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ENGINEERING GUIDE AND COMPUTER PROGRAMS
FOR DETERMINING TURBULENCE-INDUCED
VIBRATION AND RADIATION OF PLATES

by
Ralph C. Lebowitz
and
Dolores R. Wallace

DEPARTMENTS OF ACOUSTICS AND VIBRATION
AND APPLIED MATHEMATICS
RESEARCH AND DEVELOPMENT REPORT

January 1970

Report 2976
The Naval Ship Research and Development Center is a U.S. Navy center for laboratory effort directed at achieving improved sea and air vehicles. It was formed in March 1967 by merging the David Taylor Model Basin at Carderock, Maryland and the Marine Engineering Laboratory (now Naval Ship R & D Laboratory) at Annapolis, Maryland. The Mine Defense Laboratory (now Naval Ship R & D Laboratory) Panama City, Florida became part of the Center in November 1967.

Naval Ship Research and Development Center
Washington, D.C. 20034
ENGINEERING GUIDE AND COMPUTER PROGRAMS
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Ralph C. Leibowitz
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val Ship Systems Command, SHIPS 037.

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Report 2976
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>1</td>
</tr>
<tr>
<td>ADMINISTRATIVE INFORMATION</td>
<td>1</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>DISCUSSION AND RECOMMENDATIONS</td>
<td>2</td>
</tr>
<tr>
<td>CONCLUSION</td>
<td>5</td>
</tr>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>5</td>
</tr>
<tr>
<td>APPENDIX A – BOLT BERANEK AND NEWMAN MANUAL METHOD (DYER)</td>
<td>7</td>
</tr>
<tr>
<td>APPENDIX A1 – MATHEMATICAL ANALYSIS</td>
<td>12</td>
</tr>
<tr>
<td>APPENDIX A2 – SAMPLE PROBLEMS</td>
<td>28</td>
</tr>
<tr>
<td>APPENDIX A3 – METHOD FOR DETERMINING INPUT DATA</td>
<td>32</td>
</tr>
<tr>
<td>APPENDIX B – BOEING PROGRAM I (MAESTRELLO)</td>
<td>33</td>
</tr>
<tr>
<td>APPENDIX B1 – MATHEMATICAL ANALYSIS</td>
<td>37</td>
</tr>
<tr>
<td>APPENDIX B2 – METHOD FOR DETERMINING INPUT DATA</td>
<td>60</td>
</tr>
<tr>
<td>APPENDIX B3 – PROGRAM IDENTIFICATION</td>
<td>68</td>
</tr>
<tr>
<td>APPENDIX B4 – TEST RUNS</td>
<td>93</td>
</tr>
<tr>
<td>APPENDIX C – ELECTRIC BOAT PROGRAM (IZZO)</td>
<td>117</td>
</tr>
<tr>
<td>APPENDIX C1 – MATHEMATICAL ANALYSIS</td>
<td>121</td>
</tr>
<tr>
<td>APPENDIX C2 – METHOD FOR DETERMINING INPUT DATA</td>
<td>137</td>
</tr>
<tr>
<td>APPENDIX C4 – TEST RUNS</td>
<td>153</td>
</tr>
<tr>
<td>APPENDIX D – UNDERWATER SOUND LABORATORY PROGRAM (STRAWDERMAN)</td>
<td>201</td>
</tr>
<tr>
<td>APPENDIX D1 – MATHEMATICAL ANALYSIS</td>
<td>206</td>
</tr>
<tr>
<td>APPENDIX D2 – METHOD FOR DETERMINING INPUT DATA</td>
<td>229</td>
</tr>
<tr>
<td>APPENDIX D3 – PROGRAM IDENTIFICATION</td>
<td>231</td>
</tr>
<tr>
<td>APPENDIX D4 – TEST RUNS</td>
<td>243</td>
</tr>
<tr>
<td>APPENDIX E – BOEING PROGRAM II – FINITE ELEMENT (JACOBS AND LAGERQUIST)</td>
<td>266</td>
</tr>
<tr>
<td>APPENDIX E1 – MATHEMATICAL ANALYSIS</td>
<td>271</td>
</tr>
<tr>
<td>APPENDIX E2 – METHOD FOR DETERMINING INPUT DATA</td>
<td>300</td>
</tr>
<tr>
<td>APPENDIX E3 – PROGRAM IDENTIFICATION</td>
<td>306</td>
</tr>
<tr>
<td>APPENDIX E4 – TEST RUNS</td>
<td>307</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>313</td>
</tr>
</tbody>
</table>
### LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Geometry and Coordinate System for Boundary Layer Excitation</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>Coordinate Systems and Regions of Integration in the Time Domain</td>
<td>19</td>
</tr>
<tr>
<td>3</td>
<td>Modal Lattice and Constant Wave Number Contour for Simply Supported Plate</td>
<td>29</td>
</tr>
<tr>
<td>4</td>
<td>Transformation of Transverse Coordinates and Region of Integration</td>
<td>46</td>
</tr>
<tr>
<td>5</td>
<td>Transformation of Longitudinal Coordinates and Region of Integration</td>
<td>47</td>
</tr>
<tr>
<td>6</td>
<td>Transformation of Time Coordinates and Region of Integration</td>
<td>49</td>
</tr>
<tr>
<td>7</td>
<td>Variation of Modal Mean Square Displacement with Eddy Lifetime for a 36- x 6.5- x 0.04-Inch Panel</td>
<td>64</td>
</tr>
<tr>
<td>8</td>
<td>Computed Modal Mean Square Displacement for a 36- x 6.5- x 0.04-Inch Panel</td>
<td>65</td>
</tr>
<tr>
<td>9</td>
<td>Contours of Constant Turbulence Pressure Spectrum Level for Convected Semifrozen Pattern</td>
<td>65</td>
</tr>
<tr>
<td>10</td>
<td>Computed Displacement Spectral Density for a 12- x 6- x 0.062-Inch Titanium Panel</td>
<td>66</td>
</tr>
<tr>
<td>11</td>
<td>Coordinate System</td>
<td>122</td>
</tr>
<tr>
<td>12</td>
<td>Longitudinal Space-Time Correlation Function for a 2-Foot x 2.33-Foot x 3/8-Inch Steel Plate</td>
<td>154</td>
</tr>
<tr>
<td>13</td>
<td>Longitudinal Correlation Function for a 2-Foot x 2.33-Foot x 3/8-Inch Steel Plate</td>
<td>155</td>
</tr>
<tr>
<td>14</td>
<td>Computed Power Spectrum for a 2-Foot x 2.33-Foot x 3/8-Inch Steel Plate</td>
<td>156</td>
</tr>
<tr>
<td>15</td>
<td>Illustration of the Theoretical Model</td>
<td>207</td>
</tr>
<tr>
<td>16</td>
<td>Computed Response of a 3.5 x 3.5 x 0.1-Inch Steel Plate to Turbulent Boundary Layer Excitation</td>
<td>244</td>
</tr>
<tr>
<td>17</td>
<td>Computed Dimensionless Plate Velocity Power Spectrum at Dimensionless Coordinates (1/5, 1/3); 10 Percent Critical Damping</td>
<td>245</td>
</tr>
</tbody>
</table>
LIST OF TABLES

Table 1 – Comparison of the Computational Methods ......................................................... 3
Table 2 – Identification for Subprograms A, B, C, and D - Maestrello ................................. 68
Table 3 – Computer Listings for Subprograms A, B, C, and D - Maestrello .......................... 94
Table 4 – Identification for Electric Boat Program - Izzo .................................................... 139
Table 5 – Computer Listings for Electric Boat Program - Izzo ............................................. 157
Table 6 – Identification for USL Subprograms I and II - Strawderman ............................... 232
Table 7 – Computer Listings for USL Subprograms I and II - Strawderman ............................ 246
Table 8 – Sample Printouts of Boundary Layer Input Data and Computed Force Co-Power Spectral Density Matrix - Jacobs .......................................................... 308
ABSTRACT

This report is an engineering guide to the use of the Dyer method of manual computation and to several computer programs for determining turbulence-induced vibration and radiation of finite plates in air and in water. Both simple and clamped boundary conditions are treated. The Dyer method and the computer programs are presented in a series of appendixes:

A. Bolt Beranek and Newman Manual Method (Dyer)
B. Boeing Program I (Masestrello)
C. Electric Boat Program (Izzo)
D. Underwater Sound Laboratory Program (Strawderman)
E. Boeing Program II – Finite Element (Jacobs and Lagerquist)

The documentation is intended to facilitate the performance of flow-induced vibroacoustic computations as well as to furnish the groundwork for future research. It should also act as a theoretical guide for experimentalists. In the broader view, the documentation represents the initial steps of an effort to use computer programs to bridge the gap between vibroacoustic research results and design needs for structures that are subject to excitation by turbulence. Research tending to improve and extend the present program is recommended.

ADMINISTRATIVE INFORMATION

This study was sponsored by the Naval Ship Systems Command (NAVSHIPS) Code 037. Funding was provided by NAVSHIPS 0311 under Subproject S-R003 10 01, Task 11701.

INTRODUCTION

For several years, the Naval Ship Research and Development Center (NSRDC) has been concerned with computing the vibration and acoustic radiation of plates excited by fully developed turbulence. As indicated in Reference 1, the naval need for achieving accurate methods of computation exceeds the current state of the art for performing such computations.* Accurate computational methods for plates can provide a useful foundation for extension to more complex naval structures, e.g., ribbed sonar domes.

It is of interest to document the flow-induced vibroacoustic digital computer solutions that have been the subject of investigation by researchers outside NSRDC and that are germane to naval needs. These constitute a convenient reference for application and a base for

*References are listed on page 313. Technical notes are ordinarily not used as formal references in NSRDC reports. However, Reference 1 was authorized for inclusion by the Head, Department of Acoustics and Vibration and is releasable on request to him. This review of the state of the art shows that accurate methods of prediction have not yet been confirmed because of the lack of experimental data, particularly for plates in water. The rationale motivating the present study is discussed and corresponding experimental studies are recommended.
further development. Accordingly, the primary objective of this report is to present a document-

1. Theoretical methods of computation for immediate application by researchers who are interested in comparing theory and experiment, observing trends, etc.

2. Theoretical methods of computation for use as a guide in designing experiments.

3. Computational frameworks that can be modified and extended through additional re-

search to meet naval needs in an increasingly realistic manner.

4. Initial steps of an effort to use computer programs to bridge the gap between vibro-

acoustic research results and design needs for structures that are subject to excitation by turbulence.

The documentation is essentially a user's guide to the Dyer method of manual computa-

tion and to several digital computer programs for determining turbulence-induced vibration and radiation of finite plates. Simple and clamped boundary conditions are treated, and the fluid medium surrounding a plate is either air or water. The following titles identify the manual method and the computer programs treated and indicate their location in the report:

Appendix A – Bolt Beranek and Newman Manual Method (Dyer)

Appendix B – Boeing Program I (Maestrello)

Appendix C – Electric Boat Program (Izzo)

Appendix D – Underwater Sound Laboratory Program (Strawderman)

Appendix E – Boeing Program II – Finite Element – (Jacobs and Lagerquist)

Each appendix includes the appropriate notation, the mathematical development of the equations underlying the program, descriptions of input and output data and of units, computer program listings, the time required to run particular computations, flow charts and operations and rules of the computer program. Methods are also presented for determining computer program input data from either experimental or analytical results. Test runs are included to verify the results (published response curves) of the original developers of the programs and hence to indicate the successful operation of the programs at NSRDC.

The physical foundations on which the development of the equations rest are not included. The references cited in the present report direct the interested reader to appropriate literature.

The report has been organized to meet the needs of the program user.

DISCUSSION AND RECOMMENDATIONS

For convenience, the salient features of the documentation are summarized in Table 1 which identifies and compares the various methods. This table makes it easy for the potential user to identify the various features of a program that are of interest to him and to select the
## Table 1

Comparison of the Computational Methods

<table>
<thead>
<tr>
<th>Program Comparison</th>
<th>Notes on Methods</th>
<th>Theoretical Approach</th>
<th>Major Assumptions</th>
<th>Plate Boundary Conditions</th>
</tr>
</thead>
</table>
| Ball Barlow and Newman Manual Method (Cyre) | Appendix A | Cross function normal mode analysis (Lyons-Cyre theoretical model) | 1. The turbulent flow is not affected by plate motion, thus it is based on the premise that the plate velocity
| | | | 2. The boundary layer pressure field has the following characteristics:
| | | | &lt; 0.15 U, where
| | | | 3. The pressure correlation decay with time.
| | | | 4. The pressure correlation has a spatial scale similar to that of the plate motion.
| | | | 5. The pressure correlation is computed along the surface of the plate at the dimensionless free-stream velocity
| | | | 6. The pressure correlation is a homogeneous function of the spatial coordinates in a frame of reference
| | | | 7. Convective velocity as a constant given by $V_c = 0.5 V_{in}$
| Tracy Program I | Appendix B | Cross function normal mode analysis (Lyons-Cyre theoretical model) and Fourier transforms relationships between cross
| | | | 1. Same as Lines 1, 2, and 3 of Cyre assumptions, model representing turbulence as sequence of convected
| | | | 2. Lightly damped panel
| | | | 3. Orthogonal modes (except for case of coupled modes, line 1)
| | | | 4. Negligible cross-mode effect (except that it is obtained the displacement cross power spectral density the model
| | | | 5. Convective velocity as a constant given by $V_c = 0.5 V_{in}$
| | | | 6. Sample supports and that in obtaining the displacement cross power
| | | | 7. Uniform damping density, the and damped modes are included.
| Electric Lintel Program | Appendix C | Cross function normal mode analysis (Lyons-Cyre theoretical model) and Fourier transforms relationships between cross
| | | | 1. Same as lines 1, 2, 3, 5, and 7 of Cyre assumptions.
| | | | 2. Traces and subsonic damping are negligible.
| Underwater Sound Laboratory Program (Strandbeest) | Appendix D | Frequency domain analysis of spectral properties of the random variables | 1. Same as lines 1 of Cyre assumptions, boundary layer thickness near the plate is considered constant.
| | | | 2. Trench boundary layer as a homogeneous isotropic random process.
| | | | 3. Zero (or small) pressures gradient.
| | | | 4. Cavity acoustic pressure potential negligible effects on the plate, i.e., the only forces acting the plate are those associated with the turbulent boundary layer pressure fluctuations.
| | | | 5. Convective velocity as a constant given by $V_c = 0.5 V_{in}$
| | | | 6. Positive and diverging, due to the combined effect of plate and water, an assumed constant; acoustic damping effects are neglected in the description of the cavity acoustic spectral density; the only damping effects being associated with the plate
| Boeing Program II: Finite Element Method (Souza and Lagrange) | Appendix E | Finite element methods structural analysis in the frequency domain | 1. Same as lines 1, 3, and 4 of Mitsubishi assumptions.
| | | | 2. Lightly damped panel; damping properties used from, either or both
| | | | 3. Modes of the panel assumed to be concentrated at the mode points.
| | | | 4. Convective velocity is a constant given by $V_c = 0.5 V_{in}$

3
<table>
<thead>
<tr>
<th>Plot Boundary Conditions</th>
<th>Fluid Model</th>
<th>Excitation Function</th>
<th>Response Function</th>
<th>Geometry and Coordinate System for Nonlinear Models</th>
<th>Two-Phase Flow (Oscillatory Flooding)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample supports</td>
<td>Air and Water</td>
<td>Cross Correlation Function and/or Cross Spectral Density for Pressure (for determination between correlated and uncorrelated forms)</td>
<td>Space-time correlation of pressure</td>
<td>Figure 1</td>
<td>See sample problem</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>To be considered constant, mass</td>
<td>Air and water</td>
<td>Cross Correlation Function and/or Cross Spectral Density for Pressure (for determination between correlated and uncorrelated forms)</td>
<td>Space-time correlation of pressure</td>
<td>Figure 1</td>
<td>See sample problem</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>To be considered constant, density of acoustic pressure, power spectrum of acoustic pressure</td>
<td>Air and water</td>
<td>Cross Correlation Function and/or Cross Spectral Density for Pressure (for determination between correlated and uncorrelated forms)</td>
<td>Space-time correlation of pressure</td>
<td>Figure 11</td>
<td>Figure 12-14</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>To be considered constant, mass</td>
<td>Air and water</td>
<td>Cross Correlation Function and/or Cross Spectral Density for Pressure (for determination between correlated and uncorrelated forms)</td>
<td>Cross-power spectral density of turbulence pressure, i.e. cross-correlation between pressure and velocity</td>
<td>Figure 16</td>
<td>Figure 16-18</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>To be considered constant, density of acoustic pressure, power spectrum of acoustic pressure</td>
<td>Air and water</td>
<td>Cross Correlation Function and/or Cross Spectral Density for Pressure (for determination between correlated and uncorrelated forms)</td>
<td>Cross-power spectral density of turbulence pressure, i.e. cross-correlation between pressure and velocity</td>
<td>Figure 19, 24, 21</td>
<td>Figure 27, 28</td>
</tr>
</tbody>
</table>

Note: The cross-power spectral density of turbulence pressure, i.e. cross-correlation between pressure and velocity, is used in the analysis, as described in Reference 22.

Model A - non-zero mean spectrum

\[ S(f, \tau) = \frac{\sum_{i=1}^{n} x_i(t) x_i(t+\tau)}{n} \]

Model B - non-zero cross correlation

\[ S(f, \tau) = \frac{\sum_{i=1}^{n} x_i(t) x_i(t+\tau)}{n} \]

Model C - non-zero cross correlation and non-zero mean spectrum

\[ S(f, \tau) = \frac{\sum_{i=1}^{n} x_i(t) x_i(t+\tau)}{n} \]

Note: The cross-power spectral density of turbulence pressure, i.e. cross-correlation between pressure and velocity, is used in the analysis, as described in Reference 22.

The above expressions for the cross-power spectral density of turbulence pressure, i.e. cross-correlation between pressure and velocity, are based on the work presented in Reference 22. These expressions are intended to provide a general framework for the analysis of the data presented in this report.
program that most nearly meets his needs. Of course evaluation of the capability of a program for making accurate predictions with respect to naval problems requires comparison between theory and actual experiments in water.

Based on an evaluation of the computer program presented herein as well as the investigation made in Reference 1, the following recommendations are made:

1. Immediate application should be made of the programs considered to be most relevant to naval needs. A range of geometric, structural, and flow data representing naval plating under actual operating (or scaled) conditions should be submitted as input to the programs. The results of a variation in parameters should be analyzed and evaluated. Comparison of such results from different programs may yield meaningful qualitative information or trends. The conclusions drawn from such trends may provide insight into the physical nature of the problem and/or act as a guide to the design of associated experiments. Moreover, when compared with corresponding experimental results, the theoretical results will yield quantitative information on the degree of accuracy of the methods of computation. Thus, the theory in conjunction with the experimental results may lead either to modification of the existing analysis or to determination of correction factors for the theoretical results. It may also lead to determination of scaling factors for different geometries and media.

2. The methods considered to be most useful for solving naval problems should be modified and extended to include improvements that enable incorporation of the following parameters, structures, and effects:

   a. Radiation resistance
   b. Internal damping
   c. Fluid loading (added or virtual mass)
   d. Shear and rotary inertia (thick plate theory)
   e. Convection velocity
   f. Ribs
   g. Rough plates, protuberances, openings, and indentations
   h. Orthotropic plates and inhomogeneous plates
   i. Combination of plate materials (composite plates)
   j. Complex structures
   k. Boundary conditions (other than fixed or simply supported)
   l. Cross-modal coupling
   m. Surface curvature and fairness
   n. Reverberant and nonreverberant (anechoic) media

Improvements in the theory should result in the evolution of design data for selecting plate materials and geometry and structural arrangements for sonar domes and submarine hulls that will have minimum vibratory and acoustic response to turbulence excitation. In particular, design charts of vibroacoustic response as a function of structural parameters (properties and geometry) can be obtained by using a computer. These charts can be useful in establishing acoustical design criteria.
CONCLUSION

The Dyer manual method and several computer programs for determining fully developed turbulence-induced vibration and radiation of finite plates have been documented. The treatments include simple and clamped boundaries, and the environment of the plates is either air or water. Methods have also been given for determining computer input data from either experimental results or analysis. The methods are useful for immediate application by theoretical and experimental researchers and also provide a basis for modification and extension to more accurate programs that are capable of meeting naval needs in an increasingly realistic and practical manner. These achievements can be attained (1) through a better understanding of the physical foundations of the problem and hence an improved representation of the models used and the quantities to be included in the analyses, (2) through improved (new) methods of mathematical analysis as well as practical extension of presently used methods of analysis, and (3) through improvements in computer programming techniques and computer capabilities.

ACKNOWLEDGMENTS

Various personnel played significant roles in the successful completion of this project. The authors are indebted for the supervisory aid and encouragement of Messrs. G.J. Franz and G. H. Gleissner and Drs. E. H. Cuthill, F. N. Frenkiel, and F. Theilheimer. In the initial stages of the work, Dr. Theilheimer also provided mathematical assistance with respect to many phases of the computer program and so merits special appreciation.

For the provision of computer programs and additional assistance relevant to their successful operation, the authors gratefully acknowledge the help of Mr. L. Maestrello, Mrs. F. Gasche, Mr. L. Jacobs, Mr. D. Lagerquist, Mr. K. Tsurusaki, and Mr. M. C. Young of Boeing Company; Mr. Budzyk and the members of his staff at the Electric Boat Division of General Dynamics Corporation; Dr. W. A. Strawderman of the U. S. Navy Underwater Sound Laboratory; and Messrs. Dave Smith, Nelson Wolfe, Mel Eifert, and Ralph Shemovitz of Wright-Patterson Air Force Base.
APPENDIX A

BOLT BERANEK AND NEWMAN MANUAL METHOD (DYER)

APPENDIX A1 – MATHEMATICAL ANALYSIS
APPENDIX A2 – SAMPLE PROBLEM
APPENDIX A3 – METHOD FOR DETERMINING INPUT DATA
NOTATION

A
Correlation area

$A_{m_n}(r_0,\omega)$
A coefficient

$a'_{m_n}$
Total modal damping due to structure and fluid coupling

$a_{m_n}$
Modal structural damping, positive and real

B
Bending stiffness equal to $\frac{Eh^3}{12(1-\sigma^2)}$

$C_B$
Free flexural phase velocity for a thin plate equal to $\omega^2 (B/M)^{\frac{1}{2}}$

c
Velocity of sound in fluid

$c_L$
Longitudinal bar velocity (17,000 ft/sec in steel or aluminum)

$D_{pq}$
A coefficient

d
Displacement boundary thickness

E
Young's modulus

F
Force on plate due to external and sound pressure fields

f
External pressure field

$f_c$
Sound coincidence frequency

$r_{rms}/f^2$
Root-mean-square and mean-square boundary-layer pressure, respectively

$G(r,r_0,\omega)$
Green function, which is Fourier transform of impulse response $g$

$g$
Acceleration due to gravity

$g\left(\frac{r_1}{r_0},t\right)$ or $g\left(\frac{z,y,t}{x_0,y_0,t_0}\right)$
Impulse response of plate

$H_{m_n}$
A coefficient

h
Plate thickness

$I_{m_n}(r)$
Time correlation integral

k
Acoustic wave number equal to $\omega/c$

$k_{m_n}$
Wave number equal to $\sqrt{k^2 - \Gamma^2_{m_n}}$

L
Equal to $\frac{1}{2} (L_x L_y)^{\frac{1}{2}}$

$L_{x,y,z}$
Dimensions of plate along $x,y,$ and $z,$ respectively, as shown in Figure 1.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
<td>Plate structural mass per unit area</td>
</tr>
<tr>
<td>$M'$</td>
<td>Total effective mass per unit area of plate due to structure and fluid coupling</td>
</tr>
<tr>
<td>$M_1$</td>
<td>Free-space added mass per unit area</td>
</tr>
<tr>
<td>$M_2$</td>
<td>Added mass per unit area associated with coupling to sound waves in the closed liquid-filled volume</td>
</tr>
<tr>
<td>$m, n$ or $p, q$ or $\mu, \nu$</td>
<td>Mode numbers</td>
</tr>
<tr>
<td>$N(\Gamma_{mn}), \Delta N(\Gamma_{mn})$</td>
<td>Number of modes included up to wave number $\Gamma_{mn}$ and average number of modes in a frequency band $\Delta\nu$, respectively</td>
</tr>
<tr>
<td>$n(\Gamma_{mn})$</td>
<td>Modal density</td>
</tr>
<tr>
<td>$p^2_{mn}$</td>
<td>Modal mean-square pressure (a time averaged quantity)</td>
</tr>
<tr>
<td>$\overline{p^2_{mn}}$</td>
<td>Spatial average of the modal mean-square pressure $p^2_{mn}$ (a space-time average quantity)</td>
</tr>
<tr>
<td>$\overline{p^2}$</td>
<td>Equal to $p^2_{mn} \Delta N$, the average mean square pressure for all modes $\Delta N$ in a frequency band $\Delta\nu$</td>
</tr>
<tr>
<td>$p_f$</td>
<td>Sound pressure on either side of plate at $z = L_z$</td>
</tr>
<tr>
<td>$r$</td>
<td>Represents coordinate position $x, y$</td>
</tr>
<tr>
<td>$S, ds$</td>
<td>Area, differential area equal to $dxdy$</td>
</tr>
<tr>
<td>$s$</td>
<td>Radiation efficiency</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
</tr>
<tr>
<td>$U_\infty$</td>
<td>Free stream velocity of fluid external to plate</td>
</tr>
<tr>
<td>$U(t - t_0)$</td>
<td>Unit step function</td>
</tr>
<tr>
<td>$V(x,y,z,\omega)$</td>
<td>Equal to the sum of the plate modal velocities</td>
</tr>
<tr>
<td>$V_{mn}$</td>
<td>Plate modal velocity</td>
</tr>
<tr>
<td>$v$</td>
<td>Mean convection speed along the positive $x$ direction</td>
</tr>
<tr>
<td>$v_0$</td>
<td>Hydrodynamic coincidence speed</td>
</tr>
<tr>
<td>$W_{m,n}(x,y,\zeta)$</td>
<td>Normal mode for plate</td>
</tr>
<tr>
<td>$w(r,t)$ or $u(x,y,t)$</td>
<td>Displacement of neutral plane of plate</td>
</tr>
<tr>
<td>$w$</td>
<td>Weight of steel plate per square foot</td>
</tr>
<tr>
<td>$x, y, z$</td>
<td>Coordinate system for plate (see Figure 1)</td>
</tr>
<tr>
<td>$Z_{mn}$</td>
<td>Plate modal impedance</td>
</tr>
</tbody>
</table>
\( a_m \) Convection frequency equal to \( \frac{mn}{L_x} \) and interpreted as the frequency at which the turbulent field is convected past a length of plate equal to the \( m \) modal wavelength

\( \beta \) Damping coefficient, including both viscous and hysteretic damping

\( \beta_0 \) Damping coefficient, representing viscous damping of plate structure only

\( \beta_1 \) Damping coefficient, representing added viscous damping due to radiation of energy in the fluid away from plate

\( \Gamma_{mn} \) Eigenvalue for plate, taken to be real

\( \gamma, \mu \) Coordinate system for plate (see Figure 2)

\( \delta \) Dirac delta function

\( \delta_{\mu
u} \) Kronecker delta function

\( \epsilon, \eta \) Positive quantities

\( \zeta \) Equals \( z - z' \)

\( \eta \) Loss factor

\( \theta \) Mean statistical lifetime of the turbulent state

\( \kappa \) Measure of the inverse radius of the turbulence eddy

\( \nu, \Delta \nu \) Frequency and frequency band, respectively

\( \xi \) Equals \( y - y' \)

\( \rho \) Fluid density

\( \rho_s \) Density of plate steel

\( \sigma \) Poisson's ratio

\( \tau \) Equals \( t - t' \)

\( \phi_{mn}(x,y) \) Plate eigenfunction

\( \psi_{ij}(x,y,z,t) \) Velocity potential; the space \( i = 1 \) is taken to be free from boundaries except at \( z = L_z \) and the space \( i = 2 \) is a closed space with reflective boundaries

\( \Psi_{ij}(x,y,z,\omega) \) Fourier transform of \( \psi_{ij}(x,y,z,t) \)

\( \omega \) Circular frequency of vibration

\( \omega_c \) Sound coincidence circular frequency of vibration

\( \omega_{mn} \) Damped resonance frequency, positive and real
$\omega_0$  Characteristic frequency

$\langle \ldots \rangle$  Symbols for time average operation

$\ast$  Denotes complex conjugate

$\mathcal{F}$  Symbol for Fourier transformation
APPENDIX A1 - MATHEMATICAL ANALYSIS

The differential equation governing displacement of a thin plate due to turbulent boundary layer pressure excitation on the plate surface (Figure 1) is

$$B \nabla^4 w + M \frac{\partial^2 w}{\partial t^2} + \beta \frac{\partial w}{\partial t} = -f - (p_1 - p_2) z = L_z = - F(x, y, t)$$  \hspace{1cm} (A1)

The solution of Equation (A1) is

$$w(x, t) = \int_{-\infty}^{t} dt \int \int dS_0 g(x, t/t_0, t_0) F(x, t_0)$$  \hspace{1cm} (A2)

where $g$, the impulse response of the plate, is the solution to the equation

$$B \nabla^4 g + M \frac{\partial^2 g}{\partial t^2} + \beta \frac{\partial g}{\partial t} = \delta(x - x_0) \delta(y - y_0) \delta(z - z_0)$$  \hspace{1cm} (A3)

The normal mode $W_{mn}$ for the plate, which has the form

$$W_{mn}(x, y, t) = \phi_{mn}(x, y) e^{[-\sigma_{mn} t - i \omega_{mn} t]}$$  \hspace{1cm} (A4)

satisfies the homogeneous equation for the freely vibrating plate

$$B(1 - i \eta) \nabla^4 W_{mn} + M \frac{\partial^2 W_{mn}}{\partial t^2} + \beta_0 \frac{\partial W_{mn}}{\partial t} = 0$$  \hspace{1cm} (A5)

where explicit division is shown of the damping into its hysteretic and viscous components.

The solution to the nonhomogeneous Equation (A1) will be found by obtaining $g$, for inclusion in Equation (A2), in terms of a superposition of the normal modes or eigenfunctions, Equation (A4), satisfying the homogeneous Equation (A5).

Substitution of Equation (A4) in (A5) yields the following equation for the eigenfunctions $\phi_{mn}$

$$\nabla^4 \phi_{mn} - \Gamma_{mn}^4 \phi_{mn} = 0$$  \hspace{1cm} (A6a)

where

$$\Gamma_{mn}^4 = \frac{[M(\sigma_{mn} + i \omega_{mn})^2 - \beta_0 (\sigma_{mn} + i \omega_{mn})]}{B(1 - i \eta)}$$  \hspace{1cm} (A6b)

The eigenvalue $\Gamma_{mn}$ is taken to be a real quantity. Multiplying both sides of Equation (A6b) by $B(1 - i \eta)$ and equating imaginary and real numbers, we obtain
Figure 1 – Geometry and Coordinate System for Boundary Layer Excitation
\[ a_{mn} = \frac{\beta_0}{2M} + \frac{\Gamma^4_{mn} B}{2M \omega_{mn}} \]  
(A7)

\[ \omega_{mn}^2 = \frac{B}{M} \Gamma^4_{mn} - a_{mn}^2 = \frac{B}{M} \Gamma^4_{mn} \]  
(A8)

Equations (A7) and (A8) represent two simultaneous equations for \( a_{mn} \) and \( \omega_{mn} \) as functions of the eigenvalues \( \Gamma^4_{mn} \), which, as will be shown, are determined by the boundary conditions. Equation (A1) is also to obey these conditions.

