INERTIAL IMPACTION EFFICIENCY OF CYLINDRICAL COLLECTORS BY DIGITAL TECHNIQUES AND EFFECTS OF PARTICLE SIZE DISTRIBUTIONS

Arthur K. Stuempfle, et al
Edgewood Arsenal
Aberdeen Proving Ground, Maryland
October 1974
EDGEOOD ARSENAL TECHNICAL REPORT
EC-TR-74050

INERTIAL IMPACTION EFFICIENCY OF CYLINDRICAL COLLECTORS
BY DIGITAL TECHNIQUES AND EFFECTS
OF PARTICLE SIZE DISTRIBUTIONS

by
Arthur K. Stuempfle
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Chemical Laboratory

October 1974

DEPARTMENT OF THE ARMY
Headquarters, Edgewood Arsenal
Aberdeen Proving Ground, Maryland 21010

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INERTIAL IMPACTION EFFICIENCY OF CYLINDRICAL COLLECTORS BY DIGITAL TECHNIQUES AND EFFECTS OF PARTICLE SIZE DISTRIBUTIONS

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Chemical test and assessment technology

The theory of inertial impaction of particles on cylinders has been analyzed and used to develop standard inertial impaction efficiency curves. Unique exponential functions have been generated by a digital computer that accurately fit the inertial impaction theory with a maximum relative error of less than 1%. Interpolative routines have been adapted for the computer program to obtain inertial impaction efficiency predictions for all inertial parameter and velocity field scaling parameter values in the range of \(0.13 < K \leq 300\) and \(0 < \phi < 10,000\). The complete computer program with examples and solutions of test cases are presented in the appendix.

(Continued)
20. ABSTRACT

The Weibull distribution function has been adapted to analyze the effects of particle size distributions on the impaction efficiency of cylinders. Use of the mass median diameter to characterize a particle size distribution is unsatisfactory for predicting the impaction efficiency from heterogeneous aerosols. Use of the particle size distribution to predict efficiency of impaction yields good agreement between theory and experiment.
PREFACE

The work described in this report was authorized under Task 1W162116A08402, Chemical Test and Assessment Technology. This work was started in February and completed in May 1974.

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INERTIAL IMPACTION EFFICIENCY OF CYLINDRICAL COLLECTORS BY DIGITAL TECHNIQUES
AND EFFECTS OF PARTICLE SIZE DISTRIBUTIONS

I. INTRODUCTION.

The deposition of particles of intermediate size (10 to 200 μm diameter) on collectors in a flow stream is principally based on the inertial impaction mechanism. Inertial impaction of particles on collectors is of interest in diversified areas such as crop dusting, mosquito spraying, air pollution control, aircraft baggage, particulate buildup on heat exchangers, and in practically any circumstance where matter in particulate form is removed from a transport fluid. The efficiency with which the particles are removed from the flow stream is a function of the particle size, collector size, and flow field conditions. For a stationary cylinder in a moving airstream, the impaction efficiency is defined as the ratio of the cross-sectional area of the upstream envelope containing the trajectories of the particles which intersect the collector surface to the cross-sectional area of the cylindrical collector normal to the direction of flow. Estimates of the impaction efficiency for a given set of conditions can be found by computing the trajectories of the particles that challenge the collector or by graphically determining the efficiency from curves constructed from the point-by-point trajectory calculations. These methods are adequate when only a few data points are of interest but the technique becomes tedious when estimating efficiencies for a variety of possible impaction conditions. In addition, the aerosol that challenges the collector generally consists of a wide range of sizes depending on the method used for particle generation. It is recognized that, if a single parameter, such as the mass median diameter (MMD), is used to characterize the particle size distribution for computational purposes, the expected theoretical impaction efficiency can be grossly different from the experimental efficiency. These differences can be attributed in part to the turbulent nature of the flow field and to the nonlinear character of the inertial impaction efficiency curves. The objectives of this study have been to develop a computer program that rapidly calculates the inertial impaction efficiency of cylindrical collectors for any given set of laminar flow conditions and to devise a technique to assess the effects of a particular particle size distribution on impaction efficiency.

II. BACKGROUND.

The theory of inertial impaction of spherical particles on cylindrical collectors in an ideal flow field is based on a numerical solution of the equations of motion of the particles undergoing transport around the bluff body. Development of the theory and associated digital computer techniques has been described in a previous study and provides the basis for this report. Langmuir and Blowjobt proposed the following dimensionless form of the equations of motion for a particle in Cartesian coordinates.

\[
\frac{dv_x}{dr} = \frac{C_D \text{Re}}{24} \frac{1}{K} (u_x - v_x) 
\]

\[
\frac{dv_y}{dr} = \frac{C_D \text{Re}}{24} \frac{1}{K} (u_y - v_y) 
\]

where

\[ v_x, v_y \] = particle velocity components normalized by the free-stream velocity

\[ u_x, u_y \] = airstream velocity components normalized by the free-stream velocity

\[ U \] = free-stream velocity at an infinite distance from the cylinder surface
(1) \( C_D = \) drag coefficient for spherical particles in fluid

\( \text{Re} = \frac{\rho_a d_p \gamma}{\mu} \) = particle Reynolds number with respect to local relative velocity

