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 Supernonic Gas-Solid Particle Flow
 In an Axisymmetric Nozzle
 By the Method of Characteristics

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SUPERSONIC GAS-SOLID PARTICLE FLOW
IN AN AXISYMMETRIC NOZZLE
BY THE METHOD OF CHARACTERISTICS.

by

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AUGUST 1971

VON KARMAN INSTITUTE FOR FLUID DYNAMICS
# TABLE OF CONTENTS

List of Symbols ............................................................. i
Abstract ...................................................................... ii

I. INTRODUCTION ............................................................ 1
II. GOVERNING EQUATIONS ............................................. 2
III. APPLICATION OF THE METHOD OF CHARACTERISTICS ..... 5
IV. NUMERICAL SOLUTION TECHNIQUE ............................. 16
V. CONCLUSIONS AND RECOMMENDATIONS ...................... 20
REFERENCES ................................................................... 21

APPENDICES
A. Program Description .................................................. A1-1
B. Program Input ........................................................... A2-1
C. Program Listing ........................................................ A3-1
LIST OF SYMBOLS

\( C_d \) particle drag coefficient
\( C_p \) particle specific heat
\( C \) gas specific heat
\( D \) particle diameter
\( m \) density per unit volume of a particle
\( \dot{m}_p \) mass flow rate of particles
\( M \) Mach number
\( P \) pressure
\( Pr \) Prandtl number
\( R \) gas constant
\( Re \) Reynolds number
\( T \) temperature
\( u \) x-direction velocity component
\( v \) y-direction velocity component
\( W \) speed
\( x \) axial coordinate along nozzle axis
\( y \) radial coordinate measured from nozzle axis
\( \alpha \) Mach angle
\( \gamma \) isentropic exponent
\( \theta \) streamline angle with respect to nozzle axis
\( \mu \) viscosity
\( \rho \) density per unit volume

Subscripts

\( p \) particle property
\( x \) partial derivative with respect to \( x \)
\( y \) partial derivative with respect to \( y \)
ABSTRACT

A study was conducted to numerically treat a mixture composed of a gas and solid particles in a supersonic, axisymmetric nozzle. The governing equations are a set of eight first order, quasi-linear, partial differential equations. Seven of these equations are of the hyperbolic type when the flow is supersonic (based on the frozen speed of sound in the gas) and can be solved by the method of characteristics. The eighth equation (the particle continuity equation) is rewritten as an integral equation to be solved. The resulting seven compatibility equations and the seven characteristic equations (only four are distinct; the gas Mach lines and the gas and particle streamlines) are solved by the modified Euler predictor-corrector algorithm. These equations were programmed for an IBM 1130 computer. A sample nozzle calculation is given and compared with the one-dimensional calculations. These results indicate that the program is working correctly.
I. INTRODUCTION

The purpose of the present investigation was to numerically treat a mixture of a gas and solid particles in an axisymmetric, supersonic nozzle. The study was primarily intended for industrial particle micronization processes. In this process solid particles are fluidized and entrained by a high pressure air flow. The mixture is then expanded through a converging-diverging nozzle. The solid particles are accelerated during this expansion and impact a target downstream of the nozzle exit plane. If the particles have sufficient momentum they are broken upon impact. This process is repeated until the desired particle sizes are obtained.

The interest in the present study was not so much to determine two-dimensional particle velocities but rather to be able to investigate non-uniform particle distributions in each nozzle plane. One of the inherent restrictions in one-dimensional gas-solid-particle analyses is that the particles are uniformly distributed in each nozzle plane. However, experimental studies definitely show that most of the particles are concentrated near the nozzle center line. To treat the problem of nonuniform particle distribution it is necessary to formulate the problem in two dimensions.

Riethmuller\(^1\) has done an extensive literature survey on gas-solid particle flow analyses and experiments. Therefore, only articles that are directly pertinent to this analysis are referenced in this report.
II. GOVERNING EQUATIONS

The supersonic motion of most compressible fluids encountered in nozzle expansion flow can be accurately described by the governing equations of an inviscid fluid. The basic assumptions which constitute such a gas dynamic model are: (1) continuum, (2) steady, (3) inviscid, (4) adiabatic, and (5) the gas composition is frozen. The resulting partial differential equations can be treated by the well known technique of the method of characteristics. For axisymmetric or two-dimensional nozzles this method transforms the partial differential equations into a system of differential equations which are valid along certain characteristic directions in the flow field.

The transformed equations are much simpler to solve and this fact led Kliegel\(^2\) to attempt a similar analysis for a mixture of a gas and solid particles. The major assumption necessary for such a gas-solid particle analysis is that the particles behave as a continuum. Clearly, this assumption is physically unrealistic. That it yields good engineering results is another question. Kliegel has shown that the results are in good agreement with observation for rocket engine applications for moderate loading ratios and small particle sizes. Similar comparisons will be required for particle micronization processes to determine the applicability of this assumption. However, it would appear that, even for particle micronization processes this assumption should give useful engineering results for moderate loading ratios of small particles.

The equations governing the steady axisymmetric flow of a mixture of a gas and solid particles are derived by Hoffman and Thompson\(^3\). The assumptions necessary for these derivations are: (1) the gas is inviscid except for its interactions with the solid particles, (2) the mixture mass and mixture energy of the system are constant, (3) the volume
occupied by the solid particles is negligible, (4) the thermal motion of the solid particles is negligible, (5) the solid particles do not interact, (6) the drag and heat transfer characteristics of an actual particle shape and the size distribution of particles can be represented by spherical particles of a single size, (7) the internal temperature of each solid particle is uniform, (8) energy exchange between the gas and the solid particles occurs by convection only, (9) the only force acting on the solid particles is the viscous drag forces, (10) no mass transfer between the gas and the solid particles, and (11) no phase change.

Based on these assumptions, the following equations govern the gas-solid particle flow:

\[
\rho u_x + \rho v_y + u_0 x + v_0 y + \rho v/y = 0
\]  

(II-1)

\[
\rho uu_x + \rho vv_y + P_x + A_p (u - u_p) = 0
\]  

(II-2)

\[
\rho vv_x + \rho vv_y + P_y + A_p (v - v_p) = 0
\]  

(II-3)

\[
u P_x + v P_y - a^2 u_0 x - a^2 v_0 y - A B_p = 0
\]  

(II-4)

\[
\rho_p [ u P_x + v P_y - A (u - u_p) ] = 0
\]  

(II-6)

\[
\rho_p [ u P_x + v P_y - A (v - v_p) ] = 0
\]  

(II-7)

\[
\rho_p [ u T P_x + v T P_y + \frac{2}{3} A C (T_p - T) ] = 0
\]  

(II-8)

where

\[
A = \frac{3}{4} \frac{C_D}{D} \frac{\rho (u - u_p)}{\rho_p}
\]  

(II-9)

\[
G = \frac{12 k N u}{\mu C_D Re}
\]  

(II-10)
\[ B = (\gamma - 1) \left[ \frac{2}{3} C p (T_p - T) + (u - u_p)^2 + (v - v_p)^2 \right] \quad (II-11) \]

The drag coefficient and Nusselt number for a particle are calculated from equations given by Neilson and Gilchrist:

\[ C_p = 28 \text{Re}^{-0.85} + 0.48 \quad (II-12) \]

\[ N_u = 2.0 + 0.6 \text{Re}^{0.5} \text{Pr}^{0.333} \quad (II-13) \]

where the Reynolds number for a particle is defined as follows:

\[ \text{Re} = \frac{\rho D (W - W_p)}{\mu} \quad (II-14) \]
III. APPLICATION OF THE METHOD OF CHARACTERISTICS

In this section, the techniques of the method of characteristics will be employed to obtain the characteristic and compatibility equations for the flow field variables.

