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**AUTHORITY**

USAF/ESD ltr, 13 Jan 1975
LFOV OPTIMIZATION STUDY

FINAL REPORT

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Systems Development Division

For
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Lincoln Laboratory
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Under
Purchase Order No. CC-724
Prime Contractor F19628-72-C-8236
Project No. TA2633041215

This Report Covers the Period
13 September 1971 through 31 January 1972

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ABSTRACT

This report describes the study of a dual reflector microwave antenna system performed by the Westinghouse Electric Corporation for the Massachusetts Institute of Technology, Lincoln Laboratory. The objectives of the study were to: (1) develop a computer program which would define the contours of the reflectors to achieve optimum system gain over a given scan range, and (2) develop a computer program to calculate the secondary patterns and gain of the dual reflector system. The computer programs have been written. Significant improvement in performance of the dual reflector system has been demonstrated. Program results are discussed.

Accepted for the Air Force
Joseph R. Waterman, Lt. Col., USAF
Chief, Lincoln Laboratory Project Office
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1. INTRODUCTION

This document is a condensation of the final report on a 4-month study by the Westinghouse Electric Corporation under contract to the Massachusetts Institute of Technology Lincoln Laboratory.

The purpose of the study was two-fold:

a. Develop a computer program which would define the main and subreflector contours of a limited-field-of-view (LFOV) optical system to produce optimum system gain over a given scan range.

b. Develop a computer program which would calculate the secondary pattern and gain of the LFOV system with an arbitrary given surface contour.

The term, limited-field-of-view (or LFOV), refers to the application of a Gregorian optical concept to a microwave reflector antenna system. The geometry of the LFOV system is illustrated in figure 1. When used with a phased array as shown, the resulting LFOV antenna offers a substantial increase in sensitivity (gain) over the array alone at the expense of restricted scan performance.

The present study dealt with the specific problems of the LFOV mode of operation. A computer program was written which determines the reflector and subreflector contours to minimize the reduction in gain over a given scan range. A computer program was written which calculates the secondary pattern and gain for arbitrary given reflector contours.

The approach to the reflector surface optimization is as follows. The main reflector and subreflector are described as polynomials in Cartesian coordinates. The coefficients of these polynomials are treated as variables to be solved for in the optimization process. A series of output scan angles
and a series of points on the main reflector are selected, and for each point and scan angle, the amplitude and phase error are determined. The amplitude portion of the optimization operates to minimize the rms mapping error, at the subreflector, between two waveforms. One waveform is the array distribution translated upon the input scan angle to the subreflector, and the second is a signal from space, reflecting off the main reflector to reach the subreflector. The phase portion of the optimization operates to minimize the rms error in path length.

The pattern calculation program utilizes a vector diffraction technique along with some judicious geometric optics approximations to compute the phase and amplitude of the currents generated on the subreflector by the array, on the main reflector by the subreflector, and finally the far-field pattern as formed by radiation off the main reflector.

The theory and the programs are discussed in more detail in Sections 2 and 3.
2. SURFACE OPTIMIZATION METHOD

If we start with a confocal paraboloid system, where the ratio of the focal lengths of the main and subreflectors is equal to the ratio of the main reflector diameter to the array diameter, then at boresight, using geometric or ray optics, the array distribution is mapped perfectly on the projection of the main reflector. This results in patterns from the LFOV system which are similar to those of the isolated array, except for an increase in voltage gain by a factor $M$, and a decrease in the beamwidth by the same factor, $M$ being the ratio of the reflector diameter to the array diameter. As the system is scanned to other angles, however, we get errors both in phase and amplitude, the phase errors due to variations in path length for different rays, and the amplitude errors due to the rays striking the main reflector at the wrong position. Both of these errors cause distortions in the patterns and a reduction in gain. The goal of the surface optimization program is to minimize the reduction in gain over a specified scan angle.

If the array uses a different illumination function on transmit and receive, then the gain reduction will in general be different for the two cases, and the sum of the gain reduction on transmit and the gain reduction on receive has been selected as the function to be minimized.

