongational Strains Along the Mesh Directions at the Center of Plate

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5.2 Example 2: Impulsively Loaded Cylinder

The second example involves a dynamic buckling problem: determine the final equilibrium configuration of a clamped end, aluminum cylinder due to an inwardly directed, instantaneous impulse delivered over a rectangular region of its surface. The impulse imparts a uniform velocity of 7500 in/sec over an 180° arc extending the entire length of the shell. The dimensions and geometry of the problem are shown in Figure 5.7. The example employs the following REPSIL options.

- Cylindrical shell initial geometry
- Input card specified impulsive loading
- Clamped edge and symmetry edge boundaries
- Elastic-perfectly plastic, strain rate insensitive material behavior
- Damping procedure

The material properties of the aluminum are:

- Young's Modulus $E = 10.7 \times 10^6$ psi
- Poisson's Ratio $\nu = 1/3$
- Yield Stress $\sigma_y = 42,000$ psi
- Mass Density $\rho = 2.59066 \times 10^{-4}$ $\frac{\text{lb} - \text{sec}^2}{\text{in}^4}$

As already indicated, the material is assumed perfectly plastic at the yield stress and independent of the strain rate.

Advantage is taken of the symmetry of the problem to restrict the analysis to the portion of the shell on the positive side of the $y^2$, $y^3$ coordinate plane between the $y^1$, $y^3$ coordinate plane and the cross-sectional symmetry plane, located midway between the ends of the cylinder, Figure 5.7. This makes edges 1, 2 and 3 symmetry boundaries and edge 4 a clamped boundary (compare with Figures 3.1 and 3.7). Also, only 180 degrees of the circumference and half the length are prescribed as the input dimensions and the loaded region is restricted to the first 90 degrees arc. The example uses 20 mesh intervals in the circumferential direction and 12 in the axial direction, giving an almost square mesh. The thickness is divided into 4 layers. The time increment is deliberately set equal to zero in order to assure that the stable time increment determined by the program is used to solve the example.
Figure 5.7 Geometry for Example Problem 2
The damping option is used to obtain a final equilibrium configuration. Based on a preliminary solution of the example without the use of damping it is determined that most of the plastic work is accomplished by time step 270. Hence, the damping for this example is picked to begin at time step 270. The example is set to run until time step 600, since the run may not be terminated by damping before time step 600, provision is made for collecting information for a restart run every 100 time steps. Table 5.14 lists the input data for the run in the order shown in Table 3.1. Notice that while the diameter to the outer surface is specified in Figure 5.9, Card 14 calls for the radius to the middle surface. Also, because half the area associated with each mesh point along the 90° line (i.e. along $M = 12$) receives the full impulse, only half the impulse velocity is assigned to these points in Cards 16a - 16i.

Tables 5.15 - 5.23 give sample listings of the printed output. For the sake of economy, only the values of arrays $U_i(M,N)$, $Y_i(M,N)$, $SN_i(M,N)$ at $M = 2, 12, 22$ and $1 \leq N \leq 13$ (i.e. along the 0°, 90°, 180° meridians) are given. Also, the LMAT $(M,N,K)$ array is given at $K = 1, 2$. Figures 5.8 - 5.11 give examples of the plotted output. Notice that the damping operations terminate the run at time step 525, corresponding to approximately 643 microseconds.
Table 5.14 Input Data Cards for Cylindrical Shell Problem

<table>
<thead>
<tr>
<th>Card 1</th>
<th>EXAMPLE 2 FULL CYLINDER WITH IMPULSIVE LOADING</th>
</tr>
</thead>
<tbody>
<tr>
<td>Card 2</td>
<td>20 12 4 0.0</td>
</tr>
<tr>
<td>Card 3</td>
<td>600 0 100 0.0</td>
</tr>
<tr>
<td>Card 4</td>
<td>2 2 2 1</td>
</tr>
<tr>
<td>Card 5</td>
<td>0 0 270 .100000E 00 .500000E-02</td>
</tr>
<tr>
<td>Card 6</td>
<td>.107000E 08 .33333333 .420000E 05 .259066E-03 .420000E-01 1 0</td>
</tr>
<tr>
<td>Card 8</td>
<td>50 1 1 1</td>
</tr>
<tr>
<td>Card 9</td>
<td>7 100 125 200 250 300 400 500</td>
</tr>
<tr>
<td>Card 10</td>
<td>4 100 200 300 400</td>
</tr>
<tr>
<td>Card 11a</td>
<td>24 25 50 75 100 125 150 175 200 225 250 275 300</td>
</tr>
<tr>
<td>Card 11b</td>
<td>325 350 375 400 425 450 475 500 525 550 575 600</td>
</tr>
<tr>
<td>Card 12</td>
<td>90.0 1.5 6</td>
</tr>
<tr>
<td>Card 13a</td>
<td>0.0 3.0 45.0 135.0 1</td>
</tr>
<tr>
<td>Card 13b</td>
<td>0.0 3.0 45.0 135.0 0</td>
</tr>
<tr>
<td>Card 13c</td>
<td>45.0 3.0 45.0 135.0 1</td>
</tr>
<tr>
<td>Card 13d</td>
<td>45.0 3.0 45.0 135.0 0</td>
</tr>
<tr>
<td>Card 13e</td>
<td>90.0 1.5 45.0 135.0 1</td>
</tr>
<tr>
<td>Card 13f</td>
<td>90.0 1.5 45.0 135.0 0</td>
</tr>
<tr>
<td>Card 14</td>
<td>.300000E 01 .295800E 01 .360000E 03</td>
</tr>
<tr>
<td>Card 15</td>
<td>2 11 2 13 .750000E 04 13</td>
</tr>
<tr>
<td>Card 16a</td>
<td>12 2 .375000E 04</td>
</tr>
<tr>
<td>Card 16b</td>
<td>12 3 .375000E 04</td>
</tr>
<tr>
<td>Card 16c</td>
<td>12 4 .375000E 04</td>
</tr>
<tr>
<td>Card 16d</td>
<td>12 5 .375000E 04</td>
</tr>
<tr>
<td>Card 16e</td>
<td>12 6 .375000E 04</td>
</tr>
<tr>
<td>Card 16f</td>
<td>12 7 .375000E 04</td>
</tr>
<tr>
<td>Card 16g</td>
<td>12 8 .375000E 04</td>
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<tr>
<td>Card 16h</td>
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<td>Card 16i</td>
<td>12 10 .375000E 04</td>
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<tr>
<td>Card 16j</td>
<td>12 11 .375000E 04</td>
</tr>
<tr>
<td>Card 16k</td>
<td>12 12 .375000E 04</td>
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<tr>
<td>Card 16l</td>
<td>12 13 .375000E 04</td>
</tr>
</tbody>
</table>

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Table 5.15 First Page of Printed Output Summarizing the Input Data and Results of Stable Time Increment Check

**EXHIBIT 2** FULL CYLINDER WITH IMPULSIVE LOADING

20 MESHES IN THE ETAI DIRECTION
12 MESHES IN THE ETA2 DIRECTION

BENDING TIME INCREMENT = 0.689333E-05
MEMBRANE TIME INCREMENT = 0.157900E-09
INPUT TIME INCREMENT = 0.200000E-00

TIME INCREMENT USED BY REPSEL = 0.150000E-05

YOUNG'S MODULUS = 0.107000E 08
POISSON'S RATIO = 0.333333E 00
YIELD STRESS = 0.420000E 09
MASS DENSITY = 0.259000E-03
THICKNESS = 0.420000E-01

START AT TIME STEP 0
FINAL TIME STEP 600
SURFACE STRAINS EVERY 50 TIME STEP
RESTART WRITE EVERY 100 TIME STEP

LAYER = 4
NSTRN = 0
LOAD = 0
LPRESS = 0

BOUNDARY CONDITIONS
1/2/3 = CLAMPED/SYMMETRY/HINGED
EDGE1 = 2
EDGE2 = 2
EDGE3 = 2
EDGE4 = 1

PRINT OPTION CONTROL CARD
0/1 = NO PRINT/PRINT
1 = DISPLACEMENT INCREMENTS
1 = PRESSURE
1 = SURFACE NORMAL VECTOR COMPONENTS

PRINT INFORMATION AT THE FOLLOWING TIME STEPS
100 125 200 250 300 400 500
PRINT MATRIX (LMAT) AT THE FOLLOWING TIME STEPS
100 200 300 400
3-D PLOTS FOR THE FOLLOWING TIME STEPS
25 50 75 100 125 150 175 200 225 250 275 300 325 350 375 400
425 450 475 500 525 550 575 600

CONSTITUTIVE RELATION
ELASTOPLASTIC-WORK HARDENING-STRAIN RATE INDEPENDENT

STRESS-STRAIN AND STRAIN RATE PARAMETERS

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<tr>
<th>SIG0(J)</th>
<th>SSIGS(J)</th>
<th>DSR(J)</th>
<th>1/PSR(J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.200000E 04</td>
<td>3.9252536E-13</td>
<td>0.000000E 00</td>
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</tbody>
</table>

START DAMPING AFTER TIME STEP 270 TIME = 0.40500E-03
DAMPF = 0.15000E 30
GFACT = 0.90000E-02
Table S.10  Second Page of Printed Output Summarizing the Input Impulse Velocities and Sample of Third Page of Output with Initial Values of Cartesian Coordinates Array for N = 2, 12, and 22

<table>
<thead>
<tr>
<th>N</th>
<th>V1(1,1)</th>
<th>V2(1,1)</th>
<th>V3(1,1)</th>
</tr>
</thead>
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<td>0.000000000000000E+00</td>
<td>0.000000000000000E+00</td>
<td>0.000000000000000E+00</td>
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<td>12</td>
<td>0.379999999999999E+13</td>
<td>0.379999999999999E+13</td>
<td>0.379999999999999E+13</td>
</tr>
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<td>22</td>
<td>0.379999999999999E+13</td>
<td>0.379999999999999E+13</td>
<td>0.379999999999999E+13</td>
</tr>
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<td>0.000000000000000E+00</td>
<td>0.000000000000000E+00</td>
<td>0.000000000000000E+00</td>
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<tr>
<td>12</td>
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<td>0.379999999999999E+13</td>
<td>0.379999999999999E+13</td>
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<tr>
<td>22</td>
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<td>4</td>
<td>0.000000000000000E+00</td>
<td>0.000000000000000E+00</td>
<td>0.000000000000000E+00</td>
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<tr>
<td>12</td>
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<td>0.379999999999999E+13</td>
<td>0.379999999999999E+13</td>
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<td>22</td>
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<td>0.379999999999999E+13</td>
</tr>
</tbody>
</table>

**OTHER VELOCITY DISTRIBUTION**

<table>
<thead>
<tr>
<th>N</th>
<th>V1(1,1)</th>
<th>V2(1,1)</th>
<th>V3(1,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.000000000000000E+00</td>
<td>0.000000000000000E+00</td>
<td>0.000000000000000E+00</td>
</tr>
<tr>
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<td>0.379999999999999E+13</td>
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<tr>
<td>22</td>
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<td>0.379999999999999E+13</td>
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</tbody>
</table>

**INITIAL CARTESIAN COORDINATES**

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<th>( x )</th>
<th>( y )</th>
<th>( z )</th>
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<td>1.000000000000000E+00</td>
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<tr>
<td>2.000000000000000E+00</td>
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<td>2.000000000000000E+00</td>
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<tr>
<td>3.000000000000000E+00</td>
<td>3.000000000000000E+00</td>
<td>3.000000000000000E+00</td>
</tr>
</tbody>
</table>

**Sample of Third Page of Output with Initial Values of Cartesian Coordinates Array for N = 2, 12, and 22**

1.000000000000000E+00
2.000000000000000E+00
3.000000000000000E+00

**N = 2, 11 AND N = 2, 13**

**RECEIVE FULL VELOCITY**

0.379999999999999E+13
Table 5.17 Samples for M = 2, 12, and 22 of Displacement Increments
Array During Time Increment 124 to 125 and of Cartesian
Coordinates and Pressure Arrays at Time Step 125

<table>
<thead>
<tr>
<th>Array</th>
<th>x</th>
<th>y</th>
<th>z</th>
<th>Pressure</th>
</tr>
</thead>
<tbody>
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<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
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<td>0.00000</td>
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</table>

Note: The table continues with more samples for different M values.
Table 5.18: Samples at Time Step 125 of Surface Normal Components Array for M = 2, 12, and 22 and of Stress Subincrement Array for Layers \( K = 1 \) and \( 2 \)

<table>
<thead>
<tr>
<th>TIME STEP 125</th>
<th>( \sum_{\text{normal components}} ) for ( K = 1 ) and ( 2 )</th>
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</thead>
<tbody>
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<td>( y )</td>
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<tr>
<td>13</td>
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</tbody>
</table>

The table continues with similar entries for various time steps and components.
Table 5.19 Samples for $M = 7, 12, \text{ and } 22$ of Displacement Increments
Array During Time Increment 249 to 250 and of Cartesian
Coordinates and Pressure Arrays at Time Step 250

<table>
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<th>$y$</th>
<th>$z$</th>
<th>Pressure</th>
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</table>
Table 5.20  Samples at Time Step 250 of Surface Normal Components Array for \( N = 2, 12, \) and 22 and of Stress Subincrement Array for Layers \( K = 1 \) and 2

<table>
<thead>
<tr>
<th>Time Step</th>
<th>Surface Normal Components</th>
<th>Stress Subincrement Array for ( K = 1 )</th>
<th>Stress Subincrement Array for ( K = 2 )</th>
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<tbody>
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<td>22</td>
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</tbody>
</table>
Table 5.21 Samples for N = 2, 12, and 22 of Displacement Increments
Array During Time Increment 400 to 500 and of Cartesian
Coordinates and Pressure Arrays at Time Step 569

<table>
<thead>
<tr>
<th>Time Step</th>
<th>Co-Ordinate Increment at 454 and 000</th>
<th>Co-Ordinate Increment at 454 and 000</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
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<tr>
<td>2</td>
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<td>22</td>
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Table 5.22  Samples at Time Step 500 of Surface Normal Components
Array for M = 2, 12, and 22 and of Stress Subincrement
Array for Layers K = 1 and 2

<table>
<thead>
<tr>
<th>Time Step</th>
<th>Normal Components</th>
<th>Stress Subincrement</th>
</tr>
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<tbody>
<tr>
<td></td>
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</tbody>
</table>

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Table 5.23 Samples of Energy Balance and of Surface Strains Printed Output at Time Steps 250 and 500

<table>
<thead>
<tr>
<th>TIME STEP</th>
<th>TIME</th>
<th>KINETIC</th>
<th>ELASTIC</th>
<th>PLASTIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>0.3750000E-03</td>
<td>0.4823439E-02</td>
<td>0.5104436E-02</td>
<td>0.7192354E-04</td>
</tr>
<tr>
<td>500</td>
<td>0.6199778E-03</td>
<td>0.1153929E-02</td>
<td>0.3108325E-02</td>
<td>0.7205382E-04</td>
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</tbody>
</table>

**TIME STEP 250**

<table>
<thead>
<tr>
<th>ETAl</th>
<th>ETA2</th>
<th>M</th>
<th>N</th>
<th>FACE</th>
<th>ANGLE D</th>
<th>ANGLE 90</th>
<th>ANGLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>2,704</td>
<td>3,000</td>
<td>2,000</td>
<td>13,000</td>
<td>INNER</td>
<td>0.2196275E-00</td>
<td>0.1982532E-01</td>
<td>45.00</td>
</tr>
<tr>
<td>1,162</td>
<td>3,000</td>
<td>2,000</td>
<td>13,000</td>
<td>OUTER</td>
<td>0.1308712E-00</td>
<td>0.1991605E-01</td>
<td>45.00</td>
</tr>
<tr>
<td>1,162</td>
<td>3,000</td>
<td>7,000</td>
<td>13,000</td>
<td>INNER</td>
<td>0.1123653E-00</td>
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<tr>
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<td>45.00</td>
</tr>
<tr>
<td>2,123</td>
<td>1,500</td>
<td>12,000</td>
<td>7,000</td>
<td>INNER</td>
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<tr>
<td>2,123</td>
<td>1,500</td>
<td>12,000</td>
<td>7,000</td>
<td>OUTER</td>
<td>0.7085734E-00</td>
<td>0.6493766E-02</td>
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**TIME STEP 500**

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<tr>
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<th>ETA2</th>
<th>M</th>
<th>N</th>
<th>FACE</th>
<th>ANGLE D</th>
<th>ANGLE 90</th>
<th>ANGLE</th>
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</thead>
<tbody>
<tr>
<td>2,704</td>
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<td>OUTER</td>
<td>0.1308712E-00</td>
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<td>1,162</td>
<td>3,000</td>
<td>7,000</td>
<td>13,000</td>
<td>INNER</td>
<td>0.1123653E-00</td>
<td>0.3745625E-01</td>
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<td>0.7085734E-00</td>
<td>0.6493766E-02</td>
<td>135.00</td>
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</table>
Figure 5.8 Isometric and Cross-sectional Cal Comp Plots of the Deforming Middle Surface at Selected Time Steps Showing True (Unmagnified) Displacements
a. Entire history of the energy balance from 0 to 643 microseconds, showing in ascending order the kinetic energy, total energy, total energy plus damping work, and external work.

b. Blow up of the history of the energy balance from 300 to 643 microsecond, showing in ascending order the kinetic energy, total energy, and total energy plus damping work.

Figure 5.9 Cal Comp Plots of the Energy Balance for the Impulsively Loaded Cylinder
Figure 5.10 Cal Comp Plot of the History of the Deflection at Point 90° from Crown Line and Midway Between Clamped and Symmetry Edges

LOCATION

\[
\begin{align*}
\eta_1 &= 2.323 \quad (12.000) \\
\eta_2 &= 1.500 \quad (7.000)
\end{align*}
\]
Figure 5.11  Cal Comp Plots of the History of the Surface Elongational Strain: Along the Mesh Directions

a. Strains on the crown line at half the axial length.
b. Strains at 45° from the crown line and half the axial length.

Figure 3.11 (Continued)
c. Strains at 90° from the crown line and quarter the axial length.

Figure 5.11 (Continued)
6. IMPLEMENTATION OF LOADINGS AND INITIAL GEOMETRIES

REPSIL can be made to accept arbitrary impulse loadings, arbitrary time and space varying pressure loadings, and, within certain limits, arbitrary initial shell geometries. This involves a small amount of programming confined to certain REPSIL subroutine: INVEL, PRESS and INGEOM. This chapter describes the acceptable ways of implementing loadings and geometries into REPSIL so as to be compatible with the code's formulation.

6.1 Two Approaches to Implementing Loads and Geometries

Implementing a loading or an initial geometry in REPSIL basically involves assigning to each mesh point \((m,n)\) a value of initial impulse velocity or a value of pressure for each time step, or values of the initial coordinates of the middle surface. These values can be assigned in two ways: first, they can be read off a tape or cards and assigned to the mesh points by the appropriate subroutine; second, an analytical expression programmed into the appropriate subroutine can generate these values, which the subroutine then assigns to the mesh points. The first approach requires that data be generated outside the program and be read onto a tape or punched onto cards. The data itself can be generated analytically, numerically or experimentally -- this approach will accept all; therein lies its advantage, especially for ad hoc problems. Its drawbacks are the tediousness of reading onto a tape or punching onto cards large numbers of values especially when many mesh points are employed, and necessity of regenerating new data for a given physical problem whenever the number of mesh intervals is changed. The second approach has none of these drawbacks, but does require that the data be expressible analytically. These analytical expressions are most conveniently written as functions of the underlying material coordinates \((n^1, n^2)\) rather than the mesh numbers \((m,n)\), for then the forms of the expressions are not affected by a change in the number of mesh intervals. The value of such an expression at a mesh point \((m,n)\) is obtained as the value of the function at the material coordinates \((n^1, n^2)\) corresponding to the point \((m,n)\):

\[
\begin{align*}
    n^1(m) &= n^1_0 + (m-1) \Delta n^1, \\
    n^2(n) &= n^2_0 + (n-1) \Delta n^2
\end{align*}
\]  

\*(6.1)\*

* These equations are closer to the indexing of \(m\) and \(n\) used in the program than (2.1), reflecting the fact that FORTRAN does not permit zero values of array indices.*
where, for $M \times N$ mesh intervals, $1 \leq m \leq M + 1$, $1 \leq n \leq N + 1$ and

$$
\Delta n^1 = \frac{n^1 - n^0}{M}, \quad \Delta n^2 = \frac{n^2 - n^0}{N}.
$$

In the subsequent sections, in which the implementation of the loading and geometry subroutines are individually described, the two approaches are again covered, as they apply to each subroutine.

### 6.2 Subroutine INVEL

Initial impulse loadings are specified in subroutine INVEL. At the zeroth time step INVEL assigns a value of impulse velocity to each mesh point. The velocity is free to vary spatially over the shell surface, but at present is restricted to be directed along the normal - a minor restriction that is easily remedied by allowing tangential components to be defined through the middle surface basis vectors, see Section 2.3.

As stated in the previous section, two approaches can be used in programming INVEL. With the first approach, INVEL simply reads off cards or a tape a sequence of values corresponding to the normal velocity $v$ at each mesh point. Each value is immediately multiplied by the surface normal at the mesh point to give components of the velocity

$$
v^i = v n^i,
$$

which are then stored in the as yet unused three $M \times N$ arrays $\Delta u^i(m,n)$. A form of this approach is used for the subroutine INVEL presently in REPSIL, which is specified on input Card 15 and 16, see Section 3.2. (A listing of this subroutine is given in Appendix E.)

The second approach requires some analytic expressions of the form

$$
v = v(n^1, n^2)
$$

for the normal velocity as a function of the material coordinates. The material coordinates are limited to the rectangular domain specified in INGEOM, see (6.10) of Section 6.4. With this approach, (6.1) and (6.4) are both programmed into INVEL; (6.2) need not be included since the increments $\Delta n^i$ are calculated in INGEOM. INVEL determines the normal velocity at mesh points by calculating the values of the material
coordinates at the mesh point (m,n) using (6.1) and substituting these values in (6.4) to obtain

\[ v(m,n) = v(\eta^1(m), \eta^2(n)). \]  

(6.5)

As with the first approach, these values are immediately multiplied by the normal to obtain the components of the velocity at (m,n), which are then stored in the three \( M \times N \) arrays \( \Delta u^1(m,n) \).

6.3 Subroutine PRESS

Time/space varying pressure loads are specified in subroutine PRESS. Each time step this subroutine assigns a value of pressure to each mesh point.

With the first of the two approaches to programming already mentioned, at each time step PRESS reads off cards or (more usually) a tape a sequence of values of the pressure \( P \) at each mesh point and stores these values in the \( M \times N \) array \( P(m,n) \) for later use in subroutine MOTION, see Section 2.3. This approach has been used in REPSIL for a number of cases, in which the pressure data has been experimentally/numerically generated; reports on these cases are in preparation.

For the second approach, analytic expressions for the pressure as a function of the material coordinates and the time:

\[ P = F (\eta^1, \eta^2, t) \]  

(6.6)

are programmed into PRESS, along with (6.1); the material coordinates are limited to the rectangular domain (6.10) specified in INGEOM. At each time step \( \ell \), PRESS calculates the pressure \( P \) at each mesh point \((m,n)\) by determining the material coordinates of the mesh point from (6.1), the value of time at time step \( \ell \) from

\[ t(\ell) = \ell \Delta t \]  

(6.7)

and substituting these values in (6.6):

\[ P(m,n) = P(\eta^1(m), \eta^2(n), t(\ell)). \]  

(6.8)

As with the first approach, these values are stored in the \( M \times N \) array.
P \( (m,n) \). This approach is used to generate the pressure data for example problem 1 (Section 5.1); a listing of this PRESS subroutine is in Appendix E.

