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TECHNICAL NOTE

LASER RADAR TECHNOLOGY

BY

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OCTOBER 1971

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SUMMARY ABSTRACT

LIDAR (light detection and ranging, Laser Radar) is a transceiver system which measures the amount of light returned to a receiver due to backscattering of the transmitted signal from aerosols blocking the path of propagation. The term aerosol refers to any physical matter suspended in the atmosphere, other than standard atmospheric gases (O₂, N₂, CO₂, etc.). All such matter, whether solid or liquid, possesses the ability to reflect and/or absorb light energy which is incident upon it.

The extreme importance of LIDAR, as a tool for meteorological prediction and monitoring, is that light energy, returned (backscattered) to the LIDAR receiver, provides an observer with a multitude of distinct information about the volume of atmosphere which caused the light scattering. LIDAR also provides a remote ranging capability.

This paper will be devoted to an explanation of how this returned signal can be filtered for extremely detailed information concerning the portion of atmosphere with which it interacted. The study was conducted in three phases. The first phase is devoted to the LIDAR instrument itself, to include the physical parameters and governing equations, as well as the LIDAR system's advantages and shortcomings, in comparison to conventional radar systems. The second phase is devoted solely to a typical volume of the atmosphere, its physical properties, and what must be known about the physical properties. The
third phase will be devoted to linking the principles of the LIDAR to the properties of an irradiated volume of the atmosphere in an attempt to obtain a remote picture of the atmosphere through the characteristics of the returned signal. In particular, the measurement of number density of particles within a given volume, or concentration, is desired.
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In the field of meteorology, the formalism for large-scale atmospheric phenomena is well understood and documented. The worldwide grid of weather observation stations, as well as satellites, provides direct measurement or observation of the state of the atmosphere to a high degree of accuracy and reproducibility, provided the scale is large enough. It is the elusive micro and mesoscale phenomena which yet escape the interested observer. The minute scale most assuredly masks the phenomena within the larger, more gross patterns.

Thus, an indirect observation approach must be taken as is done in atomic and nuclear physics. The structure of the nucleus of an atom cannot be observed, or its properties measured, by the use of a standard laboratory microscope. The scale size is too small. Therefore, a more indirect approach is used. That is, a substance in question is bombarded with well-known elements, such as protons or electrons. The effect of the substance in question, on the known particles, is then observed in an attempt to make deductions concerning the properties or structure of the unknown. The existence of mu, pi, and k^- mesons is, for example, a result of this indirect type of observation. Similarly, the electromagnetic energy transmitted by a LIDAR system can be used to deduce indirectly the properties of a medium (the atmosphere) through which it propagates.
PHASE 1. THE LIDAR INSTRUMENT, ITS PROPERTIES AND GOVERNING EQUATIONS

The LIDAR principle is identical to that of a conventional radar system in that a pulse of energy is transmitted. For LIDAR, the transmitted signal has a wavelength in the visible or near visible region of the electromagnetic spectrum (0.2 to 1.0 microns, normally). Typically, an energy per pulse is on the order of a few joules. The duration of the pulse is usually between one and ten microseconds, although pulses shorter than one microsecond are desirable for ranging resolution. The signal, as it propagates through the atmosphere, will diverge somewhat, depending on the optical properties within the laser. Typically, LIDAR beamwidths vary from 1 to 20 milliradians. The signal, as it propagates away from the transmitter, is scattered by the medium. A small fraction of the signal will be scattered in the backward direction. It is this backscattered energy ($\Theta = 180^\circ$, Fig. 1) which is detected and measured as a function of range, or distance, at which the scattering occurred. Two quantities are now available to the observer:

a. The amount of energy returned due to scattering from a particular volume of the atmosphere.

b. The range at which this scattering occurred.