The Fourier transform pair relating the impulse response \( g \) to the Green function \( G \) is

\[ g(r_t, t_0, t_0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(r_t, t_0, \omega) e^{-i\omega(t-t_0)} d\omega \]  
(A9a)

\[ G(r_t, t_0, \omega) = \int_{-\infty}^{\infty} g(r_t, t_0, t_0) e^{i\omega(t-t_0)} dt \]  
(A9b)

Taking Fourier transforms of both sides of Equation (A3)* and noting that \( \mathcal{F} \left( \frac{d^n g}{dt^n} \right) = p^n G(\omega) \), where \( p = -i\omega \), and \( \int_{-\infty}^{\infty} \delta(t-t_0) e^{-i\omega(t-t_0)} dt = 1 \), we obtain

\[ \nabla^4 G - \Gamma^4 G = \frac{-\delta(x-x_0) \delta(y-y_0)}{B(1+i\eta)} \]  
(A10a)

where

\[ \Gamma^4 = \frac{\omega^2 M + i\omega \beta_0}{B(1+i\eta)} \]  
(A10b)

Assume that the Green function \( G \) can be expanded in terms of the eigenfunctions \( \phi_{mn} \). Then

\[ G(r_t, t_0, \omega) = \sum_{m,n} A_{mn}(t_0, \omega) \phi_{mn}(r) \]  
(A11)

To evaluate the coefficients \( A_{mn} \), substitute Equation (A11) in (A10a); using Equation (A6a), multiply by \( \phi_{pq} \) and integrate over \( S \), interchanging the summation and integral and normalizing the eigenvalues for convenience thus:

\[ \int \phi_{mn}(r) \phi_{pq}(r) dS = \delta_{mp} \delta_{nq} \]  
(A12)

*Equation (A3) modified to show explicit division of damping as in Equation (A5).
where

\[ \delta_{mp} \delta_{nq} = 0 \quad m \neq p \text{ or } n \neq q \]

\[ \delta_{mp} \delta_{nq} = 1 \quad m = p, \ n = q \]

These steps give (noting that the summation is dropped, since the final expression is true for all \( m, n \))

\[ A_{mn} = \frac{1}{\nu(1-i\eta)} \frac{\phi_{mn}(\tau_0)}{\Gamma_{mn}^4 - \Gamma^4} \]  

(A13)

Substituting Equation (A13) in (A11), we obtain

\[ G(r, \tau_0, \omega) = \frac{1}{B(1-i\eta)} \sum_{m,n} \frac{\phi_{mn}(\tau) \phi_{mn}(\tau_0)}{\Gamma_{mn}^4 - \Gamma^4} \]  

(A14)

Using Equations (A6b) and (A10b), it can be shown that

\[ \frac{1}{B(1-i\eta)} \frac{1}{\Gamma_{mn}^4 - \Gamma^4} = \frac{1}{M(p - ia)(p - (-ia + \beta_0/M))} \]

where

\[ a = \omega_{mn} - ia_{mn} \]

We also use the approximation, which ignores the hysteretic term \( \beta_0/M = 2a_{mn} \); see Equation (A7). Fourier transform tables \(^7\) can now be used to find

\[ g(r/t, \tau_0) = \sum_{m,n} \frac{\phi_{mn}(\tau) \phi_{mn}(\tau_0)}{\omega_{mn} M} e^{-q_{mn}(t-\tau_0)} \sin \omega_{mn}(t-\tau_0) U(t-\tau_0) \]  

(A15)

where the unit step function \( U(t-\tau_0) \) corresponds to \( \delta(t-\tau_0) \) in the excitation.

For a finite plate immersed in a low-density fluid (e.g., air), the radiation reaction on the plate may be neglected, i.e., \( p_1 - p_2 < < f \). Thus, for zero fluid load, Equation (A2) becomes

\[ u(r, \omega) = \int_{-\infty}^t dt_0 \int_S dS_0 g(r/t, \tau_0) f(\tau_0, t_0) \]  

(A16)

The cross correlation of the displacement is\(^6\)
\[ <w(r,t) w^*(r',t')> = \int_{-\infty}^{t} dt_0 \int_{-\infty}^{t'} dt'_0 \int_s dS_0 \int_s dS'_0 g(r, t/t_0, t_0) \]

\[ g^*(r', t'/t'_0, t'_0) <f(r_0, t_0) f^*(r'_0, t'_0)> \]  \hspace{1cm} (A17)

where Dyer used \(^2\)

\[ <f(r,t) f^*(r',t')> = A f^2 \delta(\xi - \nu \tau) \delta(\xi) e^{-\frac{|r|}{\theta}} \]  \hspace{1cm} (A18)

We assume simply supported boundaries at the plate edge

\[ w = \frac{\partial^2 w}{\partial x^2} = 0 \quad \text{at} \quad z = 0, L_x \]

\[ w = \frac{\partial^2 w}{\partial y^2} = 0 \quad \text{at} \quad y = 0, L_y \]  \hspace{1cm} (A19)

The eigenfunctions (normalized solutions of Equation (A6a) that obey Equation (A19)) and the corresponding eigenvalues, neglecting the effect of damping, are \(^9\)

\[ \phi_{mn}(r) = \frac{2}{(L_x L_y)^{1/2}} \sin \frac{m \pi r}{L_x} \sin \frac{n \pi y}{L_y} \]  \hspace{1cm} (A20)

\[ \Gamma_{mn}^2 = \left( \frac{m \pi}{L_x} \right)^2 + \left( \frac{n \pi}{L_y} \right)^2 \]  \hspace{1cm} (A21)

Equations (A18), (A20), and (A21) are now used to determine the plate vibrations, Equation (A17). Equation (A17) involves the product of two doubly infinite sums; a typical term is represented by the cross product of two modes \((m,n)\) and \((p,q)\)

\[ <w(r,t) w^*(r',t')>_{mn,pq} = \frac{A f^2 \phi_{mn}(r) \phi_{pq}(r')}{\omega_{mn} \omega_{pq} \mu^2} \int_{-\infty}^{t} dt_0 \int_{-\infty}^{t'} dt'_0 \]

\[ \int_s dS_0 \int_s dS'_0 \phi_{mn}(r_0) \phi_{pq}(r'_0) \left[ e^{-\frac{\omega_{mn}(t - t_0)}{\theta}} - \frac{\tau_0}{\theta} \right] \]

\[ \cdot \sin \omega_{mn}(t - t_0) \sin \omega_{pq}(t' - t'_0) \delta(\xi_0 - \nu \tau_0) \delta(\xi'_0) \]  \hspace{1cm} (A22)
If the slope of the plate displacement is small compared to unity, then Equation (A22) is essentially the correlation of the plate normal displacement. Because of the delta function $\delta(x) = \delta(y_0 - y_0')$, the $y_0, y_0'$ space integrations readily yield the term $\delta_n \frac{L_x}{2} \cos \alpha_m \tau_0$, where

$$\alpha_m = \frac{m \pi \nu}{L_x}$$

Thus the result for the spatial integration is

$$\cos \alpha_m \tau_0 \delta_{mnq} \delta_{nmq'}$$

and Equation (A22) may now be written

$$<w(r,t) w^*(r',t')>_{mn, pq} = \frac{AF_\alpha \phi_{mn}(r) \phi_{pq}(r')}{\omega_m \omega_p \mu^2} \int_{-\infty}^{t} dt_0 \int_{-\infty}^{t'} dt_0'$$

$$\left[ -a_m (t - t_0) - a_{pq} (t' - t_0') - \frac{1}{2} \right]$$

$$\cdot \sin \omega_m (t - t_0) \sin \omega_p (t' - t_0') \delta_{mn} \delta_{pq} \cos \alpha_m \tau_0$$

(A24)

To facilitate integration, a new coordinate system $\gamma, \mu$ is introduced, where $\gamma$ and $\mu$ are related to the coordinates $t_0, t_0'$ as follows

$$\gamma = (t' - t_0') - (t - t_0) = \tau_0 - \tau$$

$$\mu = (t' - t_0') + (t - t_0)$$

(A25)

The differentials of the $\gamma, \mu$ and $t_0, t_0'$ coordinate systems are related by means of the Jacobian

$$dt_0 \, dt_0' = \frac{\partial (t_0, t_0')}{\partial (\mu, \gamma)} \, dy \, d\mu = \begin{vmatrix} \frac{\partial t_0}{\partial \mu} & \frac{\partial t_0}{\partial \gamma} \\ \frac{\partial t_0'}{\partial \gamma} & \frac{\partial t_0'}{\partial \mu} \end{vmatrix} \, dy \, d\mu = \begin{vmatrix} 1 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{vmatrix} \, dy \, d\mu$$

$$dt_0 \, dt_0' = \frac{1}{2} \, dy \, d\mu$$

(A26)
Determination of the limits of integration for \( \mu \) and \( y \) is intricate. The limits are determined from construction of the \( \mu, y \) coordinate system as shown in Figure 2. The figure shows that the limits are \(-\mu\) to \(\mu\) for \( y \) and \(0\) to \(\infty\) for \( \mu \).

Then (letting \( P = \frac{m}{n} \) by virtue of the delta functions) Equation (A24) becomes

\[
\langle w(r, t) w^*(r', t') \rangle_{mn} = \frac{A P^2 \phi_{m,n}(r') \phi_{m,n}(r)}{4\omega_m^2 M^2} l_{m,n}(\tau) \tag{A27}
\]

where for the correlation integral \( l_{m,n}(\tau) \)

\[
l_{m,n}(\tau) = \int_{-\mu}^{\mu} \int_{-\mu}^{\mu} \left[ \frac{\kappa}{\theta} \left( \frac{y + \tau}{\theta} \right) \right] e^{\cos\alpha (y + \tau) \left( \cos \omega_m y - \cos \omega_m \mu \right)} ; \quad \tau \geq 0 \tag{A28}
\]

Because of the absolute value sign in the integrand, Equation (A28) must be integrated in separate regions, depending in part on whether \( y \) is greater or less than \(-\tau\) (i.e., \( \tau_0 > 0 \) or \( \tau_0 < 0 \)). Figure 2 shows the regions within which the integral must be evaluated. Thus, Equation (A28) becomes

\[
l_{m,n}(\tau) = \left\{ \int_0^\infty \int_0^{\tau} \left[ \frac{y + \tau}{\theta} \right] e^{\cos\alpha (y + \tau) \left( \cos \omega_m y - \cos \omega_m \mu \right)} + \int_0^{\infty} \int_{-\tau}^0 \left[ \frac{-y - \tau}{\theta} \right] e^{\cos\alpha (y + \tau) \left( \cos \omega_m y - \cos \omega_m \mu \right)} \right\} ; \quad \tau \geq 0 \tag{A29}
\]

For \( \tau < 0 \), \( \tau \) is replaced by \(-\tau\) in Equation (A29).

Equation (A29) is laborious to evaluate in generality. Dyer gives the following results for \( l_{m,n}(\tau) \) for special cases. The results are not directly applicable to underwater problems because the plate is assumed to be immersed in a low-density fluid. However, with modification, they can be used for underwater problems.

**MEAN-SQUARE RESPONSE AT COINCIDENCE**

The time integral is now specialized to \( \tau = 0 \) as well as \( \alpha_m = \omega_m n \), i.e., \( v = v_0 = C_B \)

\[
\left[ 1 + \left( \frac{mL_x}{nL_y} \right)^2 \right]^{1/2} \quad \text{where} \quad v_0 \quad \text{is the hydrodynamic coincidence speed and} \quad C_B = \omega^{1/2} \left( \frac{B}{\rho M} \right)^{1/4}.
\]
\( \gamma = (t' - t_0^0) - (t - t_0) = \tau_0 - \tau; \tau_0 = t_0 - t' \)

\( \mu = (t' - t_0^0) + (t - t_0); \tau = t - t' \)

Hence \( \gamma = \mu = 0 \) corresponds to \( \tau_0 = 4, t_0' = t' \). The line corresponding to \( \mu = 0 \) is \( t' - t_0 = t_0 - t \) which has the slope \( \left( \frac{dt_0'}{dt_0} \right)_{\mu=0} = -1 \). The line corresponding to \( \gamma = 0 \) is \( t' - t_0 = t - t_0 \) which has the slope \( \left( \frac{dt_0'}{dt_0} \right)_{\mu=0} = -1 \).

The lines are perpendicular because \( \left( \frac{dt_0'}{dt_0} \right)_{\mu=0} \cdot \left( \frac{dt_0'}{dt_0} \right)_{\mu=0} = -1 \). Since \( \left( \frac{\partial \gamma}{\partial t_0} \right)_{\mu=0} = \left( \frac{\partial \mu}{\partial t_0} \right)_{\gamma=0} = -1 \), the positive directions are those shown in Figure 2. When \( \gamma = \mu \) or \( (t' - t_0^0) - (t - t_0) = (t' - t_0) + (t - t_0) \) then \( t_0' = 2t' - t_0 \) represents a line with intercepts \( t_0 = 2t', t_0' = 2t' \) and slope \( \frac{dt_0'}{dt_0} = -1 \).

When \( \gamma = \mu \) or \( (t' - t_0') - (t - t_0) = (t' - t_0') + (t - t_0) \) then \( t_0 = t_0 \). Similarly when \( \gamma = \mu \) then \( t_0^0 = t' \).

Consider Region AA'B'B':

\[
\begin{align*}
\mu &> r - t > 0 \\
\gamma &< -\eta > 0
\end{align*}
\]

\( y + \mu = 2(t' - t_0') \) or \( y = -\mu + 2(t' - t_0') \) or \( y = -\eta = -(r - \eta) = 2(t' - t_0') \) or \( t' - t_0' = \frac{-2(y + \mu)}{2} \) a negative number.

Hence \( t_0^0 > t' \). Figure 2 shows that \( t_0^0 > t' \), \( \mu = r - t \), \( y < -r \) defines the region AA'B'B'. Since \( t_0 > y + r \) then \( y < -r \) implies \( t_0 < 0 \) for this region.
Notes for Figure 2 (Continued)

Consider Region OA'ACC
\[
\begin{align*}
\mu &= r + \epsilon; \; \epsilon > 0 \\
y &= -r + \eta; \; \eta > 0 
\end{align*}
\]
\[
y = -\mu + 2(t' - t'_0) - r + \eta = -\epsilon + 2(t' - t'_0) \quad \text{or} \quad t' - t'_0 = \frac{\epsilon - \eta}{2} \\
t'_0 > t', \; \mu = r + \epsilon, \; y = -r + \eta \quad \text{corresponds to region OA'ACC'. Since } t_0 = y + r \text{ then } y > -r \text{ implies } t_0 > 0 \text{ for region OA'ACC'}.
\]

Consider Region ACD'D
\[
\begin{align*}
\mu &= r + \epsilon; \; \epsilon > 0 \\
y &= -r + \eta; \; \eta > 0 
\end{align*}
\]
\[
y = -\mu + 2(t' - t'_0); \; -r + \eta = -\epsilon + 2(t' - t'_0); \; t' - t'_0 = \frac{\epsilon + \eta}{2} \text{ a positive number. Hence } t'_0 < t', \; \mu = r + \epsilon, \\
y = -r + \eta \quad \text{define the region. Since } t_0 = y + r \text{ then } y > -r \text{ implies } t_0 > 0.
\]

Consider Region (AD'D'' + AD'D''')
\[
\begin{align*}
\mu &= r + \epsilon; \; \epsilon > 0 \\
y &= -r - \eta; \; \eta > 0 
\end{align*}
\]
\[
y = -\mu + 2(t' - t'_0); \; t' - t'_0 = \frac{\epsilon - \eta}{2}; \; t'_0 < t', \; \mu = r + \epsilon, \; y = -r - \eta \quad \text{corresponds to region AD'D''}. \text{ Since } t_0 = y + r \text{ then } y < -r \text{ implies } t_0 < 0. \text{ If } t'_0 < t' \text{ then } y > -\mu \text{ (see Figure 2). If } t'_0 > t' \text{ then } y < -\mu \text{ which is an invalid condition since } y = -\mu \text{ is the lower limit in the integral, Equation (A28), corresponding to } t'_0 < t'. \text{ Hence region AD'D''} \text{ is excluded in the integration. From the Figure it is clear that the limits of } y \text{ depend on the upper limits of both } t_0 \text{ and } t'_0. \text{ Thus when } t_0 = t \text{ (a limit in Equation (A24)) } y = \mu \text{ and when } t'_0 = t', \; y = -\mu. \text{ The limits of } \mu \text{ in Equation (A28) are obviously from } 0 \text{ to } \mu. \text{ And } t'_0 \text{ range from the upper limit } t_0 = t, \text{ to the lower limit } t'_0 (-\mu, \text{ in Equation (A24). That is, the limits are:}}
\]
\[
-\mu \leq \mu \text{ for } y \\
0 \leq \mu \text{ for } \mu
\]

20
Then

\[ I_{mn}(0) = \frac{\theta}{a_{mn}(a_{mn} \theta + 1)} + \frac{\theta^2}{(a_{mn} \theta + 1)^2 + 4 \omega_{mn}^2 \theta^2} \left[ \frac{2 - c_{mn} \theta^2}{4 \omega_{mn}^2 \theta^2} \right], \quad \omega_{mn} \theta > 1 \]  

(A30)

\[ I_{mn}(0) = \frac{\theta}{a_{mn}}, \quad a_{mn} \theta << 1 \text{ (low damping)} \]  

(A31)

\[ I_{mn}(0) = \frac{\theta}{a_{mn}(a_{mn} \theta + 1)}, \quad \omega_{mn} \theta > 1 \text{ (high frequencies)} \]  

(A32)

**MEAN-SQUARE RESPONSE BELOW COINCIDENCE**

The time integral is specialized to \( \tau = 0 \) as well as \( \alpha_m \ll \omega_{mn} \) (i.e., \( v << v_0 \)). We get

\[ I_{mn}(0) = \frac{2 \theta^2}{1 + \omega_{mn}^2 \theta^2} \left[ \frac{\omega_{mn}^2 \theta^2 - (a_{mn} \theta + 1)}{\omega_{mn}^2 \theta^2 + (a_{mn} \theta + 1)^2} \right] + \frac{1}{\omega_{mn}^2 \theta^2}, \quad \alpha_m \ll \omega_{mn} \]  

(A33)

\[ I_{mn}(0) = \frac{2 \theta}{a_{mn}} \frac{1}{1 + \omega_{mn}^2 \theta^2}, \quad a_{mn} \theta << 1 \text{ (low damping)} \]  

(A34)

\[ I_{mn}(0) = \frac{2}{\omega_{mn}^2} \left[ 1 + \frac{1}{a_{mn} \theta} + \frac{1}{1 + \alpha_m \theta^2} \right], \quad \omega_{mn} \theta > 1 \text{ (high frequencies)} \]  

(A35)

**DISPLACEMENT CORRELATION BELOW COINCIDENCE**

Here, \( \tau \neq 0 \) and \( \alpha << \omega_{mn} \) is considered.

\[ I_{mn}(\tau) = \frac{2 \theta}{a_{mn}} \frac{\varepsilon^{-a_{mn}|\tau|}}{1 + \omega_{mn}^2 \theta^2} \cos \omega_{mn} \tau, \quad a_{mn} \theta << 1 \text{ (low damping)} \]  

(A36a)

When substituted in Equation (A27) the results give the cross correlation of (if \( \tau = 0 \)) mean-square displacement. For the latter case (\( \tau = 0 \))
\[ I_m(0) = \frac{2\theta}{a_{mn}} \cdot \frac{1}{1 + \omega_m^2 \theta^2}, \quad a_{mn} \theta < 1 \text{ (low damping)} \quad (A36b) \]

Now the radiation of boundary layer noise into the closed space shown in Figure 1 is considered, particularly the underwater case. The sound field on either side of the plate is governed by the nondissipative linear wave equation of acoustics for a homogeneous, loss- and source-free medium at rest.

\[ \nabla^2 \psi_j - \frac{1}{c^2} \frac{\partial^2 \psi_j}{\partial t^2} = 0 \quad (A37) \]

Within the closed space, let \( j = 2 \). The Fourier transform \( \Psi_2(x,y,z,\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \psi_2(x,y,z,t) e^{-i\omega t} dt \) of the velocity potential \( \psi_2 \) satisfies the Helmholtz equation

\[ \nabla^2 \Psi_2 + k^2 \Psi_2 = \frac{\partial^2 \Psi_2}{\partial x^2} + \frac{\partial^2 \Psi_2}{\partial y^2} + \frac{\partial^2 \Psi_2}{\partial z^2} + k^2 \Psi_2 = 0 \quad (A38) \]

since \( \int \left( \frac{\partial^2 \psi_2}{\partial t^2} \right) - \omega^2 \psi_2 \) and \( k = \frac{\omega}{c} \).

Except for the plate, all interior surfaces are assumed to be pressure release surfaces. Therefore, the boundary conditions are

\[ \Psi_2 = 0 \begin{cases} x = 0, & L_x \\ y = 0, & L_y \\ z = 0 \end{cases} \quad (A39) \]

Assume a general solution for Equation (A38) in the form of normal modes \( p,q \)

\[ \Psi_2(x,y,z,\omega) = \sum_{p,q} D_{p,q} \phi_{p,q}(r) \sin k_{p,q} z \quad (A40) \]

it will be shown that Equation (A40) satisfies the boundary conditions in Equation (A39). Substitution of Equation (A20) in Equation (A40) and the resultant equation in Equation (A38) gives

\[ k_{p,q}^2 = k^2 - \left[ \left( \frac{pn}{L_x} \right)^2 + \left( \frac{qn}{L_y} \right)^2 \right] = k^2 - \Gamma_{p,q}^2 \quad (A41) \]

where \( \phi_{p,q} \) and \( \Gamma_{p,q} \) are the eigenfunctions and eigenvalues given in Equations (A20) and (A21), respectively.
Equation (A37) is coupled to Equation (A1) by the continuity condition on velocity

$$\frac{\partial \psi_1}{\partial t} = \frac{\partial \psi_2}{\partial t} \bigg|_{z = L_z} = -\frac{\partial \psi_2}{\partial \xi} \bigg|_{z = L_z}$$  \hspace{1cm} (A42)

Hence

$$-\frac{\partial \psi_2}{\partial z} \bigg|_{z = L_z} = \frac{1}{2\pi} \int_0^{2\pi} \frac{\partial \omega_2}{\partial t} (x, y, z, t) e^{-i\omega t} \, dt \bigg|_{z = L_z}$$

$$= V(x, y, z, \omega) = \sum_{m,n} V_{mn} = -\sum_{m,n} H_{mn} \phi_{mn}$$  \hspace{1cm} (A43)

where $H_{mn} \phi_{mn}$ is the plate modal velocity. Using Equation (A40) in Equation (A43)

$$D_{mn} = -\frac{H_{mn}}{k_{mn} \cos k_{mn} L_z}$$  \hspace{1cm} (A44)

Substitution of Equations (A44) and (A20) in Equation (A40) gives

$$\Psi_2(x, y, z, \omega) = -\frac{2}{(L_x L_y)^{1/2}} \sum_{m,n} \frac{H_{mn}}{k_{mn} \cos k_{mn} L_z} \sin m \pi x \sin n \pi y \sin k_{mn} z$$  \hspace{1cm} (A45)

which satisfies the boundary conditions Equation (A39).

Since the pressure $p_2(x, y, z, t) = \rho \frac{\partial \psi_2(x, y, z, t)}{\partial t}$, then the modal pressure transform is

$$[P_2(x, y, z, \omega)]_{mn} = \frac{1}{2\pi} \left[ \int_{-\infty}^{\infty} \rho \frac{\partial \psi_2}{\partial \xi} e^{-i\omega t} \, dt \right]_{mn}$$

$$= -i \omega \rho \Psi_2(x, y, z, \omega)_{mn}$$

Using these equations as well as Equations (A40), (A43), and (A44), the plate modal impedance (at $z = L_z$) is obtained

$$Z_{mn}(x, y, z, \omega) = \frac{P_{mn}(x, y, z, \omega)}{V_{mn}} = \frac{P_{mn}(x, y, z, \omega)}{-H_{mn} \phi_{mn}} = \frac{-i \omega \rho \Psi_2_{mn}}{k_{mn} \tan k_{mn} L_z} = -i \omega M_2$$  \hspace{1cm} (A46)
From Equations (A41) and (A8) as well as the definition of $C_B$

$$k_{mn}^2 = k^2 - r_{mn}^2 = k^2 - \left( \frac{M}{B} \right)^{1/2} \frac{\omega}{\omega_{mn}} = \frac{\omega^2}{c^2} - \frac{\omega \omega_{mn}}{C_B^2}$$

Defining a sound coincidence frequency $\omega = \omega_c$, corresponding to $c = C_B$ then

$$k_{mn} = \pm \frac{\omega_c}{C_B} \sqrt{1 - \frac{\omega_{mn}}{\omega_c}}$$

which is real for $\omega_{mn} < \omega_c$ and imaginary for $\omega_{mn} > \omega_c$. Equation (A46) then yields

$$M_2 = \frac{\rho \tanh |k_{mn}L_z|}{|k_{mn}|}, \quad \omega_{mn} > \omega_c \quad (A47a)$$

$$M_2 = \frac{\rho \tan k_{mn}L_z}{k_{mn}}, \quad \omega_{mn} < \omega_c \quad (A47b)$$

The added mass $M_1$ of the free space external to the cavity, provided the mode number is not too low, is taken to be

$$M_1 = \frac{\rho}{|k_{mn}|}, \quad \omega_{mn} < \omega_c \quad (A48)$$

When the mode number is low, the resistive impedance (viscous damping coefficient) is taken to be

$$\beta_1 = \rho \alpha s \quad (A49)$$

where

$$s = \frac{2}{\pi m} + \frac{2}{\pi n} \frac{\left( \frac{\omega_{mn}L}{c} \right)^2}{1 + \left( \frac{\omega_{mn}L}{c} \right)^2}, \quad \omega_{mn} < \omega_c$$

$$L = \frac{1}{2} (L_xL_y)^{1/2}$$

Equation (A27) gave the modal displacement correlation for a plate in a low-density fluid. The correlation for a high-density fluid is obtained by replacing the structural mass in Equation (A27) by the total mass.
\[ M' = M + M_1 + M_2 \]  

(A50)

where \( M \) is the plate mass,  
\( M_1 \) is the free space added mass given by Equation (A48), and  
\( M_2 \) is the added mass due to the enclosed fluid given by either Equation (A47a) or (47b).

Analogously, the damping in Equation (A7) and, hence, in Equation (A27) becomes for a high-density fluid

\[
a_{mn} = \frac{\beta_0}{2M'} + \frac{\Gamma_{mn} \beta_1}{2M' \omega_{mn}} + \frac{\beta_1}{2M}
\]

(A51)

where the first two terms represent the original viscous and hysteretic damping for the plate, except that now \( M = M' \), and the last term is the added viscous damping where \( \beta_1 \) is given by Equation (A49).
APPENDIX A2 – SAMPLE PROBLEM

An example will be given to show how to determine the mean-square pressure \( \tau = 0 \) in the liquid-filled cavity \( (M + M') \) adjacent to the plate. Two alternative methods of computation will be treated.

DISCRETE FREQUENCY METHOD

The modal mean-square pressure \( P_{mn}^2 \) at the plate is determined from Equations (A46), (A27), (A28), and (A20) as a time-average quantity.

\[
P_{mn}^2 = \langle P_{mn} P_{mn} \rangle = V \left( z_{mn} \right)^2 (t) \omega_{mn}^4 M_2^2
\]

\[
= f^2 \frac{A}{L_x L_y} \left( \frac{M_2}{M'} \right)^2 \left[ \sin^2 \frac{m \pi x}{L_x} \sin^2 \frac{n \pi y}{L_y} \right] \omega_{mn}^2 I_{mn}(0)
\]

The spatial average of \( P_{mn}^2 \) is

\[
\overline{P_{mn}^2} = f^2 \frac{A}{4L_x L_y} \left( \frac{M_2}{M'} \right)^2 \omega_{mn}^2 I_{mn}(0)
\]

where* \( f^2 = (6 \times 10^{-3}) \left( \frac{1}{2} \rho U_\infty^2 \right)^2 \),

\[
A = \frac{2 \pi}{\kappa^2}, \text{ and }
\]

\[
\kappa = \frac{2}{d}
\]

For a steel structure in water, let

\[
h = \frac{1}{2} \text{ in. },
\]

\[
L_x = L_y = 5 \text{ ft },
\]

\[
L_z = 1 \text{ ft },
\]

\[
U_\infty = 20 \text{ ft/sec } \equiv 12 \text{ knots },
\]

\[
d = 0.02 \text{ ft },
\]

\[
\theta = 3 \times 10^{-2} \text{ sec },
\]

*See Appendix A3.
\[ \eta = 10^2 (Q = 100), \text{ and} \]
\[ \rho = 64.2 \text{ lb/ft}^3. \]

At coincidence \((C_B = c, \omega = \omega_c)\)
\[ C_B^4 = c^4 = \omega_c^2 \frac{B}{M} = \omega_c^2 \frac{B}{w_s/g} = \omega_c^2 \frac{B}{\rho_s k/g} \]

Hence, ignoring Poisson's ratio \(\sigma\), the sound coincidence frequency is
\[ f_c = \frac{c^2}{2\pi} \sqrt{\frac{\rho_s k/g}{E h^3/12}} = \frac{c^2}{2\pi h} \sqrt{\frac{12 \rho_s}{g E}} \]
\[ = \frac{(4910 \text{ ft/sec})^2}{2\pi(1/24 \text{ ft})} \sqrt{\frac{12(490 \text{ lb/ft}^3)}{(32.2 \text{ ft/sec}^2)(0.4175 \times 10^{10} \text{ lb/ft}^2)}} = 19,230 \text{ cps} \]

Now
\[ k_m^2 = k^2 - \Gamma_{mn}^2 \]
see Equation (A41). From page 174 of Reference 10, we see that for \(\omega_{mn} < \omega_c, C_B < c\) and \(\Gamma_{mn} > k\) (note: \(k_b\) and \(k_a\) in Reference 10 respectively become \(\Gamma_{mn}\) and \(k\) here.) Hence, for frequencies less than 10,000 cps, \(|k_{mn}|\) may be approximated by \(\Gamma_{mn}\). Further, restricting attention to frequencies greater than 200 cps, from Equations (A47b), (A48), and (A50)
\[ M' = M + M_1 + M_2 = M + \frac{\rho}{|k_{mn}|} \tan k_{mn} L_z \]

But
\[ k_{mn} = i \Gamma_{mn}, i \tan x = \tanh ix, \tanh x = -\tanh -x \]
Hence,
\[ M_2 = \frac{\rho}{i\Gamma_{mn}} \tan i\Gamma_{mn} L_z = \frac{-\rho}{\Gamma_{mn}} \tanh (-\Gamma_{mn} L_z) = \frac{\rho}{\Gamma_{mn}} \tanh \Gamma_{mn} L_z \]
\[ = \frac{\rho}{|k_{mn}|} \tanh \sqrt{\frac{h}{|B|}}^{1/2} \omega_{mn} L_z \]
for $\omega_{mn}$ sufficiently high $\tanh \sqrt{\frac{M}{B}} \omega_{mn} L_z + 1$ and $M_2 = \frac{\rho}{g |k_{mn}|}$ (Note: In the NSRDC system of units, $M_2 = \frac{\rho}{g |k_{mn}|}$ is used when making computations.) Thus

$$M' = M + \frac{2\rho}{\Gamma_{mn}} = M$$

Also, as can be calculated from Equation (A49), for $\omega_{mn}$ sufficiently high, we get $\beta_1 = 0$.

For these conditions and using Equation (A8),

$$\left( \frac{M_2}{M'} \right)^2 = \left( \frac{\rho/\Gamma_{mn} g}{M} \right)^2 = \left( \frac{\rho/g}{M} \right)^2 \left( \frac{M/\Gamma_{mn} g}{M} \right)^2 = \frac{\rho^2 / g^2}{\omega_{mn} M^2}$$

but

$$c_L = \left( \frac{E}{\rho_s \cdot \frac{1}{g}} \right)^{1/2} = \sqrt{\frac{12B (1 - \sigma^2)}{\rho_s \cdot \frac{1}{g}}} = \frac{\sqrt{12B}}{\rho_s \cdot \frac{1}{g}} \left( \frac{B}{M} \right)^{1/2} \text{ (ignoring $\sigma$)}$$

Hence,

$$\left( \frac{M}{B} \right)^{1/2} = \frac{\sqrt{12B}}{c_L h}$$

and

$$\left( \frac{M_2}{M'} \right)^2 = \frac{1}{\sqrt{12}} \frac{\rho^2 c_L h}{\omega_{mn} M^2} = \frac{1}{\sqrt{12}} \frac{\rho^2 c_L h}{\rho_s^2 h^2} = \frac{1}{\sqrt{12}} \frac{\rho^2 c_L h}{\omega_{mn} \rho_s^2 h^2}$$

Finally, from Equations (A8) and (A21) for any mode

$$\omega_{mn}^2 = \left( \frac{B}{M} \right)^{1/4} \gamma_{mn} = \left( \frac{B}{M} \right) \left[ \left( \frac{mn}{L_x} \right)^2 + \left( \frac{nn}{L_y} \right)^2 \right]$$

is calculated, and $I_{mn}(0)$ can be calculated from either Equation (A33) or (A34). In these equations, Equation (A51) is used, noting that in this problem $\beta_1 = 0$, $M' = M$, $B = E h^3 / 12$, and $\beta_0 = 0$; see page 170 of Reference 2.
Figure 3 — Modal Lattice and Constant Wave Number Contour for Simply Supported Plate
There is now sufficient data to compute all of the quantities in the equation for $P_{mn}^2$.