6.4 Subroutine INGEOM

The initial geometry of the shell is specified in subroutine INGEOM. A new subroutine INGEOM must be written for each initial geometry or at least each family of initial geometries.* The only restrictions on the admissible geometries are that the middle surface be simply connected and bounded by four smooth edges, such that none of the corners formed by intersecting edges are reentrant or straight.

Particularizing the remarks of Section 6.1 to initial geometries, the basic function of INGEOM is to set up a correspondence between mesh numbers \((m,n)\) and coordinates \(y^1\) in 3-space through which the middle surface of the shell initially passes; this correspondence should be one-to-one. As already mentioned, two approaches can be used in programming INGEOM. First, INGEOM can just comprise instructions for reading off cards or a tape a sequence of ordered triplexes \((y^1, y^2, y^3)\) and storing these in three \(M \times N\) arrays \(y^1(m,n), y^2(m,n)\) and \(y^3(m,n)\). Of course, the sequence of triplexes \(y^1\) must be chosen carefully, not only for sake of obtaining a one-to-one correspondence, but also in order that the correspondence be topologically continuous in the following sense: neighboring points in space are assigned adjacent mesh numbers in the proper order, as indicated in Figure 6.1. This approach requires that the increments in material coordinates, \(\Delta n^1\) and \(\Delta n^2\), be assigned convenient values that are subsequently used to form finite difference quotients. Also, it is important that the number of mesh intervals \(M\) and \(N\) assigned elsewhere in the program jibe with the spacing implied by subroutine INGEOM.

The second approach uses one or more sets of analytic expressions giving the Cartesian coordinates \(y^i\) of the middle surface parametrically as functions of the material coordinates \(n^n:\)

\[
y^1 = y^1(n^1, n^2), \quad y^2 = y^2(n^1, n^2), \quad y^3 = y^3(n^1, n^2) \quad (6.9)
\]

These functions are required to be continuous and one-to-one in the domain over which the material coordinates vary; the domain itself is

* REPSIL has INGEOM subroutines for a flat plate, cylindrical shell and conical shell programmed, see Section 3.2 and Appendix E.
Figure 6.1 Mapping of Material Coordinate Mesh onto the Middle Surface of Shell
limited to some rectangle in the material coordinates plane:

\[ \eta_0^1 \leq \eta^1 \leq \eta_f^1, \quad \eta_0^2 \leq \eta^2 \leq \eta_f^2. \quad (6.10) \]

Consequently, the parametric representation of the middle surface is simply a one-to-one continuous map of a rectangle in the \( \eta^1, \eta^2 \) plane into 3-space, from which automatically follow the aforementioned topological continuity and restrictions on the boundary. It should be noted that the parametric representation is not unique—many exist for a given surface. Moreover, the material coordinates need not have physical significance, such as arc length, angle, etc., although often a simple transformation can give them such meaning.

With the second approach, equations (6.9) are programmed into INGEOM, as well as (6.1) and (6.2). For a given physical problem, the extent of the shell is fixed by the limits on the material coordinates (6.10). Once the number of mesh intervals \( M \times N \) are specified, the subroutine computes the constant intervals \( \Delta \eta^a \) using (6.2) and the values of the material coordinates \( \eta^a(m) \) and \( \eta^a(n) \) at the mesh points \((m,n)\) using (6.1). Substituting these values of the material coordinates into the analytical expression (6.9), the subroutine calculates the Cartesian coordinates of the mesh points:

\[ y^1 = y^1(m,n), \quad y^2 = y^2(m,n), \quad y^3 = y^3(m,n), \quad (6.11) \]

thus mapping the rectangular \( M \times N \) mesh in the \( \eta^1, \eta^2 \) plane into a curvilinear mesh in 3-space, as pictured in Figure 6.1. As with the first approach, the subroutine stores these values in three \( M \times N \) arrays.

The second approach to programming INGEOM will be illustrated with an example of a shell having for its initial middle surface a frustum of a circular cone with an axial length \( L \), a small radius \( R_0 \) and large radius \( R_f \). Let the frustum be located relative to the \( y^1, y^2, y^3 \) axis as shown in Figure 3.8. With this orientation the coordinates of the frustum are parametrically given by

\[ y^1 = \eta^1 \sin \eta^2, \quad y^2 = \frac{\eta^1 - R_0}{R_f - R_0} L, \quad y^3 = \eta^1 \cos \eta^2, \quad (6.12) \]

with the material coordinates limited to the domain
In this representation both parameters have physical significance: \( n_1 \) being the radius from the cone axis and \( n_2 \) the angle about the axis.

This representation is not unique; for example, \( n_1 \) can be replaced by a parameter \( \eta_2 \) measuring arclength along generators through the transformation

\[
\eta_2 = n_1 \csc \alpha ,
\]

with \( \alpha \) the cone angle:

\[
\alpha = \arctan \frac{R_f - R_o}{L} ,
\]

resulting in the representation

\[
y_1 = n_1 \sin \alpha \sin n_2 , \quad y_2 = (n_1 - S_o) \cos \alpha , \quad y_3 = n_1 \sin \alpha \cos n_2 ,
\]

with domain

\[
S_o (= R_o \csc \alpha) \leq n_2 \leq S_f (= R_f \csc \alpha) , \quad 0 \leq n_2 \leq 2 \pi .
\]

Clearly, a transformation replacing \( n_2 \) by circumferential arclength cannot exist, for it is impossible for the same angle \( n_2 \) at different location along the cone axis to subtend equal arclength on the surface of a cone. For the same reason, the image of an \( M \times N \) mesh under the representation (6.15) or for that matter (6.12) will yield a curvilinear rectangular mesh with nonequal rectangles: as shown in Figures 3.8 and 6.2a, while the meridional lengths of rectangles are equal, the circumferential lengths increase with the radius. However, it is possible through a judicious transformation of \( n_1 \) or \( \eta_1 \) into say \( \eta_1 \) to obtain nonuniform meridional increments that give similar rectangles (i.e. rectangles of constant side ratio). Using differentials, the condition that constant increments \( \Delta n_1 \) and \( \Delta \eta_1 \) give rectangles with a constant meridional to circumferential side ratio of \( \nu \) can be written as

\[
R_o \leq n_1 \leq R_f \quad \& \quad 0 \leq n_2 \leq 2 \pi .
\]
Assuming that $\hat{n}^1$ is a function of $\hat{n}^1$ and using (6.15), this condition is shown to be equivalent to the differential equation

$$\frac{d\hat{n}^1}{d\hat{n}^1} = K \sin \alpha \hat{n}^1, \quad K = \kappa \frac{\Delta \hat{n}^2}{\Delta \hat{n}^1}. \quad (6.16)$$

Choosing the lower limit on $\hat{n}^1$ to be zero for the sake of convenience, the solution of this differential equation yields the transformation

$$\hat{n}^1 = S_o e^{K \sin \alpha \hat{n}^1}, \quad (6.17)$$

in the range $0 \leq K \sin \alpha \hat{n}^1 \leq \ln \frac{S_f}{S_o}$. This transformation is substituted in (6.15) to give the parametric representation

$$y^1 = S_o \sin \alpha \sin \hat{n}^2 e^{K \sin \alpha \hat{n}^1}$$

$$y^2 = S_o \cos \alpha \left( e^{K \sin \alpha \hat{n}^1} - 1 \right) \quad (6.17)$$

$$y^3 = S_o \sin \alpha \cos \hat{n}^2 e^{K \sin \alpha \hat{n}^1}$$

over the domain

$$0 \leq K \sin \alpha \hat{n}^1 \leq \ln \frac{S_f}{S_o}, \quad 0 \leq \hat{n}^2 < 2\pi \quad (6.18)$$

Since the program specified the number of mesh intervals $M$ and $N$ rather than the increments $\Delta \hat{n}^a$, the constant $K$ can be conveniently set equal to unity. Also (6.15) and (6.17) can be summarized in a single general representation by a change in the scale of $\hat{n}^1$, giving the representation

$$y^1 = S_o \sin \alpha \sin \hat{n}^2 f(\hat{n}^1 \sin \alpha)$$

$$y^2 = S_o \cos \alpha \left[ f(\hat{n}^1 \sin \alpha) - 1 \right] \quad (6.19)$$

$$y^3 = S_o \sin \alpha \cos \hat{n}^2 f(\hat{n}^1 \sin \alpha),$$

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where for a uniform meridional spacing

\[ f(n \sin \alpha) = n \sin \alpha \; ; \; 1 \leq n \sin \alpha \leq \frac{S_f}{S_0} \]  \hspace{1cm} (6.20)

and for a constant side ratio

\[ f(n \sin \alpha) = \exp(n \sin \alpha) \; ; \; 0 \leq n \sin \alpha \leq \ln \frac{S_f}{S_0} \]  \hspace{1cm} (6.21)

The programming of this representation is straightforward and is included in the listing in Appendix E. Figure 6.2 shows the difference between the two types of spacing for given M x N mesh intervals, reproduced from Cal Comp plots.

a. Constant meridional increment  b. Constant (almost square) side ratio

Figure 6.2 Comparison of the 18 x 9 Mesh Generated by Subroutine INGEOM for the Frustum of a Cone Using the Constant Meridional Increment Option and the Constant Mesh Proportions Option
REFERENCES


APPENDIX A.
FINITE DIFFERENCE OPERATORS

The partial differential equations solved by the REPSIL code employ the following partial derivatives with respect to the material coordinates:

$$\frac{\partial}{\partial n_1}, \frac{\partial}{\partial n_2}, \frac{\partial^2}{(\partial n_1)^2}, \frac{\partial^2}{\partial n_1 \partial n_2}, \frac{\partial^2}{(\partial n_2)^2}.$$ 

These partials are approximated in the code by finite difference operators of order $|\Delta n|^2$ in accuracy. These operators, which were symbolized by their corresponding partials in Section 2.3, will now be given explicitly.

Let $F(m,n)$ represent a typical mesh function (the position coordinates $y_1$ or the bending resultants $M_{AB}$, for example) defined over the domain of mesh points $m_1 < m < m_f$ and $n_1 < n < n_f$. At interior mesh points where the mesh numbers $(m,n)$ satisfy $m_1 < m < m_f$ and $n_1 < n < n_f$, central difference operators, symbolized by the superscript $c$, are used:

$$\frac{\Delta F_c^c}{\Delta n_1^1}(m,n) = \frac{F(m+1,n) - F(m-1,n)}{2\Delta n_1},$$

$$\frac{\Delta F_c^c}{\Delta n_2^2}(m,n) = \frac{F(m,n+1) - F(m,n-1)}{2\Delta n_2},$$

$$\frac{\Delta^2 F_c^c}{(\Delta n_1)^2}(m,n) = \frac{F(m+1,n) - 2F(m,n) + F(m-1,n)}{(\Delta n_1)^2},$$

$$\frac{\Delta^2 F_c^c}{(\Delta n_2)^2}(m,n) = \frac{F(m,n+1) - 2F(m,n) + F(m,n-1)}{(\Delta n_2)^2},$$

$$\frac{\Delta^2 F_c^c}{\Delta n_1 \Delta n_2}(m,n) = \frac{F(m+1,n+1) - F(m+1,n-1) - F(m-1,n+1) + F(m-1,n-1)}{\Delta n_1 \Delta n_2},$$

with $\Delta n_1$, $\Delta n_2$ (constant) increments in the material coordinates. Notice that the mixed partials operator involves the successive application of the first partial operators:
\[
\frac{\Delta^{2} F^{c}}{\Delta \eta_{1} \Delta \eta_{2}} (m, n) = \frac{\Delta F^{c}(m+1, n) - \Delta F^{c}(m-1, n)}{2\Delta \eta_{1}} - \frac{\Delta F^{c}(m, n+1) - \Delta F^{c}(m, n-1)}{2\Delta \eta_{2}}
\]

or more compactly:

\[
\frac{\Delta^{2} F^{c}}{\Delta \eta_{1} \Delta \eta_{2}} = \frac{\Delta}{\Delta \eta_{1}} \left( \frac{\Delta F^{c}}{\Delta \eta_{2}} \right)^{c} = \frac{\Delta}{\Delta \eta_{2}} \left( \frac{\Delta F^{c}}{\Delta \eta_{1}} \right)^{c}
\]

Along the boundaries where \( m = m_{i} \) or \( m_{f} \) or \( n = n_{j} \) or \( n_{f} \) central difference operators cannot be used to approximate all partials due to \( F(m, n) \) not being defined outside the domain of mesh points. Rather, some of the above central difference operators are replaced by either forward or backward difference operators. Specifically, along \( m = m_{i} \) or \( m_{f} \) only the central difference operators \( \frac{\Delta F^{c}}{\Delta \eta_{2}} \) and \( \frac{\Delta^{2} F^{c}}{(\Delta \eta^{2})^{2}} \) are employed.

The remaining operators are replaced by forward difference operators (denoted by the superscript \( f \)) along \( m = m_{i} \):

\[
\frac{\Delta F^{f}}{\Delta \eta_{1}} (m, n) = \frac{3F(m, n) - 4F(m+1, n) + F(m+2, n)}{2\Delta \eta_{1}}
\]

\[
\frac{\Delta^{2} F^{f}}{\Delta \eta_{1}^{2}} (m, n) = \frac{2F(m, n) - 5F(m+1, n) + 4F(m+2, n) - F(m+3, n)}{(\Delta \eta_{1})^{2}}
\]

and by backward difference operators (denoted by the superscript \( b \)) along \( m = m_{f} \):

\[
\frac{\Delta F^{b}}{\Delta \eta_{1}} (m, n) = \frac{3F(m, n) - 4F(m-1, n) + F(m-2, n)}{2\Delta \eta_{1}}
\]

\[
\frac{\Delta^{2} F^{b}}{\Delta \eta_{1}^{2}} (m, n) = \frac{2F(m, n) - 5F(m-1, n) + 4F(m-2, n) - F(m-3, n)}{(\Delta \eta_{1})^{2}}
\]
\[
\frac{\Delta^2_{F}^b (m,n)}{\Delta n^1 \Delta n^2} = \frac{3 \Delta F^c (m,n) - 4 \Delta F^c (m-1,n) + \Delta F^c (m - 2,n)}{2 \Delta n^1}.
\]

Notice again that the mixed partials operator involves the successive application of first partial operators (forward or backward with respect to \(n^1\) and central with respect to \(n^2\)):

\[
\frac{\Delta^2_{F}^c (m,n)}{\Delta n^1 \Delta n^2} = \frac{\Delta}{\Delta n^1} \left( \frac{\Delta F^c}{\Delta n^2} \right)^f, \quad \frac{\Delta^2_{F}^b (m,n)}{\Delta n^1 \Delta n^2} = \frac{\Delta}{\Delta n^1} \left( \frac{\Delta F^c}{\Delta n^2} \right)^b.
\]

Where possible the code takes advantage of this fact to reduce calculations.

Along the boundary \(n = n_i\) or \(n_f\) the central operators \(\frac{\Delta F^c}{\Delta n^1}\) and \(\frac{\Delta^2 F^c}{\Delta n^1 (\Delta n^2)^2}\) are retained, while the remaining operators are replaced by appropriate forward or backward operators obtained from the above operators by interchanging the roles of \(m\) and \(n\), and \(n^1\) and \(n^2\) simultaneously.

At the corners of the domain where \(m = m_i\) or \(m_f\) and \(n = n_i\) or \(n_f\) the remaining central operators are replaced by the forward or backward operators appropriate to the common boundaries; in particular, the mixed partials operators involve the successive application of the appropriate forward or backward first partial operators -- for example, at \(m = m_i\) and \(n = n_f\)

\[
\frac{\Delta^2 F}{\Delta n^1 \Delta n^2} = \frac{\Delta}{\Delta n^1} \left( \frac{\Delta F^b}{\Delta n^2} \right)^f.
\]

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APPENDIX B

COMMENTS ON ELASTOPLASTIC STRESS EVALUATIONS

In the REPSIL code stresses at time \( t \) are calculated in the manner of (finite) incremental plasticity using the incremental strains \( \Delta \epsilon^q_0 \) which occur between time \( t - \Delta t \) and time \( t \) as well as stored values of the stresses at time \( t - \Delta t \). In the finite difference analysis these calculations are made at every \((m,n)\) mesh node for each \( k \) layer. Additionally, where the strain-hardening constitutive option is used, stresses or, more precisely, substresses are calculated for each \( j \) sublayer of the mechanical sublayer model (see [2; Section 5.4.2]) and the stresses in each \( k \) layer are determined as weighted averages of the stresses in the \( j \) sublayers.

At locations where the incremental strains \( \Delta \epsilon^q_0 \) entail plastic flow, the flow parameter \( \Delta \lambda \) of (2.24) must be evaluated as the root of a quadratic equation, see (2.26). An algorithm for dealing with the various types of roots which may occur has been developed by Huffington [3]. If complex roots occur this algorithm subdivides the elastic stress increment, defined by (2.18), into \( L \) equal subincrements (for purposes of stress calculation only) in each of which elastic stress increments \( \Delta \sigma^E_0 / L \) take place. Stress calculations are performed for each of the \( L \) subincrements, consecutively. If at any stage of this process a complex root is obtained, the value of \( L \) is increased and the calculation is re-initiated. This procedure is continued until real stresses are determined for time \( t \).

Experience with this procedure has shown that under certain circumstances it may be desirable to use the subdivided increment algorithm even when real roots are obtained for the full increment. An illustration of this is depicted in Figure B.1, where for simplicity it is assumed that \( \sigma^1_1 \) and \( \sigma^2_2 \) are principal stresses \( \sigma^1_2 = 0 \). The values shown on this figure were taken from an actual computer solution. Starting from the stress state labeled "1", the components of the trial elastic stress vector \( \Delta \sigma^E \) were calculated using (2.18) and the correction stress vector \( \Delta \lambda \sigma^D_0 \) was determined by use of (2.23) and (2.28). These calculations which resulted in a real root for \( \Delta \lambda \), indicate that the actual stress increment \( \Delta \sigma \) for the full time step is represented by the vector joining points "1" and "2". Similarly calculations for the next time step result in a stress increment from point "2" to point "3" and subsequent calculations indicate continued oscillations between the third and fourth quadrants.

Suspecting that these calculations were inaccurate, the calculations for the first time step were performed using the subdivided stress increment algorithm with \( L = 2 \). At the end of the first half increment, the stress state labeled "1.5" was predicted and, for end of the second

\[ \text{Preceding page blank} \]
half step, the point labeled "2". Thus, the revised stress increment for the full cycle is presented by the vector \( \Delta \sigma^* \). Repeat applications of this algorithm using larger values of the subincrement counter \( L \) did not produce any appreciable revision of the stress increment vector \( \Delta \sigma^* \) so that this procedure appears to be rapidly convergent.

It is apparent that the inaccuracy associated with the use of \( L=1 \) in this example results from the large excursion outside the yield surface performed by the \( \Delta E \) vector. As a rational approach to arriving at an appropriate value of \( L \) to use in a given case, consider that the yield surface is surrounded by concentric ellipsoidal annuli which are labeled \( L=1,2,3,\ldots \) as one progresses radially outward from the yield surface (see Figure B.1). The elastic region within the yield surface is arbitrarily designated \( L=0 \). Then the appropriate value of \( L \) for the subincrement calculation would be determined by the designation of the ellipsoidal annulus in which the tip of the \( \Delta E \) vector is located i.e., the trial stress state \( \sigma \) given by (2.19). The analytical formulation of the criterion for selecting \( L \) may be expressed as:

\[
L = \text{INT} \left\{ \frac{Y \text{LDFAC}}{\left( \sqrt{\frac{\phi_T + \sigma_0^2}{\sigma^2}} - 1 \right)} + 1 \right\}
\]

where \( \text{INT} \) signifies the "integer part of" the quantity within the curly brackets. Note that \( \phi_T \), the function defined by (2.20) with the components of \( \sigma \) substituted, is already calculated within the STRESS subroutine to test for yielding. The coefficient YLDFAC is a parameter (not necessarily integral) which permits variation of the "thickness" of the concentric ellipsoidal annuli in stress space. The user should choose YLDFAC to suit his compromise between minimum computer time (YLDFAC=0, for which no subdivision of the elastic stress increment occurs as long as a real \( \Delta \lambda \) root is obtained) and maximum accuracy (YLDFAC=\( \infty \), corresponding to differential subintervals). Although no systematic study of the effect of varying YLDFAC has been conducted, it appears that reasonably accurate stress determinations have been obtained using YLDFAC = 1 with only nominal increases in machine time. For this value of YLDFAC the ellipsoidal annulus interfaces intersect the \( \sigma^Y \) axes at integral multiples of the uniaxial yield stress \( \sigma_0 \).

The REPSIL code now provides an optional printout of \( L \) (m,n) arrays for each \( k \) layer of the structural shell at specified time cycles. These arrays provide a definition of the regions of the structural shell within which plastic flow is occurring as well as a
qualitative indication of the magnitude of the flow activity. Where a strain-hardening constitutive model is being employed, the largest value of $L$ for any of the sublayers within the $k$th layer will be printed. Samples of the $L$ array print format are included with the example problems, see Tables 5.8, 5.12, 5.18, 5.20 and 5.22.

Figure B.1 Graphical Representation of Stress Increments
APPENDIX C.
DEFINITION OF PROGRAM VARIABLES

In this appendix we list the FORTRAN variables used in the program and give a brief description of each, identifying where possible the variable with the symbol used in the body of the report. The variables are grouped according to whether they are integer or real and whether they represent an array or not. Within each group the variables are listed alphabetically. Index notation is applied to certain sets of FORTRAN variables, with Greek indices ranging over the integers 1, 2 and Latin indices over 1, 2, 3, as before. These indices are not subscripted or superscripted. Latin indices are distinguished from letters in the FORTRAN names by not being capitalized. Input variables already described in Section 3.2 are identified by a superscript $\dagger$.

C.1 Integer Variables

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td></td>
<td>Index corresponding to surface strain locations, $1 \leq I \leq NSTRN$.</td>
</tr>
<tr>
<td>I1, I2</td>
<td></td>
<td>Dummy indices for elements of the arrays $M_{II}(I)$ and $M_{I2}(I)$.</td>
</tr>
<tr>
<td>IBCE1$\dagger$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IBCE2$\dagger$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IBCE3$\dagger$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IBCE4$\dagger$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IFLAG</td>
<td></td>
<td>Control number used in subroutine PDATA.</td>
</tr>
<tr>
<td>II</td>
<td></td>
<td>Dimension of $DAT(J)$ array, $II=2\times NSTRN + 8$.</td>
</tr>
<tr>
<td>ISR$\dagger$</td>
<td></td>
<td>Strain rate sensitivity control (input data, card 6).</td>
</tr>
<tr>
<td>J</td>
<td></td>
<td>Index corresponding to stress sublayers, $1 \leq J \leq NSFL$. Also a general dummy index.</td>
</tr>
<tr>
<td>J1, J2</td>
<td></td>
<td>Dummy indices for elements of the arrays $NI_{II}(I)$ and $NI_{I2}(I)$.</td>
</tr>
<tr>
<td>K</td>
<td>k</td>
<td>Index corresponding to layer stations, $1 \leq K \leq LAYER$.</td>
</tr>
<tr>
<td>KD</td>
<td></td>
<td>Dummy argument replacing K.</td>
</tr>
</tbody>
</table>

Preceding page blank
KEY
Control number governing mode of operation of subroutine WRTAPE: 1, write data on tape 2, read data off tape.