The flow field governing equations, Equations (II-1) through (II-8) can be written in the following general form:

\[ L_i = a_{ij}z_{jx} + b_{ij}z_{jy} + c_i = 0 \quad (i,j = 1, \ldots, 8) \quad (III-1) \]

where \( z \) represents the eight dependent variables \( u, v, p, \rho, u_p, v_p, \rho_p \), and \( T \), and the \( x \) and \( y \) subscripts denote partial differentiation. These eight equations can be combined to form a single differential operator by employing arbitrary multipliers and summing. Thus

\[ L = \sigma_i L_i = 0 \quad (i = 1, \ldots, 8) \quad (III-2) \]

where \( \sigma_i \) are the arbitrary multipliers. In this section, the convention of indicatrix summation be repeated indices will be used. The partial differential equation, Equation (III-2), can be rewritten in the form of an ordinary differential equation under certain conditions. Thus

\[ (a_{ij} \sigma_i)dz_j + c_i \sigma_i dx = 0 \quad (i,j = 1, \ldots, 8) \quad (III-3) \]

if and only if the following equations are valid:

\[ a_{ij} \sigma_i \frac{dy}{dx} = b_{ij} \sigma_i \quad (i,j = 1, \ldots, 8) \quad (III-4) \]

Equations (III-4) are eight independent equations for \( \frac{dy}{dx} \). Equations (III-4) are used to determine the unknown multipliers \( \sigma_i \) which can then be substituted into Equation (III-3) to yield the compatibility equations. Rearranging Equations (III-4) to solve for \( \sigma_i \) yields the following equations:
\[\sigma_i(a_{ij} \frac{dy}{dx} - b_{ij}) = 0 \quad (i,j = 1, \ldots, 8) \quad \text{(III-5)}\]

If this system of equations, Equations (III-5), is to have a solution other than the trivial, i.e., all \(\sigma_i = 0\), then the determinant of the coefficients of \(\sigma_i\) must be zero. Thus,

\[\begin{bmatrix} a_{ij} & dy/dx & - b_{ij} \end{bmatrix} = 0 \quad (i,j = 1, \ldots, 8) \quad \text{(III-6)}\]

Now, \(dy/dx\) can be obtained from Equation (III-6). The resulting \(dy/dx\) can then be substituted into Equations (III-4) to determine relationships in terms of the multipliers \(a_i\).

With \(a_i\) known, the final form of the compatibility equations, Equations (III-3), is obtained.

The governing equations for the flow field are repeated below for convenience.

\[\rho u_x + \rho v_y + \rho u \rho_x + v \rho_y + \rho y = 0 \quad \text{(III-7)}\]
\[\rho uu_x + \rho vu_y + P + A \rho (\rho - u \rho) = 0 \quad \text{(III-8)}\]
\[\rho uv_x + \rho vv_y + P + A \rho (v - v \rho) = 0 \quad \text{(III-9)}\]
\[u \rho_x + \rho v F_y - a^2 \rho x - a^2 \rho y - A \rho_p = 0 \quad \text{(III-10)}\]
\[\rho \rho u_x + P \rho_y + \rho \rho x + v \rho_p y + \rho \rho y / y = 0 \quad \text{(III-11)}\]
\[\rho_p \left[ u \rho_x + v \rho_y - A (u - u \rho) \right] = 0 \quad \text{(III-12)}\]
\[\rho_p \left[ u \rho_x + v \rho_y - A (v - v \rho) \right] = 0 \quad \text{(III-13)}\]
\[\rho_p \left[ c u T_p + c v T_p + \frac{2}{3} AC(T_p - T) \right] = 0 \quad \text{(III-14)}\]
The determinant, Equation (III-6) can now be written as follows:

\[
\begin{vmatrix}
\rho \frac{dy}{dx} - \rho & 0 & S_1 & 0 & 0 & 0 & 0 \\
\rho S_1 & 0 & \frac{dy}{dx} & 0 & 0 & 0 & 0 \\
0 & \rho S_1 - 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & S_1 - \epsilon^2 S_1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \rho_p \frac{dy}{dx} - \rho_p & 0 & S_2 \\
0 & 0 & 0 & 0 & 0 & \rho_p S_2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \rho_p S_2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_p \epsilon S_2 \\
\end{vmatrix} = 0
\]  

(III-15)

where \( S_1 = (u \frac{dy}{dx} - v) \) and \( S_2 = (u \frac{dy}{dx} - v_p) \).

Since the upper right quarter of the above determinant is filled with zeros, the expansion of the determinant reduces to

\[
[ G ] \times [ P ] = 0 
\]

(III-16)

where \( G \) represents the upper left quarter of the determinant and \( P \) represents the lower right quarter of the determinant as shown below:

\[
\begin{array}{c|c|c|c}
\text{Gas} & \text{Properties} & \text{Zeros} \\
\text{Particles} & \text{Zeros} & \text{Properties} \\
\hline
\end{array}
\]
Equation (III-16) can be zero by either $G$ or $P$ being zero. Setting these two determinants to zero yields the following expressions:

$$[G] = (udy/dx - v)^2 [(dy/dx)^2 \left[ u^2 - a^2 \right] - 2 u v dy/dx$$

$$+ (v^2 - a^2)] = 0 \quad (III-17)$$

and

$$[P] = \left( u_P dy/dx - v_P \right)^4 = 0 \quad (III-18)$$

Therefore, Equation (III-16) becomes:

$$\left( u_P dy/dx - v_P \right)^4 (udy/dx - v)^2 \left[ (dy/dx)^2 [ u^2 - a^2 ] - 2 u v dy/dx$$

$$+ (v^2 - a^2) \right] = 0 \quad (III-19)$$

The characteristic curves are found by solving for $dy/dx$, which, for this system of equations, are

$$dy/dx = v/u \quad (III-20)$$

$$dy/dx = \frac{uv + a^2 \sqrt{u^2 - 1}}{u^2 - a^2} \quad (III-21)$$

$$dy/dx = \frac{v}{u_P} \quad (III-22)$$

In terms of the Mach angle $\alpha$, the gas flow angle $\theta$ and the particle flow angle $\theta_P$, Equations (III-20), (III-21) and (III-22) can be rewritten, respectively, as

$$dy/dx = \tan \theta \quad (III-23)$$

$$dy/dx = \tan (\theta + \alpha) \quad (III-24)$$

$$dy/dx = \tan \theta_P \quad (III-25)$$
Thus, the characteristic curves are the gas streamlines appearing twice, the Mach lines each appearing once and the particle streamlines appearing four times. Equations (III-21) are real only when \( M > 1 \), whereas, Equations (III-20) and (III-22) are always real. Therefore, the system of governing equations, Equations (III-7) through (III-14) is hyperbolic when the flow field is supersonic. Of these eight characteristic curves, four are distinct; the two Mach lines, the gas streamlines and the particle streamlines.

In order to determine the compatibility equations, the unknown multiplier \( a_1 \) must be determined. These multipliers are determined by simultaneously solving Equations (III-4), using Equations (III-20) through (III-22) for the slope of each characteristic curve.

Substituting the coefficients \( a_{ij} \) and \( b_{ij} \) from Equations (III-7) through (III-14) into Equations (III-4) yields the following results:

\[
\sigma_1 \frac{dy}{dx} + (udy/dx - v)\sigma_2 = 0 \quad (III-26)
\]
\[
-\sigma_1 + (udy/dx - v)\sigma_3 = 0 \quad (III-27)
\]
\[
\sigma_2 \frac{dy}{dx} - \sigma_3 + (udy/dx - v)\sigma_4 = 0 \quad (III-28)
\]
\[
(udy/dx - v) (\sigma_1 - a^2 \sigma_4) = 0 \quad (III-29)
\]
\[
\sigma_5 \frac{dy}{dx} + (u_p dy/dx - v_p)\sigma_6 = 0 \quad (III-30)
\]
\[
-\sigma_5 + (u_p dy/dx - v_p)\sigma_7 = 0 \quad (III-31)
\]
\[
(u_p dy/dx - v_p)\sigma_8 = 0 \quad (III-32)
\]
\[
(u_p dy/dx - v_p)\sigma_5 = 0 \quad (III-33)
\]
The form of the general compatibility equation, Equation (III-3) is:

\[
\begin{align*}
[\rho \sigma_1 + \rho u \sigma_2] & \, du + [\rho u \sigma_3] \, dv \, + \, [\sigma_2 + \sigma_4] \\
& \, + [u_0 - a^2 u \sigma_5] \, dp \, + \, [\rho_p \sigma_5 + \rho_p u \sigma_6] \, dP_p \, + \, [p u \sigma_7] \, dp \, + \, [c_p u \sigma_8] \, dT_p \, + \, [u \sigma_9] \, dP_p \, + \, [\rho \nu \sigma_1 / \gamma + A_p (u - u_p) \sigma_2] \\
& \, + A_p (v - v_p) \sigma_3 - AB_p \sigma_4 + \rho_p v \sigma_5 / \gamma - A_p (u - u_p) \sigma_6 \\
& \, - A_p (v - v_p) \sigma_7 + \frac{2}{3} \rho_p AC(T_p - T) \sigma_8 \right] \, dx = 0 \tag{III-34}
\end{align*}
\]

