<table>
<thead>
<tr>
<th>UNCLASSIFIED</th>
</tr>
</thead>
<tbody>
<tr>
<td>AD NUMBER</td>
</tr>
<tr>
<td>AD907891</td>
</tr>
<tr>
<td>LIMITATION CHANGES</td>
</tr>
<tr>
<td>TO:</td>
</tr>
<tr>
<td>Approved for public release; distribution is unlimited.</td>
</tr>
<tr>
<td>FROM:</td>
</tr>
<tr>
<td>Distribution authorized to U.S. Gov't. agencies only; Test and Evaluation; AUG 1972. Other requests shall be referred to Army Safeguard System Command, Huntsville, AL 35807.</td>
</tr>
<tr>
<td>AUTHORITY</td>
</tr>
<tr>
<td>USASSC ltr 2 May 1974</td>
</tr>
</tbody>
</table>

THIS PAGE IS UNCLASSIFIED
A GROUND-LIGHTNING ENVIRONMENT
FOR ENGINEERING USAGE

By: N. CIANOS    E. T. PIERCE

Prepared for:
McDONNELL DOUGLAS ASTRONAUTICS COMPANY
HUNTINGTON BEACH, CALIFORNIA  92647

DISTRIBUTION STATEMENT "B"

DISTRIBUTION LIMITED TO U. S. GOVERNMENT AGENCIES
ONLY SINCE THIS IS TEST AND EVALUATION DATA
(6 MAR 73). OTHER REQUEST FOR THIS DOCUMENT
MUST BE REFERRED TO:
COMMANDER
U. S. ARMY SAFEGUARD SYSTEM COMMAND
ATTN:  SSC-AO
P. O. BOX 1500
HUNTSVILLE, ALABAMA  35807

STANFORD RESEARCH INSTITUTE
Menlo Park, California  94025  U.S.A.
A GROUND-LIGHTNING ENVIRONMENT FOR ENGINEERING USAGE

By: N. Cianos  E. T. Pierce

Prepared for:
McDONNELL DOUGLAS ASTRONAUTICS COMPANY
HUNTINGTON BEACH, CALIFORNIA  92647

CONTRACT L.S.-2817-A3

SRI Project 1834

Approved by:
D. A. Johnson, Director
Radio Physics Laboratory

R. L. Leadabrand, Executive Director
Electronics and Radio Sciences Division

DISTRIBUTION STATEMENT "B"

DISTRIBUTION LIMITED TO U. S. GOVERNMENT AGENCIES ONLY SINCE THIS IS TEST AND EVALUATION DATA (6 MAR 73). OTHER REQUEST FOR THIS DOCUMENT MUST BE REFERRED TO:

COMMANDER
U. S. ARMY SAFEGUARD SYSTEM COMMAND
ATTN: SSC-AD
P. O. BOX 1500
HUNTSVILLE, ALABAMA  35807

Copy No. 71
ABSTRACT

The objectives of the report are firstly to survey and to colligate information on the physical characteristics of lightning, and secondly to show how this information can be used by an engineer concerned with estimating the lightning sensitivity of equipment.

Data on lightning incidence are first examined. Expressions are derived relating lightning incidence to the widely available monthly thunderstorm-day statistic, to the diurnal variation of activity, and to structure height. By using these expressions the number of lightning strikes to be expected over any period of time can be estimated, on a climatological basis, for a structure of known height located anywhere in the world.

The physical parameters of lightning are then discussed; the parameters considered include—but are not limited to—peak current, time to peak current, rate of current rise, magnitude and duration of continuing currents, total charge transfer, number of strokes, and time between strokes. Median values and statistical distributions for the parameters are deduced; the statistics can usually be conveniently expressed in terms of a log-normal law.

Several models for lightning are derived and expressed in convenient analytical forms. It is emphasized that caution in the derivation process is necessary so as to obtain models that are both physically plausible and internally self-consistent. Two types of models are identified—basic models developed solely from the physical properties of lightning, and applied models modified appropriately for use with equipment, the lightning sensitivity of which is partially defined. Basic models are
presented for typical and severe flashes; in the latter case the criterion of the severity for a lightning parameter is taken as approximately the two-percent point on the statistical distribution. An example of an applied model is also given.

The main emphasis of the report is on the direct effects of flashes to ground. However, discussions are also given of the physical characteristics of intracloud discharges, and of the static and electromagnetic fields generated by lightning.
## CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>iii</td>
</tr>
<tr>
<td>LIST OF ILLUSTRATIONS</td>
<td>vii</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>xi</td>
</tr>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>xiii</td>
</tr>
<tr>
<td>I  INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II  THE CLIMATOLOGY OF LIGHTNING INCIDENCE</td>
<td>5</td>
</tr>
<tr>
<td>A. Flash Incidence</td>
<td>5</td>
</tr>
<tr>
<td>1. The Thunderstorm-Day Statistic and Its Meaning</td>
<td>5</td>
</tr>
<tr>
<td>2. Rough Estimates of Flash Incidence per Thunderstorm Day</td>
<td>6</td>
</tr>
<tr>
<td>3. Relationships Between Flash Incidence and Thunderstorm-Day Statistics</td>
<td>7</td>
</tr>
<tr>
<td>B. The Duration of Thunderstorms</td>
<td>10</td>
</tr>
<tr>
<td>C. Proportion of Flashes Going to Ground</td>
<td>12</td>
</tr>
<tr>
<td>D. Diurnal Variations</td>
<td>13</td>
</tr>
<tr>
<td>E. Influence of Structure Height</td>
<td>17</td>
</tr>
<tr>
<td>F. Application of Climatological Data to a Hypothetical Case</td>
<td>22</td>
</tr>
<tr>
<td>III THE STATISTICAL CHARACTERISTICS OF FLASHES TO GROUND</td>
<td>25</td>
</tr>
<tr>
<td>A. General</td>
<td>25</td>
</tr>
<tr>
<td>B. Mechanisms of Charge Transfer by a Flash to Earth</td>
<td>29</td>
</tr>
<tr>
<td>1. Return Strokes</td>
<td>29</td>
</tr>
<tr>
<td>a. Current</td>
<td>29</td>
</tr>
<tr>
<td>b. Time to Peak Current and Rate of Current Rise</td>
<td>33</td>
</tr>
<tr>
<td>c. Time to Half-Value and Intermediate Currents</td>
<td>38</td>
</tr>
<tr>
<td>2. Number of Return Strokes, Intervals Between Return Strokes, and Continuing Currents</td>
<td>42</td>
</tr>
</tbody>
</table>
ILLUSTRATIONS

1  Relationship Between Thunderstorm Days per Month and Monthly Flash Density ........................................... 9
2  Relationship Between Geographical Latitude, Thunderstorm Days, and Proportion of Flashes to Ground ............... 14
3  Diurnal Variation of Thunderstorm Activity .................. 16
4  Relationship Between Structure Height and Number of Lightning Strikes .................................................. 21
5  Schematic Illustration of Processes and Currents Occurring during a Flash to Ground ................................. 26
6  Statistics of Peak Currents ...................................... 31
7  Statistics of Time to Peak Current ............................. 35
8  Statistics of Rate of Current Rise .............................. 37
9  Diagram Illustrating Lack of Association Between Peak Current and Rise Time of Current .......................... 39
10  Statistics of Decay Time to Half-Peak-Current Value .... 40
11  Statistics of Number of Strokes per Flash .................. 43
12  Statistics of Duration of Intervals Between Return Strokes ................................................................. 44
13  Statistics of Duration of Continuing Currents .............. 45
14  Statistics of Amplitudes of Continuing Currents .......... 46
15  Distribution of Charge Transfer in Continuing Currents .............................................................................. 47
16  Statistics of Total Charge Transfer in a Flash to Ground .................................................................................. 48
17  Statistics of Total Duration for a Flash to Ground ........ 50
18  Analytic Forms of Return-Stroke Currents (Main and Intermediate Plotted Separately) .............................. 55
19  Analytic Forms of Complete Return-Stroke Currents (Main Succeeded by Intermediate) ........................... 56
20  Action Integrals for Lightning Currents ........................ 59
21  Distribution of the Number of Return Strokes per Flash .................................................................................. 61
<table>
<thead>
<tr>
<th>Page</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>Distribution of Duration of Flashes to Earth</td>
<td>63</td>
</tr>
<tr>
<td>23</td>
<td>Distribution of Time Interval Between Strokes</td>
<td>64</td>
</tr>
<tr>
<td>24</td>
<td>Distribution of Peak Currents for First Return Stroke and Subsequent Strokes</td>
<td>65</td>
</tr>
<tr>
<td>25</td>
<td>Distribution of Charge per Flash</td>
<td>66</td>
</tr>
<tr>
<td>26</td>
<td>Distribution of Time to Peak Current</td>
<td>67</td>
</tr>
<tr>
<td>27</td>
<td>Distribution of Rates of Rise</td>
<td>68</td>
</tr>
<tr>
<td>28</td>
<td>Distribution of Time to Current Half-Value</td>
<td>69</td>
</tr>
<tr>
<td>29</td>
<td>Distribution of Duration of Continuing Currents</td>
<td>70</td>
</tr>
<tr>
<td>30</td>
<td>Distribution of Amplitude of Continuing Current</td>
<td>71</td>
</tr>
<tr>
<td>31</td>
<td>Distribution of Charge in Continuing Current</td>
<td>72</td>
</tr>
<tr>
<td>32</td>
<td>Time History of Typical (Basic) Lightning Models</td>
<td>76</td>
</tr>
<tr>
<td>33</td>
<td>Time History of Return Stroke and Intermediate Currents for Typical (Basic)</td>
<td>78</td>
</tr>
<tr>
<td>34</td>
<td>Time History of Action Integrals for Typical (Basic) Lightning Models</td>
<td>79</td>
</tr>
<tr>
<td>35</td>
<td>Time History of Severe (Basic) Lightning Model</td>
<td>82</td>
</tr>
<tr>
<td>36</td>
<td>Time History of Return Stroke and Intermediate Currents for Severe (Basic)</td>
<td>84</td>
</tr>
<tr>
<td>37</td>
<td>Time History of Action Integrals for Severe (Basic) Lightning Model</td>
<td>85</td>
</tr>
<tr>
<td>38</td>
<td>Time History of Severe (Applied) Lightning Model</td>
<td>88</td>
</tr>
<tr>
<td>39</td>
<td>Time History of Return-Stroke and Intermediate Currents for Severe (Applied)</td>
<td>89</td>
</tr>
<tr>
<td>40</td>
<td>Time History of Action Integrals for Severe (Applied) Lightning Model</td>
<td>90</td>
</tr>
<tr>
<td>41</td>
<td>A Comparison of Analytical and Experimental Results for the Action Integral</td>
<td>91</td>
</tr>
<tr>
<td>42</td>
<td>Statistics of Duration of Intervals Between K Changes</td>
<td>96</td>
</tr>
<tr>
<td>43</td>
<td>Distribution of Peak Currents for K Changes</td>
<td>98</td>
</tr>
<tr>
<td>44</td>
<td>Comparison of Current Forms in Return Strokes and in K Changes</td>
<td>99</td>
</tr>
<tr>
<td>45</td>
<td>Structure of the Fields Radiated by Lightning as a Function of Time and Frequency</td>
<td>103</td>
</tr>
<tr>
<td>Page</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>------</td>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>46</td>
<td>Amplitude Spectrum of Return-Stroke Signals Radiated by Lightning</td>
<td>101</td>
</tr>
<tr>
<td>47</td>
<td>Peak Received Amplitude for Signals Radiated by Lightning</td>
<td>105</td>
</tr>
<tr>
<td>A-1</td>
<td>Relationship Between Annual Thunderstorm Day (T_y) and Flash Density (T_y) Values</td>
<td>121</td>
</tr>
</tbody>
</table>
TABLES

| 1 | Relationship Between Thunderstorm Days and Flash Densities | 11 |
| 2 | Relationship Between Thunderstorm Days and Daily Duration of Storm Activity | 12 |
| 3 | Range of Values for Lightning Parameters | 60 |
| 4 | Properties of Statistical Distributions for Lightning Parameters | 62 |
| 5 | Typical Lightning-Model Parameters (Basic Models) | 77 |
| 6 | Severe-Lightning-Model Parameters (Basic Model) | 83 |
| 7 | Severe-Lightning-Model Parameters (An Applied Model) | 87 |
| 8 | Time Schedule for Return Strokes | 110 |
| 9 | Static Magnetic Fields from Close Lightning | 111 |
| 10 | Comparison of Basic Models for Lightning | 117 |
| 11 | Comparison of Lightning Models Used in Applications | 119 |
| A-1 | Monthly Thunderstorm-Day Data for Selected U.S. Stations | 122 |
| A-2 | Lightning-Flash Densities per Month for Selected U.S. Stations | 123 |
ACKNOWLEDGMENTS

We are much indebted to Professor Martin A. Uman for discussions of his recent experimental data, and for permission to incorporate some of his results in this report before their formal publication.

Some use has been made of analyses supported by the Office of Naval Research under Contract N00014-71-C-0106 and published as yet only in summary form.

Throughout the course of this work we have benefited from technical interchanges with the staff of McDonnell Douglas Astronautics; A. P. Venditti, C. K. Jackson, R. C. Twomey, and N. Thomas were particularly helpful. We have also profited from the contributions of various individuals to discussions at meetings; in this connection we mention especially R. P. Massey (Bell Telephone Laboratories), R. J. Rettberg (Gulf Radiation Technology), C. E. Jackson and G. O. Folkins (Sandia Laboratories, Albuquerque), and J. D. Robb (Lightning and Transients Research Institute).

This report was prepared for McDonnell Douglas Astronautics Company under contract to Bell Telephone Laboratories and the U.S. Army Safeguard System Command.
I INTRODUCTION

Designers and users of engineering systems are often faced with problems due to lightning. The objective of this report is to supply a comprehensive survey of the physical characteristics of lightning, and to indicate to the engineer—aware of the special sensitivities of his particular system—how this information can be applied in minimizing lightning hazards. The major emphasis of the report is on effects produced in installations directly struck by discharges to ground. However, some consideration is given to flashes that do not reach the earth (mainly intracloud discharges). Also, estimates are provided of the fields—electric and magnetic—radiated by close lightning.

The body of the report consists of three main sections. Firstly (Section II), an analysis is presented whereby lightning incidence at any time of day and any month of the year can be assessed at any location for which thunderstorm-day climatological data are available. Also, the degree to which lightning incidence is modified by the presence of tall structures is indicated. Lightning occurrence is a vital factor in determining the economics of avoiding lightning hazards. Elaborate protective measures are obviously far more justifiable for high-lightning-exposure areas such as Florida, than they are for flat deserts where thunderstorms rarely occur. Even in low-exposure regions, however, especially sensitive sites such as explosives factories and missile installations may require particular consideration.

A substantial part of the report (Section III) deals with the characteristics of lightning flashes to ground. Statistical distributions are given for many of the parameters likely to create problems for the engineer; median and extreme values are specified. The information presented
is based on a critical survey of data available in the literature, and in some instances this critical process has led to a partial rejection of some previously accepted values. The degree of interconnection of the parameters is discussed; this can be a very important point in the assessment of lightning hazards. If, for example, the range of current variation is narrowly limited, then there is a close connection between the total charge passing and the time occupied by the discharge. If, on the other hand, large current surges can occur that are of very short duration compared with that of the flash, the interrelationship between peak current and charge passing is very slight.

The remaining major section of the report (Section IV) demonstrates how the data given can be applied to developing models representative of average and severe lightning flashes. It is also shown how the lightning sensitivity can be assessed for a specific engineering system installed at a particular geographical location. The lightning incidence—with appropriate modifications for structural considerations—is determined for the particular location over the periods during which the system is exposed; one aspect of the hazard is thus defined. The engineer must himself determine to which lightning parameters his system is sensitive and the relative degree of sensitivity to each parameter; this assessment is often difficult. After the significant parameters have been identified, the statistical data on the lightning characteristics can be used to determine the frequency of occurrence of values of the parameters to which the equipment is sensitive. These values must be set by the engineer familiar with the design of his equipment. The chances of the selected values being achieved, combined with the information on lightning incidence, make it possible to estimate the lightning sensitivity of the installed system. If the system is influenced by only one parameter—for example, the peak current—the estimate can be quite precise. If, however, several parameters are involved, with differing associations
of system sensitivity and differing interconnections in the lightning phenomenology, the estimate can be quite uncertain. It is almost self-evident that the greater the number of parameters involved, the less the precision of the estimate.
II THE CLIMATOLOGY OF LIGHTNING INCIDENCE

A. Flash Incidence

1. The Thunderstorm-Day Statistic and Its Meaning

The thunderstorm day is the only parameter related to lightning incidence for which worldwide data extending over many years are available. It is thus by far the best source of information on monthly, seasonal, and annual variations of global thunderstorm and lightning activity. Thunderstorm-day data have been tabulated by the World Meteorological Association\(^1\,^2\) and also presented cartographically.\(^2\,^4\)

The thunderstorm-day parameter is normally defined as a local calendar day on which thunder is heard. The parameter has the notable deficiency that it does not contain any information on the intensity or duration of the storm;\(^5\) a calendar day is recorded as a thunderstorm day irrespective of whether the number of close lightning flashes is one or many.

Thunder is very rarely heard at distances exceeding 20 to 25 km from the lightning channel,\(^1\,^5\,^7\) and because of various reasons, the average practical limit of audibility seems to be about 15 km. One of these reasons is acoustic refraction; Fleagle\(^8\) has estimated that according to the various atmospheric refractive circumstances normally encountered, the range of audibility of thunder lies between 5 and 25 km.

* References are listed at the end of the report.
An audible range of 15 km implies that a thunderstorm day involves the occurrence of at least one discharge within an area of about 700 km\(^2\) (\(\pi \times 15^2\)) surrounding the observing station. This, however, is not necessarily the area corresponding to the thunderstorm-day statistic, since it is easily shown from simple analysis that, assuming uniform lightning incidence and an audible range of 15 km, the average distance of an audible flash from the observing station is only 10 km. Crichlow et al.\(^5\) consider that the area corresponding to a thunderstorm day is \(1260 \text{ km}^2\); this figure seems likely to be an overestimate since it was obtained by the two dubious assumptions of an audible range as high as 20 km, and the occurrence of discharges on every thunderstorm day at the limit (20 km) of this range.