KJ
Index corresponding to the Jth stress sublayer of the Kth layer, \( 1 \leq KJ \leq KJ\text{MAX} \).

KJ\text{MAX}
Total number of stress sublayers in the entire thickness of shell, \( KJ\text{MAX} = \text{NSFL}\star\text{LAYER} \).

KN
Total number of stress sublayers in the first \((K-1)\) layers, \( KN = \text{NSFL}\star(K-1) \).

L
Number of subincrements into which the elastic stress increment \( \Delta_{\text{E}} \) is divided for plastic flow calculations described in Appendix B, \( L \leq 100 \).

LAYER\$ K or \( K_{\text{max}} \)
Total number of layers into which the thickness is divided (input data, card 2).

LC
Counter for the elastic stress subincrement used in plastic flow calculations described in Appendix B, \( 1 \leq LC \leq L + 1 \).

LINK
Control number used in subroutines STRAIN and PDATA.

LNNK
Counter for the maximum number of subincrements \( L \) occurring in a layer, supplying entries for the array \( LMAT(M,N,K) \).

LOAD\$
Control number governing mode of loading (input data, card 5).

LPRESS\$
Last time step at which pressure is nonconstant (input data, card 5).

M m
Mesh number in the \( \eta \) direction, \( 1 \leq M \leq MM \).

M1
Maximum value of \( M \) at which strains and stresses are calculated, a function of the edge conditions along boundary 3:

\[
M1 = \begin{cases} 
MS, \text{ symmetry edge} \\
MM, \text{ clamped or hinged edge.}
\end{cases}
\]

MASH\$
Number controlling mesh proportion of conical shell (input data, card 14).
Final time step (input data, card 3).

Maximum value of $N$ at which the displacement increments are not modified by clamped edge conditions along boundary 3:

$$MB_1 = \begin{cases} \text{MR, clamped edge} \\ \text{MS, symmetry or hinged edge.} \end{cases}$$

Dummy argument replacing $M$.

Time step at which damping operations begin (input data, card 5).

Number of mesh intervals in the $n^1$ direction (input data, card 2).

Maximum value of $N$ for points receiving uniform initial velocity $VR$ (input data, card 15).

Minimum value of $N$ for points receiving uniform initial velocity $VR$ (input data, Card 15).

Total number of mesh points in the $n^1$ direction.

Values of $N$ at mesh points bracketing the location at which the displacement $U_i$ is determined, see subroutine STRAIN.

MR = $MN - 2$.

Time steps at which restart data is collected by subroutine WRTAPE.

MS = $MN - 1$.

Mesh number in the $n^2$ direction, $1 < N < NN$.

Maximum value of $N$ at which strains and stresses are calculated, a function of the edge conditions along boundary 2:

$$N_1 = \begin{cases} \text{NS, symmetry edge} \\ \text{NN, clamped or hinged edge} \end{cases}$$

Number of time steps at which 3D plots are drawn (input data, card 11).
Minimum value of $N$ at which the displacement increments are not modified by clamped edge conditions along boundary 4:

$$N_{B1} = \begin{cases} 2, & \text{hinged edge} \\ 3, & \text{clamped edge}. \end{cases}$$

Maximum value of $N$ at which the displacement increments are not modified by clamped edge conditions along boundary 2:

$$N_{B2} = \begin{cases} NR, & \text{clamped edge} \\ NS, & \text{symmetry or hinged edge} \end{cases}$$

Initial time step (input data, card 3).

$$NCYC = NCYCLE - 1.$$  

Time step.

Dummy argument replacing $N$.

Number of time steps between surface strain prints, replaces NPRINT in program.

Maximum value of $N$ for points receiving uniform initial velocity $V_R$ (input data, card 15).

Minimum value of $N$ for points receiving uniform initial velocity $V_R$ (input data, card 15).

Counter indexing the array JCYNLP(J).

Number of times the array LMAT(M,N,K) is printed (input data, card 10).

Number of mesh intervals in the $n^2$ direction (input data, card 2).

Total number of mesh points in the $n^2$ direction.

Counter indexing the array NC3DP(J).

Counter indexing the array NCYCH(J).

Plotting tape unit number.

Input on card 8 as the number of time steps between surface strain prints, replaced by NDELP in program. In program NPRINT gives the time step, at which surface strains are printed.
Values of \( N \) at mesh points bracketing the location at which the displacement \( U_1 \) is determined, see \( MQ_1, MQ_2 \).

\( NR = NN - 2 \)

Number of time steps between collection of restart data (input data, card 3).

\( NS = NN - 1 \)

Total number of stress sublayers in each layer - a plasticity modelling control (input data, card 6).

Total number of locations at which surface strain are computed (input data, card 12).

Number of time steps at which JCHK(J) controlled data and energy-work data are printed (input data, card 9).

Number of mesh points receiving initial velocities \( V \) different from uniform initial velocity \( VR \) (input data, card 15).

**C. Integer Arrays**

Print control (input data, card 8).

Time steps at which array \( LMAT(M,N,K) \) is printed (input data, card 10).

Matrix of maximum stress subincrements for the \( K \)th layer, see description of \( L \) and \( LMK \), and Appendix B.

Values of \( M \) at mesh points bracketing the \( I \)th surface strain location, \( M11(I) < PM(I) < M12(I) = M11(I) + 1 \).

Time steps at which 3D plots are obtained (input data, card 11).

Time steps at which JCHK(J) controlled data and energy-work data are printed (input data, card 9).

Control selecting surface on which \( I \)th surface strain location is situated (input data, card 13).

Values of \( N \) at mesh points bracketing the \( I \)th surface strain location, \( N11(I) < PN(I) < N12(I) = N11(I) + 1 \).
### C.3 Real Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A11, A12, A22</td>
<td>Covariant components of middle surface metric.</td>
</tr>
<tr>
<td>AA</td>
<td>Coefficient of quadratic equation for $\Delta \lambda$.</td>
</tr>
<tr>
<td>ALFN</td>
<td>Alphanumeric print indicating surface on which strain are determined.</td>
</tr>
<tr>
<td>ANGEL</td>
<td>$\theta$ or $\text{ANGLB}(I)$ or $\text{ANGLE}(I)$ in radians.</td>
</tr>
<tr>
<td>AR11, AR12, AR22</td>
<td>Contravariant components of middle surface metric.</td>
</tr>
<tr>
<td>AVGRAD</td>
<td>Cylinder radius or average cone radius, zero for flat plate.</td>
</tr>
<tr>
<td>B11, B12, B22</td>
<td>Coefficient of quadratic equation for $\Delta \lambda$.</td>
</tr>
<tr>
<td>BM11, BM12, BM21, BM22</td>
<td>Mixed components of 2nd fundamental tensor.</td>
</tr>
<tr>
<td>BT</td>
<td>Trace of 2nd fundamental tensor.</td>
</tr>
<tr>
<td>C</td>
<td>Sound speed, $c = \sqrt{\frac{E}{\rho(1-\nu^2)}}$.</td>
</tr>
<tr>
<td>CI</td>
<td>Program constant used in equations of motion, $C_1 = C_2/(4 + C_2)$.</td>
</tr>
<tr>
<td>CIOLD</td>
<td>Old value of $C_1$ in subroutine DESTEP.</td>
</tr>
<tr>
<td>C2</td>
<td>Program constant used in damping work calculations.</td>
</tr>
<tr>
<td>CA</td>
<td>$\Delta \eta^1 \Delta \eta^2$ Program constant used in kinetic energy and work calculations.</td>
</tr>
<tr>
<td>CB</td>
<td>$\Delta \eta^1 \Delta \eta^2 \Delta \xi / E$ Program constant used in elastic energy calculations.</td>
</tr>
<tr>
<td>CINEP</td>
<td>Kinetic energy removed by previous KEA operation.</td>
</tr>
</tbody>
</table>
CINER \( T_- \) Kinetic energy at time \( t - \frac{1}{2} \Delta t \).

CINES \( T_+, T_++ \) Kinetic energy at time \( t + \frac{1}{2} \Delta t \) or at time \( c + \frac{3}{2} \Delta t \) in subroutines DAMP and DESTEP.

CINES1 \( T_- \) Previous value of CINER in subroutines DAMP and DESTEP.

CINET \( T \) Kinetic energy at current time \( t \).

CM,CN Weighing factor accounting for reduced areas at boundarys in summing for the kinetic energy and work.

\[
\begin{align*}
CS111 & \quad CS112 \\
CS122 & \quad CS211 \\
CS212 & \quad CS222
\end{align*}
\]

\[ CS_{\alpha \beta \gamma} \Gamma_{\beta \gamma} \alpha \] Christoffel symbols for the middle surface.

\[
\begin{align*}
CSM1 & \quad CSM2 \\
CSM\alpha &
\end{align*}
\]

Normal components of \( F_{NT\alpha i} \).

D \( B^2 - AC \) Discriminant of quadratic equation for \( \Delta \lambda \).

\[
\begin{align*}
D1 & \quad D_1 \\
D2 & \quad D_2 \\
D3 & \quad D_3
\end{align*}
\]

Components of the displacement of the middle surface particle with material coordinate \( \text{ETAD1, ETAD2} \).

DA \( a \) Determinant of the surface metric \( a_{\alpha \beta} \).

\[
\begin{align*}
DA11 & \quad DA12 \\
DA22 \end{align*}
\]

One half the incremental change in \( a_{\alpha \beta} \) during the time interval from \( t - \Delta t \) to \( t \).

DAMPF$ D $ Viscous damping coefficient (input data, card 5).

\[
\begin{align*}
DB11 & \quad DB12 \\
DB22
\end{align*}
\]

Incremental change in \( b_{\alpha \beta} \) during the time interval from \( t - \Delta t \) to \( t \).

DELB \( \Delta t_B \) Critical bending time increment for stability.

DELGAM \( \Delta t^2/\Gamma_0 \) Program constant used to calculate \( \text{TEMP}(M,N) \).

DELIN \( \Delta t \) Temporary storage of input \( \text{DELTAT} \).

DELM \( \Delta t_M \) Critical membrane time increment for stability.
DELMIN Minimum of DELB and DELM rounded off.

DELR $\left(\frac{\Delta t^*}{\Delta t}\right)^2$ Factor used in computing $\Delta u^i$ and $T$ in subroutine DESTEP, see (2.53 and 2.54).

DELS $\left(\frac{\Delta t^*}{\Delta t}\right)^2 \frac{2T_o D\Delta t}{2T_o + D\Delta t^*}$ Factor used in computing $\Delta u^i$ and $T$ in subroutine DESTEP, see (2.53 and 2.54).

DELSQ $\Delta t^2$ DELSQ = DELTAT**2.

DELTAS $\Delta t$ Time increment (calculated by program or input on card 3).

DEPS11 DEPS12 DEPS21 DEPS22 Mixed components of the strain increment $\Delta \varepsilon_{\alpha\beta}$

DETA1 $\Delta \eta^\alpha$ Increments in the $\eta^1$ and $\eta^2$ material coordinates, respectively.

DETA2 $\frac{1}{2} \left(1 - \frac{1}{K^2}\right)$ Factor used in calculating DELB, see (2.3).

DEFACT$^S$ Factor for terminating damping operations (input data, card 5).

DG $g$ Determinant of $g_{\alpha\beta}$

DN $\Delta n$ Normal component of increment in $n^i$.

DNL1 $\Delta n_1$ Tangential components of increment in $n^1$.

DNL2 $\Delta n_2$ Contravariant form of $\Delta n^\alpha$, above; $\Delta n^\alpha = a^{\alpha\beta} \Delta n_\beta$

DSG11L DSG12L DSG21L DSG22L Subincrements into which $\Delta \sigma_{\alpha\beta}$ below is divided.

DSIG11 DSIG12 DSIG21 DSIG22 Mixed components of elastic stress increment.

DSQOLD $\Delta t^2$ Old values of DELSQ in subroutine DESTEP.
Corrections applied to the normal components of $\Delta u^i$ along clamped edges and at clamped corners, respectively.

Normal component of $\Delta u^i$.

Factors used in calculating DELB, see (2.3).

Young's modulus (input data, card 6).

Work increment in the time interval $t - \Delta t$ to $t$.

Value of $ENS$ at the time $t - \Delta$.

Proportional to work increment in the time interval $t - 1/2 \Delta t$ to $t + 1/2 \Delta t$.

Deviator strain rate used in strain rate dependence law.

Intermediate components of strain used in calculating surface strains.

Material coordinates $\eta^1$ and $\eta^2$ of middle surface particles.

Material coordinates of the point at which the displacements $U^i$ are calculated (input data, card 12).

Proportional to bending resultant $FM_{a\beta} (M,N)$.

Factor used in calculating DELB.

Fraction of stress subincrement.

Proportional to stress resultant $FN_{xi} (M,N)$.

Poisson's ratio (input data, card 6).

Shear Modulus.
Gil
Used in subroutine STRESS for the metric $g_{\alpha\beta}$ of the lamella $\zeta$ distance from the middle surface and in subroutine STRAIN for the initial metric $G_{\alpha\beta}$ on the bounding surface.

GAMMAR
See EPSR1 above.

GAMZ $\Gamma_0 = \rho h$ Mass per initial middle surface area.

GR $\frac{1}{\sqrt{G_{11}G_{22}}}$ Time constant computed from $G_{\alpha\beta}$.

GR11 $g_{\alpha\beta}$ Inverse of metric $g_{\alpha\beta}$.

GR12 $g_{\alpha\beta}$ Inverse of metric $g_{\alpha\beta}$.

GR22 Twice the shear modulus $G$.

GTWO $\frac{E}{1+\nu}$ Twice the shear modulus $G$.

H $1/2 h$ Half the shell thickness, $H = \frac{\text{THICKN}}{2}$.

LENGTH$^b$ Length of plate, cylinder, or cone (input data, card 14).

PHIT $\phi$ Yield function.

P1 $\pi$ The mathematical constant $\pi$.

PLAST $W_p$ Plastic work.

PM1 $\text{PM1} = \text{MI1(I)}$ Conversion from integer to floating point form.

PM2 $\text{PM2} = \text{MI2(I)}$

PN1 $\text{PN1} = \text{NI1(I)}$

PN2 $\text{PN2} = \text{NI2(I)}$

PRAT $\frac{E}{1-\nu}$ Material constant.

Q11 $\frac{Q_{\alpha\beta}}{a^2 \Delta \zeta}$ Proportional to membrane components of stress resultant.

Q12 $\frac{Q_{\alpha\beta}}{a^2 \Delta \zeta}$ Proportional to membrane components of stress resultant.

Q22 $\frac{Q_{\alpha\beta}}{a^2 \Delta \zeta}$ Proportional to membrane components of stress resultant.

Q11, QM2, QN1, QN2 Interpolation coefficient used to compute the displacement components $D_i$.

QM $m, n$ Mesh numbers corresponding to material coordinates ETAD1 and ETAD2.

QN $m, n$ Mesh numbers corresponding to material coordinates ETAD1 and ETAD2.

QM1, QM2 Interpolation coefficient used to compute the displacement components $D_i$.

QN1, QN2 Interpolation coefficient used to compute the displacement components $D_i$.

RA $\frac{1}{a}$ RA = $\frac{1}{a}$.

RADIUS$^g$ Small radius of conical shell (input data, card 14).

RADIUS$^g$ Initial radius of cylindrical shell (input data, card 14).
RADF§ Large radius of conical shell (input data, card 14).

RD11 \( \frac{1}{(\Delta t)^2} \) Program constants used in computing second order finite difference derivatives, see Appendix A.

RD12 \( \frac{1}{4\Delta t^4\Delta n^2} \)

RD22 \( \frac{1}{(\Delta n^2)^2} \)

RG \( \frac{1}{g} \) RG = \( \frac{1}{\partial g} \)

RHC§ \( \rho \) Initial mass density (input data, card 6).

RN1 \( n_i \) Cartesian components of unit normal \( \mathbf{S} \alpha_i (M,N) \) at the previous time \( t - \Delta t \).

RN2 \( n_i^* \)

RN3

RRA \( \frac{1}{a^2} \) RRA = \( \frac{1}{SRA^2} \)

RSUM Factor used in rounding off DELMIN.

RTD1 \( \frac{1}{2\Delta t} \) Program constants used in computing first order finite difference derivatives, see Appendix A.

RTD2 \( \frac{1}{2\Delta n^2} \)

SA,SB \( \alpha, \beta \) Constants for computing surface strain in \( \theta \) direction.

SIG11 SIG12 SIG21 SIG22

\( \sigma_{\alpha}^{\beta} \) Mixed components of stress at time \( t \).

SIG11D SIG12D SIG21D SIG22D

\( D_{\alpha}^{\beta} \) Mixed components of plastic flow corrector stress.

SIG11I SIG12I SIG21I SIG22I

\( \sigma_{\alpha}^{\beta} \) Mixed components of stress at time \( t - \Delta t \).

SIG11L SIG12L SIG21L SIG22L

\( T_{\alpha}^{\beta} \) Mixed components of trial stress.

SIGYSQ \( \sigma_o^2 \) Square of yield stress or square of yield stress magnified by rate sensitivity factor.

SIGZ§ \( \sigma_o \) Uniaxial yield stress (input data, card 6).
SNN $n \cdot n_-$ Scalar product of normals at times $t$ and $t - \Delta t$.

SRA $a^\frac{1}{2}$ SRRA $= \sqrt{DA}$.

SRDEL $1/\sqrt{1-S^2}$ Time constant used in subroutine STRAIN.

SRG $g^\frac{1}{2}$ SRG $= \sqrt{DG}$.

SS11 $\sigma_\alpha \sigma_\beta$ Weighted sum of the mixed components of sublayer stress for given layer, giving the layer stress.

SS12 SS$\alpha \beta$ $\sigma_\alpha \sigma_\beta$

SS21 SS$\alpha \beta$ $\sigma_\beta$

SS22

STREN $V$ Elastic strain energy.

SUM Factor used in rounding off DELMIN.

SUMA11 $\sum_k \sigma_\alpha \beta$ Sum of components of layer stress $SS1MN(k)$, etc. over all layers.

SUMA12

SUMA22

SUMB11 $\sum_k \sigma_\alpha \beta \zeta$ Sum of first moment of components of layer stress $SS1MN(k)$, etc. over all layers.

SUMB12

SUMB22

SUMC11 $\sum_k \sigma_\alpha \beta \zeta^2$ Sum of second moments of components of layer stress $SS1MN(k)$, etc. over all layers.

SUMC12

SUMC22

$T_\Delta \frac{1 + 15 \nu}{8(\Delta n_1 \Delta n_2)}$ Factor used in calculating DELB, see (2.3).

TA $\Delta \zeta$ Layer thickness, $TA = \text{THICKN} / \text{ZAYER}$.

TAMBDA $\Delta \lambda$ Factor measuring amount of plastic flow.

TB $a^\lambda \Delta \zeta$ Program constant used in calculating resultants.

TDAMP $W_D$ Damping work.

THETA $\theta$ Angle subtended by cylindrical and conical panels (input data, card 14).

THICKN $h$ Thickness of shell (input data, card 6).

TIME $t$ Current time, $TIME = NCYCLE * \text{DELTAT}$.

TNRG $W$ Total external work (due to pressure).

TRL $\frac{1}{(2\Delta n_1 \Delta n_2)}$ $TRD = 2 \cdot RD12$. 

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| U11 | U12 | U13 | \( U_{ai} \) | \( U_{a} \) | First finite difference derivatives of the displacement increment \( U_i \) (M,N) with respect to material coordinates \( \text{ETA}_a \).
|-----|-----|-----|------------|------------|--------------------------------------------------|
| U21 | U22 | U23 | \( U_{ai} \) | \( U_{a} \) | Second finite difference derivatives of the displacement increment \( U_i \) (M,N) with respect to material coordinates \( \text{ETA}_a \).
|-----|-----|-----|------------|------------|--------------------------------------------------|
| U11 | U12 | U13 | \( U_{ai} \) | \( U_{a} \) | Components of displacement increment during the time interval \([t - \Delta t, t]\).
|-----|-----|-----|------------|------------|--------------------------------------------------|
| U11 | U12 | U13 | \( U_{ai} \) | \( U_{a} \) | Components of displacement increment during the time interval \([t, t + \Delta t]\).
|-----|-----|-----|------------|------------|--------------------------------------------------|
| V11 | V12 | V13 | \( \bar{v} \) | \( v \) | Initial velocity at mesh points not receiving uniform initial velocity \( \bar{V}_r \) (input data, card 16).
|-----|-----|-----|------------|------------|--------------------------------------------------|
| V11 | V12 | V13 | \( \bar{v} \) | \( v \) | Force due to the pressure \( P \) (M,N) in the equations of motion.
|-----|-----|-----|------------|------------|--------------------------------------------------|
| VM1 | VM2 | VM3 | \( \frac{\partial^2 (\alpha_{\beta_1} \bar{v}^{i})}{\partial \eta \partial \xi\beta} \) | \( \frac{\partial^2 (\alpha_{\beta_1} \bar{v}^{i})}{\partial \eta \partial \xi\beta} \) | Force due to bending resultants \( F_{M\alpha\beta}(M,N) \) in the equations of motion.
|-----|-----|-----|------------|------------|--------------------------------------------------|
| VN1 | VN2 | VN3 | \( \frac{\partial^2 \alpha_i}{\partial \eta \partial \xi\alpha} \) | \( \frac{\partial^2 \alpha_i}{\partial \eta \partial \xi\alpha} \) | Force due to stress resultants \( F_{N\alpha i}(M,N) \) in the equations of motion.
|-----|-----|-----|------------|------------|--------------------------------------------------|
| VR1 | VR2 | VR3 | \( \bar{v} \) | \( v \) | Uniform initial velocity (input data, card 15).
|-----|-----|-----|------------|------------|--------------------------------------------------|
| WIDTH5 | | | \( \text{WIDTH}_5 \) | | Half width of plate (input data, card 12).
|-----|-----|-----|------------|------------|--------------------------------------------------|
| Y11 | Y12 | Y13 | \( Y_{ai} \) | \( Y_{a} \) | First finite difference derivatives of the position vector \( Y_i \) (M,N) with respect to material coordinate \( \text{ETA}_a \), forming local basis for middle surface.
|-----|-----|-----|------------|------------|--------------------------------------------------|
Second finite difference derivatives of the position vector $Y_i (M,N)$ with respect to material coordinates ETAs.

Parameter governing the "thickness" of stress ellipsoidal annuli, see Appendix B (input data, card 2).

Dual or reciprocal basis to the basis $Y_{ai}$.

Floating point representation of LAYER.

$ZETAK = 2 \times ZETA(K)$. 