\( \rho_a = \) fluid density

\( \mu = \) fluid viscosity

\( d_p = \) particle diameter

\( K = \frac{\rho_a d_p^2 \mu}{18 \mu R} \) = inertial parameter of particle

\( \rho = \) particle density

\( R = \) cylinder radius

\( \tau = \frac{U}{R} \) = time scale

The inertial parameter, \( K \), is a measure of the inertia of the particle and relates to the magnitude of the external force required to cause a change in its direction of motion. If Stokes' law of resistance is assumed, the inertial parameter represents the ratio of the "stopping" distance of a particle projected with velocity \( U \) into still air to the radius of the cylinder. A second dimensionless parameter proposed by Langmuir and Blodgett is the velocity field scaling parameter, \( \phi \), originally defined in terms of the free-stream particle Reynolds number. The \( \phi \) parameter is used to calculate the magnitude of the instantaneous Reynolds number of the particle at any point in the flow field and can be related to the Reynolds number of the cylindrical collector as follows:

\[
\phi = \frac{\text{Re}_o^2 18 \rho_a^2 \mu}{\text{Re}_c \rho} = \frac{9 \rho_a}{\rho} \left( \text{Re}_c \right)
\]

where

\( \text{Re}_o = \frac{d_p \rho_a \mu}{\mu} = \) free-stream Reynolds number of the particle

\( \text{Re}_c = \) free-stream Reynolds number of the cylindrical collector

The magnitude of the \( \phi \) parameter at any point in the particle path is a measure of the deviation from Stokes' law due to the forces acting on the particle.

The starting point for a mathematical description of the fluid flow field around the collector is the Navier-Stokes equation. A solution to the general Navier-Stokes equation is possible after a number of limiting assumptions are made with respect to the fluid and flow conditions. One form of solution can be found if the fluid is assumed to be ideal; i.e., constant density, irrotational, and without viscosity. The flow pattern around the cylinder under steady-state conditions is dependent on the Reynolds number of the collector. Where the Reynolds number of the cylinder is one thousand or greater (\( \text{Re}_c \geq 10 \) in air), the flow can be considered ideal in the absence of turbulence and the flow field is adequately described by potential theory for an incompressible fluid. The streamlines for flow outside the cylinder can be obtained and the airstream velocity components can be written in dimensionless terms as simple functions of the reduced position coordinates\(^1\); namely,

\[
u_x = 1 + \frac{y^2 - x^2}{(x^2 + y^2)^2}
\]

\[
u_y = -\frac{2xy}{(x^2 + y^2)^2}
\]
Figure 1. Coordinate System for Cylinder in Potential Flow Field
where the Cartesian coordinates have been normalized with respect to the cylinder radius.

A numerical solution of the differential equations of motion given above requires the instantaneous drag force on the particle to be estimated as the trajectory is developed. The steady-state drag coefficient of the sphere is a function of the Reynolds number, and the reliable experimental data as tabulated by Fuchs have been interpolated to obtain these estimations. The probable error in the drag coefficient for solid spheres is claimed by Fuchs to be less than 4% over the Reynolds number range of 0.01 to 500. These data are nearly identical to the experimental data cited by Schlichting as adopted and reported by Hussein and Tabakoff. Over the Reynolds number range of 0.10 to 7.0, the earlier approximations used by Langmuir and Blodgett differ from the current drag coefficient data.

The Cartesian coordinate system used in calculating the trajectory of particles around the cylindrical collector is identical to that of Brun, Lewis, Perkins, and Serafini and is shown in figure 1. The motion of the particles is in a plane perpendicular to the cylinder axis which is the origin of the coordinate system. Theoretically, the maximum angle of impingement, $\theta_M$, is the angle beyond which no deposition occurs by the inertial mechanism.

Several simplifying assumptions have been made in the derivations and in computing the trajectory of a particle. These assumptions include the following:

1. At an infinite distance upstream of the cylinder, the particles have horizontal and vertical velocity components equal to the free-stream air.
2. The particles are spherical, monometric (uniform size), and monodisperse (single particles), and they do not evaporate or deform.
3. Gravitational, electrostatic, and any other external forces are negligible.
4. The particle radius is negligible with respect to the cylinder radius. (For interception effect considerations, see previous publication.)
5. The boundary layer about the cylinder surface does not affect the particle trajectory.
6. The airflow around the cylinder is described as ideal and without circulation and is unaffected by the presence of the particles.
7. The instantaneous drag force coefficient for the particle is given by the steady-state data and is not subjected to acceleration effects.
8. All particles that strike the collector adhere to it.

Results of the digital computer trajectory calculations to determine the efficiency of impaction of spherical particles on cylindrical collectors in an ideal flow field as a function of the $K$ and $\phi$ scaling parameters are shown in figure 2. The impaction data for inertial parameter values less than one ($K \leq 1$) are displayed in figure 3. In addition to the impaction efficiency, the maximum-angle-of-impingement data are plotted in figure 4. The theory has been experimentally verified under laminar flow conditions for a wide range of inertial parameter and velocity field scaling parameter values. In general, however, the theory is inappropriate for accurately predicting the impaction efficiency of particles possessing small inertial parameter values on collectors in fluid flows that exhibit turbulence intensity levels above 7.5% or when the Reynolds number of the collector is less than approximately one thousand ($\phi \approx 10$ in air). Further, the collection efficiency of a cylinder that is challenged by a distribution of particle sizes is not accurately predicted by a single particle parameter such as the mass median diameter. For many practical circumstances, though, the theory is quite sufficient, and this study has been undertaken so that impaction efficiency predictions can be rapidly made by computer with minimum input data requirements or computer programming background.
Figure 2. Inertial Impaction Theory

\[ \Phi = \frac{18 \rho_a^2 \overline{UR}}{\mu p} \]

\[ \kappa = \frac{\rho dp^2 \overline{U}}{18 \mu R} \]
Figure 3. Impaction Efficiency for Cylinder for $K \leq 1.0$

WHERE

$\Phi = \frac{18 \rho a^2 U R}{\mu D}$

$K = \frac{\rho d p^2 U}{18 \mu R}$
Figure 4. Maximum Angle of Impingement on Cylinders

WHERE
\[ \Phi = \frac{18 \rho_a L U e}{\mu p} \]
\[ K = \frac{d_p^2 L u}{18 \mu R} \]
III. METHODS AND RESULTS.