There are two distinct reasons for expecting that by modifying the reflector geometry we can reduce the gain loss. First we have the general mathematical fact that to approximate a function with minimum peak error across a given region with a limited number of variables, the solution is not to minimize the error and error derivatives at the center of the region, but rather to introduce variations which allow the errors to increase near the center of the region while reducing them at the edges. (In the case of
polynomials of fixed degree, this corresponds to the use of Tchebycheff polynomials rather than Taylor series.) The second reason is the fact that only a portion of the subreflector is illuminated at any given scan angle. This allows each part of the subreflector to be optimized for only the range of scan angles over which it is used. The bottom edge of the subreflector, for example, is used only at the extreme positive elevation scan angle. It is clearly not optimum to set this part of the reflector for zero error at boresight, where it is not even used.

The problem is best treated by a combination of analytical and numerical techniques. Since the ray reflected from a point on a reflector is affected by both the location of the point and the slope of the reflector at that point, both must be determined at each point considered, and it becomes virtually impossible to ensure reflector continuity unless an analytical representation of the surface is adopted, which can be differentiated to obtain the slopes. A sixth-degree polynomial has been chosen, with only even terms used in the horizontal plane because of the symmetry of the problem, and 15 coefficients are then required to describe each reflector surface. To compute the intersections of the rays on the reflector surface requires solution of simultaneous sixth-degree equations, which is a problem suited to numerical methods. For any given output scan angle, there is an input scan angle which maximizes the gain, and a reference path length which minimizes the rms path length variations. There is no need to force correlation of these values from one scan angle to another, so an analytical expression has not been adopted here. Instead, we choose 16 scan angles distributed reasonably uniformly throughout one half of the scan region (the other half giving identical gain reduction because of symmetry), and treat the reference path length, the azimuth input scan angle, and the elevation input scan angle for each of these angles as independent variables. Along with the 15 variables representing the coefficients needed to describe each reflector surface, this gives 78 variables, all of which interact, which must be adjusted simultaneously to minimize the gain reduction. If for each ray the error
can be expressed as a linear function of the variables, then taking the partial
derivative of the total error expression with respect to each of the variables
and setting it equal to zero gives a set of linear simultaneous equations
which can be solved analytically to given an exact value for each variable
which minimizes the error. In practice, the error is not a linear function
of the variables, but if we start at a point not too far from the optimum and
use the position and slope of the curve at that point as a linear approximation,
we still get the correct answer provided the nonlinearity is negligible over the
region from the starting point to the optimum point. Even though this con-
dition is not met, if the nonlinearity is not too great, the answer we get will
be closer to the optimum than was the starting point. Then we can use the
answer as a new starting point and repeat the computation, thus converging
on the correct answer through iteration. Even if the nonlinearity is so great
or the initial estimate so far from the optimum that convergence does not
occur naturally, it can be forced by artificially placing a limit on the size of
step which can be taken, although convergence is not as rapid in this case.
A combination of several methods has been adopted to achieve stable con-
vergence, including judicious choice of starting geometry, initial solution
with some of the variables inhibited, adding a parabolic error around the
starting point which limits step size, and changing a few of the variables to
program parameters which are held fixed on any given run. The program
also includes an evaluation subroutine to accurately assess the results of the
optimization.

The program was initially set up in two dimensions, and considerable
data obtained on the effect of varying geometry. After the usefulness of the
technique was demonstrated in two dimensions, the program was expanded
to handle the problem in three dimensions. Only a small amount of data has
been obtained from the three-dimensional program to date, because of time
limitations.

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3. PATTERN CALCULATION METHOD

The antenna system analyzed uses a phased array to illuminate a double reflector system; a subreflector and a main reflector. The array is steered about the subreflector to move the final beam. This process is simulated on the computer and allows determination and evaluation of the beam pattern; the gain, beamwidth, and sidelobes near the peak.

The vector diffraction technique used consists of dividing the array, subreflector, and main reflector into a large number of regions, each represented by a point in the center plus an elementary pattern. Each point on the subreflector receives radiation from all points on the array which is then reradiated to all points on the main reflector. The superposition of all these fields at the main reflector allows computation of the pattern. For each direction in space corresponding to a pattern point, the field from each point on the main reflector is summed, to give the total field in that direction.