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### C.4 Real Arrays

<table>
<thead>
<tr>
<th>Array</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A111(I)</td>
<td>Initial $a_{\alpha\beta}$ at mesh point $(MI1(I), NI1(I))$.</td>
</tr>
<tr>
<td>A121(I)</td>
<td>$\alpha\beta1(I)$</td>
</tr>
<tr>
<td>A221(I)</td>
<td></td>
</tr>
<tr>
<td>A112(I)</td>
<td>Initial $a_{\alpha\beta}$ at mesh point $(MI1(K), NI2(I))$.</td>
</tr>
<tr>
<td>A122(I)</td>
<td>$\alpha\beta2(I)$</td>
</tr>
<tr>
<td>A222(I)</td>
<td></td>
</tr>
<tr>
<td>A113(I)</td>
<td>Initial $a_{\alpha\beta}$ at mesh point $(MI2(I), NI1(I))$.</td>
</tr>
<tr>
<td>A123(I)</td>
<td>$\alpha\beta3(I)$</td>
</tr>
<tr>
<td>A223(I)</td>
<td></td>
</tr>
<tr>
<td>A114(I)</td>
<td>Initial $a_{\alpha\beta}$ at mesh point $(MI2(I), N2(I))$.</td>
</tr>
<tr>
<td>A124(I)</td>
<td>$\alpha\beta4(I)$</td>
</tr>
<tr>
<td>A224(I)</td>
<td></td>
</tr>
<tr>
<td>ANGLB(I)</td>
<td>Angle specifying direction of surface strains $E_\theta$ at location $I$</td>
</tr>
<tr>
<td>ANGLE(I)</td>
<td>(input data, card 13).</td>
</tr>
<tr>
<td>ASA(I), ASB(I)</td>
<td>$2\alpha^2$ Constant for computing surface strain in $\theta$ direction.</td>
</tr>
<tr>
<td>B111(I)</td>
<td>Initial $b_{\alpha\beta}$ at mesh point $(MI1(I), NI1(I))$.</td>
</tr>
<tr>
<td>B121(I)</td>
<td>$\beta\alpha1(I)$</td>
</tr>
<tr>
<td>B221(I)</td>
<td></td>
</tr>
<tr>
<td>B112(I)</td>
<td>Initial $b_{\alpha\beta}$ at mesh point $(MI1(I), NI2(I))$.</td>
</tr>
<tr>
<td>B122(I)</td>
<td>$\beta\alpha2(I)$</td>
</tr>
<tr>
<td>B222(I)</td>
<td></td>
</tr>
<tr>
<td>B113(I)</td>
<td>Initial $b_{\alpha\beta}$ at mesh point $(MI2(I), NI1(I))$.</td>
</tr>
<tr>
<td>B123(I)</td>
<td>$\beta\alpha3(I)$</td>
</tr>
<tr>
<td>B223(I)</td>
<td></td>
</tr>
<tr>
<td>B114(I)</td>
<td>Initial $b_{\alpha\beta}$ at mesh point $(MI2(I), N2(I))$.</td>
</tr>
<tr>
<td>B124(I)</td>
<td>$\beta\alpha4(I)$</td>
</tr>
<tr>
<td>B224(I)</td>
<td></td>
</tr>
<tr>
<td>BSA(I), BSB(I)</td>
<td>$2\beta^2$ Constant for computing surface strain in $\theta$ direction.</td>
</tr>
<tr>
<td>CSA(I), CSB(I)</td>
<td>$2\alpha\beta$ Constant for computing surface strain in $\theta$ direction.</td>
</tr>
<tr>
<td>DAT(J)</td>
<td>Array storing energy, displacement and strain plotting data each time step.</td>
</tr>
<tr>
<td>DEPS1(K)</td>
<td>$\Delta\varepsilon_{11}$ Elongational components in the $\eta^1$ and $\eta^2$ directions and shear component of strain increment in the Kth layer.</td>
</tr>
<tr>
<td>DEPS2(K)</td>
<td>$\Delta\varepsilon_{22}$</td>
</tr>
<tr>
<td>DGAMMA(K)</td>
<td>$\Delta\varepsilon_{12}$</td>
</tr>
</tbody>
</table>
Interpolation coefficient used to compute components of surface strain at location I.

Constant in strain rate dependence law (input data, card 7).

Elongational surface strains in the \( \theta \) directions at location I.

Elongational components in the \( \eta^1 \) and \( \eta^2 \) directions and shear component of strain on the negative bounding surface at mesh point \((M,N)\).

Elongational surface strains in the \( \eta^1 \) and \( \eta^2 \) directions at location I.

Elongational components in the \( \eta^1 \) and \( \eta^2 \) directions and shear component of strain on the positive bounding surface at mesh point \((M,N)\).

Material coordinates of the Ith surface strain location (input data, card 13).

Components of the bending resultant tensor at the mesh point \((M,N)\).

Components of the stress resultant tensor at the mesh point \((M,N)\).

See EPSSL1(M,N) above.

See EPSU1(M,N) above.

Time constants computed from the initial surface metric \( G_{\alpha\beta} \) at strain location I.

The pressure or augmented pressure at mesh point \((M,N)\).

Mesh numbers corresponding to material coordinates ETA1(I) and ETA2(I).

Constant in strain rate dependent law (input data, card 7).
SE(J)  Slopes of Jth segments of polygonal approximation to strain hardening stress-strain curve.

SEPS(J)  Strains at corners of polygonal approximation to strain hardening stress-strain curve (input data, card 7).

SIG1(M,N,KJ)  \( \sigma_{11} \)  Normal and tangential components of stress in the \( \eta^1 \) and \( \eta^2 \) directions at mesh point (M,N), layer K, sublayer J.

SIG2(M,N,KJ)  \( \sigma_{22} \)

TAU(M,N,KJ)  \( \sigma_{12} \)

SIGZSQ(J)  Square of yield stress at Jth sublayer.

SN1(M,N)  \( n^1 \)  Cartesian components of unit normal to middle surface at mesh point (M,N).

SN2(M,N)  \( n^2 \)

SN3(M,N)  \( n^3 \)

SS1MN(K)  Normal and tangential components of stress in layer K, obtained from the mixed component form SSa8 by raising index.

SS2MN(K)  \( \sigma_{11} \)

SS2MN(K)  \( \sigma_{22} \)

SS1MN(K)  \( \sigma_{12} \)

SSMNK(K)  \( \sigma \)

SSIG(J)  Stresses at corners of polygonal approximation to strain hardening stress-strain curve (input data, card 7).

STMN(K)  See SS1MN(K) above.

TAU(M,N,KJ)  See SIG1(M,N,KJ) above.

TEMP(M,N)  \( \Delta t^2 / (a_0 h) \)  Value of time constant at mesh point (M,N).

U1(M,N)  Cartesian components of displacement increment undergone by the mesh point (M,N) in the time interval \( \Delta t \).

U2(M,N)  \( \Delta u^1 \)

U3(M,N)  \( \Delta u^2 \)

WT(J)  Weighing factors used in summing the sublayer stress to obtain layer stress, see SSa8.

Y1(M,N)  Cartesian coordinate of the mesh point (M,N) on the middle surface.

Y2(M,N)  \( y^1 \)

Y3(M,N)  \( y^2 \)

ZETA(K)  Distance along the normal from middle surface to midpoint of Kth layer.

ZETASQ(K)  Square of ZETA(K).
APPENDIX D
REPSIL PLOTTING PROGRAM

This is an independent program, separate from the REPSIL program. It was written to satisfy the requirement for a visual display of the output from REPSIL and PETROS structural response programs. This plotting program is useful in quickly interpreting results that in tabular form would be extremely difficult if not impossible, to understand.

The program makes use of the Cal Comp Standard Plotting Package SCOOP which is standard software for a large number of computing systems. If SCOOP is not available it can easily be adapted to any particular plotting system.

D.1 Description of Main Program and Subroutines

The main program reads the plotting data tape and controls the flow of information. If the variable IFLAG equals one, data is read and stored in the plotting data arrays. If it equals two, data is read and the program calls subroutine PLOT3D. When IFLAG equals 99999 the program calls subroutine GRAPH.

If the number of cycles are greater than MAXAR-2 an ERROR PRINT will occur indicating the need for enlargement of the following data arrays: TIM(MAXAR), U1(MAXAR), U2(MAXAR), U3(MAXAR), CIN(MAXAR), STC(MAXAR), TNR(MAXAR), DAMPLT(MAXAR), and EPSSI(N), EPSS2(N), where subscript N is equal to NSTRN*MAXAR. If memory is available MAXAR and the plotting data arrays can easily be enlarged.

Subroutine PLOT3D reads one control card and plots a cross-section, profile and isometric view of the middle surface of the shell. The coordinate data points Y1(M,N), Y2(M,N) and Y3(M,N) are mapped into XP, YP for each M and N to form the isometric view of the surface. Every line lying on the surface is plotted, not just those which are visible (cf. Figure 5.8).

The surface is plotted by connecting successive (M,N) mesh points with straight-line segments for each M over all N on the mesh and, in turn, connecting successive (N,N) mesh points for each N over all M.

Input parameters on the control card allow considerable flexibility in plotting the three views. The deflection can be magnified in order to bring out the details of the deformation pattern. The scale to which the plots are drawn can be adjusted in two ways. First, the scaling factor SF is found automatically, all three dimensions are scaled independently in order to fit the surface into a cube. Then the maximum scale of the three dimensions is selected as the desired scaling. If the maximum is less than one, SF is set to one. Second, the user specifies the scaling factor SF on the input control card.
It is recommended that the first way be used with a cube size, SOFC, of 3.5 inches for every new surface to automatically find SF. See Figures 5.3 and 5.8 for examples of the plotted output.

Subroutine GRAPH produces two graphs. First, time vs displacements U1, U2, U3 at the coordinate point (ETADI, ETAD2), see Figure 5.5 and 5.10. The second graph is energy balance information (time vs kinetic energy, strain energy, total energy and total damping work) which is useful for detection of numerical instabilities and as an indication of when the solution may be terminated, see Figures 5.4 and 5.9.

Subroutine STRGRA produces NSTRN graphs of time vs longitudinal strain in the \( \eta^1 \) and \( \eta^2 \) directions at the coordinate point (ETAG1, ETAG2) on the inner or outer surface of the shell. The results can be compared with experimental measurements recorded by strain gauge mounted on the surface, see Figures 5.6 and 5.11. See REPSIL input, Section 3.2 for the description of NSTRN.

Subroutine SEDSHL will plot a dashline instead of a solid line.

Subroutine MAXMIN finds the maximum and minimum point for two or more arrays of data on the Y-axis for a given plot.

### D.2 Input Plotting Control Card

<table>
<thead>
<tr>
<th>Variables</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>DEFLM, SOFC, SF, NPT</td>
<td>3E10.4, I5</td>
</tr>
</tbody>
</table>

DEFLM  Deflection magnification factor.

SOFC  Cube size, automatically finds scale factor SF.

SF  

- \( \leq 0 \), Scaling factor SF is found automatically.
- \( > 0 \), Desired scale factor.

Example: Scale 1/2, punch a real number 2 in columns 21 to 30.

Scale 1/10, punch a real number 10 in columns 21 to 30.

NPT  Mesh point location in the \( \eta^2 \) direction at which a cross-section is desired.

Following are some examples of the plotting control card.

Example 1

DEFLM = 1.0, SOFC = 3.5, SF = 0.0, NPT = 13

Example 2

DEFLM = 3.0, SOFC = 0.0, SF = 2.0, NPT = 33
Example 3

\[
\text{DFFLM} = 100.0, \quad \text{SOFC} = 0.0, \quad \text{SF} = 10.0, \quad \text{NPT} = 10
\]

**D.3 Description of Variables**

<table>
<thead>
<tr>
<th>Name</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>A11, A12, A21</td>
<td>Constants used in the mapping functions XM, YM to rotate and transform ([Y1, Y2, Y3]) into ([XP, YP])</td>
</tr>
<tr>
<td>A22, A23</td>
<td></td>
</tr>
<tr>
<td>CIN(1500)</td>
<td>The array of numbers, corresponding to kinetic energy used in the energy balance plot.</td>
</tr>
<tr>
<td>DAMPLT(1500)</td>
<td>The array of numbers, corresponding to total damping work used in the energy balance plot.</td>
</tr>
<tr>
<td>DAT(20)</td>
<td>Array used in the plotting data tape input list.</td>
</tr>
<tr>
<td>EPSS1(9000)</td>
<td>The array of numbers, corresponding to the strains in (\eta^1) direction.</td>
</tr>
<tr>
<td>EPSS2(9000)</td>
<td>The array of numbers, corresponding to the strains in (\eta^2) direction.</td>
</tr>
<tr>
<td>ETAD1, ETAD2</td>
<td>See Section 3.2 and Appendix C.</td>
</tr>
<tr>
<td>ETAG1(6), ETAG2(6)</td>
<td>See Section 3.2 and Appendix C.</td>
</tr>
<tr>
<td>HEAD1(4)</td>
<td>First line of the title that appears on the first isometric plot.</td>
</tr>
<tr>
<td>HEAD2(3)</td>
<td>Second line of the title that appears on the first isometric plot.</td>
</tr>
<tr>
<td>HEAD3(2)</td>
<td>Title that appears on every isometric plot.</td>
</tr>
<tr>
<td>I1</td>
<td>Number of mesh points in the (\eta^1) direction.</td>
</tr>
<tr>
<td>I2</td>
<td>Number of mesh points in the (\eta^2) direction.</td>
</tr>
<tr>
<td>IBCE3</td>
<td>Boundary condition value.</td>
</tr>
<tr>
<td>IBUP(1000)</td>
<td>Array used by the Cal Comp basic software package</td>
</tr>
<tr>
<td>IPEN</td>
<td>Control for pen during movement.</td>
</tr>
</tbody>
</table>
IFLAG

A flag. When it equals 1, the program will read data for the time wise plots. When it equals 2, the program will read data for the cross-section, profile and isometric views. When it equals 99999, the program will plot the time wise plots.

ML

Same as IL.

MAXAR

Size of plotting data arrays.

MAXA

MAXA = MAXAR - 2

NCYCL

The number of time steps for each curve on the time wise plots.

NCYCLE

Time cycle counter.

NETAG(6)

Zero or one, corresponding to inner or outer surface used in the title of the strain plots.

NPLST

Magnetic tape input unit number.

NSTRN

Number of strain plots.

PA(4)

Array that stores the dash line pattern.

PM(6), PN(6)

The arrays which stores the mesh location (M,N) that appear in the title on the strain plots.

QM, QN

Mesh location (M,N) that appear in the title on the vector displacement plot.

SF

Scale factor.

SOFC

Size of cube.

STC(1500)

The array of numbers corresponding to strain energy used in the energy balance plot.

TE1, TE2, TE3

Values used in the mapping functions.

TI

TI = TIME x 1.0 x 10^6

TIM (1500)

The array of numbers corresponding to time used in the time wise plots.

TIME

Current values of time.

TNR (1500)

The array of numbers corresponding to total energy used in the energy plot.
The arrays of numbers corresponding to the vector displacement U₁, U₂, U₃.

The arrays of numbers corresponding to the initial shape, Y₁(M,N), Y₂(M,N), Y₃(M,N).

The array of XP values for plotting the cross-section and profile views.

The array of YP values for plotting the cross-section and profile views.

Board coordinate point on the plotting paper in inches, measured from the lower left corner of the page.

Length of X-axis in inches.

X-axis mapping function used in isometric view.

Maximum X-coordinate.

Minimum X-coordinate.

X, Y coordinates on the plotting surface.

Plotting page length.

See Appendix C.

Y-axis mapping function used in isometric view.

Maximum and Minimum Y-coordinate.

See XP.

Maximum and Minimum Z-coordinate.

D.4 FORTRAN Listing of Plotting Program

A complete FORTRAN listing is given in the following order.

1. MAIN Program
2. PLOT 3D
3. GRAPH
4. STRGRA
5. MAXMIN
6. SCDSHL
C REPSIL PLOTTING PROGRAM (CALCOMP SOFTWARE PACKAGE SCOOP) MAIN 1
COMMON ETAG1(6),ETAG2(6),PM1(6),PN(6),NETAG(6),NSTRN,MAXX MAIN 2
COMMON Y1(25,35),Z(25,35),X(25,35),Y2(25,35),Z(25,35) MAIN 3
COMMON DAT(20),NCYCLE,TIME,IBCE3,ETAD1,ETAD2,QM,QN,NSTRN,NCYC
COMMON TIM(1500),U1(1500),U2(1500),U3(1500),CIN(1500),STC(1500), MAIN 4
NTR(1500),DAMPLT(1500),EPS1(9000),EPS2(9000) MAIN 5
C MAIN 6
NCVCL=0 MAIN 7
NPLT=3 MAIN 8
REWIN(NPLOT) MAIN 9
MAX=1500 MAIN 10
MAXX=MAXX-2 MAIN 11
C MAIN 12
READINPLOT) IBCE3,ETAD1,ETAD2,QM,QN,NSTRN MAIN 13
READINPLOT) (ETAG1(1),ETAG2(1)),PM1(1),PN(1),NETAG(1),I=1,NSTRN MAIN 14
READINPLOT) NCYCLE,TIM,NCYCLE*DAT(1),TIM(*DAT(1)) MAIN 15
NCYCL=NCYCL+1 MAIN 16
TIME=TIME+1 MAIN 17
C SAVE INITIAL SHAPE NEEDED FOR DEFLECTION MAGNIFICATION IN PLOT3D MAIN 18
DO 5 M=1,N MAIN 19
DO 5 N=1,M MAIN 20
X(M,N)=Y1(M,N) MAIN 21
Y(M,N)=Y2(M,N) MAIN 22
Z(M,N)=Z(M,N) MAIN 23
5 CONTINUE MAIN 24
CALL PLOT3D(N,1) MAIN 25
C MAIN 26
10 READINPLOT) IFLAG MAIN 27
IF(IFLAG .EQ. 99999)GOTO 30 MAIN 28
GOTO(20,25),IFLAG MAIN 29
C MAIN 30
20 READ (NPLT) NCYCLE,(DAT(I),I=1,II) MAIN 31
NCYCL=NCYCL+1 MAIN 32
TIM(NCYCL)=DAT(1) MAIN 33
U1(NCYCL)=DAT(2) MAIN 34
U2(NCYCL)=DAT(3) MAIN 35
U3(NCYCL)=DAT(4) MAIN 36
CIN(NCYCL)=DAT(5) MAIN 37
STC(NCYCL)=DAT(6) MAIN 38
TNR(NCYCL)=DAT(7) MAIN 39
DAMPLT(NCYCL)=DAT(8) MAIN 40
DO 22 J=1,9,112 MAIN 41
J=NCYCL+MAXA*(1-9)/2 MAIN 42
EPS1(J)=DAT(11) MAIN 43
EPS2(J)=DAT(12) MAIN 44
22 CONTINUE MAIN 45
IF(INCYCL .GE. MAXA)GOTO 28 MAIN 46
GOTO 10 MAIN 47
C MAIN 48
25 READINPLOT) NCYCLE,TIME,M1,N1,(Y1(M,N),Y2(M,N),Y3(M,N),M=1,M1), MAIN 49
IN=1,N1 MAIN 50
CALL PLOT3D(M1,N1) MAIN 51
GOTO 10 MAIN 52
C MAIN 53
28 WRITE(6,100) NCYCL MAIN 54
30 CALL GRAPH MAIN 55
CALL EXIT MAIN 56
C MAIN 57
100 FORMAT(15H1ERROR NCYCL = 15,29H ENLARGE PLOTTING DATA ARRAYS) MAIN 58
END MAIN 59
158
SUBROUTINE PLOT3D (11,12)

DIMENSION IBUF(1000),X1(90),X2(90),HEAD1(4),HEAD2(3),HEAD3(2)
COMMON /PLOT/ YNP,XDIST,PA(4)
DATA I/0/11,707107,A12,707107/A2/-408248/A22/-408248/
DATA IA3/-1497/
DATA (HEAD1(IK),IK=1,4)/10ME CROSS SECT,10M DEFLEC,10MION AT N /
DATA (HEAD2(IK),IK=1,3)/10MEFLEC,10MEFLEC,10MEION /
DATA (HEAD3(IK),IK=1,2)/10M MICROSECO,10M4/
DATA (PA(J),J=1,4)/.1,.1,.1,.1/
COMMON USE MAIN

C
XM(1,2)=SF*(A1+A2+A22+A23+A3)
YM(1,2)=SF*(A21+A1+A22+A23+A3)

C
IF(I.EQ.0)GOTO 10
I=I+1
XBARN=-4.25
YBAR=10.0
YNP=YNP+10.0
IF(I.LE.3)GOTO 30
CALL PLOT (XDIST,YNP,3)
GOTO 20
10 XPAGE=200.0
READ (5,11) DEFLM,SOFC,SF,H12
XDIST=6.75
11 FORMAT(3E10.4,15)
C
12 CONTINUE
IF(SF .NE. 0.0)GOTO 13
X=$=(XMAX-XMIN)/SOFC
Y=$=(YMAX-YMIN)/SOFC
Z=$=(ZMAX-ZMIN)/SOFC
SF=AINT(SF)
SF=SF/SF
13 CALL PLOTS (IBUF,1000,XPAGE)
C
CALL SYMBOL (1.0,.1,.1,.1,HEAD1,.0,.32)
CALL NUMBER (999,.999,.1,HEAD2,.0,.215)
CALL SYMBOL (1.0,.0,.8,.1,HEAD3,.0,.21)
CALL NUMBER (999,.999,.1,DEFLM,.0,.1)
CALL SYMBOL (1.0,.0,.6,.1,HEAD3,.1,.8)
CALL NUMBER (999,.999,.1,SOFC,.0,.5)
C
20 YBAR=4.3
CALL PLOT (XBAR,YBAR,-3)
CALL PLOT (0.0,-1.0,3)
CALL PLOT (0.0,1.0,2)
CALL PLOT (0.0,0.0,3)
CALL PLOT (1.0,0.0,2)

K=0
DO 90 M=2,11
K=K+1
X1(K)=SF*X(M+N12)
X2(K)=SF*Z(M+N12)

90 CONTINUE
X1(K+1)=0.0
X1(K+2)=1.0
X2(K+1)=0.0
X2(K+2)=1.0
CALL SCOSM (X1,X2,K,1,PA,4)
CALL LINE (X1,X2,K,1,2)

K=0
DO 95 M=2,11
K=K+1
X1(K)=SF*(X1,N12)+DEFLM*(Y1(N12)-X1,M12)
X2(K)=SF*(Z(N12)+DEFLM*(Y3(N12)-Z(N12)))

95 CONTINUE
CALL LINE (X1,X2,K,1,4)

CALL PLOT (XBAR,YBAR,2,12)
CALL PLOT (XBAR,YBAR,2)

L=2
DO 85 J=1,2
K=0
DO 82 N=1,12
K=K+1
X1(K)=SF*Y(L,N)
X2(K)=SF*Z(L,N)