Results of the point-by-point trajectory calculations define the inertial impaction theory as shown in figures 2 and 3 and provide the impaction efficiency as a function of the $K$ and $\phi$ scaling parameters. These data serve as the standards for which unique exponential functions have been obtained to fit the curves. The analytical expressions for the standard $\phi$ curves can then be interpolated to determine the impaction efficiency for any nonstandard $K$ and $\phi$ value. A digital computer program has been developed to perform these calculations and only a minimum of input data is necessary for operation.

The standard velocity field scaling parameter curves selected for fit include $\phi = 0; 1; 2.2; 5; 10; 22; 50; 100; 200; 500; 1,000; 2,210; 5,000; 10,000$. Each curve has been subdivided into four inertial parameter ranges and each portion, respectively, has been fitted by the same form of the unique exponential functions. Any differences that occur between the computed efficiency along a standard $\phi$ curve and the point-by-point impaction efficiency data are in the third or higher significant digit position and the maximum relative error observed has been less than 1%.

Lagrange's interpolation formula has been used to compute impaction efficiency values for velocity field scaling parameters enclosed within the aforementioned standard $\phi$ values that are equal to or greater than one. The formulation used in this study has been defined as

$$\frac{\sum_{i=1}^{M} E_{\phi_i} \prod_{j=1, j \neq i}^{M} \left( \ln \phi - \ln \phi_j \right)}{\prod_{j=1}^{M} \left( \ln \phi_i - \ln \phi_j \right)}$$

(7)

where

$$E_{\phi_i} = \frac{1.0}{\exp\left\{\exp\left[A_N(\ln K)^N + A_{N-1}(\ln K)^{N-1} + \ldots + A_2(\ln K)^2 + A_1\ln K + A_0\right]\right\}}$$

(8)

and

$$0.13 < K \leq 64$$

$$\phi_i, \phi_j = 1; 2.2; 5; 10; \ldots 5,000; 10,000$$

$$i, j = 1, 2, \ldots, 13$$

$$M = 1, 2, \ldots, 13$$

$$N = 4-7$$
If $64 < K \leq 300$,

$$\phi_i, \phi_j = 1; 5; 22; 100; 500; 2,210; 10,000$$

$$i, j = 1, 2, ..., 7$$

$$M = 7$$

where $K$ = inertial parameter value of interest; $0.13 < K \leq 300$.

One advantage of Lagrange's interpolation formula over other interpolative routines is that equidistance values of the independent variable are not necessary to obtain a reliable value of the dependent variable. For $\phi$ values between 0 to 1, where the potential flow assumption may not be valid, a linear interpolation has been employed.

Comparisons have been made between impaction efficiency values obtained by the interpolative method, and the point-by-point trajectory technique and relative errors are less than 1% when the absolute impaction efficiency is greater than 5%. For impaction efficiencies less than 5%, the largest relative error observed has been within 10% of the point-by-point trajectory calculated efficiency value and is due principally to round-off (four-place accuracy). The trajectory calculations had been terminated at the inertial parameter value of 64. However, the interpolative routine extends this range to $K$ values of 300 where the impaction efficiency curves asymptotically approach the theoretical limiting efficiency of 100% for point-mass particles. Consequently, for all practical purposes, the interpolative routine provides an accurate prediction of the impaction efficiency of spherical particles on cylindrical collectors over an extensive range of inertial parameter values. A complete and detailed description of the digital computer program developed in this study is contained in the appendix.

Experimental verification of the inertial impaction theory has been accomplished over a wide range of parameter values. The theory turns out to be remarkably accurate for the monometric (single-size) particles used in the experimental efforts especially in view of the assumptions that have been made regarding fluid properties and flow conditions.

In practical applications, however, the aerosol that impacts on a collector will seldom consist of particles of uniform size. The size distribution of the aerosol is a function of the methods and techniques used in generation of the particles. A single empirical or theoretical equation does not exist that can universally predict the particle size distribution resulting from the dispersion of liquids by spray nozzles, hot and cold gas atomizers, and explosive disseminators or the dispersion of solids from mechanical dispersers.

A vast literature has been developed by various authors in their attempts to obtain mathematical distribution functions that characterize experimentally generated particle size distributions. Familiar examples include the Nukiyama-Tanasawa equation for drop sizes in sprays generated by air atomization, the Rosin-Rammler equation for pulverized coal, Roller's formula for powder materials, the normal distribution function for symmetrically distributed particles of narrow size range such as plant spores, and the log-normal distribution for a large number of condensation, natural, and mechanically generated aerosols. However, the intent of this study is not to generalize on particle size distributions or delve into their specific merits and methods but rather to examine the necessity of accounting for the particle size distribution when predicting the impaction efficiency of a particular cylindrical collector.
A simple technique used to obtain a size distribution has been to microscopically observe and measure a representative sample of an aerosol population. The number of particles that lie between radius \( r \) and \( r + dr \) can be found as a function of the radius. The fraction of the total number of particles that lie in an interval is given as 
\[
\frac{dn}{r} = \pi r^2 dr
\]
where \( \int_0^\infty f(r)dr = 1 \).