Finally, from Equations (A46) and (A40) and using $k_{mn} = -i \Gamma_{mn}$

$$P_{mn}(x,y,z,\omega) = -i\omega p \Psi_{mn}(x,y,z,\omega) = -i\omega p D_{mn} \phi_{mn} \sin(-i\Gamma_{mn}^2)$$

$$= \omega p D_{mn} \phi_{mn} \sinh \Gamma_{mn}^2$$

Hence, $P_{mn}^2 = \sinh \Gamma_{mn}^2$; therefore, the modal mean-square pressure for a location away from the excited plate would be reduced by the factor

$$\left[ \frac{\rho_{mn}^2}{\rho_{mn}^2} \right]_2 < L_z \left[ \frac{\sinh \Gamma_{mn}^2}{\sinh \Gamma_{mn}^2 L_z} \right]$$

**MODAL DENSITY METHOD**

We now determine $P^2$, the mean-square pressure as measured by frequency analysis in bands of width $\Delta \nu$. The modal density of a plate found by considering the area included by the quarter circle of radius $\Gamma_{mn}$ (Figure (3)) is

$$n(\Gamma_{mn}) = \frac{dN(\Gamma_{mn})}{d\Gamma_{mn}} = \frac{\Gamma_{mn} L_x L_y}{2 \pi}$$

see page 135 of Reference 10. Hence, using Equation (A8) and the relation $c_L = \sqrt{\frac{12}{B} \left( \frac{M}{h} \right)}^{1/2}$, previously shown, the number of modes included up to wave number $\Gamma_{mn}$ is

$$dN(\Gamma_{mn}) = \Gamma_{mn} \frac{L_x L_y}{2 \pi} \left( \frac{M}{B} \right)^{1/2} \frac{\Gamma_{mn} L_x L_y}{4 \pi} d\omega = \frac{\sqrt{3} L_x L_y}{c_L h} d\nu$$

where $d\omega = 2\pi d\nu$.

Thus, the average number of modes $\Delta N$ in a band $\Delta \nu$ is approximately

$$\Delta N = \frac{\sqrt{3} L_x L_y \Delta \nu}{c_L h}$$

Using the relation $c_L h = \sqrt{3} L_x L_y \frac{\Delta \nu}{\Delta N}$ and the relation for $\left( \frac{M}{M'} \right)$, previously derived, the equation for the modal mean-square pressure per mode is
For all modes \( \Delta N \) in the band, the average mean-square pressure is

\[
\overline{p^2} = \frac{f^2 A}{4L_x L_y} \frac{1}{\sqrt{1/2}} \frac{\rho^2 \sqrt{3} L_x L_y}{\omega_m \rho_s^2 h^2} \frac{\Delta \nu}{\Delta N} \omega_m^2 \omega_m(0)
\]

where \( \omega_m \) is now considered to be a continuous frequency variable.

Since all data are known (see Discrete Frequency Method), then for a given frequency bandwidth \( \Delta \nu \), \( \overline{p^2} \) can be computed. The results of the computation are given for \( \Delta \nu = 1 \) in terms of spectrum level, \( SL = 10 \log_{10} \frac{\overline{p^2}}{p_0^2} \) db, where reference pressure \( p_0 = 1 \) \( \mu \) bar = 1 dyn/cm\(^2\) in Reference 2, Figure 7.
APPENDIX A3 – METHOD FOR DETERMINING INPUT DATA

Dyer used the following estimates of input data for the boundary-layer pressure field

\[ f_{r.m.s.} = 6 \times 10^{-3} \frac{1}{2} \rho U_\infty^2 \]

based on measurements by Willmarth\textsuperscript{12}

\[ \kappa d = 2 \]

\[ \theta = 30 \frac{d}{U_\infty} \]

\[ \nu = 0.8 U_\infty \]

\[ \omega_0 = \kappa \nu \]

based on a comparison of Equations (A18) and its Fourier transform, which is the spectral density

\[ s(\omega) = 2 \int_{-\infty}^{\infty} \langle f(r,t) f^*(r',t') \rangle \exp(-i\omega \tau) d\tau \]

with measurements.\textsuperscript{12,13}

To determine the cross correlation or mean square pressures, geometric and structural data must also be given as in the sample problem.
APPENDIX B

BOEING PROGRAM 1 (MAESTRELLA)

APPENDIX B1 – MATHEMATICAL ANALYSIS
APPENDIX B2 – METHOD FOR DETERMINING INPUT DATA
APPENDIX B3 – PROGRAM IDENTIFICATION
APPENDIX B4 – TEST RUNS
NOTATION

\( A \) \quad \text{Constants}
\( a, b \) \quad \text{Length of panel sides}
\( a_{\text{mm}} \) \quad \text{Plate modal damping}
\( a_{v, u} \) \quad \text{Amplitude of temporal fluctuation for unsteady convection}
\( B \) \quad \text{Bending stiffness}
\( c \) \quad \text{Speed of sound in fluid}
\( E \) \quad \text{Young’s modulus}
\( F \) \quad \text{Equals} \ \frac{\delta^*}{U_c}
\( g \) \quad \text{Plate input-response function}
\( g_i \) \quad \text{A function}
\( h \) \quad \text{Panel thickness}
\( I_{\nu}, K_{\nu} \) \quad \text{Integrals}
\( i \) \quad \text{Equals} \ \sqrt{-1}
\( J_{\xi} \) \quad \text{Jacobian}
\( K_{\omega} \) \quad \text{Bending-wave speed}
\( K_{1, K_{\nu}, K_{\gamma}} \) \quad \text{Constants; wave numbers}
\( L_{\xi} \) \quad \text{Characteristic length of pressure field for semifrozen flow} (length over which a given turbulent pressure pattern remains distinguishable)
\( M \) \quad \text{Plate mass}
\( m, n \) \quad \text{Mode numbers}
\( N \) \quad \text{Constant, for piston radiation} \ N = 4
\( P \) \quad \text{Panel perimeter}
\( P_r \) \quad \text{Total length of panel ribs}
\( PWL_{mn} \) \quad \text{Sound power level for} \ mn \ \text{mode}
\( p(x,y,t) \) \quad \text{Turbulence wall pressure fluctuation}
\( \overline{p^2} \) \quad \text{Mean-square pressure}
\( R(\xi, \eta, \tau) \) \quad \text{Correlation coefficient for pressures}
\( R(\tau) \) \quad \text{Normalized autocorrelation for pressures}
\( t, t' \) \quad \text{Time; times at which displacements are measured at points} \ x,y \ \text{and} \ x', y', \text{respectively}
$t_0, t'_0$  Times at which pressure measurements are made at points $x_0, y_0$ and $x'_0, y'_0$, respectively

$U_c$  Broadband convection velocity

$U, U_\infty$  Free-stream velocity

$u$  Local flow velocity

$u(t - t_0)$  Unit step function

$u_f$  Frictional velocity

$V_{mn}$  Volume displacement for a mode

$x$  Distance from leading edge of plate

$(x,y), (x',y')$  Points on the panel at which displacements are measured

$(x_0, y_0), (x'_0, y'_0)$  Points on panel at which turbulence pressures are measured

$Y(x,y,t)$  Panel displacement

$\overline{Y}$  Root-mean-square of displacement

$\overline{Y^2}$  Mean square displacement

$Y_{mn}$  Eigenfunction or orthogonal mode of plate oscillation

$y$  Distance from wall normal to flow direction

$\beta_{ac}$  Acoustic-damping coefficient

$\beta_c$  Critical damping

$\beta_{st}$  Structural damping coefficient

$\Gamma_{mn}$  Eigenvalue

$\delta$  Boundary layer displacement thickness

$\delta_{mn}$  Total damping ratio

$\eta$  Equals $y - y'$, lateral partial separation

$\eta'$  Equals $y_0 + y'_0$

$\theta, \theta_1, \theta_2$  Eddy lifetime for steady convection, i.e., time in which value of correlation coefficient obtained from envelope of correlation maxima (maxima-maximorum) drops to $1/e$

$\overline{\theta_1, \theta_2}$  Eddy lifetime for unsteady convection

$\nu, \nu_w$  Kinematic viscosity of fluid near wall

$\xi$  Equals $x - x'$, longitudinal partial separation

$\xi'$  Equals $x_0 + x'_0$

$\rho$  Density of fluid

$\rho_w$  Density of fluid near wall
\( \sigma_1, \sigma_2 \)  
Standard deviation of distribution

\( \tau \)  
Equals \( t - t' \), time delay

\( \tau_0 \)  
Equals \( t_0 - t'_0 \)

\( \tau'_0 \)  
Equals \( t_0 + t'_0 \)

\( \tau_w \)  
Local wall-shear stress

\( \phi_{mn}(x,y) \) or \( \phi(x) \phi(y) \)  
Plate eigenfunctions

\( \omega \)  
Circular frequency equal to \( 2\pi f \)

\( \omega_1 \)  
Circular frequency of temporal fluctuation

\( \omega_{mn} \)  
Plate modal frequency

\( \nabla^4 \)  
Equals \( \frac{\partial^4 y}{\partial x^4} + \frac{\partial^4 y}{\partial x^2 \partial y^2} + \frac{\partial^4 y}{\partial y^4} \)

\( < \cdots > \)  
Symbol for time-average operation
APPENDIX B1 - MATHEMATICAL ANALYSIS

Two models were taken from experimental data\textsuperscript{14,15} to represent the cross correlation of the turbulence wall-pressure fluctuations in a broad frequency band. The models, designated as Model A-convected semifrozen pattern and Model B-unsteady convection-will be considered in turn.

MODEL A-CONVECTED SEMIFROZEN PATTERN ($U_c = \text{CONSTANT}$)

The normalized cross-correlation function of the pressure fluctuations for this model in a moving frame of reference is represented by the sum of two Gaussian distributions corresponding to measurements made with a series of constant time delays and variable spatial separation; see Reference 16 Figure 4. The convection velocity, obtained from Equation (B1), is defined by

$$\frac{\partial R(\xi,\tau)}{\partial \xi} = 0, \quad \text{when} \quad U_c = \frac{\xi_{\text{max}}}{\tau}$$  \hspace{1cm} (B2)

Alternatively, the normalized cross-correlation function may also be represented by

$$R(\xi,\eta,\tau) = e^{-\frac{|\xi|}{U_c\theta}} \left\{ \begin{array}{l} A_1 e^{-\frac{-(\xi - U_c\tau)^2 + \eta^2}{2\sigma_1^2}} + A_2 e^{-\frac{-(\xi - U_c\tau)^2 + \eta^2}{2\sigma_2^2}} \end{array} \right\}$$  \hspace{1cm} (B3)

corresponding to measurements made with constant spatial separation and variable time delay; see Reference 16 Figure 3. The convection velocity obtained from Equation (B3) is defined by
\[ \frac{\partial R(\xi, \tau)}{\partial \tau} = 0, \quad \text{when} \quad U_e = \frac{\xi}{\tau_{\max}} \quad \text{(B4)} \]

Since most data were taken at constant spatial separation, the autocorrelation function obtained from Equation (B3) by setting $\xi = \eta = 0$ is
\[ R(\tau) = A_1 e^{-\frac{-U^2 \tau^2}{2\sigma^2_1}} + A_2 e^{-\frac{-U^2 \tau^2}{2\sigma^2_2}} \quad \text{(B5)} \]
and the corresponding power spectrum is (use Fourier transform Pairs 708 of Reference 7)
\[ P(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R(\tau) e^{-i\omega \tau} d\tau = \frac{A_1 \sigma_1}{U_e \sqrt{2\pi}} e^{-\frac{2U^2}{U_e}} + \frac{A_2 \sigma_2}{U_e \sqrt{2\pi}} e^{-\frac{2U^2}{U_e}} \quad \text{(B6)} \]

Comparison of theory and experiment shows that the high-frequency resolution of Equation (B3) is poor; see Reference 14 Figure 1. Hence a more satisfactory empirical representation of the cross correlation of the pressures is constructed as follows from the autocorrelation obtained from the measured power spectral density, with decay in a moving frame of reference. This representation corresponds to a broader range of frequency components than the previous one; see Reference 14 Figure 2. The measured dimensionless power spectral density can be represented by
\[ \frac{P(\omega) e^{-K_1 \omega F}}{\tau_{\omega}^2 \delta^*} = A_1 e^{-K_1 \omega F} + A_2 e^{-K_2 \omega F} + A_3 e^{-K_3 \omega F} \quad \text{(B7)} \]
where
\[ A_1 = 1.6 \]
\[ A_2 = 7.2 \]
\[ A_3 = 12.0 \]
\[ F = \delta^*/U \]
\[ K_1 = 0.470 \]
\[ K_2 = 3.0 \]
\[ K_3 = 14.0 \]
Using Fourier Pair 632 of Reference 7, the normalized autocorrelation is found

\[
R(\tau) = \frac{R(\tau, \text{unnormalized})}{R(0)}
\]

\[
= \left[ \frac{A_1 K_1}{K_1^2 + (\tau/F)^2} + \frac{A_2 K_2}{K_2^2 + (\tau/F)^2} + \frac{A_3 K_3}{K_3^2 + (\tau/F)^2} \right]
\]

\[
= \left( \frac{A_1}{K_1} + \frac{A_2}{K_2} + \frac{A_3}{K_3} \right)
\]

\[
\frac{-|\xi|}{U_c \theta}
\]

By introducing the spatial decay \( e \) and moving frame of reference \((\xi - U_c \tau)\), the normalized cross-correlation function of the pressure becomes (by analogy with Equation (B3)) with respect to symmetry about the \( \xi \) and \( \tau \) axes:

\[
R(\xi, \eta, \tau) = e^{\frac{-|\xi|}{U_c \theta}} \left[ \sum_{y=1}^{3} \frac{A_y K_y}{K_y^2 + (1/PU_c)^2 (|\xi - U_c \tau|^2 + \eta^2)} \right]
\]

\[
\sum_{y=1}^{3} \frac{A_y}{K_y}
\]

By changing the decay rate from \( \frac{|\xi|}{U_c \theta} \) to \( \frac{|\tau|}{\theta} \) in Equation (B9), (the Taylor hypothesis), the functional form of \( R(\xi, \eta, \tau) \) still fits the experimental results.\(^{17} \) Hence either form of the exponential may be used to describe the decay of the wall pressure correlation.

**MODEL B-UNSTEADY CONVECTION**

\((U_c \neq \text{CONSTANT})\)

For unsteady convection, the normalized cross correlation has the form

\[
R(\xi, \eta, \tau) = A_1 \left( e^{\frac{-|\tau|}{\theta_1}} + a_1 e^{\sin \omega_1 |\tau|} \right) e^{\frac{-|\xi - U_c \tau|^2 + \eta^2}{2 \sigma_1^2}}
\]

\[
+ A_2 \left( e^{\frac{-|\tau|}{\theta_2}} + a_2 e^{\cos \omega_2 |\tau|} \right) e^{\frac{-|\xi - U_c \tau|^2 + \eta^2}{2 \sigma_2^2}}
\]
and the normalized autocorrelation ($\eta = \xi = 0$) becomes

$$R(\tau) = \frac{-|\tau|}{\theta_1} + \frac{-|\tau|}{\theta_2} + \frac{-U^2\tau^2}{2\sigma^2_1}$$

$$+ \frac{-U^2\tau^2}{2\sigma^2_2}$$

(B11)

**STRUCTURAL RESPONSE TO TURBULENCE EXCITATION**

The displacement cross correlation of a plate due to the cross correlation of a random pressure $\langle p(x_0, y_0, t_0) p(x_0', y_0', t_0') \rangle$ has been given by Dyer;\(^2\) see Appendix A

$$<Y(x, y, t) Y(x', y', t')> = \int_{-\infty}^{t} dt_0 \int_{-\infty}^{t'} dt'_0 \int_{0}^{a} dx_0 \int_{0}^{b} dy_0 \int_{0}^{\alpha} dx'_0 \int_{0}^{\beta} dy'_0$$

$$g(x, y, t; x_0, y_0, t_0) g(x', y', t'; x_0', y_0', t_0')$$

$$\cdot \langle p(x_0, y_0, t_0) p(x_0', y_0', t_0') \rangle$$

(B12)

Using Equation (B9) with $\frac{|\xi|}{U_e} = |\tau| = |t_0 - t'_0|$, the unnormalized cross correlation for *semifrozen* flow becomes

$$\langle p(x_0, y_0, t_0) p(x_0', y_0', t_0') \rangle = \frac{-|t_0 - t'_0|}{\theta}$$

$$\sum_{\nu=1}^{3} A_{\nu} K_{\nu}$$

$$\sum_{\nu=1}^{3} \frac{A_{\nu} K_{\nu}}{K_{\nu}^2 + (1/FU_e)^2 \left\{ (l(x_0 - x'_0) - U_e(t_0 - t'_0))^2 + (y_0 - y'_0)^2 \right\} \right}$$

(B13)

As in Appendix A1, the homogeneous equation for plate vibrations has the form
\[ B \varphi^4 Y(x, y, t) + M \frac{\partial^2 Y(x, y, t)}{\partial t^2} + (\beta_{ae} + \beta_{st}) \frac{\partial Y(x, y, t)}{\partial t} = 0 \]  

(B14)

where

\[ Y_{mn}(x, y, t) = \phi_{mn}(x, y) e^{-i \omega_{mn} t} \]  

(B15)

is the normal mode solution to Equation (B14). Substitution of Equation (B15) in Equation (B14) yields

\[ \varphi^4 \phi_{mn}(x, y) - \Gamma_{mn} \phi(x, y) = 0 \]  

(B18)

where

\[ \Gamma_{mn}^4 = \frac{-M(-a_{mn} + i \omega_{mn})^2 - (\beta_{ae} + \beta_{st}) (-a_{mn} + i \omega_{mn})}{B} \]  

(B17)

Equating imaginary and real quantities in Equation (B17),

\[ a_{mn} = \frac{\beta_{ae} + \beta_{st}}{2M} \]

and

\[ \omega_{mn}^2 = \frac{B}{M} \Gamma_{mn}^4 \]  

(B19)

\*Forced response of a mode to a sinusoidal force or 3-dB method was used to measure damping. For small damping, if \( \beta = \beta_{ae} + \beta_{st} \), the decay ratio \( \frac{\beta}{\beta_c} = \frac{1}{2} \frac{\omega_c - \omega}{\omega_{mn}} = \frac{1}{2} \frac{\Delta \omega}{\omega_{mn}} \) where \( \omega_c \) and \( \omega \) are circular frequencies of vibration, when response differs from extreme value by ratio of \( \sqrt{2} \) (equivalent to 3dB). In Reference 14, Equation (13) Maestrello defined the total damping ratio by \( \delta_{mn} = \frac{\Delta \omega}{\omega_{mn}} \). Hence

\[
\frac{\beta}{\beta_c} = \frac{1}{2} \frac{\omega_c - \omega}{\omega_{mn}} = \frac{\delta_{mn}}{2 \omega_{mn}} = \frac{a_{mn}}{2 M \omega_{mn}} \]  

and

\[ a_{mn} = \frac{\delta_{mn} \omega_{mn}}{2} \]

see Reference 18, pages 14-16.
For low damping, the impulse response function was found (see Appendix A1) to be

\[ g(x, y, t', x_0, y_0, t_0) = \sum_{m} \frac{\phi_{mn}(x, y) \phi_{mn}(x_0, y_0)}{\omega_{mn} M} \]

\[ \cdot \left[ e^{-a_{mn}(t-t_0)} \sin \omega_{mn}(t-t_0) \right] u(t-t_0) \]  \hspace{1cm} (B20)

Substituting Equations (B13) and (B20) in Equation (B12) and ignoring cross-coupling terms,

\[ <Y(x, y, t) Y(x', y', t')> = \sum_{\nu=1}^{3} \frac{A_{\nu}}{K_{\nu}} \sum_{mn} \frac{\phi_{mn}(x, y) \phi_{mn}(x', y')}{\omega_{mn}^2} \]

\[ \cdot \int_{-\infty}^{t} dt_0 \int_{-\infty}^{t'} dt_0' \int_{0}^{a} dx_0 \int_{0}^{b} dy_0 \int_{0}^{a} dx_0' \int_{0}^{b} dy_0' e^{-a_{mn}(t-t_0')} \]

\[ \cdot [ \sin \omega_{mn}(t'-t_0') u(t'-t_0') ] e^{-a_{mn}(t-t_0)} [ \sin \omega_{mn}(t-t_0) u(t-t_0) ] \]

\[ -|t_0-t_0'| \]

\[ \sum_{\nu=1}^{3} \frac{\phi_{mn}(x_0, y_0) \phi_{mn}(x_0', y_0') A_{\nu} K_{\nu} e^{-a_{mn}(t-t_0')}}{K_{\nu}^2 + (1/FU_c)^2 \left[ (x_0 - x_0') - U_c (t_0 - t_0') \right]^2 + (y_0 - y_0')^2} \] \hspace{1cm} (B21)

Also (see Appendix A1)

\[ \phi_{mn} = \frac{2}{(ab)^{1/2}} \sin \frac{mn x}{a} \sin \frac{mn y}{b} \] \hspace{1cm} (B22)

By trigonometric manipulation,

\[ \phi_{mn}(x_0, y_0) \phi_{mn}(x_0', y_0') = \frac{1}{ab} \left[ \cos \frac{mn(x_0 - x_0')}{a} - \cos \frac{mn(x_0 + x_0')}{a} \right] \]

\[ \cdot \left[ \cos \frac{n \pi (y_0 - y_0')}{b} - \cos \frac{n \pi (y_0 + y_0')}{b} \right] \] \hspace{1cm} (B23)
The space integral in Equation (B21) may then be written

\[ l_\nu = \int_0^b dy_0 \int_0^b dx_0 \int_0^a dx_0' \int_0^a dx_0'' \]

\[
\frac{1}{ab} \left[ \cos \left( \frac{\nu \pi (x_0 - x_0')}{a} \right) - \cos \left( \frac{\nu \pi (x_0 + x_0')}{a} \right) \right] \left[ \cos \left( \frac{\nu \pi (y_0 - y_0')}{b} \right) - \cos \left( \frac{\nu \pi (y_0 + y_0')}{b} \right) \right]
\]

\[ K_\nu^2 + \left( \frac{1}{FUc} \right)^2 \left\{ \left[ (x_0 - x_0') - Uc(t_0 - t_0') \right]^2 + (y_0 - y_0')^2 \right\} \]

\[ = \int_0^b dy_0 \int_0^b dx_0 \int_0^a dx_0' \int_0^a dx_0'' f(y_0, y_0', x_0, x_0') \]  \hspace{1cm} (B24)

To simplify computation of Equation (B21), the variable of integration is changed, thus reducing the number of operations for the integral.

Let

\[
\eta = y_0 + y_0' \quad \eta' + \eta = 2y_0 \quad y_0 = \frac{\eta' + \eta}{2} = g_1(\eta, \eta', \xi = 0, \xi' = 0)
\]

\[
\eta = y_0 - y_0' \quad \eta' - \eta = 2y_0' \quad y_0' = \frac{\eta' - \eta}{2} = g_2(\eta, \eta', \xi = 0, \xi' = 0)
\]

\[
\xi = x_0 + x_0' \quad \xi' + \xi = 2x_0 \quad x_0 = \frac{\xi' + \xi}{2} = g_3(\eta = 0, \eta' = 0, \xi, \xi')
\]

\[
\xi = x_0 - x_0' \quad \xi' - \xi = 2x_0' \quad x_0' = \frac{\xi' - \xi}{2} = g_4(\eta = 0, \eta' = 0, \xi, \xi')
\]

In Appendix A1, following Dyer, the transformation of two variables in a multiple integral had the following form. Let

\[ x = \phi(u, v), \quad y = \psi(u, v) \]

Then

\[
\iint f(x, y) \, dx \, dy = \iint f(\phi(u, v), \psi(u, v)) \left| \begin{array}{cc} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{array} \right| du \, dv
\]

A similar formulation may be made for any number of variables of transformation.\textsuperscript{19,20}
\[ l_v = \int f(x) \, dx = \int_{g^{-1}(x)} f(g(t)) \, J_g(t) \, dt \]  

(B25)

where \( x = g(t) \) and \( t = g^{-1}(x) \) (the inverse function). In this case, from Equation (B24)

\[
f(x) = f(y_0, y'_0, x_0, x'_0)
\]

\[
\frac{1}{ab} \left[ \cos \frac{mn(x_0 - x'_0)}{a} - \cos \frac{mn(x_0 + x'_0)}{a} \right] \left[ \cos \frac{nn(y_0 - y'_0)}{b} - \cos \frac{nn(y_0 + y'_0)}{b} \right]
\]

\[
K_v^2 + (1/Fv_0)^2 \left[ (x_0 - x'_0) - U_c(t_0 - t'_0) \right]^2 + (y_0 - y'_0)^2
\]

also

\[
f[g(t)] = f[g_1(\eta, \eta', \xi, \xi'), g_2(\eta, \eta', \xi, \xi'), g_3(\eta, \eta', \xi, \xi'), g_4(\eta, \eta', \xi, \xi')]
\]

\[
= f\left( \frac{\eta' + \eta}{2}, \frac{\eta' - \eta}{2}, \frac{\xi' + \xi}{2}, \frac{\xi' - \xi}{2} \right)
\]

Substitute in \( f(x) \)

\[
x_0 - x'_0 = \xi, x_0 + x'_0 = \xi', y_0 - y'_0 = \eta, y_0 + y'_0 = \eta'
\]

to obtain

\[
f[g(t)] = \frac{1}{ab} \left[ \cos \frac{mn \xi}{a} - \cos \frac{mn \xi'}{a} \right] \left[ \cos \frac{nn \eta}{b} - \cos \frac{nn \eta'}{b} \right]
\]

\[
K_v^2 + (1/Fv_0)^2 \left[ \xi - U_c(t_0 - t'_0) \right]^2 + \eta^2
\]

(B26)

also

\[
J'_v(t) = \frac{\partial (g_1, g_2, g_3, g_4)}{\partial (\eta, \eta', \xi, \xi')}
\]

\[
= \begin{bmatrix}
\frac{\partial g_1}{\partial \eta} & \frac{\partial g_1}{\partial \eta'} & \frac{\partial g_1}{\partial \xi} & \frac{\partial g_1}{\partial \xi'} \\
\frac{\partial g_2}{\partial \eta} & \frac{\partial g_2}{\partial \eta'} & \frac{\partial g_2}{\partial \xi} & \frac{\partial g_2}{\partial \xi'} \\
\frac{\partial g_3}{\partial \eta} & \frac{\partial g_3}{\partial \eta'} & \frac{\partial g_3}{\partial \xi} & \frac{\partial g_3}{\partial \xi'} \\
\frac{\partial g_4}{\partial \eta} & \frac{\partial g_4}{\partial \eta'} & \frac{\partial g_4}{\partial \xi} & \frac{\partial g_4}{\partial \xi'} \\
\end{bmatrix}
= \begin{bmatrix}
1 & 1 & 0 & 0 \\
2 & 2 & 0 & 0 \\
1 & 1 & 0 & 0 \\
2 & 2 & 0 & 0 \\
0 & 0 & 1 & 1 \\
0 & 0 & 1 & 1 \\
0 & 0 & 1 & 1 \\
0 & 0 & 1 & 1 \\
\end{bmatrix}
\]

(B27)
To obtain the new limits, transform the limits in the $y_0, y'_0$ plane (for the transverse coordinates) to corresponding quantities in the $\eta, \eta'$ plane and the limits in the $x_0, x'_0$ plane (for the longitudinal coordinates) to corresponding quantities in the $\xi, \xi'$ plane. The transformation is shown in Figures 4 and 5. From these figures, the limits for the $\mathcal{F}$ to $T$ planes are set to

\[
x = g(t) = (y_0, y'_0, x_0, x'_0): 0 \leq y_0 \leq b, 0 \leq y'_0 \leq b, 0 \leq x_0 \leq a, 0 \leq x'_0 \leq a
\]

\[
t = g^{-1}(x) = (\eta, \eta', \xi, \xi'): -b \leq \eta \leq b, |\eta| \leq |\eta'| \leq 2b - |\eta|, -a \leq \xi \leq a, |\xi| \leq |\xi'| \leq 2a - |\xi|
\]

(Note: For $\eta \geq 0$, $\eta' = 2b - \eta$, respectively, or equal $2b - |\eta|$ along upper branch. For $\eta \leq 0$, $\eta' = \pm \eta$, respectively, or equal $|\eta|$ along lower branch. Similarly for $\xi'$.)

Substituting Equations (B26) and (B27) in Equation (B25) and using the limits for $\xi, \xi', \eta, \eta'$ derived above, we get

\[
l_{\nu} = \frac{1}{4ab} \int_{-a}^{a} d\xi \int_{-b}^{b} d\eta \left[ \frac{2a-|\xi|}{|\xi|} - d\xi \right] \left[ \frac{2b-|\eta|}{|\eta|} - d\eta \right] \left[ \frac{m\nu \xi}{a} - \cos \frac{mn \xi}{a} \right] \left[ \frac{m\nu \eta}{b} - \cos \frac{mn \eta}{b} \right] \frac{K_0^2 + (1/FU_c)^2}{(\xi - U_c(t_0 - t'_0))^2 + \eta^2}
\]

Integration with respect to $\eta'$ and $\xi'$, in turn, reduces the integral to

\[
l_{\nu} = \frac{2}{ab} \int_{-a}^{a} \left[ \int_{0}^{b} \frac{(a - |\xi|) \cos \frac{m\nu \xi}{a} + a \sin \frac{m\nu |\xi|}{a}}{K_0^2 + (1/FU_c)^2} \left[ \frac{(b - |\eta|) \cos \frac{mn \eta}{b} + b \sin \frac{mn |\eta|}{b}}{\xi - U_c(t_0 - t'_0))^2 + \eta^2} \right] d\eta \right] d\xi
\]

where

\[
\int_{-b}^{b} \rightarrow 2 \int_{0}^{b}
\]

since the trigonometric functions are even functions of their argument and are thus unaffected by the signs of $\xi, \eta$.

Equation (B21) is rewritten

\[
< Y(x, y, t) Y(x', y', t') > = \frac{p^2}{\omega_{mn}^2} \sum_{\nu = 1}^{3} \frac{A_{\nu}}{K_{\nu}} \sum_{m,n} \phi_{mn}(x, y) \phi_{mn}(x', y')
\]

(Equation continued on top of page 48.)
Figure 4A - Original Coordinates and Region of Integration

Figure 4B - Transformed Coordinates and Region of Integration

Note to Figure 4:

Line $f_1$ where $y_o' = 0$ transforms to $y' = -y$
Line $f_2$ where $y_o = b$ transforms to $y' = -y' = 2b$ because $y' = y_o - y_o' = y_o' \iff y_o = y_o'$
Line $f_3$ where $y_o' = b$ transforms to $y' = y = 2b$
Line $f_4$ where $y_o = 0$ transforms to $y' = y'$

Thus these limit points of Equation (214) for the region $y_o, y_o'$ denoted by $T_1$ in Figure 4a transform into the lines enclosing the region $T$, shown in Figure 4b. The limits for $\eta, \eta'$ obtained from the later are based on the following relationships.

Along $f_2$: $0 \leq y_o \leq b$

\[
\begin{align*}
\eta &= \eta' = y_o' = y_o \\
\eta' &= y_o + y_o' = y_o + y_o
\end{align*}
\]

Hence $\eta'$ ranges from 0 to $b$ with slope $\frac{dy'}{d\eta} = 1$.

Where $\eta_{1,2} = \eta = y_o = y_o'$ then $\eta_{1,2} = 0$;

Where $\eta_{1,2} = \eta = y_o - y_o' = y_o$ then $\eta_{1,2} = y_o$.

Along $f_2$: $0 \leq y_o' \leq b$

\[
\begin{align*}
\eta &= \eta' = y_o = y_o' = b + y_o
\end{align*}
\]

Hence $\eta'$ ranges from $b$ to $2b$ with slope $\frac{dy'}{d\eta} = 1$.

Where $\eta_{1,2} = 0, y_o = y_o' = y_o = y_o$ then $\eta_{1,2} = 0$;

Where $\eta_{1,2} = b, y_o = y_o' = y_o = y_o$ then $\eta_{1,2} = b$.

Along $f_2$: $0 \leq y_o \leq b$

\[
\begin{align*}
\eta &= \eta' = 2b + \frac{dy'}{d\eta} = 1
\end{align*}
\]

Hence $\eta'$ ranges from 0 to $2b$ with slope $\frac{dy'}{d\eta} = 1$.

Where $\eta_{1,2} = 0, y_o = y_o' = y_o = y_o$ then $\eta_{1,2} = 0$;

Where $\eta_{1,2} = b, y_o = y_o' = y_o = y_o$ then $\eta_{1,2} = b$.