82 CONTINUE
X1(K+1)=0.0
X1(K+2)=1.0
X2(K+1)=0.0
CALL SCDSNL (X1, X2, K1, 1, PA, X1, X2, K1, 1, PA, X1, X2, K1, 1, PA)
CALL LINE (X1, X2, K1, 1, PA, X1, X2, K1, 1, PA)
K = 0
DO 83 N = 1, 12
K = K + 1
X1(K) = SF(Y1(L, N) + DEFLM(Y2(L, N) - Y1(L, N)))
X2(K) = SF(Z1(L, N) + DEFLM(Z2(L, N) - Z1(L, N)))
CONTINUE
CALL LINE (X1, X2, K, 1, 1, 4)
IF (IBGE3 = NE. 2) GOTO 86
L = 1
CONTINUE
85 CONTINUE
DRAW AXIS
CALL PLOT (4, 25, -3, 0, -3)
CALL PLOT (0, 0, 1, 0, 3)
CALL PLOT (-A12, -A22, 2)
CALL PLOT (0, 0, 0, 0, 3)
CALL PLOT (A11, A21, 2)
CALL PLOT (0, 0, 0, 1, 1)
IPEN = 3
DO 40 N = 1, 12
TE1 = X(M, N) + DEFLM(Y1(M, N) - X(M, N))
TE2 = Y(M, N) + DEFLM(Y2(M, N) - Y(M, N))
TE3 = Z(M, N) + DEFLM(Z3(M, N) - Z(M, N))
XP = XMITE1, TE2
YP = YMITE1, TE2, TE3
CALL PLOT (XP, YP, IPEN)
IF (IPEN .LT. 3) GOTO 40
IPEN = 2
GOTO 35
CONTINUE
35 CONTINUE
DO 50 N = 1, 12
IPEN = 3
DO 60 M = 2, 11
TE1 = X(M, N) + DEFLM(Y1(M, N) - X(M, N))
TE2 = Y(M, N) + DEFLM(Y2(M, N) - Y(M, N))
TE3 = Z(M, N) + DEFLM(Z3(M, N) - Z(M, N))
XP = XMITE1, TE2
YP = YMITE1, TE2, TE3
CALL PLOT (XP, YP, IPEN)
IF (IPEN .LT. 3) GOTO 60
IPEN = 2
GOTO 45
CONTINUE
45 CONTINUE
RETURN
END
SUBROUTINE GRAPH
DIMENSION SYM1(4),SYM2(3),ETA1(3),ETA2(1),AI(4),X2(4)
COMMON (USE MAIN)
COMMON /PLOT1/ YNP,XDIST,PA(4)
COMMON/ PLOT2/ YNP,XDIST,PA(4)
DATA(SYM1(1),1=1,4)/10HCOMPONENT1/10HOF VECTOR,10HDSPLACEMENT,2HNT/
;/ 
DATA(ETA1(1),1=1,3)/10H, A1 = 10H (4H )/,
1 (ETA2(1),1=1,3)/10HETA2 = 10H (4H )/,
DATA(SYM2(1),1=1,3)/10HENERGY BALANCE, POUND, 8HD-INCHES/
C
XDIST=8.0 
CALL PLOT (XDIST-YNP-,3)
XBAR=0.0 
YNP=2.0 
XL=8.0 
YL=0.0 
N=NCYCL 
CALL SCALE (TMIN,XL,N,1) 
XMIN=TMIN(N+1) 
XS=TMIN(N+2) 
C ------------- GRAPH ONE (VECTOR DISPLACEMENT) ---------------
CALL MAXMIN (U1,YMIN,YMAX,1,N+1) 
CALL MAXMIN (U2,YMIN,YMAX,1,N+2) 
CALL MAXMIN (U3,YMIN,YMAX,1,N+2) 
XI(1)=YMIN 
XI(2)=YMAX 
YMIN=XI(3) 
YS=XI(4) 
U1(N+1)=YMIN 
U1(N+2)=YS 
U2(N+1)=YMIN 
U2(N+2)=YS 
U3(N+1)=YMIN 
U3(N+2)=YS 
CALL PLOT (XBAR,YNP-,3)
CALL AXIS (0.0,0.0,4HTIME,-5.0,0.0,XMIN,XS) 
CALL AXIS (0.0,0.0,SYM1,32,YL,90.0,YMIN,YS) 
CALL LINE (TMIN,U1,N,1,0.0) 
TX=TMIN/1/XS*2 
TY=(U1(N)-YMIN)/YS 
CALL SYMBOL (TX,TY,1.2HUI,0.0,2) 
CALL LINE (TMIN,U2,N,1,0.0) 
TY=(U2(N)-YMIN)/YS 
CALL SYMBOL (TX,TY,1.2HU2,0.0,2) 
CALL LINE (TMIN,U3,N,1,0.0) 
TY=(U3(N)-YMIN)/YS 
CALL SYMBOL (TX,TY,1.2HU3,0.0,2) 
CALL SYMBOL (2.1-1.0,1.0LOCATION,0.0,8) 
CALL SYMBOL (3.0-1.15,4.1H,0.0,1) 
CALL SYMBOL (5.35-1.15,4.1H,0.0,1) 
CALL SYMBOL (3.1-0.0,1.0,ETA1,0.0,24) 
CALL NUMBER (3.8,999,1,ETA1,0.0,3) 
CALL NUMBER (4.8,999,1,GM,0.0,3) 
CALL SYMBOL (3.1-1.1,1.0,ETA2,0.0,24) 
CALL NUMBER (3.8,999,1,ETA2,0.0,3) 
CALL NUMBER (4.8,999,1,GM,0.0,3) 
C ------------- GRAPH TWO (ENERGY BALANCE) ---------------
XBAR=0.0 
YNP=10.0
CALL MAXMIN (CIN, YMIN, YMAX, N+1)
CALL MAXMIN (STC, YMIN, YMAX, 1, N+2)
CALL MAXMIN (TNR, YMIN, YMAX, 1, N+2)
CALL MAXMIN (DAMP1, YMIN, YMAX, 1, N+2)
X(1)=YMIN
X(2)=YMAX
CALL SCALE (X, YL, 2, 1)
YMIN=X(3)
YS=X(4)
CIN(N+1)=YMIN
CIN(N+2)=YS
STC(N+1)=YMIN
STC(N+2)=YS
TNR(N+1)=YMIN
TNR(N+2)=YS
DAMP1(N+1)=YMIN
DAMP1(N+2)=YS
CALL PLOT (XBAR, YNP, -3)
CALL AXIS (0.0, 0.0, 8, 28, XL, YL, 0.0, YMIN, 05)
CALL LINE (TIM, CIN, N, 0, 0)
CALL LINE (TIM, STC, N, 0, 0)
CALL LINE (TIM, TNR, N, 0, 0)
CALL LINE (TIM, DAMP1, N, 0, 0)
DO 100 I=1, NSTRN
J=MAXAR(I)-1
CALL STRGRA (TIM, EPS1, EPS2, ETAG1(I), ETAG2(I), PM(I), PNN(I),
INETAG(I), J, N)
100 CONTINUE
RETURN
END

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GRAP 61
GRAP 62
GRAP 63
GRAP 64
GRAP 65
GRAP 66
GRAP 67
GRAP 68
GRAP 69
GRAP 70
GRAP 71
GRAP 72
GRAP 73
GRAP 74
GRAP 75
GRAP 76
GRAP 77
GRAP 78
GRAP 79
GRAP 80
GRAP 81
GRAP 82
GRAP 83
GRAP 84
GRAP 85
GRAP 86
GRAP 87
GRAP 88
GRAP 89
GRAP 90
GRAP 91
GRAP 92
SUBROUTINE STRGAA (XY, 2, ETA1, ETA2, PH1, PH2, NETA, J, N)
X IS THE NAME OF THE ARRAY CONTAINING THE X COORDINATE DATA.
Y IS THE NAME OF THE ARRAYS CONTAINING THE Y COORDINATE DATA.
ETA1, ETA2 LOCATION POINT USED IN THE TITLE.
PH1, PH2 MESH LOCATION USED IN THE TITLE.
NETA IS THE CONTROL FOR INNER OR OUTER USED IN THE TITLE.
J IS THE LOCATION OF THE FIRST DATA POINT IN ARRAYS Y AND Z FOR
EACH STRAIN PLOT.
N IS THE NUMBER OF DATA POINTS.
DIMENSION X(1), Y(1), Z(1), SYM1(3), SYM2(3), SYM3(2), SYM4(2), X1(4),
Y2(4),
COMMON /PLOT0/, YNP, XDIST, PA(4)
DATA(SYM1(K), K=1,3)/10META1 = 10H
1 (SYM2(K), K=1,3)/10META2 = 10H
2 (SYM3(K), K=1,2)/10META2 COMP04HMENT/0
3 (SYM4(K), K=1,2)/10META1 COMP04HMENT/
C IF(I.EQ.0)GOTO 10
I=1+1
YNP=YNP+10.0
YBAR=10.0
IF(I.LE.3)GOTO 29
YNP=YNP-10.0
CALL PLOT (XDIST, YNP, -3)
GOTO 20
10 XDIST=13.0
CALL PLOT (XDIST, YNP, -3)
XL=8.0
YL=6.0
CALL SCALE (X, XL, N+1)
XMIN=X(N+1)
XS=X(N+2)
20 XBAR=0.0
YNP=0.0
YBAR=0.0
I=1
25 CALL MAXMIN (Y, YMIN, YMAX, J, N+1)
CALL MAXMIN (Z, YMIN, YMAX, J, N+2)
X(1) = YMIN
X(2) = YMAX
CALL SCALE (X, YL, 2, 1)
YMIN=X(3)
YS=X(4)
JH=J+N
JS=J+N+1
Y(JH)=YMIN
Y(JS)=YS
Z(JS)=YNP
Z(JS)=YMIN
Z(JS)=YS
CALL PLOT (XBAR, YBAR, -3)
CALL AXIS (0.0, 0.0, 4THTIME, -4, XL, 0.0, XMIN, XS)
CALL AXIS (0.0, 0.0, 10HSTRAIN (1.10, YL, 90.0, YMIN, YS)
CALL SYMBOL (1.6, 1.0, -1, 10HLOCATION, 0.0, 8)
CALL SYMBOL (2.7, 1.15, -4.1H, 0.0, 1)
CALL SYMBOL (5.0, -1.15, 4.1H, 0.01)
IF(INETA .NE. 0)GOTO 30
CALL SYMBOL (5.3, -1.0, 1.5SHINER, 0.0, 5)
GOTO 35
30 CALL SYMBOL (5.3, -1.0, 1.5SHINER, 0.0, 5)
35 CALL SYMBOL (2.6, -0.9, -1, SYM1, 0.0, 24)
CALL NUMBER (3.5,999...1,ETA1,0.0,0,3)
CALL NUMBER (4.5,999...1,PN1,0.0,0,3)
CALL SYMBOL (2.0,-1.1...1,SYM2,0.0,24)
CALL NUMBER (3.5,999...1,ETA2,0.0,0,3)
CALL NUMBER (4.5,999...1,PN1,0.0,0,3)
X1(1)=2.9
X2(1)=-1.3
X1(2)=3.4
X2(2)=-1.3
X1(3)=0.0
X2(3)=0.0
X1(4)=1.0
X2(4)=1.0
CALL LINE (X1,X2;2;1;0;0)
CALL SYMBOL (3.6,-1.3...1,SYM4,0.0,14)
X2(1)=-1.5
X2(2)=-1.5
CALL SCOSNL (X1,X2;2;1;PA;4)
CALL SYMBOL (3.6,-1.5...1,SYM3,0.0,14)
CALL LINE (X,Y;1;N;1;0;0)
CALL SCOSNL (X,Z;1;N;1;PA;4)
RETURN
END
SUBROUTINE MAXMIN (A,AMIN,AMAX,J,N,KEY)
DIMENSION A(1)
GO TO (10,20),KEY
10 J1=J
   J2=J+N-1
   AMIN=A(J1)
   AMAX=A(J2)
20 DO 100 I=J1,J2
   AMAX=MAXI(AMAX,A(I))
   AMIN=MINI(AMIN,A(I))
100 CONTINUE
RETURN
END
SUBROUTINE SCOSHL(X,Y,M,INC,PA,N)

C(1) X,Y ARE THE NAMES OF THE ARRAYS CONTAINING THE X AND Y
C(2) COORDINATES, AND THE SCALING PARAMETERS. (SEE 'SCALE').
C(3) M IS THE NUMBER OF POINTS IN THE X AND Y ARRAYS. THIS DOES
C(4) NOT INCLUDE THE TWO EXTRA LOCATIONS FOR THE SCALING PARAMETERS.
C(5) INC IS THE INCREMENT THAT THE SCOSHL SUBROUTINE IS TO USE
C(6) IN GETTING DATA FROM THE X AND Y ARRAYS. (SEE 'SCALE').
C(7) PA IS THE NAME OF THE LINEAR ARRAY WHICH CONTAINS THE ELEMENTS
C(8) OF THE PATTERN WHICH IS TO BE REPEATED UNTIL THE CURVE
C(9) IS DRAWN.
C(10) PA(I), FOR I=1,3,5,..., CORRESPOND TO THE DASHES IN
C(11) THE PATTERN, WHILE
C(12) PA(I), FOR I=2,4,6,..., CORRESPOND TO THE SPACES
C(13) BETWEEN THE DASHES.
C(14) N IS THE NUMBER OF ELEMENTS IN THE PATTERN DESCRIPTION.
C(15) DIMENSION PAI(1),X(1),Y(1)

NP=NP+INC+1
NQ=NP+INC
DX=1.0/NP
FX=X(NP)
DY=1.0/NQ
FY=Y(NQ)
K=0
DLN=0.0
DX2=(X(I)-FX)*DX
DY2=(Y(I)-FY)*DY
I=1
J=1
OTO=PA(I)
CALL PLOT(DX2,DY2,3)
GOTO 30

20 DS1=DT2-DY1
DCO=DX2-DX1
DLN=SOR(T(DS1+DCO+DCO))
K=MOD(J,2)
IF(DML.F.GE.DTD)GOTO 30
OY1=OY1+OTO*DCO/DLN
OY1=OY1+OTO*DS1/DLN
CALL PLOT(DX1,DY1,3-X)
J=J+1
IF(J.GT.N)J=1
OTO=PA(J)
GOTO 20

30 IF(K.NE.0)CALL PLOT(DX2,DY2,2)
OX1=DX2
DY1=DY2
I=I+INC
OTO=OTO+1
IF(I.GT.M)RETURN
DX2=(X(I)-FX)*DX
DY2=(Y(I)-FY)*DY
OTO=OTO+DLN
GOTO 30
END
APPENDIX E.
FORTRAN LISTING OF THE REPSIL PROGRAM

The listing is given in the following order, consisting of Main program and 22 subroutines.

1. MAIN Program
2. START
3. INVEL
4. POSITN
5. DGEOM
6. GRAD
7. STRESS
8. RESULT
9. MOTION
10. WRTAPE
11. STRAIN
12. BOUNDU
13. ABINIT
14. SYMTRY
15. KINET
16. PWORK
17. DAMP
18. DESTEP
19. PDATA
20. PRESS*
21a. INGEOM (Flat Plate)
21b. INGEOM (Full Cylinder)
21c. INGEOM (Cone)

There are three nonstandard FORTRAN statements used:

1. COMMON (USE MAIN) is used instead of repeating long COMMON statements in the subroutines that appear in the MAIN program.

2. CALL SKIPFILE (t,n) is used in subroutine PDATA to move tape t forward n file marks. The next READ or WRITE statement will begin with the information after the file mark.

3. CALL BACKFILE (t,n) is used in subroutine PDATA to move tape t backward n file marks. The next READ statement will begin with the information after the file mark, the WRITE will erase the file mark.

*Subroutine PRESS gives the pressure loading applied in the flat plate example, Section 5.1.
6,DEPS1[1:4], DEPS2[4], DGAHA[4], ZETA[4], SS1N[4], SS2N[4], SSNN[4],
7,TEMP[23,34], P[23,34], EPSL[23,34], EPSU[23,34],

CALL START

C CHECK IF THIS IS A RESTART

IF(NCONT .EQ. 0) GOTO 15
CALL NRTAPE (2)
NPRINT=(NCYCLE-MOD(NCYCLE,NDELP))\NDELP
GOTO 49

C SET CYCLE NUMBER = 0

NCYCLE=0
TIME=0.0
CINES=0.0
TDA=0.0
CINEP=0.0
ENS=0.0
C=0.0
D=0.0

C SET SYMMETRY BOUNDARY CONDITIONS FOR EDGES 1, EDGES 2, EDGES 3

IF(I1C1E2 .NE. 2) GOTO 17
C 16 M=2,M1
Y1(M,NN)=Y1(M,NN)
Y2(M,NN)=Y2(M,NN)
Y3(M,NN)=Y3(M,NN)

170
16 CONTINUE
17 DO 20 M=1,NN
   Y1(I,N)=Y1(I,3,N)
   Y2(I,N)=Y2(I,3,N)
   Y3(I,N)=Y3(I,3,N)
   IF(I>BCE3 .NE. 2)GOTO 20
   Y1(MM,N)=Y1(MM,N)
   Y2(MM,N)=Y2(MM,N)
   Y3(MM,N)=Y3(MM,N)
20 CONTINUE

C SET INITIAL DISPLACEMENT,PRESSURE,STRAIN AND STRESS = 0
   DO 28 K=1,KJMAX
   SIG1(M,N,K)=0.
   SIG2(M,N,K)=0.
   SIG3(M,N,K)=0.
   EPSL1(M,N)=0.0
   EPSL2(M,N)=0.0
   EPSL3(M,N)=0.0
   GAMMAL(M,N)=0.0
   EPSU1(M,N)=0.0
   EPSU2(M,N)=0.0
   EPSU3(M,N)=0.0
   GAMMAU1(M,N)=0.0
   DO 28 K=1,KJMAX
   SIG1(M,N,K)=0.
   SIG2(M,N,K)=0.
   SIG3(M,N,K)=0.
   EPSL1(M,N)=0.0
   EPSL2(M,N)=0.0
   EPSL3(M,N)=0.0
   GAMMAL(M,N)=0.0
   EPSU1(M,N)=0.0
   EPSU2(M,N)=0.0
   EPSU3(M,N)=0.0
   GAMMAU1(M,N)=0.0

28 CONTINUE

C IF(LOAD) 29,30,29
29 CALL PRESS
30 DELGAM=DELSQ/GAMZ
C WRITE INITIAL CARTESIAN COORDINATES, PRESSURE
   WRITE(6,300)
   DO 46 N=2,N1
   WRITE(6,400)N,Y1(M,N),Y2(M,N),Y3(M,N),P(M,N),N=1,N1
46 WRITE(6,400)N,Y1(M,N),Y2(M,N),Y3(M,N),P(M,N),N=1,N1

C CALL DGEGM
CALL STRAIN
   IF(LOAD) 31,31,43
31 CALL INVEL
C DO 35 N=2,NS
   DO 35 N=2,NS
   U1(M,N)=DELTAT*U1(M,N)
   U2(M,N)=DELTAT*U2(M,N)
   U3(M,N)=DELTAT*U3(M,N)
35 CONTINUE

C CALL BOUNDU
CALL KINET
CINES=2.0*CINET
TNRG=CINES
C IF(LOAD) 42,45,43
C CALL PHORK
C DO 44 N=2,NS
44 CONTINUE
C 44 CONTINUE
CC 44 N=3,NS
U1(N,N)=U1(N,N)-P(N,N)*SN1(N,N)*TEMP(N,N)
U2(N,N)=U2(N,N)-P(N,N)*SN2(N,N)*TEMP(N,N)
U3(N,N)=U3(N,N)-P(N,N)*SN3(N,N)*TEMP(N,N)

44 CONTINUE
CALL BOUNDU
CALL PWOK
ENR=ENS
CALL KINET
TNKG=CINET

45 IF(NCYCCH(1).EQ.0)NNN=2
49 CALL PDATA (1)

C
END INITIALIZATION

50 NCYCLE=NCYCLE+1
TIME=TIME+DELTAT

C
CHECK FOR FINAL STEP

IF(INCYCLE-MAXC) 60,60,70

C
CHECK IF CALL PRESS IS NEEDED

60 IF(LPRESS-NCYCLE) 64,62,62
62 CALL PRESS
64 CALL POSTIN
CALL DEGN
CALL STRAIN
CALL NOTION
CALL PDATA (2)
CALL DMP

C
CHECK FOR RESTART DUMP

IF(INCYCLE-MRITE) 90,66,90

66 CALL WTAPE (1)
MRITE=MRITE+MRITE
CALL PDATA (3)
GOTO 90

70 IF(INCYCLE-LT. 25)CALL EXIT
CALL PDATA (4)
CALL PDATA (6)
CALL EXIT

C

300 FORMAT(1H1,32X,9H:INITIAL CARTESIAN COORDINATES,32X,8HPRESSURE/3X,MAIN 159
10N,N,3X,1N,N,9X,THY1(N,N),10X,THY2(N,N),10X,THY3(N,N),20X,6HP(N,N)) MAIN 160
400 FORMAT(214,4(2X,E23.167/(26,102X,E23.16))) MAIN 161
END
SUBROUTINE START
DIMENSION TITLE(8)
COMMON (USE MAIN)
READ(5,100) TITLE
READ(5,105) MESH,NMESH, LAYER, YLDFA
READ(5,105) MAXC,NCONT, NWRITE, DELTAT
READ(5,110) IBCE1, IBCE2, IBCE3, IBCE4
READ(5,115) LOAD, LPRESS, MDAMP, DAMPF, DFACT
READ(5,120) E, FNU, SIGZ, RHO, THICK, NSFL, ISR
IF(NSFL .EQ. 1 .AND. ISR .EQ. 0) GOTO 700
IF(NSFL .EQ. 0) GOTO 700
READ (5,125) (SSIG(J), SEPS(J), DSR(J), PSR(J), J=1,NSFL)
700 IF(NSFL .LT. 1) ISR=1
IF(NSFL .LT. 1) NSFL=1
READ(5,110) NPRINT, (JCHK(J), J=1,3)
READ(5,110) NUMC, (NCYC(J), J=1, NUMC)
READ(5,110) NLPRIN, (JCYNP(J), J=1, NLPRIN)
READ(5,110) N3D, (NC3P(J), J=1, N3D)
SSIG(1)=SIGZ
SEPS(1)=SIGZ/E
KJMAX=LAYER*NSFL
SE(1)=E
SIGZSQ(1)=SIGZ(1)**2
DO 795 J=1, NSFL
IF(I3R .LT. 1) GOTO 794
IF(DSR(J) .GT. 0.0 .AND. PSR(J) .GT. 0.0) GOTO 793
791 WRITE(6,792)
792 FORMAT(/'ERROR IN STAIN HARDENING OR STRAIN RATE DATA')
CALL EXIT
793 PSR(J) = 1.0/PSR(J)
794 IF(J .EQ. 1) GOTO 795
IF(SEPS(J), LE, SEPS(J-1)) GOTO 791
SE(J1=SSIG(J)-SSIG(J-1))/SEPS(J-1)
MT(J-1)=SE(J-1)-SE(J)/E
SIGZSQ(J)=SIGZSQ(J-1)**2
795 CONTINUE
MT(NSFL)=SE(NSFL)/E
C CHECK NUMBER OF MESH POINTS NEEDED FOR ETA1, ETA2 DIRECTIONS
MM=MESH+2
NN=MESH+1
NI=NN
C CHECK BOUNDARY CONDITIONS FOR EDGE2
IF(IBCE1 .NE. 2) GOTO 4
NN=MESH+2
NI=NN-1
C 4 M1=MM
MB1=MM-2
MB2=MM-3
C CHECK BOUNDARY CONDITIONS FOR EDGE3
IF(IBCE3 .NE. 2) GOTO 5
MM=MESH+3
M1=MM-1
MB1=MM-1
5 MS=MM-1
NS=MM-1
MA=MM-2
NR=MM-2
IF(IBCE3 .EQ. 3) MB1=MS
IF(ICE = EQ. 3) MB1 = 2
IF(ICE = NE. 1) MB2 = NS