The mean area of a well-developed thunderstorm has been variously estimated\(^6,7\) as 300 km\(^2\) to 500 km\(^2\); this area, again, is not necessarily the same as that involved in the thunderstorm-day statistic. Brooks\(^1\) in an early treatment has estimated that the observation of a thunderstorm day corresponds to a thundery area of 500 km\(^2\) in the vicinity of the observing station; no information that has emerged subsequently justifies any substantial modification of this number.

2. **Rough Estimates of Flash Incidence per Thunderstorm Day**

Crude assessments of the flash incidence on a thunderstorm day are quite easily made. If \(A\) is the thundery area, \(D\) the storm duration, and \(F\) the average flashing rate, then the flash density per unit area in the day is \(DF/A\).

Most observations of thunderstorms\(^10\) suggest that the average rate of flashing in a thunderstorm cell is about three per minute (one every 20 seconds) irrespective of locality. The lifetime of a single cell is somewhat less than an hour, and during this time the flashing
rate varies from less than one per minute to a maximum approaching ten per minute, this maximum being attained fairly early in the lifetime of the cell. Temperate thunderstorms usually include only one active cell, but in tropical storms several cells are normally involved; they often become active consecutively. Thus the average flashing rate may not be grossly changed but the duration of the storm is more extended. Typically, single-cell storms last for about an hour, but the average duration of a tropical thunderstorm is some three hours. If we take $A$ as $500 \text{ km}^2$, $F$ as one every twenty seconds, and $D$ as ranging from one to three hours, the corresponding flash incidence is $10^{-4} \text{ km}^{-2} \text{s}^{-1}$, and the flash densities assuming one storm per thunderstorm day are approximately between 0.4 and 1.0 per square kilometer.

Various other estimates exist or may be derived for flash incidence and flash densities. Golde\textsuperscript{3} gives a global average of 0.16 flashes to earth per km$^2$ per thunderstorm day; since perhaps 20 percent of all discharges go to ground the corresponding figure for all flashes is 0.8 per km$^2$. Both estimates\textsuperscript{11} and measurements\textsuperscript{14} suggest that about 100 lightning flashes occur each second over the whole world; with an average flashing rate of three per minute this corresponds to 2000 active thunderstorms; if each storm has an area of $500 \text{ km}^2$ the flash incidence is $10^{-4} \text{ km}^{-2} \text{s}^{-1}$. Aiya\textsuperscript{12} finds that Indian storms produce one discharge per km$^2$ over a duration of three hours; the corresponding flash incidence is approximately $9 \times 10^{-5} \text{ km}^{-2} \text{s}^{-1}$. However, Horner\textsuperscript{18} using various methods, estimates that in tropical thundery areas the flash incidence is only $10^{-5} \text{ km}^{-2} \text{s}^{-1}$.

3. Relationships Between Flash Incidence and Thunderstorm-Day Statistics

The increasing use of lightning-flash counters is enabling the connection between flash density and the thunderstorm-day parameter to
be much more precisely identified than has hitherto been possible.\textsuperscript{16}

Counters respond to a preset threshold value of lightning signal; this value corresponds to an "effective range" of the counter that can be determined. Strong flashes originating beyond the effective range are counted but are compensated for by the nonregistering of weak discharges occurring within the effective range. Two designs of counters have been extensively employed—the CCIR model, and the Pierce-Golde-ERA-CIGRE* instrument.

It has often been assumed without any justification that flash density per month or per annum is directly proportional to the number of monthly, $T_m$, or yearly, $T_y$, thunderstorm days. However, almost all thorough studies of the available data indicate that in most circumstances, empirical relationships between flash density, $c_m$ and $c_y$, approach a proportionality with $T_m^2$ and $T_y^2$ rather than with $T_m$ and $T_y$.

For example, the flash density has been variously given in investigations based on counter data\textsuperscript{17-19} as proportional to $T_m^{1.74}$, $T_m^{1.5}$, and $T_y^{1.5}$.

Two especially thorough analyses of lightning-flash-counter data have been made. Pierce\textsuperscript{16},\textsuperscript{20} from his analysis has developed the empirical relationship

$$c_m^2 = aT_m^m + aT_m^n$$

\textbf{1}

* These names are those—in chronological order—of the individuals and organizations that have made major contributions to the design of this counter. Descriptions of the counter are to be found in the literature under various combinations of these names.

† The inclusion of more than two significant figures in the exponent is certainly not justified, given the uncertainties of the basic data.
where $\sigma_m$ is the monthly flash density (flashes km$^{-2}$), and the constant $a$ has the value $3 \times 10^{-2}$. This relationship is plotted in Figure 1. Note that for months of fairly high activity ($T_m \geq 5$) $\sigma_m$ is approximately

![Figure 1: Relationship between Thunderstorm Days Per Month and Monthly Flash Density](image)

proportional to $T_m^2$. It also follows from Eq. (1), for most locations of substantial activity, that $\sigma_y$ approaches a proportionality with an exponent of $T_y$ that is not much less than 2. This is because at most places thunderstorm activity tends to be concentrated in a few months of the year. Also plotted on Figure 1 is the relationship

$$\sigma_m = 0.06 T_m^{1.5}$$

(2)
This is based on an extensive analysis by workers at Westinghouse. Over the practically important range of $2 \leq T_m \leq 10$ there is not much difference between Eqs. (1) and (2).

Equations (1) and (2) are quite consistent with most of the approximate information of Section II-A-2. For example, if, following Aiya, we assume that a day of thundery activity in India produces one discharge per km, it follows that during the months of the main thunderstorm season--$T_m \approx 20$--Aiya's results would indicate $T_m \approx 20$. Substituting $T_m = 20$ in Eq. (1) gives $T_m \approx 12$ in reasonable agreement with Aiya. However, $T_m = 12$ agrees even better with the deduction of Horner that at Singapore $T_m = 14$ when $T_m = 20$.

In the Appendix, $T_m$ data are listed for eight selected U.S. stations. Values of $T_m$ are calculated from Eqs. (1) and (2). The calculations are extended in the Appendix to annual values ($T_m$ and $T_y$) and a comparison is made with Japanese and European data. All the results are in fair agreement, but the Westinghouse work [Eq. (2)] yields lower values of flash densities than do the other analyses. As a general guide, Table 1 indicates the approximate correspondence between thunderstorm days and flash densities.

B. The Duration of Thunderstorms

In some countries--notably those in Eastern Europe--thundery activity is reported in terms of the duration of thunderstorms, and the thunderstorm-day parameter is not used. Popolansky and Laitinen have investigated the interconnections between storm durations and thunderstorm days. From a further development and manipulation of their analysis the approximate relationships

$$D_m = 0.9 T_m^{0.4}$$  (3)
Table 1

RELATIONSHIP BETWEEN THUNDERSTORM DAYS AND FLASH DENSITIES

<table>
<thead>
<tr>
<th>Number of Thunderstorm Days per Month</th>
<th>Flash Density per km² per Month</th>
<th>Number of Thunderstorm Days per Annum</th>
<th>Flash Density per km² per Annum</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.2</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>25</td>
<td>4</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>50</td>
<td>10</td>
</tr>
<tr>
<td>15</td>
<td>6</td>
<td>80</td>
<td>30</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
<td>100</td>
<td>50</td>
</tr>
</tbody>
</table>

and

\[
\bar{D}_m = 0.9 \bar{D}_Y^{0.3} \tag{4}
\]

can be deduced, where \(\bar{D}_m\) and \(\bar{D}_Y\) are the average duration (hours) of thundery activity in a thunderstorm day on monthly and annual bases, respectively.

Table 2 lists some calculations from Eqs. (3) and (4). We note that the results in Table 2 are in quite reasonable agreement with the general conclusion that a thunderstorm day normally involves only one active storm. However, in temperate regions of slight thundery activity the duration of the storm is only about an hour, while in tropical regions the duration approaches three hours.

The results of Section III-A-3 show that flash incidence depends on a power of the thunderstorm-day parameter that is almost always greater than unity and usually approaches two. Comparison of Tables 1 and 2
Table 2

RELATIONSHIP BETWEEN THUNDERSTORM DAYS AND DAILY DURATION OF
STORM ACTIVITY

<table>
<thead>
<tr>
<th>Number of Thunderstorm Days per Month</th>
<th>Number of Days per Thunderstorm Day of Thundery Activity* (hours)</th>
<th>Number of Thunderstorm Days per Year</th>
<th>Average Duration per Thunderstorm Day of Thundery Activity † (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.2</td>
<td>10</td>
<td>1.8</td>
</tr>
<tr>
<td>5</td>
<td>1.7</td>
<td>25</td>
<td>2.4</td>
</tr>
<tr>
<td>10</td>
<td>2.3</td>
<td>50</td>
<td>2.9</td>
</tr>
<tr>
<td>15</td>
<td>2.7</td>
<td>80</td>
<td>3.3</td>
</tr>
<tr>
<td>20</td>
<td>3.0</td>
<td>100</td>
<td>3.6</td>
</tr>
</tbody>
</table>

* Monthly basis.
† Yearly basis.

suggests that this behavior is almost equally due to increased storm duration and increased storm intensity in the more thundery regions.

C. Proportion of Flashes Going to Ground

It is well known that the fraction of discharges in a thunderstorm that reach the ground is extremely variable. There are changes even within the course of a single storm, with a tendency for the central mature phase of the storm to produce the greatest proportion of flashes to ground. There are great differences between individual storms, and the type of storm—whether frontal or air-mass (heat)—appears to have some influence although the experimental evidence is conflicting.22,23 Also, the nature of the local topography is important; flashes to ground are more common in mountainous regions than over flat land.16 Finally,
it seems well established that—on a climatological basis—the proportion of discharges to earth increases with increasing geographical latitude. \(^2^6\)

It is plausible that the relative likelihoods of intracloud discharges and flashes to earth are primarily controlled by the separation of the charge centers in the thundercloud and by their altitudes relative to ground. We may anticipate that the greater the height of the lower (negative) main charge center, the less the proportion of discharges to earth. The differences between phases of a single thunderstorm and between individual storms are probably all partially attributable to the charge-center distributions, and this in turn is influenced by the local topography and by the latitude.

Pierce\(^2^6\) has represented the latitudinal variation by

\[
p = 0.1 \left[ 1 + \left( \lambda / 30 \right)^2 \right] \tag{5}
\]

where \(p\) is the proportion of discharges that go to ground and \(\lambda\) is the geographical latitude in degrees. A more complicated relationship,

\[
p = 0.05 + \frac{\sin \lambda + 0.05}{\left( T_m + 3 \right)^{1/2}} \tag{6}
\]

has been given by the Westinghouse workers.\(^1^8\) Comparisons between Eqs. (5) and (6) are plotted in Figure 2.

D. Diurnal Variations

It is well established that for land stations the peak of thundery activity usually occurs in the late afternoon and evening hours, while the minimum is at about 0800 Local Mean Time (LMT). The diurnal cycle, however, even at a specific locality, shows monthly and seasonal changes.
There are also wide local variations, even within quite limited geographical areas, associated with differences in the influences generating the storms; these influences include the relative importance of frontal or air-mass (heat) effects, and the significance of local topography. When heat and orographic effects are dominant the diurnal variation has
a marked peak in the local afternoon; however, when frontal storms are important the peak is much less pronounced and its time of occurrence tends to be rather later. The differences in the character of storms produce some rather general systematic variations in the diurnal cycle. In temperate latitudes there is substantial evidence that the amplitude of the cycle tends to diminish, and the time of the maximum to become later, with a move from continental interiors toward coastal fringes. The same trends in amplitude and time-of-peak activity occur with decreasing geographic latitude as the equator is approached.

An especially thorough study of the diurnal variation has been made by the Westinghouse group. Two curves derived from their data, for the United States in July, are given in Figure 3(a). The composite curve for the mountain states (Arizona, New Mexico, Colorado, Utah, Nevada, Utah, Wyoming, and Montana) and the prairie states (Texas, Oklahoma, Kansas, Nebraska, and the Dakotas) may be compared with that for the other parts of the continental United States. The curve for the mountain and prairie states is of larger amplitude and has an earlier maximum than the other graph; this is consistent with the greater predominance of heat and orographic storms for the mountain and prairie region than for the rest of the United States. Separation of the mountain and prairie sets of data produces an even more extreme variation for the former area. The two curves are for July; this is the time of year for the entire continental United States at which the afternoon maximum is highest and occurs at the earliest time. At other seasons the curves are flatter and the maximum is displaced toward later times.

Also shown in Figure 3(a) is a curve due to Maxwell and Stone. This is an average representation for several global land areas and has been reproduced in other reports.
FIGURE 3  DIURNAL VARIATION OF THUNDERSTORM ACTIVITY
Our knowledge of the diurnal variation over the oceans is somewhat uncertain. The orientation of the present report is toward sensitivity to lightning strikes, and for this purpose information on oceanic storms is not normally of much practical significance. However, estimates of oceanic storm activity are often of importance—for example, in the prediction of atmospheric noise levels. Most previous researches have considered that the diurnal change in thunderstorm activity over the oceans is so slight that the variation can be ignored. Some data derived by Solov'jev from sferics measurements over the North Atlantic, and represented in Figure 3(b), support this viewpoint. However, the Westinghouse group, reversing their former opinion, now consider that the diurnal variation over the oceans is very pronounced and approximately in anti-phase to that over land [see Figure 3(b)]. They base their opinion on a combination of theoretical meteorological arguments and six-hour data obtained from the National Weather Records Center. The oceanic curve of Figure 3(b) is quite extreme; its amplitude is almost identical with that of the curve for the mountain and prairie states in Figure 3(a).

E. Influence of Structure Height

Flashes to ground are normally initiated by a leader streamer in the thundercloud. The leader travels downward, distributing charge along its channel in the process, the charge—usually negative—being drawn from the cloud. As the charged leader approaches the ground, the field strengths adjacent to the earth's surface increase and the field morphology changes, under the influence of the charge on the downward-moving leader. Ultimately the field distribution near ground level is sufficiently severe for an upward-moving leader to be induced. It unites with the original downward leader, and the instant of union represents the true commencement of the high-current lightning return-stroke.
The length of the upward leader does not exceed a few meters for a discharge to flat, open terrain. However, structures or other protruberances from the earth's surface cause a field concentration at their tips. Consequently, upward leaders are induced while the downward leader is still some distance from the tip of the structure, and the length of the upward leader can be as much as a few hundred meters before the two leaders unite. It is interesting to note that as the height of the structure increases, since the ultimate breakdown is primarily controlled by the local field configuration between the tip of the structure and the downward leader, the actual altitude of the structure above the ground plane should become of less significance. Statistical data exist to support this conclusion. For very tall buildings, upward leaders are sometimes initiated even before the charges in the thundercloud have developed sufficiently for leader breakdown to take place within the cloud; such incidents are usually described as triggered lightning. A recent review by Pierce concludes that instances of triggered lightning are not common for structures of less than 150 m in height, but that as the height increases above this threshold the proportion of the lightning strikes that are triggered also increases and very rapidly.

For structures less than 100 m in height, and which do not therefore trigger lightning, the experimental evidence indicates that the number of lightning strikes increases according to a power of h, the structure height, that lies between one and two. This result is somewhat in conflict with the well-known concept of the "cone of protection" provided by a lightning conductor; this concept would suggest a strike dependence on $h^2$. The apparent discrepancy is mainly because the cone-of-protection concept does not recognize the existence of upward leaders. Also, even if upward leaders did not occur, the "attractive distance" of an elevated conductor is not necessarily identical with its protective range.
Several models of the charge distribution along a leader channel exist, and by using these models various calculations related to the development of upward leaders from the ground can be made.\(^\text{37}\) For example, given the height and general configuration of a building, its attractive range can be deduced.\(^\text{35}\) However, great reliance should not be placed in such calculations. The development of upward leaders is most critically controlled by the charge distribution on the portion of the downcoming leader channel that approaches closest to the ground. Because of effects due to space charge, irregularities in leader velocity, and so on, the charge distribution on this final part of the leader channel is not well known, and is indeed probably very variable. Thus the models are most deficient at their most crucial stage.

It is appropriate to point out that it is widely asserted that upward leaders develop more easily if the succeeding stroke is to be of high current. In other words the attractive range—and consequently protective influence—of a lightning conductor increases with increasing return-stroke current magnitudes. We do not believe this assertion is proven. It is based on two assumptions. These, as given by Golde,\(^\text{39}\) are:

(1) "The electric gradient under a leader channel is a function of the charge on the leader channel."

(2) "This [the charge on the leader channel] is proportional to the amplitude of the current in the return stroke to which it gives rise."

Assumption (1) is unexceptionable, but we know of no evidence confirming the proportionality suggested in Assumption (2). Indeed, there are strong physical arguments against such a simple relationship. It is believed that the leader channel consists of a conducting arc core a few centimeters in diameter surrounded by a sheath, several meters in extent, of space charge generated by corona.\(^\text{6}\) Most of the leader charge resides in this sheath. The return stroke follows the core in its rapid ascent,
presumably immediately neutralizing only the charge on the core, and carrying a current related primarily to this core charge. The total charge (core and sheath) deposited along the leader and responsible for the induction of upward leaders is several coulombs. However, the charge represented by the current-time product for the first few microseconds of the return stroke (corresponding to an ascent of about a kilometer) is only on the order of one-tenth of a coulomb. Thus any simple relationship between total charge on the leader channel and return-stroke current seems unlikely.

Pierce, from a combination of theory and empiricism, has deduced expressions for the attractive area, $A_a$, corresponding to a structure of height $h$. The results of this work have only been briefly reported; much of this research, which has been conducted under Contract N00014-71-C-0106 for the Office of Naval Research, remains unpublished. The attractive range (radius), $r_a$, is given approximately in meters by

$$r_a = 80/\sqrt{h} \left\{ \exp(-Ah) - \exp(-Bh) \right\} + 400 \left\{ 1 - \exp(-Ch^2) \right\}$$ (7a)

where $h$ is also in meters. In Eq. (7a) the constants $A$, $B$, and $C$ have the values, respectively, of $2 \times 10^{-2}$, $5 \times 10^{-2}$, and $10^{-4}$. The attractive area $A_a$ of the structure is of course given by

$$A_a = \pi r_a^2$$ (7b)

In the same analysis Pierce has considered the contribution to the lightning hazard, for high structures, produced by triggered lightning. This contribution can be included by multiplying $A_a$ by the factor $F_T$, where

$$F_T = \left\{ 1 + 2 \left( 9 - \frac{1500}{h} \right) \right\}$$ (8)
The second term in the large brace represents the influence of triggered lightning. It is negligible for $h < 100$ m, begins to become of some significance at $h \approx 150$ m, and thereafter increases rapidly with increasing $h$ to attain a limiting value of about 500. Equation (8) has both some theoretical and empirical justification. However, the available data are limited to structures with $h$ less than 400 m; accordingly, Eq. (8) should be applied with some caution in the case of taller structures and configurations.