Along $f_2$: $0 \leq y_o' \leq b$

\[
\begin{align*}
\eta &= \eta' = 2b + \frac{dy'}{d\eta} = 1
\end{align*}
\]

Hence $\eta'$ ranges from 0 to $2b$ with slope $\frac{dy'}{d\eta} = 1$.

Where $\eta_{1,2} = 0, y_o = y_o' = y_o = y_o$ then $\eta_{1,2} = 0$;

Where $\eta_{1,2} = b, y_o = y_o' = y_o = y_o$ then $\eta_{1,2} = b$.

Along $f_2$: $0 \leq y_o \leq b$

\[
\begin{align*}
\eta &= \eta' = \frac{dy'}{d\eta} = -1
\end{align*}
\]

Hence $\eta'$ ranges from 0 to $b$ with slope $\frac{dy'}{d\eta} = -1$.

Where $\eta_{1,2} = 0, y_o = y_o' = y_o = y_o$ then $\eta_{1,2} = 0$;

Where $\eta_{1,2} = b, y_o = y_o' = y_o = y_o$ then $\eta_{1,2} = b$.

Along $f_2$: $0 \leq y_o' \leq b$

\[
\begin{align*}
\eta &= \eta' = \frac{dy'}{d\eta} = -1
\end{align*}
\]

Hence $\eta'$ ranges from 0 to $b$ with slope $\frac{dy'}{d\eta} = -1$.

Where $\eta_{1,2} = 0, y_o = y_o' = y_o = y_o$ then $\eta_{1,2} = 0$;

Where $\eta_{1,2} = b, y_o = y_o' = y_o = y_o$ then $\eta_{1,2} = b$. 

46
Figure 5 - Transformation of Longitudinal Coordinates and Region of Integration

Not to Figure 5:

Line $f_1$ where $x_0 = 0$ transforms to $\xi' = \xi$

Line $f_2$ where $x_0 = a$ transforms to $\xi' = \xi = 2a$

Line $f_3$ where $x_0 = a$ transforms to $\xi' = \xi = 2a$ because $\xi = x_0 - x_0' = \phi^{-1}(y_0, y_0', x_0, x_0')$

Line $f_4$ where $x_0 = 0$ transforms to $\xi' = -\xi$

Thus the limit points of Equation (B.24) for the region $x_0, x_0'$ denoted by $\Delta^2$ in Figure 5a transform into the lines enclosing the region $T_2$ shown in Figure 5b. The limits for $\xi, \xi'$ obtained from the latter are based on the following relationships:

Along $f_1$: $0 < x_0 \leq a$

$\xi' = x_0 + x_0' = x_0$

Hence $\xi'$ ranges from $0$ to $a$ with slope $\frac{d\xi'}{d\xi} = 1$.

Along $f_2$: $0 \leq x_0' \leq a$

$\xi' = x_0 + x_0' = a + x_0'$

Hence $\xi'$ ranges from $a$ to $2a$ with slope $\frac{d\xi'}{d\xi} = 1$.

Along $f_3$: $0 < x_0 \leq a$

$\xi' = x_0 + x_0' = x_0 + a$

Hence $\xi'$ ranges from $a$ to $2a$ with slope $\frac{d\xi'}{d\xi} = 1$.

Along $f_4$: $0 < x_0' \leq a$

$\xi' = -\xi$ and $\frac{d\xi'}{d\xi} = -1$

Hence $\xi'$ ranges from $0$ to $a$ with slope $\frac{d\xi'}{d\xi} = -1$.
\[ \int_{m}^{t} dt_0 \int_{m}^{t'} dt'_0 \left\{ e^{-a_m(t-t_0)} \sin \omega_m(t-t_0) u(t-t_0) e^{-a_m(t'-t'_0)} \sin \omega_m(t'-t'_0) u(t'-t'_0) \right\} \]

\[
\sum_{\nu=1}^{3} \left[ A_{K} \int_{0}^{1} dx_0 \int_{0}^{1} dy_0 \int_{0}^{1} dx'_0 \int_{0}^{1} dy'_0 \frac{\phi_m(x_0, y_0) \phi_m(x'_0, y'_0) e^{-|t_0-t'_0|}}{K^2_o + (1/FU_o)^2} \left[ \left( \frac{(x_0 - x'_0) - U_o(t_0 - t'_0)}{2} \right)^2 + (y_0 - y'_0)^2 \right] \right]
\]

where the space integrals are given by Equation (B28).

Having transformed the space coordinates, now transform the time coordinates. First, change the variable of integration and find the Jacobian

\[
\tau'_0 = t_0' + t'_0 \\
2t'_0 = \tau_0 + \tau'_0 \\
t'_0 = \frac{\tau_0 + \tau'_0}{2} = g_1(\tau_0, \tau'_0)
\]

\[
\tau_0 = t_0 - t'_0 \\
2\tau'_0 = \tau_0 - \tau'_0 \\
t'_0 = \frac{\tau_0 - \tau'_0}{2} = g_2(\tau_0, \tau'_0)
\]

As for the space variables, now obtain the limits of \( \tau_0, \tau'_0 \) in the transformed \((T_{cd})\) plane. The transformation of the limits in the \(t'_0, \ldots\) plane to corresponding quantities in the \( \tau_0, \tau'_0 \) plane is given in Figures 6a and 6b where the limits from the \( \Xi_{cd} \) to \( T_{cd} \) plane are shown to be

\[
\Xi_{cd} = \left\{ (t_0, t'_0) : -\infty \leq t_0 \leq t, -\infty \leq t'_0 \leq t' \right\}
\]

\[
T_{cd} = \left\{ (\tau_0, \tau'_0) : \tau_0 - 2t' \leq \tau_0 \leq 2t - \tau'_0, -\infty \leq \tau'_0 \leq t + t' \right\}
\]
The limits for the transformed variables \( t_0', r_0' \) are now obtained from Figure 6b by subdividing the rectangular region into three regions A, B, C, over which the integrations are to be performed. From the figure and letting \( c = \infty, d = \infty \), we have:

For Region A:
\[
\begin{align*}
\{t_0' - 2t' \leq t_0' \leq 2t - t_0' \} & \quad \text{becomes} \quad -\infty \leq t_0' \leq t + t' \\
\{t - d \leq t_0' \leq t + t' \} & \quad \text{becomes} \quad -\infty \leq t_0' \leq t + t' 
\end{align*}
\]

For Region B:
\[
\begin{align*}
\{t_0' - 2t' \leq t_0' \leq 2t - t_0' \} & \quad \text{becomes} \quad -\infty \leq t_0' \leq t + t' \\
\{t - c \leq t_0' \leq t - d \} & \quad \text{becomes} \quad -\infty \leq t_0' \leq t - d
\end{align*}
\]

Since the limits for \( t_0' \) coincide, they will yield a zero contribution to the total integral. Therefore these limits are not considered.

For Region C:
\[
\begin{align*}
\{-c - d \leq t_0' \leq t' - c \} & \quad \text{becomes} \quad -\infty \leq t_0' \leq -c - d
\end{align*}
\]

Again, these limits are not considered.

Since only the limits for Region A contribute to the total integral, they alone are to be considered.
The limits for $T_{cd}$ shown in Figure 6 correspond to integrations for Region A only, which is the only region that need be treated. To perform the integration in Region A, it is convenient to subdivide the region in two, $Q$ and $Q'$ as shown in Figure 6c. The limits are then re-established so that the first is integrated with respect to $r_0'$ in Region $Q$ (the inner integral) and then with respect to $r_0$ in Region $Q'$; similar action is taken for Region $Q'$. The limits are (letting $t-t'=\tau$)

In Region $Q$: \[ -\infty \leq r_0' \leq r_0 + 2t' \]
\[ -\infty \leq r_0 \leq t - t' = \tau \]

In Region $Q'$: \[ -\infty \leq r_0' \leq 2t - r_0' \]
\[ t - t' = \tau \leq r_0 \leq \infty \]

The transformed time integral is now obtained from Equation (B25)

\[
I' = \int f(x) \, dx = \int_{\mathbb{R}^d} \int f[g(t)] \, J_\phi(t) \, dt
\]  

using Equation (B29) to obtain

\[
f[g(t)] = \frac{1}{\pi} e^{-\frac{1}{2} (t'-t_0'^2)} \sin \omega_m(t - t_0) \sin \omega_m(t' - t_0') \sin \omega_m(t' - t_0) \sin \omega_m(t - t_0')
\]

\[
= \frac{1}{\pi} e^{-\frac{1}{2} (t'-t_0'^2)} \sin \omega_m(t + t' - t_0 - t_0') \cos \omega_m(t - t_0) \cos \omega_m(t' - t_0') \cdot u(t-t_0) \cdot u(t'-t_0')
\]

\[
= \frac{1}{\pi} e^{-\frac{1}{2} (t'-t_0'^2)} \sin \omega_m(t + t' - t_0') \cos \omega_m(t - r_0) \cos \omega_m(t' - t_0') \cdot u\left[\frac{t-(r_0^2-r_0')}{2}\right] \cdot u\left[\frac{t'-(r_0^2-r_0')}{2}\right]
\]  

(B34)
and Equation (B31) to obtain

\[ J_{g}(t) = \frac{1}{2} \]  \hspace{1cm} (B35)

The time integral \( I' \) then becomes

\[
I' = I'_1 + I'_2 = \frac{1}{2} \int_{-\infty}^{\tau} \left[ \int_{-\infty}^{\tau_0 + 2t'} f[g(t)] \, d\tau' \right] \, d\tau_0
\]

For \( Q \)

\[
= \frac{1}{4} \int_{-\infty}^{\tau} \left\{ \left[ \int_{-\infty}^{\tau_0 + 2t'} \cos \omega_{mn}(t' + \tau_0 - \tau) - \int_{-\infty}^{\tau_0 + 2t'} \cos \omega_{mn}(t + \tau_0 - \tau') \right] \right\} \frac{-|\tau|}{\theta} \, d\tau_0
\]

For \( Q' \)

\[
= \frac{1}{4} \int_{-\infty}^{\tau} \left\{ \left[ \int_{-\infty}^{2t - \tau_0} \cos \omega_{mn}(t + \tau_0 - \tau') \right] \right\} \frac{-|\tau|}{\theta} \, d\tau_0
\]

(For convenience the \( u \)'s are omitted.)

Using Formula 414 of Reference 21 to obtain the first and third integrals (the second and fourth integrations are simply performed) and noting that \( t + t' - \tau_0 - 2t' = t - \tau_0 \) and \( t + t' - 2t + \tau_0 = -t + \tau_0 = \tau_0 - \tau \), we get

\[
I' = I'_1 + I'_2 = -\frac{1}{4} \int_{-\infty}^{\tau} e^{-a_{mn}(\tau - \tau_0)} \left[ a_{mn} \cos \omega_{mn}(\tau - \tau_0) - \omega_{mn} \sin \omega_{mn}(\tau - \tau_0) \right] \frac{-|\tau|}{\theta} \, d\tau_0
\]

\[
+ \frac{1}{4a_{mn}} \int_{-\infty}^{\tau} e^{-a_{mn}(\tau_0 - \tau)} \cos \omega_{mn}(\tau - \tau_0) \frac{-|\tau_0|}{\theta} \, d\tau_0
\]

51
We define

\[ f_1 (\xi) = (a - |\xi|) \cos \frac{m \pi \xi}{a} + a \sin \frac{m \pi |\xi|}{a} \]

\[ f_2 (\eta) = (b - |\eta|) \cos \frac{n \pi \eta}{b} + b \sin \frac{n \pi |\eta|}{b} \]  

(B38)

Then

\[
<Y(z, y, t) Y(z', y', t')> = \frac{p^2}{\sum_{\nu=1}^{3} (A_{\nu})^2 M^2} \sum_{m,n} \frac{\phi_{mn}(x, y) \phi_{mn}(x', y')}{\omega_{mn}^2} 
\]

\[
\cdot \sum_{\nu=1}^{3} A_{\nu} K_{\nu} \frac{2}{ab} \int_{a}^{b} f_{1}(\xi) f_{2}(\eta) \frac{d\eta d\xi [l_{1}' + l_{2}']}{K_{\nu}^2 + (1/\omega)^2 \left[ \left( \xi - U_{c} (t_{0} - t_{0}') \right)^2 + \eta^2 \right]} \]  

(B39)

or

\[
<Y(z, y, t) Y(z', y', t')> = \frac{p^2}{\sum_{\nu=1}^{3} (A_{\nu})^2 M^2} \sum_{m,n} \frac{\phi_{mn}(x, y) \phi_{mn}(x', y')}{2ab \omega_{mn}^2} [l_{1}' + l_{2}'] \]  

(B40)

where

\[
l_{1} = \sum_{\nu=1}^{3} A_{\nu} K_{\nu} \int_{a}^{b} f_{1}(\xi) \left[ \int_{0}^{b} f_{2}(\eta) \, d\eta \right] \, d\xi \frac{4 l_{1}'}{K_{\nu}^2 + (1/\omega)^2 \left[ \left( \xi - U_{c} (t_{0} - t_{0}') \right)^2 + \eta^2 \right]} \]

\[
= \frac{\omega_{mn}}{a_{mn}^2 + \omega_{mn}^2} \int_{a}^{b} f_{1}(\xi) \left[ \int_{0}^{b} f_{2}(\eta) \sum_{\nu=1}^{3} A_{\nu} K_{\nu} \right] \]

\[ \left[ \int_{-\infty}^{\tau} e^{-a_{mn}(\tau - t_{0})} \frac{\sin \omega_{mn}(\tau - t_{0}) + \omega_{mn} a_{mn} \cos \omega_{mn} (\tau - t_{0})}{\omega_{mn}} e^{-\frac{|\tau - t_{0}|}{\theta}} \right] \frac{d\tau_{0}}{K_{\nu}^2 + (1/\omega)^2 \left[ \left( \xi - U_{c} (t_{0} - t_{0}') \right)^2 + \eta^2 \right]} \]

\[ \frac{d\eta}{d\xi} \]  

(B41)
\[ l_2 = \sum_{\nu=1}^{3} A_\nu K_\nu \int_{a}^{b} f_1(\xi) \left[ \int_{0}^{b} f_2(\eta) \, d\eta \right] \, d\xi \frac{4l_2^2}{K_\nu^2 + (1/FU)_c^2 \left[ (\xi - U_c(\xi_0 - t_0') \right]^2 + \eta^2]} \]

\[ = \frac{\omega_{mn}}{a_{2mn} + \omega_{2mn}} \int_{a}^{b} f_1(\xi) \left[ \int_{0}^{b} f_2(\eta) \sum_{\nu=1}^{3} A_\nu K_\nu \right] \]

\[ \left[ \int_{-\infty}^{\infty} \frac{e^{-a_{mn}(t_0 - \tau)} \left\{ \sin \omega_{mn}(\tau_0 - \tau) + \frac{\omega_{mn}}{a_{mn}} \cos \omega_{mn}(\tau_0 - \tau) \right\} e^{-|\tau_0|}}{K_\nu^2 + (1/FU)_c^2 \left[ (\xi - U_c(\xi_0 - t_0') \right]^2 + \eta^2] d\tau_0 \right\} d\xi \right] \]

(B42)

The time integral in \( l_1 \) and \( l_2 \) can be further simplified by changing the limits of integration. In Equation (B41), for \( l_1 \) let

\[ \bar{k}_1 = \int_{-\infty}^{\infty} \frac{e^{-a_{mn}(\tau - \tau_0)} \left\{ \sin \omega_{mn}(\tau_0 - \tau) + \frac{\omega_{mn}}{a_{mn}} \cos \omega_{mn}(\tau_0 - \tau) \right\} e^{-|\tau_0|}}{K_\nu^2 + (1/FU)_c^2 \left[ (\xi - U_c(\xi_0 - t_0') \right]^2 + \eta^2] d\tau_0 \]

let

\[ \bar{x} = \tau - \tau_0 \quad \tau_0 = \tau - \bar{x} \quad d\bar{x} = -d\tau_0 \]

when

\[ \tau_0 = \tau, \bar{x} = 0 \]

\[ \tau_0 = -\infty, \bar{x} = \infty \]

substituting the previously mentioned quantities in \( \bar{k}_1 \) and noting that \( \int_{-\infty}^{0} = \int_{0}^{\infty} \)

\[ \bar{k}_1 = \int_{0}^{\infty} \frac{e^{-a_{mn}\bar{x}} \left\{ \sin \omega_{mn}\bar{x} + \frac{\omega_{mn}}{a_{mn}} \cos \omega_{mn}\bar{x} \right\} e^{-|\tau - \bar{x}|}}{K_\nu^2 + (1/FU)_c^2 \left[ (\xi - U_c(\xi_0 + U_c\bar{x})^2 + \eta^2 \right] d\bar{x} \quad (B43) \]
Similarly, for the time integral in \( l_2 \), set \( \bar{x} = \tau_0 - \tau, \tau_0 = \tau + \bar{x} \) \( d\bar{x} = d\tau_0 \), and note that when \( \tau_0 = \infty, \bar{x} = \infty \) whereas when \( \tau_0 = \tau, \bar{x} = 0 \). The substitutions yield

\[
\begin{align*}
\bar{K}_2 &= \int_0^\infty \frac{e^{-a_m n \bar{x}} \left\{ \sin \omega_m n \bar{x} + \frac{\omega_m n}{a_{mn}} \cos \omega_m n \bar{x} \right\} e^{-|\tau + \bar{x}|}}{K^2 \nu + (1/F U_c)^2 \left[ (\xi - U_c \tau) - U_c \bar{x} \right]^2 + \eta^2} d\bar{x} \\
&= (B44)
\end{align*}
\]

When Equations (B43) and (B44) are substituted into Equations (B41) for \( l_1 \) and (B42) for \( l_2 \), respectively, Equation (B40) becomes

\[
\begin{align*}
< Y(x,y,t) \cdot \nabla X(z',y',t') > &= \frac{p^2}{2ab} \sum_{\nu = 1}^3 \left\{ \frac{A_\nu K_\nu}{\omega_m n (a^2 + \omega_m^2)} \int_0^\infty \frac{\phi_{mn}(\tau,y) \phi_{mn}(z',y') e^{-|\tau - x|}}{\theta} d\tau \right\} \\
&+ \int_a^b \int_0^b \int_0^\infty \int_0^\infty g_1(\bar{z}) \sum_{\nu = 1}^3 \left\{ \frac{A_\nu K_\nu e^{-|\tau + \bar{x}|}}{\theta} \right\} d\bar{z} d\tau d\eta d\bar{x} \\
&= (B45)
\end{align*}
\]

where

\[
\begin{align*}
g_1(\bar{z}) &= e^{-a_m n \bar{z}} \left\{ \sin \omega_m n \bar{z} + \frac{\omega_m n}{a_{mn}} \cos \omega_m n \bar{z} \right\} \\
&= (B46)
\end{align*}
\]

and it is important to note that

\[
\begin{align*}
\bar{x} &= \tau - \tau_0; \ d\bar{x} = d\tau_0 \ for \ the \ first \ terms \ in \ the \ time \ integral \\
\bar{x} &= \tau_0 - \tau; \ d\bar{x} = d\tau_0 \ for \ the \ second \ terms \ in \ the \ time \ integral \\
&= (B47)
\end{align*}
\]
Let

\[ y' = \frac{n \eta}{b} \text{ or } \eta = \frac{b y'}{n \pi}; \quad z' = \frac{m \xi}{a} \text{ or } \xi = \frac{a z'}{m \pi} \]  

(B48)

then

\[ f_1(\xi) = \frac{\frac{m \eta}{a}}{\xi} \cos \left( \frac{m \pi |\xi|}{a} \right) + \frac{1}{m \pi} \sin \left( \frac{m \pi |\xi|}{a} \right) \]

(B49)

\[ f_1(\bar{z}) = \frac{\frac{a \bar{z}}{m \pi}}{\bar{a}} \cos \bar{a} + \frac{1}{m \pi} \left( -|\bar{z}| \cos \bar{a} + \sin |\bar{z}| \right) \]

Note that \( f_1(\xi) = a f_{11}(\bar{z}) \) and \( f_2(\eta) = b f_{21}(\bar{y}) \)

also when

\[ \xi = -a, \quad \bar{z} = -m \pi \]
\[ \xi = a, \quad \bar{z} = m \pi \]
\[ \eta = 0, \quad \bar{y} = 0 \]
\[ \eta = b, \quad \bar{y} = n \pi \]

(B50)

and

\[ d\eta = \frac{b}{n \pi} d\bar{y}, \quad d\xi = \frac{a}{m \pi} d\bar{z}, \quad d\eta d\xi = \frac{ab}{m \pi^2} d\bar{y} d\bar{z} \]

(B51)

Substituting Equations (B48) through (B51) in Equation (B45), the final result for the displacement cross correlation is
\[
\langle Y(x,y,t) Y(x',y',t') \rangle = \frac{p^{2ab}}{2\pi^2} \sum_{\nu=1}^{\frac{3}{M^2}} \phi_{mn}(xy) \phi_{mn}(x',y') \frac{\omega_{mn}(a^2_{mn} + \omega^2_{mn})}{\omega_{mn}(a^2_{mn} + \omega^2_{mn})}
\]

\[
\cdot \int_{-\infty}^{\infty} f_{11}(\tilde{z}) \left[ \int_{0}^{\infty} f_{21}(\tilde{y}) \left[ \int_{0}^{\infty} g_{1}(\tilde{z}) \sum_{\nu=1}^{3} \left\{ \frac{A_{\nu} K_{\nu}}{K_{\nu}^2 + (1/FU_c)^2} \left[ \left( \frac{\tilde{a}_{\nu}}{m} - U_c \tau \right) + U_c \tilde{x} \right]^2 + \left( \frac{\tilde{b}_{\nu}}{n} \right)^2 \right] \right] dy \right] d\tilde{z}
\]

(Note that in Reference 15, Equation (28), \( f_{11}(\tilde{z}) \rightarrow f(\tilde{z}), f_{21}(\tilde{y}) \rightarrow f(\tilde{y}) \) and \( g_{1}(\tilde{z}) \rightarrow g(\tilde{z}) \).)

If the second form of decay is used (for constant partial separation and variable time delay) for the cross correlation of the wall pressure for semifrozen flow, then in Equation (B13)

\[
\frac{-l_{01} - l_{01}}{\theta} = \frac{-l_{x} - x_{01}}{\theta} = \frac{-l_{x} - x_{01}}{\theta} = \frac{-l_{x} - x_{01}}{\theta} = \frac{-l_{x} - x_{01}}{\theta}
\]

so that following a procedure similar to that used in deriving Equation (B52), for this case

\[
\langle Y(x,y,t) Y(x',y',t') \rangle = \frac{p^{2ab}}{2\pi^2} \sum_{\nu=1}^{\frac{3}{M^2}} \phi_{mn}(xy) \phi_{mn}(x',y') \frac{\omega_{mn}(a^2_{mn} + \omega^2_{mn})}{\omega_{mn}(a^2_{mn} + \omega^2_{mn})}
\]

\[
\cdot \int_{-\infty}^{\infty} f_{11}(\tilde{z}) \left[ \int_{0}^{\infty} f_{21}(\tilde{y}) \left[ \int_{0}^{\infty} g_{1}(\tilde{z}) \sum_{\nu=1}^{3} \left\{ \frac{A_{\nu} K_{\nu}}{K_{\nu}^2 + (1/FU_c)^2} \left[ \left( \frac{\tilde{a}_{\nu}}{m} - U_c \tau \right) + U_c \tilde{x} \right]^2 + \left( \frac{\tilde{b}_{\nu}}{n} \right)^2 \right] \right] dy \right] d\tilde{z}
\]

(B53)
where

\[ f_{11}(\overline{z}) = \frac{\left( \frac{a}{mn} \overline{z} \right)}{b} = \frac{\cos \overline{z} + \frac{1}{mn} (\overline{z} \cos \overline{z} + \sin \overline{z})}{e^{\frac{-a}{mn} \overline{z} \theta}} \]

\[ f_{21}(\overline{z}) = \frac{\left( \frac{b}{mn} \overline{z} \right)}{b} = \cos \overline{y} + \frac{1}{nn} (\sin |\overline{y}| - |\overline{y}| \cos |\overline{y}|) \]

\[ g_1(\overline{z}) = e^{-\frac{a}{mn} \overline{z}} \left[ \sin \omega_m \overline{z} + \frac{\omega_m}{a_{mn}} \cos \omega_m \overline{z} \right] \]

Equations (B52) and (B53) have similar numerical results.

If the pressure covariance is used for unsteady convection (Model B) given in Equation (B10), the response of a panel including modal couplings is given by

\[ \langle Y(z',y',t') \rangle = \int_{-\infty}^{t} dt_0 \int_{-\infty}^{t'} dt'_0 \int_{a_0}^{a} dz_0 \int_{b_0}^{b} dy_0 \int_{0}^{b} dy'_0 \]

\[ \cdot \left[ \frac{\phi_{mn}(z,y) \phi_{mn}(z_0,y_0)}{\omega_{mn} M} e^{-\frac{a}{mn} (t-t_0)} \{ \sin \omega_m (t-t_0) \} u(t-t_0) \right] \]

\[ \cdot \left[ \frac{\phi_{pq}(z',y') \phi_{pq}(z'_0,y'_0)}{\omega_{pq} M} e^{-\frac{a}{pq} (t'-t'_0)} \{ \sin \omega_p (t'-t'_0) \} u(t'-t'_0) \right] \]

\[ \cdot \left( \frac{\cos \omega_1 (t-t_0)}{e^{\frac{\cos \omega_1 (t-t_0)}{2}}} \right) \cdot \sin \omega_1 |t_0 - t'_0| \]

\[ \frac{\left( (x_0 - x'_0) - \frac{v}{2c_0} (t_0 - t'_0) \right)^2 + \frac{1}{2c^2_0} |(x_0 - x'_0) - \frac{v}{2c_0} (t_0 - t'_0) |^2}{e^{-\frac{|t_0 - t'_0|}{2\sigma_2^2}}} + A_2 \left[ e^{\frac{-|t_0 - t'_0|}{\sigma_2}} \cos \omega_2 (t_0 - t'_0) \right] \]

\[ - \left\{ \int (x_0 - x'_0) - \frac{v}{2c_0} (t_0 - t'_0) \right\}^2 + \frac{1}{2c^2_0} |(x_0 - x'_0) - \frac{v}{2c_0} (t_0 - t'_0) |^2 \}

\[ e^{2\sigma_2^2} + A_2 \left[ e^{\frac{-|t_0 - t'_0|}{\sigma_2}} \cos \omega_2 (t_0 - t'_0) \right] \]

Equations (B54)
Using the modal volume displacement

\[ \text{Vol}_{mn} = \sqrt{2} \int_{0}^{b} \int_{0}^{a} \phi(x)\phi(y) \, dx \, dy \]  \hspace{1cm} (B55)

the modal acoustic power radiated in a reverberant field can be calculated

\[ \text{PWL}_{mn} = \frac{N\omega^2 \rho c K^2}{2\pi} \frac{2P_r + P_p}{P_p} \left[ \int_{0}^{a} \int_{0}^{b} |\phi_m(x)| |\phi_n(y)| \, dx \, dy \right] \]  \hspace{1cm} (B56)

Finally, using Equation B9, the normalized power spectrum of the turbulence pressures is computed for case of \( \eta = 0 \). The normalized correlation coefficient is defined as

\[ R_{y}(\xi, \eta, \tau) = \frac{Y(x, y, t) Y(x', y', t')}{[Y(x, y, t)^2 Y(x', y', t')^2]^{1/2}} \]

The normalized power spectrum is then

\[ P(K_1, \omega) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R(\xi, 0, \tau) e^{-(K_1 \xi + \omega \tau)} d\xi d\tau \]  \hspace{1cm} (B57)

\[ = \frac{1}{(2\pi)^2} \frac{-|\xi|}{U_c^\theta} \sum_{\gamma=1}^{3} \frac{A_{\gamma}K_{\gamma}^F}{K_{\gamma}} \left[ \int_{-\infty}^{\infty} \frac{e^{-i\omega \tau} d\tau}{K_{\gamma}^2 F^2 + \left( \frac{\xi}{U_c} \right)^2} \right] e^{-iK_1 \xi} d\xi \]  \hspace{1cm} (B58)

From Pair 632.2 of Reference 7, with \( p = i\omega \) and \( \beta = K_{\gamma}^F \), the quantity in brackets equals

\[ \frac{\pi}{K_{\gamma}^F} e^{-K_{\gamma}^F |\omega| - i\xi \omega / U_c} \]  \hspace{1cm} . Hence

\[ P(K_1, \omega) = \frac{F}{4\pi} \sum_{\gamma=1}^{3} \frac{A_{\gamma}}{K_{\gamma}} \sum_{\gamma=1}^{3} A_{\gamma} e^{-|\omega| K_{\gamma}^F \int_{-\infty}^{\infty} \frac{-|\xi|}{U_c^\theta} e^{-iK_1 \xi} d\xi} \]  \hspace{1cm} (B59)

where \( K_1' = \frac{\omega}{U_c} + K_1 \).

59
Using Pair 444 of Reference 7, with $p = iK_1^* = i\left(\frac{\omega}{U_c} + K_1^*\right)$, and $\beta = \frac{1}{U_r\theta}$, the integral equals

$$\frac{-2\cdot U_c \theta}{-(K_1^*)^2 - (U_c \theta)^2} = \frac{2U_c \theta}{1 + \theta^2(\omega + K_1^* U_c)^2}$$

Hence

$$P(K_1, \omega) = \frac{\theta}{1 + \theta^2(\omega + K_1 U_c)^2} \frac{\sum_{y=1}^{3} A_y e^{-i \omega y \zeta}}{(F U_c / 2\pi) \left(\sum_{y=1}^{3} \frac{A_y}{K_y}\right)}$$

(B60)

which agrees with Reference 16, Equation (6).

**COMPUTER PROGRAMS**

Computer programs for either Equation (B52) or (B53) and Equation (B60) are designated as Subprograms A and B, respectively. Reference 22 presents equations similar to Equations (B52) and (B53) and gives Fourier transforms to yield the displacement cross spectral density; (see Reference 22, Equation (27)). Since the method of derivation is similar to the method presented in this report, the details are omitted here.* The corresponding computer subprograms are designated as Subprograms C (modal coupling excluded) and D (modal coupling included). The latter subprograms treat simple and clamped supports and uncoupled and coupled modes.

**APPENDIX B2 — METHOD FOR DETERMINING INPUT DATA**

The following data are furnished to the computer

**Flow data:** $U_c, \tau_w, \delta^* = F U, \omega, \theta, A_y, K_y$, where $\nu = 1, 2, 3$

**Panel data:** $a, b, \delta, E, M, \phi_m, \omega_m, \xi, \eta, \tau, m, n, \bar{Y}, N, p c K_a^2, \frac{2P_r + P_p}{P_p}, x, y, x', y'$

The method for determining the data is now described. Either the data are arbitrarily selected by the user, i.e., the values are chosen to represent the range of interest, or the selections may correspond to experimental values for a parameter.