READ(5, 130) ETAD1, ETAD2, NS
C
READ(5, 151)(ETAG1(I), ETAG2(I), ANGL1(I), ANGL2(I), NETAG(I), I=1, NS) N
C
CALL INGEO
QM = 2.0 + ETAD1/DETA1
QM = 1.0 + ETAD2/DETA2
QM = QM1
QM = QM1 + 1
QM = QM2
QM = QM1 - QM2
QM = FLOAT(QM1)
QM = FLOAT(QM2) - QM
QM = FLOAT(QM2) - QM
DO 20 I = 1, NS
PN1(I) = 2.0 + ETAG1(I)/DETA1
PN1(I) = 1.0 + ETAG2(I)/DETA2
NI1(I) = PN1(I)
IM1(I) = NI1(I) + 1
NI2(I) = PN1(I)
IM2(I) = NI2(I)
PN2 = NI2(I)
OM1(I) = PN1(I) - PN1
OM2(I) = PN2 - PN1
OM1(I) = PN1(I) - PN1
OM2(I) = PN2 - PN1
C
20 CONTINUE
C
TIME INCREMENT BY VON NEUMAN
C = SQRT((1.0/FNU)**2)
DELM = 2.0 / C + SQRT(1.0/DETA1**2 + 1.0/DETA2**2)
ZAYER = LAYER
FLAYER = LAYER / 2.0
DETA1 = 0.0333333333333333 - (1.0/(1.0/FLAYER))
DX = 1.0 / DETA1**4
DY = 1.0 / DETA2**4
T2 = (1.0 + 15.0*FNU)/18.0**DETA1**2 + DETA2**2
DELM = 1.0 / (2.0*THICK**2*SQRT(DETA1*(DX + DY + T2))
DELM = DELM/(DELM + DELM)
SUM = 1.0
SUM = SUM + 0.0
IF(ISUM.LT. DELM) GOTO 30
GOTO 25
C
30 DELM = AINT(DELMIN*SUM)/SUM
DEL = DELTAT
IF(DEL < .0) DELMIN = AINT(DELMIN)
DELTAT = DELMIN
C
PROGRAM CONSTANTS
H = 0.5*THICK
GAM = THICK*W
NDL = NPRINT
MR = WRITE
WRITE = NCONT + WRITE
DO 3 K = 1, LAYER
ZETAK = H + (1.0 - (2.0*FLOAT(K) - 1.0)/ZAYER)
C
174
ZETASQIK() = ZETA()**2

3 CONTINUE
TA=TA(2,0/2*LAYE)
DELSQ=DELTAT**2
Q=5.0E/1.0*FNU)
GTOW=GTOW+1.0
PRA=1.0/1.0*DETA1
RTO2=1.0/1.0*DETA2
PD1=1.0/1.0*DETA1**2
RD2=1.0/1.0*DETA2**2
RDI2=0.25/(DETA1*DETA2)
CA=DETA1*DETA2
CB=DETA1*DETA2/E
RD=2.0RDI2

C DAMPING CONSTANTS
C2=2.0O*DETA*DAMPF/GAMZ
C1=C2/(4.0*C2)

WRITE(6,130)
WRITE(6,140) TITLE
WRITE(6,150) MESH,DETA1,NMESH,DETA2
WRITE(6,160) DEL8,DEL9,DEL10,DELTAT
WRITE(6,160) E,FNU,SIGZ,RHO,THICKN
WRITE(6,170) NCONT,MAXC,NPRINT,WRITE
WRITE(6,170) LAYER,NPRINT,LOAD,LPRESS
WRITE(6,170) IBCE1,IBCE2,IBCE3,IBCE4
WRITE(6,170) JCHK(1),I=1,3
WRITE(6,170) NCYCH(j),j=1,NMYC)
WRITE(6,170) JCYNLJ),J=1,NUMYN)
WRITE(6,170) NC30P(I),I=1,N30)
IF(ISR .EQ. -1) WRITE(6,404)
IF(ISFL .EQ. 1 .AND. ISR .EQ. 1) WRITE(6,405)
IF(ISFL .EQ. 1 .AND. ISR .EQ. 0) WRITE(6,410)
IF(ISFL .EQ. 1 .AND. ISR .EQ. 1) WRITE(6,420)
IF(ISFL .EQ. 1 .AND. ISR .EQ. 0) WRITE(6,420)

WRITE(6,821) (J,SIGM0J),J=1,NSFL)
WRITE(6,110) TIME=DELTAT=FLOAT(MDAMP)
WRITE(6,200) MDAMP,TIME,DAMPF,DFACT
RETURN

15 FORMAT(E10.4,15)
100 FORMAT(EA10)
105 FORMAT(E15.6)
110 FORMAT(E16.15)
115 FORMAT(E15.2E12.6)
120 FORMAT(E15.2E12.6)
125 FORMAT(E15.7)
130 FORMAT(E10.4,15)

C 140 FORMAT(H15.53,15HRL REPSIL COOE//Z4X,6A10/)
150 FORMAT(3X,14,29H MESHES IN THE ETA1 DIRECTION,3X,1HDETA =E12,6)
11H/38X,14,29H MESHES IN THE ETA2 DIRECTION,3X,8HDETA =E12,6,1HISTAR
174

2/)
160 FORMAT(55X,17HYOUNG'S MODULUS =E12,6)
1 /3X,17HPOISSON'S RATIO =E12,6,10X,17HYIELD STRESS =E12,6STAR 177
2 /3X,17HMASS DENSITY =E12,6,10X,17THICKNESS =E12,6STAR 178

170 FORMAT(15X,18HSTART AT TIME STEP5/55X,18HFINAL TIME STEP ,15/
1 55X,21MSURFACE STRAINS EVERY 10 TIME STEP/        STAR 181
2 55X,21HRESTART WRITE EVERY 10 TIME STEP/        STAR 182
175 FORMAT(43X,7HSTRAIN =15,15X,8XHSTRAIN =15/   STAR 183
1 43X,7HLOAD =15,15X,8XHLOAD =15/                 STAR 184
180 FORMAT(44X,19HBOUNDARY CONDITIONS/47X,1H/2/3 = CLAMPED/ SYM.       STAR 185
1HINGED/7X,THEEDGE =14/5X,THEEDGE =14/5X,THEEDGE =14/       STAR 186
1 57X,THEEDGE4 =14/                                    STAR 187
185 FORMAT(50X,25HPRTN OPTION CONTROL CARD/52X,20H0/1 = NO PRINT/PRINT       STAR 188
17/50X,14,24H DISPLACEMENT INCREMENTS/        STAR 189
2 50X,14,32H CARTESIAN COORDINATES, PRESSURE/       STAR 190
3 50X,14,33H SURFACE NORMAL VECTOR COMPONENTS/       STAR 191
190 FORMAT(24X,45HPRINTER INFORMATION AT THE FOLLOWING TIME STEPS/124X,       STAR 192
1(1615))                                           STAR 193
195 FORMAT(24X,38H3-D PLOTS FOR THE FOLLOWING TIME STEPS/124X,(1615))       STAR 194
200 FORMAT(46X,29H5 START DAMPING AFTER TIME STEPS/5X,6HTIME =E10.4/       STAR 195
140X,7HDAMPF =E10.4,16X,7HDFACT =E10.4)              STAR 196
300 FORMAT(47X,25HBENDING TIME INCREMENT= E12.6/47X,25HMEMBRANE TIME       STAR 197
1INCEREMENT= E12.6/47X,25HINPUT TIME INCREMENT= E12.6/44X,31HTIM       STAR 198
2E INCREMENT USED BY REPSI= E12.6/)                STAR 199
400 FORMAT(37X,32HCONSTITUTIVE RELATION ELASTIC)   STAR 200
405 FORMAT(24X,7HCONSTITUTIVE RELATION       ELASTOPLASTIC-NO WORK HARDSTAR 201
1LENGTH-STRAIN RATE DEPENDENT)                    STAR 202
410 FORMAT(24X,7HCONSTITUTIVE RELATION       ELASTOPLASTIC-WORK HARDENSTAR 203
1LENGTH-STRAIN RATE INDEPENDENT)                  STAR 204
415 FORMAT(24X,8HCONSTITUTIVE RELATION       ELASTOPLASTIC-WORK HARDSTAR 205
1LENGTH-STRAIN RATE DEPENDENT)                    STAR 206
420 FORMAT(37X,7HCONSTITUTIVE RELATION       ELASTOPLASTIC-WORK HARDENSTAR 207
1LENGTH-STRAIN RATE DEPENDENT)                    STAR 208
500 FORMAT(24X,45HPRTN L MATRIX (LMAT) AT THE FOLLOWING TIME STEPS/       STAR 209
1(124X,1615))                                      STAR 210
820 FORMAT(35X,31HSTRESS-STRAIN APPROXIMATION HAS,13,10H SUBLAYERS)       STAR 211
821 FORMAT(46X,40HSTRESS STRAIN AND STRAIN RATE PARAMETERS/       STAR 212
130X,1HJ,9X,7HSSIG(J),10X,7HSSIG(J),6MIRSIG(J),6X,8HPSR(J)/       STAR 213
2X,15X,162E15.7,5X,2E15.7))                           STAR 214
END                                                STAR 215

176
SUBROUTINE INVEL
COMMON (USE MAIN)
C
EVALUATE THE INITIAL VELOCITY AT TIME=0 FOR ALL MESH POINTS
READ(5,100) M,N,M,F,N,F,VR,NO
WRITE(6,200) M,N,M,F,N,F,VR
C
DO 30 M=1,M,F
DO 30 N=1,N,F
U1(M,N)=VR*SH1(M,N)
U2(M,N)=VR*SH2(M,N)
U3(M,N)=VR*SH3(M,N)
30 CONTINUE
IF(NW)5F0,50,40
40 WRITE(6,300)
DO 45 K=1,N,F
READ(5,500) R,N,V
WRITE(6,400) R,N,V
U1(R,N)=VR*SH1(R,N)
U2(R,N)=VR*SH2(R,N)
U3(R,N)=VR*SH3(R,N)
45 CONTINUE
IF(NW)5F0,50,40
50 RETURN
C
100 FORMAT(15,E12.6,15)
200 FORMAT(H1,E13,1H3,9H) AND (H13,1H3,9H) RECEIVE
FULL VELOCITY, (VR)= .E12.6/
300 FORMAT(52X,2THOTHER VELOCITY DISTRIBUTION/57X,1HM,4X,1HN,8X,1HV/)
400 FORMAT(54X,2F15.2X,E12.6)
500 FORMAT(2F15,E12.6)
END
SUBROUTINE POSIT
COMMON (USE MAIN)
DO 50 M = 1, NM
DO 50 N = 1, NM
Y1(M,N) = Y1(M,N) + U1(M,N)
Y2(M,N) = Y2(M,N) + U2(M,N)
Y3(M,N) = Y3(M,N) + u3(M,N)
50 CONTINUE
IF (NCYCLE .NE. NCYC(H MMM) ) GOTO 75
WRITE DISPLACEMENT INCREMENTS
IF (JCHK(11) I 63, 65)
5 NCYC = NCYCLE - 1
WRITE (6, 509) NCYC, NCYCLE
WRITE (6, 9915)
DO 60 M = 2, NM
60 WRITE (6, 9917) M, Y1(M,N), u2(M,N), u3(M,N), N = 1, NM
WRITE CARTESIAN COORDINATES, PRESSURE
IF (JCHK(21) I 75, 75)
65 WRITE (6, 99991) NCYCLE, TIME
WRITE (6, 903) NCYCLE, NCYCLE, NCYCLE
DO 70 M = 2, NM
70 WRITE (6, 902) M, Y1(M,N), Y2(M,N), Y3(M,N), P(M,N), N = 1, NM
75 RETURN
C
509 FORMAT (1H1, 2IX, 36D DISPLACEMENT INCREMENTS BETWEEN T.S. 14, 4H AND 14, 4)
9915 FORMAT (1H, 9X, 1HM, 4X, 1HM, 10X, 7H1(M,N), 18X, 7H2(M,N), 18X, 7H3(M,N))
9917 FORMAT (16, 15, 3D2, 2X, 21, 16) (/111, 3D(2X, E23, 16))
99991 FORMAT (10H TIME STEP, 15, 6X, 5H TIME, 16, 6)
903 FORMAT (3D2, 1HM, 3D1HM, 19X, 7H1(M,N), 14, 1H), 13X, 7H2(M,N), 14, 1H), 13X, 7H3(M,N), 14, 1H)
2, 15X, 6H(N,N))
902 FORMAT (214, 4D2, 2X, 21, 16) (/18, 4D(2X, E23, 16))
END
1
CB1 = R1*U11 + R2*U12 + R3*U13 + C111*DNL1 + C121*DNL2 + B11
CB2 = R1*U21 + R2*U22 + R3*U23 + C122*DNL1 + C122*DNL2 + B12
CB12 = R1*U12 + R2*U12 + R3*U13 + C112*DNL1 + C122*DNL2 + B12
CD = 89, 1, LAYER
CPPS11(K) = DA11 - ZETA(K) * DB11
CPPS21(K) = DA22 - ZETA(K) * DB22
CGAMMA1(K) = DA12 - ZETA(K) * DB12
CALL STRESS(M, N, K)
89 CONTINUE
EPSLI(M, N) = EPSLI(M, N) + DA11 + DB11
EPSL2(M, N) = EPSL2(M, N) + DA22 + DB22
GAMMAL(M, N) = GAMMAL(M, N) + DA12 + DB12
EPSU1(M, N) = EPSU1(M, N) + DA11 - DB11
EPSU2(M, N) = EPSU2(M, N) + DA22 - DB22
GAMMAU(M, N) = GAMMAU(M, N) + DA12 - DB12
CALL RESULT(M, N)
90 CONTINUE
IF(NCYCLE .EQ. 0) GOTO 820
CALL SYMPY
100 STREN = CB1*STREN
   IF(BGE2 .EQ. 2) STREN = 2.0*STREN
C
WRITE LNATRIX
IF(NCYCLE .NE. JCYNLP) GOTO 820
MLP = MLP + 1
WRITE(6, 811) NCYCLE, TIME
DO 800 K = 1, LAYER
   WRITE(6, 812) K, (M, N = 2, M1)
   WRITE(6, 812) K, (M, N = 2, M1)
   WRITE(6, 813) M, (N = 2, M1), SM(N, M), SM2(N, M), SM3(N, M), N = 1, M1
800 WRITE(6, 820) M, (N = 2, M1), SM(N, M), SM2(N, M), SM3(N, M), N = 1, M1
700 RETURN
C
011 FORMAT(10H1TIME, STEP, 15, 6X, 4HTIME, 1PE16.7, 1H., 10X, 1 40SUBDIVISONS OF TIME INCREMENT IN STRESS/)
812 FORMAT(10H1TIME, STEP, 15, 6X, 3MMATIM, M1, 12, 11:1///M N M = 4013/) 813 FORMAT(13, 5X, 4013)
900 FORMAT(10H1TIME, STEP, 15, 6X, 5M TIME, E16.8)
910 FORMAT(12X, 32HSURFACE NORMAL VECTOR COMPONENTS/3X, 1 1MH, 3X, 1HN, 3X, BSN1(M, N), lTX, BSN2(M, N), lTX, BSN3(M, N), lTX, BSN3(M, N))
920 FORMAT(214, 312X, E29.16/16, 312X, E29.16))
END
SUBROUTINE GRAD (MD, NO)

COMMON (USE MAIN)

MD = NO

IF (MD.EQ.0) GOTO 3

Y11 = RTD1(Y1(N+1,N)-Y1(N-1,N))
Y12 = RTD1(Y2(N+1,N)-Y2(N-1,N))
Y13 = RTD1(Y3(N+1,N)-Y3(N-1,N))
U11 = RTD1(U1(N+1,N)-U1(N-1,N))
U12 = RTD1(U2(N+1,N)-U2(N-1,N))
U13 = RTD1(U3(N+1,N)-U3(N-1,N))

Y11 = RTD1(Y1(N+1,N)-2.*V1(N,N)+Y1(N-1,N))
Y12 = RTD1(Y2(N+1,N)-2.*V2(N,N)+Y2(N-1,N))
Y13 = RTD1(Y3(N+1,N)-2.*V3(N,N)+Y3(N-1,N))
U11 = RTD1(U1(N+1,N)-2.*U1(N,N)+U1(N-1,N))
U12 = RTD1(U2(N+1,N)-2.*U2(N,N)+U2(N-1,N))
U13 = RTD1(U3(N+1,N)-2.*U3(N,N)+U3(N-1,N))

IF (V.EQ.1) GOTO 1

IF (V.EQ.2) GOTO 2

Y12 = RTD1(Y1(N+1,N)+Y1(N-1,N)-V1(N+1,N)+V1(N-1,N))

Y12 = RTD1(Y2(N+1,N)+Y2(N-1,N)-V2(N+1,N)+V2(N-1,N))

Y12 = RTD1(Y3(N+1,N)+Y3(N-1,N)-V3(N+1,N)+V3(N-1,N))

U12 = RTD1(U1(N+1,N)+U1(N-1,N)-U1(N+1,N)+U1(N-1,N))

U12 = RTD1(U2(N+1,N)+U2(N-1,N)-U2(N+1,N)+U2(N-1,N))

U12 = RTD1(U3(N+1,N)+U3(N-1,N)-U3(N+1,N)+U3(N-1,N))

GOTO 4

1 Y12 = RTD1(Y1(N+1,N)+Y1(N-1,N)-V1(N+1,N)+V1(N-1,N))

Y12 = RTD1(Y2(N+1,N)+Y2(N-1,N)-V2(N+1,N)+V2(N-1,N))

Y12 = RTD1(Y3(N+1,N)+Y3(N-1,N)-V3(N+1,N)+V3(N-1,N))

U12 = RTD1(U1(N+1,N)+U1(N-1,N)-U1(N+1,N)+U1(N-1,N))

U12 = RTD1(U2(N+1,N)+U2(N-1,N)-U2(N+1,N)+U2(N-1,N))

U12 = RTD1(U3(N+1,N)+U3(N-1,N)-U3(N+1,N)+U3(N-1,N))

GOTO 4

2 Y12 = RTD1(Y1(N+1,N)-Y1(N-1,N)-V1(N+1,N)-V1(N-1,N))

Y12 = RTD1(Y2(N+1,N)-Y2(N-1,N)-V2(N+1,N)-V2(N-1,N))

Y12 = RTD1(Y3(N+1,N)-Y3(N-1,N)-V3(N+1,N)-V3(N-1,N))

U12 = RTD1(U1(N+1,N)-U1(N-1,N)-U1(N+1,N)-U1(N-1,N))

U12 = RTD1(U2(N+1,N)-U2(N-1,N)-U2(N+1,N)-U2(N-1,N))

U12 = RTD1(U3(N+1,N)-U3(N-1,N)-U3(N+1,N)-U3(N-1,N))

GOTO 4

3 Y11 = RTD1(Y1(N-1,N)-4.*V1(N-1,N)+3.*V1(N,N))
Y12 = RTD1(Y2(N-1,N)-4.*V2(N-1,N)+3.*V2(N,N))
Y13 = RTD1(Y3(N-1,N)-4.*V3(N-1,N)+3.*V3(N,N))
U11 = RTD1(U1(N-1,N)-4.*U1(N-1,N)+3.*U1(N,N))
U12 = RTD1(U2(N-1,N)-4.*U2(N-1,N)+3.*U2(N,N))
U13 = RTD1(U3(N-1,N)-4.*U3(N-1,N)+3.*U3(N,N))

Y11 = RTD1(Y1(N-1,N)-5.*V1(N-1,N)+4.*V1(N-2,N)-Y1(N-3,N))
Y12 = RTD1(Y2(N-1,N)-5.*V2(N-1,N)+4.*V2(N-2,N)-Y2(N-3,N))
If (in eq. 1) goto 5

If (in eq. 2) goto 6

Y12 = RTD2*(Y1(N+1)-Y1(N))
Y22 = RTD2*(Y2(N)-Y2(N-1))
U21 = RTD2*(U1(N)-U1(N-1))
U22 = RTD2*(U2(N)-U2(N-1))
U23 = RTD2*(U3(N)-U3(N-1))
U24 = RTD2*(U4(N)-U4(N-1))
U25 = RTD2*(U5(N)-U5(N-1))
U26 = RTD2*(U6(N)-U6(N-1))

IF (in eq. 3) goto 7

1 = -4.0*(Y1(N-1)+Y1(N-1)+3.0*RTD1*Y21)
Y12 = -4.0*(Y1(N+1)-Y1(N)-Y1(N+1))
Y22 = -4.0*(Y2(N+1)-Y2(N)-Y2(N+1))
U21 = -4.0*(U1(N+1)-U1(N)-U1(N+1))
U22 = -4.0*(U2(N+1)-U2(N)-U2(N+1))
U23 = -4.0*(U3(N+1)-U3(N)-U3(N+1))
U24 = -4.0*(U4(N+1)-U4(N)-U4(N+1))
U25 = -4.0*(U5(N+1)-U5(N)-U5(N+1))
U26 = -4.0*(U6(N+1)-U6(N)-U6(N+1))

IF (in eq. 4) goto 7

Y12 = RTD2*(Y1(N+2)-Y1(N+1)-Y1(N+1))
Y22 = RTD2*(Y2(N+2)-Y2(N+1)-Y2(N+1))
U21 = RTD2*(U1(N+1)-U1(N)+U1(N+1))
U22 = RTD2*(U2(N+2)-U2(N+1)-U2(N+1))
U23 = RTD2*(U3(N+2)-U3(N+1)-U3(N+1))
U24 = RTD2*(U4(N+2)-U4(N+1)-U4(N+1))
U25 = RTD2*(U5(N+2)-U5(N+1)-U5(N+1))
U26 = RTD2*(U6(N+2)-U6(N+1)-U6(N+1))

IF (in eq. 5) goto 7

Y12 = RTD2*(Y1(N+2)-Y1(N+1)-Y1(N+1))
Y22 = RTD2*(Y2(N+2)-Y2(N+1)-Y2(N+1))
U21 = RTD2*(U1(N+1)-U1(N)+U1(N+1))
U22 = RTD2*(U2(N+2)-U2(N+1)-U2(N+1))
U23 = RTD2*(U3(N+2)-U3(N+1)-U3(N+1))
U24 = RTD2*(U4(N+2)-U4(N+1)-U4(N+1))
U25 = RTD2*(U5(N+2)-U5(N+1)-U5(N+1))
U26 = RTD2*(U6(N+2)-U6(N+1)-U6(N+1))

IF (in eq. 6) goto 7

Y12 = RTD2*(Y1(N+2)-Y1(N+1)-Y1(N+1))
Y22 = RTD2*(Y2(N+2)-Y2(N+1)-Y2(N+1))
U21 = RTD2*(U1(N+1)-U1(N)+U1(N+1))
U22 = RTD2*(U2(N+2)-U2(N+1)-U2(N+1))
U23 = RTD2*(U3(N+2)-U3(N+1)-U3(N+1))
U24 = RTD2*(U4(N+2)-U4(N+1)-U4(N+1))
U25 = RTD2*(U5(N+2)-U5(N+1)-U5(N+1))
U26 = RTD2*(U6(N+2)-U6(N+1)-U6(N+1))
Y23 = RTD2*Y3(M,N-2) - 4.0*Y3(M,N-1) + 3.0*Y3(M,N-1)
U21 = RTD2*U1(M-2,N-2) - 4.0*U1(M-1,N-1)
U22 = RTD2*U2(M,N-2) - 4.0*U2(M,N-1) + 3.0*U2(M,N-1)
U23 = RTD2*U3(M,N-2) - 4.0*U3(M,N-1) + 3.0*U3(M,N-1)
Y21 = RTD2*(2.0*Y1(M,N) - 5.0*Y1(M,N-1) + 4.0*Y1(M,N-2) - Y1(M,N-3))
Y22 = RTD2*(2.0*Y2(M,N) - 5.0*Y2(M,N-1) + 4.0*Y2(M,N-2) - Y2(M,N-3))
Y23 = RTD2*(2.0*Y3(M,N) - 5.0*Y3(M,N-1) + 4.0*Y3(M,N-2) - Y3(M,N-3))
U21 = RTD2*(2.0*U1(M,N) - 5.0*U1(M,N-1) + 4.0*U1(M,N-2) - U1(M,N-3))
U22 = RTD2*(2.0*U2(M,N) - 5.0*U2(M,N-1) + 4.0*U2(M,N-2) - U2(M,N-3))
U23 = RTD2*(2.0*U3(M,N) - 5.0*U3(M,N-1) + 4.0*U3(M,N-2) - U3(M,N-3))