Finally, it is informative to consider to what extent Eqs. (7) and (8) agree with actual data. These data points (shown in Figure 4) were derived from early results listed by Beck$^{39}$ and some more recent supplementary references.$^{33}$ The data are almost entirely for northern temperate
latitudes. The points have been adjusted for the "effective" structure height, * to a geographic latitude, \( \lambda \), of 42-degrees north, and to an annual thunderstorm day level, \( T_y \), of 32. With \( T_y = 32 \), Figure A-1 in the Appendix indicates \( \sigma_y \approx 7 \), while from Eqs. (5) and (6) the proportion of flashes to ground is approximately 0.3; it follows that the data points on Figure 4 correspond to an unperturbed lightning-flash density \( \sigma_{yg} \) of approximately 2.1 discharges to earth per \( \text{km}^2 \) per annum. The annual number of discharges \( N_{yh} \) to a structure of height \( h \) is given by

\[
N_{yh} = \sigma_{yg} A_y F_y a_T
\]

where \( A_y \) and \( F_y \) are defined by Eqs. (7) and (8). The curve shown on Figure 4 is a plot of \( N_{yh} \) vs. \( h \), derived from Eq. (9) with \( \sigma_{yg} \approx 2.1 \); it is a reasonable fit to the data points.

F. Application of Climatological Data to a Hypothetical Case

As an illustration of how the climatological information can be applied we will consider the case of a tower 100 m high located at Grand Forks, North Dakota. Suppose the lightning strike incidence has to be assessed for

1. The whole year
2. The month of July
3. Between 1530 and 1630 LMT on a day in July.

For all these three cases the attractive area of the tower remains the same; from Eq. (7) we have \( r_a \approx 356 \text{ m} \) and \( A_a \approx 0.398 \text{ km}^2 \). For the factor

* In the case, for example, of towers at the summits of sharply peaked mountains the effective height is substantially greater than the actual height of the structure.
$F_T$, Eq. (8) gives $F_T \approx 1.02$, showing that triggered lightning is of little significance. However with its inclusion we have—in round numbers—an effective attractive area of $0.4 \text{ km}^2 (-0.398 \times 1.02)$.

The annual number of thunderstorm days $T_t$ is 24 (see Appendix) and with this value $\sigma_y$ is given by Figure A-1 as approximately 4 per $\text{km}^2$ per annum. Alternatively, we may obtain $\sigma_y$ by summing the values of $\sigma_m$ for each month; the figures obtained respectively from the Pierce [Eq. (1)] and Westinghouse [Eq. (2)] relationships are 4.4 and 3.2 (Table A-2). Thus, 4 seems a reasonable approximate value to use for $\sigma_y$. Since the latitude $\lambda$ is 47°55' N, the proportion $p$ of flashes to earth is given by Eq. (5) as about 0.36; the more complicated relationship of Eq. (6) yields—after appropriate analysis—$p \approx 0.32$. Taking $p = 0.34$, $\sigma_y = 4$, and an effective attractive area of $0.4 \text{ km}^2$, gives an annual flash occurrence of about 0.5. Thus we may expect a lightning discharge to the structure once every two years.

For the month of July, we have $T_m = 6.4$ and estimates of $\sigma_m$ as approximately 1.3 (Pierce) and 1.0 (Westinghouse). Equation (6) gives $p = 0.26$, as compared with 0.36 from Eq. (5). With $\sigma_m = 1.2$, $p = 0.3$, and an effective attractive area of $0.4 \text{ km}^2$, the ground-flash incidence is approximately 0.14. In other words, there is a one-in-seven chance of the 100-m-tall tower being struck by lightning during the month of July.

Figure 3(a) indicates that in July about nine percent of the thunderstorm activity occurs in the hour centered around 1600 LMT. It follows that the daily ground-discharge incidence within this hour is

$$(1/31)(0.14)(0.09) \approx 4 \times 10^{-4}.$$
Thus if an engineer happened to be working on the 100-m tower for two and a half hours between 1530 and 1630 LMT in the month of July, he would—from climatological statistics—have a one-in-a-thousand chance of being exposed to the hazards associated with a lightning flash to the tower.
A. General

As indicated in Section II-C, about one-fourth of all lightning flashes occur between the thundercloud and the ground. The duration of the flash is typically a few tenths of a second. During this time interval the lightning discharge has several distinct stages. An initial leader phase is followed by the first return stroke, which is then usually succeeded by further return strokes. The intervals between the return strokes may include continuing currents or may be relatively quiescent. After the last return stroke a final-stage current sometimes flows to earth in a continuing fashion. Figure 5 schematically illustrates these processes and the corresponding currents flowing to the ground during the flash.

Figure 5(a) illustrates the flash to ground as seen by an observer. With time-resolved photography* the various stages of the flash can be separated as shown in Figure 5(b); the ground currents for these various stages are represented in Figure 5(c).

During the initial phase of a lightning discharge, a downward-moving leader, the front section of which often brightens intermittently in a stepwise fashion, lowers charge from the cloud toward the ground. The average time occupied by this stepped leader stage is approximately 50 ms. During this time an ionized channel is formed by the charge being drained from the cloud. A few coulombs of charge are deposited along

* Summaries of the high-time-resolution photographic methods for lightning studies are given in standard texts.6,40-42
FIGURE 5  SCHEMATIC ILLUSTRATION OF PROCESSES AND CURRENTS OCCURRING DURING A FLASH TO GROUND
the channel. There is no charge transfer to earth and consequently no channeled current flow between the cloud and ground.

When the leader makes contact with the ground—or more precisely with an upward leader from the earth (Section II-E)—a violent current surge known as the return stroke occurs. The current surge is associated with a bright luminous front moving upward at perhaps $10^8$ m/s along the channel formed by the stepped leader. The current usually reaches a peak of approximately 20 kA within 1 or 2 $\mu$s and decays to half its value in about 40 $\mu$s. Often the return stroke is followed by an intermediate current of about 1 kA and lasting for a few milliseconds. There may be only one return stroke in the flash. More often there are several return strokes. It is believed that immediately after a return stroke, leader-streamers from the top of the channel probe into the cloud. If one of these probing leaders encounters a concentration of charge a recoil-streamer occurs backward along the channel represented by the probing leader and the decaying return stroke. These recoils have associated luminous and electrical effects known as K changes. Usually the K recoil is insufficiently strong to reenergize the entire original channel down to ground level, and does not therefore involve any charge transfer to earth. Sometimes, however, very energetic K recoils cover the entire distance from cloud to ground along the channel still partly preionized by the previous return stroke; these particularly energetic recoils are termed dart leaders. A dart leader has a duration of approximately a millisecond and deposits perhaps a coulomb of charge along the channel.

When the dart leader makes contact with the ground, another return stroke follows. These subsequent strokes have a peak current of about half that

* There is of course some diffuse current transfer by point-discharge (corona) processes.
of the first stroke. The time to peak and time to half-value, however, do not differ appreciably from those of the first stroke.

The average time between the strokes or the return-stroke interval is 50 to 60 ms. On occasion, the return-stroke interval includes a continuing current of about 100 A and lasting for some 100 or 200 milliseconds as shown in Figure 5(c) after the second stroke. The dart-leader/return-stroke combinations may be repeated until the last return stroke; this is followed at times by a final-stage continuing current of about 100 A and lasting about 100 ms.

The above discussion has briefly summarized the typical stages of development for a lightning flash that transfers negative charge to ground. Further details may be found in standard texts; that by Uman is especially recommended.

Discharges that carry positive charge to earth ("positive flashes") are not common. In the case of flashes to open ground, several investigations indicate that something less than 10 percent involve the passage of positive return-stroke currents. There is some evidence that the proportion of positive flashes increases with the height of the structure concerned. Thus, Berger and Vogelsanger find that about 15 percent of the discharges initiated by downward leaders to the towers on Monte San Salvatore are positive, while Davis and Standring, from a study of flashes to tethered balloons, conclude that over a third of the discharges they recorded involved the passage of positive current surges.

The most important respects in which positive flashes differ from negative discharges is that positive flashes often involve protracted

* This is defined as the time between the start of a return stroke and the start of the succeeding return stroke.
leader and final stages, and rarely contain more than one stroke. Indeed, a positive first stroke is more likely to be succeeded by a second negative stroke (thus making the discharge complex) than by a second positive stroke. Positive discharges to ground probably occur both from the upper positive charge in a thundercloud and from the localized positive center at the cloud base. The former kind of flash would involve lengthy leader stages and substantial continuing currents; the latter type would take place relatively easily to high structures.

B. Mechanisms of Charge Transfer by a Flash to Earth

As mentioned above, charge is transferred to earth during the return-stroke, the intermediate, and the continuing-current stages of a flash. This subsection reviews and discusses the measured characteristics of these stages. The leader stages are not considered since they involve no charge transfer to earth. For a summary of the leader characteristics, see Uman.

1. Return Strokes

a. Current

There is a considerable amount of information on the peak currents flowing in return strokes, with typical currents of 20 kA being observed. However, there are relatively few data at the high end of the statistical amplitude distribution where the statistics of the extremes are sparse. The general statistics obtained by several investigators and outlined, for example, in Uman indicate that perhaps five percent or less of the return strokes have currents exceeding 70 kA. Szpor on the other hand, somewhat in opposition to the consensus viewpoint, has measured peak currents of 250 kA, and suggests that currents exceed 150 kA in five percent of cases and exceed 200 kA for one percent of the
strokes. Szpor contends that the usual quoted statistics (as summarized in Uman) are significantly low as a result of errors in instrumentation techniques and in the methods used to account for the distribution of currents within the objects struck by lightning. It should be noted that a large portion of the measurements have been obtained on tall metal towers as used for transmission lines, or on high-rise buildings such as the Empire State Building. On applying corrections to these measurements, Szpor has shown that the corrections tend to increase the deduced currents, so that the corrected results are in good agreement with Szpor's own data.

In an extensive Japanese investigation, limited, however, to currents exceeding 10 kA, Tsurumi et al. have measured peak currents of 100 kA with frequencies of occurrence between five and ten percent. These results are intermediate between those of Szpor and the general consensus.

Uman has recently perfected the technique of deducing lightning-current characteristics from observations of the electric and magnetic fields radiated from close discharges. This approach, originated by Norinder, has the great advantage that the flashes being investigated are characteristic of discharges to open ground, and do not therefore possess the abnormalities associated with flashes to tall structures. Uman's data suggest that currents of between 50 to 120 kA occur for five percent of strokes. These measurements are in partial agreement with Szpor's results. The significance of the recent work is that the frequency of occurrence of the peak currents is higher than had previously been believed. The results of some of these measurements are presented in Figure 6.

There is substantial direct—and some indirect—evidence that the currents in subsequent return strokes are, on the average, only
about half that of the first stroke. Some actual data due to Berger\(^5\) for first- and subsequent-stroke currents are compared in Figure 6; it can be seen that the latter currents are perhaps 50 percent of the former. Measurements of parameters, indirectly related to peak currents, such as radiated magnetic fields,\(^{23}\) intensity of luminosity,\(^{40}\) and charge transfer per stroke,\(^{54}\) all indicate that the subsequent stroke effects are one-half or less of those due to the first stroke.

Positive currents should be mentioned principally because of the widely held misconception that the current in positive strokes is "usually of very high value."\(^{51}\) This misconception may be traced to a misleading emphasis in some of Berger's papers on positive "current-giants."\(^5,57\) If all Berger's data (Refs. 44, 53, 56-59, and elsewhere) for positive currents are examined, there is an apparent separation into a large number of quite small currents and a few instances of high currents. This dual distribution was identified for discharges to balloons by Davis and Standring.\(^45\) They suggested that the high-current strokes occurred between the balloon and the concentrated small positive charge at the cloud base; the proximity of the two termination points of the discharge--and consequently the low channel resistance--produced the high current. This interpretation still seems plausible.

We have already noted (Section III-A) that tall structures experience more positive flashes than does open ground. Also, the indications--as discussed above--are that the taller the structure, the greater the likelihood of high-current positive strokes. Thus, measurements such as those of Berger obtained effectively on tall structures are likely to be especially misleading as regards positive flashes. Even, however, if the data\(^48,53,58-69\) are taken at face value, the proportion of positive strokes giving peak currents exceeding 50 kA is only a few percent, and is indeed somewhat less than the corresponding proportion.
for negative strokes. Less than 10 percent of all flashes to open ground are positive, and while negative discharges are usually multiple, containing several return strokes, positive flashes are normally one-stroke affairs. Thus it seems quite an unnecessary refinement to consider positive strokes separately.

b. **Time to Peak Current and Rate of Current Rise**

The time to peak current and the rate of current rise are important since the design of some equipment (for example, lightning arrestors) is dependent on these times.

The work of Berger and his colleagues on the form of the current surge in the return stroke has been widely quoted and used. Nevertheless, there have been some reservations in the lightning community as to its reality, especially as regards the first stroke. Berger's results indicate that the rise time for the first stroke is about 10 μs, in contrast to that of subsequent strokes which is about 1 μs or even less. It has been suspected that much of the apparent current rise during the first stroke has been influenced by the development of upward leaders from the towers on Monte San Salvatore used to study the lightning. As many as 80 percent of the flashes to the 70-m-high towers are initiated by upward leaders, and it is consequently believed that the effective height of the towers is increased to almost 300 m by the configuration of the mountain.

Some recent measurements by Fisher and Uman appear to justify the mistrust of Berger's results. Their work indicates that there is little difference in the measured rise times, for first and subsequent strokes, of the electric field radiated by a close flash. The average rise times were 3.7 μs and 3 μs for the first and subsequent strokes, respectively, with corresponding standard deviations of 1.6 μs and 1.1 μs.
It is not immediately obvious, without considerable analysis, how to translate precisely the electric-field rise times to current rise times. If \( i_t \) is the current at any instant \( t \), \( L_t \) is the length of the channel being energized by \( i_t \), and \( d \) is the distance, then the radiated field \( E_t \) at time \( t \) is given approximately by

\[
E_t = \frac{1}{4\pi \varepsilon_0} \left\{ \int_0^t \frac{M dt}{d^3} + \frac{M_t}{cd^2} + \frac{(dM_t/dt)_t}{c^2d} \right\}
\]

where \( M = 2i_t L, \) and \( \varepsilon_0 \) is the permittivity of free space. Equation (10) assumes no losses in propagation over the ground. In the ideal circumstances of no losses and the instantaneous development of a channel, the changes in \( E_t \) [assuming that the complications due to the relative magnitudes and phasing of the three components in Eq. (10) can be distinguished] should be directly related to \( i_t \). Under most practical circumstances that can be envisaged the rise for \( E_t \) should be slower than that for \( i_t \). Thus, measured rise times for \( E_t \) would be greater than, but would define a limit to, actual rise times for \( i_t \). The measurements of Fisher and Uman indicate that this is indeed so. A comparison of the distribution of electric-field rise times as measured by them, to the distributions of time-to-peak-current measurements, shown in Figure 7, illustrates the fact that the former rise times are generally greater than the latter. Hence, it would not seem unreasonable on the basis of Fisher and Uman's results to take the rise times of the first and subsequent strokes as being the same, with both being certainly less than 4 \( \mu s \).

There is a good deal of confusion in the literature regarding the definition of current rise time and rate of rise. We prefer to define rise time as the total rise time between the first detectable onset of the current surge and the time of peak current; others—largely because of instrumentation and recording limitations—have used the time
RISE TIMES MEASURED BETWEEN

- FISHER-UMAN: 0 to peak
- UMAN (NASA): 0 to peak
- McCANN: 10 to 90%
- HYLTVN-CAVALLIUS-STROMBERG
- HAGENGUTH-ANDERSON: 10 to 90%

**FIGURE 7** STATISTICS OF TIME TO PEAK CURRENT
between 10 percent and 90 percent of the peak current value. Concerning rate of rise, we prefer the parameter of average rate of rise measured over the total rise time (peak current/total rise time). The average rate of rise is also often quoted for the 10-to-90-percent (of peak value) interval—i.e., 80 percent of peak current/time from 10 to 90 percent of peak current. Also, when oscillographic techniques are used, the maximum rate of rise during the increasing current phase is sometimes reported.

Some measured distributions of rates of current rise are shown in Figure 8. It must be emphasized that the peak current, time to peak current, and rate of current rise (no matter how the two latter are defined) are not entirely independent parameters. Sometimes the interrelationship is quite basic; thus, peak current is equal to the product of total time to peak current and the average rate of current rise. Unfortunately many of the statistical distributions presented in the literature fail to note which parameters are independent or measurable properties and which are dependent or derived from the measured parameters. For example, the peak current and the total time to peak current can readily be, and often are, measured experimentally; then the rate of rise is derived from these quantities.

There is no obvious physical reason why there should be any interrelationship between peak currents and time of current rise,

---

* This difference in definition does not in actuality involve much difference in rise times. For example, with the typical double-exponential current models used later in this report the total rise time is 1.5 \( \mu s \), and the 10-to-90-percent rise time is 0.9 \( \mu s \).