---

*The mathematical form of the model representing turbulence excitation pressures used in Reference 22 is slightly different from that used by Maestrello in his earlier works.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
</table>
| \(A, K
| Prescribed constants used in Equation (B7) |
| \(a, b, h\) | Prescribed quantities |
| \(E\) | A prescribed quantity |
| \(M\) | A prescribed quantity |
| \(m, n\) | Prescribed data |
| \(N\) | Determined as described on page 434 of Reference 15. Maestrello assumed piston radiation for which \(N = 4\). |
| \(\frac{2P_r + P_p}{P_p}\) | Equal to 1 for a plate without ribs |
| \(\overline{P^2}\) | \(\int_0^\infty P(\omega) \, d\omega\), where \(P(\omega)\) is obtained from Equation (B7). This quantity can also be measured directly. |
| \(U_c\) | A parameter whose values are prescribed by the user, \(U_c = 0.8 \, U_\infty\); see Reference 15 page 427. A method for measuring this quantity by experimentation is given by Equations (B2) and (B4) or alternately for \(K, \omega\) space by equations given in Reference 15, page 414. |
| \(x, y, x', y'\) | Prescribed points; the cross correlation of the displacements are computed for these points. |
| \(\overline{F}\) | Determined by taking the square root of the calculated mean-square displacement. |
| \(\delta^* (\sim FU)\) | \(0.37/8 \, (U_\infty \, \nu/\nu)^{-1/5}\); see Reference 25, Equations (21.6) and (21.8). Tables 1.1 and 2.1 of that reference give values of \(\nu\) in air and water. Using this equation, values of \(\delta^*\) for a given fluid can be prescribed over a range of \(U_\infty\). |
| \(\delta_{mn}\) | Total damping ratio obtained in accordance with the methods of Section 3.4 of Reference 14 and further described in the footnote to statement preceding Equation (B18). Note that in Reference 15 Figure 19, \(a_{mn} = \omega_{mn}/2\), whereas determination of \(a_{mn}\) in Figure 16 of the same reference is made from the formula \(a_{mn} = 0.5(\omega_{mn})^{1/3}\), which is based on El Baroudis data. The latter was considered acceptable for thin plates. In general it is preferable to use the former method in making a theoretical calculation. Value of \(a_{mn}\) may be prescribed by the user to determine the effects of damping variations. |

\*Our equation agrees with that given by Jacobs in Equation (59), of Reference 34 letting \(U_c = 0.8 \, M_c(M = \text{Mach Number} \text{ and } c = \text{sound velocity})\) and noting that \(U\) should be in milliseconds in that equation. \*Our equation differs from that given in Reference 14, Equation 7, by a factor of \(10^{-3}\), see also Appendix E.
Corresponds to the time in which the value of the measured correlation coefficient of the fluctuating pressures at the wall, obtained from the envelope of the correlation maxima, drop to 1. The plots of \( \theta \) versus Mach number for broad- and narrow-band frequencies are given in Reference 14, Figure 5. From Figure 5:

\[
\theta = 1.37 \times 10^{-3} U_c + 1.15
\]

where \( \theta \) is in milliseconds and \( U_c \) in feet per second. Thus, \( \theta = \theta(U_c) \) may also be prescribed for various values of \( U_c \); \( \theta \) may also be obtained from the plot of \( U_c\theta/\delta \) versus the Mach number given in Reference 15, Figure 6.

Prescribed data

\( \xi, \eta, \tau \)

\( \rho CK^2 \)

The numerical value of this quantity for air has been computed and included in the program. For a water medium, it is necessary to modify this value in the ratio of \( (\rho C)_{\text{water}} / (\rho C)_{\text{air}} \) by adding an instruction to the program.

\( \tau_w \)

Determination of this quantity is based on the law of the wall described in detail and shown in Reference 23, Figure 1. The Maestrello data given in Reference 24, Figure 2, lie along the universal curve representing this law. (Note that for incompressible flow, the vertical coordinate of this figure equals \( u/\nu \)).

\[
yu^{\prime}_{w} / u^{\prime}_{w} = \text{constant} = \text{inverse slope of the curve, which is measured. Hence if the velocity profile of the users data agrees with the universal curve, then for a known value of } \nu_{w}, \text{ selecting a value of } u \text{ corresponding to a value of } y \text{ yields the value of } u^{\prime}_{w}. \text{ By definition } \tau_{w} = \rho_{w} u^{2}_{w}. \]

\( \phi_{mn}(xy) \phi_{m'n'}(x',y') \)

The data required for the computer program are calculated by the digital computer for a range of prescribed values of \( m, n, x, y, x', \) and \( y' \).

Prescribed in Equation (B7) to obtain \( P(\omega) \)

\( \omega \)

\( \omega_{mn} \)

For a plate of given geometry and structural properties, this quantity is computed by hand for a plate with rigid or fixed boundaries using Equation (A7) of Reference 14. For a simply supported plate, use Equation (IV.5.16) of Reference 10.
APPENDIX B3 – PROGRAM IDENTIFICATION

A program is presented for computing mean-square displacement (Subprogram A) of a simply supported rectangular panel excited by a turbulent boundary layer and the modal acoustic-power radiation of the plate in a reverberant field. The program also computes the turbulent pressures on the plate (Subprogram B) and the cross spectral density of the displacement for simple or clamped boundaries without modal cross coupling (Subprogram C) and with modal cross coupling (Subprogram D); see Tables 2 and 3. Computer running times for sample problems are as follows:

<table>
<thead>
<tr>
<th>Subprogram</th>
<th>Computer</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>IBM 7090 at NSRDC</td>
<td>The Simpson rule of integration for 4 modes (m = 1, 3, 5, 7), with (n = 1) and 1 convection velocity (U_c). Approximately 28 min</td>
</tr>
<tr>
<td>B</td>
<td>IBM 7090 at NSRDC</td>
<td>48 frequencies at 8 spatial separations. Approximately 2 min</td>
</tr>
<tr>
<td>C</td>
<td>IBM 360/91 at Applied Physics Laboratory, Johns Hopkins University</td>
<td>Gaussian quadrature (9 point) for 4 modes (m = 1, 3, 5, 7), with (n = 1) and 1 (U_c). Approximately 9 sec</td>
</tr>
</tbody>
</table>
Note 1: For Curves I and II, use ordinate scales to left and right, respectively.

Note 2: NSRDC Curve I was obtained by selection of a value of $p^2 = 13.72$ lb/in. which yielded a curve approximating that of Maestrello Curve I. Maestrello's actual value of $p^2$ (Figure 7, Reference 16) is unknown. NSRDC Curve I was then manually adjusted to give the same value of $p^2$ as that given by Maestrello at $\frac{2\pi L}{\lambda} = 1$. (This should be equivalent to using the same value of $p^2$ as Maestrello.)

The replot is shown as Curve II. Observe that in view of the scaling errors involved in copying the Maestrello curve, the curves shown in the replot are in good agreement.

Figure 7 - Variation of Modal Mean Square Displacement with Eddy Lifetime for a 36- x 6.5- x 0.04-Inch Panel
Figure 8 – Computed Modal Mean Square Displacement for a 36- x 6.5- x 0.04-Inch Panel

Compare with Figure 19 of Reference 15. The trend of curves there is similar to curves drawn here, but differences appear to be due either to a normalization factor of $4\pi^2 u^2$ or to the use of different values of $p^2$ than those used here.

Figure 9 – Contours of Constant Turbulence Pressure Spectrum Level for Convected Semifrozen Pattern

Compare with Figure 5 of Reference 15.
Figure 10 – Computed Displacement Spectral Density for a 12- x 6- x 0.062-Inch Titanium Panel

Compare with Figure 20 of Reference 22.
Notes for Figures 10a and 10b:

To compute the power spectrum, without coupling, Subprogram C first uses the Warburton method to determine the eigenfunctions and eigenvalues. Hence the user must submit the coefficients for the boundary conditions he desires. The program comes in two parts: the first solves for the peak response at $\omega_0 = \omega_{mn}$ and the second solves for the spectrum, varying $\omega$ around $\omega_{mn}$.

Figure 10a shows a narrow bandwidth for some modes, such as (5.1). This means that if the user were to run the program only for $\omega = \omega_{mn}$ (as in Figure 10b which ignores the first part of the solution) he could bypass the peak response. The curve shown in Figure 10a is a result of both programs, where DW, the interval used in the computer program, was 100, for the case $\omega \neq \omega_{mn}$. The curve was then extrapolated to its peak value at $\omega = \omega_{mn}$.

Input data for Figures 10a and 10b were obtained from Maestrello, he computed his modal frequencies for $\Delta \rho = 0.06$ and 14 psi and used experimentally obtained values of damping. The relations $U_e = 0.8 U_i$ and $p^s = 12 \pi$ were used. Input dimensions of length are in units of feet, whereas by means of a conversion factor in the program, the corresponding output is in inch units.

In Figure 10b the static pressure $\Delta \rho = 14$ psi is introduced into the program by modifying the value of $\omega_{res}$ obtained at $\Delta \rho = 0.06$ psi = 0 psi from the Warburton equation for clamped boundaries. Since $\omega_{res} = \sqrt{E/2}$ and approximately $k = E$, then $\omega_{res}$ is proportional to the square root of $E$ and therefore the stiffness $k$. If originally we have $E_{\Delta \rho = 0} = 0.21 \times 10^8$ psi, then modifying $E$ in proportion to the change in stiffness which was measured with a strain gage on the panel under static loading, we get $E_{\Delta \rho = 14} = 0.33 \times 10^8$ psi. We then recompute the value of $\omega_{res}$ corresponding to this value of $E$. 

Figure 10b — Results Ignoring the First Part of the Solution
APPENDIX B

TABLE 2

Identification of Subprograms A, B, C, and D – Maestrello

This table includes identification input and output data units (in foot – pound – seconds), flow charts, order of data, and sample data. Computer subprogram listings are given at the end of this appendix as Table 3.

<table>
<thead>
<tr>
<th>Subprogram</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subprogram A:</td>
<td>Computation of Plate Vibration and Acoustic Response for Model A (Semifrozen Convection)</td>
</tr>
<tr>
<td>Subprogram B:</td>
<td>Computation of Cross Spectral Density of Turbulent Pressures for Model A (Semifrozen Convection)</td>
</tr>
<tr>
<td>Subprogram C:</td>
<td>Computation of Cross Spectral Density of Displacement</td>
</tr>
<tr>
<td>Subprogram D:</td>
<td>Generalization of Subprogram C to Include Modal Cross Coupling</td>
</tr>
</tbody>
</table>
### TABLE 2 A

Input Required for Subprogram A to Compute the Plate Vibration and Acoustic Response for the Semifrozen Convection (Model A) and Corresponding Output

(Units are given in foot-pound-seconds)

<table>
<thead>
<tr>
<th>Data</th>
<th>Description</th>
<th>Type</th>
<th>Symbol Used in Program</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_c$</td>
<td>Broadband convection velocity</td>
<td>Decimal</td>
<td>UC(I)</td>
</tr>
<tr>
<td>$\bar{p}^2$</td>
<td>Mean-square wall-pressure fluctuations, which vary with $U_c^2$</td>
<td>Decimal</td>
<td>PB2*DPB2(I)</td>
</tr>
<tr>
<td>$(F_U)^2$</td>
<td>Quantity $\left(\frac{8^*U_c}{U}\right)^2$ squared where:</td>
<td>Decimal</td>
<td>FUCSQ</td>
</tr>
<tr>
<td></td>
<td>$\delta^* = \text{boundary-layer displacement thickness}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$U = \text{free stream velocity}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_1, K_2, K_3$</td>
<td>Universal constants:</td>
<td>Decimal</td>
<td>AK</td>
</tr>
<tr>
<td></td>
<td>$K_1 = 0.470$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$K_2 = 3.0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$K_3 = 14.0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_1, A_2, A_3$</td>
<td>Universal constants:</td>
<td>Decimal</td>
<td>AN</td>
</tr>
<tr>
<td></td>
<td>$A_1 = 1.6$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$A_2 = 7.2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$A_3 = 12.0$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(*PB2 would represent a unique value of $\bar{p}^2$ if $\bar{p}^2$ were independent of $U_c$. It enters the program (i.e., data cards) once only. Since $\bar{p}^2$ actually varies with $U_c$, a correction factor DPB2(I) is entered with every value of $U_c$. Thus $\bar{p}^2$ as a function of $U_c$ is accounted for by the quantity PB2* DPB2(I).)
<table>
<thead>
<tr>
<th><strong>Data</strong></th>
<th><strong>Description</strong></th>
<th><strong>Type</strong></th>
<th><strong>Symbol Used in Program</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Plate Characteristics</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h$</td>
<td>Panel thickness</td>
<td>Decimal</td>
<td>$H$</td>
</tr>
<tr>
<td>$\gamma^2$</td>
<td>Square of plate mass</td>
<td>Decimal</td>
<td>$FM2$</td>
</tr>
<tr>
<td>$a, b$</td>
<td>Lengths of panel sides</td>
<td>Decimal</td>
<td>$ZUP, YUP$</td>
</tr>
<tr>
<td>$\delta_{m,n}$</td>
<td>Total damping ratio</td>
<td>Decimal</td>
<td>$DAMP$</td>
</tr>
<tr>
<td>$\omega_{m,n}$</td>
<td>Modal frequencies of the plate</td>
<td>Decimal</td>
<td>$OMEGA$</td>
</tr>
<tr>
<td><strong>Additional Quantities</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Range of plate mode numbers for which calculations are desired</td>
<td>First $m$ mode number, last $m$ mode number, interval between $m$ mode number, total number of $m$'s</td>
<td>Integer</td>
<td>MLOW, MUP, DM, MSTEPS</td>
</tr>
<tr>
<td>Same information as previously described, with respect to $n$ mode numbers</td>
<td>Integer</td>
<td>NLOW, NUP, DN, NSTEPS</td>
<td></td>
</tr>
<tr>
<td>$r$</td>
<td>Time delay</td>
<td>Decimal</td>
<td>$TAU$</td>
</tr>
<tr>
<td>Number of values of $U_c$ to be calculated</td>
<td>Integer</td>
<td>$KUC$</td>
<td></td>
</tr>
<tr>
<td>Set of reference points against which mean-square displacement and acoustic power will be tabulated</td>
<td>$2U_c \lambda_{m,n} \delta \lambda_{m,n} \omega_{m,n}$</td>
<td>Decimal</td>
<td>$PARAM$</td>
</tr>
<tr>
<td>Number of values of $PARAM$ specified</td>
<td>Integer</td>
<td>$NP$</td>
<td></td>
</tr>
<tr>
<td>$x_0, y_0$</td>
<td>Coordinates of a point on plate at which mean-square displacement and acoustic power are calculated</td>
<td>Decimal</td>
<td>$XO, YO$</td>
</tr>
<tr>
<td>$x_0', y_0'$</td>
<td>any point on plate different from $x_0, y_0$</td>
<td>Decimal</td>
<td>$XOP, YOP$</td>
</tr>
</tbody>
</table>
### TABLE 2A (Continued)

<table>
<thead>
<tr>
<th>Data</th>
<th>Description</th>
<th>Type</th>
<th>Symbol Used in Program</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Calculated Output in Inch-Pound-Seconds</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi_{m,n}$</td>
<td>Value of eigenfunctions of mean-square displacement</td>
<td>Decimal</td>
<td>EIGEN</td>
</tr>
<tr>
<td></td>
<td>A value of EIGEN is computed for each mode $(m,n)$ with three values of total damping; $1/10 \alpha_{m,n} : \alpha_{m,n} : 10\alpha_{m,n}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_{m,n}$</td>
<td>Values of total damping associated with each mode $(m,n)$</td>
<td>Decimal</td>
<td>FA$(m,n,1)$ for computation; A$(m,n, DAMP)$ in output</td>
</tr>
<tr>
<td>$V_{m,n}$</td>
<td>Volume under each eigenfunction</td>
<td>Decimal</td>
<td>VOL</td>
</tr>
<tr>
<td>$\sqrt{2Y}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$l(m,n)$</td>
<td>Triple integral of Equation 1.13: integral of cross correlation</td>
<td>Decimal</td>
<td>1XY2</td>
</tr>
<tr>
<td>$\overline{Y^2}$</td>
<td>Mean-square displacement for values of $1/10 \alpha_{m,n} : \alpha_{m,n} : 10\alpha_{m,n}$</td>
<td>Decimal</td>
<td>ANS</td>
</tr>
</tbody>
</table>
Output is printed for a given convection velocity $\mathcal{U}_c$ for the modes of interest. The last four columns of the output respectively represent $\overline{Y^2}$ for $1/10 \ a_{m,n}$, $\overline{Y^2}$ for $a_{m,n}$, and $\overline{Y^2}$ for $10 \ a_{m,n}$ for chosen PARAM. Since this program was written, Maestrello has developed methods of computing damping, but here he chooses the magnitude of $a_{m,n}$ according to the $\omega \theta$ region of the curve that interests him. For example, Figure 7 plots $\overline{Y^2}$ against $\omega \theta$. In the region $\omega \theta = 10^{-1}$ to 1.0 he uses $a_{m,n} / 10$; however, for $\omega \theta = 1.0$ to 40, he uses $a_{m,n}$. Also, $\overline{Y^2}$ corresponds to the value of PARAM closest to $\omega \theta$ where $\theta$ corresponds to $\mathcal{U}_c$.  

72
TABLE 2A (Continued)

1. READ AND WRITE INPUT DATA
2. CALCULATE VARIABLES USED INSIDE LOOPS
3. CALCULATE
   \[ \frac{ab}{2} \sum_{i=1}^{3} \frac{1}{H_i} \]
4. DOUBLE DO LOOP ON 10
   \[ m \text{ and } n \text{ Vary} \]
   \[ x_0 = A/2, y_0 = R/2 \]
   \[ m \text{ IS EVEN} \]
   \[ n \text{ IS EVEN} \]
   \[ y_0 = A/2 - A/(2m) \]
   \[ y_0 = R/2 - R/2n \]
   \[ \text{DO on 45} \]
   \[ \text{Set up for 3 dams} \]
   \[ \frac{1}{10}, 1, 10 \]
   \[ \text{CALCULATE for each dam} \]
   \[ \sum_{\text{ mn }=\text{ unit }} \frac{\phi_m(x,y) \phi_n(x',y')}{\omega_m \omega_n} \left( \frac{X_m^2 + Y_m^2}{2} \right) \]
   \[ 45 \text{ CONTINUE} \]
   \[ 10 \text{ CONTINUE} \]

5. WRITE
   Values ju = calculated

SUBROUTINE VOLUM
Calculates 1 per \( m, n \)

\[ V_{mn} = \frac{16 ab 144 \sin\left(\frac{2m+1}{2}\frac{x}{2}\right) \sin\left(\frac{2n+1}{2}\frac{y}{2}\right)}{(2n+1)\sinh(2m+1)\frac{x}{2} - (2m+1)\sin(2n+1)\frac{y}{2}} \]

WRITE answers
RETURN
TABLE 2A (Continued)

1. \text{DO 777} \quad \text{Var} \ V_j \text{ CONST}
\text{DO 778} \quad \text{DPB2(KU)}
\text{WRITE} \quad U_j \text{ AND CONST}

2. \text{DO 779} \quad m \text{ Values}

3. \text{DO 780} \quad \text{Set up } \theta \text{ 's}
\text{781} \quad \text{PARAM } \theta/m/U_j \text{ + } \theta

Set upper and lower limits for z, y, x integrals determine no. of steps for each integral and the step size for each

4. \text{DO 781} \quad \text{Calculate coefficients for}
Simpson Rule

5. \text{Set variable of integration - lower limit on } z
\text{Calculates function of } z \text{ values}

6. \text{Set variable of integration - lower limit on } y
\text{Calculates function of } y \text{ values}

7. \text{Set up and initialize to zero, the array for summing on outside integral}

8. \text{Set variable of integration - lower limit on } x
\text{Calculates function of } x \text{ varying } y \text{ and } z \text{ and summing up parts previously calculated}

9. \text{WRITE}
\text{Answers of}
\text{triple integral}

10. \text{COMPUTE AND WRITE}
\text{ANS + IXYZ + EIGEN AND CONST}

779 \quad \text{CONTINUE}

777 \quad \text{CONTINUE}

STOP
<table>
<thead>
<tr>
<th>COLUMN HEADINGS FOR INPUT FORMS ON DATA CARDS</th>
</tr>
</thead>
<tbody>
<tr>
<td>TAU</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>H</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>XO</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>YO</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>XOP</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>YOP</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>FUCSQ</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>PB2</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>FM2</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>AK(1)</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>AK(2)</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>AK(3)</td>
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<td>-----</td>
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<td>AN(1)</td>
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<tr>
<td>-----</td>
</tr>
<tr>
<td>AN(2)</td>
</tr>
<tr>
<td>-----</td>
</tr>
</tbody>
</table>
Table 2A (Continued)

<table>
<thead>
<tr>
<th>AN(1)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
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<tr>
<td></td>
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<tr>
<td>TITLE</td>
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<td></td>
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</tr>
<tr>
<td>KUC</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
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<tr>
<td>UC(1)</td>
<td>10</td>
<td>DPB2(1)</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
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<td></td>
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<td></td>
<td></td>
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</tr>
</tbody>
</table>

Number of cards needed for U_C, DPB2 arrays is equal to KUC

<table>
<thead>
<tr>
<th>ZUP</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>YUP</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>70</td>
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<td></td>
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</tr>
<tr>
<td>H</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
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<td></td>
</tr>
<tr>
<td>MLOW</td>
<td>10</td>
<td>MUP</td>
<td>20</td>
<td>DM</td>
<td>30</td>
<td>MSTEPS</td>
<td>40</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NLLOW</td>
<td>10</td>
<td>NUP</td>
<td>20</td>
<td>DN</td>
<td>30</td>
<td>NSTEPS</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OMEGA(1,1)</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>70</td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OMEGA(2,1)</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

With the order of cards (1,1), (2,1), ..., (M,1), (1,2), ..., (M,2), ..., (1,N), ..., (m,N) to complete OMEGA(m,n) array.
TABLE 2A (Continued)

<table>
<thead>
<tr>
<th>DAMP (1,1)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>DAMP (2,1)</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>70</td>
</tr>
</tbody>
</table>

With the order of cards (1,1), (2,1), (m,1), (1,2), (m,2), (1,n), (m,n) to complete the DAMP (m,n) array.

<table>
<thead>
<tr>
<th>PARAM (1)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
</tr>
</thead>
</table>

The number of cards needed to complete PARAM array is equal to NP.
### TABLE 2B

Input Required for Subprogram B to Compute the Cross Spectral Density of Turbulence Pressures for the Semifrozen Convection (Model A) and Corresponding Output

(Units are given in foot-pound-seconds)

<table>
<thead>
<tr>
<th>Data</th>
<th>Description</th>
<th>Type</th>
<th>Symbols Used in Program</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow Characteristics</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>Eddy Lifetime</td>
<td>Decimal</td>
<td>THETA</td>
</tr>
<tr>
<td>$\xi$</td>
<td>$x - x'$ (Plate separation in $x$ direction)</td>
<td>Decimal</td>
<td>SI</td>
</tr>
<tr>
<td>$\tau_w$</td>
<td>Wall shear stress</td>
<td>Decimal</td>
<td>TW</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Frequency</td>
<td>Decimal</td>
<td>FR</td>
</tr>
<tr>
<td>$\delta^*$</td>
<td>Boundary-layer displacement thickness</td>
<td>Decimal</td>
<td>DSTAR</td>
</tr>
<tr>
<td>$K_1, K_2, K_3$</td>
<td>Universal constants: $K_1 = 0.470, K_2 = 3.0, K_3 = 14.0$</td>
<td>Decimal</td>
<td>D</td>
</tr>
<tr>
<td>$A_1, A_2, A_3$</td>
<td>Universal constants: $A_1 = 1.6, A_2 = 7.2, A_3 = 12.0$</td>
<td>Decimal</td>
<td>A</td>
</tr>
<tr>
<td>$U_c$</td>
<td>Broadband convection velocity</td>
<td>Decimal</td>
<td>UC</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Calculated Output in Foot-Pound-Seconds</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$R(r)$</td>
<td>Normalized autocorrelation of the pressure; see Equation (B8)</td>
<td>Decimal</td>
</tr>
<tr>
<td>$R(\xi,n,r)$</td>
<td>Normalized cross correlation of the pressure; see Equation (B9)</td>
<td>Decimal</td>
</tr>
<tr>
<td>$P(\omega) U$</td>
<td>Dimensionless power spectrum of wall-pressure fluctuations; see Equation (B7)</td>
<td>Decimal</td>
</tr>
<tr>
<td>$\frac{\overline{r^2 \delta^*}}{\tau_w}$</td>
<td>Power spectrum; see Equation (B7)</td>
<td>Decimal</td>
</tr>
<tr>
<td>$P(K_1,\omega)$</td>
<td>Normalized cross-power spectral density, using longitudinal space-time cross correlation of semifrozen model</td>
<td>Decimal</td>
</tr>
<tr>
<td>$P(\omega)$</td>
<td>Power spectrum, corresponding to $P(K_1,\omega)$</td>
<td>Decimal</td>
</tr>
</tbody>
</table>
TABLE 2B (Continued)

<table>
<thead>
<tr>
<th>AUTOCORRELATION</th>
<th>CROSS CORRELATION</th>
<th>POWER SPECTRUM</th>
</tr>
</thead>
<tbody>
<tr>
<td>EQUATION (88)</td>
<td>EQUATION (89)</td>
<td>EQUATION (87)</td>
</tr>
</tbody>
</table>

Figure 9

START

READ DATA
For Auto and
Cross Correlation

Calculate Variables
used inside DO Loops
and \( \sum A_i \)

DO Loop on TAU

WRITE RAUTG

DO Loops on 35 (TAU)
30 (SI)

COMPUTE Rcross

WRITE Rcross

DO on (FR)

READ FREQUENCIES

WRITE
\( P(\omega), \frac{P(\omega) U}{\omega^2} \delta^* \)

DO Loops on 50, \( K, U_\omega, \gamma \)

SET UP DATA GRID
\( \omega \) Versus \( K, U_\omega \)

COMPUTE
\( P(K_1, \omega) \)

WRITE
\( P(K_1, \omega), P(\omega) \)

STOP
<table>
<thead>
<tr>
<th>TABLE 2B (Continued)</th>
</tr>
</thead>
<tbody>
<tr>
<td>COLUMN HEADINGS FOR INPUT FORMS ON DATA CARDS</td>
</tr>
<tr>
<td>TITLE</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>A(1)</td>
</tr>
<tr>
<td>D(1)</td>
</tr>
<tr>
<td>THETA</td>
</tr>
<tr>
<td>S1(1)</td>
</tr>
<tr>
<td>FR(1)</td>
</tr>
<tr>
<td>TW</td>
</tr>
</tbody>
</table>

As many cards as needed to complete array FR (25)
### TABLE 2C

Input Required for Subprogram C to Compute Cross Density Displacement and Corresponding Output

(Units are given in foot-pound-seconds)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Program Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Flow and Plate Characteristics and Additional Quantities</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ARG(I)</td>
<td>Gaussian quadrature</td>
<td>ARG(I)</td>
</tr>
<tr>
<td>WGT(I)</td>
<td>Weights and arguments; NW of them; in this problem 9 points</td>
<td>WGT(I)</td>
</tr>
<tr>
<td>NW</td>
<td>Number of points in Gaussian integration</td>
<td>NW</td>
</tr>
<tr>
<td>K</td>
<td>Universal constants; NAK of each; in this case, 4 (supersonic)</td>
<td>AK(I)</td>
</tr>
<tr>
<td>A</td>
<td>Number of A's and K's; 4 (supersonic)</td>
<td>AN(I)</td>
</tr>
<tr>
<td>NAK</td>
<td></td>
<td>NAK</td>
</tr>
<tr>
<td>TITLE</td>
<td>Alphanumeric title -- labels the output; User's choice of words</td>
<td>TITLE(I)</td>
</tr>
<tr>
<td>(x)</td>
<td>(x) Coordinate position on plate</td>
<td>X</td>
</tr>
<tr>
<td>(x')</td>
<td>Second (x) coordinate: (\xi = x - x')</td>
<td>XP</td>
</tr>
<tr>
<td>(y)</td>
<td>(y) Coordinate position on plate</td>
<td>Y</td>
</tr>
<tr>
<td>(y')</td>
<td>Second (y) coordinate: (\eta = y - y')</td>
<td>YP</td>
</tr>
<tr>
<td>(a)</td>
<td>Plate size - (x) direction</td>
<td>A</td>
</tr>
<tr>
<td>(b)</td>
<td>Plate size - (y) direction</td>
<td>B</td>
</tr>
<tr>
<td>(U_e)</td>
<td>Free-stream velocity</td>
<td>UE</td>
</tr>
<tr>
<td>(M)</td>
<td>Plate mass</td>
<td>FM</td>
</tr>
<tr>
<td>(\delta^*)</td>
<td>Boundary layer displacement thickness</td>
<td>DEL</td>
</tr>
<tr>
<td>(\overline{p^2})</td>
<td>Mean-square wall pressure fluctuation</td>
<td>PB2</td>
</tr>
<tr>
<td>(U_c)</td>
<td>Convection velocity</td>
<td>UC</td>
</tr>
<tr>
<td>(\theta)</td>
<td>Eddy lifetime</td>
<td>TH</td>
</tr>
<tr>
<td>MLOW</td>
<td>First mode of interest for (m)</td>
<td>MLOW</td>
</tr>
<tr>
<td>MUP</td>
<td>Last mode of interest for (m)</td>
<td>MUP</td>
</tr>
<tr>
<td>DM</td>
<td>Increment between (m) modes</td>
<td>DM</td>
</tr>
<tr>
<td>NLOW</td>
<td>Same as described but for (n) modes</td>
<td>NLOW</td>
</tr>
<tr>
<td>NUP</td>
<td>Same as described but for (n) modes</td>
<td>NUP</td>
</tr>
<tr>
<td>DN</td>
<td>Same as described but for (n) modes</td>
<td>DN</td>
</tr>
</tbody>
</table>
### TABLE 2C (Continued)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Program Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_m$</td>
<td>Warburton $\frac{1}{k}$ for $\theta (x)$ to define plates with fixed edges, for $m,n$ modes, respectively.</td>
<td>GM, KM</td>
</tr>
<tr>
<td>$k_m$</td>
<td></td>
<td>GN, KN</td>
</tr>
<tr>
<td>$m$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega_{mn}$</td>
<td>Plate modal frequencies</td>
<td>WMN(M,N)</td>
</tr>
<tr>
<td>$\alpha_{mn}$</td>
<td>Acoustical damping, obtained experimentally</td>
<td>FA(M,N)</td>
</tr>
</tbody>
</table>

Following Symbols Are Used Only in Program for $\omega \neq \omega_{mn}$

| WI     | Lower limit of frequency to vary around a given $\omega_{mn}$ | WI |
| WF     | Upper limit for $\omega \neq \omega_{mn}$ | WF |
| DW     | Increment size for intervals between WI and WF | DW |

**Calculated Output in Inch-Pound-Seconds**

\[
P(\omega) = \frac{a b m^2 n^2 \phi(x) \phi(x') \phi(y) \phi(y')}{\sum_i (A_i e^{(-K_i \omega \delta^*/U_c)} \frac{p^2 \delta^*}{U_c})}
\]

\[
Y(x,y; x', y'; \omega) = \sum A_i e^{(-K_i \omega \delta^*/U_c)} \frac{p^2 \delta^*}{U_c}
\]

Equation 27 of Reference 21: cross spectral density, assuming panel modes well separated.

Equation 27 $\times \omega^4$

\[
(\text{AMN}^2 + (\omega_{mn} - \omega)^2) (\text{AMN}^2 + (\omega_{mn} + \omega)^2)
\]

*POFW, ANS, PWRSD have real and imaginary parts printed out.*
TABLE 2C (Continued)

Column Headings for Input Forms on Data Cards

<table>
<thead>
<tr>
<th>NW</th>
<th>NAK</th>
<th>20</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARG(1)</td>
<td>20</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>WGT(1)</td>
<td>20</td>
<td>80</td>
<td></td>
</tr>
</tbody>
</table>

And as many cards as needed to complete the ARG(NW) array (NW cards).

And as many cards as needed to complete the WGT(NW) array (NW cards).