For the gas streamline characteristic curve, \((udy/dx - v) = 0\). Thus, by substituting this relationship into Equations ("II-26) through (III-33), the following results are obtained:

\[
\begin{align*}
\sigma_1 &= 0 \\
\sigma_2 &= \frac{u \sigma_3}{\gamma} \tag{III-35} \\
\sigma_5 &= \sigma_7 = \sigma_8 = 0
\end{align*}
\]

Substituting Equation (III-35) into Equation (III-34) and regrouping terms into coefficients of the two arbitrary multipliers, \(\sigma_3\) and \(\sigma_4\), yields the following result:

\[
\begin{align*}
[\rho u du + \rho v dv + dp + A_p (u - u_p) dx + A_p (v - v_p) dy] \frac{u \sigma_3}{\gamma} \\
&\, + [udp - a^2 u dp - AB_p dx \sigma_4] = 0 \tag{III-36}
\end{align*}
\]

Since the multipliers \(\sigma_3\) and \(\sigma_4\) are arbitrary, their coefficients must equal zero. Equating the coefficients of these multipliers
to zero yields the following compatibility equations which are valid along gas streamlines:

\[ \rho u u + \rho v v + d p + A p \left( u - u_p \right) dx + A p \left( v - v_p \right) dy = 0 \quad (III-37) \]

\[ u d p - a^2 u d p - A B p \ dx = 0 \quad (III-38) \]

Noting that

\[ u u + v d v = W d W \]

Equation (III-37) can be rewritten as

\[ \rho W d W + d p + A p \left[ \left( u - u_p \right) dx + \left( v - v_p \right) dy \right] = 0 \quad (III-39) \]

Equations (III-38) and (III-39) are valid only along gas streamlines, i.e., along curves defined by

\[ \frac{d y}{d x} = \tan \theta \quad (III-40) \]

Along the Mach lines, the slope \( dy/dx \) is determined from either Equations (III-21) or (III-24) and \( (udy/dx - v) \neq 0 \). Therefore, by combining Equations (III-26) through (III-33) with Equation (III-31), the relationships for the multipliers \( \sigma_i \) for determining the compatibility equations which are valid along the Mach lines are obtained and are as follows:

\[ \sigma_1 = a^2 \sigma_4 \]

\[ \sigma_2 = \frac{- a^2 \sigma_4 \ dy/dx}{(udy/dx - v)} \]

\[ \sigma_3 = \left[ \frac{(udy/dx - v)^2 - a^2(dy/dx)^2}{(udy/dx - v)} \right] \sigma_4 \]

\[ \sigma_5 = \sigma_6 = \sigma_7 = \sigma_8 = 0 \quad (III-41) \]
Since no relationship is obtained for \( c_n \), the multiplier is arbitrary. The compatibility equations valid along the Mach lines are obtained by substituting Equations (III-41) into Equation (III-34), the general compatibility equation, and equating the coefficient of \( c_n \) to zero, which yields

\[
(- vdu + udv) \pm \frac{\sqrt{M^2 - 1}}{\rho} dp + \frac{v}{y} (udy - vdx) \\
- A(\rho_p/\rho)(u - u_p)dy + A(\rho_p/\rho)(v - v_p)dx \\
- AB(\rho_p/\rho a^2)(udy - vdx) = 0
\]

(III-42)

Equation (III-42) can be rewritten in terms of \( \theta \) and \( \alpha \) as

\[
d\theta \pm (\cot \alpha/\rho a^2) dp \pm \left[ \frac{\sin \theta}{y} - \frac{A_p}{\rho W} (1 + B/\alpha^2) \right] x
\\
\left[ \frac{\sin \alpha}{\cos (\theta \pm \alpha)} \right] dx + A(\rho_p/\rho W^2) \left[ u_p dy - v_p dx \right] = 0
\]

(III-43)

where the gas Mach lines are given by

\[
dy/dx = \tan (\theta \pm \alpha)
\]

(III-44)

In Equations (III-43) and (III-44), the upper signs refer to left-running Mach lines and the lower signs refer to right-running Mach lines.

For the particle streamline characteristic curve, \( (u_p dy/dx - v_p) = 0 \). Thus, by substituting this relationship into Equations (III-26) through (III-33), the following results are obtained:

\[
\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = \sigma_5 = 0
\]

(III-45)
Substituting Equation (III-45) into Equation (III-34) and regrouping terms into coefficients of the three arbitrary multipliers, \( \sigma_6, \sigma_7 \) and \( \sigma_8 \) yields the following result:

\[
\begin{align*}
[ u_p \frac{du_p}{dx} - A(u - u_p) ]\sigma_6 + [ v_p \frac{dv_p}{dx} - A(v - v_p) ]\sigma_7 \\
+ [ cu_p \frac{dT_p}{dx} + \frac{2}{3} AC(T_p - T) ]\sigma_8 &= 0 \\
(III-46)
\end{align*}
\]

Again, the multipliers \( \sigma_6, \sigma_7 \) and \( \sigma_8 \) are arbitrary. Therefore, their coefficients must be zero. Equating the coefficients of these three multipliers to zero yields the following compatibility equations which are valid along the particle streamlines:

\[
\begin{align*}
\frac{udu_p}{dx} - A(u - u_p) &= 0 \\
(III-47) \\
\frac{udv_p}{dx} - A(v - v_p) &= 0 \\
(III-48) \\
\frac{cudT_p}{dx} + \frac{2}{3} AC(T_p - T) &= 0 \\
(III-49)
\end{align*}
\]

where the particle streamline is given by

\[
\frac{dy}{dx} = \frac{v_p}{u_p} \\
(III-50)
\]

It should be pointed out that only seven compatibility equations are obtained in this analysis, i.e., no equation is obtained for solving for the particle density, \( \rho_p \). This can be explained by noting that \( \sigma_5 \) is zero for all characteristic curves. Therefore, the fifth governing equation, Equation (III-11), the particle continuity equation does not enter the analysis as treated by the method of characteristics. If Equation (III-11) is removed from the set of governing equations, seven characteristic curves and seven compatibility equations are obtained, as should be. In order to have a complete set of equations it is necessary to include Equation (III-11) or some equivalent equation in order to have a relationship for \( \rho_p \). In the present analysis an integral equivalent equation for particle continuity
is employed. The equation used is as follows:

\[
\dot{m}_p = 2\pi \int_0^y \rho_p u_p dy - v_p \frac{dy}{dx} \text{ } dx
\]  

Equation (III-51) is used to calculate an average particle density at each point in the flow field.

It should be reemphasized that treating the particles as a continuum is an approximation in the present two-dimensional analysis just as in previous one-dimensional analyses.