The manner in which the energy is detected and ranged is straightforward. A narrow column of the atmosphere is illuminated, and the light is scattered in all directions, as well as absorbed by the medium. Whatever fraction of the original signal that is returned to the detector is passed through a narrow band filter to reject unwanted stray light of different wavelengths, such as sunlight. The signal is then focused internally on a series of photomultipliers. The
photomultiplier tubes convert the focused light into an electrical signal which is then amplified to a usable level, calibrated, and portrayed in some form of scope display or digital readout. Frequently, the returned signal is so weak that individual photons are counted. The problem of signal amplitude measuring due to scattering from many different ranges is one of counting the number of pulses from the photomultipliers in a series of gated intervals.

The amount of power returned - i.e., the fraction of the incident power scattered in the backward direction - is dependent upon:

a. The intensity of the incident signal.

b. The wavelength of the incident signal.

c. The physical and optical properties of the scattering media.

LIDAR, rather than a conventional radar, is essential for observing small particulate matter and gases as indicated by the equation which yields the efficiency for scattering a signal. The scattering cross section ($\sigma$) is defined as:

$$\sigma = \frac{64\pi^6 \alpha^6}{\lambda^4} \left| \frac{m^2 - 1}{m^2 + 2} \right|^2 \tag{1}$$

for scattering particles with radii sizes less than the wavelength of the incident signal,

where $\alpha = \text{radius of scatterer}$

$\lambda = \text{wavelength of signal}$

$m = \text{index of refraction of scatterer}$.

If $\lambda = 10 \text{ cm}$ (conventional radar)
and \( a = 10 \text{ mm} \) (raindrop)

\[
\frac{a^6}{\lambda^4} = 10^{-4} \text{ cm}^2.
\]

This ratio is easily detectable by conventional radar.

If \( \lambda = 10 \text{ cm} \) (conventional radar)
and \( a = 10^{-4} \text{ mm} \) (dust particle)

\[
\frac{a^6}{\lambda^4} = 10^{-34} \text{ cm}^2.
\]

This ratio would not be detected by conventional radar since the dust particle is so small.

If \( \lambda = 10^{-3} \text{ cm} \) (LIDAR)
and \( a = 10^{-4} \text{ mm} \) (dust particle)

\[
\frac{a^6}{\lambda^4} = 10^{-18} \text{ cm}^2.
\]

The dust particle is now detectable due to the much shorter wavelength of the LIDAR.

Thus, a LIDAR possesses the capability of detecting extremely minute atmospheric particles, as well as gases.

PHASE 2. PROPERTIES OF ATMOSPHERIC SCATTERERS

It was shown in PHASE 1 that, for a given signal intensity \( I_0 \), the amount returned \( I_R \) due to scattering at a range \( R \) would be considerably less than \( I_0 \). The actual amount returned is dependent on the
scattering angle ($\Theta$), the wavelength ($\lambda$), and the properties of the scatterers. Therefore, if the transmitted intensity ($I_0$) is known, and the returned intensity ($I_R$) is known, some deduction concerning the properties of the scatterers can be made. However, the computation is complex.

The atmosphere is composed of many different particles and gases of many different sizes. The scattering cross section (efficiency of a particle to scatter light) was theorized by Mie (1908). His hypothesis was:

$$\sigma(\Theta) = \frac{2\pi}{k^2} \{i_1(\Theta) + i_2(\Theta)\}$$

(2)

where $\sigma(\Theta)$ is the scattering cross section for a direction $\Theta$; $i_1(\Theta)$ and $i_2(\Theta)$ are the amplitude scattering functions (Appendix I); and $k = \frac{2\pi}{\lambda}$.