<table>
<thead>
<tr>
<th>TITLE (from column 1 through 80)</th>
<th>12</th>
<th>24</th>
<th>36</th>
<th>48</th>
<th>60</th>
<th>72</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>XP</td>
<td>Y</td>
<td>YP</td>
<td>12</td>
<td>24</td>
<td>36</td>
<td>48</td>
</tr>
<tr>
<td>AK(1)</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>AK(NAK)</td>
<td>12</td>
</tr>
<tr>
<td>AN(1)</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>AN(NAK)</td>
<td>12</td>
</tr>
<tr>
<td>A</td>
<td>B</td>
<td>FA</td>
<td>UE</td>
<td>DEL</td>
<td>PB2</td>
<td>12</td>
<td>24</td>
</tr>
<tr>
<td>UC</td>
<td>TH</td>
<td>12</td>
<td>24</td>
<td>80</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LOW</th>
<th>MUP</th>
<th>DM</th>
<th>NLOW</th>
<th>NUP</th>
<th>DN</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
</table>

84
TABLE 2C (Continued)

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>GM(1)</td>
<td>GM(2)</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>12</td>
<td>24</td>
<td>36</td>
<td>48</td>
<td>60</td>
</tr>
</tbody>
</table>

... and as many cards (6 nos/card) to complete GM(MUP)

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>GN(1)</td>
<td>GN(2)</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>12</td>
<td>24</td>
<td>36</td>
<td>48</td>
<td>60</td>
</tr>
</tbody>
</table>

... and as many cards (6 nos/card) to complete GN(NUP)

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>KM(1)</td>
<td>KM(2)</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>12</td>
<td>24</td>
<td>36</td>
<td>48</td>
<td>60</td>
</tr>
</tbody>
</table>

... as many to complete KM(MUP) (MUP cards)

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>KN(1)</td>
<td>KN(2)</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>12</td>
<td>24</td>
<td>36</td>
<td>48</td>
<td>60</td>
</tr>
</tbody>
</table>

... as many to complete KN(NUP) (NUP cards)

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>WMN(1,1)</td>
<td>WMN(2,1)</td>
<td>...</td>
<td>WMN(MUP)</td>
<td>WMN(MUP,1)</td>
</tr>
<tr>
<td>10</td>
<td>36</td>
<td>60</td>
<td>80</td>
<td>10</td>
</tr>
</tbody>
</table>

WMN(MUP,2), and so on, until MUP x NUP cards are used to complete WMN (MUP, NUP) array

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>FA(1,1)</td>
<td>FA(MUP,1)</td>
<td>...</td>
<td>FA(MUP,NUP)</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

FA(1,2), and so on, in WMN array, i.e., use MUP x NUP to complete FA(MUP,NUP) array

For case \( \alpha \neq c_{max} \) only: 1 card for each \( (M,i) = (1,1), (1,2), \ldots (1,N), (2,1), \ldots \) so on

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>W1</td>
<td>DW</td>
<td>WF</td>
<td>36</td>
<td>12</td>
</tr>
<tr>
<td>24</td>
<td>24</td>
<td>24</td>
<td>24</td>
<td>24</td>
</tr>
</tbody>
</table>

For case \( \alpha = c_{max} \): 1 card for each \( (M,i) = (1,1), (1,2), \ldots (1,N), (2,1), \ldots \) so on

85
**SUBPROGRAM D**

Subprogram D represents a generalization of Subprogram C to include modal-cross coupling. Data descriptions of the subprograms are identical, except that Subprogram D requires a second set of modal input data. These data are:

**TABLE 2D**

Generalization of Subprogram C to Include Modal Cross Coupling

(Data descriptions of Subprograms C and D are identical except that Subprogram D requires a second set of modal input data as indicated below)

<table>
<thead>
<tr>
<th>Symbol for Cross-Coupled Modes</th>
<th>Symbols for Uncoupled Modes*</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLOW for p modes</td>
<td>MLOW</td>
</tr>
<tr>
<td>PUP for p modes</td>
<td>MUP</td>
</tr>
<tr>
<td>DP</td>
<td>DM</td>
</tr>
<tr>
<td>QLOW for q modes</td>
<td>NLOW</td>
</tr>
<tr>
<td>QUP for q modes</td>
<td>NUP</td>
</tr>
<tr>
<td>DQ</td>
<td>DN</td>
</tr>
<tr>
<td>GP</td>
<td>GM</td>
</tr>
<tr>
<td>GQ</td>
<td>GN</td>
</tr>
<tr>
<td>KP</td>
<td>KM</td>
</tr>
<tr>
<td>KQ</td>
<td>KN</td>
</tr>
<tr>
<td>WPQ(P,Q) for p modes</td>
<td>WMN (M,N)</td>
</tr>
<tr>
<td>APQ(P,Q) for q modes</td>
<td>FA (M,N)</td>
</tr>
</tbody>
</table>

*Analogous to data in Subprogram C.

Note: The output will now include the additional eigenvalues EIGN(P,Q) for the (P,Q) modes. DENCOM is the denominator of Equation (26). Reference 22.
TABLE 2D (Continued)

READ AND WRITE INPUT DATA

CALCULATE CONSTANT

\[ \frac{144(22.7L^2)^2}{\pi^2 M^2} \]

CALCULATE BOUNDARY CONDITIONS

DO 30 M = MLOW, MUP, DM

CALCULATE BOUNDARY CONDITIONS \( x \) - USE COS AND COSH TERMS

BOUNDARY CONDITION ON \( x \) - USE THE SIN AND SINH TERMS

DO 45 N = NLOW, NUP, DN

CALCULATE BOUNDARY CONDITIONS \( y \) - USE COS AND COSH TERMS

BOUNDARY CONDITION ON \( y \) - USE THE SIN AND SINH TERMS

CALCULATE EIGEN (M,N)

WRITE OUT

45 CONTINUE

50 CONTINUE

CALCULATE EIGEN (P,Q)

DO 545 P = PLOW, PUP, DP

CALCULATE BOUNDARY CONDITIONS \( x' \) - USE COS AND COSH

BOUNDARY CONDITION ON \( x' \) - USE SIN AND SINH

DO 546 Q = QLOW, QUP, DQ

CALCULATE BOUNDARY CONDITION \( y' \) - USE COS AND COSH

BOUNDARY CONDITION ON \( y' \) - USE SIN AND SINH

CALCULATE EIGEN (P,Q) WRITE OUT

545 CONTINUE

550 CONTINUE
TABLE 2D (Continued)

\[ \begin{align*}
\text{DO } & 778 \quad M = \text{MLOW, MUP, DM} \\
\text{DO } & 7781 \quad P = \text{PLOW, PUP, DP} \\
\text{DO } & 776 \quad M = \text{MLOW, MUP, DN} \\
\text{DO } & 7761 \quad Q = \text{QLOW, QUP, DQ} \\
\end{align*} \]

\[ \begin{align*}
\text{READ} \\
\omega_f, n_{0f}, n_f \\
\text{CALCULATE} \\
\frac{n_f}{n_{0f}}, n_{0f}, n_f^2, \\
\frac{n_f}{n_{0f}}, n_{0f}^2, \text{ and } d^2 \\
\text{DO } 150 \quad M = 1, NWW \\
\text{Initialize answer to 0} \\
\text{Calculate DENCOM} \\
\end{align*} \]

\[ \begin{align*}
\left[ \begin{array}{c}
\lambda_{mn} + i\omega \\
\lambda_{mn} - i\omega
\end{array} \right] \\
\left[ \begin{array}{c}
\frac{1}{\frac{\lambda_{mn} + \lambda_{pq}}{2}} \\
\frac{1}{\frac{\lambda_{mn} + \lambda_{pq}}{2}}
\end{array} \right] + \omega_{mn} \omega_{pq} \left( \frac{1}{\frac{\lambda_{mn} + \lambda_{pq}}{2}} - \frac{1}{\frac{\lambda_{mn} + \lambda_{pq}}{2}} \right)
\end{align*} \]

\[ \begin{align*}
\left[ \begin{array}{c}
\frac{1}{\frac{\lambda_{pq} - \lambda_{pq}}{2}} \\
\frac{1}{\frac{\lambda_{pq} - \lambda_{pq}}{2}}
\end{array} \right] + \omega_{pq} \omega_{mn} \left( \frac{1}{\frac{\lambda_{pq} - \lambda_{pq}}{2}} - \frac{1}{\frac{\lambda_{pq} - \lambda_{pq}}{2}} \right)
\end{align*} \]

\[ \begin{align*}
\left( \frac{\lambda_{mn} + \lambda_{pq}}{2} \right) + \omega_{mn} \omega_{pq} \left( \frac{1}{\frac{\lambda_{mn} + \lambda_{pq}}{2}} - \frac{1}{\frac{\lambda_{mn} + \lambda_{pq}}{2}} \right)
\end{align*} \]

\[ \begin{align*}
\frac{1}{\frac{\lambda_{pq} - \lambda_{pq}}{2}} + \omega_{pq} \omega_{mn} \left( \frac{1}{\frac{\lambda_{pq} - \lambda_{pq}}{2}} - \frac{1}{\frac{\lambda_{pq} - \lambda_{pq}}{2}} \right)
\end{align*} \]

\[ \begin{align*}
P(\omega) - \frac{p^2 \beta}{16} \sum_{k=1}^{4} a_k e^{-\frac{i\omega\beta}{4z}}
\end{align*} \]
TABLE 2D (Continued)

CALCULATE SPACE INTEGRAL USING GAUSSIAN QUAD

<table>
<thead>
<tr>
<th>DO 300 J1 = 1, NXO</th>
</tr>
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<tbody>
<tr>
<td>CALCULATE BOUNDARY CONDITIONS ON z USE COS AND COSH</td>
</tr>
<tr>
<td>NO</td>
</tr>
<tr>
<td>M is Even</td>
</tr>
<tr>
<td>USE SIN AND SINH for BOUNDARY CONDITIONS</td>
</tr>
<tr>
<td>YES</td>
</tr>
<tr>
<td>P is Even</td>
</tr>
<tr>
<td>USE SIN AND SINH for BOUNDARY CONDITIONS</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DO 300 K1 = 1, NXOP</th>
</tr>
</thead>
<tbody>
<tr>
<td>CALCULATE BOUNDARY CONDITIONS ON z' USE 'COS AND COSH</td>
</tr>
<tr>
<td>NO</td>
</tr>
<tr>
<td>P is Even</td>
</tr>
<tr>
<td>USE SIN AND SINH for BOUNDARY CONDITIONS</td>
</tr>
<tr>
<td>YES</td>
</tr>
<tr>
<td>Q is Even</td>
</tr>
<tr>
<td>USE SIN AND SINH for BOUNDARY CONDITIONS</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DO 400 J1 = 1, NYO</th>
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<tbody>
<tr>
<td>CALCULATE BOUNDARY CONDITIONS ON y USE COS AND COSH</td>
</tr>
<tr>
<td>NO</td>
</tr>
<tr>
<td>N is Even</td>
</tr>
<tr>
<td>CALCULATE BOUNDARY CONDITIONS USE SIN AND SINH</td>
</tr>
<tr>
<td>YES</td>
</tr>
<tr>
<td>Q is Even</td>
</tr>
<tr>
<td>CALCULATE BOUNDARY CONDITIONS USE SIN AND SINH</td>
</tr>
</tbody>
</table>

ANSWER = SPACE INTEGRAL * CONSTANT * $P(\omega) * \text{EIGEN} * \text{EIGN} * \text{DECOM} * \sin \omega$

<table>
<thead>
<tr>
<th>WRITE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega = \omega + d\omega$</td>
</tr>
<tr>
<td>150 CONTINUE</td>
</tr>
<tr>
<td>7761 CONTINUE</td>
</tr>
<tr>
<td>776 CONTINUE</td>
</tr>
<tr>
<td>7781 CONTINUE</td>
</tr>
<tr>
<td>778 CONTINUE</td>
</tr>
</tbody>
</table>

STOP
TABLE 2D (Continued)

Column Headings for Input Forms on Data Cards

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<tr>
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</thead>
<tbody>
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<td>10</td>
<td>30</td>
<td>50</td>
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<td>80</td>
<td></td>
</tr>
<tr>
<td>NW</td>
<td>NAK</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>20</td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ARG(1)</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

And as many cards as needed to complete the ARG(NW) array (NW cards).

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<td>20</td>
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<td></td>
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</tr>
<tr>
<td>WGT(1)</td>
<td></td>
<td></td>
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</tbody>
</table>

And as many cards as needed to complete the WGT(NW) array (NW cards).

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<table>
<thead>
<tr>
<th></th>
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<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>80</td>
</tr>
<tr>
<td>TITLE</td>
<td>(from column 1 through 80)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>12</td>
<td>24</td>
<td>36</td>
<td>48</td>
<td>60</td>
<td>72</td>
<td>80</td>
</tr>
<tr>
<td>X</td>
<td>XP</td>
<td>Y</td>
<td>YP</td>
<td></td>
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<td></td>
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</tbody>
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<td>12</td>
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<td>36</td>
<td>48</td>
<td>60</td>
<td>72</td>
<td>80</td>
</tr>
<tr>
<td>AK(1)</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
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</tbody>
</table>

<p>| | | | | | | |</p>
<table>
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<th></th>
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</thead>
<tbody>
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<td>12</td>
<td>24</td>
<td>36</td>
<td>48</td>
<td>60</td>
<td>72</td>
<td>80</td>
</tr>
<tr>
<td>AN(1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>24</td>
<td>36</td>
<td>48</td>
<td>60</td>
<td>72</td>
<td>80</td>
</tr>
<tr>
<td>A</td>
<td>B</td>
<td>FM</td>
<td>UE</td>
<td>DEL</td>
<td>PB2</td>
<td></td>
</tr>
</tbody>
</table>

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
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<td>12</td>
<td>24</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UC</td>
<td>TH</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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</tbody>
</table>

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
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<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>MLOW</td>
<td>MUP</td>
<td>DN</td>
<td>MLOW</td>
<td>NUP</td>
<td>DN</td>
<td></td>
</tr>
</tbody>
</table>
TABLE 2D (Continued)

<table>
<thead>
<tr>
<th></th>
<th>PLOW</th>
<th>PUP</th>
<th>DP</th>
<th>GLOW</th>
<th>QUP</th>
<th>DQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>24</td>
<td>36</td>
<td>48</td>
<td>60</td>
<td>72</td>
<td>80</td>
</tr>
<tr>
<td>GM(1)</td>
<td>GM(2)</td>
<td>GM(3)</td>
<td>And so on, six numbers per card, to have MUP values of GM.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>24</td>
<td>36</td>
<td>48</td>
<td>60</td>
<td>72</td>
<td>80</td>
</tr>
<tr>
<td>GN(1)</td>
<td>GN(2)</td>
<td>And so on, six numbers per card, to have NUP values of GN.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>24</td>
<td>36</td>
<td>48</td>
<td>60</td>
<td>72</td>
<td>80</td>
</tr>
<tr>
<td>GQ(1)</td>
<td>GQ(2)</td>
<td>And so on, six numbers per card, to have QUP values of GQ.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>24</td>
<td>36</td>
<td>48</td>
<td>60</td>
<td>72</td>
<td>80</td>
</tr>
<tr>
<td>KM(1)</td>
<td>KM(2)</td>
<td>And so on, six numbers per card, for MUP values of KM.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>24</td>
<td>36</td>
<td>48</td>
<td>60</td>
<td>72</td>
<td>80</td>
</tr>
<tr>
<td>KP(1)</td>
<td>KP(2)</td>
<td>And so on, six numbers per card, for PUP values of KP.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>24</td>
<td>36</td>
<td>48</td>
<td>60</td>
<td>72</td>
<td>80</td>
</tr>
<tr>
<td>KN(1)</td>
<td>KN(2)</td>
<td>And so on, six numbers per card, for NUP values of KN</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>24</td>
<td>36</td>
<td>48</td>
<td>60</td>
<td>72</td>
<td>80</td>
</tr>
<tr>
<td>KQ(1)</td>
<td>KQ(2)</td>
<td>And so on, six numbers per card, for QUP values of KQ.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>24</td>
<td>36</td>
<td>48</td>
<td>60</td>
<td>72</td>
<td>80</td>
</tr>
<tr>
<td>GP(1)</td>
<td>GP(2)</td>
<td>And so on, six numbers per card, to have PUP values of GP.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>WMN(1,1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>WMN(MUP,1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>WMN(1,7)</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

91
TABLE 2D (Continued)

<table>
<thead>
<tr>
<th>WMN (MUP, 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>And so on, one number per card, to have MUP x NUP values of WMN</td>
</tr>
<tr>
<td>WPQ(1, 1)</td>
</tr>
<tr>
<td>One number per card, from WPQ(1, 1) to WPQ(PUP, 1), WPQ(1, 2) to WPQ(PUP, 2).</td>
</tr>
<tr>
<td>And so on, to WPQ(PUP, QUP), i.e., PUP x QUP cards.</td>
</tr>
<tr>
<td>FA(1, 1)</td>
</tr>
<tr>
<td>One number per card, MUP x NUP cards. Cycle as for WMN.</td>
</tr>
<tr>
<td>APQ(1, 1)</td>
</tr>
<tr>
<td>One number per card, PUP x QUP cards. Cycle as for WPQ.</td>
</tr>
<tr>
<td>WI</td>
</tr>
<tr>
<td>One card for each cross mode of interest; cycling for (m, n, p, q).</td>
</tr>
</tbody>
</table>
APPENDIX B4 – TEST RUNS

Results obtained from the computer programs of Table 2 are indicated in Figures 7 – 10. Figures 7 and 8 are obtained from Subprogram A, Figure 9 from Subprogram B, and Figure 10 from Subprogram C. The figures show test runs for modal mean square displacement, turbulence pressure spectrum levels, and displacement spectral density.
TABLE 3

Computer Listings for Subprograms A, B, C, and D – Maestrello

Table 3A – Computer Listing for Subprogram A
(Semifrozen Convection – Model A)

<table>
<thead>
<tr>
<th>Line</th>
<th>Description</th>
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<tbody>
<tr>
<td>31BFCT</td>
<td>TURAD</td>
</tr>
<tr>
<td>C</td>
<td>REORGANISED PROGRAM FOR COMPUTING TRIPLE</td>
</tr>
<tr>
<td>C</td>
<td>INTEGRAL</td>
</tr>
<tr>
<td>C</td>
<td>USING SIMPSONS RULE</td>
</tr>
<tr>
<td>DIMENSION VOL(20*10)</td>
<td>40</td>
</tr>
<tr>
<td>DIMENSION F3(3)*AN5(3)</td>
<td>50</td>
</tr>
<tr>
<td>DIMENSION IXYZ(10)+50+1*IV(20)+IY(11)</td>
<td>0010</td>
</tr>
<tr>
<td>DIMENSION G3(11)+30+1*YY(11)+TEMP1(11)+G5(4000)</td>
<td>0070</td>
</tr>
<tr>
<td>DIMENSION PARAM(101)*THETA(101)</td>
<td>0.800</td>
</tr>
<tr>
<td>DIMENSION AK(3)*AN(3)+TITLE(20)</td>
<td>0090</td>
</tr>
<tr>
<td>DIMENSION OMEGA(20)+DAMP(20*10)</td>
<td>0100</td>
</tr>
<tr>
<td>DIMENSION F1(20<em>10)+F2(20</em>10)+EIGEN(20<em>10)+FUDGE(20</em>10)</td>
<td>0110</td>
</tr>
<tr>
<td>DIMENSION UC(20)+DPB2(20)</td>
<td>0120</td>
</tr>
<tr>
<td>REAL IXYZ<em>IY</em>IY</td>
<td>0130</td>
</tr>
<tr>
<td>INTEGER DM+DN</td>
<td>0140</td>
</tr>
<tr>
<td>READ(5+13) TAU</td>
<td>0150</td>
</tr>
<tr>
<td>13 FORMAT(F10.0) WRITE(6,801) TAU</td>
<td>0160</td>
</tr>
<tr>
<td>801 FORMAT(1H15/S5 TAU=EI5(6))</td>
<td>0180</td>
</tr>
<tr>
<td>READ(5+10) H<em>XI</em>YO<em>AD</em>YOP+FUCSQ*P2+FM2+AK,</td>
<td>0190</td>
</tr>
<tr>
<td>1AN TITLE</td>
<td>200</td>
</tr>
<tr>
<td>103 FORMAT(I4(F10+0/1)(10A6)) WRITE(6,201) TITLE</td>
<td>0210</td>
</tr>
<tr>
<td>201 FORMAT(20<em>A6) WRITE(6,203) FUCSQ</em>PB2+FM2+AK,AN<em>XI</em>YO*</td>
<td>0230</td>
</tr>
<tr>
<td>1XOP*YOP</td>
<td>0250</td>
</tr>
<tr>
<td>203 FORMAT(I4HOFUC SQUARED= EI6+8* X 8H RHO-BAR 9H SQUARED=EI6+8* 1 11H M-SQUARED=EI6+8*/ 24HOK1=EI6+8+4H K1=EI6+8/ 34H A1=EI6+8+4H A2=EI6+8+4H A3=EI6+8/ 44H X0=EI6+8+5H X0=EI6+8+5H X0P=EI6+8+5H YOP=EI6+8)</td>
<td>0290</td>
</tr>
<tr>
<td>READ(5+102) KUCs(UC(11)+DPB2(11)+1 = 1*KUC)</td>
<td>0320</td>
</tr>
<tr>
<td>102 FORMAT(I10/(2F10.0)) WRITE(6,204) UC(11)+DPB2(11)+1 = 1*KUC</td>
<td>0330</td>
</tr>
<tr>
<td>204 FORMAT(I1H2HUC13X4HDPB2/(1H 2E1681))</td>
<td>0350</td>
</tr>
<tr>
<td>99 READ(5+1) ZUP+YUP+H+MLow+NUP+MD+MSTEPS+NLOW+NUP+DN+NSTEPS</td>
<td>0360</td>
</tr>
</tbody>
</table>
TABLE 3A (Continued)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>FORMAT(3(F10.0,1x,4F10.3))</td>
</tr>
<tr>
<td>2</td>
<td>WRITE(6,2) NLOW+NUP+DN+NSTEPS</td>
</tr>
<tr>
<td>2</td>
<td>WRITE(6,2) MLOW+MUP+DN+NSTEPS</td>
</tr>
<tr>
<td>112</td>
<td>A TOTAL OF 15+7H STEPS/9H N FROM 15+</td>
</tr>
<tr>
<td>24</td>
<td>A TOTAL OF 15+7H STEPS/9H N FROM 15+</td>
</tr>
<tr>
<td>202</td>
<td>READ(5,101) ((OMEGA(M*N),M=1,20),N=1,3)</td>
</tr>
<tr>
<td>101</td>
<td>WRITE(6,205) ((OMEGA(M*N),M=MLOW+MUP+DN),</td>
</tr>
<tr>
<td>405</td>
<td>WRITE(6,205) ((OMEGA(M*N),M=MLOW+MUP+DN),</td>
</tr>
<tr>
<td>205</td>
<td>FORMAT(7H0MEGA*/(8E16.8))</td>
</tr>
<tr>
<td>206</td>
<td>FORMAT(6H0DAMP*/(8E16.8))</td>
</tr>
<tr>
<td>207</td>
<td>READ(5,207) NP*(PARAM(IP),IF = 1*NP)</td>
</tr>
<tr>
<td>208</td>
<td>FORMAT(110/(6F10.0))</td>
</tr>
<tr>
<td>209</td>
<td>WRITE(U,C=3.14159265)</td>
</tr>
<tr>
<td>210</td>
<td>WRITE(U,C=3.14159265)</td>
</tr>
</tbody>
</table>

95
TABLE 3A (Continued)

\[
\begin{align*}
C_4 &= C_2^2 \\
C_5 &= \frac{A_6 + B_8 + 2B_{12} + B_{17}}{12}\, \text{with} \, C_4 \text{ and } F_2 \text{ as defined} \\
C_5 &= C_2^3 \times 2 \times 12^2 \\
C_6 &= \frac{C_4^2}{A_2} \\
D_0 &= 10 \\
X_0 &= A/2 \\
Y_0 &= B/2 \\
M &= M_{LOW}M_{UP}D_{M} \\
X_{N} &= N \\
X_{O} &= A/2 - A/(X_{M}^2) \\
Y_{O} &= B/2 - B/(X_{N}^2) \\
\text{IF}(M = N) \text{ GO TO 43} \\
\text{IF}(N = M) \text{ GO TO 44} \\
X_{O} &= A/2 + A/(X_{M}^2) \\
Y_{O} &= B/2 + B/(X_{N}^2) \\
\text{DAMP}(M \# N) &= \text{DAMP}(M \# N)/10 \\
\text{FA}(M \# N) &= \text{DAMP}(M \# N)/2 \times \text{OMEGA}(M \# N) \\
\text{FUDGE}(M \# N) &= \text{FA}(M \# N)/2 \times \text{OMEGA}(M \# N) \\
\text{IF}(M = N) \text{ GO TO 45} \\
\text{CONTINUE} \\
\text{WRITE}(6,100) \\
\text{FORMAT(33E0X0=X0,Y0,Y0P=AND THEY VARY WITH} \\
\text{THE Mode NUMBERS}) \\
\text{DO 46 L = 1, 5} \\
\text{WRITE}(6,133) \{\text{EIGEN}(M \# N) \times \text{MLOWsNUP} = \text{DM} \times N = \text{NLOWsNUP} = \text{DN}\} \\
\text{CONTINUE} \\
\text{WRITE}(6,133) \{\text{EIGEN}(M \# N) \times \text{MLOWsNUP} = \text{DM} \times N = \text{NLOWsNUP} = \text{DN}\} \\
\text{CONTINUE} \\
\text{WRITE}(6,133) \{\text{EIGEN}(M \# N) \times \text{MLOWsNUP} = \text{DM} \times N = \text{NLOWsNUP} = \text{DN}\} \\
\text{CONTINUE} \\
\text{WRITE}(6,133) \{\text{EIGEN}(M \# N) \times \text{MLOWsNUP} = \text{DM} \times N = \text{NLOWsNUP} = \text{DN}\} \\
\text{CONTINUE} \\
\text{WRITE}(6,133) \{\text{EIGEN}(M \# N) \times \text{MLOWsNUP} = \text{DM} \times N = \text{NLOWsNUP} = \text{DN}\} \\
\text{CONTINUE} \\
\text{WRITE}(6,133) \{\text{EIGEN}(M \# N) \times \text{MLOWsNUP} = \text{DM} \times N = \text{NLOWsNUP} = \text{DN}\} \\
\text{CONTINUE} \\
\text{WRITE}(6,133) \{\text{EIGEN}(M \# N) \times \text{MLOWsNUP} = \text{DM} \times N = \text{NLOWsNUP} = \text{DN}\} \\
\text{CONTINUE} \\
\end{align*}
\]
CALL VOLUM(A+$BL+MLOW+MUP+OM+RLOW+WUP+D+$VOL)
     NUPM=NUP-1
     DO 777 KUR=KUC
     CONST=SUMP(BK(FI))
     WRITE(6,303) UC(KU),CONST
     303 FORMAT(4H UC=$E16.8$ MN, CONST*$E16.8$)
     DO 778 N=MLOW+MUP+OM
     XM=M
     DO 779 IP=1,NP
     779 THETA(IP)=PARAM(IP)/XM$UC(KU)*A
     WRITE(6,780) (THETA(IP),IP=1,NP)
     780 FORMAT(4H THETA$/(E16.8$)
     IF(THETA(IT),GT,100.0)GO TO 999
     XL=O+:
     DX=1.0+OMEGA(M,NUPM)/2.0+14159265
     IUP=5/X,THETA(NP)+1:
     IF(IUP,GT,3599) IUP=3599
     IF(IUP-IUP/2)*501+500
     500 IUP=IUP+1
     501 IUP=IUP+1
     502 DX=(THETA(2)-THETA(1))/2
     ZLOW=-A
     JUP=2*K+1
     ZJUP=JUP-1
     ZJ=2*A/JJUP
     ZUP=10*K+1
     UKUP=KUP+1
     DY=K/YKUP
     DO 11 I=1,IUP
     510 G5(I)=G5(I)*2
     11 IF(I.EQ.401) GO TO 11
     IF(I.NE.1.AND.I.NE.IUP) GO TO 510
     GO TO 511
     511 G5(I)=G5(I)/2
     12 GO TO 11
TABLE 3A (Continued)

11 CONTINUE
21 Z=ZLOW
DO 21 J=1,JUP
XMMM
D1=XM*C2*Z/A
D2=COS(D1)
G2(J+M)=D2+1./(XM*C2)*ABS(XM)*ABS(D1)
IF(JsNE,1.*AND.*JsNE,JUP) GO TO 520
GO TO 521
520 G2(J+M)=G2(J+M)*(1.*X)
521 IF(JsJ/2.*EQ.0) GO TO 525
GO TO 21
525 G2(J+M)=G2(J+M)*(1.*2.1)
21 2=Z*DO
Y=YLLOW
DO 31 K=1,KUP
DO 30 M=LOW,MUP+DN
XNM
D3=XM*C2*Y/B
D4=COS(D3)
G3(K+M)=D3+1.*(XM*C2)*ABS(XM)*ABS(D3)
IF(KsNE,1.*AND.*M=KUP) GO TO 530
GO TO 531
530 G3(K+M)=G3(K+M)*(1.*2.1)
531 IF(KsK/2.*EQ.0) GO TO 533
GO TO 30
533 G3(K+M)=G3(K+M)*(1.*2.1)
30 CONTINUE
31 YY=YY+DY
YY=YLLOW
DO 39 K=1,KUP
YY(K)=YY
39 YY=YY+DY
DO 40 L=1,3
DO 40 MM=LOW,MUP+DN
DO 40 IT=1,MP
40 CONTINUE
98
TABLE 3A (Continued)

40  IYZ(M+N+I+L+I)=0.
KAPP=0
X*XLW
DO 160 I=1,UP
IF(TAU+EO=0.0) GO TO 630
E1=UC(KU)*(TAU-X)
GO TO 632
630 E1=UC(KU)*X
632 CONTINUE
DO 50 N=NLOW+NUP+DN
50  IYZ(M+N)=0.
Z*ZLOW
DO 120 J=1,UP
EZ=(Z-E1)*(Z-E1)
E4=B1+E2
E5=B2+E2
E6=B3+E2
DO 60 K=1,KUP
60  TEMP(K)=B4/(E4+YY(K))+B5/(E5+YY(K))+B6/(E6+YY(K))
DO 70 N=NLOW+NUP+DN
70  IY(N)=0.
DO 90 K=1,KUP
DO 90 N=NLOW+NUP+DN
90  IY(N)=IY(N)+G3(K+N)*TEMP(K)
DO 100 N=NLOW+NUP+DN
100  IY(N)=IY(N)*DY/3.
TEMP=IY(N)
110  IYZ(M+N)=IYZ(M+N)+G2(J+N)*TEMP
120  Z=Z+DZ
DO 130 N=NLOW+NUP+DN
130  IYZ(M+N)=IYZ(M+N)*DZ/3.
1001 CONTINUE
DO 140 N=NLOW+NUP+DN
TEMP=IYZ(M+N)
XM=M
XN=N

99
<table>
<thead>
<tr>
<th>140</th>
<th>CONTINUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>141</td>
<td>FORMAT(3H1,MN=15//45X6HI(MN)/3X$8BMN)#EIGEN#CONST//</td>
</tr>
<tr>
<td>142</td>
<td>12X$8BMN#EIGEN#THETA$6XI1HDAMP=1$1/10$5X7HDAMP=1$1 9X8HDAMP=10</td>
</tr>
<tr>
<td>143</td>
<td>28X11HDAMP=1$1/10$5X7HDAMP=1$1 9X8HDAMP=10$7X5HPRAM)</td>
</tr>
<tr>
<td>144</td>
<td>GO TO 100</td>
</tr>
<tr>
<td>145</td>
<td>DO 601</td>
</tr>
<tr>
<td>146</td>
<td>IF(THETA(IT)$=GT$100.+) GO TO 999</td>
</tr>
<tr>
<td>147</td>
<td>WRITE(6,141) MHN</td>
</tr>
<tr>
<td>148</td>
<td>601 CONTINUE</td>
</tr>
</tbody>
</table>
TABLE 3A (Continued)

778 CONTINUE
777 CONTINUE
999 STOP
END
$IBFCT VOLUME
SUBROUTINE VOLUME(A,B,MLOW,MUP,DM,NLOW,NUP,DN,VOL)
INTEGER DM,DN
DIMENSION VOL(20,10)
PI=3.14159265
DO 10 N=NLOW,NUP,DN
XM=N
DO 10 M=MLOW,MUP,DM
XM=M
VOL(M,N)=0.
10 CONTINUE
WRITE(6,20)((VOL(M,N),M=MLOW,MUP,DM,N=NLOW,NUP,DN)
$"VOLUME UNDER EIGENFUNCTION= //
20 FORMAT(285E16.8)
RETURN
END

101
Table 3B – Computer Listings for Subprogram B
(Semifrozen Convection – Model A)

$184FC ACCROS
DIMENSION RAUTO(100),RCROSS(8*100),POW(25),CAK(3) 00 *
DIMENSION RCROSS(8*100),CROSS(8*100) 00 02
DIMENSION POW(25) 00 30
DIMENSION SI(10),TAU(100),FR(25),DA(3),ADD(3),DA 00 40
X(3),XCORD(25),TITLE(12) 0 * 50
DIMENSION P(2500) 0 60
DIMENSION SKNE(500),W(1000) 0* 70
1000 FORMAT(12A6) 80
1001 FORMAT(3F8.4) 90
1002 FORMAT(3F8.4) 0100
1003 FORMAT(3F8.4) 0110
1010 FORMAT(3F8.4) 0120
1011 FORMAT(3F8.4) 0130
1014 FORMAT(3F8.4) 0140
1015 FORMAT(3F8.4) 0150
1016 FORMAT(3F8.4) 0160
2000 FORMAT(1H12A6) 0170
2001 FORMAT(3H12A6) 0180
2002 FORMAT(3H12A6) 0190
2003 FORMAT(3H12A6) 0200
2004 FORMAT(3H12A6) 0210
2005 FORMAT(3H12A6) 0220
2006 FORMAT(3H12A6) 0230
2007 FORMAT(3H12A6) 0240
2008 FORMAT(3H12A6) 0250
2009 FORMAT(3H12A6) 0260
2010 FORMAT(3H12A6) 0270
2011 FORMAT(3H12A6) 0280
2012 FORMAT(3H12A6) 0290
2013 FORMAT(3H12A6) 0300
2014 FORMAT(3H12A6) 0310
2015 FORMAT(3H12A6) 0320
2016 FORMAT(3H12A6) 0330
2017 FORMAT(3H12A6) 0340
2018 FORMAT(3H12A6) 0350
2019 FORMAT(3H12A6) 0360
C LIST OF VARIABLES AND CONSTANTS
C RAUTO = R(TAU)
C RCROSS = R(SI*ETA*TAU)