IF (M.N.E.NM) GOTO 7

Y121 = RTD2*Y1(M-2,N-2) - 4.0*Y1(M-1,N-2) + Y1(M-2,N-1)
U121 = RTD2*(Y1(M-2,N-2) - 4.0*Y1(M-1,N-2) + Y1(M-2,N-1))
Y122 = RTD2*(Y2(M-2,N-2) - 4.0*Y2(M-1,N-2) + Y2(M-2,N-1))
U122 = RTD2*(Y2(M-2,N-2) - 4.0*Y2(M-1,N-2) + Y2(M-2,N-1))
Y123 = RTD2*(Y3(M-2,N-2) - 4.0*Y3(M-1,N-2) + Y3(M-2,N-1))
U123 = RTD2*(Y3(M-2,N-2) - 4.0*Y3(M-1,N-2) + Y3(M-2,N-1))

RETURN

END
SUBROUTINE STRESS(MD,MD,KD) 

COMMON(USE MAIN) 

K=MD 

. + MD 

K=KD 

SS11=0.0 

SS12=0.0 

SS21=0.0 

SS22=0.0 

LMMK=0 

KN=(K-1)*NSFL 

ZETAK=2.0*ZETA(K) 

G11 =A11-ZETAK*B11 

G12 =A12-ZETAK*B12 

G22 =A22-ZETAK*R22 

DG=G11*G22-G12**2 

SRE=SQR(DG) 

RG=1.0/SG 

GR11 = RG * G22 

GR12 = RG * G12 

GR22 = RG * G11 

DEPS11 = GR11*DEPS11(K) + GR12*DGAMMA(K) 

DEPS12 = GR12*DEPS12(K) + GR11*DGAMMA(K) 

DEPS21 = GR21*DEPS21(K) + GR22*DGAMMA(K) 

DEPS22 = GR22*DEPS22(K) + GR12*DGAMMA(K) 

DS1G11 = ?RAT*(DEPS11 + FNU*DEPS22) 

DS1G12 = GTWO*DEPS12 

DS1G21 = GTWO*DEPS21 

DS1G22 = PRAT*(DEPS22 + FNU*DEPS11) 

IF(ISR.GT.0).EPSDOT=SQR(DEPS11*(DEPS11-DEPS22)+DEPS22**2 

!+3.0*(DEPS12*DEPS21)/DELTAT 

DO 1 J=1,NSFL 

K=N* + J 

SIG11I=G11I*SIG11(M,N,KJ)+G12I*TAU(M,N,KJ) 

SIG12I=G12I*SIG11(M,N,KJ)+G22I*TAU(M,N,KJ) 

SIG21I=G21I*SIG21(M,N,KJ)+G11I*TAU(M,N,KJ) 

SIG22I=G22I*SIG12(M,N,KJ)+G12I*TAU(M,N,KJ) 

SIGYSQ=SIGYSQ.I 

IF(ISR.GT.0).SIGYSQ=SIGYSQ.I*(1.0+(EPSDOT/DSR(J))**PSR(J))**2 

I=1 

LC=1 

SIG11L=SIG11I+SIG11 

SIG12L=SIG12I+SIG12 

SIG21L=SIG21I+SIG21 

SIG22L=SIG22I+SIG22 

IF(ISR.LT.0).GOTO 2 

PHI=SIG11L*(SIG11L-SIG22L)+SIG22L**2+3.0*SIG12L*SIG21L-SIGYSQ 

IF(PHIT.LE.0.0).GOTO 2 

L=INT(YLDFAC*(SORT(PHIT+SIGYSQ)/SIGYSQ)-1.0))+1 

100 SIG11L=SIG11I 

SIG12L=SIG12I 

SIG21L=SIG21I 

SIG22L=SIG22I 

IF(L.EQ.1).GOTO 3 

LC+1 

FLOATL=1.0/FLOAT(L) 

DS1G11L = DS1G11L*FLOATL 

DS1G22L = DS1G22L*FLOATL 

184
DSG21L = DSG21*FLOATL
DSG22L = DSG22*FLOATL
101 SIG11L = SIG11L+DSG11L
SIG12L = SIG12L+DSG12L
SIG21L = SIG21L+DSG21L
SIG22L = SIG22L+DSG22L
PHIT = SIG11L*(SIG11L-SIG22L)+SIG22L**2+3.0*SIG12L*SIG21L-SIGYSQ
IF(PHIT .GT. 0.0) GOTO 3
C ELASTIC
SIG11 = SIG11L
SIG12 = SIG12L
SIG21 = SIG21L
SIG22 = SIG22L
GOTO 9
C PLASTIC
SIG11D = (2.0-FNU)*SIG11-(1.0-2.0*FNU)*SIG22
SIG12D = 3.0*(1.0-FNU)*SIG12
SIG21D = 3.0*(1.0-FNU)*SIG21
SIG22D = (2.0-FNU)*SIG22-(1.0-2.0*FNU)*SIG11
AA = SIG11D**2-SIG11D*SIG22D+SIG22D**2+3.0*SIG12D*SIG21D
B = -(SIG11D*(2.0*SIG11D-SIG22D)+SIG22L*(2.0*SIG22D-SIG11D)+9.0*(SSTRE
SIG12L*SIG21L*SIG12L))
D = B**2-4.*AA*PHIT
IF(L .GT. LMNK) LMNK=L
IF(AA) 16,4
8 WRITE(6,10)
10 FORMAT(1H 4X,14AA NEGATIVE AT)
GOTO 12
4 IF(LT.0.0 OR. B.GT.0.0) GOTO 16
TAMBD = (-B-SQRT(D))/(2.*AA)
SIG11 = SIG11L-TAMBA*SIG11D
SIG12 = SIG12L-TAMBA*SIG12D
SIG21 = SIG21L-TAMBA*SIG21D
SIG22 = SIG22L-TAMBA*SIG22D
9 LC = LC+1
IF(LC=L1) 101,101,102
16 L = L+1
IF(L=100) 100,100,103
103 WRITE(6,104)
104 FORMAT(1H 4X,36HSTRESS CALCULATION UNSATISFACTORY AT)
12 WRITE(6,105) NCYCLE,M,N,K,L,LC
105 FORMAT(1H 4X,9HTIME STEP,14.5X,2HM=,12.5X,2HNM=,12.5X,2HK=,12.5X,2STRE.
1HM=13.5X,3HLC=1,L3)
WRITE(6,106) EPSL1(M,N),EPSL2(M,N),GAMMAL(M,N),EPSU1(M,N),
LEPSU2(M,N),GAMMAU(M,N),DEPS1(K),DEPS2(K),DGAU(M,K),
2 SIG1(M,N,K),SIG2(M,N,K),TAUM(N,K),SIG11,
DSIG1L,DSIG12,DSIG21,DSIG22,SIG11,SIG12,SIG21,SIG22,SIG11L,SIG12L,SIG21L,SIG22L
SRE
3 SIG11L,SIG12L,SIG21L,SIG22L,AA,B,PHIT
SRE
106 FORMAT(1H 4X,9X,7HEPS1 =E15.8,3X,7HEPS2 =E15.8,3X,7HEPAML =E15.8,SRE
1/1H,9X,THEPS1L =E15.8,3X,7THEPS2L =E15.8,3X,8HGAMMA =E15.8,
11
1/1H,9X,7HDS1G1 =E15.8,3X,7HDS2G2 =E15.8,3X,7HDS1G1L =E15.8,3X,7HDS2G2L =E15.8,3X,7HDS1G1STRE
12
2H,9X,7HDS1G1L =E15.8,3X,7HDS1G2L =E15.8,3X,7HDS2G1L =E15.8,3X,7HDS2G2L =E15.8,3X,7HDS1G1STRE
13
15.8,3X,7HDS1G2L =E15.8,1H,9X,7HDS1L =E15.8,3X,7HDS1L =E15.8,3X,7HDS1LSTRE
14
5L,8,3X,7HDS1G2L =E15.8,1H,9X,7HDS1G1L =E15.8,3X,7HDS1G2L =E15.8,3X,7HDS1G2L =E15.8,3X,7HDS1G1LSTRE
15
6,3X,7HDS1G2L =E15.8,3X,7HDS2G2L =E15.8,1H,9X,7HDS1G1D =E15.8,3X,7HDS2G2L =E15.8,3X,7HDS2G1STRE
16
7HDS1G1D =E15.8,3X,7HDS2G2L =E15.8,3X,7HDS2G1D =E15.8,1H,9X,7HDS2G1STRE
17
8 =E15.8,3X,7PH =E15.8,3X,7HSTRE
SRE
118
NC30P(N3D)=NCYCLE
SRE
119
CALL POATA (2)
SRE
120
CALL PDATA (3)
CALL PDATA (4)
CALL EXIT
102 SS11=SS11*SIG11*WT(J)
SS12=SS12*SIG12*WT(J)
SS21=SS21*SIG21*WT(J)
SS22=SS22*SIG22*WT(J)
SIG11(M,N,KJ)=GR11*SIG11*GR12*SIG12
SIG22(M,N,KJ)=GR22*SIG22*GR12*SIG21
TAU (M,N,KJ)=GA12*SIG22*GA11*SIG21
803 CONTINUE
IF(IN .EQ. 2 .OR. M .EQ. M1) SRG=0.5*SRG
IF(IN .EQ. 1 .OR. N .EQ. N1) SRG=0.5*SRG
STREN=STREN+(SS11+SS22)*2-(1.+FNU)*Z.*(SS11*SS22-SS12*SS21)*SRG
SS1NN(K)=GR11*SS11+GR12*SS12
SS2NN(K)=GR22*SS22+GR12*SS21
STNN(K)=GA12*SS22+GR11*SS21
LMAT(N,N,K) = LMNK
RETURN
END
SUBROUTINE RESULT(MD, ND) 
COMMON (USE MAIN) 
C THE CALCULATION OF THE STRESS AND MOMENT RESULTANTS FOLLOW 
M=MD 
N=ND 
TB=TA*SRA 
SUMA11 = SS1MN(1) 
SUMA22 = SS2MN(1) 
SUMA12 = STMN(1) 
SUMB11 = SS1MN(1)*ZETA(1) 
SUMB22 = SS2MN(1)*ZETA(1) 
SUMB12 = STMN(1)*ZETA(1) 
SUMC11 = SS1MN(1)*ZETASQ(1) 
SUMC22 = SS2MN(1)*ZETASQ(1) 
SUMC12 = STMN(1)*ZETASQ(1) 
IFILAYER .EQ. 1)GOTO 5 
DO 4 K=2, LAYER 
SUMA11 = SUMA11 + SS1MN(K) 
SUMA22 = SUMA22 + SS2MN(K) 
SUMA12 = SUMA12 + STMN(K) 
SUMB11 = SUMB11 + SS1MN(K)*ZETA(K) 
SUMB22 = SUMB22 + SS2MN(K)*ZETA(K) 
SUMB12 = SUMB12 + STMN(K)*ZETA(K) 
SUMC11 = SUMC11 + SS1MN(K)*ZETASQ(K) 
SUMC22 = SUMC22 + SS2MN(K)*ZETASQ(K) 
SUMC12 = SUMC12 + STMN(K)*ZETASQ(K) 
4 CONTINUE 
IF ILayer .NE. 1)GOTO 6 
RESU 27 
Q11=SUMA11-BT*SUMB11 
Q22=SUMA22-BT*SUMB22 
Q12=SUMA12-BT*SUMB12 
F11=SUMB11-(BT-BM11)*SUMC11-BM12*SUMC12 
F22=SUMB22-(BT-BM22)*SUMC22-BM21*SUMC12 
F12=SUMB12-0.5*(BT*SUMC12-0.5*(BM21*SUMC11+BM12*SUMC22) 
IF(IBCE4 .NE. 3)GOTO 1 
IF(IN .NE. 1)GOTO 1 
IF(IBCE2 .NE. 3)GOTO 2 
IF(IN .NE. NN)GOTO 2 
FM11=MN)GOTO 3 
FI1=0.0 
RESU 37 
RESU 38 
RESU 39 
RESU 40 
RESU 41 
RESU 42 
3 CSM1=CS11*F11+CS12*F22+2.0*CS12*F12 
CSM2=CS211*F11+CS22*F22+2.0*CS22*F12 
FNT11=Q11*Y11+Q12*Y21+CSM1*SN1(M,N) 
FNT12=Q11*Y12+Q12*Y22+CSM1*SN2(M,N) 
FNT13=Q11*Y13+Q12*Y23+CSM1*SN3(M,N) 
FNT21=Q12*Y11+Q22*Y21+CSM2*SN1(M,N) 
FNT22=Q12*Y12+Q22*Y22+CSM2*SN2(M,N) 
FNT23=Q12*Y13+Q22*Y23+CSM2*SN3(M,N) 
FM11(M,N) = TB*F11 
FM22(M,N) = TB*F22 
FM12(M,N) = TB*F12 
FM11(M,N) = TB*FNT11 
FM21(M,N) = TB*FNT21 
FM13(M,N) = TB*FNT13 
FM22(M,N) = TB*FNT22 
FM23(M,N) = TB*FNT23 
RETURN 
RESU 51 
RESU 52 
RESU 53 
RESU 54 
RESU 55 
RESU 56 
RESU 57 
RESU 58 
RESU 59 
RESU 60 
RESU 61
SUBROUTINE MOTION
COMMON (U$E, MAIN)

C
VF1=0.0
VF2=0.0
VF3=0.0
IF(LOAD) 10, 30, 10
10 ENS=0.0
CALL PWORK

C
DO 130 M=2, NS
DO 130 N=2, NS
VM1=RD11*(FM11(M+1, N)*SN1(M+1, N)-2.0*FM11(M, N)*SN1(M, N))
1 +FM11(M-1, N)*SN1(M-1, N)) + TRD * (FM12(M+1, N+1)*SN1(M, N-1)
2 (M+1, N+1)-FM12(M+1, N-1)*SN1(M+1, N-1)-FM12(M-1, N+1)*SN1(M-1, N+1)
3 ) +FM12(M-1, N-1)*SN1(M-1, N-1)+RD22*(FM22(M, N+1)*SN1(M, N+1)
4 -2.0*FM22(M, N)*SN1(M, N)+FM22(M, N-1)*SN1(M, N-1))
VM2=RD11*(FM11(M+1, N)*SN2(M+1, N)-2.0*FM11(M, N)*SN2(M, N))
1 +SN2(M-1, N)) + TRD * (FM12(M+1, N+1)*SN2(M+, N+1)-FM12(M+1, N-1)
2 ) +SN2(M+1, N-1)-FM12(M+1, N+1)*SN2(M+1, N+1)+FM12(M-1, N-1)*SN2(M, N-1)
3 -1, N-1)*RD22*(FM22(M, N+1)*SN2(M, N+1)-2.0*FM22(M, N)*SN2(M, N)+FM22(M, N-1)
4 ) +SN2(M-1, N+1)*SN2(M, N))
VM3=RD11*(FM11(M+1, N)*SN3(M+1, N)-2.0*FM11(M, N)*SN3(M, N)+FM11(M-1, N))
1 +SN3(M-1, N)) + TRD * (FM12(M+1, N+1)*SN3(M+1, N)+FM12(M-1, N-1)
2 ) +SN3(M+1, N-1)+FM12(M-1, N+1)*SN3(M-1, N+1)-FM12(M-1, N-1)*SN3(M, N-1)
3 -2.0*FM22(M, N)*SN3(M, N)+FM22(M, N-1)*SN3(M, N-1)+RD22*(FM22(M, N+1)*SN3(M, N+1)
4 -2.0*FM22(M, N)*SN3(M, N)+FM22(M, N-1)*SN3(M, N-1))
VM1=RT01*(FMN1(M+1, N)-FN11(M+1, N)-FN11(M-1, N)) +RTD21*FM21(M, N+1)-FM21(M, N-1)
VM2=RT01*(FMN1(M+1, N)-FN12(M+1, N)+FN12(M-1, N)+FN12(M, N-1)
VM3=RT01*(FMN1(M+1, N)-FN13(M+1, N)-FN13(M-1, N))+RTD21*FN23(M, N+1)-FN23(M, N-1)
IF(LOAD) 40, 50, 40
40 VF1=-SN1(M, N)*P(M, N)
VF2=-SN2(M, N)*P(M, N)
VF3=-SN3(M, N)*P(M, N)
C
50 U1R=U1(M, N)
U2R=U2(M, N)
U3R=U3(M, N)
U1S=U1R+(VM1+VN1+VF1)*TEMP(M, N)
U2S=U2R+(VM2+VN2+VF2)*TEMP(M, N)
U3S=U3R+(VM3+VN3+VF3)*TEMP(M, N)
IF(TDAMP.EQ.0.0)GOTO 115
C
VISCOUS DAMPING C1
U1S=U1S-(U1S+U1R)*CL
U2S=U2S-(U2S+U2R)*CL
U3S=U3S-(U3S+U3R)*CL
115 U1(M, N)=U1S
U2(M, N)=U2S
U3(M, N)=U3S
C
130 CONTINUE
CALL BOUND
CALL KINET
IF(LOAD) 65, 75, 65
65 CALL PWORK
EN=0.5*CAW*ENS
INF=ENS
IF(IBC2.NE.2)GOTO 74
EN=2.0*ENS
74 TNRG=TNRG+ENS
75 PLAST=TNRG+CINET-STREN-TDAMP
C
IF(NCYLE .NE. NCYCL(NNN)) GOTO 140
WRITE(6,99991) NCYCLE,TIME,CINET,STREN,PLAST,TNRG
NNN=NNN+1
140 RETURN
C
99991 FORMAT(10H TIME STEP,E15.3X,5HTIME,E16.8X,8HUNELASTIC,E15.8X,8HPLASTIC=E15.8X,14H TOTAL ENERG.,E15.8)
END
SUBROUTINE WRrAPE (KEY)
WRITE + READ CONTINUATION RUN TAPE (KEY=1, WRITE KEY=2, READ) WRRA 1
COMMON (USE MAIN) WRRA 2
GOTO I0,20,KEY WRRA 3
10 WRITE (1) NCYCLE, TIME, CINES, CINEP, TNRG, TDAMP, D1, D2, D3, ENR WRRA 4
   WRITE (1) (U1(M,N), U2(M,N), U3(M,N), V1(M,N), V2(M,N), Y3(M,N),
   EPSL1(M,N), EPSL2(M,N), GAMMA(M,N), EPSU1(M,N), EPSU2(M,N),
   2GAMMAU(M,N), P(M,N), N=1, NN), M=1, MM)
   WRITE (1) (SIG1(M,N,K), SIG2(M,N,K), TAU(M,N,K),
   EPSL1(M,N), EPSL2(M,N), GAMMA(M,N), EPSU1(M,N), EPSU2(M,N),
   2GAMMAU(M,N), P(M,N), N=1, NN), M=1, MM)
   WRITE (1) (SN1(M,N), SN2(M,N), SN3(M,N), TEMP(M,N), N=1, NN), M=1, MM)
   WRITE (1) (G11(N), G12(N), G12(N), ASA(N), BSA(N), CSA(N), ASB(N),
   1BSB(N), CSB(N), N=1, NSTN)) WRRA 10
   WRITE (6, 100) NCYCLE, TIME
   RETURN WRRA 11
20 REWIND 1 WRRA 12
25 READ (1) NCYCLE, TIME, CINES, CINEP, TNRG, TDAMP, D1, D2, D3, ENR WRRA 13
   READ (1) (U1(M,N), U2(M,N), U3(M,N), V1(M,N), V2(M,N), Y3(M,N),
   EPSL1(M,N), EPSL2(M,N), GAMMA(M,N), EPSU1(M,N), EPSU2(M,N),
   2GAMMAU(M,N), P(M,N), N=1, NN), M=1, MM)
   READ (1) (SIG1(M,N,K), SIG2(M,N,K), TAU(M,N,K),
   EPSL1(M,N), EPSL2(M,N), GAMMA(M,N), EPSU1(M,N), EPSU2(M,N),
   2GAMMAU(M,N), P(M,N), N=1, NN), M=1, MM)
   READ (1) (G11(N), G12(N), G12(N), ASA(N), BSA(N), CSA(N), ASB(N),
   1BSB(N), CSB(N), N=1, NSTN)) WRRA 20
   IF (NCYCLE.NE.NCONT) GOTO 25 WRRA 21
   WRITE (6, 200) NCYCLE, TIME
   RETURN WRRA 22
C 100 FORMAT(/H TAPE 1 WRITTEN, NCYCLE=I, TIME=E15.8) WRRA 23
   200 FORMAT(/H INFORMATION READ FROM TAPE 1 FOR CYCLE=I, TIME=E15.8, WRRA 24
   1.8)
END WRRA 25
SUBROUTINE STRAIN

PRINT STRAINS ON INNER OR OUTER SURFACE

DIMENSION EPSANG(6), EPSANB(6)
COMMON (USE MAIN)
DATA P1/3.141592653589793/
IFINCYCLE .GT. 0 GO TO 25

-------- INITIAL ENTRY --------

DO 20 I=1,NSTRN
A11=DM2(I)*DM2(I)*A11(I)*DN1(I)*A12(I)
A12=DM2(I)*DM2(I)*A12(I)*DN1(I)*A11(I)
1 +DM1(I)*DM2(I)*A123(I)*DN1(I)*A124(I)
A22=DM2(I)*DM2(I)*A221(I)*DN1(I)*A222(I)
L +DM1(I)*DM2(I)*A223(I)*DN1(I)*A224(I)
B11=DM2(I)*DN2(I)*B111(I)*DN1(I)*B112(I)
1 +DM1(I)*DN2(I)*B113(I)*DN1(I)*B114(I)
B12=DM2(I)*DN2(I)*B121(I)*DN1(I)*B122(I)
1 +DM1(I)*DN2(I)*B123(I)*DN1(I)*B124(I)
B22=DM2(I)*DN2(I)*B221(I)*DN1(I)*B222(I)
1 +DM1(I)*DN2(I)*B223(I)*DN1(I)*B224(I)
IFINETAG(I).EQ.1 GO TO 10
G11 = A11-2.0*H*BN1
G12 = A12-2.0*H*BN12
G22 = A22-2.0*H*BN22
GOTO 15
10 G11 = A11+2.0*H*BN11
G12 = A12+2.0*H*BN12
G22 = A22+2.0*H*BN22
15 G11(I)=1.0/G11
G122(I)=1.0/G22
GR=SQR(G11(I)*G122(I))
DELTA=G12*GR
G1122(I)=2.0*GR
SRDEL=1.0/SQR(1.0-DELTA**2)
ANGEL=ANGL(I)*PI/180.0
SA=SRDEL*SIN(ANGEL)
SB=COS(ANGEL)-DELTA*SA
AS1(I)=2.0*SA*SA
BS1(I)=2.0*SB*SB
CS1(I)=2.0*SA*SB
ANGEL=ANGL(I)*PI/180.0
SA=SRDEL*SIN(ANGEL)
SB=COS(ANGEL)-DELTA*SA
ASB(I)=2.0*SA*SA
BSB(I)=2.0*SB*SB
CSB(I)=2.0*SA*SB
20 CONTINUE
GOTO 25