The Mie solution describes the ability of a particle to scatter incident light in any direction as a function of the radius, index of refraction of the scattering particle, and the wavelength of the incident light. An indication of the completeness of the solution is the fact that it contains the two classical cases of scattering:

a. When a particle's diameter is much less than the wavelength, the Mie solution reduces to the familiar case (as in conventional radars) of Rayleigh scattering, where a particle's ability to scatter light is inversely proportional to the fourth power of the laser transmitting wavelength.
b. When a particle's diameter is orders of magnitude greater than the wavelength, the Mie solution approaches geometric scattering, where its ability to scatter light is nearly independent of wavelength and is proportional to the particle size or geometric area of the particle.

\[ \sigma = \frac{64\pi a^6}{\lambda^4} \left| \frac{m^2 - 1}{m^2 + 2} \right|^2 \]  

(3)

In these two regions, the variation of the cross section (\(\sigma\)) with \(a\) and \(\lambda\) is quite smooth. However, if \(a\) is nearly equal to \(\lambda\), \(\sigma\) is a highly fluctuating quantity and the generalized version of Mie's equation is required. From electromagnetic theory, this equation can be derived rigorously for spherical scatterers of arbitrary size. A somewhat detailed derivation is provided in Appendix I.

An important assumption was made within Equation 2; that is, that the scattering particles are spheres. The Mie theory, when applied to scattering from nonspherical particles, indicates that the shape of the particle is not nearly as important as its size and optical properties.

The most important quantity in determining the cross section, other than radius and wavelength, is the complex index of refraction. This is a quantity, associated with the particle, which defines its ability to scatter light; i.e., it indicates how well a given particle will reflect and/or absorb incident light.
Now, given an arbitrary particle radius ($a$), its index of refraction ($m$), an incident signal intensity ($I_0$), and wavelength ($\lambda$), the amount of power scattered in a direction ($\theta$) can be computed.

However, the typical atmosphere consists of scattering particles of various sizes and indices of refraction. Therefore, the total scattering cross section for an illuminated volume of the atmosphere will be the sum of the scattering cross sections for all the different types of particles (different indices of refraction) and the sum, over the size distribution (the number of particles for each type). The cross section for all particles whose index of refraction is $m$ is:

$$
\sigma_m = \int_{a_1}^{a_2} n(a)\sigma_m(a) da
$$

(4)

$$
\sigma_T = \int_{m_1}^{m_2} \sigma_m dm
$$

(5)

PHASE 3. LINKAGE OF LIDAR PRINCIPLES TO SCATTERING PRINCIPLES

The returned power is a function of the incident power and the backscattering coefficient. The LIDAR actually measures the volumetric backscattering coefficient ($\sigma_T$); i.e., the signal can be processed to yield $\sigma_T$. However, these values are not useful.

The process of diffusion in the atmosphere, as a function of time and space, is represented as concentration $M$. 
The ranging and real time capability of LIDAR will automatically provide a number which is a function of time and space, but this questionable number must be edited in such a way that it is an accurate measure of the concentration of particles (M) at any particular time and space coordinate.

Let a tracer be introduced into the atmosphere. This tracer shall fluoresce at a designated wavelength, one which the LIDAR receiver is tuned to detect. If the properties of the tracer are such that it mixes with the atmosphere and remains in suspension, then the concentration of the tracer will be the same as that of the included atmospheric sample. Since the size distribution of the tracer is known, $a_m$ in Equation 4 is known. The index of refraction of the tracer is known; hence, Equation 5 is a constant.

For these conditions the quantity actually being measured by the LIDAR is the total number of tracer particles, $n_T(a)$. Hence, the concentration of the tracer cloud is known. If observation of diffusion of the tracer were the only requirement, this method would be acceptable.

However, disseminating a tracer into a cloud each time a concentration measurement is desired is inconvenient. A tracer would be totally unacceptable if random observation of air pollution is desired.

If a tracer is not used, assumptions must be made concerning the

\[ M = \frac{4}{3\pi} \int_{a_1}^{a_2} a^3 n(a) da \]
size distribution of particles and integrated index of refraction. For a typical atmospheric sample, some particles would be falling out; hence, the size distribution is also a function of time. The effect of this and variation of index of refraction and size distribution on the concentration are currently being studied at the National Center for Atmospheric Research (NCAR). No conclusive results have yet been determined.