102
TABLE 3B (Continued)

C POW = P(W)/U/(TAU**2*DELTA-STAR) 0370
C DSTAR = DELTA-STAR 0380
C D(I) = K(I) 0390
C FR = FREQUENCY 0400
PI = 3.1415926 0410
READ(S+1000) (TITLE[I]=1:12) 0420
WRITE(6*2000) (TITLE[I]=1:12) 0430
READ(S+1001) (A[I]=1:3) 0440
READ(S+1002) (D[I], I=1:3) 0450
READ(S+1003) THETA*DSTAR*UC 0460
WRITE(S+2001) (A[I]=1:3) 0470
WRITE(S+2002) (D[I]=1:3) 0480
WRITE(S+2003) THETA*DSTAR*UC 0490
READ(S+1010) (SI[I]=1:8) 0500
WRITE(S+2010) (SI[I]=1:8) 0510
C COMPUTE AUTOCORRELATION 0520
C SI = 0 ETA = 0 0530
AKC = 0.0 0540
DO 10 I=1:3 0550
CAK(I) = A(I)/D(I) 0560
10 AKC = AKC * CAK(I) 0570
U = UC/0.8 0580
F = DSTAR/U 0590
FDUC = (1./F*UC)**2 0600
GO TO 400 0610
TAU(I) = 0.0 0620
DO 25 J=1:100 0630
RAUT = 0.0 0640
DO 20 I =1:3 0650
ADD(I) = (A(I)*D(I))/D(I)**2+(TAU(J)/F)**2) 0660
20 RAUT = ADD(I) + RAUT 0670
25 CONTINUE 0690
WRITE(S+2004) (TAU(J)*RAUT(J), J=1:100) 0700
C COMPUTE CROSS CORRELATION 0720
C ETA = 0.0 0730
TABLE 3B (Continued)

```
DO 16 I = 1, 8
16 SI(I) = SI(I)/12
DO 35 J = 1,100
DO 13 I = 1,8
RCRO(I,J) = 0.0
DO 30 IM = 1,3
AD(IM) = (A(IM)**2+FDUC*(SI(I)-UC*TAU(J)**2)
30 RCRO(I,J) = RCRO(I,J)+AD(IM)
CROSS(I,J) = RCRO(I,J)*EXP(ABS(SI(I)))/UC*THETA)
RCROSS(I,J) = CROSS(I,J)/AKC
TAU(J+1) = TAU(J) + 0.00001
33 CONTINUE
35 CONTINUE
WRITE(6,2007) (J,(RCROSS(I,J))I=1,8),J=1,100)
COMMENT Fig.2 EOTN6 & SEMI-FRPZEN CASE
400 CONTINUE
READ(5,1012) (FR(I),I=1,25)
WRITE(6,2012) (FR(I),I=1,25)
READ(5,1016) TW
WRITE(6,2016) TW
TW = TW**2
WRITE(6,2020)
DO 45 J = 1,25
POW(J) = 0.0
DO 40 I = 1,3
DA(I) = A(I)*EXP(-D(I)*6.2832*FR(J)*F)
POW(J) = POW(J) + DA(I)
40 CONTINUE
XCORD(J)=(6.2832*FR(J)*DSTAR)/U
PDW(J) = POW(J)*DSTAR*TW/U
45 CONTINUE
WRITE(6,2022) (XCORD(J),PDW(J),J=1,25)
GO TO 500
500 CONTINUE
WRITE(6,2007)
500 CONTINUE
WRITE(6,2007)
500 CONTINUE
```

104
TABLE 3B (Continued)

C  *FR  VS  P(K-ONE±2*PI*FR)
   WI = -48000,
   SKF = WI
   WF = 480000,
   DW = 200000,
   M = (WF-WI)/DW + 1.
   DSK = DW
   N = M
   SK = SKF
   DO 50 I = 1*M
   SKNE(I) = SK
   WW = WI
   DO 49 K = 1*N
   W(K) = WW
   FAC = [THETA*F*UC]/((1+THETA*THETA*(W(K)+SKNE(I))**2)*2*PI)
   ARF = 0.0
   DO 55 J = 1:3
   FAR = A(J)*EXP(-ARS(W(K))*D(J)*F)
   ARF = ARF + FAR
55 CONTINUE
   PKW = FAC*ARF/AKC
   PW = PKW*PI*(1+THETA*THETA*(W(K)+SKNE(I))**2)/THETA
   WRITE(6*2041) 1+PKW_PW_W(K)+SKNE(I)
49 CONTINUE
50 CONTINUE
500 CONTINUE
STOP
END
Table 3C - Computer Listings for Subprogram C
(Semifrozen Convection - Model A)

<table>
<thead>
<tr>
<th>COMMENT</th>
<th>REMOVE CARD 1030...READ(5*3)*100</th>
<th>FOR CASE W = WMN</th>
</tr>
</thead>
<tbody>
<tr>
<td>COMMENT</td>
<td>I030ossREAD(5#3*)WI9OW*WF</td>
<td></td>
</tr>
<tr>
<td>COMMENT</td>
<td>NO IBFCT CARD FOR RUN AT APL</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>MULTIPLE INTEGRAL PROGRAM NO.</td>
<td>FOR LM BY FG</td>
</tr>
<tr>
<td>C</td>
<td>USES GAUSSIAN QUADRATURE FOR FOUR INTEGRALS</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>GENERAL CASE WITH NO COUPLING</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>ANSWER IS IN INCHES SQUARED PER SEC,</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>IMPlicit REAL*8(A-H,T0-Z)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>INTEGER DM*ON</td>
<td></td>
</tr>
<tr>
<td></td>
<td>REAL KM*RM</td>
<td></td>
</tr>
<tr>
<td></td>
<td>COMPLEX <em>16 ARGCOM</em>2<em>FV</em>SUM<em>5</em>FV6<em>SUM6</em>ANS<em>ANSINT</em>PWRS0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>DIMENSION GM(20),GN(10),KN(10),KM(20)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>DIMENSION TITLE(20),AX(4),AN(4),WMN(20),101,FA(20),101,FC(20),101</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IEGEN(20),101,WGT(21),ARG(21)</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>READ AND WRITE INPUT DATA</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>NO. OF GAUSSIAN POINTS AND NO OF TERMS IN SUM OF A AND K</td>
<td></td>
</tr>
<tr>
<td></td>
<td>READ(59)NW*NAK</td>
<td></td>
</tr>
<tr>
<td></td>
<td>READ(5*33)(ARG(I),I=1,NW)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>READ(5*33)(WGT(I),I=1,NW)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>33 FORMAT(020.8)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PI=3.14159265358979323</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P12=2.*PI</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PI3=PI**3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>READ(5*11)(TITLE(I),I=1,20)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>FORMAT(20A4)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>WRITE(6,21)(TITLE(I),I=1,20)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>FORMAT(1H1,20A4)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>READ(5*31)(X,Y,P)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>READ(5*31)(AK(I),I=1,NAK)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>READ(5*31)(AN(I),I=1,NAK)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>READ(5<em>31)(FM</em>UE+dDEL*PB2)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>FORMAT(16F12.6)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>FM2<em>FM</em>FM</td>
<td></td>
</tr>
<tr>
<td></td>
<td>READ(5<em>31)UC</em>TH</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ALPH1=.02*750/DEL</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ALPH2=.3*DEL</td>
<td></td>
</tr>
</tbody>
</table>

106
TABLE 3C (Continued)

```
TH=1./(ALPHI*UC*DEL)
WRITE(6,71)A*B*FM2*UE*DEL*PB2*UC*TH
7 FORMAT(15H4A=F10.4*5H A=F10.4*15H MASS SQUARED= E15.6*
11H U R E= E15.6/FH DEL=E15.6*17H P BAR SQUARED= E15.6*
26H UC=E15.6*4H TH=E15.6/*
READ(5,9)MLOW*NUP+DM+MLOW*NUP+DN
9 FORMAT(16I5)
READ(5,3)GM(M),M=1,NUP)
READ(5,3)GN(N),N=1,NUP)
READ(5,3)KM(M),M=1,NUP)
READ(5,3)KN(N),N=1,NUP)
DO 2000 M=1,NUP
2000 GM(M)=GM(M)*P1
DO 2001 N=1,NUP
2001 GN(N)=GN(N)*P1
READ(5,12)((WW(N(N),M=1,NUP),N=1,NUP)
12 FORMAT(F10.2)
WRITE(6,13)((WW(N(N),M=1,NUP),N=1,NUP)
13 FORMAT(7H00MEGA= /jIX#,)=
READ(5,12)((PA(N(N),M=1,NUP),N=1,NUP)
WRITE(6,18)((PA(N(N),M=1,NUP),N=1,NUP)
18 FORMAT(8H0A(M*N~a /(BE14*6))
C134o/(A*8)
17 FORMAT(7H CONSTANT= E15.6)
CONST=144*A*32.2*32.2*4*A*A*8*B, (P13*FM2)
WRITE(6,17)CONST
DO 50 M=1,NLOW+NUP+DN
XM=XM
XMPI=XMPI
GMX2=GMX(M)*(X/A..S)
GMXPA2=GMX(M)*(XP/A..S,
SXMC2A=DCOS(GMXA)
IF(M-M/2*2.E0~O)SXMC2A=DSIN(GMXA2)+KM(M)*/DSINH(GMXA2)
SXOPCA=DCOSH(GMXA2)+KM(M)3*DCOS(GMXA2)
IF(M-M/2*2.E0~O)SXOPCA=DSINH(GMXA2)
S INXXP=SXMC2A*SX0PCA
DO 45 N=NLOW+NUP+DN
```

107
TABLE 3C (Continued)

\[
\begin{align*}
XN & = N \\
XNPi & = XN \times \pi \\
OGA & = WMN(MN) \\
FC(MN) & = OGA / FA(MN) \\
FUDGE & = XM \times XM \times XN \\
GNYP & = GN(N) \times (Y/B - .5) \\
GNYP2 & = GN(N) \times (Y/P - .5) \\
EIGEN(MN) & = C1 / FUDGE \times SINXP \times (DCOS(GNYP2) + KN(N) \times DCOSH(GNYP2)) \\
I(DCOS(GNYPB2) + KN(N) \times DCOSH(GNYPB2)) & = 0 \times 10^{-8} \\
IF(N - N/2 \times 2 \times EQ \times I)EIGEN(MN) & = C1 / FUDGE \times SINXP \\
I(DSIN(GNYPB2) + KN(N) \times DSINH(GNYPB2)) & = 0 \times 10^{-8} \\
WRITE(6,16)XYMNEIGEN(MN),FA(MN) & = 0 \times 10^{-8} \\
CONTINUE & = 0 \times 10^{-16} \\
WRITE(6,19) & = 0 \times 10^{-19} \\
XNPi & = XN \times \pi \\
XNPi = XN \times \pi \\
BNP & = BN \times \pi \\
READ(533,1,3,1,4,1,3,1,3,1,3,1,3,1,3,1,3) & = 0 \times 10^{-13} \\
GW & = GW \times GW \\
NYO & = NYO \times NYO \\
OGA & = WMN(MN) \\
OGA2 & = OGA \times OGA
\end{align*}
\]
TABLE 3C (Continued)

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>FAL1</td>
<td>FAL1 = FA(M+N)</td>
</tr>
<tr>
<td>FAL12</td>
<td>FAL1 * FAL</td>
</tr>
<tr>
<td>W</td>
<td>W=W</td>
</tr>
<tr>
<td>DO 150 M1=1+NW</td>
<td></td>
</tr>
<tr>
<td>ANSINT=0.</td>
<td></td>
</tr>
<tr>
<td>W2=W*W</td>
<td></td>
</tr>
<tr>
<td>DEN=(FAL12*(OGA-W)*<em>2)+(FAL12</em>(OGA+W)**2)</td>
<td></td>
</tr>
<tr>
<td>SUMAK=0.</td>
<td></td>
</tr>
<tr>
<td>DO 120 IS=1+NAK</td>
<td></td>
</tr>
<tr>
<td>SUMAK+SUMAK+AN(IS)*DEXP(-AK(IS)<em>W</em>DEL/UE)</td>
<td></td>
</tr>
<tr>
<td>POFW=SUMAK<em>PB2</em>DEL/UE</td>
<td></td>
</tr>
<tr>
<td>SUM6=0.</td>
<td></td>
</tr>
<tr>
<td>DO 500 I1=1+NXO</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>11=I1/NW</td>
</tr>
<tr>
<td>11R=I1-NW+1</td>
<td></td>
</tr>
<tr>
<td>XOP=PI*(1.5+5*ARG(K1R)+FLOAT(I10))</td>
<td></td>
</tr>
<tr>
<td>XOA=GM(M)*XOP/UE</td>
<td></td>
</tr>
<tr>
<td>SFXO=DCOS(XOP)*ROI(M)*DCOSH(XOA)</td>
<td></td>
</tr>
<tr>
<td>IF(M-M/2*2.EQ.0)SFZO=DSIN(XOP)*ROI(M)*DSINH(XOA)</td>
<td></td>
</tr>
<tr>
<td>SUM4=0.</td>
<td></td>
</tr>
<tr>
<td>DO 300 KL=1+NXOP</td>
<td></td>
</tr>
<tr>
<td>KL=K1-I1/NW</td>
<td></td>
</tr>
<tr>
<td>K1R=K1-NW+1</td>
<td></td>
</tr>
<tr>
<td>XOP=PI*(1.5+5*ARG(K1R)+FLOAT(K10))</td>
<td></td>
</tr>
<tr>
<td>XOXP=XOP+XOP</td>
<td></td>
</tr>
<tr>
<td>E1<em>DEXP=AJMP</em>ALPH1*OARS(XOXOP)</td>
<td></td>
</tr>
<tr>
<td>ARGCOM=DCMPLX(DO+-W/A/UC<em>TMPI</em>XOXOP)</td>
<td></td>
</tr>
<tr>
<td>E2*DEXP=ARGCOM</td>
<td></td>
</tr>
<tr>
<td>XOXP=GM(M)*XOP/XM1+5</td>
<td></td>
</tr>
<tr>
<td>SFXOP=DCOS(XOP)*ROI(M)*DCOSH(XOP)</td>
<td></td>
</tr>
<tr>
<td>IF(M-M/2*2.EQ.0)SFXOP=DSIN(XOP)*ROI(M)*DSINH(XOP)</td>
<td></td>
</tr>
<tr>
<td>FV4=SFXOP*E1'E2</td>
<td></td>
</tr>
<tr>
<td>SUM4=SUM4+FV4*WGT(KL)<em>PI/2</em></td>
<td></td>
</tr>
<tr>
<td>300 CONTINUE</td>
<td></td>
</tr>
<tr>
<td>FV6=SUM4*SFXO</td>
<td></td>
</tr>
<tr>
<td>SUM6=SUM6+FV6*WGT(I1R)<em>PI/2</em></td>
<td></td>
</tr>
<tr>
<td>500 CONTINUE</td>
<td></td>
</tr>
</tbody>
</table>
TABLE 3C (Continued)

SUM5 = 0
DO 470 J1 = 1, NYO
J10 = J1 / NW
J1R = J1 - NW * J10
YO = P1 * YO + 5 * ARG(J1R) + FLOAT(J10))
Y0B = GN(J1) * YO / XNP1 * 5
SFY0 = DCOS(YO8) + KN(J1) * DCOSH(Y0B)
IF (N/N + 2.2 * EQ) SFY0 = DSIN(YO8) + KN(J1) * DSINH(Y0B)
SUM3 = 0
DO 200 L1 = 1, NYOP
L10 = L1 / NW
L1R = L1 - NW * L10
Y0P = P1 * L1 + 5 * ARG(L1R) + FLOAT(L10))
Y0P + YO = Y0P
BNP1 B = XNP1 * UCTH
E3 = DEXP(-B / XNP1 * ALPH2 * DAMS(Y0P1))
Y0P = GN(J1) * (Y0P / XNP1 + 5)
SFY0P = DCOS(Y0P1) + KN(J1) * DCOSH(Y0P1)
IF (N/N + 2.2 * EQ) SFY0P = DSIN(Y0P1) + KN(J1) * DSINH(Y0P1)
FV3 = SFY0P * E3
SUM3 = SUM3 + FV3 * WGT(J1R) * PI/2
200 CONTINUE
FV5 = SUM3 + SFY0
SUM5 = SUM5 + FV5 * WGT(J1R) * PI/2
400 CONTINUE
ANSINT = SUM6 + SUM5
ANS = ANSINT * CONST * POFW * EIGEN(M1N) / DEN * C1
PWRSD = ANS * W**4
WRITE(6, 20) Mn + POFW * EIGEN(M1N) * DEN
20 FORMAT(1X, 4E15, 6E15)
WRITE(6, 21) ANSINT, ANS, PWRSD
21 FORMAT(1X, 4E15, 6E15)
W = W + DW
150 CONTINUE
776 CONTINUE
778 CONTINUE
STOP

END
Table 3D - Computer Listings for Subprogram D
(Semifrozen Convection - Model A)

C  MULTIPLE INTEGRAL PROGRAM NO.  FOR LM BY FG 0000
C  USES GAUSSIAN QUADRATURE ON FOUR INTEGRALS 0010
COMMENT  GENERAL CASE WMN AND WPO 0020
C  ANSWER IS IN INCHES SQUARED PER SEC.  0030
C
IMPLICIT REAL*8(A-H,O-Z)  0 50
INTEGER GLOW*QUP*DO*PLOW*PUP*Q  0 60
INTEGER DM*DN  70
REAL KP*KQ  80
REAL KN*KM  90
COMPLEX*16 DENCOM  0100
COMPLEX *16 COM1*COM2*XNMTR  0110
COMPLEX *16 ARGCOM*E2*PV4*SUM4*FV5*SUM5*FV6*SUM6*ANSXANSINT*PWRSD  0120
DIMENSION APO(20*10),WPO(20*10),GP(20*10),QP(20*10),KPQ(1*10)  0130
DIMENSION GM(20),GN(10),KN(10),KM(20)  0140
DIMENSION TITLE(20),AN(4),WMN(20,10),FA(20,10),FC(20*10)  0150
1E1GEN(20*10),WGT(21),ARG(21)  0160
C
C  READ AND WRITE INPUT DATA  0180
C  NO. OF GAUSSIAN POINTS AND NO OF TERMS IN SUM OF A AND K  0190
C
READ(5,33)(ARG(I),I=1,NW)  0200
READ(5,33)(WGT(I),I=1,NW)  0210
33 FORMAT(D20#8)  0220
P12*W2**PI  0230
P13*PI**3  0240
READ(5,33)(TITLE(I),I=1,20)  0250
1 FORMAT(20H4)  0260
WRITE(6,21)(TITLE(I),I=1,20)  0270
2 FORMAT(I10)  0280
READ(5,33)(AK(I),I=1,NAK)  0290
WRITE(6,23)(AK(I),I=1,NAK)  0300
2 FORMAT(I10)  0310
READ(5,33)(AN(I),I=1,NAK)  0320
READ(5,33)(AM(I),I=1,NAK)  0330
READ(5,33)(BU*UX*DEL*PB2)  0340
3 FORMAT(6F12#6)  0350
FM2*FM*FM  0360
TABLE 3D (Continued)

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>READ(5,3)UC*TH</td>
<td></td>
</tr>
<tr>
<td>ALPH1=0.02*UC/DEL</td>
<td></td>
</tr>
<tr>
<td>ALPH2=0.02*DEL</td>
<td></td>
</tr>
<tr>
<td>TH=1./(ALPH1<em>UC</em>DEL)</td>
<td></td>
</tr>
<tr>
<td>WRITE(6,7)A+XFM2+UC<em>DEL</em>PB2<em>UC</em>TH</td>
<td></td>
</tr>
<tr>
<td>7 FORMAT(3HOA=FIO<em>4</em>5HF10.4) MASS AVERAGE= E15.6</td>
<td></td>
</tr>
<tr>
<td>11H U SUB E= E15.6/5H DEL=E15.6/17H P BAR SQUARED= E15.6</td>
<td></td>
</tr>
<tr>
<td>26H UC=E15.6*H TIME=E15.6/1</td>
<td></td>
</tr>
<tr>
<td>READ(5,9)MLOWMUPDM+NLONUPDN</td>
<td></td>
</tr>
<tr>
<td>READ(5,9)PLLOWPUP+PLOWMUP+Q</td>
<td></td>
</tr>
<tr>
<td>9 FORMAT(165)</td>
<td></td>
</tr>
<tr>
<td>READ(5,3)GM(M),N=MUP</td>
<td></td>
</tr>
<tr>
<td>READ(5,3)GP(P),P=P1PUP</td>
<td></td>
</tr>
<tr>
<td>READ(5,3)GN(N),N=N1NUP</td>
<td></td>
</tr>
<tr>
<td>READ(5,3)KP(P),P=P1PUP</td>
<td></td>
</tr>
<tr>
<td>READ(5,3)KN(N),N=N1NUP</td>
<td></td>
</tr>
<tr>
<td>READ(5,3)KO(Q),Q=Q1QUP</td>
<td></td>
</tr>
<tr>
<td>DO 2000 M=MUP</td>
<td></td>
</tr>
<tr>
<td>2000 GM(M)=GM(M)*PI</td>
<td></td>
</tr>
<tr>
<td>DO 2001 N=N1NUP</td>
<td></td>
</tr>
<tr>
<td>2001 GN(N)=GN(N)*PI</td>
<td></td>
</tr>
<tr>
<td>DO 2002 P=P1PUP</td>
<td></td>
</tr>
<tr>
<td>2002 GP(P)=GP(P)*PI</td>
<td></td>
</tr>
<tr>
<td>DO 2003 Q=Q1QUP</td>
<td></td>
</tr>
<tr>
<td>2003 GO(Q)=GO(Q)*PI</td>
<td></td>
</tr>
<tr>
<td>READ(5,12)WMN(M,N),M=MUP,N=N1NUP</td>
<td></td>
</tr>
<tr>
<td>WRITE(6,12)UMN(M,N),M=MUP,N=N1NUP</td>
<td></td>
</tr>
<tr>
<td>12 FORMAT(10,2)</td>
<td></td>
</tr>
<tr>
<td>WRITE(6,13)WMN(M,N),M=MLOWMUP+DM,N=NLONUP+DN</td>
<td></td>
</tr>
<tr>
<td>13 FORMAT(12HOMEGA(M,N)=/(1X,8BD14.5))</td>
<td></td>
</tr>
<tr>
<td>WRITE(6,23)HOMEGA(P,O)P=PO+QLOW+QUP+QDP</td>
<td></td>
</tr>
<tr>
<td>23 FORMAT(12HOMEGA(P,O)=/(1X,8BD14.5))</td>
<td></td>
</tr>
<tr>
<td>READ(5,12)FMN(M,N),M=MUP,N=N1NUP</td>
<td></td>
</tr>
<tr>
<td>WRITE(6,18)FMN(M,N),M=MUP,N=N1NUP</td>
<td></td>
</tr>
<tr>
<td>18 FORMAT(8E0A(M,N)=/(8E14.6))</td>
<td></td>
</tr>
</tbody>
</table>
### TABLE 3D (Continued)

```plaintext
READ(5,12)((APQ(P,Q)*P=1*PUP)@Q=1*QUP)
WRITE(6,18)((FA(M,N),H=MLOW+UPUP,DM=1*NLOW+UPUP+DN)
WRITE(6,28)((PQ(P,Q)*P=PLOW*UPUP,DP=O=QLOW*QUP,DP)
28 FORMAT(8HOA(PsQ)m /(X98E14*61)
C1*2/DGRT(A*B)
17 FC*MAT(7H CONST= E15+6)
CONST=144*32*2*32*2 **B=B/(PI*3*FM2)
WRITE(6,17)CONST
DO 50 N=MLOW+UPUP+DM
X=M
XPI=XPI*PI
GMX=A2(GMXA2)*(X/A+5)
SMC2A=DCOS(GMXA2)+K(M)*DCOSH(GMXA2)
IF(M-M/2*2*EQ.0)SMC2A=DSINH(GMXA2)+K(M)*DSINH(GMXA2)
DO 45 N=MLOW+UPUP+DN
X=X+X
XPI=XPI*PI
FA(M,N)*OGA/FA(M,N)
GNB2=GN(N)*(X/B+5)
FUDGE=X*GMX
EIGEN(P+*C=1/FUDGE*SMC2A*(DCOS(GNCB)+KN(N)*DCOSH(GNCB))
1(DSINH(GNCB)+KN(N)+DSINH(GNCB))
WRITE(6,16)XY+MNEIGEN(M,N),FA(M,N)
16 FORMAT(1H02F12*6,214#6El3&6)
CONTINUE
50 CONTINUE
DO 550 P=PLOW*PUP+DP
X=P
XPI=XPI*PI
GMXPA2=GP(P)*X(A+5)
SMC2A=DCOS(GMXPA2)+K(P)*DCOSH(GMXPA2)
IF(P-P/2*2*EQ.0)SMC2A=DSINH(GMXPA2)+K(P)*DSINH(GMXPA2)
DO 545 Q=QLOW+QUP+DO
X=Q
XPI*XQ+PI
118
```

118
TABLE 3D (Continued)

OGP=WPQ(IP,0)
OGP2=OGP*OGP
FUDGE=XP*X0
GNYPB2=QG(Q)*DCSIGNYPB2+KO(Q)*DCOSIGNYPB2)
IF(Q-0.1>0.0)IEIGN(PQ)/C1/FUDGE*SIGMA(Q)*DCOSIGNYPB2)
1H(GNYPB2)
RITE(6+26)XP*XP*PO+IEIGN(PQ),APQ(F1,0)

26 FORMAT(1H2F12+6*2I4*6E13+6)
545 CONTINUE
550 CONTINUE
WRITE(6+19)

19 FORMAT(1H1,3X,1HM,3X,1HM,11X,4HMDFM,12X,3HWMN,12X,3HDEN,
1/18X,1H0,10X,5HANS INT R, 7X,1H0,10X,5HANS I*18X,7HPWRSD 11)
2R,8X,7HPWRSD 11/)
UCHM=UCHM
DO 776 W*MLOW*MUP*DM
XMP1=XMP1
AMP1=A/(XMP1*UCHM)
DO 778 P=MLOW+PUP*DP
XP=P
XMP1=AMP1
DO 776 N=MLOW*NUP*DN
XN=N
XNP1=XN+PI
BNP1=B/(XNP1*UCHM)
DO 776 G=MLOW+QUP*DO
X0=0
X0P=X0+PI
READ(5+3)W,DFM+WF
NW=(WF-WF)/DFM+1.
X0P=NWP
NXOP=NWP
OGA=MNN(M,N)

1110
1120
1130
1140
1150
1160
1170
1180
1190
1200
1210
1220
1230
1240
1250
1260
1270
1280
1290
1300
1310
1320
1330
1340
1350
1360
1370
1380
1390
1400
1410
1420
1430
1440
1450
1460
1470
<table>
<thead>
<tr>
<th>TABLE 3D (Continued)</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>OGA2<em>OGA</em>OGA</td>
</tr>
<tr>
<td>FAL1=FAL1*FAL1</td>
</tr>
<tr>
<td>OGP=W01+Q</td>
</tr>
<tr>
<td>OGP2=OGP*OGP</td>
</tr>
<tr>
<td>FAL2=AP1*(P+Q)</td>
</tr>
<tr>
<td>FAL2=FAL2+FAL2</td>
</tr>
<tr>
<td>W=I</td>
</tr>
<tr>
<td>DO 150 MM1=1*MMW</td>
</tr>
<tr>
<td>ANSINT=0.</td>
</tr>
<tr>
<td>MMW=MMW</td>
</tr>
<tr>
<td>DENS=(((FAL1+FAL2)/2.*2+OGA2/4.*OGP2/4.)**2</td>
</tr>
<tr>
<td>1-OGA2*OGP2/4.)<em>OGA</em>OGP</td>
</tr>
<tr>
<td>COM1=FAL1*DCMPLX10.0+1.0</td>
</tr>
<tr>
<td>COM2=FAL2*DCMPLX10.0+1.0</td>
</tr>
<tr>
<td>FALL12=(FAL1+FAL2)/2.</td>
</tr>
<tr>
<td>XNMRTR=(COM1*(OGA<em>OGP+FALL12)+OGA</em>OGP*(FALL12+FALL12-</td>
</tr>
<tr>
<td>1/2*(OGA2/4.0+OGP2/4.)**2+COM1<em>COM1-OGA2)+COM2</em>COM2-</td>
</tr>
<tr>
<td>2*(1/2*(OGA<em>OGP+FALL12)+OGA</em>OGP*(FALL12+FALL12+OGA2/4.0+OGP2/4.))/COM2*COM2</td>
</tr>
<tr>
<td>DENC0M=XNMRTR/DEN</td>
</tr>
<tr>
<td>SUMAK+0.</td>
</tr>
<tr>
<td>DO 120 IS=1*NAK</td>
</tr>
<tr>
<td>120 SUMAK=SUMAK+AN(IS)*DEXP(-AK(IS)<em>W</em>DEL/UE)</td>
</tr>
<tr>
<td>SUM6=0.</td>
</tr>
<tr>
<td>DO 500 I=1*NXO</td>
</tr>
<tr>
<td>500 IF(M-M/2<em>2.E0)SFXO=DSIN(XOA-1/2.0+K1</em>(W)-1/2.0+K1*(SIN(XOA-1/2.0+K1*(W))</td>
</tr>
</tbody>
</table>
TABLE 3D (Continued)

\[ \text{XOP} = \text{PI} \times 1.5 \times 5 \times \text{ARG} (\text{KIR}) \times \text{FLOAT} (\text{K10}) \]
\[ \text{XOXP} = \text{XO} \times \text{XOP} \]
\[ \text{E1} \times \text{DEXPI} - \text{A} / \text{D} \times \text{ALP} \times \text{DARS} (\text{XO} \times \text{XM} - \text{XOP} / \text{XXP}) \]
\[ \text{ARCCOM} = \text{DCMP} (\text{XO} \times \text{DO} - \text{W} / \text{A}/(\text{IC} \times \text{PI}) \times (\text{XO} / \text{XM} - \text{XOP} / \text{XXP})) \]
\[ \text{F2} \times \text{CDEXP} (\text{ARCCOM}) \]
\[ \text{XOP} = \text{GP} (\text{P}) \times (\text{XOP} / \text{XXP}) - 5 \]
\[ \text{SFXOP} = \text{DCOS}(\text{XOP}) + \text{KP}(\text{P}) \times \text{DCOSH}(\text{XOPA}) \]
\[ \text{IF} \times (\text{P} - \text{PI} / 2 \times \text{EQ} + 0) \times \text{SFXOP} \times \text{DSIN}(\text{XOPA}) + \text{KPI} \times \text{DSINH}(\text{XOPA}) \]
\[ \text{FV} 4 \times \text{SF} \times \text{OP} \times \text{E1} \times \text{E2} \]
\[ \text{SUM} 4 \times \text{SUM} 4 \times \text{FV} 4 \times \text{WGT}(\text{K1R}) / 2 \times \text{PI} \]

300 CONTINUE

\[ \text{FV} 6 \times \text{SUM} 6 \times \text{FV} 6 \times \text{WGT} (\text{I1R}) / 2 \times \text{PI} \]

500 CONTINUE

\[ \text{FV} 6 \times \text{SUM} 4 \times \text{SF} \times \text{XO} \]
\[ \text{SUM} 6 \times \text{SUM} 6 \times \text{FV} 6 \times \text{WGT}(\text{I1R}) / 2 \times \text{PI} \]

DO 400 \text{J1} = 1, \text{NYOP}

\[ \text{J10} = (\text{J1} - 1) / \text{NW} \]
\[ \text{J1R} = (\text{J1} - 1) / \text{NW} \times \text{J10} \]
\[ \text{YO} = \text{PI} \times 1.5 \times 5 \times \text{ARG} (\text{J1R}) \times \text{FLOAT} (\text{J1Q}) \]
\[ \text{YO} = \text{GN}(\text{N}) \times (\text{YO} / \text{XNP} - 5) \]
\[ \text{SFYO} = \text{DCOS} (\text{YOB}) + \text{KN}(\text{N}) \times \text{DCOSH} (\text{YOB}) \]
\[ \text{IF} \times (\text{N} - \text{N} / 2 \times \text{EQ} + 0) \times \text{SFYO} \times \text{DSINH} (\text{YOB}) + \text{KN}(\text{N}) \times \text{DSINH} (\text{YOB}) \]
\[ \text{SUM} 3 = 0 \]

DO 200 \text{L1} = 1, \text{NYOP}

\[ \text{L10} = (\text{L1} - 1) / \text{NW} \]
\[ \text{L1R} = (\text{L1} - 1) / \text{NW} \times \text{L10} \]
\[ \text{YO} = \text{PI} \times 1.5 \times 5 \times \text{ARG} (\text{L1R}) \times \text{FLOAT} (\text{L1Q}) \]
\[ \text{YO} = \text{YOP} = \text{YOP} \]
\[ \text{BNP} = \text{B} / \text{XNP} \times \text{UCTH} \]
\[ \text{E1} \times \text{DEXPI} - \text{A} / \text{D} \times \text{ALP} \times \text{DARS} (\text{YO} / \text{XNP} - \text{YOP} / \text{XQ}) \]
\[ \text{YO} = \text{GQ} (\text{Q}) \times (\text{YOP} / \text{XQP} - 5) \]
\[ \text{SFYO} = \text{DCOS} (\text{YOB}) + \text{KQ}(\text{Q}) \times \text{DCOSH} (\text{YOB}) \]
\[ \text{IF} \times (\text{Q} - \text{Q} / 2 \times \text{EQ} + 0) \times \text{SFYO} \times \text{DSINH} (\text{YOB}) + \text{KQ}(\text{Q}) \times \text{DSINH} (\text{YOB}) \]
\[ \text{FV} 3 \times \text{SF} \times \text{OP} \times \text{E} 3 \]
\[ \text{SUM} 3 \times \text{SUM} 3 \times \text{FV} 3 \times \text{WGT}(\text{L1R}) / 2 \times \text{PI} \]

200 CONTINUE

\[ \text{FV} 3 \times \text{SUM} 3 \times \text{SFYO} \]

TABLE 3D (Continued)

\[ \text{SUM} 5 \times \text{SUM} 5 \times \text{FV} 5 \times \text{WGT}(\text{J1R}) / 2 \times \text{PI} \]

400 CONTINUE

\[ \text{ANSINT} = \text{SUM} 4 \times \text{SUM} 5 \]
\[ \text{ANSINT} \times \text{CONST} \times \text{POFW} \times \text{EIGEN}(\text{MN}) \times \text{EIGEN}(\text{P}) \times \text{DENCOM} \times \text{C1} \times \text{C1} \]
\[ \text{POWRSD} = \text{ANSINT} \times \text{ANSINT} \times \text{ANSINT} \times \text{ANSINT} \times \text{ANSINT} \]
\[ \text{WRITE} (\text{J1} \times 20) \times \text{MN} \times \text{W} \times \text{POFW} \times \text{EIGEN}(\text{MN}) \times \text{DEN} \]
\[ \text{WRITE} (520) \times \text{P} \times \text{XNHTR} \times \text{EIGEN}(\text{P}) \times \text{DENCOM} \]

20 FORMAT (1X, 2I4, 6E15.6)

21 FORMAT (1X, 2I4, 6E15.6)

W=W+DW

150 CONTINUE

7761 CONTINUE

7776 CONTINUE

7778 CONTINUE

STOP

END
APPENDIX C

ELECTRIC BOAT PROGRAM (IZZO)

APPENDIX C1 – MATHEMATICAL ANALYSIS
APPENDIX C2 – METHOD FOR DETERMINING INPUT DATA
APPENDIX C3 – PROGRAM IDENTIFICATION
APPENDIX C4 – TEST RUNS
NOTATION

$A$  
Correlation area of turbulence over which the mean square pressure $p^2$ is constant

$A_{mn}, A_{rs}$  
Coefficient used in series representation of deflection

$A_n, A'_n$  
Normal acceleration of plate

$a_i$  
Speed of sound in water

$a_m, b_m, c_m, d_m$  
Constants

$a_{mn}$  
Modal damping function

$B$  
Equal to $\eta / 2$

$h$  
Bending stiffness

$c_{mn}, D_{mn}$  
Constants defined in Equation (C27)

$c_{rs}$  
Coefficient in Equation (C34)

$g(\ )$  
Greens function (impulse response of plate at point $r_0$ due to forcing function $P$ at $r_0$) defined by Equation (C10)

$h$  
Plate thickness

$l, i$  
Refers to properties on the side of the plate where the fluid is in motion (i.e., turbulent) and where the fluid is stagnant, respectively, as shown in Figure 11.