† With a typical double-exponential current model these three definitions of rate of rise would yield respective values of 13, 16, and 77 kA/\( \mu s \).
FIGURE 8  STATISTICS OF RATE OF CURRENT RISE
although it is often asserted that such a connection exists, and that in particular, high peak currents are associated with short rise times. In order to investigate this point a scatter diagram of data from McCann,\textsuperscript{6,3} Uman,\textsuperscript{4,9} and Berger\textsuperscript{2,3} is plotted in Figure 9. Berger's results are presented only for the subsequent strokes since the rise times of the first strokes appear to be biased as discussed above. Two sets of values for Uman's results are shown, where one set of data has been formally reported\textsuperscript{43} and the other set is an adjusted version of the same data. Uman suggests that the peak currents may be as much as 30 percent less than originally reported.\textsuperscript{51} This factor has also been incorporated in the current measurements of Figure 6. The main feature of Figure 9 is the uncorrelated scatter in the various data sets. Thus we may state with some confidence that peak currents and time of current rise are essentially independent parameters. Another interesting feature of the data is apparent in the distributions of Figures 6, 7, and 8, where the spread between the individual distributions illustrated on each graph is much less for the current and time-to-peak curves than it is for the rate-of-rise curves. Hence, for this analysis it seems appropriate to take the peak current and time to peak current as independent parameters, with the average rate of current rise being a derived quantity.

Finally, regarding positive strokes, the scanty evidence suggests that the rise times are, if anything, longer than for negative strokes.\textsuperscript{6,44} Since most equipment is more sensitive to the faster surges, and since--as previously discussed--positive strokes are quite rare, they will not be considered separately.

c. Time to Half-Value and Intermediate Currents

After crest current, the current surge decays to half its peak value in about 40 \( \mu \)s and typically continues to decay to a few
FIGURE 9  DIAGRAM ILLUSTRATING LACK OF ASSOCIATION BETWEEN PEAK CURRENT AND RISE-TIME OF CURRENT
kiloamps within a few hundred microseconds. Statistical distributions for the times to half-values are readily available, and some of these data (for negative strokes) are illustrated in Figure 10.

\[\text{HYLTEN-CAVALLIUS AND STRÖMBERG}\]

\[\text{McCANN}\]

\[\text{HAGENGUTH AND ANDERSON}\]

FIGURE 10  STATISTICS OF DECAY TIME TO HALF-PEAK CURRENT VALUE

The decay phase following fairly-high-current positive strokes is a matter of some uncertainty (and consequently of considerable interest). The work of Berger and Vogelsanger\(^{44,59}\) on Monte San Salvatore indicates that for positive strokes with peak currents exceeding some
30 kA the current decay after the peak is quite slow, with times to half-value of about 1 ms; this is an order of magnitude greater than shown by the data of Figure 10. The peak current statistical distributions above 30 kA do not differ substantially for positive and negative strokes, so the protracted decay for the former type implies that the total charge transfer is also greater by about an order of magnitude for large current positive than for similar negative strokes. Some of the actual Monte San Salvatore measurements indicate that this is indeed so.\textsuperscript{59} However, simultaneous optical observations show that the positive strokes are associated with an extreme development of upward leaders; these approach about 1 km in length.\textsuperscript{59} It seems most unlikely that such lengthy upward leaders can be induced from open ground; consequently, any application of the San Salvatore results, on positive strokes, to normal lightning environments is very dubious. More information on the current-time characteristics of positive flashes to open country is obviously needed. The techniques of Uman\textsuperscript{49} could be used, but in view of the rarity of positive flashes an extensive observational program would be necessary.

Reverting to conventional negative strokes, after the initial decay following the peak current, there is usually a low-level current of a few kiloamperes that persists for several milliseconds; this current is conveniently termed the "intermediate" current.\textsuperscript{64} Few direct observations of intermediate currents have been made. Experimenters have often tended to concentrate on the short-duration, high-current, initial surge in the return stroke, and have consequently frequently used techniques incapable of recording the intermediate currents. However, in addition to the direct measurements there is a very substantial amount of indirect deduction regarding the characteristics of the intermediate currents. Much of this indirect information has been obtained because it is necessary to postulate certain properties for the intermediate currents in order to explain experimental observations of
the extremely-low-frequency (ELF) component of atmospheric waveforms.\textsuperscript{98,97} Incidentally, since experimental observations\textsuperscript{98} show that most atmospherics contain both VLF (generated by the initial return-stroke current surge) and ELF components (produced by intermediate currents), it follows that an intermediate current usually succeeds a return stroke.

2. Number of Return Strokes, Intervals Between Return Strokes, and Continuing Currents

There is a considerable amount of statistical information on the number of return strokes per flash.\textsuperscript{25} These data are incorporated in Figure 11. There appears to be a tendency for the number of return strokes per flash to increase with decreasing geographic latitude;\textsuperscript{24} the average number changes from about three at a latitude of 60 degrees to six at the equator.

Information on the time intervals between strokes is also extensive.\textsuperscript{25} The data are summarized in Figure 12. The various sources of data are in better agreement than is the case for the number of strokes. The typical time between strokes is about 50 or 60 ms. Sometimes the time intervals between strokes contain continuing currents; more often the time intervals are undisturbed. It has been found\textsuperscript{74} that if an interval exceeds about 100 ms and does not include a continuing current, the succeeding stroke is unlikely to follow the same channel as its predecessor; in these instances there is possibly some justification for regarding the succeeding stroke as starting a new flash.

The early work of Pierce\textsuperscript{22} indicated that continuing currents were present in 26 percent of intervals. Brook, Kitagawa, and Workman\textsuperscript{54} say that their results are in agreement with this value; however, an examination of the data contained in their paper and its companion—Kitagawa, Brook, and Workman\textsuperscript{74}—shows only 6 to 7 percent of intervals as containing continuing current.
The duration of the intervals that include continuing current is substantially larger than the usual. Pierce\textsuperscript{22} finds an average duration for such intervals of 145 ms with 5 percent exceeding 400 ms. These values are in good agreement with the results of the New Mexico workers,\textsuperscript{54,74} which show continuing currents persisting for from 40 to 500 ms with an average of 150 ms. Some data for these durations are available and are plotted in Figure 13.
The continuing currents passing in the long continuing phase have been estimated by Brook, Kitagawa, and Workman\textsuperscript{5} as lying between 40 and 130 A with an average of 80 A flowing for 150 ms; this corresponds to the passage of 12 C. Williams and Brook\textsuperscript{75} give rather larger values; their range is 40 to 500 A with an average of 180 A flowing for 170 ms, corresponding to a charge of about 30 C; this figure for charge transfer agrees well with a Japanese estimate\textsuperscript{76} of 24 C. Some other results\textsuperscript{44} for continuing currents are in good agreement with those of the New Mexico workers.\textsuperscript{54,75} However, Hagenguth and Anderson's data\textsuperscript{64} give currents that are considerably greater than even those of Williams and Brook.
Distribution for the amplitudes of continuing current are shown in Figure 14 and a distribution of charge transfer in continuing currents is given in Figure 15.

3. Final Stage

After the last return stroke there is often a continuing current phase usually referred to as the F stage. Malan finds that this stage is present for about 30 percent of all flashes; Pierce recognized its existence in 50 percent of discharges; but the New Mexico workers detect its presence in only 25 percent of flashes. Both Pierce and Malan found that the smaller the number of strokes in a flash the more likely
it is for the flash to contain a final phase of continuing current. Typically,\textsuperscript{22} 60 percent of one-stroke flashes produce a final continuing current phase; for discharges with five strokes or more the proportion is only 30 percent.

When a final stage is present its median duration is about 100 ms, with 5 percent exceeding 500 ms.\textsuperscript{22} The magnitudes of the continuing currents in the final stage do not differ significantly from those occurring in continuing-current intervals between return strokes.\textsuperscript{54}
4. **Total Charge Transfer in a Flash to Ground**

Current-time measurements for objects that are struck can be integrated to give estimates of the total charge transfer. Typically this is some 10 to 30 C. Three sets\textsuperscript{44,83,84} of data statistically distributed are shown in Figure 16. Some indication of the charge transfer for flashes to open country can be derived from electrostatic-field measurements although these yield electric-moment changes rather than charges, and in to translate the moment information into charge estimates it is necessary to make a reasonable assessment of the height of the charges involved. Mean values of moment recorded in various
investigations are 150 C-km for discharges without a continuing current, and 350 C-km for discharges with a continuing current;\textsuperscript{54} and 150 C-km,\textsuperscript{22,70} 220 C-km,\textsuperscript{78} and 260 C-km,\textsuperscript{79} these being averages of all flashes. The investigators have variously assumed heights of from 3 to 5 km as being involved, giving average charge estimates of 15 to 35 C; this is in good agreement with the current-measurement information.

There are a few sources that indicate average charge transfers of as large as 100 and 200 C. These are the papers by Hatakeyama;\textsuperscript{80} Meese and Evans;\textsuperscript{81} and, subsequent to Meese and Evans, by Nelson.\textsuperscript{82} Pierce\textsuperscript{83} has pointed out, from internal evidence in the paper by Meese and Evans, that the validity of their method of deducing charge transferred from their magnetic field measurements is very suspect. The work
of Nelson is an improvement but still contains some dubious features. Of the 17 flashes he studied, six gave charge transfers exceeding 200 C; it is difficult to accept a result so at variance with the consensus of previous work unless the measuring technique is above reproach; this does not seem to be the case.

5. **Time Occupied by a Flash to Ground**

Extensive information on the duration of a discharge to earth is given by Pierce, Malan, and Mackerras. These data are incorporated into Figure 17, which shows that the typical flash has a duration of a few tenths of a second.

We note from Figure 17 that the Pierce and Malan data are in excellent agreement but that considerably longer durations are listed by Mackerras; the respective median values are approximately 200, 200, and 600 ms. Other work gives median durations of 500 to 600 ms and thus tends to support Mackerras. The reasons for the discrepancies are not entirely clear. However, they are certainly at least partially real in origin, and caused by systematic differences (average number of strokes per flash, for example) in storm characteristics among various geographical areas.

6. **Interrelation of Parameters**

Various parameters associated with lightning have been considered above. It is sometimes vital, in practical determinations of the lightning hazard to a particular system, to assess to what extent

---

* Note that these data were derived from electric-field measurements and thus include a leader stage of some 50 ms in average duration. The actual time occupied in charge transfer to ground is less by this leader duration.
FIGURE 17  STATISTICS OF TOTAL DURATION FOR A FLASH TO GROUND
the parameters are interdependent. It is also important, in generating models, to define those parameters considered as basic and those which, because of an interdependence, are essentially derived. We may broadly categorize the interdependence of parameters as closely interrelated, loosely interrelated, or essentially independent.

Concerning the return stroke, the peak current is a most important parameter, for which much experimental data exists. This parameter is of fundamental importance in determining equipment sensitivity, and should be regarded as basic when models are being defined. Time to peak, time to half-value, and the characteristics of intermediate currents are all parameters that seem to be essentially independent of each other and of the peak current. On the other hand, rate of rise is closely related to peak current and time to peak and should be regarded as a derived quantity. The charge transfer per stroke is another derived quantity since it represents the integration of the current-time variation over the duration of the stroke.

Subsequent strokes have a general similarity with each other, and—in time history only—with the first stroke. Here there is perhaps a loose interrelationship. Also the intensity of the first stroke is on the average twice that of subsequent strokes; this is another loose interrelationship. It seems physically plausible that the length of an interval between strokes, and whether or not the interval contains a continuing current, should influence the characteristics of the stroke terminating the interval. However, the evidence on this point is scanty and conflicting, indicating that any interrelationship can only be slight. The presence of a continuing current and the duration of an interval have the special interconnection that intervals exceeding about 100 ms must include a continuing current.
The number of strokes in a flash and the character of the current-time variations within the strokes appear to be essentially independent of each other. The existence of a continuing current in a flash, either in an interval between strokes or in the final stage, depends only slightly on the number of strokes; the reduced chance of a continuing current in the final stage for multi-stroke discharges is compensated for by the greater availability of intervals in which a continuing current may occur.

The charge, current, and duration of a continuing-current phase—whether in a stroke interval or in the final stage—appear to have somewhat similar statistics. Since charge is the time integral of current, this implies some connection between current magnitudes and duration of current flow. Many systems are sensitive essentially to the total charge passing in a continuing current rather than to the size or duration of the current. In developing models it is often preferable therefore to regard the charge transferred in continuing currents as basic, and to then adjust the corresponding currents and times to plausible values; this is contrary to the procedure for return strokes in which the current time history is regarded as basic and the charge is derived.

The total charge transfer is of course the sum of the charge transferred in return strokes (including intermediate currents) and in continuing currents; it is thus related to the number of strokes and to the presence or absence of continuing currents. There is some belief that the charge available in the thundercloud may be drained to earth either by discrete strokes or in continuing-current phases; this concept would imply that there is an inverse relationship between the number of strokes and the relative magnitude of continuing currents. There is some evidence supporting such a dependency. Total charge transfer is a derived quantity in modeling if the properties of the
return stroke currents, intermediate currents, and continuing currents are defined. However, since total charge passing is sometimes an independently measured experimental quantity, a type of feedback adjustment between current time characteristics and total charge transfer is often appropriate.

Finally, the total duration of the charge transfer is the sum of the return-stroke intervals (with or without continuing currents) and of the time occupied by the final stage; it is thus related to the number of strokes, mean stroke interval, and presence or absence of a final continuing-current phase. In modeling—as with total charge—total duration will normally be a quantity derived from the time histories of the separate stages. Again, however, since independent evidence of total duration is available, feedback adjustments—as in the case of total charge—should be made often in order to optimize self-consistency.

C. Analytical and Statistical Models for Lightning Flashes to Ground

For many applications analytical and statistical models for the characteristics of a lightning flash to ground are needed. The basic characteristics of these discharges have been discussed in Section III-B. In the first portion of the following discussion analytical models are presented to describe the nature of the currents transferred to earth in a cloud-to-ground discharge, and in a later section statistical models for these characteristics are presented.

1. Analytic Models

The current surge of the return stroke is usually represented as

\[ I(t) = I_0 \left[ \exp(-\alpha t) - \exp(-\beta t) \right] \]  

(11)
a form originally publicized by Bruce and Golde, based on empirical results. Oetzel recognized that Eq. (11) can be related to the lumped-circuit parameters of an LRC circuit; more recently Plooster has been somewhat successful in using this expression to simulate numerically spark discharges in air and also the lightning flash. In addition, Pierce, Arnold, and Dennis, have shown how the parameters $I_o$, $\alpha$, and $\beta$ can be defined from a knowledge of the time to peak current, the time to half-current value, and the peak current. Typically, the time to peak current is $\sim 1.5 \mu s$ and the time to half-value is $40 \mu s$, giving

$$\alpha = 1.7 \times 10^4 s^{-1} \quad \text{and} \quad \beta = 3.5 \times 10^6 s^{-1}$$

The corresponding value of peak current, $I_p$, is approximately $0.97 I_o$. Since $I_p$ is usually $20 \text{kA}$ or so, then $I_o$ is about $21 \text{kA}$.

As previously mentioned (Section III-B), the peak current of the subsequent strokes of the flash is about half that of the first stroke. However, the times to the peak current and current half-value are similar for all strokes. Thus an average model for the subsequent strokes can be given as

$$I(t) = \frac{I_o}{2} [\exp(-\alpha t) - \exp(-\beta t)] \quad (12)$$

A deficiency in Eqs. (11) and (12) is that the currents become small—that is, much less than $1 \text{kA}$—for times greater than a few hundred microseconds. In practice, an intermediate current of the order of $1 \text{kA}$

* Computer users often complain that the derivative is not zero at $t = 0$. The point seems trivial, since there are myriads of mathematical devices whereby the demands on the computer program at early times can be accommodated.
or so usually flows for a few milliseconds. The intermediate current can be represented by the addition of another expression. This is

\[ I_i(t) = I_1 \left[ \exp(-\gamma t) - \exp(-\delta t) \right] \]  

(13)

where \( \gamma = 10^3 \text{ s}^{-1} \), \( \delta = 10^4 \text{ s}^{-1} \), and \( I_1 = 2 \text{ kA} \). The peak value of \( I_i(t) \) is about \( 0.7 I_1 \). For comparison purposes, the double exponential terms of Eqs. (11) and (13) are plotted in Figure 18.

FIGURE 18  ANALYTIC FORMS OF RETURN-STROKE CURRENTS (main and intermediate plotted separately)

Thus the models for the first return stroke and subsequent strokes are, respectively,
\[ I_0 \left[ \exp(-\alpha t) - \exp(-\beta t) \right] + I_1 \left[ \exp(-\gamma t) - \exp(-\delta t) \right] \]  \hspace{1cm} (14)

and

\[ \frac{I_0}{2} \left[ \exp(-\alpha t) - \exp(-\beta t) \right] + I_1 \left[ \exp(-\gamma t) - \exp(-\delta t) \right] \]  \hspace{1cm} (15)

For \( I_0 = 20 \text{ kA} \) and \( I_1 = 2 \text{ kA} \), Expressions (14) and (15) are plotted in Figure 19.

The continuing-current phase can simply be modeled as a steady current, \( I_c \), flowing for a duration, \( T_c \). In general, \( I_c \) is about 150 A, and \( T_c \) is about 150 ms. When a continuing-current phase follows a return stroke it can conveniently be modeled by adding \( I_c \) to either Eq. (11) or
Eq. (12), whichever is appropriate. The further refinement of including an intermediate current between the main surge and the continuing current may of course be accomplished by adding \( I_c \) to Expressions (14) or (15).

The charge transfer can easily be obtained for each of the currents above by simply integrating the expressions with respect to time. The charge transferred for each current is respectively given as

\[
Q_R \approx \begin{cases} 
I_0 \left( \frac{\beta - \alpha}{\alpha \beta} \right) & \text{First stroke} \\
\frac{I_0}{2} \left( \frac{\beta - \alpha}{\alpha \beta} \right) & \text{Subsequent stroke} \\
I_1 \left( \frac{\delta - \nu}{\delta \nu} \right) & \text{Intermediate current} \\
I_c T & \text{Continuing current}
\end{cases}
\]

For the typical values of the lightning-model parameters given above, \( Q_R \) is approximately 1 C for the first stroke and 0.5 C for the subsequent stroke. The charge \( Q_I \) is about 2 C. Then, for strokes containing an intermediate current, the charge transfer per stroke is 3 C for the first and 2.5 C for the subsequent strokes. These values are in good agreement with Brook, Kitagawa, and Workman, who found an average charge transfer of 2.5 C per stroke. The charge transfer for the continuing current is \( Q_c \approx 23 \) C, which is higher by a factor of two than that given by Brook and his associates but is in reasonable agreement with Williams and Brook's measurements of 31 C and with Ishikawa's estimate of 25 C.