Two other methods could possibly circumvent this problem area. One would be the use of Raman scattering. This method totally ignores size distribution and focuses on particle identity. The only shortcoming of this type of system is laser technology itself. More power output than is now available from commercial lasers would be needed in order to produce any useful ranging capability. The other solution would be to use a standard, but wavelength tuneable, LIDAR system. This would provide more information concerning the indices of refraction of scattering particles. Again, however, improved laser technology (power output) will be required.

A LIDAR using a tracer seeded atmosphere would be orders of magnitude better than any diffusion monitoring system which now exists.
APPENDIX

DERIVATION OF SCATTERING EQUATIONS FROM MAXWELL'S EQUATIONS

There are four fundamental equations of electromagnetism--Maxwell's Equations. The equations are completely general and apply to all electromagnetic phenomena in media which are at rest with respect to coordinate system used. They are valid for nonhomogeneous, nonlinear, and even nonisotropic media.

These equations are:

\[ \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \]  
(7)

\[ \nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J} \]  
(8)

\[ \nabla \cdot \mathbf{B} = 0 \]  
(9)

\[ \nabla \cdot \mathbf{D} = \rho \]  
(10)

Some other useful relationships are:

\[ \mathbf{D} = \varepsilon \mathbf{E} \]  
(11)

\[ \mathbf{B} = \mu \mathbf{H} \]  
(12)

\[ \mathbf{J} = \sigma \mathbf{E} \]  
(13)
\[ \frac{\partial E}{\partial t} + \nabla \cdot J = 0 \]  

(14)

where

\[ \begin{align*}
\vec{E} & = \text{electric field intensity} \\
\vec{D} & = \text{electric displacement} \\
\vec{J} & = \text{current density} \\
\vec{H} & = \text{magnetic field intensity} \\
\vec{B} & = \text{magnetic induction} \\
\rho & = \text{free charge density} \\
\varepsilon & = \text{dielectric constant} \\
\mu & = \text{magnetic permeability} \\
\sigma & = \text{conductivity of medium} \\
t & = \text{time}
\end{align*} \]

From these eight equations the motion of an electromagnetic wave through some medium can be determined. The intensity of a beam at a particular set of coordinates \((x,y,z)\) and at a particular time \((t)\) are to be determined. Equations 7 through 8 are rearranged and combined along with Equations 11 through 14 to yield the wave equation. Solutions to the wave equation provide the magnitude of \(E\) and \(H\) fields as a function of \(x, y, z,\) and \(t\). However, solving Maxwell's equations in an \(x, y, z,\) and \(t\) coordinate system is a very difficult and complex procedure. Therefore, "Plane Wave Solutions" are defined as those solutions to Maxwell's equations depending only upon one space and time coordinate.

Let \(\xi\) be a space coordinate such that
\[ \mathbf{E} = \mathbf{E}(\xi,t) \]
\[ \mathbf{H} = \mathbf{H}(\xi,t) \]

Figure 2. Coordinate System Illustration.

where \( \mathbf{i} \) = unit vector in \( x \)-direction

\( \mathbf{j} \) = unit vector in \( y \)-direction

\( \mathbf{k} \) = unit vector in \( z \)-direction

\( \mathbf{n} \) = unit vector in \( \xi \)-direction

The following mathematical operators reflect the change of the \( \xi \) axis with respect to the \( x \), \( y \), and \( z \) axes:
\[
\frac{\partial}{\partial x} = \frac{\partial \xi}{\partial x} \frac{\partial}{\partial \xi} \\
\frac{\partial}{\partial y} = \frac{\partial \xi}{\partial y} \frac{\partial}{\partial \xi} \\
\frac{\partial}{\partial z} = \frac{\partial \xi}{\partial z} \frac{\partial}{\partial \xi}
\]

Figure 3. The \(\xi\) Coordinate System.