The relative frequency distribution curve is found by plotting the resulting data representing \( dn \), and the various distribution functions mentioned above can provide a mathematical expression of the function, \( f(r) \).

Distribution functions can be mathematically expressed and used in a number of ways depending on the problem requirements. In some instances, the cumulative fraction of particles having radii greater or less than some radius is convenient. Thus, respectively,
\[
F(r) = \int_r^\infty f(r)dr \quad \text{or} \quad F(r) = \int_0^r f(r)dr
\]

In addition to the number distribution, the mass distribution function can be determined and is useful in many practical applications. The mass fraction of particles having radii between \( x \) and \( x + dx \) is written as 
\[
df = f(x)dx
\]
and \( \int_0^\infty f(x)dx = 1 \). The mass distribution function is related to the number distribution function as
\[
f(x) = \omega m_r f(r)
\]
where
\[
m_r = \text{mass of particle with radius } r
\]
\[
\omega = \text{proportionality constant}
\]

With respect to impaction of particles on collectors, one is usually interested in the mass distribution function because the dose acquired by the collector is dependent on the aerosol mass that is deposited over the sampling period. In order to estimate the mass collected, an average particle size is generally computed and applied to the inertial impaction theory. The objectives of using an average diameter or other measure of central tendency are to provide a single number that will simply describe the behavior of the entire aerosol population and to eliminate an extraordinary number of tedious calculations. The problem is that the inertial parameter value computation makes use of the square of the particle diameter and, further, the impaction curves are a nonlinear function of the inertial parameter. Consequently, an accurate estimation of impaction efficiency or mass deposit can only be expected when a single particle size estimator is used if the particle size distribution is over a very narrow range.

In order to demonstrate the effect that a particle size distribution has on the impaction efficiency, experimental data are required for comparison purposes. Most of the experimental work that has been performed in the past to verify the inertial impaction theory has been conducted with monometric (single-size) aerosols. One notable exception is that of Landahl and Herrmann. Several authors refer to these data but they consider the results to be of limited value due to the use of heterogeneous aerosols, especially at small values of the inertial parameter.
Landahl and Herrmann studied the deposition of aerosols on wires and cylinders with particles generated by an impinger-type atomizer and by a "modified Brink's nozzle." The size distributions were obtained by cascade impactors, and microscopic observations and representative distributions of the clouds produced are illustrated in the following table:

<table>
<thead>
<tr>
<th>Percent</th>
<th>Small particle cloud</th>
<th>Medium particle cloud</th>
<th>Large particle cloud</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>(0.8)</td>
<td>1.8</td>
<td>5</td>
</tr>
<tr>
<td>10</td>
<td>1.0</td>
<td>2.9</td>
<td>8</td>
</tr>
<tr>
<td>50</td>
<td>4.0</td>
<td>13</td>
<td>28</td>
</tr>
<tr>
<td>90</td>
<td>12</td>
<td>40</td>
<td>58</td>
</tr>
<tr>
<td>98</td>
<td>(20)</td>
<td>55</td>
<td>75</td>
</tr>
</tbody>
</table>

*Percent of mass less than stated diameter.

These data are not easily fitted or well represented by the aforementioned distribution functions. However, a general statistical distribution function that has found wide applicability recently is the Weibull distribution function. \(^{13-15}\) This distribution function has been successfully used in processes involving limits and maxima/minima problems that include lifetime and failure rate distributions of electrical systems. It has also been used for describing bounded particle size distributions.

The cumulative distribution function of the Weibull distribution has been applied to the Landahl and Herrmann data and has been expressed in the following form:

\[ F(x) = 1 - \exp \left[ \left( \frac{x - \gamma}{\eta} \right)^\beta \right] \]  \( ^{(9)} \)

where

- \( F(x) \) = cumulative mass fraction
- \( x \) = particle diameter (\( \mu m \))
- \( \gamma \) = minimum size; location parameter (\( \mu m \))
- \( \eta \) = characteristic size; scale parameter (\( \mu m \))
- \( \beta \) = shape parameter

The trial-and-error method of estimating the Weibull parameters has been described by Nelson\(^{15}\) and a digital computer program has been developed\(^{16}\) to estimate the parameters of the distribution function for the
small, medium, and large particle clouds. The parameter approximations obtained by the principle of maximum likelihood method turn out to be as follows:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Small cloud</th>
<th>Medium cloud</th>
<th>Large cloud</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$</td>
<td>5.3053</td>
<td>17.9678</td>
<td>34.4356</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.9532</td>
<td>1.1183</td>
<td>1.7461</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.5919</td>
<td>0.5269</td>
<td>-1.7375</td>
</tr>
</tbody>
</table>

Evaluating the cumulative mass function for the experimental particle diameters to assess the Weibull distribution fit yields the following comparisons:

<table>
<thead>
<tr>
<th>Experimental F(x)</th>
<th>Small cloud</th>
<th>Medium cloud</th>
<th>Large cloud</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x$ F(x) calculated</td>
<td>$x$ F(x) calculated</td>
<td>$x$ F(x) calculated</td>
</tr>
<tr>
<td>5 %</td>
<td>$\mu m$ 5</td>
<td>$\mu m$ 5</td>
<td>$\mu m$ 5</td>
</tr>
<tr>
<td>10 %</td>
<td>1.0 9</td>
<td>2.9 10</td>
<td>8 10</td>
</tr>
<tr>
<td>50 %</td>
<td>4.0 52</td>
<td>13 50</td>
<td>28 51</td>
</tr>
<tr>
<td>90 %</td>
<td>12 90</td>
<td>40 92</td>
<td>58 91</td>
</tr>
<tr>
<td>98 %</td>
<td>20 98</td>
<td>55 97</td>
<td>75 98</td>
</tr>
</tbody>
</table>

As observed, an adequate fit is obtained for all clouds considering the limited available data and the wide size distributions that are involved. It must be emphasized again that the objective is not to find the “correct” distribution function to describe a particular particle size distribution but rather to approximate the function as simply as possible and to test whether the resultant distribution significantly affects the impaction efficiency predictions.