The energy transferred by the lightning discharge is also of practical interest. A measure of the energy is given in terms of the action integral, which is
\[ J(T) = \int_{0}^{T} i^2 dt \]  

(16)

Again, the above current models are used to obtain \( J(T) \) for the individual components of current.

For the return-stroke model, since \( \beta \gg \alpha \) the action integral can be approximated as

\[ J_R(T) = \frac{I_0^2(1 - e^{-2\alpha T})}{2\alpha} \]  

(17)

for the first stroke. For the subsequent strokes, \( I_0^2 \) is replaced by \( I_0^{2/4} \). The contribution from the intermediate current is slightly more complicated but can be given as

\[
J_I(T) \approx \begin{cases} 
I_1^2(3.7 \times 10^{-4}) \left[ 1 - 1.4 e^{-2\alpha T} \right] & \text{for } T > 1 \text{ ms} \\
I_1^2(3.7 \times 10^{-4}) \left[ 1 - 1.4 e^{-2\alpha T} + 0.5 e^{-11\alpha T} - 0.1 e^{-20\alpha T} \right] & \text{for } T \leq 1 \text{ ms}
\end{cases}
\]

(18)

and the continuing current component is simply given as

\[
J_C = \begin{cases} 
I_c^2 & \text{for } T < T_c \\
I_c^2 & \text{for } T \geq T_c
\end{cases}
\]

(19)

For the typical values of \( I_0, I_1, I_c, \) and \( T_c \), the components \( J_R, J_I, \) and \( J_C \) are shown, respectively, in Figure 20. For a lightning flash containing three strokes with one continuing-current phase, the total action integral for the flash is on the order of \( 10^4 \text{ A}^2\text{ -s} \) which is in good agreement with Berger's measurements (see also Section IV-E).
2. Statistical Models

The above models describe the principal features of the lightning discharges. These models are in good agreement with the measured average characteristics. Often, however, the typical parameters quoted are not adequate for use in some applications in which a system may be especially sensitive to a given parameter such as the peak current in the return stroke or the charge transferred by the flash. For this type of analysis, statistical models for the various parameters are needed. Typical, and reliably reported extreme values of the parameters are listed in Table 3. Note that the apparent observed extremes are somewhat determined by the limitations of experimental and recording techniques and by questions of definition; these constraints apply especially to the minimum values. Statistical distributions for the parameters are shown in Figures 21 through 31. Distributions for the leader processes are not included in these figures since the leaders do not transfer charge to the ground. The general leader statistics are given in Uman. 6
### Table 3

**RANGE OF VALUES FOR LIGHTNING PARAMETERS**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Minimum</th>
<th>Typical</th>
<th>Maximum</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of return strokes per flash</td>
<td>1</td>
<td>2 to 4</td>
<td>26</td>
<td></td>
</tr>
<tr>
<td>Duration of flash (s)</td>
<td>0.03</td>
<td>0.2</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Time between strokes (ms)</td>
<td>3</td>
<td>40 to 60</td>
<td>100</td>
<td>Without continuing current</td>
</tr>
<tr>
<td>Peak current per return stroke (kA)</td>
<td>1</td>
<td>10 to 20</td>
<td>250</td>
<td></td>
</tr>
<tr>
<td>Charge per flash (C)</td>
<td>1</td>
<td>15 to 20</td>
<td>400</td>
<td>Includes continuing current</td>
</tr>
<tr>
<td>Time to peak current (μs)</td>
<td>&lt;0.5</td>
<td>1.5 to 2</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Rate of rise (kA/μs)</td>
<td>&lt;1</td>
<td>20</td>
<td>210</td>
<td></td>
</tr>
<tr>
<td>Time to half-value (μs)</td>
<td>10</td>
<td>40 to 50</td>
<td>250</td>
<td></td>
</tr>
<tr>
<td>Duration of continuing current (ms)</td>
<td>50</td>
<td>150</td>
<td>500</td>
<td></td>
</tr>
<tr>
<td>Peak continuing current (A)</td>
<td>30</td>
<td>150</td>
<td>1600</td>
<td></td>
</tr>
<tr>
<td>Charge in continuing current (C)</td>
<td>3</td>
<td>25</td>
<td>330</td>
<td></td>
</tr>
</tbody>
</table>

The distributions in general represent a reasonable consensus of the sets of experimental curves presented in Section III-B. With the exception of Figure 21 describing the number of strokes per flash, all the distributions are modeled as log-normal curves.

The log-normal distribution is a normal distribution with a variate that changes logarithmically. Then the probability of the parameter, $x$, exceeding a given value, $P(x > X)$, is
\[ P(x > X) = \frac{1}{\sqrt{2\pi} \sigma} \int_{X}^{\infty} \exp \left( -\frac{(\log x - \log m)^2}{2\sigma^2} \right) d(\log x) \quad (20) \]

where \( \sigma \) is the log of the standard deviation relative to the median value \( m \). The log-normal distribution is to be expected for any process that consists of a number of independent contributing factors.

Also shown on each curve are the values of the variate for the 2-percent and 10-percent extremes and the median. For convenience, these values are tabulated in Table 4.

D. Examples of Procedures Used in Developing Models

The following two examples illustrate the methods used in selecting the parameters for a lightning flash to ground. For these examples a
Table 4

PROPERTIES OF STATISTICAL DISTRIBUTIONS FOR LIGHTNING PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Percentage of Occurrence</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2%</td>
</tr>
<tr>
<td>Number of return strokes</td>
<td>10 to 11</td>
</tr>
<tr>
<td>Duration of flash (ms)</td>
<td>850</td>
</tr>
<tr>
<td>Time between strokes (ms)</td>
<td>320</td>
</tr>
<tr>
<td>Return stroke current† (kA)</td>
<td>140</td>
</tr>
<tr>
<td>Charge transfer per flash (C)</td>
<td>200</td>
</tr>
<tr>
<td>Time to peak current (μs)</td>
<td>12</td>
</tr>
<tr>
<td>Rates of current rise (kA/μs)</td>
<td>100</td>
</tr>
<tr>
<td>Current half-value time (μs)</td>
<td>170</td>
</tr>
<tr>
<td>Duration of continuing current (ms)</td>
<td>400</td>
</tr>
<tr>
<td>Continuing current (A)</td>
<td>520</td>
</tr>
<tr>
<td>Charge in continuing current (C)</td>
<td>110</td>
</tr>
</tbody>
</table>

* Note that not all of the parameters are independent. Some judgment must be made in using the values for consistency.

† Values for first stroke.

typical lightning flash in temperate latitudes is considered. For most of the parameters the 50-percent or median values are used. However, some judgment must be used in selecting the parameters in order to have a consistent model.

Beginning with the number of strokes per flash (Figure 21), the median number is two to three strokes. With these numbers of strokes the chance of occurrence of a continuing current phase in the complete
discharge, either between strokes or in the final stage, is 50 to 60 percent. Since this chance of occurrence is borderline we will consider two examples—one with and one without continuing currents. We will assign three strokes to the flash without continuing current and two strokes to the other discharge. This apparently arbitrary procedure enables the criterion of average total charge transfer to be more plausibly satisfied.

For each case, the first stroke of the flash has a peak current of 20 kA and the subsequent strokes have peak currents of 10 kA. The time to peak current (Figure 26) is the same for all strokes and also for the time to half-value (Figure 28), with the respective values being 1.5 μs and 40 μs.
For the three-stroke case there are two intervals between strokes, the duration of each being 60 ms. A decision must be made in the two-stroke case as to whether the continuing current is to follow the first stroke or the second. It is noted in Section III-B that on the average perhaps 16 percent of the stroke intervals contain continuing currents while some 35 percent of the flashes have a final stage-continuing current. Since the final-stage continuing current is the more likely to occur, the continuing current of the two-stroke case is taken after the second stroke. The time between the two strokes is assumed to be 60 ms.

Next, the intermediate currents must be considered. For the case of no continuing currents it would seem reasonable that all three strokes are followed by an intermediate current. However, some consideration
must be given to the total charge transferred in the flash in order to determine the magnitude of these currents. From Figure 25, the usual charge transfer in a flash is ~15 C. In the model, the three strokes contribute about 2 C of charge to the flash, while the remaining charge is transferred by the intermediate currents. If the value of \( I_1 \) in Eq. (13) is taken as 4 kA, then the intermediate current of each stroke contributes 4 C of charge to the flash. Then the total charge transferred by the three-stroke model is about 14 C, which is in good agreement with the statistics.

For the two-stroke case, an intermediate current is taken after the first stroke but not after the second. The second stroke is followed with a final stage-continuing current. Some judgment must be used in
selecting the parameters for a consistent model. Since most of the charge is transferred by the final stage, the intermediate current has $I_i = 2$ kA. Then the first and second strokes together transfer about 3.5 C of charge. Plausible values of the parameters for the final-stage current are 150 A with a duration of 100 ms, giving the charge transferred in the final stage as 15 C, which is somewhat less than that in Figure 31 but certainly not unreasonable. The total charge transfer in the flash is 19 C.

The duration of the flashes is about 120 ms for the three-stroke case and 160 ms for the two-stroke case. These values are in good agreement with Figure 22, bearing in mind that much of the duration data summarized in Figure 22 includes a leader stage of about 50 ms.
It has been pointed out in Section III-B that the number of strokes per flash—and in consequence the duration and probably total charge transfer—are greater for tropical thunderstorms than for thunderstorms in temperate regions. A reasonable approximation to a model for an equatorial flash to ground is merely to add the two models discussed above, with the three-stroke case being succeeded by the two-stroke model. The resulting equatorial model would have five strokes each separated by 60 ms, a final phase of continuing current, a total charge transfer of 33 C, and a duration (without leader stage) of 340 ms.
In summary, the above examples illustrate how models can be developed from the data of Section III-B and the analysis of Section III-C. But, as emphasized above, it is important to exercise some judgment in using the information, so as to construct a consistent model.
FIGURE 28  DISTRIBUTION OF TIME TO CURRENT HALF VALUE
FIGURE 29  DISTRIBUTION OF DURATION OF CONTINUING CURRENTS
FIGURE 30 DISTRIBUTION OF AMPLITUDE OF CONTINUING CURRENT
FIGURE 31 DISTRIBUTION OF CHARGE IN CONTINUING CURRENT
IV APPLICATIONS TO ENGINEERING SYSTEMS

A. Sensitivity of Systems

It has already been indicated (Section I) that different types and designs of engineering equipment will differ in their relative response to the various parameters of lightning. In the simplest instance, equipment will be affected by only one parameter. More usually there will be a sensitivity to several lightning parameters, and the degrees of sensitivity to the individual parameters will differ. An additional complication is that the various lightning parameters will not normally be entirely independent, but will be interrelated to an extent that differs among separate sets of parameters.

In general, engineering equipment appears to be most sensitive to the peak current—and consequent magnetic forces involving impulsive and explosive effects—attained in the return stroke. This parameter may be regarded as the most fundamental of all lightning parameters from the viewpoint of the engineer. The magnitude of the peak return-stroke current (occurring probably in the first stroke) is, as far as we can judge, principally determined by charge-distributing processes—especially those in the atmosphere adjacent to ground level—occurring before the initial leader makes contact with the earth. Subsequent occurrences and their associated parameters may depend slightly on the peak current in the first stroke, but there is no interconnection in the reverse direction; in other words, the peak current is an essentially independent parameter.

Equipment that may be affected by voltages developed by coupling and inductive effects is sensitive to the rate of current rise. The extremes of this parameter are determined by those of the peak current and
only to a secondary degree by the time history of the current rise (see, for example, Table 3). Thus, specification of an extreme current in a lightning model often implies simultaneous specification of a severe value of $\frac{di}{dt}$.

Total charge transfer is probably the second most important lightning characteristic, after peak current, to which engineering equipment is sensitive. Total charge transfer gives some measure of the degree of electrical erosion and heating that can occur during a flash. For the larger values of charge transfer most of the charge passes in the form of continuing currents and has therefore little dependence on the return-stroke characteristics. Thus, in developing models for severe environments, specification of peak currents does not entail any appreciable definition of charge transfer; the two parameters must be considered separately.

The action integral $\int i^2 dt$ controls some types of effects—for example, those produced by melting. Because the action integral depends on the square of the current, its value during the return-stroke phase is quite closely linked to that of the peak current. For a steady continuing current the action integral is the product of charge and current; consequently, there is also a close connection between the action integral and the total charge transfer. If severe values of peak current and total charge transfer are chosen, the action integrals will also be severe.

Several of the lightning parameters not hitherto considered have some influence on the sensitivity of engineering systems, although this influence is usually slight relative to that of peak current and total charge transfer. The number of strokes and time intervals between strokes can affect equipment where capacitor charging by successive strokes is important. The charge transferred in a stroke is sometimes significant; this charge not only involves the peak current but is also quite dependent
on time to half-value and the characteristics of the intermediate current. Regarding continuing currents, the charge involved—with its important contribution to total charge transfer—seems to be the significant parameter; the magnitude and duration of the continuing currents are separately unimportant. Finally, the whole time history, during the discharge, of the currents and action integrals—and therefore the flash duration—can be significant. For example, there may be a steady dissipation of heat from an object that is struck so that the manner in which the energy indicated by the action integral is applied is important.

In developing lightning models for engineering usage a duality of approach can be identified. Both philosophies have their advantages and disadvantages. Analytic and statistical models can be produced founded solely on the physical characteristics of lightning; examples of such basic models are those presented in Section III-D and further discussed below (Sections IV-B and IV-C). Alternatively, models custom-tailored to the particular sensitivities of specific systems can be produced; these models are adjusted to be physically accurate and plausible as regards the lightning parameters that affect the equipment, but are less realistic concerning the parameters irrelevant to the system performance. An example of such an applied model is given below (Section IV-D).

B. Typical Lightning Model (A Basic Model)

In Section III-D, two basic models for a typical lightning flash in temperate latitudes have been developed to illustrate the use of the analytical representations and statistical information presented for the various parameters of the flash. This subsection summarizes the characteristics of these two models. The models are based entirely on the physical realities of lightning; they do not consider equipment sensitivities.
The characteristics of the typical lightning flash are shown in Table 5. Table 5(a) describes a model having three strokes without any continuing current interval between the strokes and without any final stage current. However, each return stroke is followed by an intermediate current stage of a few kiloamps flowing for a few milliseconds. Table 5(b) describes a model having two strokes. An intermediate current follows the first stroke and a final-stage continuing current follows the second stroke. Figure 32 schematically illustrates a time history for each
### Table 5

**TYPICAL LIGHTNING-MODEL PARAMETERS (BASIC MODELS)**

**Table 5**

(a) For a Flash Without any Continuing Currents

<table>
<thead>
<tr>
<th>Stroke Order</th>
<th>Peak Current (kA)</th>
<th>Current Between Strokes (ms)</th>
<th>Model Current, $I_o$ (kA)</th>
<th>$I^m_1$ (kA)</th>
<th>Charge (C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>~1.0</td>
<td>21</td>
<td>4</td>
<td>~4</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>~0.5</td>
<td>11</td>
<td>4</td>
<td>~4</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>~0.5</td>
<td>11</td>
<td>4</td>
<td>~4</td>
</tr>
</tbody>
</table>

Totals: Charge transferred = 14 C
Duration = 0.12 s
Action integral = $3.9 \times 10^4$ A-s

(b) For a Flash Having Continuing Currents

<table>
<thead>
<tr>
<th>Stroke Order</th>
<th>Peak Current (kA)</th>
<th>Current Between Strokes (ms)</th>
<th>Model Current, $I_o$ (kA)</th>
<th>$I^m_1$ (kA)</th>
<th>Charge (C)</th>
<th>Continuing Current</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>~1.0</td>
<td>21</td>
<td>2</td>
<td>~2</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>~0.5</td>
<td>11</td>
<td>0</td>
<td>0</td>
<td>Final stage</td>
</tr>
</tbody>
</table>

Totals: Charge transferred = 19 C
Duration = 0.16 s
Action integral = $2.0 \times 10^4$ A-s

* The time history for all strokes is defined by Eq. (11) with $\alpha = 1.7 \times 10^4$ s$^{-1}$ and $\beta = 3.5 \times 10^6$ s$^{-1}$. The time to peak current is 1.5 $\mu$s for all strokes. The time to half-value is 40 $\mu$s for all strokes.

† The time history for all intermediate currents is defined by Eq. (13), with $\gamma = 10^3$ s$^{-1}$ and $\delta = 10^4$ s$^{-1}$.

‡ Final stage-continuing current = 150 A; duration = 100 ms; charge transfer = 15 C.
model. Note that the axes are not to scale. And Figure 33 shows the return-stroke current and the intermediate current drawn to scale. The

![Diagram of current-time history for return strokes and intermediate currents]

**FIGURE 33 TIME HISTORY OF RETURN STROKE AND INTERMEDIATE CURRENTS FOR TYPICAL (basic) LIGHTNING MODELS**

The action integral for each component of the flashes is shown in Figure 34, which illustrates several interesting and important features. First of all, the first stroke of each flash has the largest component value for the action integral. But in the case of Table 5(a) the three strokes contribute to about half of the total action integral \((3.9 \times 10^4 \text{ A}^2\text{-s})\), while the intermediate currents contribute to the other half. For the case of Table 5(b) the major contribution to the action integral \((2.0 \times 10^4 \text{ A}^2\text{-s})\) is from the main surge of return-stroke current, while the contribution from the intermediate current and continuing current is much less \((0.4 \times 10^4 \text{ A}^2\text{-s})\). From this it is seen that the current surge of the return strokes is the most significant component in determining the value of the action integral.
From Table 5 it is also seen that the two cases are comparable in terms of their duration, the amount of charge passing, and the energy transferred as measured by the action integral.

There is an important difference, though. This difference is in the way the charge is transmitted to earth. For the case of Table 5(a) the charge passes in three short time intervals on the order of a few milliseconds each. In the case of Table 5(b), most of the charge is transferred in the continuing current. An object that is struck by lightning of either case may have a different response to each, and consequently its sensitivity must be assessed for both models. For example, a lightning arrester may be able to sustain the impulsive currents of the three-stroke model but might be affected by the second case [Table 5(b)], in which the long continuing current could cause excessive heating.

The models of Table 5 and their analytic representations can be used as guides in engineering calculations of the average effects of a
lightning strike in temperate latitudes. For tropical environments it is probably more appropriate (see Section III-D) to use a model consisting of the case of Table 5(a) followed by that of Table 5(b).