25 LINK=1

CHECK FOR SURFACE STRAIN PRINT
IFINCYCLE-NPRINT) 40,30,30
NPRINT=NPRINT+NDELP
LINK=2
30 DO 40 I=1,NSTRN
11=NI1(I)
J1=NI1(I)
I2=NI2(I)
40 CONTINUE
JZ = N12(I)
IF(NETAG(I) .EQ. 1) GOTO 44
EPSR1 = (DN2(I) * EPSU1(I1, J1) + DN1(I) * EPSU1(I1, J2))
1 + DM1(I) * EPSU1(I2, J1) + DN1(I) * EPSU1(I2, J2)) * G11111)
EPSR2 = (DN2(I) * EPSU2(I1, J1) + DN1(I) * EPSU2(I1, J2))
1 + DM1(I) * EPSU2(I2, J1) + DN1(I) * EPSU2(I2, J2)) * G12211)
GAMMAR = (DN2(I) * GAMMAU1(I1, J1) + DN1(I) * GAMMAU1(I1, J2))
1 + DM1(I) * GAMMAU1(I2, J1) + DN1(I) * GAMMAU1(I2, J2)) * G11211)
GOTO 45
EPSR1 = (DN2(I) * EPSL1(I1, J1) + DN1(I) * EPSL1(I1, J2))
1 + DM1(I) * EPSL1(I2, J1) + DN1(I) * EPSL1(I2, J2)) * G11111)
EPSR2 = (DN2(I) * EPSL2(I1, J1) + DN1(I) * EPSL2(I1, J2))
1 + DM1(I) * EPSL2(I2, J1) + DN1(I) * EPSL2(I2, J2)) * G12211)
GAMMAR = (DN2(I) * GAMMAL(I1, J1) + DN1(I) * GAMMAL(I1, J2))
1 + DM1(I) * GAMMAL(I2, J1) + DN1(I) * GAMMAL(I2, J2)) * G11211)
CONTINUE

C COMPONENTS OF VECTOR DISPLACEMENT
D1 = D1 + QM2 * (QM2 * U1(MQ1, MQ1) + QM1 * U1(MQ2, MQ2))
1 + QM1 * QM2 * U1(MQ2, MQ1) + QM1 * U1(MQ2, MQ2))
D2 = D2 + QM2 * (QM2 * U2(MQ1, MQ1) + QM1 * U2(MQ2, MQ2))
1 + QM1 * QM2 * U2(MQ2, MQ1) + QM1 * U2(MQ2, MQ2))
D3 = D3 + QM2 * (QM2 * U3(MQ1, MQ1) + QM1 * U3(MQ2, MQ2))
1 + QM1 * QM2 * U3(MQ2, MQ1) + QM1 * U3(MQ2, MQ2))
GOTO (71, 50), LINK
WRITE(6, 60) N CYCLE, TIME
DO 70 1 = 1, MSTAN
ALFN = SHINNER
IF(NETAG(I) .EQ. 1) GOTO 67
ALFN = SHUTER
WRITE(6, 65) ETAG1(I), ETAG2(I), PM(I), PN(I), ALFN, EPSS1(I), EPSS2(I),
1 ANGLE(I), EPSANG(I), ANGLB(I), EPSANB(I)
CONTINUE
70 CONTINUE
71 RETURN

C
60 FORMAT(//: ON TIME STEP, 15.3, 5 HTIME, E16.8//: 14X, 19H SURFACE STRAINS)
1, 3X, 19H STRAIN GAGE READING/2X, 4THET1, 4X, 6THET2, 6X, 1HM, 7X, 1HN, 5X, 3X,
24H FACE, 8X, HANGLE 0, 10X, HANGLE 90, 6X, 5HANGLE, 18X, 5HANGLE/
65 FORMAT(1H, F7.3, 1X, F7.3, 2X, F7.3, 1X, F7.3, 2X, A5, 1X, 2X, E15.8)
12(2X, F6.2, 2X, E15.8))
END
SUBROUTINE BOUNDU

** COMPUTATION OF U1, U2, U3 AT THE BOUNDARY

COMMON (USE MAIN)

IF(IICE4 .EQ. 3) GOTO 121

DO 120 M = 2, M1

DUSN1 = (U1(M, 2) - 0.25*U1(M, 3)) + SN1(M, 1)
1. + (U2(M, 2) - 0.25*U2(M, 3)) + SN2(M, 1)
2. + (U3(M, 2) - 0.25*U3(M, 3)) + SN3(M, 1)

U1(M, 2) = U1(M, 2) - SN1(M, 1) * DUSN
U2(M, 2) = U2(M, 2) - SN2(M, 1) * DUSN
U3(M, 2) = U3(M, 2) - SN3(M, 1) * DUSN

120 CONTINUE

121 IF(IICE2 .NE. 1) GOTO 123

DO 122 M = 2, M1

DUSN1 = (U1(M, NS) - 0.25*U1(M, NR)) + SN1(M, NN)
1. + (U2(M, NS) - 0.25*U2(M, NR)) + SN2(M, NN)
2. + (U3(M, NS) - 0.25*U3(M, NR)) + SN3(M, NN)

U1(M, NS) = U1(M, NS) - SN1(M, NN) * DUSN
U2(M, NS) = U2(M, NS) - SN2(M, NN) * DUSN
U3(M, NS) = U3(M, NS) - SN3(M, NN) * DUSN

122 CONTINUE

123 IF(IICE3 .NE. 1) GOTO 125

DO 124 N = NB1, NB2

DUSN1 = (U1(NS, 2) - 0.25*U1(NS, 3)) + SN1(NS, M)
1. + (U2(NS, 2) - 0.25*U2(NS, 3)) + SN2(NS, M)
2. + (U3(NS, 2) - 0.25*U3(NS, 3)) + SN3(NS, M)

DUSN21 = (U1(NS, 2) - 0.25*U1(NS, 3)) + SN1(NS, M)
1. + (U2(NS, 2) - 0.25*U2(NS, 3)) + SN2(NS, M)
2. + (U3(NS, 2) - 0.25*U3(NS, 3)) + SN3(NS, M)

U1(NS, 2) = U1(NS, 2) - SN1(NS, M) * DUSN1 + SN1(NS, M) * DUSN2
U2(NS, 2) = U2(NS, 2) - SN2(NS, M) * DUSN1 + SN2(NS, M) * DUSN2
U3(NS, 2) = U3(NS, 2) - SN3(NS, M) * DUSN1 + SN3(NS, M) * DUSN2

124 CONTINUE

125 IF(IICE2 .NE. 1) GOTO 127

DO 128 N = NB1, NB2

DUSN1 = (U1(NS, NS) - 0.25*U1(NS, NR)) + SN1(M, NS)
1. + (U2(NS, NS) - 0.25*U2(NS, NR)) + SN2(M, NS)
2. + (U3(NS, NS) - 0.25*U3(NS, NR)) + SN3(M, NS)

DUSN21 = (U1(NS, NS) - 0.25*U1(NS, NR)) + SN1(M, NS)
1. + (U2(NS, NS) - 0.25*U2(NS, NR)) + SN2(M, NS)
2. + (U3(NS, NS) - 0.25*U3(NS, NR)) + SN3(M, NS)

U1(NS, NS) = U1(NS, NS) - SN1(M, NS) * DUSN1 + SN1(M, NS) * DUSN2
U2(NS, NS) = U2(NS, NS) - SN2(M, NS) * DUSN1 + SN2(M, NS) * DUSN2
U3(NS, NS) = U3(NS, NS) - SN3(M, NS) * DUSN1 + SN3(M, NS) * DUSN2

127 CONTINUE

SET SYMMETRY BOUNDARY CONDITIONS FOR EDGE1, EDGE2, EDGE3

50 IF(IICE2 .NE. 2) GOTO 30

DO 25 M = 2, M1

U1(M, NN) = U1(M, NR)
U2(M, NN) = U2(M, NR)
U3(M, NN) = U3(M, NR)
U2(M, NS) = 0.0

25 CONTINUE

30 DO 5 N = 1, NN

U1(1, N) = 0.
U1(1, N) = U1(3, N)
U2(1, N) = U2(3, N)
U3(1, N) = U3(3, N)

5 CONTINUE

C

194
IF(IBCE3 .NE. 2)GOTO 5
UL(MS,N) = 0.0
UI(MR,N) = UI(MR,N)
U2(MR,N) = U2(MR,N)
U3(MR,N) = U3(MR,N)
CONTINUE
RETURN
END
SUBROUTINE ABINIT(N,N)
COMMON (USE MAIN)
DO 200 I=1,NSTRA
IF(N.EQ.MI1(I)) .AND. (N.EQ.MI1(I)) .OR. N.EQ.MI2(I)) GOTO 205
IF(N.EQ.MI2(I)) .AND. (N.EQ.MI1(I)) .OR. N.EQ.MI2(I)) GOTO 220
GOTO 200
205 IF(N .EQ. MI2(I)) GOTO 210
   A11(I)=A11
   A12(I)=A12
   A21(I)=A22
   B11(I)=B11
   B12(I)=B12
   B21(I)=B22
   GOTO 200
210 A112(I)=A11
   A122(I)=A12
   A222(I)=A22
   B112(I)=B11
   B122(I)=B12
   B222(I)=B22
   GOTO 200
220 IF(N .EQ. MI2(I)) GOTO 225
   A113(I)=A11
   A123(I)=A12
   A223(I)=A22
   B113(I)=B11
   B123(I)=B12
   B223(I)=B22
   GOTO 200
225 A114(I)=A11
   A124(I)=A12
   A224(I)=A22
   B114(I)=B11
   B124(I)=B12
   B224(I)=B22
200 CONTINUE
RETURN
END
SUBROUTINE SYTBY
COMMON (USE MAIN)
DO 10 N=1, N1
SN1(l,N)=SN1(3,N)
SN2(l,N)=SN2(3,N)
SN3(l,N)=SN3(3,N)
FN11(l,N)=FN11(3,N)
FN21(l,N)=FN21(3,N)
FN31(l,N)=FN31(3,N)
FN41(l,N)=FN41(3,N)
FN51(l,N)=FN51(3,N)
FN61(l,N)=FN61(3,N)
FN71(l,N)=FN71(3,N)
FN81(l,N)=FN81(3,N)
FN91(l,N)=FN91(3,N)
FN101(l,N)=FN101(3,N)
FN111(l,N)=FN111(3,N)
FN121(l,N)=FN121(3,N)
FN131(l,N)=FN131(3,N)
FN141(l,N)=FN141(3,N)
FN131(l,N)=FN131(3,N)
FN121(l,N)=FN121(3,N)
FN221(l,N)=FN221(3,N)
FN331(l,N)=FN331(3,N)
10 CONTINUE
IF (IBCE3 .NE. 2) GOTO 30
DO 20 N=1, N1
SN1(M,N)=SN1(M,N)
SN2(M,N)=SN2(M,N)
SN3(M,N)=SN3(M,N)
FM11(M,N)=FM11(M,N)
FM21(M,N)=FM21(M,N)
FM12(M,N)=FM12(M,N)
FM13(M,N)=FM13(M,N)
FM12(M,N)=FM12(M,N)
FM21(M,N)=FM21(M,N)
FM22(M,N)=FM22(M,N)
FM11(M,N)=FM11(M,N)
FM12(M,N)=FM12(M,N)
FM21(M,N)=FM21(M,N)
FM22(M,N)=FM22(M,N)
FM33(M,N)=FM33(M,N)
20 CONTINUE
30 IF (IBCE2 .NE. 2) GOTO 50
DO 40 M=1, MM
SN1(M,NN)=SN1(M,NN)
SN2(M,NN)=SN2(M,NN)
SN3(M,NN)=SN3(M,NN)
FM11(M,NN)=FM11(M,NN)
FM12(M,NN)=FM12(M,NN)
FM13(M,NN)=FM13(M,NN)
FM21(M,NN)=FM21(M,NN)
FM22(M,NN)=FM22(M,NN)
FM11(M,NN)=FM11(M,NN)
FM12(M,NN)=FM12(M,NN)
FM21(M,NN)=FM21(M,NN)
FM22(M,NN)=FM22(M,NN)
FN22(M,NN)=FN22(M,NN)
FN33(M,NN)=FN33(M,NN)
40 CONTINUE
50 RETURN
END
SUBROUTINE KINET
COMMON (USE MAIN)
CINET=0,
DO 20 M=2,MS
CM=1.0
IF(M.EQ.2 .OR. (IBECE3.EQ.2 .AND. M.EQ.NS))CM=0.5
DO 20 N=2,NS
CN=1.0
IF(IBCE2 .EQ. 2 .AND. N .EQ. NS)CN=0.5
20 CINET=CINET+(U1(M,N)**2+U2(M,N)**2+U3(M,N)**2)/TSMPL(M,N)*CM*CN
IF(IBCE2 .NE. 2)GOTO 25
CINET=2.0*CINET
25 CINER=CINES
CINES=CA*CINET
CINET=0.5*(CINES+CINER)
RETURN
END
SUBROUTINE PWORK
COMMON (USE MAIN)
DO 20 M=2,MS
CM=1.0
IF(M.EQ.2 OR (IBCE3.EQ.2 AND M.EQ.MS))CM=0.5
DO 20 N=2,NS
CN=1.0
IF(IBCE2 .EQ. 2 .AND. N .EQ. NS)CN=0.5
CN=UI(M,N)*SN1(M,N)+U2(M,N)*SN2(M,N)+U3(M,N)*SN3(M,N)
ENS=ENS-CN*CWIM(M,N)
20 CONTINUE
RETURN
END
SUBROUTINE DAMP

COMMON (USE MAIN)

C CHECK FOR START OF DAMPING
IF(MCYCLE .LT. MDAMP) RETURN
DO 15 M=1,MM
DO 15 N=1,NN
PI(N,M)=0.0
15 CONTINUE

IF(CINES-CINER).LE.0 .AND. NDAMP RETURN
DO 20 M=1,MM
DO 20 N=1,NN
UL(M,N)=0.0
U2(M,N)=0.0
U3(M,N)=0.0
20 CONTINUE

CINEP=CINET
CINES1=CINER
CINES=0.0
MCYCLE=MCYCLE+1
TIME=TIME+DELTA
CALL MOTION
IF(CINES .LE. CINES1) GO TO 39
CALL DESTEP
39 CALL PDATA(2)
40 TDAMP=TDAMP*C2*CINES
45 RETURN
50 WRITE(6,100) MCYCLE,MAXC,MAXC=MCYCLE
55 MCYCLE=MCYCLE
55 NCYCLE=MCYCLE
CALL PDATA (2)
50 GOTO 45
100 FORMAT(IHI,10X,30HRUN SEL: TERMINATED TIME STEP,15)
END
SUBROUTINE DESTEP
C
CHANGE DELTAT
COMMON (USE MAIN)
DELTAT=SQRT(CINES/CINES)*DELTAT
DSQOLD=DEL SQ
C10LD=C1
DELSQ=DELTAT**2
C1=2.0*DELTAT*DAMP/GAMZ
C2=C2/(4.0+C2)
DELR=DELSQ/DSQOLD
DELS=DELR*(1.0-C1)/(1.0-C10LD)
CINES=CINES*DELS**2/DELR
CINET=0.5*(CINES+CINER)
PLAST=THRG-CINET-STREN-TDAMP
DO 10 M=2,N
DO 10 N=1,N1
TEMP(M,N)=DEL*R*TEMP(M,N)
UI(M,N)=DELS*UI(M,N)
U2(M,N)=DELS*U2(M,N)
U3(M,N)=DELS*U3(M,N)
10 CONTINUE
RETURN
END
SUBROUTINE PDATA(LINK)
  DIMENSION DAT(20)
  COMMON (USE MAIN)
  PDATA SELECTS AND WRITES DATA ON TAPE(NPLOT), FOR THE REPSIT
  PLOTTING PROCRAK
  C
  C
  GOTO (10,40,50,60), LINK
  10 NN3D=1
  II=2*NSTRN+8
  12 CALL SKIPFILE (NPLOT,1)
   READNPLOT (IFLAG)
   IF(IFLAG .EQ. 99999) GOTO 14
   READNPLOT (IFLAG)
   IF(IFLAG .NE. NCONT+1) GOTO 12
  14 CALL BACKFILE (NPLOT,1)
   GOTO 50
  15 WRITE(NPLOT) IBCE3,ETAD1,ETAD2,QR,ON,NSTRN
   WRITE(NPLOT) (ETAG1(I),ETAG2(I),PM(I),PN(I),NCONTAG(I)),I=1,NSTRN
   WRITE(NPLOT) NCYCLE,TIME,M1,N1,((Y1(M,N),Y2(M,N),Y3(M,N)),M=1,N1),
   IN=1,N1
  C
  DO 25 I=1,II
   DAT(I)=0.0
  25 CONTINUE
  30 DAT(3)=-TNRG
   DAT(6)=TNRG
   DAT(7)=TNRG
   DAT(8)=TNRG
  35 IFLAG=1
   WRITE(NPLOT) IFLAG
   WRITE(NPLOT) NCYCLE,(DAT(I),I=1,II)
   GOTO 100
  C
  40 DAT(1)=TIME
   DAT(2)=D1
   DAT(3)=D2
   DAT(4)=D3
   DAT(5)=CINE1
   DAT(6)=STREN+CINET
   DAT(7)=TNRG
   DAT(8)=DAT(6)+TAMP
   J=9
  45 CONTINUE
   IFLAG=1
   WRITE(NPLOT) IFLAG
   WRITE(NPLOT) NCYCLE,(DAT(I),I=1,II)
   CHECK FOR 3D PLOT
   IF(NCYCLE .NE. N3D) NC3DP(N3D) GOTO 100
   NN3D=NN3D+1
   IFLAG=2
   WRITE(NPLOT) IFLAG
   WRITE(NPLOT) NCYCLE,TIME,M1,N1,((Y1(M,N),Y2(M,N),Y3(M,N)),M=1,N1),
   IN=1,N1
   GOTO 100

202
C
  50 END FILE NPLT
     GOTO 100
C
  60 IFLAG=99999
     WRITE(NPLT) IFLAG
  100 RETURN
     END
SUBROUTINE PRESS
COMMON (USE MAIN)
COMMON /IPR/ RSQ(23,34)
DO 10 M=2,MS
DO 10 N=2,NS
T1=(SQRT(RSQ(M,N)+225.0)-15.0)/144000.0
P(M,N)=0.0
IF(TIME .LT. T1)GOTO 10
P(M,N)=5504737.5*EXP(-13000.0*(TIME-T1))/(RSQ(M,N)+225.0)
10 CONTINUE
RETURN
END
SUBROUTINE INGEOM

     FLAT PLATE

COMMON (USE MAIN)
COMMON /IPR/ RSO(23,34)
REAL LENGTH

READ (5,100) LENGTH,WIDTH
DETA1=WIDTH/FLOAT(NMESH)
DETA2=LENGTH/FLOAT(NMESH)

DO 10 M=2,N1
    DO 10 N=1,N1
    Y1(M,N)=FLOAT(M-2)*DETA1
    Y2(M,N)=FLOAT(N-1)*DETA2
    Y3(M,N)=0.0
    RSO(M,N)=Y1(M,N)**2+(Y2(M,N)-LENGTH)**2
10 CONTINUE
RETURN

100 FORMAT(2E10.4)
END
SUBROUTINE INGEON
  CYLINDER
  COMMON (USE MAIN)
  REAL LENGTH
  DATA DTOR, 1.745329251994329E-01/
  C
  READ (9, 100) LENGTH, RADIUS, THETA
  DETAL = THETA * RADIUS / DTOR / FLOAT(MESH)
  DETA2 = LENGTH / FLOAT(MESH)
  DO 10 M = 2, N1
  ETA1 = FLOAT(N - 2) * DETAL
  ETA2 = FLOAT(N - 1) * DETA2
  Y1(M, N1) = RADIUS * SIN(ETA1 / RADIUS)
  Y2(M, N1) = ETA1
  Y3(M, N1) = RADIUS * COS(ETA1 / RADIUS)
  10 CONTINUE
  C
  ETA1 = ETA1 * RADIUS / DTOR
  UD 20 I = 1, NSTRN
  ETA2(I) = ETA2(RADIUS / DTOR)
  20 CONTINUE
  RETURN
  C
  100 FORMAT (E12.6)
  END
C SUBROUTINE INGEOH
CONICAL SHELL
C
COMMON ( U90 MAIN )
REAL LENGTH
DATA DTOR/ 1.17432925194326E-01/
C
RAD1S,1002 LENGTH, RADI, RADF, THEA, RASH
AMPA=ATAN2(RADF-RADI)/LENGTH
SNHALF=SIN(ALPHA)
CSHALF=COS(ALPHA)
CHALF=LENGTH/ERADF-RADI)
ETA1F=RADI+THEA+DTOR
INISH. =. 0 . GOTO 1
ETA21 .0 .
ETA2F=RADC SCALF*LOG(RADF/RADI)
GOTO 2
1 ETA21=RADI+CSHALF
ETA2F=RADC SCALF
2 DBTAI=ETA1F/ FLOAT (MESH)
DBTAI=ETA1F-ETA21)/FLOAT (INMESH)
DHALF=ETA1F/RADI
DHALF=ETA2F+CSHALF/RADI
MHH=1
IBTHETA =LT. 180.0 GOTO 5
MHH=MH+2
MHH=MH
1 IB2=NH .HM. M9+2) #U=1+MH/2
9 DO 10 MM=1,NN
CH2=FLOATMM-I+0CH2=CH12
FM2=FM1
IB4=ASH . LT. 01 FINK=EXP (CH2)
DO 10 MM=1,NN
CH2=FLOATMM-2+0CH2
Y1MM=YN*RIM (CH12+FINK)
Y2MM=YN*RIM (CH2+FINK-1.0)
Y1MM=YN*RIM (CH2+FINK)
10 CONTINUE
IBTHETA =LT. 180.0 GOTO 50
IB2=NH .LV. M9+2 GOTO 30
NH=NU+1
10MM=NU . NH. M9+2) HH=1+KU
C "O MM=1,NN
L "O M=M1,NN
MM=MM+2-N
Y1MM=YN+Y3(MK,N)
Y2MM=YN-Y1(MK,N)
20 Y1MM=Y1(MK,N)
30 DO 40 MM=1,NN
40 MM=NN+H
Y1MM=Y1(MK,N)
Y2MM=Y2(MK,N)
40 Y1MM=Y1(MK,N)
C CBRAD=CSCALF* RADI
ETHAD1=ETA1F*RADI+ DТОR
INISH . =. 011 ETA2=CSRAD*LOG(1.+ ETA2D/CSRAD)
DO 45 I=1,NSYAN

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ETAG1=ETAG 1*RAD1*DM
IF (NASH = 0* I=E TAG2=ETAG2= CSCRAD*ALOG1.*ETAG2/CSCRAD)
49 CONTINUE
50 RETURN
C 100 FORMAT(4X,10,4X,15)
END