Given the above distribution functions and approximate values for the Weibull parameters, one can solve equation 9 for the particle size as a function of the cumulative mass fraction, $F(x)$: namely,

$$x = \gamma + (\eta - \gamma) \left[ \ln \left( \frac{1}{1-F(x)} \right) \right]^{1/\beta}$$

(10)

Subsequently, by selecting appropriate values for $F(x)$, the average particle size associated with a given particle mass interval can be determined by difference. For example, if 30% of the cloud mass has particle sizes below 7.4 $\mu m$ and 31% of the cloud mass has particles less than 7.6 $\mu m$ in diameter, then 1% of the cloud mass is represented by particles with an average diameter of approximately 7.5 $\mu m$. The impaction efficiency for each size interval can then be computed and an average efficiency can be obtained for the entire distribution.
A computer program has been written to perform the tedious computations for each 1% mass
interval, and the average distribution impaction efficiency is found based on 96% of the cloud mass. In addition,
the program provides the impaction efficiency based strictly on the mass median diameter of the cloud. The only
supplementary input requirements necessary for use of the distribution function option to the computer
program are the Weibull parameters $\eta, \beta, \gamma$.

The sample distribution functions have been used to compute average impaction efficiencies for
comparison with the experimentally obtained efficiencies of Landahl and Herrmann. The results of these
computations are included in table 1. The velocity field scaling parameter values for the Landahl and Herrmann
tests are very low and theoretically the potential flow assumptions regarding the fluid flow field should not
apply. Further, Landahl and Herrmann state that the airflow was turbulent, but the degree of turbulence is not
indicated. In addition, the illustrative particle size distributions provided by Landahl and Herrmann do not
correspond with their reported efficiency data except for the 13-μm-MMD case. Nevertheless, examination of
table 1 clearly shows that the theoretical impaction efficiencies for the distribution functions more closely
predict the observed impaction efficiencies than the mass median diameter computed efficiencies for almost all
circumstances. This is especially true for the 13-μm-MMD case where the assumed particle size distribution
apparently matches the experimental mass median diameter and the theoretical efficiencies turn out to be
considerably more accurate than the mass median diameter prediction. Note that the small- and large-cloud
particle size distributions have mass median diameters that exceed the experimental mass median diameters and,
therefore, tend to overestimate the theoretical efficiencies from the distribution function. Thus, to summarize the
results, when the particle size distribution is used to compute the average impaction efficiency for the cloud, the
prediction corresponds much more closely to the observed efficiency than that of a simple mass median diameter
prediction. The predictions are relatively accurate when the particle size distribution is known as evidenced by
the 13-μm-MMD test case.

IV. DISCUSSION.

The important conclusion that can be drawn from the theoretical and experimental data presented in
table 1 is that use of the mass median diameter to describe a particle size distribution will seldom result in an
accurate prediction of the impaction efficiency for the cloud. In general, at large values of the inertial parameter,
K, the mass median diameter overestimates the impaction efficiency of the distribution, whereas, for small values
of the parameter where the theoretical efficiency for the mass median diameter is less than 1%, the efficiency
can be underestimated by orders of magnitude. These differences are due to the nonlinear character of the
inertial impaction theory. However, when the particle size distribution is considered in the predictions, the
agreement between theory and experiment is much better.

Calculations were performed using the typical particle size distributions previously given to
determine the range of inertial parameter values over which the mass median diameter predictions were within
±10% of the average impaction efficiencies for the distributions. In general, if the mass median diameter yields a
K value between 1 and 2, for all $\phi$ values, the effect of the distribution on efficiency need not be considered.
When the inertial parameter value based on the mass median diameter lies outside this range, the average
distribution efficiency will differ from the mass median diameter predicted efficiency by more than 10%. To
accurately predict the impaction efficiency for heterogeneous aerosols when the K value for the mass median
diameter is less than one, the particle size distribution must be taken into account. Unfortunately, data are not
available to determine the degree of monomodality of the distribution that is required in order that the mass
median diameter or other average particle diameter will adequately represent the population over the entire
inertial impaction region of interest.
Table 1. Experimental and Theoretical Impaction Efficiencies

<table>
<thead>
<tr>
<th>D Cylinder diameter cm</th>
<th>U (cm/sec)</th>
<th>0.9 mph = 40.2</th>
<th>3 mph = 134.1</th>
<th>8 mph = 357.6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>MMD (μm)</td>
<td>MMD (μm)</td>
<td>MMD (μm)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>15</td>
<td>3.7</td>
<td>13</td>
</tr>
<tr>
<td>0.008</td>
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a Landahl and Herrmann experimental efficiency.
b Average efficiency from typical particle size distributions.
c Efficiency from mass median diameter of typical distribution, 4, 13, and 28 μm, respectively.
It is also interesting that the predicted efficiencies are close to the experimentally measured efficiencies even though the velocity scaling parameter values are much less than 10 in some cases. In general, the lower φ values would lead to reduced impaction parameter values because the flow field would tend to exhibit viscous flow properties. On the other hand, no account of interception effects has been made for the interpolation routine, and interception could occur since the particle to collector diameter ratios are substantial. The mass median diameter to collector diameter ratio ranges from $4.1 \times 10^{-4}$ to $3.4 \times 10^{-1}$. The effect of interception would be to substantially increase the values of the mass median diameter efficiency. However, with such large diameter ratios, the collection efficiency may not equal the theoretical impaction efficiency due to particle bounce-off and re-entrainment from the collector surface.