C. Severe-Lightning Model (a Basic Model)

In Section IV-B a typical lightning model (basic model) is summarized; in this, only typical values of the lightning parameters are used. Additionally, these parameters are slightly interadjusted to obtain a consistent model. Often, however, typical models are not entirely adequate for analyzing some problems, particularly those problems involving severe or critical conditions. For example, in the design of a lightning arrester, it may be determined that the arrester can withstand the typical flash but may be affected by a flash having higher peak currents or greater values of charge transfer. As a means for assessing the critical lightning environment, a severe model can be developed that is based on physical realities. Again the model parameters are chosen from the statistical distributions shown in Figures 21 through 31, using the methods for selecting the parameters that have already been described in Section III-D.

In the following discussion an example of a severe lightning model (basic model) is summarized; the critical lightning parameters have been selected as the number of strokes per flash, the peak current in the first and second strokes, the charge transferred in a continuing current between strokes, the charge passing in the continuing current of the final stage, the total charge transfer (the latter two parameters being interadjusted), and the time between the first and second strokes. For consistency, the limit of severity of the critical parameters is chosen as approximately the 2-percent extreme of the statistical distributions.
The severe lightning model thus consists of ten strokes with a peak current of 140 kA in the first stroke and 70 kA in the second stroke. The time interval between these strokes is 10 ms. The remaining subsequent strokes are chosen as having a peak current of 30 kA—on the high side of the median but not so much so as to be termed severe. The time between strokes—except for a continuing current interval—is taken as the typical value of 60 ms.

For all the strokes, the time to peak current is 1.5 μs. The average rate of current rise can be evaluated from the peak current and the time to peak current. Hence, the rate of rise is about 100 kA/μs for the first return stroke, 50 kA/μs for the second stroke, and 20 kA/μs for the remaining subsequent strokes. The fall time to half the peak current is chosen as the typical value—40 μs—for each stroke.

An intermediate current of approximately 1 kA that lasts for a few milliseconds follows the first, second, fourth, sixth, and eighth strokes. A continuing current of 400 A follows the fifth stroke and lasts 300 ms. The charge transferred by this current is taken as the critical (2-percent) value, and is 120 C. A final stage—continuing current of 200 A follows the last stroke and lasts 160 ms, corresponding to a charge transfer of 32 C. Since the total charge of 200 C transferred in the flash is taken as the critical (2-percent) value, and since the total charge transferred before the final-stage current is 168 C, the final-stage transfer is selected as 32 C; plausible values of current and time have been chosen to correspond to this charge transfer.

The distribution of the total charge transfer in the flash is 28 C from the return strokes, 20 C from the intermediate currents, 120 C from the continuing current, and 32 C from the final-stage current. The total duration of the flash is 0.9 s. In addition, the total action integral over the flash is $10^6 A^2\cdot s$, a value in good agreement with Berger's results.\(^5\)
A summary of the severe-lightning-model parameters is shown in Table 6, while a time history of the flash is illustrated (not to scale) in Figure 35. The time behavior of the individual components of the flash is also shown to scale in Figure 36, and the time history of the contributions to the action integral is given in Figure 37.

D. Example of a Severe Lightning Model (An Applied Model)

The preceding subsection dealt with typical models based only on the physical characteristics of lightning. We now proceed to an example of a method for developing applied models to investigate equipment problems in which the lightning sensitivity is at least partially defined. For our application we postulate the following assumptions, in their order of importance, and list the related requirements:

* It should again be emphasized that this is a severe model developed for a specific applied requirement. A severe model based only on lightning characteristics, as in Section IV-C, would be different, containing, for example, more strokes.
### Table 6
SEVERE-LIGHTNING-MODEL PARAMETERS (BASIC MODEL)

<table>
<thead>
<tr>
<th>Stroke Order</th>
<th>Peak Current (kA)</th>
<th>Charge Between Strokes (C)</th>
<th>Time Between Strokes (ms)</th>
<th>Model Current, ( I_0 ) (kA)</th>
<th>Continuing Current Model‡</th>
<th>Charge (C)</th>
<th>Continuing Current</th>
<th>Time History</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>140</td>
<td>(~8)</td>
<td>10</td>
<td>144</td>
<td>4</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>70</td>
<td>(~4)</td>
<td>60</td>
<td>72</td>
<td>4</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>(~2)</td>
<td>60</td>
<td>31</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>(~2)</td>
<td>60</td>
<td>31</td>
<td>4</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>30</td>
<td>(~2)</td>
<td>300</td>
<td>31</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>30</td>
<td>(~2)</td>
<td>60</td>
<td>31</td>
<td>0</td>
<td>0</td>
<td></td>
<td>Continuing current‡</td>
</tr>
<tr>
<td>7</td>
<td>30</td>
<td>(~2)</td>
<td>60</td>
<td>31</td>
<td>4</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>30</td>
<td>(~2)</td>
<td>60</td>
<td>31</td>
<td>4</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>30</td>
<td>(~2)</td>
<td>60</td>
<td>31</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>30</td>
<td>(~2)</td>
<td>60</td>
<td>31</td>
<td>0</td>
<td>0</td>
<td></td>
<td>Final stage§</td>
</tr>
</tbody>
</table>

Totals: Charge transferred = 200 C

Duration = 0.9 s

Action integral = \( 10^6 \) A²·s

* The time history for all strokes is defined by Eq. (11) with \( \alpha = 1.7 \times 10^4 \) s⁻¹ and \( \beta = 3.5 \times 10^5 \) s⁻¹. The time to peak current is 1.5 \( \mu \)s for all strokes. The time to half-value is 40 \( \mu \)s for all strokes.

† The time history of all intermediate currents is defined by Eq. (13) with \( \gamma = 10^3 \) s⁻¹ and \( \delta = 10^4 \) s⁻¹.

‡ Continuing current = 400 A; duration = 300 ms; charge transfer = 120 C.

§ Final-stage continuing current = 200 A; duration = 160 ms; charge transfer = 32 C.
(1) **Assumptions Regarding Equipment**

(a) The equipment is sensitive to high peak currents.

(b) The equipment is sensitive to high values of total charge transfer, some of which may pass in continuing currents.

(c) The equipment may be sensitive to high values of \( \frac{di}{dt} \).

(d) The equipment may be sensitive to high values of the total action integral.

(e) The equipment may be sensitive to high values of charge transfer in a single stroke.
(f) The equipment may be influenced by the total time history in the flash but is believed to be relatively insensitive to number of strokes, stroke intervals, duration of discharge, and other parameters except insofar as they are involved in (a) to (e) above.

(2) Requirements for Lightning Model for Equipment Application—Produce a severe lightning model that can be analytically represented; that involves a peak current selected as 200 kA and a total charge transfer selected as 200 C; and that gives values on the order of 100 kA/μs for dI/dt, $10^6$ A²·s for the total action integral, and 10 to 20 C for the charge transfer per stroke. The model should be internally self-consistent, and plausibly
compatible with the known characteristics of lightning. Subject to the above constraints the model should be as simple as possible.

The sequence of the model development is the following:

(1) A total charge transfer of 200 C is not easily achieved in a model with less than three strokes. Accordingly, we select three strokes as representing the simplest case that is still reasonably plausible physically. The continuing current is then most likely to succeed the last stroke, and the stroke intervals will therefore be typical at 60 ms each.

(2) The high peak current of 200 kA is most likely in the first stroke. The typical time to peak of 1.5 \( \mu \)s yields an average rate of rise of 130 kA/\( \mu \)s, thus giving a high value of \( \frac{dI}{dt} \) (Assumption 1c above). A typical time-to-half-current value—40 \( \mu \)s—is also indicated.

Peak currents in subsequent strokes will be half—namely, 100 kA—that in the first stroke, with an associated average rate of rise of 65 kA/\( \mu \)s.

(3) The charge transfer represented by the main surges of Item 2 are 12 and 6 C, for first and subsequent strokes respectively. Addition of plausible intermediate currents to the first and second strokes adds 8 C in each case, thus giving a high value of charge transfer per stroke (Assumption 1e above).

(4) The total charge transfer from the return strokes (main and intermediate currents) is 40 C, leaving 160 C to be accounted for by the continuing current. This charge is obtained by interadjustment of continuing-current magnitude and duration, with 400 A and 400 ms being the most likely values.

(5) From Items 1 through 4, and their associated analytical representations the value of the total action integral is about \( 1.9 \times 10^6 \) A\(^2\)-s; this is a high value of the action integral (Assumption 1d above). The total duration of the flash is approximately 0.5 s, a quite plausible value.

The characteristics of the severe model are summarized in Table 7. A time history is shown in Figure 38 and the return-stroke currents and
Table 7

SEVERE-LIGHTNING-MODEL PARAMETERS (AN APPLIED MODEL)

<table>
<thead>
<tr>
<th>Stroke Order</th>
<th>Peak Current (kA)</th>
<th>Charge (C)</th>
<th>Time Between Strokes (ms)</th>
<th>Model Current, I₀ (kA)</th>
<th>Intermediate Current Model †</th>
<th>Continuing Current</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>~12</td>
<td>60</td>
<td>206</td>
<td>9</td>
<td>~8</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>~6</td>
<td>60</td>
<td>103</td>
<td>9</td>
<td>~8</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>~6</td>
<td>60</td>
<td>103</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Totals: Charge transferred = 200 C
Duration = 0.5 s
Action integral = 1.9 × 10⁶ A²-s

* The time history for all strokes is defined by Eq. (11), with
  \( \alpha = 1.7 \times 10^4 \) s⁻¹ and \( \beta = 3.5 \times 10^6 \) s⁻¹. The time to peak current is
  1.5 \( \mu \)s for all strokes. The time to half-value is 40 \( \mu \)s for all strokes.

† The time history for all intermediate currents is defined by Eq. (13) with \( \gamma = 10^3 \) s⁻¹ and \( \delta = 10^4 \) s⁻¹.

‡ Final-stage continuing current = 400 A; duration = 400 ms; charge transfer = 160 C.
intermediate currents are plotted in Figure 39. The action integrals for the components of the flash are shown in Figure 40. As before (Sections IV-B and IV-C), the first stroke has the largest component \((1.2 \times 10^6 \text{ A}^2\text{s})\) and the major contribution to the integral is from the three current surges of the stroke \((1.8 \times 10^6 \text{ A}^2\text{s})\).

E. Comparison of Analytical and Experimental Results for the Action Integral

For each model described in Sections IV-B and IV-C, the total action integral has been evaluated; these results are now compared with experimentally determined values. Unfortunately the only experimental measurements available are those given by Berger.\(^{53}\) As has already been indicated, these measurements are characteristic of flashes to high structures and are therefore perhaps not entirely representative of
discharges to open ground. It seems correct therefore to deduce a statistical distribution for the total action integral not solely on the basis of the experimental data as done previously (Figures 21 to 31), but also by using the model information. This procedure is especially appropriate for the total action integral, since it is a quantity derived from the entire current time history during the flash, and the models of Sections IV-B and IV-C are designed to give plausible representations of this current time history.

A statistical distribution for the action integral derived from Berger's data is shown in Figure 41. As with the previous lightning
parameters it is not surprising that it is also a log-normal curve. The analytical values for the action integral as determined by the models are also plotted, where points $B_1$ and $B_2$ are the typical or 50-percent values, and $B_3$ is the severe or 2-percent case. As can be seen, the severe model is in very good agreement with Berger's results, while the typical models are within less than an order of magnitude of his data. The action integral for the applied model is also plotted (Point A). Its frequency of occurrence (~0.7 percent) is assumed to be approximately equal to the frequency of occurrence of the peak return-stroke currents since the action integral is largely determined by the return-stroke currents. Again, the model is in very good agreement with the measured data.
The points $B_1$, $B_2$, and $B_3$ can be used to derive a possible model for the action-integral statistics; this model is plotted in Figure 41. For 80 percent of the suggested distribution the model lies within an order of magnitude of Berger's results, and the agreement is best at the more important high-amplitude end. Much more experimental information is required, however, preferably obtained by techniques involving discharges to open country; this will be difficult, and solutions to the technical problems entailed are not immediately obvious. In the meantime, it appears appropriate to adopt the suggested log-normal statistical distribution of the total action integral as supplemental to the distributions already presented in Figures 21 through 31. All the distributions
are then reasonably self-consistent and in good agreement with experimental data.

F. Example of Assessment of Lightning Hazard

A combination of the climatological data presented in Section II, the statistical information of Section III, and the equipment-sensitivity considerations discussed above, enables the lightning hazard to be estimated for specific circumstances.

Suppose we consider the example of Section II-F—namely, a 100-m-high tower located at Grand Forks, North Dakota. Its effective attractive area is $0.4 \text{ km}^2$ and it is struck on the average by about one flash in every two years (Section II-F). Suppose equipment is installed at the tip of the tower (the likely point to be struck); suppose further that this equipment is somewhat similar to that considered in Section IV-D and is sensitive either to peak currents exceeding 200 kA or to total charge transfers exceeding 200 C. About 0.7 percent of flashes to ground (since the peak current tends to be in the first stroke) fit the former criterion (Figure 24), while the latter condition involves perhaps 2 percent of the discharges (Figure 25). Since peak current and total charge transfer are almost unrelated, the percentages, being small, are essentially additive. Thus about 2.7 percent (or, in round figures, one in forty) of the discharges to the tower would influence the equipment. If the lifetime of the equipment on the tower is eight years, it follows that there is a 10-percent chance of the installation being affected by lightning during this period.

It is perhaps appropriate to point out that in most practical circumstances the lightning hazard is much more controlled by the effective attractive area for lightning, or in other words the degree of exposure, than it is by the extremes of the lightning statistics. It follows that
lightning hazards are usually more easily minimized by decreasing the
degree of exposure (avoiding installations on high structures, supplying
protective lightning-conducting shields, and so on) than by fitting
auxiliary devices such as lightning arrestors. If, for instance, our
equipment of the last paragraph had been mounted flush with a flat ground
surface, and it covered an area of $4 \text{ m}^2$, then, even without any additional
protective arrangements, the chance of the equipment being affected by
lightning during the eight-year life period would be only 1 in $10^6$. 
Most flashes that do not reach the ground are of the intracloud type occurring entirely within the thundercloud. Cloud-to-cloud and air flashes are rare. It is also probable that their electrical characteristics do not differ significantly from those of intracloud discharges; consequently, their separate consideration is unnecessary.

Intracloud flashes and discharges to earth have considerable similarities—notably, the total duration, the total charge involved as indicated by observations of electric-moment changes, and the length of the discharge channel.

It is believed that an intracloud discharge consists basically of a probing leader–streamer that attempts to bridge the gap between—and thus discharge—the main charge centers in the thundercloud. The leader–streamer advances fairly steadily, carrying a comparatively continuous current. Occasionally the leader encounters a localized concentration of opposite charge; such an encounter generates a streamer recoiling along the channel preionized by the leader and causing the phenomena known as K-effects (Section III-A). However—and this is the salient difference between flashes to ground and intracloud discharges—there is never any encounter with a charged surface of extent and conductivity comparable with that of the earth. Consequently, no very rapid neutralization of charge can occur, so that the K-current surges attain peaks only an order of magnitude less than those of the return strokes.

There is a reasonable amount of information in the literature regarding K changes. Some of this is conflicting. However, within orders of magnitude, K phenomena involve a time of a millisecond, a recoil
length of a kilometer along the channel, an average current of a kilo-
ampere, and a charge of a coulomb. They occur at intervals of some 10
ms, and tap local charge centers 100 m apart.

Statistical distributions of intervals between K changes have been
given by Takagi$^{98}$ and by Kitagawa and Brook$^{94}$; these are plotted in
Figure 42. We note that, fortuitously enough, the median value (6 ms)
of the interval is 10 percent of that (60 ms) between return strokes
(Figure 23). Furthermore, the slopes of the distributions for return-
stroke intervals and K-change intervals are quite similar (Figures 12,
23, and 42). It follows that the distribution of K-change intervals is
well described by the log-normal law of Eq. (20), with the median being 6 ms. The standard deviation expressed in dB relative to the median is the same for both return-stroke and K-change intervals.

The current flowing in a K recoil must be almost entirely deduced from indirect observations. There are a few direct measurements indicating that the peak currents in strikes involving aircraft (these will usually be intracloud discharges) are normally less than a few thousand amperes. However, the only extensive information regarding K effects is indirect. Arnold and Pierce have presented some statistics originally obtained by Kitagawa that relate the size of the VLF pulse radiated by a K change to that originating in a return stroke. Since the statistics of the peak currents in return strokes are well established, this information of Kitagawa's can be manipulated so as to derive a distribution for peak currents in K changes. The result obtained is shown in Figure 43. Note that—as in the case of the intervals—we again have a very convenient relationship. The median K current is 2 kA—that is, 10 percent of the median in first-return strokes (Figure 24). Also the slopes of the distributions are similar, thus indicating that the log-normal representation applies with a standard deviation of 8 dB relative to the median.

The form of the current-time curve in a K change is even more uncertain than the peak reached by the current. However, Pierce, in a study supported by the Office of Naval Research and published only in summary, has conducted a critical review of K effects. He has been able to reconcile much of the conflict in the information regarding K changes derived from examinations of the time variations of the luminosity, electrostatic field-change, and radiated field. Pierce has deduced that the current, $i_{K't'}$, in a K change can be represented as a function of time by
where $i_t$ is in kiloamperes. Equation (21) is plotted in Figure 44 for comparison with the return-stroke currents; the peak K current is about 2 kA. Pierce has also derived an expression for the velocity $v_t$ of the recoil streamer generating the K effects. This is
\[
V_t^K = 2 \times 10^7 \exp(-4 \times 10^4 t) + 10^6 \exp(-2 \times 10^3 t)
\]  
(22)

where the unit of velocity is m/s.

We are now in a position to deduce a typical model for an intra-cloud discharge in temperate latitudes. We take the total change in electric moment as 120 C-km. The duration of the flash is 180 ms; thus it may be assumed that 30 K changes, each separated on the average by 6 ms, occur in the discharge. It can be shown, by using Eqs. (21) and (22), that the electric-moment alteration due to each K change is 1 C-km.
Thirty K changes thus produce 30 C-km, leaving 90 C-km to be accounted for by continuing currents. The steady current in the probing leader advances the leader over a total vertical distance of 4 km in 180 ms and thus at a mean speed of about $2.2 \times 10^4$ m/s. The average channel length involved is 2 km, so that 90 C-km change in electric moment implies a charge of 22.5 C and a continuing current of approximately 125 A for the time of 180 ms. Note that this value of continuing current is quite close to the median—140 A—of Figure 30.