In summary, the governing partial differential equations have been transformed to a system of differential equations valid only along certain characteristic curves. These are:

**Gas Streamlines**

\[
\frac{dy}{dx} = \tan \theta
\]  

\[
\rho W dW + d\rho + \frac{A\rho}{\rho_p} \left[ (u - u_p) dx + (v - v_p)dy \right] = 0
\]  

\[
u d\rho - \rho_p a^2 d\rho - A\rho_p dx = 0
\]

**Gas Mach Lines**

\[
\frac{dy}{dx} = \tan \left( \theta \pm \alpha \right)
\]  

\[
ds \pm \frac{(\cot \alpha / \rho W^2) d\rho}{\sin \theta} \pm \frac{A\rho}{\rho W} \left( 1 + B/a^2 \right) \frac{\sin \theta}{\cos (\theta \pm \alpha)} dx
\]

\[
\pm A\left( \rho_p / \rho W^2 \right) \left[ u_p dy - v_p dx \right] = 0
\]  

(III-52)  

(III-53)  

(III-54)  

(III-55)  

(III-56)
Particle Streamlines

\[ \frac{dy}{dx} = \tan \theta p \]  
(III-57)

\[ u_p \frac{du}{dx} - A(u - u_p) dx = 0 \]  
(III-58)

\[ u_p \frac{dv}{dx} - A(v - v_p) dx = 0 \]  
(III-59)

\[ u_p c dT_p + \frac{2}{3} AC(T_p - T) dx = 0 \]  
(III-60)

and the particle continuity equation

\[ \hat{m}_p = 2\pi \int_0^y \rho_p (u_p \frac{dy}{dx} - v_p) dx \]  
(III-61)
IV. NUMERICAL SOLUTION TECHNIQUE

The formulation of the problem of treating a supersonic flow field consisting of a mixture of a gas and solid particles represents only one part of the present investigation. In order to make use of the equations derived in the previous section, it is necessary to translate the equations of motion of Section III into a numerical algorithm. This section outlines the solution procedure which was programmed for the IBM 1130 computer at the von Karman Institute.

As was discussed in the previous section, the gas Mach lines (two of the characteristic curves) are real only when the gas Mach number (ratio of the gas speed and the frozen speed of sound) is greater than one. Therefore, the present scheme is only applicable in regions of the flow which are supersonic. This means that in order to calculate the flow in a nozzle by the method of characteristics one must know the flow properties in the region downstream of, but near, the gas sonic line. In order to obtain the necessary initial conditions for a gas-solid particle flow the complete subsonic and transonic flow field must be solved. Since this problem has not been solved to date, it is necessary to use approximate methods to obtain the gas and solid particle properties from which to start the solution procedure. It is clearly evident that much more work must be done in the subsonic and transonic flow regions before the present supersonic analysis can be effectively utilized, i.e., to get realistic results one must specify realistic initial conditions.

The numerical algorithm does not depend on the initial conditions. Therefore, as better methods for predicting the initial conditions become available, they can be input into the present analysis directly as input data.

The numerical algorithm used is a modified Euler, predictor-corrector technique where averaged coefficients are
used. The transformed equations, Equations (III-52) through (III-61) are forward differenced. To illustrate the technique, Figure 1 presents an interior mesh point grid. The four distinct characteristic curves are shown with the point numbering scheme as used in the computer program. Point 3 is the point where the solution is sought. Points 1 and 2 are previously obtained solution points or given points on the initial value line. The properties at points 4 and 5 are known (by interpolation) once the location of each point is determined. The solution procedure is as follows:

(1) Predict the x and y direction of Point 3 by solving the two equations, Equations (III-55). In finite difference form this yields:

\[
\begin{align*}
x_3 &= (y_2 - y_1 - x_2 H_{23} + x_1 s_{13})/(s_{13} - H_{23}) \\
y_3 &= y_1 + s_{13}(x_3 - x_1)
\end{align*}
\]

where

\[
\begin{align*}
H_{23} &= \frac{1}{2} [ \tan (\theta_2 + \omega_2) + \tan (\theta_3 + \omega_3) ] \\
s_{13} &= \frac{1}{2} [ \tan (\theta_1 - \omega_1) + \tan (\theta_3 + \omega_3) ]
\end{align*}
\]

It should be noted that the second term within the brackets in the above two equations are set equal to the first term for the predictor.

(2) Solve for the gas flow angle and the pressure by simultaneously solving the two equations, Equations (III-56).

(3) Calculate the position of Points 4 and 5 by the particle streamline equation, Equation (III-57) and the gas streamline equation, Equation (III-52), respectively.
(4) Interpolate for the properties at points 4 and 5.

(5) Calculate the gas velocity and the gas density at Point 3 from Equations (III-53) and (III-54).

(6) Calculate the particle velocity, particle flow angle, particle temperature, and particle density from Equations (III-58) through (III-61).

(7) Now that all properties are known (predicted values), evaluate all equation coefficients, average the coefficients, and repeat steps (1) through (6).

This algorithm is second order accurate. To iterate again accomplishes nothing. If the accuracy is not sufficient change the grid spacing by increasing the number of points on the initial value line and redo the calculations.

The same technique is used on the wall boundary except that on the wall the solution point is obtained by the intersection of the left characteristic and the wall contour. Since the gas flow angle is the wall angle, the flow angle is known. This reduces the unknowns by one, which is necessary since the right characteristic compatibility equation cannot be used.

The same procedure exists at the centerline. Here the gas flow angle is zero (known, a priori), and the left characteristic compatibility equation is not used.

To check the program out, a sample case was executed for which previous one-dimensional results were available. The initial value line properties were taken from the one-dimensional results. The results were then compared at the nozzle exit, 9 cm. downstream of the initial value line. The following table presents the results as obtained from the two programs. The loading ratio was 1.5,
i.e., the particle flow rate was 1.5 times the gas flow rate.
Particle diameter was 500 microns.

<table>
<thead>
<tr>
<th>Present Analysis</th>
<th>2 - d</th>
<th>1 - d</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas Velocity</td>
<td>469.0 m/sec</td>
<td>467.6 m/sec</td>
</tr>
<tr>
<td>Particle Velocity</td>
<td>213.2 m/sec</td>
<td>214.7 m/sec</td>
</tr>
<tr>
<td>Gas Temperature</td>
<td>159.7 °K</td>
<td>159.7 °K</td>
</tr>
<tr>
<td>Particle Temperature</td>
<td>295.7 °K</td>
<td>291.9 °K</td>
</tr>
<tr>
<td>Gas Mach number</td>
<td>1.850</td>
<td>1.845</td>
</tr>
<tr>
<td>Pressure</td>
<td>1.86×10^6 n/m²</td>
<td>1.88×10^6 n/m²</td>
</tr>
</tbody>
</table>

The run time for the two-dimensional calculation on the IBM 1130 was approximately 9 minutes.

The above comparison indicates that the program is working correctly and that it can be used effectively for particle-gas problems.
V. CONCLUSIONS AND RECOMMENDATIONS

The results of the comparison of the present two-dimensional, supersonic analysis with the one-dimensional analysis indicate that the program is working correctly and that it can be a useful tool for particle micronization processes. The present analysis has the advantage that non-uniform particle distributions can be treated if the necessary initial data are available. The program has been written in modular form so that any required or desired program changes are relatively simple.

Further studies are needed in two areas. These are: (1) a working subsonic and transonic analysis to take advantage of the potential applications of this analysis and (2) much more effort is required to obtain governing equations that are more physically realistic. The use of the particle continuum assumption, obviously, has restrictions. Experimental measurements are necessary to determine the region where this assumption breaks down. It should be emphasized that one-dimensional analyses suffer from the same problem and, therefore, if for particle micronization processes the continuum assumption does not provide engineering results neither approach can be used to predict particle behaviour in the nozzle.
REFERENCES


Figure 1 Gas-solid particle characteristic mesh
APPENDICES
APPENDIX A

PROGRAM DESCRIPTION

The computer program was written in the FORTRAN IV language for the IBM 1130 computer at the von Karman Institute. The program consists of the MAIN program and four subroutines. These subroutines are designated FL0WP, M0CP, M0CB and C0EFP. The structure of the program is illustrated in Figure A-1. A brief description of the program is as follows:

MAIN - This program calls the flow field logic program or CALLS EXIT as determined by whether SWITCH 10 on the console is "on" or "off".