The computer program prepared for this study and presented in the appendix enables impaction efficiency predictions to be made over practically the entire region of interest of inertial impaction on cylinders. Four optional computational methods are provided for the user depending on the amount of input information available. Option 1 provides the impaction efficiency of a cylinder and assumes unit density particles and an ambient temperature of 20°C. The only input requirements are the mean windspeed, cylinder diameter, and particle diameter in centimeter-gram-second units. Option 2 makes use of all the variables required to compute the K and φ parameters defined by equations 3 and 4, but the program provides the necessary calculations based on these input values. Option 3 assumes that the user has computed the K and φ parameter values from equations 3 and 4 and desires only the predicted efficiency. Thus, only the K and φ input values are required. Option 4 considers the Weibull distribution function and requires all input data from option 2 except for the particle diameter for which the Weibull parameter estimates of $q$, $\beta$, and $\gamma$ are substituted. The program computes the average impaction efficiency of the distribution, as well as the efficiency based on the mass median diameter of the distribution. Option 4 has been separated from the main program so that the reader could substitute his own distribution function, if required, with only a few minor program changes. A detailed explanation, sample input cards, test cases with sample outputs, and the complete program are provided in the appendix for the convenience of the reader.

The approximation methods developed in this study to calculate the impaction efficiency of a cylinder for any K and φ value have reduced the computational time for the point-by-point trajectory calculations used to develop the inertial impaction theory from 5.6 seconds to 13 milliseconds per data point on a digital computer comparable to a Univac 1108 computer without sacrificing the accuracy of the prediction.

V. CONCLUSIONS.

1. Standard inertial impaction efficiency curves for cylinders developed from point-by-point trajectory calculations have been fitted by unique exponential functions.

2. Lagrange’s interpolation formula has been adopted and applied to compute the impaction efficiency of a cylinder for any parameter values of $0.13 \leq K \leq 300$ and $0 \leq \phi \leq 10,000$.

3. The accuracy of the impaction efficiency computations by use of the digital computer program is within 1% of the point-by-point efficiency predictions.
4. The cumulative distribution function of the Weibull distribution has been incorporated into the digital computer program and has been used to fit the experimental particle size distributions of Landahl and Herrmann.

5. The average impaction efficiency for the particle size distributions and the mass median diameter impaction efficiencies have been compared to the experimental data obtained by Landahl and Herrmann for heterogeneous aerosols.

6. Use of a mass median diameter to characterize a particle size distribution does not accurately predict the experimentally obtained impaction efficiency. For large values of the inertial parameter, the mass median diameter overestimates the efficiency and, at small inertial parameter values, the mass median diameter significantly underestimates the impaction efficiency.

7. Consideration of the particle size distribution in impaction efficiency predictions results in good agreement between theory and experiment.
LITERATURE CITED


I. INTRODUCTION.

The computer program IMPEF and its subroutines LAGRNG, WEIBUL, and POLY have been written for the Edgewood Arsenal Univac 1108, time-sharing, multiprocessor system.

IMPEF has four executable options. Three of these enable the user to input various particle and fluid flow conditions to obtain an inertial impaction efficiency value. The fourth option computes the average impaction efficiency for a particle size distribution generated from the Weibull distribution function and provides the impaction efficiency for the mass median diameter (MMD) of the distribution.

II. MATHEMATICAL BASIS OF PROGRAM IMPEF.

The first step in the development of the approximation methods has been to employ the point-by-point particle trajectory method* to obtain the inertial impaction efficiency curves for 14 selected velocity field scaling parameter values chosen as standards ($\phi = 0; 1; 2.2; 5; 10; 22; 50; 100; 200; 500; 1,000; 2,210; 5,000; 10,000$).

Polynomial equations have been developed that accurately fit the standard $\phi$ curves. Each curve has been sectioned into four parts representing four intervals of the inertial parameter, $K (0.13-0.22; 0.22-0.5; 0.5-1.0; 1.0-64.0)$. The polynomial coefficients are stored in the subroutine POLY. If the $K$ parameter lies between 0 and 0.13, the impaction efficiency is assumed equal to zero based on results obtained from the point-by-point trajectory model computations.

In order to approximate the impaction efficiency for any nonstandard $\phi$ value over the range $1 \leq \phi < 10,000$, Lagrange’s interpolation formula has been applied after assuming a functional relationship exists among the efficiencies at a given $K$ value. The advantage of Lagrange’s interpolation routine is that equidistant values of the independent variable are not necessary to obtain a reliable result. A linear interpolation has been used for those cases where the $\phi$ value falls between 0 and 1.

Lagrange’s routine is utilized with alternating standard $\phi$ curves ($1; 5; 22; 100; 500; 2,210; 10,000$) to compute efficiencies for inertial parameter values in the extrapolated range of $64 < K < 300$.

The adaptation of Lagrange’s interpolation formula used in the LAGRNG subroutine of the program IMPEF is written as:

$$E_{\phi} = \frac{1}{M} \sum_{i=1}^{M} E_{\phi_i} \prod_{j=1, j \neq i}^{M} \frac{(\ln \phi - \ln \phi_j)}{(\ln \phi_i - \ln \phi_j)}$$

---

where
\[
E_{\phi_i} = \frac{1.0}{\exp\{\exp(A_0(\ln K)^N + A_{-1}(\ln K)^{N-1} + \cdots + A_2(\ln K)^2 + A_1 \ln K + A_0)\}}
\]
and \( K = \) inertial parameter value of interest; \( 0.13 < K \leq 300 \).