In Figure 3, the point \(P\) has two sets of space coordinates: (1) \(P = P(x,y,z)\) and (2) \(P = P(\xi)\);

that is \(\vec{\xi} = \vec{i} + \vec{y}^\dagger + \vec{z}\)
or \[ \zeta = (n \cdot \hat{i})x + (n \cdot \hat{j})y + (n \cdot \hat{k})z \]
or \[ \zeta = (n_x)x + (n_y)y + (n_z)z \]

where \[ n_x = \frac{\partial \zeta}{\partial x} \]
\[ n_y = \frac{\partial \zeta}{\partial y} \]
\[ n_z = \frac{\partial \zeta}{\partial z} \]
hence \[ \frac{\partial}{\partial x} = \frac{\partial \zeta}{\partial x} \frac{\partial x}{\partial \zeta} = n_x \frac{\partial}{\partial \zeta} \]
\[ \frac{\partial}{\partial y} = n_y \frac{\partial}{\partial \zeta} \]
\[ \frac{\partial}{\partial z} = n_z \frac{\partial}{\partial \zeta} \]
similarly \[ \nabla = \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \]
\[ = \hat{i} n_x \frac{\partial}{\partial \zeta} + \hat{j} n_y \frac{\partial}{\partial \zeta} + \hat{k} n_z \frac{\partial}{\partial \zeta} \]
\[ = (n_x \hat{i} + n_y \hat{j} + n_z \hat{k}) \frac{\partial}{\partial \zeta} \]
\[ = \frac{\partial}{\partial \zeta} \]
and \[ \nabla \times E = \frac{\hat{\zeta} \times E}{\partial \zeta} \]
and \[ \nabla \cdot \mathbf{E} = \mathbf{n} \cdot \frac{\partial \mathbf{E}}{\partial \xi} \]

Maxwell's equations may now be written in a simpler and more workable form.

\[ \nabla \times \frac{\partial \mathbf{E}}{\partial \xi} + \nabla \times \frac{\partial \mathbf{H}}{\partial \tau} = 0 \tag{15} \]

\[ \nabla \times \frac{\partial \mathbf{H}}{\partial \xi} - \epsilon \frac{\partial \mathbf{E}}{\partial \tau} - \sigma \mathbf{E} = 0 \tag{16} \]

\[ \mathbf{n} \cdot \frac{\partial \mathbf{H}}{\partial \xi} = \frac{\partial}{\partial \xi} (\mathbf{n} \cdot \mathbf{H}) = 0 \tag{17} \]

Since \( \rho \) is equal to 0 in a conductive media - i.e., no free charge - then \( \nabla \cdot \mathbf{D} \) is equal to 0. Therefore

\[ \nabla \cdot \frac{\partial \mathbf{E}}{\partial \xi} = \frac{\partial}{\partial \xi} (\mathbf{n} \cdot \mathbf{E}) = 0 \tag{18} \]

Differentiating Equation 15 with respect to \( \xi \)

\[ \nabla \times \frac{\partial^2 \mathbf{E}}{\partial \xi^2} + \mu \frac{\partial^2 \mathbf{H}}{\partial \xi \partial \tau} = 0. \tag{19} \]

Crossing \( \mathbf{n} \) into Equation 19

\[ \frac{1}{\mu} \left\{ \nabla \times \left[ \nabla \times \frac{\partial^2 \mathbf{E}}{\partial \xi^2} \right] \right\} + \left\{ \nabla \times \frac{\partial^2 \mathbf{H}}{\partial \xi \partial \tau} \right\} = 0. \tag{20} \]

Differentiating Equation 16 with respect to \( \tau \)
\[ \frac{\partial^2 \mathcal{E}}{\partial \xi^2} \frac{\partial^2 \mathcal{B}^+}{\partial \xi^2} - \frac{\partial \mathcal{E}}{\partial t^2} \frac{\partial \mathcal{B}^+}{\partial t} = 0. \quad (21) \]

Combining Equations 20 and 21

\[ \frac{1}{\mu} \left( \mathbf{n} \times \frac{\partial^2 \mathcal{E}}{\partial \xi^2} \right) + \frac{\partial \mathcal{E}}{\partial t^2} + \frac{\partial \mathcal{B}^+}{\partial t} = 0. \]