In this analysis the vector properties of the fields are retained, thus vertically and horizontally polarized parts of the patterns are determined. The difficulty of this type of analysis is in the number of operations which must be performed. This is somewhat relieved by using the directional properties of each region and assigning a pattern to it.

Blockage is included optically in this analysis. Thus in those situations where the array lies between the subreflector and main reflector, the contribution to the field at the main reflector is deleted. Similarly, if the subreflector interferes with fields from the main reflector in the pattern point direction, then these are also deleted.
3.1 SYSTEM GEOMETRY

The system, consisting of an array illuminating a subreflector which reradiates to a main reflector, is shown in figure 1. The array and reflectors are defined by grid points located on each surface, and the vector normals at these grid points. The reflectors are described by sixth order polynomial in both vertical and horizontal planes, although they must be symmetric in the horizontal plane and thus the odd powers in this plane are zero. This permits analysis of some simple shapes in addition to the optimized reflector shape. The normals to the surfaces are determined from the partial derivatives.

3.2 ARRAY ILLUMINATION

The program provides for up to four simultaneous pattern solutions corresponding to transmit, receive, elevation, and azimuth difference distributions. For transmit, a uniform illumination is provided. For receive and the difference distributions, function routines are used to simplify redefining these tapered distributions.

The array is steered in azimuth and elevation at the user's discretion. The vector representation of the fields is determined at the array, and assumed to radiate out. These fields can be described as being vertically or horizontally polarized. From then on there is no attempt to break these up except to print the final polarized field patterns.

3.3 RADIATED FIELDS

The fields emanating from the array are summed at the subreflector grid points maintaining the wave and vector properties and with appropriate attention. The incident fields at the subreflector are then reflected to the main reflector.

A pattern factor is used to modify these fields when reradiated in directions other than the main phase front. This is done to minimize the number of points necessary to define the subreflector. Since this concept is borrowed
from the geometric optics solution, the direction of reflection off the sub-
reflector is assumed to be that defined by geometric optics.

The subreflector fields are then transmitted to the main reflector from
which the far field pattern can be determined.

Blockage between the two reflectors by the array is accounted for by
deleting these rays.

The far field patterns are found by computing the orthogonal main re-
rector fields projected on a reference plane. The absolute gain in dB is
printed out for elevation, azimuth, or diagonal pattern cuts in polar coordi-
nates.
4. RESULTS

To assess the performance of the optimization program, the evaluation subroutine was used to determine the gain reduction with scan angle for a confocal paraboloid geometry. An optimization run was then made and the results evaluated at the same scan angles. Figure 2 shows a comparison of the results. It can be seen that the optimization program has produced a reduction in peak two-way loss over the 7 by 8 degree scan region from 13.885 dB to 3.328 dB. Most of the improvement is due to optimization of the subreflector surface, a peak two-way loss of 3.49 dB being obtainable by varying only the subreflector surface. The changes in the reflector surface to obtain this improvement are small compared to the dimensions of the system, but amount to several wavelengths. Changes in the main reflector are about one-half wavelength maximum. Parameters chosen for this comparison were a 45 wavelength diameter array, a 136 wavelength diameter main reflector, and 205 wavelength spacing between reflectors.

Both the main reflector and subreflector surfaces are defined by 6th degree polynomials of the form:

\[
\]

The coefficients, \(A(N)\), for both surfaces after optimization are listed in table 1. Results from the two dimensional program suggest that a more detailed search, varying the other parameters available to the program, would probably bring the two-way loss below 3 dB. The spacing between reflectors is a critical parameter, the losses varying inversely as at least the second power of the spacing, so that the desire for a large spacing for best results must be balanced against the mechanical considerations.
Figure 2. Comparison of Confocal Paraboloids with Optimized Reflectors
TABLE I
REFLECTOR SURFACE POLYNOMIAL COEFFICIENTS