If \( 0.13 < K \leq 64 \),
\[
\phi_i, \phi_j = 1; 2.2; 5; 10; 22; 50; 100; 200; 500; 1,000; 2,210; 5,000; 10,000
\]
i, j = 1, 2, ..., 13
\( M = 1, 2, ..., 13 \)
\( N = 4, 5, 6 \) or 7

If \( 64 < K \leq 300 \),
\[
\phi_i, \phi_j = 1; 5; 22; 100; 2,210; 10,000
\]
i, j = 1, 2, ..., 7
\( M = 7 \)

\( A_M \)'s are the coefficients obtained from the fit of the standard \( \phi \) curves and are given in the subroutine POLY.

III. EXPLANATION OF INPUT FOR PROGRAM IMPEF WITH EXAMPLES.

Option One.

Computes the velocity field scaling parameter, \( \phi \), the inertial parameter, \( K \), and the inertial impaction efficiency assuming an ambient temperature of \( 20^\circ C \) and unit density particles. Requires input of particle diameter, cylinder diameter, and free-stream wind velocity.

### Format for Input Cards

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<td>U, D, PDM</td>
<td>Free-stream velocity, Collector diameter, Particle diameter</td>
<td>cm/sec, cm, microns</td>
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Sample Input

TEST CASE FOR OPTION ONE

PARTICLE DENSITY = 1.0000+00 GM/CC
FLUID DENSITY = 1.2047-03 GM/CC
FLUID VISCOSITY = 1.8100-04 POISE
COLLECTOR DIAMETER = 1.4000 CM
FREE-STREAM VELOCITY = 424.650 CM/SEC
PARTICLE DIAMETER = 55.000 MICRONS

VELOCITY FIELD SCALING PARAMETER $\phi$ = 42.9023
INERTIAL PARAMETER $K$ = 5.6326
INERTIAL IMPACTION EFFICIENCY = .7353169

Option Two.

Computes the velocity field scaling parameter, $\phi$, the inertial parameter, $K$, and the inertial impaction efficiency for the applicable input conditions. Requires input of particle diameter and density, collector diameter, fluid density and viscosity, and free-stream wind velocity at the applicable temperature.
### Format for Input Cards

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<td>PDM</td>
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### Sample Input

```
312.5 1.20 75.0
1.0430 1.2047E-03 1.81E-4
```

### Sample Output

```
TEST CASE FOR OPTION TWO
PARTICLE DENSITY = 1.0430+00 GM/CC  FLUID DENSITY = 1.2047-03 GM/CC
FLUID VISCOSITY = 1.8100-04 POISE  COLLECTOR DIAMETER = 1.2300 CM
FREE-STREAM VELOCITY = 312.500 CM/SEC
PARTICLE DIAMETER = 75.000 MICRONS
VELOCITY FIELD SCALING PARAMETER, PHI = 25.9453
INERTIAL PARAMETER, K = 9.3793
INERTIAL IMPACTION EFFICIENCY = .8221419
```
Option Three.

Computes the inertial impaction efficiency for a given set of $K$ and $\phi$ input parameter values.

Format for Input Cards

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Sample Input

```
8.4367 23.9025

TEST CASE FOR OPTION THREE

Sample Output

TEST CASE FOR OPTION 3
VELOCITY FIELD SCALING PARAMETER, $\phi$ = 8.4367
INERTIAL PARAMETER, $K$ = 23.9025
INERTIAL IMPACTION EFFICIENCY = .9223695
POTENTIAL FLUID FLOW MAY NOT APPLY

Appendix

26
Option Four.

Computes a particle size distribution generated from the Weibull distribution function, the average inertial impaction efficiency for the distribution, and the efficiency for the mass median diameter of the distribution. Requires the Weibull parameter values of the distribution, particle density, collector diameter, fluid density and viscosity, and free-stream wind velocity.

**Format for Input Cards**

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**Sample Input**