However, for any vectors \( \mathbf{A}, \mathbf{B}, \) and \( \mathbf{C} \)

\[ \mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \cdot \mathbf{C}) \mathbf{B} - (\mathbf{A} \cdot \mathbf{B}) \mathbf{C} \]

or

\[ \frac{1}{\mu} \left( \mathbf{n} \cdot \frac{\partial^2 \mathcal{E}}{\partial \xi^2} \mathbf{n} - \mathbf{n} \cdot \frac{\partial \mathcal{E}}{\partial t^2} \mathbf{n} \right) + \frac{\partial \mathcal{E}}{\partial t^2} + \frac{\partial \mathcal{B}^+}{\partial \xi} = 0 \quad (22) \]

but

\[ \mathbf{n} \cdot \frac{\partial \mathcal{E}}{\partial \xi} = \frac{\partial \mathcal{B}^+}{\partial \xi} (\mathbf{n} \cdot \mathbf{E}) = 0 \]

Therefore

\[ \mathbf{n} \cdot \frac{\partial^2 \mathcal{E}}{\partial \xi^2} = \frac{\partial^2}{\partial \xi^2} (\mathbf{n} \cdot \mathbf{E}) = 0 \]

and Equation 22 becomes

\[ \frac{\partial \mathcal{E}}{\partial t^2} - \mu \frac{\partial \mathcal{B}^+}{\partial t} + \gamma \frac{\partial \mathcal{E}}{\partial t} = 0. \quad (23) \]

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A plane wave solution of Equation 23 is

\[ \psi = e^{i(km - i\omega t)} \]

where

\[ k = \frac{\omega}{c} = \frac{2\pi}{\lambda} \]

and

\[ m = \sqrt{\mu \varepsilon - \frac{4\pi \mu_0}{\omega}} \]

Now that a representation for the propagation of an electromagnetic wave through a medium has been obtained, boundary conditions are imposed; that is, the effect of a sharp boundary between homogeneous media is to be considered. For the atmosphere, these boundaries are the gases and aerosol scatterers. An expression for the change in the electromagnetic wave (scattering) caused by these boundaries can be formulated by considering the solutions to the wave equation with specific boundary conditions.

The "disturbances" \((u,v)\) of a wave can be expressed in terms of the plane wave solutions of Equation 23. This solution, as shown by Mie, gives the components of the \(E\) and \(H\) fields at any point inside and outside a scattering particle.

\[
 u = \left\{ \frac{-i}{kr} \right\} e^{-i(kr - i\omega t)} \sum_{n=1}^{\infty} a_n \frac{2n+1}{n(n+1)} P_n(\cos \theta) \]

(24a)
Consider a simple example of this scattering phenomena, as illustrated in Figure 4. An isolated particle is irradiated by a plane electromagnetic wave (as previously defined) of intensity $I_0$. For propagation of this wave through a homogeneous medium $(\varepsilon_1, \mu_1, \sigma_1)$, its character is preserved; however, the presence of a scattering particle with properties differing from the surrounding medium $(\varepsilon_2, \mu_2, \sigma_2)$ distorts the wave.

The disturbance of the incident wave is classified according to:

a. That portion of the incident energy which is scattered or
reflected by an angle $\theta$.

b. That portion of the incident energy which is absorbed by the scattering particle.

The scattering capability of an arbitrary scattering particle is defined by its optical properties; i.e., its complex index of refraction,

$$m = \sqrt{\mu - \frac{4\pi i\omega}{\omega}}$$

where the complex term defines the absorption properties of the scatterer.