<table>
<thead>
<tr>
<th>N</th>
<th>Main Reflector A (N)</th>
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</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.122785</td>
<td>-0.040360</td>
</tr>
<tr>
<td>2</td>
<td>0.218099</td>
<td>0.729146</td>
</tr>
<tr>
<td>3</td>
<td>0.218613</td>
<td>0.697714</td>
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<tr>
<td>4</td>
<td>0.000459</td>
<td>-0.078354</td>
</tr>
<tr>
<td>5</td>
<td>0.000121</td>
<td>-0.146562</td>
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<td>6</td>
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<tr>
<td>7</td>
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<td>10</td>
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</tr>
<tr>
<td>11</td>
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<tr>
<td>12</td>
<td>0.0</td>
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</tr>
<tr>
<td>13</td>
<td>0.0</td>
<td>-0.729649</td>
</tr>
<tr>
<td>14</td>
<td>0.0</td>
<td>-0.663223</td>
</tr>
<tr>
<td>15</td>
<td>0.0</td>
<td>-0.226827</td>
</tr>
</tbody>
</table>

The pattern program has been compared with other programs in relation to prediction of gain and pattern shape. Comparing results with the geometric optics technique on confocal reflectors gave similar pattern shapes. The gain and sidelobe agreement averaged 0.3 dB and 0.75 dB respectively.

Patterns have been computed to show the effects of optimization program. In these cases, equal reflectors and arrays are used, with the same number of points on each.

The array transmit illumination for both confocal paraboloids and the optimized reflectors is uniform. The array receive illumination for both cases is tapered and is given by:
\[ A(R) = 1.394 + \left[ 0.618 \cos (3.35R) \right] \left[ 1.0 - 0.2205 \cos (3.35R) \right] \]

where \( R \) is the radial distance from the center of the aperture. Patterns are shown in figures 3 through 10 for both the confocal and optimized reflectors. The improvement in gain is indicated in figure 11.
Figure 4. Confocal Paraboloidal Reflectors - Elevation Transmit Patterns
Figure 6. Conical Paraboloidal Reflectors - Elevation Receive Patterns
Figure 9. Optimized Reflector - Azimuth Receive Patterns
Figure 10. Optimized Reflector - Elevation Receive Patterns
Figure 11. Two-Way Gain Comparison (Predicted Gain of Optimized Reflector Over Confocal Parabola as Found From Pattern Program)
A new and very powerful tool has been developed for the analysis of a particular class of microwave antennas. While the analysis was limited to a particular application, the resulting computer program can be readily modified to accommodate different configurations and applications.

The analytical tool, the computer program, is, again, a tool. It must be exercised with understanding, and, when properly applied, will provide a heretofore unobtainable insight into the performance characteristics of the Gregorian microwave optical system and its variations.

The accompanying pattern calculation program demonstrates the power of vector diffraction technique as a means of computing the secondary radiation patterns of complex microwave antennas. The application of the technique is possible only through the use of the digital computer. Even so, considerable effort was necessary to achieve reasonable computer run time per pattern.

The combination of the reflector surface optimization method and the pattern calculation program will enable the antenna designer to optimize the design within specified physical constraints and accurately predict the performance of a class of microwave antennas.

As suggested by Lincoln Laboratory, the Gregorian optical arrangement can also be utilized as a wide bandwidth feed for a lens array antenna. The geometry is illustrated in figure 12. For moderate instantaneous bandwidths, the steering angle of the lens array will vary an amount approximately equal to the bandwidth. That is, for a 20 percent bandwidth and 40 degrees steering, the variation is about 8 degrees. By using a time-delay steered array as the feed, the steering angle variation can be compensated
Figure 12. Time Delay Steering
and the wide instantaneous bandwidth performance of the lens array can be improved. Because of the magnification, $M$, of the optical system, the feed array required only $1/M^2$ as many elements as the lens array.

The methods and concepts developed for the LFOV mode are applicable in general to the wide bandwidth mode. Some additional work is necessary to adapt the computer programs to a different geometry and optimization requirements.
This report describes the study of a dual reflector microwave antenna system performed by the Westinghouse Electric Corporation for the Massachusetts Institute of Technology, Lincoln Laboratory. The objectives of the study were to: (1) develop a computer program which would define the contours of the reflectors to achieve optimum system gain over a given scan range, and (2) develop a computer program to calculate the secondary patterns and gain of the dual reflector system. The computer programs have been written. Significant improvement in performance of the dual reflector system has been demonstrated. Program results are discussed.