FL0WP - This subroutine controls the logic in calculating the flow field in an axisymmetric nozzle. All input data and all output options are accomplished by this subroutine. Subroutine FL0WP calls M0CP for the solution at an interior mesh point or M0CB for the solution at the wall boundary or the centerline boundary. Two arrays store the solution along a back left characteristic and the left characteristic being solved.

M0CP - This subroutine uses the modified Euler predictor corrector technique with averaged coefficients to solve for an interior mesh point. Subroutine C0EFP is called to evaluate all the necessary coefficients used in the finite difference equations.

M0CB - This subroutine is nearly identical to the above subroutine except the solution point is on the boundary.

C0EFP - This subroutine evaluates the coefficients of the finite difference equations and is called from both M0CP and M0CB.
Figure A-1  Program Structure
APPENDIX B

PROGRAM INPUT

The input data for the program has been kept to a minimum to keep the program simple. The following data are required for program execution:

**CARD 1** (FORMAT 3F10.0)

1-10  GAMMA, ratio of the specific heats for the gas (air, \( \gamma = 1.4 \))
11-20  RGAS, the gas constant (air, \( R = 29.27 \))
21-30  PR, the gas Prandtl number (air, \( Pr = 0.7 \))

**CARD 2** (FORMAT 4F13.0)

1-10  DIAM, particle diameter (meters)
11-20  RHOP, particle density \( (\text{kg/m}^3) \)
21-30  CPART, particle specific heat
31-40  PFLRT, particle flow rate \( (\text{kg/sec}) \)

**CARD 3** (FORMAT 12,8X,I2,OX,I2,8X,I1)

1-2  NPTSL, the number of points for which data is specified on the initial value line (MAX=5)
11-12  IMAX, the maximum number of left characteristics to be calculated
21-22  IOUT, the number of the left characteristic where print out begins. If all characteristics are desired IOUT=0.
31  NU, not used

The following two cards are repeated for each of the NPTSL points. Two cards are required per point. The first point input is the wall point, the last point is the centerline point.

**CARD 4** (FORMAT 7F10.0)

1-10  x-location of the point (meters)
11-20  y-location of the point (meters)
21-30  Gas Mach number
31-40  Gas velocity (m/sec)
41-50  Gas flow angle (degrees)
51-60  pressure (n/m^2)
61-70  gas density \( (\text{kg/m}^3 \cdot \rho_c) \)
CARD 5 (FORMAT 5F10.0)

1-10  particle velocity (m/sec)
11-20  particle flow angle (degrees)
21-30  particle density (kg/m³)
31-40  particle temperature (°K)
41-50  particle stream function (0.0 for centerline, 1.0 at limiting streamline).
APPENDIX C

PROGRAM LISTING

This appendix contains the complete program listing. All subroutines are stored on disc 6. For execution, it is only necessary to compile the MAIN program. The two subroutines \texttt{MOCP} and \texttt{MOCB} are stored in the \texttt{LOCAL} mode.
MAIN PROGRAM

COMMON STATEMENTS

ITP3A, RP3A, TEP3A, PSI3A
COMMON GAMMA, GC, RGAS, DIAM, RHOP, PR, CPART

10 CONTINUE
PAUSE
CALL DATSW (10, K10)
GO TO (20, 30), K10
20 CALL FLOWP
GO TO 10
30 CALL EXIT

END
SUBROUTINE FLOWP

THIS PROGRAM CONTROLS THE LOGIC NECESSARY TO CALCULATE
THE FLOW FIELD IN A SUPERSONIC AXI symmetric (NU=1) OR
TWO-DIMENSIONAL (NU=0) NOZZLE BY THE METHOD OF CHARACTERIS

THE ARRAY SLV IS USED TO STORE THE INITIAL-VALUE LINES.
THE FIRST SUBSCRIPT IS THE POINT NUMBER, THE SECOND DENOTE
THE FIELD PROPERTY DEFINED BY THE FOLLOWING:

1 % POSITION
2 Y POSITION
3 GAS MACH NUMBER
4 GAS VELOCITY
5 GAS FLOW ANGLE
6 GAS PRESSURE
7 GAS DENSITY
8 PARTICLE VELOCITY
9 PARTICLE FLOW ANGLE
10 PARTICLE DENSITY
11 PARTICLE TEMPERATURE
12 PARTICLE STREAM FUNCTION

THE SAME SCHEME IS USED FOR THE TWO CHARACTERISTIC ARRAYS
CHARA AND CHARL. TWO ARRAYS ARE NEEDED SINCE IT IS NECESS
TO SAVE ONE LEFT CHARACTERISTIC WHILE THE NEXT ONE IS BEEN
CALCULATED.

** NOTE **
THE INPUT PARAMETER NPTSL GOVERNS THE SIZE OF THE THREE
ARRAYS. THESE MUST BE SLV(NPTSL,12), CHARA(2*NPTSL,12)
AND CHARL(2*NPTSL,12).

TYPE, DIMENSION AND COMMON STATEMENTS

REAL *I-8
DIMENSION SLV(5,12), CHARA(10,12), CHARL(10,12)
COMMON NU, CONST, THETI, PELRT, PI, X3A, Y3A, E3A, V3A, T3A, P3A, Q3A, VF, E3, 
1TP3A, TP3A, EPS1A, PS13A
COMMON GAT1A, GC, RGAS, D1A, RHOP, PC, CPART

SET PROGRAM CONSTANTS AND INITIALIZE COUNTERS

GC=9.81
$\pi = 3.1415926$
\[ \text{RAD} = 180.0 / \pi \]
\[ \text{LINE} = 9 \]
\[ \text{ETER} = 100.0 \]
\[ \text{[CL]} = 1 \]
\[ \text{IMEX} = 1 \]
\[ j = 0 \]
\[ j = 0 \]
\[ j = 1 \]

**READ INPUT GAS DATA**

**READ (2,1091) GAMMA,GAS,PR**

**READ INPUT PARTICLE DATA**

**READ (2,1092) DIAM,HLPIC,PART,PFLT**

**READ INPUT CONTROL FLAGS**

**READ (2,1092) NPSTL,IMAX,OUT,IN**

**READ INITIAL-VALUE LINE POINTS AND FLOW PROPERTIES**

\[ \text{DO 10 K=1,NPSTL} \]
\[ \text{READ (2,1095) (SLV(K,IP),IP=1,12)} \]
\[ \text{SLV(K,5)=SLV(K,5)/RAD} \]
\[ \text{SLV(K,9)=SLV(K,9)/RAD} \]
\[ \text{CONTINUE} \]

**STORE FIRST POINT OF INITIAL-VALUE LINE IN AUXILIARY ARRAY**

\[ \text{DO 20 K=1,12} \]
\[ \text{CHARA(K,K)=SLV(1,K)} \]
\[ \text{CONTINUE} \]
\[ \text{THEH=SLV(1,5)} \]
\[ \text{CONCH=SORT(GAMMA*GAS*GC)} \]
\[ \text{WRITE (1,1686)} \]
\[ \text{GO TO 210} \]

**INCREMENT COUNTER**

\[ \text{1=1+1} \]
\[ \text{GO TO (40,50), INDEX} \]

\[ \text{3: IS THE NUMBER OF POINTS ON THE LEFT CHARACTERISTIC} \]
\[ \text{BEING CALCULATED} \]
C

40 J=J+1
   MM=J+1
   GO TO 60

C

50 J=NP+1-1
   ""1=J

THE COUNTER J K DETERMINES WHETHER THE LEFT CHARACTERISTIC
BEING CALCULATED STARTS FROM THE INITIAL-VALUE LINE OR FROM
THE CENTERLINE

50 KK=1+1-NP

LOOP FOR CALCULATING THE PROPERTIES ALONG A LEFT CHARACTER

10 L=1,J
   R=L+1
   ""1=L+1
   S=L-1
   IF(H) 7°,7°,12°
   7° CONTINUE
   IF(KK) 9°,9°,11°
   9° INDEX=2
   9° JP=1+1
      DO 10 IP=1,12
         CHAR(L,IP)=SLV(JP,IP)
         10 CONTINUE
      GO TO 120