```
34.4356  1.7461  -1.7975
357.6    0.9    
1.043e0  1.2047e-3  1.81e-4

4 TEST CASE FOR OPTION FOUR
```

Appendix 27
TEST CASE FOR OPTION FOUR

PARTICLE DENSITY = 1.04300 G/M/CC  FLUID DENSITY = 1.2047-03 GM/CC
FLUID VISCOSITY = 1.8100-04 POISE  COLLECTOR DIAMETER = .9000 CM
FREE-STREAM VELOCITY = 357.600 CM/SEC
VELOCITY FIELD SCALING PARAMETER, PHI = 22.2678
WEIBULL PARAMETERS: SCALE = 34.4356 SHAPE = 1.7461 LOCATION = -1.7375
AVERAGE INERTIAL IMPACTION EFFICIENCY = .4666792

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<th>INTERVAL AVERAGE (MICRONS)</th>
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Appendix
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<th>INTERVAL AVERAGE (MICRONS)</th>
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CLAUDE DELLEGRINO IMPEF

1 C MAIN PROGRAM IMPEF
2 DIMENSION PDI(38), APDI(37), RKS(17), E(I7), ID(79)
3 IPR = 0
4 1 READ (5, 38, END=28) IOP, ID
5 IF (IOP.LT.10 OR IOP.GT.4) GO TO 26
6 GO TO (5+9+11)* IOP
7 2 READ (5, 35) U+O+PDM
8 RHOC = 1.0
9 RHOA = 1.2947E-3
10 RMD = 1.81E-4
11 3 READ (PDM+PDM+PDM), RN+O+RNU+O+PDM
12 PHI = (RHOA/RHOA+RMD+O)/RMD
13 GO TO 6
14 4 READ (5, 36) RHO+RHOA+RNU+O+PDM
15 GO TO 3
16 5 READ (5, 38) PHI, RK
17 6 IF (IPRT = 7.7 + 6) WRITE (6+32) ID
18 GO TO 9
19 7 WRITE (6+33) ID
20 IF (IOP.EQ.1) GO TO 10
21 WRITE (6+41) RH0+RHOA+RMD+O+J
22 WRITE (6+42) PDM
23 10 IF (RK.GT.3.13 AND RK.LE.300.) GO TO 12
24 IF (RK.LE.0.0 OR RK.ST.300.) GO TO 27
25 EFF = 3.0
26 GO TO 22
27 11 READ (5, 37) RHO+RHOA+RNU+O+D
28 PHI = (RHOA/RMD+O+D)/(RNU+RHO)
29 12 IF (PHI.LT.0.0 OR PHI.GT.1000.0) GO TO 28
30 IF (PHI.LE.1.0) GO TO 13
31 CALL NUMTRA (PHI)
32 13 IF (IOP.NE.4) GO TO 21
33 WRITE (6+32) ID
34 WRITE (6+41) RH0+RHOA+RMD+O+J
35 WRITE (6+43) PHI
36 CALL WEIBUL (CHLD, RKMD, PDI, APDI, RKS)
37 SUME = 0.0
38 GO TO 16 I = 2 + 97
39 IF (I = 3 OR I = 13) GO TO 14
40 CALL LAGMNG (PHI, RKS(I), E(I))
41 GO TO 15
42 14 E(I) = 0.0
43 15 SUME = SUME + E(I)
44 16 CONTINUE
45 AVGE = SUME/26.
46 WRITE (6+44) AVGE
47 WRITE (6+31)
48 WRITE (6+45)
49 LINES = 12
50 LINES = LINES + 2
51 IF (I.EQ.501) GO TO 17
52 WRITE (6+46) I, PDI(I), APDI(I), RKS(I), E(I)
53 IF (LINES.LT.55) GO TO 20

Appendix 32
SUBROUTINE WEIBUL (CHLD,RXMD,PD,APD,RKS)

DIMENSION PD(98),APD(97),RKS(97)

READ (5,2) ETA,BETA,GAMMA

WRITE (6,31) ETA,BETA,GAMMA

POWER=1./BETA

DIFF=ETA-GAMMA

DO 1 INC=2,98

PD(INC)=GAMMA+DIFF*(ALOG10.0/11.0-(FLOAT(INC)/100.0)**POWER)

1 CONTINUE

RETURN

END

Appendix
CLAUDE PELLEGRINO - LAGRNG

1 SUBROUTINE LAGRNG (PHI, RK, EFF)
2 DIMENSION EN(13), ED(13), DIF(13), XI(13), EL(7), ENL(7), OIFL(7), I(2) I
3 101(3)
4 DATA X/-.78539816, 6.70775526, 2.125663947, .83457913, 
5 2.91034382/22357020, 1.18125693, 1.0265407, 1.681568921, 1.767131
6 341628.39796.864518228.2921, -2.774318228.2921, -2q986.1752.45785.7457,
7 2.690203004.330206.731, -1.864390.07171702.87170/10098.27530.
8 51523.10928610.97745.479.28178.8921, 4.7671, 4.5409640.1501.681568.921.7671/
9 30/22.05.1, 0.
10 KIN=1
11 IF (.LT.RNK(KIN)) GOTO 2
12 KIN:=-1
13 IF (KIN=-3) 1=1+2
14 RKX=ALOG(RK)
15 IF (PHI.LE.1) G0 TO 5
16 EFF=0.0
17 INC:1
18 IF (MK.GT.64.0) INC:2
19 DO 4 N=214+INC
20 CALL POLY(N, KIN, RKX, E)
21 IF (INC.EQ.2) GOTO 3
22 EFF=EFF+EN(N) I EN(N)+E
23 GO TO 4
24 I2N:2
25 EFF=EFF+EN(N) I EN(N)+E
26 CONTINUE
27 RETURN
28 5 DO 8 N=1+2
29 CALL POLY(N, KIN, RKX, E)
30 Y(N)=E
31 CONTINUE
32 RETURN
33 EFFY(I)*(1.-PHI)+Y(2) I PHI
34 ENTRY NUMTR(PHI)
35 PHIL=ALOG(PHI)
36 DO 7 I=113
37 DIF(I)=(PHIL-X(I))
38 IF (I.GT.7) GO TO 7
39 DIF(I) I (PHIL-X(I))
40 CONTINUE
41 DO 9 I=113
42 ENU=1.0
43 ELN=1.0
44 DO 8 J=13
45 IF (I.EQ.J) GO TO 8
46 ENM=DIF(I)+EN
47 IF (I.GT.J) OR (J.GT.7) GO TO 8
48 ELN=I DIF(I)+EN
49 CONTINUE
50 ENM=EN
51 IF (I.GT.7) GO TO 9
52 ELN=EN
53 CONTINUE
54 RETURN
55 END
114 \[E = 1 / \left(\exp\left(\exp\left(\text{ALLREI}\right)\right)\right)\]

115 RETURN

116 END

RETURN

END POL

Appendix

POLY3114

POLY3115

POLY3116