Our typical intracloud discharge may thus be defined in terms of current-time history as consisting of a steady continuing current of 125 A lasting for 180 ms. Superimposed on this steady current are 30-K surges whose current-time history is given by Eq. (21) and for which the median peak current is 2 kA. With 30 surges it is to be expected that one surge will crest at 12 kA (the 3-percent point of Figure 43). The continuing current characteristics are quite similar for intracloud discharges and flashes to earth. However, the current surges in the latter case are appreciably more severe.
VI RADIATED AND STATIC FIELDS

A. General

It is well known that lightning discharges produce electric and magnetic fields that vary with time, frequency, and distance. The characteristics of these fields are important since they can contribute to the sensitivity of a given system. For example, the radiated fields can couple into and interrupt or produce errors in computers, cause failures in electronic circuitry (particularly solid-state devices) and produce noise in communications equipment.

As already described (Section III and Section V), a lightning flash consists of various stages, with each stage having characteristic currents. Only the high-magnitude currents have been considered hitherto and—as will appear later--these currents are dominantly responsible for the radiated fields only at frequencies less than about 100 kHz. At HF and VHF a multiplicity of small current sparks, for which no plausible models exist, are the main contributors to the radiated signals.

The fields produced by lightning are different for each stage, and very complex in many respects. A considerable amount of effort has gone into their understanding and interpretation, since a large portion of our knowledge of lightning processes is based on a variety of electric- and magnetic-field measurements. In general, the fields produced by lightning consist of the far fields or radiated components, and the near fields or static and induction components. Analytically, these far- and near-field components are shown in Eq. (10). The first term of $E_t$ is the static field, the second is the induction term, and the third is the radiation field. It can be seen from Eq. (10) that the relative...
contribution of these components to the total field is dependent on the frequency and distance from the source. If $E_t$ were expressed in the frequency domain, then the three components would be equal when $d = c/(2\pi f)$. Generally speaking, for distances greater than 15 km and frequencies exceeding 3 kHz, the radiated fields are dominant. Reviews of the radiated fields have been presented by Horner and by Oetzel and Pierce. Extensive references to electrostatic field measurements will be found in standard texts. The following subsection summarizes the characteristics of the radiated and static-field components.

B. Radiation (Far) Fields

The structure of the electromagnetic radiation from lightning varies with frequency and time, as schematically illustrated in Figure 45. For this analysis, only the fields for close (within ~100 km) lightning are presented; propagational degradation is therefore not considered. The electric fields for a typical ground and cloud flash are shown, illustrating the relative magnitudes of the fields at various frequencies. At very low frequencies, VLF (3 to 30 kHz), the pulses are discrete and are generated principally by the return stroke and/or reooil streamers (K changes). As the frequency increases, the number of pulses per flash also increases, with a maximum of about $10^4$ per discharge for frequencies between 30 to 300 MHz (VHF); the disturbance accompanying the flash is then quasi-continuous. These pulses appear to be associated with the initial leader, including its steps, and also with the electrical breakdown processes accompanying probing leaders moving within the cloud. These probing leaders can occur, for a flash to earth, between return strokes or after the final stroke; for an intracloud discharge their presence is possible at almost any stage of the discharge. We note the interesting feature that the signals at HF and VHF associated with return strokes and K changes are not strong, and are indeed partly "quenched"
following the occurrence of return strokes and K changes. It is believed that this quenching is due to a temporary absence of probing leaders. As frequency is further increased beyond the VHF range, there is a sharp decrease in the number of pulses until at centimetric wavelengths (~GHz) the pulses are again well separated and associated with the macroscopic features of the return strokes.

Peak pulse amplitudes are reached, for a flash to earth, at frequencies of about 5 kHz. With increasing frequency up to about $10^4$ MHz there is a general decrease in amplitude which approximately follows an inverse frequency dependency. However, over substantial sections of the spectrum between 10 kHz and $10^4$ MHz, there are probably appreciable deviations from this simple law. Figures 46 and 47 illustrate the spectral
characteristics of the radio emissions from close lightning. The results—which present information from several sources—have been normalized to a distance of 10 km. Figure 46 represents the amplitude spectrum, $S(f)$, of the return-stroke signals, while Figure 47 is the peak amplitude, $e_p$, for a receiver of bandwidth 1 kHz and for all lightning-generated emissions. The connection between $S(f)$ and $e_p$ is complicated, as described by Horner, who has derived a relation between $S(f)$ and $e_p$ for a narrow-bandwidth receiver. For illustrative purposes, though, Figures 46 and 47 can be approximately interconnected if the
ordinate scale of Figure 46 is multiplied by $10^3$, as shown by the solid curve in Figure 47. It should be noted that for frequencies below 100 kHz where the signals are discrete pulses, the field is scaled linearly with bandwidth. In addition, Figure 46 represents the spectrum of the return-stroke pulse; at 5 kHz this exceeds the spectrum of a K-change pulse by more than an order of magnitude. But at 100 kHz the two spectra are more comparable.

Analytic models relating the frequency dependence of the radiated fields to the physical processes are not well developed for the entire spectrum. However, at frequencies less than 100 kHz, several models
describing the radiation from the return stroke are available. Despite these shortcomings, a model relating the field strength over a wide range of frequencies is often required. An empirical relation between the peak field strength, $e_p$, and the frequency, based on an equation previously derived for the VLF range, is

$$\log \frac{e_p}{e_0} = -\log \frac{f}{f_m}$$

for $f \geq 1$ kHz

where $e_p = 2 \times 10^6 \mu V/m$ in a 1-kHz bandwidth and the frequency mode is $5 \times 10^{-3}$ MHz, the frequency, $f$, being expressed in megahertz. Then $e_p$ is the field strength in $\mu V/m$ in a 1-kHz bandwidth. Equation (23) is shown in Figure 47. It should be noted that the model for $e_p$ represents the average or typical field strength. As with the previous lightning parameters described in Sections III through V, $e_p$ has a statistical behavior generally obeying a log-normal distribution with a standard deviation of about 6 dB relative to the mean. Again, Eq. (23) is only an analytical tool and does not imply any physical justification. The radiation fields are scaled linearly with distance for distances exceeding 10 km, as square root of the bandwidth for frequencies greater than 100 kHz, and linearly with bandwidth for frequencies less than 100 kHz.

The behavior of the radio emissions from close lightning may be summarized by stating that a multitude of subsidiary sparks of many different types are involved: the larger the current peak in a given type of spark, the longer the energized channel, the lower the frequency at which peak signal is radiated, and the less frequent the occurrence of the particular kind of spark. High-current channels tend to be orientated vertically (especially the return stroke); minor subsidiary discharges are, however, much more randomly disposed.
It cannot be overemphasized that a lightning flash involves this multiplicity of sparks and consequently a protracted and complicated generation of radio signals. It is still a common misconception that the lightning discharge occurs as one single, large spark, and that therefore all radio emissions are produced almost simultaneously as in the case of the nuclear electromagnetic pulse (EMP). In reality the time histories of the lightning emissions and of the EMP are quite different, and comparisons of equipment response to the two types of signals should entail intelligent recognition of this fact. For example, at HF the EMP would generate one very large pulse, while a typical lightning flash might create ten thousand small pulses. Designs of equipment can be conceived that would be uninfluenced by the single large pulse but could be affected by the repetition of the small pulses.

Figures 46 and 47 have been normalized to a distance of 10 km, and it has been indicated that a scaling inversely with distance is appropriate for greater distances. It is, however, a very dangerous procedure to apply an inverse scaling to distances appreciably less than 10 km. For example, at VLF the lengths of the radiating channels are an appreciable fraction of 10 km and there is consequently no orderly change in magnitude with distance at the closer ranges. Also, it appears that the subsidiary sparks providing most of the radiation at HF and VHF are usually located within the thundercloud; consequently, they are seldom very close to ground equipment. Finally, as already indicated and further discussed in the next section, the static (near) field will dominate at close distances and low frequencies, so that scaling of the radiation field alone can be very misleading.
C. Static (Near) Fields

When the distance to the lightning discharge becomes small (less than about 15 km), the static and inductive field components or near fields become important, largely because these fields in general are much greater than the field contribution from the radiation component. As previously mentioned, Eq. (10) describes the basic character of these fields; however, as the distance to the flash becomes comparable to its dimensions (~3 km), Eq. (10) is no longer a valid approximation in describing the fields, largely because the moment, \( M_t \), is difficult to define at close distances. However, approximations to the near fields can be obtained as summarized below.

Beginning with the magnetic field, the peak fields are produced during the current surge of the return strokes. For distances very close to the lightning channel, the magnetic field can be approximated by a long current-carrying conductor given as

\[
H \approx \frac{I}{2\pi D} \text{ A/m}
\]

(24)

where the current, \( I \), is in units of amperes and the distance from the channel, \( D \), is in meters. Then, for a typical stroke having a peak current of 20 kA and at a distance of \( D = 100 \) m, the magnetic field is about 32 A/m. As the distance \( D \) increases, the channel length, \( L \), must be taken into consideration, and the field is approximately given as

\[
H = \frac{I}{2\pi D} \frac{L}{\left(L^2 + D^2\right)^{1/2}}
\]

(25)

In most models of the ascending return stroke it is assumed that the current is uniform between ground level and the tip of the advancing return stroke. There is no direct evidence supporting this deduction of
uniformity but there are some indications that, if anything, the current averaged along the return-stroke channel is less than that entering the base of the channel. Analytic representations\textsuperscript{30} of the fields produced by a return stroke usually consider that a channel of increasing length is energized uniformly by a current having the double exponential time characteristic [Eq. (11)].

The conventional approach leads to the interesting implication that the peak return-stroke current is never experienced at appreciable heights above the ground. This point is easily illustrated by defining a time schedule for the return stroke. The velocity $V_r$ of the ascending return stroke in m/s is approximately represented\textsuperscript{6,30} by

$$V_r = 10^8 \exp(-2.5 \times 10^4 t)$$

for a first return stroke, and may be taken as constant at $10^8$ m/s for a subsequent stroke. Integration of Eq. (26) yields a limiting channel length of 4 km. Using Eq. (11) for the time history of a typical current surge and the velocity information allows us to construct Table 8. We note that the channel length is only about 150 m at the time of peak current. When $D$ is much less than $L$, Eq. (25) approaches the limiting form of Eq. (24) but for $D \gg L$ we have

$$H \approx \frac{IL}{2\pi D^2}$$

It can be seen that with Eq. (24) the time variation of $H$ is similar to that of $I$. However, at appreciable distances [Eq. (27)] the time variation of $H$ involves that of both $I$ and $L$, so that the maximum in $H$ is reached considerably later. We note (Table 8), for instance, that the product $IL$, and therefore $H$, are much greater at the half-value time (40 $\mu$s) than they are at the time (1.5 $\mu$s) of peak current. Table 9
<table>
<thead>
<tr>
<th>Channel Length (km)</th>
<th>Time After Start (ms)</th>
<th>Charge Involved (C)</th>
<th>Current (kA)</th>
<th>Line Charge Density (C/km)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>First Stroke</td>
<td>First Stroke</td>
<td>Subsequent Stroke</td>
</tr>
<tr>
<td>0.5</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>1.0</td>
<td>0.05</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>1.5</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>2.0</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>5.0</td>
<td>0.47</td>
<td>0.47</td>
<td>0.47</td>
<td>0.47</td>
</tr>
<tr>
<td>10.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>20.0</td>
<td>1.6</td>
<td>1.6</td>
<td>1.6</td>
<td>1.6</td>
</tr>
<tr>
<td>40.0</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Table 8: Time Schedule for Return Strokes
Table 9

STATIC MAGNETIC FIELDS FROM CLOSE LIGHTNING

<table>
<thead>
<tr>
<th>Peak Current (kA)</th>
<th>Static Magnetic Field (A/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10 m from Flash</td>
</tr>
<tr>
<td>10</td>
<td>$1.6 \times 10^2$</td>
</tr>
<tr>
<td>20</td>
<td>$3.2 \times 10^2$</td>
</tr>
<tr>
<td>30</td>
<td>$4.8 \times 10^2$</td>
</tr>
<tr>
<td>70</td>
<td>$1.1 \times 10^3$</td>
</tr>
<tr>
<td>100</td>
<td>$1.6 \times 10^3$</td>
</tr>
<tr>
<td>140</td>
<td>$2.2 \times 10^3$</td>
</tr>
<tr>
<td>200</td>
<td>$3.2 \times 10^3$</td>
</tr>
</tbody>
</table>

lists the maximum magnetic fields as calculated from Eq. (24) for distances of 10 and 100 m and various values of peak current. It can be seen from Table 8 that at the half-value of peak-current time (40 $\mu$s) the channel length for a first stroke is about 2.5 km. The last column of Table 9 shows the fields calculated at this time for a distance of 10 km and corresponding to the indicated values of peak currents.

Unlike the magnetic fields, where the peak fields are determined by the return-stroke currents, the electric fields are a function of the interplay between the charge drawn from the thundercloud, the charge deposited along the leader, and the redistribution of charge during the various current stages of the flash. Again, the fields can be simply modeled. The electric-field change produced by the leader is approximately given as
where $D$ is the distance (along the ground) from the leader channel and $L$ is the channel length (typically $\sim 3$ to $4 \text{ km}$). As the leader moves toward the ground, its height above the ground is $h$; then when the leader makes electrical contact to the ground, $h = 0$, and Eq. (28) can be approximated as

$$E \approx - \frac{\rho_L}{2\pi \varepsilon_0} \left[ \frac{1}{\sqrt{h^2 + D^2}} - \frac{1}{\sqrt{L^2 + D^2}} - \frac{(L - h) L}{(L^2 + D^2)^{3/2}} \right]$$

$$\varepsilon_0 = \frac{1}{36\pi} \times 10^{-9} \text{ f/m}$$

The line charge density, $\rho_L$, along the leader channel will not usually be uniform. Various models for the distribution of leader charge exist, but there is no generally accepted representation. However, it is agreed that the average charge density deposited along the leader is on the order of $10^{-3} \text{ C/m}$ or $1 \text{ C/km}$. Using this value for $\rho_L$, at a distance of $100 \text{ m}$ the electric field produced is about $2 \times 10^5 \text{ V/m}$ just before the leader makes contact with the ground. This calculation is in reasonable agreement with observations of the electrostatic fields due to very close lightning flashes. These show that the field variations approach $100 \text{ kV/m}$ in amplitude. In a realistic ground environment it should be remembered that there will be screening space charges near the earth due to corona; these space charges are not considered in Eqs. (28) and (29). The existence of the space charges usually limits the steady field at the ground below a thundercloud existing between flashes to about $10 \text{ kV/m}$, and appreciably larger fields are only recorded transiently during close discharges before readjustment of the space charge can occur. For
comparison purposes it may be noted that the fair-weather electrostatic field is about 100 V/m.

After electrical contact is made with the ground, the ascending return stroke then produces an electric field approximately given as

$$E_R = -\frac{\rho_R}{2\pi \varepsilon_0} \left[ \frac{1}{D} - \frac{1}{\sqrt{h_R^2 + D^2}} \right]$$

(30)

where $h_R$ is the length of the return-stroke channel measured from ground level. We assume again that the charge density $\rho_R$ along the return stroke is uniform. Values of $\rho_R$ can be obtained by dividing the charge passing into the return-stroke channel by the channel length; the charge follows, of course, from integrating the current-time curve. Some values of $\rho_R$ derived in this manner are included in the last columns of Table 8. We note that $\rho_R$ is in general less than the typical value—1 C/km—of $\rho_L$. This is consistent with the generally accepted concept that the return stroke in its initial ascent neutralizes only the charge residing on the leader core. One consequence is that the return stroke in its first main current surge produces a somewhat smaller change in field than that involved in the leader stage; however, the return-stroke change occurs much more rapidly.

It should be noted that for a point on the ground immediately below (zero distance) a vertical leader the field change monitored with the leader stage will be entirely negative (according to the usual convention of atmospheric electricity), and the field change accompanying the return stroke will be positive. Because of the well-known reversal-distance effect, the leader field-change will become more positive as distance increases, with the effect becoming initially apparent for the part of the leader just preceding the return stroke. At distances exceeding some 10 km the entire field change in all stages of the discharge—leader,
return strokes, continuing currents, and final stage—is positive. Typically, at 10 km the total field change summed for all stages is about 1000 V/m.\textsuperscript{22}

The values for electrostatic fields given above are all for average conditions. For the more extreme end of the statistical distribution, calculated electrostatic fields due to either the leader or the return-stroke stages (current \textasciitilde 200 kA) would be larger by an order of magnitude, indicating fields approaching 10\textsuperscript{6} V/m at 100 m. It is doubtful, however, that such high fields are ever actually attained, since local breakdown, in the form first of corona and then of upward leaders, would inhibit the development of the high ground-level fields by providing screening space charge.

In summary, the fields produced by close lightning are very large in comparison to the radiation fields described in the previous section. Thus, in applications for determining system sensitivity, it is very important to distinguish between the near- and far-field effects on the system and to determine which of the fields is the more important.
VII DISCUSSION

The information presented on lightning has been almost entirely deduced from ground observations. Consequently, all the models for flashes to earth represent essentially the current-time and charge-transfer histories at the point where the discharge contacts the ground. Thus they can be confidently applied in the case of ground-located equipment. However, the models do not accurately represent the situation for objects such as aircraft and rockets that may be struck while in free flight. Fortunately, the deviations are all such (under most practical conditions that can be envisaged) as to reduce the severity of the lightning exposure. Thus if the models are applied to flight conditions the bias is toward excessive caution rather than overconfidence.