CALCULATE FLUID PROPERTIES ON THE CENTERLINE

110 ICL=2
   VELP1=CHAR(2,2)
   VELP2=CHAR(1,2)
   TP1=CHAR(2,9)
   TP2=CHAR(2,9)
   RP1=CHAR(2,10)
   RP2=CHAR(2,10)
   TP1=CHAR(2,11)
   TP2=CHAR(2,11)
   PSI1=CHAR(2,12)
   PSI2=CHAR(2,12)
   CALL PRP(2,CHAR(2,1),CHAR(1,1),CHAR(2,2),CHAR(1,2),CHAR(2,3)
   1,CHAR(1,3),CHAR(2,4),CHAR(1,4),CHAR(2,5),CHAR(1,5),CHAR(2,6)
   2,CHAR(1,6),CHAR(2,7),CHAR(1,7),VELP1,VELP2,TP1,TP2,RP1,RP2,
   STP1,STP2,PSI1,PSI2)
- A3-6 -

```
12° CONTINUE
   GO TO 18°
13° N=L
   GO TO 15°

14° ""=1-1
15° VELP1=CHARL(K, 3)
       VELP2=CHARA(I, 3)
       TP1=CHARL(K, 7)
       TP2=CHARA(I, 7)
       RP1=CHARL(2, 18)
       RP2=CHARA(2, 11)
       TEP1=CHARL(K, 11)
       TEP2=CHARA(I, 11)
       PS11=CHARL(K, 12)
       PS12=CHARA(I, 12)
       PZ=L-0
       IF(K<1) 16°, 17°, 18°
       
       CALCULATE FLOW PROPERTIES AT AN INTERIOR MESH POINT

16° CALL IOPR (CHARL(K, 1), CHARA(I, 1), CHARL(K, 2), CHARA(I, 2), CHARL(K, 3
   1), CHARA(I, 3), CHARL(K, 4), CHARA(I, 4), CHARL(K, 5), CHARA(I, 5), CHARL(K, 6
   2), CHARA(I, 6), CHARQ(7), CHARA(I, 7), VELP1, VELP2, TP1, TP2, RP1, RP2,
   TEP1, TEP2, PS11, PS12)
   GO TO 18°

       CALCULATE FLOW PROPERTIES ON THE NOZZLE WALL

17° CALL IOPR (1, CHARL(K, 1), CHARA(I, 1), CHARL(K, 2), CHARA(I, 2), CHARL(K, 3
   1), CHARA(I, 3), CHARL(K, 4), CHARA(I, 4), CHARL(K, 5), CHARA(I, 5), CHARL(K, 6
   2), CHARA(I, 6), CHARQ(7), CHARA(I, 7), VELP1, VELP2, TP1, TP2, RP1, RP2,
   TEP1, TEP2, PS11, PS12)

       STORE THE SOLUTION OBTAINED FROM SUBROUTINE 'IOPR' IN
       THE AUXILIARY ARRAY

18° CHARA(I, 1)=X3A
       CHARA(I, 2)=Y3A
       CHARA(I, 3)=Z3A
       CHARA(I, 4)=W3A
       CHARA(I, 5)=T3A
```
CHARA(1,6)=P3A
CHARA(1,7)=R3A
CHARA(1,3)=VELP3
CHARA(1,9)=TP3A
CHARA(1,10)=RP3A
CHARA(1,11)=TEP3A
CHARA(1,12)=PS13A
200 CONTINUE
210 CONTINUE

C MOVE DATA FROM AUXILIARY ARRAY TO THE LEFT CHARACTERISTIC ARRAY

DO 240 K=1,11
DO 220 LL=1,12
CHARL(K,LL)=CHARA(K,LL)
220 CONTINUE
IF(I-IOUT) 240,230,230

C PRINT OUT SOLUTION ALONG THE LEFT CHARACTERISTIC

230 CONTINUE
IF(LINE=62) 234,232,232
232 WRITE (1,1008)
WRITE (1,1006)
LINE=0
234 XCH=CHARL(K,1)*HETER
YCH=CHARL(K,2)*HETER
TDEG=CHARL(K,5)*RAD
RKG=CHARL(K,7)/GC
PRES=CHARL(K,6)/(GC*HETER*HETER)
TEMP=CHARL(K,6)/(RGAS*CHARL(K,7))
TPDEG=CHARL(K,9)*RAD
WRITE (1,1004) XCH,YCH,CHARL(K,3),CHARL(K,4),TDEG,PRES,RKG
1,TEMP
WRITE (1,1007) CHARL(K,8),TPDEG,CHARL(K,10),CHARL(K,11)
LINE=LINE+2
240 CONTINUE
IF(I-IOUT) 260,250,250
259 WRITE (1,1005)
LINE=LINE+3

C STORE WALL POINT IN THE LEFT CHARACTERISTIC ARRAY

260 NW=W+1
DO 270 LL=1,12
CHARL('W',LL)=CHARA('W',LL)
270 CONTINUE
27° CONTINUE
IF(1-I.IAX) 30,30,280
280 RETURN

C
C FORMAT STATEMENTS
C
1001 FORMAT (4F10.6)
1002 FORMAT (12,3X,12,8X,12,8X,11)
1003 FORMAT (7F10.2/5F10.2)
1004 FORMAT (3(F10.5,1X),2(F10.4,1X),F11.4,3X,F10.4,1X,F10.2,8X,'GAS')
1005 FORMAT (/)
1006 FORMAT (26X,'MACH',2X,'FLOW'/6X,'X',10X,'Y',7X,'NUMBER',4X,'VELOC
ITY',6X,'ANGLE',2X,'PRESSURE',5X,'DENSITY',3X,'TEMPERATURE'/4X,
2'(C')',7X,'(C')',17X,'(1/SEC)',6X,'(DEG)',4X,'(KG/SQ CH)',3X,
3'(KG/CJ )',4X,'(DEG K')/)
1007 FORMAT (33X,2(F10.4,1X),14X,F10.4,1X,F10.2,6X,'PARTICLE')
1008 FORMAT (///)
C
END
SUBROUTINE COEFP (Y, E, T, P, RHO, VELP, TEP, RHOPP, TP, S, H, Q, APU, APV, IARU, ARV, CE, G, F, FP, GP, BP, C, DP)

THIS SUBROUTINE CALCULATES THE COEFFICIENTS AT EACH POINT WHICH ARE USED IN SOLVING THE DIFFERENTIAL EQUATIONS FOR THE METHOD OF CHARACTERISTICS SOLUTION TECHNIQUE

TYPE, DIMENSION : 40 COMMON STATEMENTS

REAL KGAS, MU, NU1

COMMON GAMMA, GO, R, DIAM, RHOP, PR, CPART

SOS=V/E
N=1.0/SQRT(E**2-1.0)
A=ATAN(D)
STMA=SIN(T-A)
STPA=SIN(T+A)
CTMA=COS(T-A)
CTPA=COS(T+A)
S=STMA/CTMA
H=STPA/CTPA
Q=SOS(A)/(SIN(A)*RHO/G0*V**2)
UP=VELP*COS(TP)
VP=VELP*SIN(TP)
DU=V*COS(T)-UP
DV=V*SIN(T)-VP
TEMP=P/(R*RHO)
MU=0.0900143*SQRT(TEMP)/(1.0+105.9/TEMP)
KGAS=10.0*4.0*MU
REY=RHO*DIAM*ABS(V-VELP)/(MU*GO)
CX=0.48+28.0/(REY**0.85)
NU=2.0+6.0*SQRT(RLY)*PR**0.333
CP=12.0*GAS*NU1/(MU*CX*REY)
R=(TAP*M**1.0)*(2.0*CP*(TEP-1*EP)/3.0*DIU+DV)-DU+DV
AP=0.75*CX*RHO*ABS(V-VELP)/(DIAM*RHOP*GO)
APU=AP+DP/UP
APV=AP*DV/UP
ARU=AP*RHOPP*DU
ARV=AP*RHOPP*DV
CE=2.0*AP*CP*(TEMP-TEP)/(3.0*UP*CPART)
F=-AP*GO*RHOPP*(RHO*V*E*STPA)*(1.0*B/(SOS*SOS))
G=-AP*GO*RHOPP*(RHO*V*E*-STMA)*(1.0*B/(SOS*SOS))
IF(Y=0.,X=0.00001) 20, 20, 10
10 STY=SIN(T)/Y
F=F+STY/(E*STPA)
G=G+STY/(E*STMA)
20 FP=-AP*RHOPP*GO*VELP/(V*V*RHO)*(COS(TP)+H-SIN(TP))
GP=-AP*RHOPP*GO*VELP/(V*V*RHO)*(COS(TP)+S-SIN(TP))
RP=RHO*V/GO
C=SOS*SOS/GO
DP=AP*8*RHOPP/(C*V*COS(T))
RETURN
END
SUBROUTINE HOCP (X1, X2, Y1, Y2, E1, E2, V1, V2, T1, T2, P1, P2, R1, R2, 
VELP1, VELP2, TP1, TP2, RP1, RP2, TEP1, TEP2, PSI1, PSI2)

This subroutine is a method of characteristics subprogram used to calculate the flow properties at an interior mesh point.