From the disturbances, as defined by Maxwell's equations (Equations 24a and 24b), a set of usable amplitude scattering functions ($S_1(\theta), S_2(\theta), S_3(\theta), S_4(\theta)$) are defined.

\[
S_1(\theta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} \left( a_n P_n(\cos \theta) + b_n R_n(\cos \theta) \right) \quad (25a)
\]

\[
S_2(\theta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} \left( a_n P_n(\cos \theta) + b_n R_n(\cos \theta) \right) \quad (25b)
\]

For a scattering medium which is spherical and homogeneous, the functions $S_3(\theta)$ and $S_4(\theta)$ are zero.

\[
\tau_n(\cos \theta) = \frac{1}{\sin \theta} P_n^1(\cos \theta) \quad \text{Associated Legendre Polynomials}
\]

\[
\pi_n(\cos \theta) = \frac{d}{d\theta} P_n^1(\cos \theta)
\]
\( \psi_n(z) = zj_n(z) \)
\( \zeta_n(z) = zh_n^{(2)}(z) \)
\( \psi'_n(z) = \frac{d}{dz} (zj_n(z)) \)
\( \zeta'_n(z) = \frac{d}{dz} (zh_n^{(2)}(z)) \)

\[
\begin{align*}
\alpha_n &= \frac{\psi'_n(y)\psi_n(x) - m\psi_n(y)\psi'_n(x)}{\psi_n(y)\zeta_n(x) - m\psi_n(y)\zeta'_n(x)} \\
\beta_n &= \frac{m\psi'_n(y)\psi_n(x) - \psi_n(y)\psi'_n(x)}{m\psi_n(y)\zeta_n(x) - \psi_n(y)\zeta'_n(x)}
\end{align*}
\]

where

\[
x = ka = \frac{2\pi a}{\lambda}
\]
\[
y = mka
\]

Consider an atmospheric model consisting of one scattering sphere in a medium. For incident radiation of intensity \( I_0 \), the radiation scattered in a direction \( \theta \) (as illustrated in Figure 4) is given by Mie as:

\[
I(\theta) = I_0 \left( \frac{i_1(\theta) + i_2(\theta)}{k^2 a^2} \right)
\]  \hspace{1cm} (26)
where

\[ k = \frac{2\pi}{\lambda} \]

\[ i_1(\theta) = |S_1(\theta)|^2 \]

\[ i_2(\theta) = |S_2(\theta)|^2 \]

Hence, Equation 26 is a general representation of the intensity of radiation scattered into an angle \( \theta \), due to the presence of a scattering medium as defined by \( i_1(\theta) \) and \( i_2(\theta) \). The quantities \( i_1(\theta) \) and \( i_2(\theta) \) are called the intensity functions.

The efficiency of an object for scattering radiation back to the source is expressed as the radar cross section \( (\sigma) \) of the object.

\[ \sigma(\theta) = \frac{2\pi}{k^2}\{i_1(\theta) + i_2(\theta)\} \]
REFERENCES


Abstract

Laser Radar (LIDAR) is a transceiver system which measures the amount of light returned to a receiver due to backscattering of the transmitted signal from the intervening media (gases, droplets, and aerosols). Besides providing a ranging capability, LIDAR is an important tool for the monitoring of atmospheric diffusion processes.

The usefulness of such a system is most evident in light of the fact that backscattered energy which is detectable to the receiver provides an observer with a multitude of information about the small volume of atmosphere which caused the scattering. Since several highly variable atmospheric parameters define the amount and manner of light scattering, hence the amount of energy returned, numerical models of the atmosphere are constructed which account for the fluctuations of the returned signal, in terms of the controlling atmospheric parameters. Thus the amplitude of the signal returned is an anomalous measurement of the state of the atmospheric volume which caused the scattering.

In order to relate the atmospheric variables to the LIDAR system variables, an exact equation which describes the motion and interaction of the signal beam with the atmosphere must be developed and modeled. The development of equations of motion which describe the propagation of an arbitrary electromagnetic wave through a media was first accomplished by Mie in 1908 and later by Van de Hulst. This derivation, beginning with Maxwell's equations, and appropriate solutions to the wave equations for the purpose of a radar system, is provided. A discussion of the implications and significance of the scattering equations and parametric relationships is then undertaken, in an attempt to isolate concentration of the atmospheric volume as a measurable quantity in terms of system variables.
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