Objects in flight may either be struck by flashes within the cloud or to ground, or may, by their presence, trigger such discharges. Whatever the manner in which the flash is initiated, the object becomes part of the lightning channel, with the lightning current having exit and entry points (usually located at extremities) on the object. We note from Table 8 that—according to conventional views of lightning—the crest of the current surge in the return stroke as measured at ground level is not experienced more than 150 m above the earth. Thus, for example, a rocket in free flight at altitudes greater than this, and that becomes part of a flash to earth, will presumably never encounter a current as large as that occurring at ground level. A somewhat similar situation will occur for intracloud discharges (Section V). The length of the K recoils is limited, so that an aircraft encountering an intracloud discharge will experience many fewer K surges than the number (30) specified.
in the model of Section V; furthermore, of the K surges experienced, most will have currents less than the peak occurring early in the recoil.

A major objective of this report has been the development of physically plausible models for lightning. Gordon\textsuperscript{107} has presented some models previously, and it is of interest to compare the present work with his. The comparison is not readily made in some respects, since Gordon, probably for analytical convenience, chose to regard every return stroke as consisting of a main surge followed by a slowly decaying low-level continuing current. He considered the stroke to be essentially terminated when the value of the continuing current dropped below one ampere; it is doubtful that this criterion is applicable, since there is evidence that the conductivity of the channel does not persist for currents less than 10 amperes.\textsuperscript{64} In our modeling we have regarded each return stroke as containing a main surge that is sometimes succeeded by an intermediate current; the continuing current has subsequently been modeled as a separate entity essentially independent of the return-stroke history.

Table 10 presents the comparison between our models and those of Gordon. Most of the entries tabulated under Gordon have been deduced from his analytical representations. Note that the typical models are founded on average and/or median values of the lightning parameters. For the "severe" models SRI chose the critical parameter values on the basis of their being passed in 2 percent of cases; Gordon's criterion was 10 percent. The extreme model due to Gordon may be compared with the limits indicated in Table 3 and in Figures 21 through 31.

We do not intend to discuss the discrepancies in the various entries of Table 10 in any detail. However, it is important to point out those differences that are most likely to be of practical significance. The times to peak given by SRI and largely based on the recent work of Uman and others (Section III) are appreciably less than those listed by Gordon,
Table 10

COMPARISON OF BASIC MODELS FOR LIGHTNING

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Typical Models (Criterion ~60 Percent)</th>
<th>Severe Models</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SRI (Temperate)</td>
<td>SRI (Severity Criterion ~2 Percent)</td>
</tr>
<tr>
<td></td>
<td>No Continuing Current [Table 5(a)]</td>
<td>With Continuing Current [table 5(b)]</td>
</tr>
<tr>
<td>Number of strokes</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>Time intervals between strokes (ms)</td>
<td>60</td>
<td>10 to 300</td>
</tr>
<tr>
<td>Peak current--first stroke (kA)</td>
<td>20</td>
<td>140</td>
</tr>
<tr>
<td>Time to peak--all strokes (µs)</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Time to half-current--all strokes (µs)</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Peak current--subsequent strokes (kA)</td>
<td>10</td>
<td>30 to 70</td>
</tr>
<tr>
<td>Amplitude of continuing current (A)</td>
<td>150</td>
<td>~1 to 110</td>
</tr>
<tr>
<td>Duration of continuing current (ms)</td>
<td>100</td>
<td>400 and 200</td>
</tr>
<tr>
<td>Charge passing in continuing current (C)</td>
<td>0</td>
<td>300 and 160</td>
</tr>
<tr>
<td>+ 1n s rₚₚ stroke (C)</td>
<td>0.5, 3</td>
<td>120 and 32</td>
</tr>
<tr>
<td>L. tₚₚ charge in flash (C)</td>
<td>4.5 to 5</td>
<td>2 to 12</td>
</tr>
<tr>
<td>Flash duration (s)</td>
<td>0.12</td>
<td>200</td>
</tr>
<tr>
<td></td>
<td>0.16</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>~0.13</td>
</tr>
</tbody>
</table>
but the values of the peak currents are comparable. It follows that we feel that use of the Gordon models is likely to underestimate the potential lightning threat to equipment sensitive to rate of current rise. Times to half-value are somewhat longer in the Gordon rather than the SRI approach. However, this parameter is not of intrinsic practical importance; rather its significance is in determining the charge transfer per stroke. The additional intermediate current incorporated in the SRI approach results indeed in the stroke-charge transfer being if anything greater than with the Gordon models; this is not what would be expected from the differences in times to half-value. The most serious discrepancies in Table 10 relate to the continuing currents; we believe that the Gordon models considerably underestimate the charge transferred in these currents, and its often major contribution to the total charge passing in the discharge. Application of the Gordon models to equipment sensitive to charge transfer in low-level currents may well lead to a sense of euphoria unjustified by practical realities.

Finally, in Table 11, we compare some information used by McDonnell-Douglas in equipment applications, with the SRI Severe (Applied) Model of Table 7. The McDonnell-Douglas work was founded on the range of parameter values specified in the last column of Table 11. This range is in reasonably good agreement with Table 3 and the statistics of Figures 21 through 31. The analytical model in the first column of Table 11 was based on developments of Gordon's work. We note that its main deviations from the SRI Severe Model are in a much shorter interval between strokes and a much longer time to peak current; less important differences are in the distribution of total charge transfer passing in the continuing currents and flowing in return strokes. We have already emphasized our belief in typical times to peak of 1 or 2 $\mu$s; hence, we feel that misleading results would be obtained if the analytical model of Column 1, Table 11 were applied to equipment sensitive to rate of rise. Equally,
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Severe Analytical Model First Used by McDonnell-Douglass (Developed from Gordon)</th>
<th>SRI Severe (Applied) Lightning Model (Table 7)</th>
<th>Range of Values Specified in STS 108</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of strokes</td>
<td>Adopted Value: 4, Chance of Adopted Value being Exceeded or Not: 50.0 (%)</td>
<td>Adopted Value: 3, Chance of Adopted Value being Exceeded or Not: 60.0 (%)</td>
<td>1 to 34</td>
</tr>
<tr>
<td>Time intervals between strokes (ms)</td>
<td>5, Chance of Adopted Value being Exceeded or Not: 99.6 (%)</td>
<td>60, Chance of Adopted Value being Exceeded or Not: 60.0 (%)</td>
<td>5 to 500</td>
</tr>
<tr>
<td>Peak current--first stroke (kA)</td>
<td>200, Chance of Adopted Value being Exceeded or Not: 0.7 (%)</td>
<td>200, Chance of Adopted Value being Exceeded or Not: 0.7 (%)</td>
<td>2 to 200</td>
</tr>
<tr>
<td>Time to peak--all strokes (µs)</td>
<td>-10, Chance of Adopted Value being Exceeded or Not: 99.3 (%)</td>
<td>1.5, Chance of Adopted Value being Exceeded or Not: 50.0 (%)</td>
<td>1 to 15</td>
</tr>
<tr>
<td>Time to half-current--all strokes (µs)</td>
<td>-100, Chance of Adopted Value being Exceeded or Not: 90.0 (%)</td>
<td>40, Chance of Adopted Value being Exceeded or Not: 50.0 (%)</td>
<td>10 to 120</td>
</tr>
<tr>
<td>Peak current--subsequent strokes (kA)</td>
<td>200, Chance of Adopted Value being Exceeded or Not: 99.3 (%)</td>
<td>100, Chance of Adopted Value being Exceeded or Not: 0.7 (%)</td>
<td>None specified</td>
</tr>
<tr>
<td>Amplitude of continuing current (A)</td>
<td>Order of 100 A, Chance of Adopted Value being Exceeded or Not: 70.0 (%)</td>
<td>400, Chance of Adopted Value being Exceeded or Not: 4.0 (%)</td>
<td>None specified</td>
</tr>
<tr>
<td>Duration of continuing current (ms)</td>
<td>~550, Chance of Adopted Value being Exceeded or Not: 99.5 (%)</td>
<td>400, Chance of Adopted Value being Exceeded or Not: 2.0 (%)</td>
<td>None specified</td>
</tr>
<tr>
<td>Charge passing in continuing current (C)</td>
<td>56, Chance of Adopted Value being Exceeded or Not: --</td>
<td>160, Chance of Adopted Value being Exceeded or Not: ~0.7 (%)</td>
<td>--</td>
</tr>
<tr>
<td>Charge per stroke (C)</td>
<td>43 (including continuing current), Chance of Adopted Value being Exceeded or Not: 28.0 (%)</td>
<td>20 (max), Chance of Adopted Value being Exceeded or Not: 2.0 (%)</td>
<td>None specified</td>
</tr>
<tr>
<td>Total charge in flash (C)</td>
<td>175, Chance of Adopted Value being Exceeded or Not: 97.0 (%)</td>
<td>200, Chance of Adopted Value being Exceeded or Not: 98.0 (%)</td>
<td>5 to 200</td>
</tr>
<tr>
<td>Flash duration (s)</td>
<td>0.6, Chance of Adopted Value being Exceeded or Not: 98.0 (%)</td>
<td>0.5, Chance of Adopted Value being Exceeded or Not: 90.0 (%)</td>
<td>0.01 to 1.5</td>
</tr>
</tbody>
</table>
if the equipment were affected by short intervals between strokes, use of the SRI Severe (Applied) Model would be inappropriate. The SRI Severe (Applied) Model was designed (Section IV) for application to a system principally sensitive to the two parameters of peak current and total charge transfer. We note that the values adopted for these parameters in the McDonnell-Douglas First Analytical Model and in the SRI Severe (Applied) Model are in very good agreement. It follows that if either of the two models is used in calculations involving equipment, mainly influenced by the parameters of peak current and total charge transfer, then the results obtained should not differ greatly.
Appendix

THUNDERSTORM-DAY DATA

Table A-1 lists the monthly values of thunderstorm days ($T_m$) for a selection of localities within the continental United States. The data were either obtained directly from tables\(^1\),\(^2\) or interpolated from maps\(^3\),\(^4\).

Table A-2 gives estimates of the lightning-flash densities, $\sigma_m$ (flashes per km\(^2\) per month) corresponding to the values of $T_m$ listed in Table A-1. The estimates were derived by using Eq. (1),

$$\sigma_m^2 = a T_m^m + a T_m^{-2}$$

where $a = 3 \times 10^{-2}$ due to Pierce\(^5\) and Eq. (2),

$$\sigma_m = 0.06 T_m^{1.5}$$

developed by Westinghouse\(^6\); these two derivations are identified respectively by P and W.

Equations (1) and (2) do not yield simple relationships between the annual flash densities ($\sigma_y$) and number of thunderstorm days ($T_y$), since the connection depends on the distribution of thundery activity (individual values of $T_m$) through the year. However, some idea of the association between $\sigma_y$ and $T_y$ is often desirable. Accordingly, values of $T_y$ and $\sigma_y$ [obtained by using Eqs. (1) and (2)] are tabulated in Table A-2 and plotted on Figure A-1 for the selected U.S. stations. Also shown on Figure A-1 are two straight-line relationships that can be derived—using
# Table A-1

MONTHLY THUNDERSTORM-DAY DATA FOR SELECTED U.S. STATIONS

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>47°33' N 47°30' N 47°30' N 38°43' N 38°51' N 28°33' N 38°49' N 30°18' N</td>
<td>97°5' W 111°20' W 93°33' W 77°02' W 81°20' W 104°42' W 97°42' W</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>January</td>
<td>&lt;0.5 0.0 0.0 0.4 &lt;0.5 1 0 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>February</td>
<td>0 0.0 0.5 0.7 1 &lt;0.5 0 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>March</td>
<td>&lt;0.5 0.0 0.5 3.1 1 3 1 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>April</td>
<td>1 0.3 1.0 6.8 2 6 3 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>May</td>
<td>2 3.2 4.0 8.8 5 8 10 7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>June</td>
<td>3 6.7 6.9 8.6 7 16 12 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>July</td>
<td>2 6.4 9.0 9.0 9 23 18 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>August</td>
<td>2 5.0 6.0 7.3 6 18 15 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>September</td>
<td>1 1.3 2.0 4.8 3 11 7 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>October</td>
<td>&lt;0.5 1.0 0.5 3.9 1 3 2 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>November</td>
<td>0 0.0 0.0 1.3 &lt;0.5 1 0 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>December</td>
<td>&lt;0.5 0.0 0.0 0.4 &lt;0.5 1 0 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Days per year (T&lt;sub&gt;y&lt;/sub&gt;)</td>
<td>11 23.8 30.5 55 35 91 68 42</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table A-2
LIGHTNING-FLASH DENSITIES PER MONTH FOR SELECTED U.S. STATIONS

<table>
<thead>
<tr>
<th>Month</th>
<th>Spokane P</th>
<th>Grand Forks P</th>
<th>Great Falls P</th>
<th>Whiteman P</th>
<th>Washington, D.C. P</th>
<th>Orlando P</th>
<th>Colorado Springs P</th>
<th>Austin, Texas P</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>W</td>
<td>W</td>
<td>W</td>
<td>W</td>
<td>W</td>
<td>W</td>
<td>W</td>
<td>W</td>
</tr>
<tr>
<td>January</td>
<td>0.1</td>
<td>0.01?</td>
<td>0</td>
<td>0</td>
<td>0.1</td>
<td>0.01?</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>February</td>
<td>0</td>
<td>0</td>
<td>0.02</td>
<td>0.03</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
<td>0.01?</td>
</tr>
<tr>
<td>March</td>
<td>0.1?</td>
<td>0.01?</td>
<td>0.1</td>
<td>0.02</td>
<td>0.4</td>
<td>0.3</td>
<td>0.4</td>
<td>0.3</td>
</tr>
<tr>
<td>April</td>
<td>0.2</td>
<td>0.1</td>
<td>0.2</td>
<td>1.4</td>
<td>1.1</td>
<td>0.3</td>
<td>1.2</td>
<td>0.9</td>
</tr>
<tr>
<td>May</td>
<td>0.3</td>
<td>0.2</td>
<td>0.4</td>
<td>0.3</td>
<td>2.3</td>
<td>0.8</td>
<td>1.9</td>
<td>1.4</td>
</tr>
<tr>
<td>June</td>
<td>0.4</td>
<td>1.4</td>
<td>1.0</td>
<td>1.5</td>
<td>1.1</td>
<td>1.5</td>
<td>1.6</td>
<td>1.1</td>
</tr>
<tr>
<td>July</td>
<td>0.3</td>
<td>0.2</td>
<td>1.3</td>
<td>1.0</td>
<td>2.5</td>
<td>1.6</td>
<td>2.4</td>
<td>1.7</td>
</tr>
<tr>
<td>August</td>
<td>0.3</td>
<td>0.2</td>
<td>0.8</td>
<td>0.7</td>
<td>1.2</td>
<td>0.9</td>
<td>1.2</td>
<td>0.9</td>
</tr>
<tr>
<td>September</td>
<td>0.2</td>
<td>0.1</td>
<td>0.2</td>
<td>0.1</td>
<td>0.3</td>
<td>0.6</td>
<td>0.4</td>
<td>0.3</td>
</tr>
<tr>
<td>October</td>
<td>0.1?</td>
<td>0.01?</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
<td>0.5</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>November</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>December</td>
<td>0.1?</td>
<td>0.01?</td>
<td>0.1</td>
<td>0.01</td>
<td>0.1</td>
<td>0.1</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>o&lt;sub&gt;y&lt;/sub&gt; per year</td>
<td>2.1</td>
<td>1.1</td>
<td>4.4</td>
<td>3.2</td>
<td>6.6</td>
<td>4.5</td>
<td>12.4</td>
<td>8.7</td>
</tr>
<tr>
<td>Thunderstorm days per year (T&lt;sub&gt;y&lt;/sub&gt;)</td>
<td>11</td>
<td>23.8</td>
<td>30.5</td>
<td>55</td>
<td>33</td>
<td>91</td>
<td>68</td>
<td>42</td>
</tr>
</tbody>
</table>

* Pierce model.
† Westinghouse model.
an effective counter range of 15 km—from some Japanese\textsuperscript{17} and some European\textsuperscript{19} results. The Japanese relationship has the equation

\[ \sigma_y = 0.02 T_y^{1.74} \quad (A-1) \]

or, more realistically,

\[ \sigma_y \approx 0.02 T_y^{1.7} \quad . \]

The European data are described by

\[ \sigma_y = 0.007 T_y^{1.999} \quad (A-2) \]
or—even more realistically—by

\[ \sigma_y \approx 0.007 T_y^2. \]

Over the range of \( T_y \) that is of most practical importance (20 to 100), the Japanese, European, and Pierce estimates of \( \sigma_y \) do not differ greatly. However, the Westinghouse equation gives rather lower values of \( \sigma_y \) than do the other relationships.
REFERENCES


51. M. A. Uman, University of Florida (private communication).


108. Mr. A. P. Venditti, McDonnell-Douglas Astronautics (private communication).
The objectives of the report are firstly to survey and to colligate information on the physical characteristics of lightning, and secondly to show how this information can be used by an engineer concerned with estimating the lightning sensitivity of equipment.

Data on lightning incidence are first examined. Expressions are derived relating lightning incidence to the widely available monthly thunderstorm-day statistic to the diurnal variation of activity, and to structure height. By using these expressions the number of lightning strikes to be expected over any period of time can be estimated, on a climatological basis, for a structure of known height located anywhere in the world.

The physical parameters of lightning are then discussed; the parameters considered include—but are not limited to—peak current, time to peak current, rate of current rise, magnitude and duration of continuing currents, total charge transfer, number of strokes, and time between strokes. Median values and statistical distributions for the parameters are deduced; the statistics can usually be conveniently expressed in terms of a log-normal law.

Several models for lightning are derived and expressed in convenient analytical forms. It is emphasized that caution in the derivation process is necessary so as to obtain
<table>
<thead>
<tr>
<th>KEY WORDS</th>
<th>LINK A</th>
<th>LINK B</th>
<th>LINK C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ground-lightning environment</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lightning</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lightning modeling</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lightning climatology</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intracloud lightning</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Radiated and static fields from lightning</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
models that are both physically plausible and internally self-consistent. Two types of models are identified—basic models developed solely from the physical properties of lightning, and applied models modified appropriately for use with equipment, the lightning sensitivity of which is partially defined. Basic models are presented for typical and severe flashes; in the latter case the criterion of the severity for a lightning parameter is taken as approximately the two-percent point on the statistical distribution. An example of an applied model is also given.

The main emphasis of the report is on the direct effects of flashes to ground. However, discussions are also given of the physical characteristics of intracloud discharges, and of the static and electromagnetic fields generated by lightning.