The gasdynamic model is rotational and a perfect gas is assumed. This subroutine uses the modified-Euler predictor-corrector technique, where averaged coefficients are used.

Points 1 and 3 are on the right characteristic, points 2 and 3 are on the left characteristic, points 5 and 3 are on the gas streamline, points 4 and 3 are on the particle streamline where point 3 is the desired solution point.

Type, dimension and common statements

COMMON NU, CONS, TETHY, PFLRT, P1, X3A, Y3A, E3A, V3A, T3A, P3A, R3A, VELP3, 
TP3A, RP3A, TEP3A, PSI3A
COMMON G, G0, R, DIAM, RHOP, PR, CPART
I=-1 
ICL=1

Calculate the coefficients at point 1

CALL COEP (Y1, E1, V1, T1, P1, R1, VELP1, TEP1, RP1, TP1, S1, H, Q1, APU, APV, 
ARU, ARV, F1, FP, GP1, B, C, D)

Average the coefficients for the predictor

S13=S1 
Q13=Q1 
G13=G1 
GP13=GP1

Calculate the coefficients at point 2

CALL COEP (Y2, E2, V2, T2, P2, R2, VELP2, TEP2, RP2, TP2, S, H2, Q2, APU, APV, 
ARU, ARV, CE, G2, F2, FP2, GP, R, C, D)
IF(Y2 < 0.000001) 70, 70, 80

Centerline approximation
AVERAGE COEFFICIENTS FOR THE PREDICTOR

80 H23=H2, Q23=Q2, F23=F2, FP23=FP2

90 I=I+1

INTERIOR MESH POINT SOLUTION

X3A=(Y2-Y1-X2*H23+X1*S13)/(S13-H23)
Y3A=Y1+S13*(X3A-X1)
GC TO (150,140), ICL

SOLUTION WHEN POINT 2 IS ON THE CENTERLINE

140 PA=(-2.0*T1+Q23*P2+2.0*Q13*P1-2.0*G13*(Y3A-Y1)-2.0*GP13*(X3A-X1))
   1-F23*(Y3A-Y2)+FP23*(X3A-X2))/(2.0*Q13+Q23)
   T3A=T1+013*(P3A-P1)+G13*(Y3A-Y1)+GP13*(X3A-X1)
   F2=F2+T3A/Y3A
   ICL=1
   GO TO 210

150 PA=(Q23*P2+Q13*P1+T2-T1-F23*(Y3A-Y2)+FP23*(X3A-X2)-G13*(Y3A-Y1)-
   2*GP13*(X3A-X1))/(Q23+Q13)
   T3A=T2-Q23*(P3A-P1)-F23*(Y3A-Y2)+FP23*(X3A-X2)

CALCULATE THE POSITION OF POINT 5 FROM THE GAS
STREAMLINE CHARACTERISTIC EQUATION.

210 JXY=1
   JX12=X1-X2
   IF(ABS(JX12)<0.00001) 230,240,240

230 DXDY=(X2-X1)/(Y2-Y1)
   DX12=Y1-Y2
   JXY=2
   GO TO 250

240 DVDX=(Y1-Y2)/DX12

250 CONTINUE
   IF(1) 260,260,270

260 T5=T3A

270 TT35=(SIN(T3A)/COS(T3A)+SIN(T5)/COS(T5))=0.5.
GO TO (280,290), IXY
280 X5=(Y3A-Y2+X2*DYDX-X3A*TT35)/(DYDX-TT35)
Y5=Y3A+TT35*(X5-X3A)
DX=X5-X2
GO TO 300

C
290 Y5=(Y3A+TT35*(X1-X3A)-TT35*DXDY*Y1)/(1.+TT35*DXDY)
X5=X1+(Y5-Y1)*DXDY
DX=Y5-Y2
C
CALCULATE THE DERIVATIVES OF THE PROPERTIES BETWEEN
POINTS 1 AND 2.
C
300 SM12=(E1-E2)/DX12
SV12=(V1-V2)/DX12
ST12=(T1-T2)/DX12
SP12=(P1-P2)/DX12
SR12=(R1-R2)/DX12
SVPL2=(VELP1-VELP2)/DX12
STP12=(TP1-TP2)/DX12
SRP12=(RP1-RP2)/DX12
STE12=(TEP1-TEP2)/DX12
C
LINEARLY INTERPOLATE FOR THE PROPERTIES AT POINT 5
C
E5=E2+SM12*DX
V5=V2+SV12*DX
T5=T2+ST12*DX
P5=P2+SP12*DX
R5=R2+SR12*DX
VELP5=VELP2+SPV12*DX
TP5=TP2+STP12*DX
RP5=RP2+SRP12*DX
TEP5=TEP2+STE12*DX
IF(I) 302,302,306
C
CALCULATE THE POSITION OF POINT 4 FROM THE PARTICLE
STREAMLINE CHARACTERISTIC EQUATION.
C
302 TP3A=TP5
TP4=TP5
304 TT34=(SIN(TP3A)/COS(TP3A)+SIN(TP4)/COS(TP4))*0.5.
GO TO (306,307), IXY
306 X4=(Y3A-Y2+X2*DYDX-X3A*TT34)/(DYDX-TT34)
Y4=Y3A+TT34*(X4-X3A)
DX=X4-X2
GO TO 308
LINEARLY INTERPOLATE FOR THE PROPERTIES AT POINT 4

307  \( Y_4 = (Y_3 + TT34 \times (X_1 - X_3) - TT34 \times DXY \times Y_1) / (1.0 - TT34 \times DXY) \)
     \( X_4 = X_1 + (Y_4 - Y_1) \times DXY \)
     \( DXX = Y_4 - Y_2 \)

CALCULATE THE COEFFICIENTS AT POINT 5

CALL COEFP \((Y_5, E_5, V_5, T_5, P_5, R_5, VELP_5, TEP_5, RP_5, S, H, N, APU, APV,\)
 \( \text{ARU}_5, \text{ARV}_5, CE, G_2, F, FP, GP, B_5, C_5, D)\)

CALCULATE THE COEFFICIENTS AT POINT 4

CALL COEFP \((Y_4, E_4, V_4, T_4, P_4, R_4, VELP_4, TEP_4, RP_4, S, H, N, APU_4, APV_4,\)
 \( \text{ARU}_4, \text{ARV}_4, CE_4, G_2, F, FP, GP, B, C, D)\)

IF \((1) \) 320, 320, 330

AVERAGE COEFFICIENTS FOR THE PREDICTOR

320  C35 = C5
     B35 = B5
     D35 = D5
     ARU35 = ARU5
     ARV35 = ARV5
     APU43 = APU4
     APV43 = APV4
     CE43 = CE4
     GO TO 340

AVERAGE COEFFICIENTS FOR THE CORRECTOR

330  B35 = (B3 + B5) \times 0.5.
     C35 = (C3 + C5) \times 0.5.
ARU35 = 0.5*(ARU3 + ARU5)
ARV35 = 0.5*(ARV3 + ARV5)
A'RV35 = 0.5*(APU4 + APV3)
APV3 = 0.5*(APV4 + APV3)
CE3 = 0.5*(CE4 + CE3)

C C C C C C
CALCULATE THE VELOCITY AND DENSITY FROM THE STREAMLINE
COMPATIBILITY EQUATIONS

C
340 V3A = V5*(P5 - P3A - ARU35*(X3A - X5) - ARV35*(Y3A - Y5))/B35
R3A = R3*(P3A - P3)/C35 - D35*(X3A - X5)
TE3 = PI*SQRT(TE3P)
E3A = V3A/SOS
U3P = VELP4*COS(TP4) + APU43*(X3A - X4)
V3P = VELP4*SIN(TP4) + APV43*(X3A - X4)
VEL3P = SQRT(U3P**2 + V3P**2)
YDOT3 = VP3/VEL3P
TP3A = ATAN(YDOT3)
TEP3A = TEP4 + CE43*(X3A - X4)
PS13A = PS14
RP3A = CF1RT*(PS13A - PS12)/PI - RP2*VELP2*COS(TP2)*0.5*(Y3A**2 - Y4**2)
1*(X3A - X2)**2*RP2*VELP2*SIN(TP2)/(0.5*UP3*(Y3A**2 - Y4**2) - Y3A*VELP3)
2*SIN(TP3A)*(X3A - X2))
C
C
C
AVERAGE COEFFICIENTS FOR THE CORRECTOR

C
H23 = (H2 + H3)*0.5.
Q23 = (Q2 + Q3)*0.5.
F23 = (F2 + F3)*0.5.
S13 = (S1 + S3)*0.5.
Q13 = (Q1 + Q3)*0.5.
G13 = (G1 + G3)*0.5.
FP23 = 0.5*(FP2 + FP3)
GP13 = 0.5*(GP1 + GP3)
C
GO TO 90
C
410 RETURN
END
- A3=15 -

SUBROUTINE "OCR (L0, X1, X2, Y1, Y2, E1, E2, V1, V2, T1, T2, P1, P2, R1, R2, 
VELP1, VELP2, TP1, TP2, RP1, RP2, TEP1, TEP2, PS11, PS12)
COMMON : XI, THET1, THET2, PFLRT, P1, X3A, Y3A, E3A, V3A, T3A, P3A, R3A, VELP3, 
TP3A, RP3A, TEP3A, PS3A
COMMON G, R, R, T1A1, R1P2, PR, CPART
I=1
GO TO (16, 20), 10

C C C C
C REDCFRE DATA TRANSFERRED THROUGH CALL STATEMENT FOR 
C ALL POINT SOLUTION
C C C C
10 X5=X1
Y5=Y1
E5=E1
V5=V1
T5=T1
P5=P1
R5=R1
VELP5=VELP1
TP5=TP1
RP5=RP1
TEP5=TEP1
PS5=PS11
TANT5=SIN(THET1)/COS(THET2)
X4=X1
Y4=Y1
E4=E1
V4=V1
T4=T1
P4=P1
R4=R1
VELP4=VELP1
TP4=TP1
RP4=RP1
TEP4=TEP1
PS4=PS11
GO TO 42

C C C C
C REDCFRE DATA TRANSFERRED THROUGH CALL STATEMENT FOR 
C CENTERLINE SOLUTION
C C C C
20 X5=X2
Y5=Y2
E5=E2
V5=V2
T5=T2
P5=P2

GO TO 42
VELP5 = VELP2
TP5 = TP2
TP5 = TP2
PS15 = PS12
X4 = X2
Y4 = Y2
'4 = E2
Y4 = V2
T4 = T2
T4 = T2
YELP4 = VELP2
TP4 = TP2
TP4 = TP2
PS14 = PS12
CALL COEP (Y1, E1, V1, T1, P1, R1, VELP1, TEP1, RP1, TP1, S1, H, Q1, APU, APV, L1A1, A1V, C1, E, F, FP, GP1, N, C, D)
S13 = S1
G13 = G1
GP13 = GP1
GO TO (50, 90), 13
46 CALL COEP (Y2, E2, V2, T2, P2, R2, VELP2, TEP2, RP2, TP2, S, H2, Q2, APU, APV, L2A2, A2V, C2, E2, F2, FP2, GP2, N, C, D)
H2 = H2
Q2 = Q2
F2 = F2
FP2 = FP2
GO TO (100, 180), 18
100 X3A = (Y3 - Y2 + H2*3*#2 - X5*Ta + Ta)/((H23 - T3A + T2))/12
Y3A = Y2 + H2*3*(X3A - X2)
T3A = T2 + T2
Y3A = (Y3A + P2 - T3A - F23*Y3A - Y2 + FP2 + (X3A - X2))/12
GO TO 221

CALL POINT SOLUTION

120 X3A = X1 - Y1/S13
Y3A = P, 0

CENTER LINE SOLUTION

130 X3A = X1 - Y1/S13
Y3A = P, 0
T3A=r, r
P3A=((13*r1-T1+G13*Y1-AP13*(X3A-X1))/Q13

10 CONTINUE
10(1) 310, 310, 330
310 CALL COEP5 (Y5, F5, V5, T5, P5, R5, VELP5, TEP5, RP5, TP5, S, u, q, AP5, APV5, ARU5, ARV5, C5, G2, F, TP, SP, T5, Q5, O)

CALCULATE THE COEFFICIENTS AT POINT 4

CALL COEP5 (Y4, E4, V4, T4, P4, R4, VELP4, TEP4, RP4, TP4, S, u, q, AP4, APV4, ARU4, ARV4, C4, G2, F, TP, SP, T4, Q4, O)

330 R35=((S3+C5)*R5)
C35=C3+C5
D35=R5
ARU35=ARU5
ARV35=ARV5
APU35=APU4
APV35=APV4
CF43=CF4

GO T 340
340 C35=(S3+C5)*R5
C35=C3+C5
D35=R5
ARU35=ARU5
ARV35=ARV5
APU35=APU3
APV35=APV3
CF43=CF4+C5)

P3A=r,5+(S3A+S5)/C35-Q35*(X3A-X5)
TF=P3A/(13*33)
SOC=2*(Y3A/CF3)
F3A=Y3A/CF3
WP3=VELP4+G3S(TP4)+APU3*(X3A-X4)
VP3=VELP4+G13*(TP4)+APU3*(X3A-X4)
VELP3=SQRT((RP3**2+VP3**2)
Y3OT3=VP3/RP3
TP3A=ATAH(Y3OT3)
TEP3A=TP3A+CF3*(X3A-X4)
PS13A=PS14
GO T0 (342, 344), 10
342 T3PA=(PELT*(PS13A-PS12)/PI-VP2+VELP2*GOS(TP2)*Q,5*(Y3A**2-Y2**2)
1+(X3A-X2)+VP2+VELP2*G13*(TP2))/Q,5*UP3*(Y3A**2-Y2**2)-Y3A*VELP3
2*G13*(TP3A)*(X3A-X2))
GO TO 346
1.VELP1+SI:(TP1))/(0.5*UP3*Y1**2)
346 CONTINUE
IF(1) 350, 350, 410
GO TO (390, 370), 10
C C CENTERLINE APPROXIMATION
C 370 STY=SI:(T1)/Y1
   N=1.0/STY^2(F3A**2-1.0)
   A=ATA**2(N)
   ST'A=SI:(C3A-A)
   C3=C3+STY/(F3A*ST'A)
   GO TO 400
C 390 '':23=((1+''3)*0.5
   N23=((2+''3)*0.5.
   F23=F2+F3)*0.5
   FP23=C.5*(FP2+FP3)
   GO TO (90, 400), 19
400 S13=(C1+G3)*0.5
   G13=(N1+G3)*0.5
   C13=(N1+C3)*0.5
   GP13=0.5*(GP1+GP3)
   GO TO 90
410 RETURN
END
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<tr>
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