$I_{mn}(r)$  
Time correlation integral defined by Equation (C24)

$K$  
Constant

$K'(\omega_{mn})$  
Modal amplitude factor

$k_{mn}, k_{mn}$  
Constants defined in Equation (C27) and (C28), respectively

$L(\ )$  
Linear differential operator defined in Equation (C12)

$L_x, L_y$  
Lateral dimensions of plate along the $x$ and $y$ axes, respectively

$M$  
Mass per unit area of plate

$m, n; p, q; r, s$  
Mode numbers

$P_i$  
Acoustic pressure

$\overline{p^2}$  
Mean square pressure at surface beneath turbulent boundary layer
\( \rho(r,t) \) Surface pressure beneath turbulent boundary layer

\( R_A(\cdot) \) Space-time correlation of plate accelerations

\( R_p(r,r^*,r) \) Space-time correlation of acoustic pressures

\( R_p(r_0,r^*,r) \) Space-time correlation of turbulence pressures

\( R_A(\cdot) \) Space-time correlation of plate velocities

\( R_A(\cdot) \) Space-time correlation of plate displacements

\( r, r^*, r_0, r_0^* \) Radius vectors defined in Figure 11

\( \overline{r_0(\overline{x_0}, \overline{y_0}, \overline{z_0})}, \overline{r_0^*} \) Space-time coordinates of the forcing function \( p \)

\( S, S_0, S_0^* \) Surface area of plate \( (dS_0 = dx_0 dy_0 \text{ etc}) \)

\( S_p(r,r^*,\omega) \) Cross-spectral density of acoustic pressures

\( S_p(r,\omega) \) Power spectrum of acoustic pressures at a point \( r \) along the normal through the center of the plate

\( S_p(r,\omega) \) Cross-spectral density of plate displacements

\( T \) Kinetic energy of plate

\( t, t^*, t_0, t_0^* \) Time variables

\( U(t-t_0) \) Unit step function

\( U_c \) Average convective speed of turbulent pressure field (or pattern)

\( U_0 \) Ship speed; free stream velocity

\( u \) Velocity component, at any point \( y \) in the boundary layer, parallel to the \( x \) axis; see Figure 11

\( V \) Potential energy of plate

\( V_n, V_n^* \) Normal velocity of plate

\( w \) Plate displacement

\( X_m(\varphi), Y_n(y) \) Mode shapes of the plate along \( x \) and \( y \), respectively

\( x, x^*, x_0, x_0^* \)

\( y, y^*, y_0, y_0^* \) Space coordinates defined in Figure 11

\( Z_n, Z_n^*, z, z^* \) Normal displacement of plate
\[ a \]
- Equal to \( mn \, U \, e' \, L \)

\[ \beta_0 \]
- Plate viscous damping (resistance coefficient)

\[ \beta_1 \]
- Radiation damping coefficient

\[ \gamma, \mu \]
- Functions of the time variables as defined by Equation (C 22)

\[ \delta, \delta^* \]
- Boundary layer thickness and boundary layer displacement thickness, respectively

\[ \delta( ) \]
- Dirac delta function

\[ \delta_{mn}, \delta_{pq} \]
- Kronecker delta equal to 1 for \( m = n \) or \( p = q \)

\[ \delta_{mn} \]
- Kronecker delta equal to 1 for \( mn = rs \) and 0 for \( mn \neq rs \)

\[ \eta \]
- Loss factor (plate hysteretic damping)

\[ \theta \]
- Temporal decay factor of turbulent boundary layer associated with eddy decay

\[ \kappa \]
- Measure of the inverse radius of the turbulence eddy

\[ \lambda_{mn} \]
- Eigenvalue

\[ \mu \]
- Poisson's ratio

\[ \rho \]
- Density of plate material

\[ \rho_0 \]
- Fluid density

\[ r, r', r_0 \]
- Delay times equal to \( t' - t, t' - t + \frac{r_0 - r'_0}{a_i} \), and \( t'_0 - t_0 \), respectively

\[ \Phi(\omega_{mn}) \]
- Spectrum shape factor

\[ \phi_{mn}, \phi_{pq} \]
- Eigenfunction

\[ \omega \]
- Circular frequency, equal to 2\( \pi f \)

\[ \omega_{mn} \]
- Undamped natural modal frequency

\[ * \]
- Symbol representing the complex conjugate

\[ < > \]
- Symbol representing cross (space-time) correlation function
APPENDIX C1 - MATHEMATICAL ANALYSIS

Equations are now derived for the space-time correlation and spectral density of the acoustic pressure in the near and far field on both sides of a turbulence excited vibrating plate.\textsuperscript{26}

The Rayleigh formulation\textsuperscript{27} of the velocity potential in the acoustic field resulting from a vibrating plane is

\[
\phi(r, \theta) = -\frac{1}{2\pi} \int \frac{dS_0}{r} V_n \left( r_0, \frac{t - r_0}{a_i} \right) \tag{C1a}
\]

The corresponding acoustic pressure is

\[
P_i(r, t) = -\rho_i \frac{\partial \phi}{\partial t} = \frac{\rho_i}{2\pi} \int \frac{dS_0}{r_0} \frac{\partial V_n}{\partial t} \left( r_0, \frac{t - r_0}{a_i} \right) \tag{C1b}
\]

Figure 11 shows the coordinate system used for this formulation. The space-time correlation of the acoustic pressures are

\[
\langle P_i(r, t) P_i(r', t') \rangle = R_p(r, r', t) = \langle \frac{\rho_i}{2\pi} \int \frac{dS_0}{r_0} \frac{\partial V_n}{\partial t} \left( r_0, \frac{t - r_0}{a_i} \right) \rangle.
\]

\[
\frac{\rho_i}{2\pi} \int \frac{dS_0'}{r_0'} \frac{\partial V_n}{\partial t} \left( r_0', \frac{t' - r_0'}{a_i} \right) \tag{C2}
\]

where \( t' = t + r \).

The integration and ensemble averaging processes may be interchanged to give

\[
R_p(r, r', t) = \frac{\rho_i^2}{4\pi^2} \int \int \frac{dS_0}{r_0} \frac{dS_0'}{r_0'} < \frac{\partial V_n}{\partial t} \left( r_0, \frac{t - r_0}{a_i} \right) \frac{\partial V_n}{\partial t} \left( r_0', \frac{t' - r_0'}{a_i} \right) > \tag{C3}
\]

Now

\[
< \frac{\partial^2 Z_n}{\partial t^2} \frac{\partial^2 Z_n'}{\partial t'^2} > = < \frac{\partial V_n}{\partial t} \frac{\partial V_n'}{\partial t'} > = < A A >
\]

121
Figure 11 - Coordinate System
may be written as

\[
\frac{\partial^4}{\partial r^4} R_z \left( r_0, t - \frac{r_0}{a_i}; r_0', t' - \frac{r_0'}{a_i} \right) = \frac{\partial^2}{\partial r^2} \left( r_0, t - \frac{r_0}{a_i}; r_0', t' - \frac{r_0'}{a_i} \right) \\
= R_A \left( r_0, t - \frac{r_0}{a_i}; r_0', t' - \frac{r_0'}{a_i} \right)
\]

(C4)

Hence

\[
R_p(r, r', r) = \frac{\rho_i^2}{4\pi^2} \int \int \int \frac{dS_0}{r_0} \frac{dS_{0'}}{r_{0'}} \frac{\partial^4}{\partial r^4} R_z \left( r_0, t - \frac{r_0}{a_i}; r_0', t' - \frac{r_0'}{a_i} \right) (C5)
\]

Assuming a stationary random process, we can shift the time origin by an amount \( r_0 / a_i \) without changing the results of averaging. We get

\[
\eta_p(r, r', r) = \frac{\rho_i^2}{4\pi^2} \int \int \int \frac{dS_0}{r_0} \frac{dS_{0'}}{r_{0'}} \frac{\partial^4}{\partial r^4} R_z \left( r_0, t - \frac{r_0}{a_i}; r_0', t' + r \right) (C6)
\]

where \( r' = r + \frac{r_0 - r_0'}{a_i} \).

The Wiener-Khintchine relations between the cross-correlation and cross-spectral density of the acoustic pressures are

\[
S_p(r, r', \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dr \ R_p(r, r', r) e^{-i\omega r} \quad (C7a)
\]

\[
R_p(r, r', r) = \int_{-\infty}^{\infty} d\omega \ S_p(r, r', \omega) e^{i\omega r} \quad (C7b)
\]

123
Substituting Equation (C5) in (C7a), we obtain*  

\[ S_p(r, r', \omega) = \frac{j \rho^2}{4\pi^2} \int \int_{S} \frac{dS_0}{r_0} \frac{dS_0'}{r_0'} S_s(r, r', \omega) \]  

(C8)

Equation (C6), (C7), and (C8) describe the acoustic pressure resulting from the vibration of a plane surface in a semi-infinite medium. The equations are a function of \( R_z \) only and are applicable to any type of plate with any type of boundaries.

The method for determining plate response to turbulence excitation is identical to that of Dyer except that more general boundary conditions are included here. However, because the notation is somewhat different, the relevant equations are outlined for the benefit of the computer program user. The reader is referred to Appendix A1 for a more detailed development. The differential equation of motion of the linear system \( L z = - p \) has the solution

\[ Z(r_0, t) = \int_{-\infty}^{t} dt_0 \int_{S} dS_0 g(r_0, t; r_0, t_0) p(r_0, t_0) \]  

(C9)

where

\[ L g = -\delta(r - r_0) \delta(t - t_0) = -\delta(x - x_0) \delta(y - y_0) \delta(t - t_0) \]  

(C10)

For turbulence excitation (random pressures), the ensemble average or cross correlation of the plate displacements can then be expressed in terms of the correlation of the turbulent pressure forces by

\[ \langle z(r_0, t) Z^*(r_0', t')\rangle = R_z(r_0, r_0', r) = \int_{-\infty}^{t} dt_0 \int_{-\infty}^{t'} dt_0' \int_{S} dS_0 \int_{S} dS_0' g(r_0, t; r_0, t_0) p(r_0, t_0) g^*(r_0', t'; r_0', t_0') \]  

\[ = \int_{-\infty}^{t} dt_0 \int_{-\infty}^{t'} dt_0' \int_{S} dS_0 \int_{S} dS_0' g(r_0, t; r_0, t_0) g^*(r_0', t'; r_0', t_0') \]  

\[ R_{tp}(r_0, r_0', r) \]  

(C11)

*  

\[ J \left( \frac{\partial^4 R_z}{\partial t^4} \right) = \omega^4 S_z \]
For thin plate vibration

\[ \dot{\varphi}(1 - i\eta) \varphi^4(t) + M \frac{\partial^2}{\partial t^2} \phi + \beta_0 \frac{\partial}{\partial t} = \Phi(t) \]  

(C12)

The Green function satisfying Equation (C12) has been shown to be represented by

\[ g(t_0, t; t_0, t_0) = \sum_{m, n} \frac{\phi_{m, n}(t_0) \phi_{m, n}(t_0)}{\omega_{m, n}} e^{-\alpha_{m, n}(t - t_0)} \sin \omega_{m, n}(t - t_0) \phi_{m, n}(0) U(t - t_0) \]  

(C13)

where \( \phi_{m, n} \), the orthonormal set of eigenfunctions, satisfies the conditions:

\[ L \phi_{m, n} = 0 \]  

(C14)

and

\[ \int_S \phi_{m, n} \phi_{p, q} dS = \delta_{m, p} \delta_{n, q} \]  

(C15)

and where*

\[ a_{m, n} = \frac{\omega_{m, n}}{\eta} \left[ \left( 1 + \eta \frac{\beta}{\omega_{m, n} M} + \eta^2 \right)^{1/2} - 1 \right] \]  

(C16)

\[ \omega_{m, n} = \left( \frac{\beta}{M} \right)^{1/2} \lambda_{m, n}^2 \]  

(C17)

*Equation (A7) of Appendix A can be written in the present notation as

\[ a_{m, n} = \frac{\omega_{m, n}}{\eta} \left\{ \left[ \frac{\beta \eta}{2 \omega_{m, n}} + \frac{\lambda_{m, n}^4 \eta^2}{2 \omega_{m, n}^2} + 1 \right]^2 \right\}^{-1} \]

Expanding this equation and assuming, with Dyer (Equation (12) of Reference 2), that \( \eta \leq 1/3 \) and \( \frac{\beta}{2 \omega_{m, n} M} \leq 1/3 \)

as well as using Equation (A8) in the expansion, we obtain Equation (C16).
When radiation damping is also included, we write

\[ a_{mn} = \frac{\beta_0}{2M} + \frac{\eta}{2} \omega_{mn} + \frac{\beta_1}{2M} \]  

(C18)

In this analysis, it is assumed that \( \beta_0 \) and \( \beta_1 \) are negligible so that

\[ a_{mn} = \frac{\eta}{2} \omega_{mn} = B \omega_{mn} \]  

(C19)

Using Dyer's equation for the pressure correlation

\[ R_{tp}(r, r) = \hat{p}^2 A \delta[(x_0 - x_0') - U_x t'] \delta(y_0 - y_0') e^{-\frac{|r|}{\theta}} \]  

(C20)

Using Equations (C12), (C13), and (C20), we obtain the working expression for the displacement correlation function for a plate excited by a turbulent boundary layer. It is applicable to arbitrary boundary conditions provided expressions can be obtained for the eigenvalues and normalized eigenfunctions.

\[ R_{d}(r_0, r_0', r) = \sum \sum \frac{A\hat{p}^2}{\omega_{mn} \omega_{pq} M^2} \phi_{mn}(r_0) \phi_{pq}(r_0') \int_{-\infty}^{t} dt_0 \int_{-\infty}^{t'} dt_0' \int_{S} dS_0 \int_{S'} dS_0' \left[ -a_{mn}(t - t_0) - a_{pq}(t' - t_0') - \frac{r_0}{\theta} \right] \]

\[ \cdot \sin \omega_{mn}(t - t_0) \sin \omega_{pq}(t' - t_0') \delta[(x_0 - x_0') - U_x t'] \delta(y_0 - y_0') \]  

(C21)

Performing a spatial integration of Equation (C21), then introducing the transformation used by Dyer

\[ \gamma = (t' - t_0') - (t - t_0) = \tau_0 - \tau \]

\[ \mu = (t' - t_0') + (t - t_0) \]  

(C22)

*If values or relations for \( \beta_0, \beta_1 \) are known, we can include these terms in the analysis and program.
followed by a temporal integration yields

\[ R_z(r_0, r_0', r) = \sum_m \sum_n \frac{A p^2}{4 \omega_m^2 \omega_n^2} \phi_m(r_0) \phi_n(r_0') I_m(r) \]  

(C23)

where

\[ I_m(r) = \int_0^\infty \int_{-\mu}^{\mu} dy e^{-\frac{|y+r|}{\theta}} \cos \alpha (y+r) \{ \cos \omega_m y - \cos \omega_n y \}, \ r \geq 0 \]

(C24)

where \( \alpha = m \pi U_e / L_x \cdot \)

Since \( \theta \) is small, then for \( |r_0| > 0 \), \( e^{-\frac{|r_0|}{\theta}} \to 0 \) so that from Equation (C21) \( R_z(r_0, r_0', r') \to 0 \).

Hence, with small error, we need consider only the value \( r_0 = (r_0' - r_0) = 0 \). With this approximation we perform the integration in Equation (C21) to obtain the displacement correlation below coincidence. To render the analysis tractable for the integration, it is also assumed that \( U_e \theta \ll L_x \ll \omega_{mn} \), i.e. the correlation length of the pressure field is much smaller than the length of the plate and the convection speed of the turbulence is small compared to the modal wavelength, and \( a_{mn} \theta \ll 1 \) (low damping); see (A22), (A23), (A36), and (A28b) of Appendix A1. The result for the displacement correlation function, which is independent of plate boundary conditions, is*

*It is important to note that although the same symbols \( \phi_{mn}(r_0) \) and \( \phi_{mn}(r_0') \) are used,

\[ \phi_{mn}(r_0) = \frac{(L_x L_y)^{1/2}}{2} \phi_{mn}(r_0') \quad \text{Eq (C25)} \quad \phi_{mn}(r_0') = \frac{(L_x L_y)^{1/2}}{2} \phi_{mn}(r_0) \quad \text{Eq (C25)} \quad \phi_{mn}(r_0) = \frac{(L_x L_y)^{1/2}}{2} \phi_{mn}(r_0') \quad \text{Eq (C21)} \]

The value of the normalized eigenfunction \( \phi_{mn} \) used in Equation (C21) agrees with that used by Dyer. That the value of \( \phi_{mn} \) used in Equation (C25) differs from Dyer's results by a factor \( \frac{(L_x L_y)^{1/2}}{2} \) can be seen by comparing Equations (C35) and (A20) for the case of a simply supported plate. Thus, we see that Equations (C25) and Equation (A27), where \( I_m(r) \) is given by Equation (A36), are in agreement.
\[
R_A(r_0, r_0', r) = \sum_{m} \sum_{n} \frac{2A\theta p^2}{L_x L_y \omega_m^2 \nu_m M^2 (1 + \omega_m^2 \theta^2)} 
\cdot \phi_{mn}(r_0) \phi_{mn}(r_0') e^{-\frac{a_{mn} r_1}{r}} \cos \omega_m r.
\] (C23)

For the plate mode shape, assume that
\[
\phi_{mn}(r) = X_m(x) Y_n(y)
\] (C26)

where

\[
X_m(x) = a_m \cos \alpha_m x + b_m \sin \alpha_m x + c_m \cosh \alpha_m x + d_m \sinh \alpha_m x
\]
\[
Y_n(y) = a_n \cos \alpha_n y + b_n \sin \alpha_n y + c_n \cosh \alpha_n y + d_n \sinh \alpha_n y
\] (C26a)

From Equations (C4) and (C6), we see that \( R_p(r, r', r) \) is a function of

\[
\frac{\partial^4}{\partial r^4} R_p(r_0, r_0', r) = R_A(r_0, r_0', r).
\]

Page 99 of Reference 28 shows that the correlation of the plate acceleration for \( r > 0 \) can be expressed as*

\[
R_A(r_0, r_0', r) = \sum_{m} \sum_{n} \mathcal{K}_{mn} \phi_{mn}(r_0) \phi_{mn}(r_0') e^{-\frac{a_{mn} r}{r} \left[C_{mn} \cos \omega_m r + D_{mn} \sin \omega_m r\right]}
\]

\[; r > 0\] (C27)

where

\[
\mathcal{K}_{mn} = \frac{2A\theta p^2}{L_x L_y \omega_m^2 \nu_m M^2 (1 + \omega_m^2 \theta^2)}
\]
\[
C_{mn} = \omega_m^4 - 6 \omega_m \omega_n a_m^2 + a_m^4
\]
\[
D_{mn} = 4a_m \omega_m (a_m^2 - \omega_n^2)
\]

*Footnote on following page.
Footnote to preceding page. Using Equation (C25) and the relationship between $R_A$ and $R_z$ we have

$$R_A = \frac{\partial^4}{\partial r^4} R_z = K \frac{\partial^4}{\partial r^4} \left[ e^{-a|\omega_m^*|r} \right] \cos \omega_m^* r$$

where

$$K = \sum \sum_{n=0}^{\infty} \frac{2 \Delta \rho^2}{L_x L_y \omega_m^2 a_{mn}^2 \phi_m^2 (1 + \omega_m^2 \phi_m^2)} \phi_m^2 (r)^2 \phi_m^2 (r)^2$$

Let $a_{mn} = a$, $\omega_m = b$. Then

$$f(\tau) = e^{-a|r|} \cos b r = e^{ar} \cos b r$$

for $\tau < 0$

$$= e^{-ar} \cos b r$$

for $\tau > 0$

or

$$= e^{-ar} \cos b r = u \cdot v$$

where $a = -a$ for $\tau < 0$

$$a = a$$

for $\tau > 0$

$$u(r) = e^{-ar} \text{ and } v(r) = \cos br$$

The Leibniz theorem is obtained by differentiating $u v$ with respect to $r$, $n$ times. When $n = 4$, corresponding to the fourth derivative, the theorem gives

$$f^{(4)}(r) = u^{(4)} v + 4 u^{(3)} v' + 6 u^{(2)} v'' + 4 u v''' + u^{(4)} v'$$

$$= a^4 e^{-ar} \cos br + 4(-a^3) e^{-ar} (-b \sin br) + 6a^2 e^{-ar} (-b^2) \cos br$$

$$+ 4(-a) e^{-ar} b^3 \sin br + b^4 e^{-ar} \cos br$$

$$= (a^4 - 6a^2 b^2 + b^4) f(r) + 4ab(a^2 - b^2) g(r)$$

$$= (a^4 - 6a^2 b^2 + b^4) f(r) + 4ab(a^2 - b^2) g(r)$$

$$= (a^4 - 6a^2 b^2 + b^4) f(r) - 4ab(a^2 - b^2) g(r)$$

$$= (a^4 - 6a^2 b^2 + b^4) f(r) - 4ab(a^2 - b^2) g(r)$$

where $f(r) = e^{-ar} \cos br$ and $g(r) = e^{-ar} \sin br = e^{-a|r|} \sin br$.

The results agree with those in Appendix II of Reference 29 determined there by use of Heaveside functions. We note that the first derivative of $f(r)$ has a finite discontinuity at the origin so that the second and higher order derivatives will have infinite discontinuities at this point.
Reference 29 also shows that Equation (C27) satisfies the Wiener-Khintchine relations, Equations (C1a) and (C1b), thereby establishing the derived expression for $R_a(t_0, t_0', r)$ as a valid correlation function.*

\[
\frac{d^4 f(r)}{dt^4} \quad \text{and therefore } R_a \text{ satisfies the Wiener-Khintchine relations, consider the Fourier cosine transform } \frac{d^4 f(r)}{dt^4}. \quad \text{Because } \frac{d^4 f(r)}{dt^4} \text{ is an even function, the Fourier sine transform of this function vanishes. Since the fourth derivative is a continuous function, we integrate by parts to find}
\]

\[
\int_0^\infty \frac{d^4 f(r)}{dt^4} \cos \omega t \, dr = \left[ \frac{d^3 f(r)}{dt^3} \cos \omega t \, dr + \omega \frac{d^2 f(r)}{dt^2} \sin \omega t - \omega^2 \frac{d f(r)}{dt} \cos \omega t \right]_0^\infty - \omega^3 \int_0^\infty \frac{d f(r)}{dt} \sin \omega t \, dr
\]

\[
f(r) \text{ and all of its derivatives are zero at } r = \infty. \quad \text{Also, odd derivatives of } f(r) \text{ over the range } -\infty < r < \infty \text{ are odd functions of } r \text{ so that the value of these derivatives at the point of discontinuity (i.e., origin } r = 0) \text{ is zero. Hence, this equation reduces to}
\]

\[
\int_0^\infty \frac{d^4 f(r)}{dt^4} \cos \omega t \, dr = -\omega^3 \int_0^\infty \frac{d f(r)}{dt} \sin \omega t \, dr - \omega^4 \int_0^\infty f(r) \cos \omega t \, dr
\]

after integration by parts of the bracketed integral. Substituting $f(r) = e^{-a|r|} \cos \beta r$ in the last integral we obtain

\[
\int_0^\infty \frac{d^4 f(r)}{dt^4} \cos \omega t \, dr = \frac{\omega^4}{2 \left( a^2 + (b + \omega)^2 \right)} + \frac{\omega^4}{2 \left( a^2 + (b - \omega)^2 \right)}
\]

The inverse Fourier cosine transform of this expression is

\[
\frac{1}{2\pi} \int_0^\infty \left[ \frac{\omega^4}{2 \left( a^2 + (b + \omega)^2 \right)} + \frac{\omega^4}{2 \left( a^2 + (b - \omega)^2 \right)} \right] \cos \omega t \, d\omega
\]

\[
= \left( a^2 - 6ab^2 + b^4 \right) f(t) + 4ab(a^2 - b^2) g(t)
\]

which we have previously shown to be $\frac{d^4 f(t)}{dt^4}$ for $r > 0$. Thus, the terms $\frac{d^4 f(t)}{dt^4}$ and $\frac{a}{2 \left( a^2 + (b + \omega)^2 \right)}$ + $\frac{\omega^4}{2 \left( a^2 + (b - \omega)^2 \right)}$ are Fourier pairs. These results are obtained with somewhat more rigor in Reference 29.
Substitution of Equation (C27) in (C8) yields the following expression for the cross correlation of acoustic pressures*

\[ R_p(r, r', r) = \sum \sum m n K_{mn} \int \int \frac{dS_0}{r_0} \frac{dS'_0}{r'_0} \phi_{mn}(r_0) \phi_{mn}(r'_0) e^{-a_{mn}r'} \]

\[ \cdot \left[ C_{mn} \cos \omega_{mn}r' + D_{mn} \sin \omega_{mn}r' \right] \]

where

\[ K_{mn} = \frac{\rho_f^2}{4 \pi^2} K_{mn} \]

and

\[ r' = r + \frac{(r_0 - r'_0)}{a_i} \]

Equations (C28) and (C7) represent working expressions for determining anywhere in the field the desired statistical properties of the acoustic pressure resulting from the vibrations of a turbulence-excited finite plate of arbitrary boundary conditions. The mode shapes of the plate \( \phi_{mn}(r) \) in these equations implicitly represent the dependence of the acoustic field on the boundary conditions.

The method of analysis used by Young\(^{(30)}\) (the Ritz method) is used to determine the eigenfunctions and eigenvalues of vibrating rectangular plates with continuous spring-type boundary conditions. This treatment allows for various combinations of clamped and free boundaries.

The Ritz method consists of equating the maximum potential energy of the plate

\[ V = \frac{b}{2} \int \int_{\text{plate area}} \left[ \left( \frac{\partial^2 \omega}{\partial x^2} \right)^2 + \left( \frac{\partial^2 \omega}{\partial y^2} \right)^2 + 2\mu \frac{\partial^2 \omega}{\partial x \partial y} \frac{\partial^2 \omega}{\partial y^2} + 2(1 - \mu) \left( \frac{\partial^2 \omega}{\partial x^2} \right)^2 \right] \, dx \, dy \]  

\[ (C29) \]

*Note that whereas in the statement below Equation (C26a) the term \( \frac{r_0 - r'_0}{a_i} \) was implicitly included in the variables \( r_0, r'_0 \) of the function \( R_A(r_0, r'_0, r) \), in Equation (C27) the term \( \frac{r_0 - r'_0}{a_i} \) is linked with \( r \) to form \( r' \).
to the maximum kinetic energy of the plate

$$T = \omega^2 \frac{\rho h}{2} \int_\text{plate area} w^2 \, dx \, dy$$  \hspace{1cm} (C30)

to obtain an expression for the frequency of the vibrating system

$$\omega^2 = \frac{9}{\rho h} \frac{V}{\int_\text{plate} w^2 \, dx \, dy}$$  \hspace{1cm} (C31)

The natural frequencies are determined by finding expressions for \( w \) that satisfy the boundary conditions and minimize Equation (C31). The Ritz method consists of assuming the deflection \( w(x, y) \) as a linear series of "admissible" functions (see Reference 30) and adjusting the coefficients in the series so as to minimize Equation (C31).

For a rectangular plate with edges parallel to the \( x \)- and \( y \)-axes, the series approximation for the displacement function is taken in the form

$$w(x, y) = \sum_{m=1}^{p} \sum_{n=1}^{q} A_{mn} X_m(x) Y_n(y)$$  \hspace{1cm} (C32)

and substituted in Equation (C31). We get

$$\omega^2 = \frac{9}{\rho h} \frac{V}{\int_\text{plate} \sum \sum A_{mn} X_m(x) Y_n(y) \, dx \, dy}$$

or

$$V = \frac{\omega^2 \rho h}{2} \int_\text{plate} \sum \sum A_{mn} X_m(x) Y_n(y) \, dx \, dy$$

This expression is minimized by setting the partial derivative with respect to each coefficient equal to zero. This yields

$$\frac{\partial V}{\partial A_{rs}} = \frac{\omega^2 \rho h}{2} \frac{\partial}{\partial A_{rs}} \int_\text{plate} w^2 \, dx \, dy = 0$$  \hspace{1cm} (C33)
where $A_{rs}$ is any of the coefficients $A_{mn}$. Equation (C33) represents a system of linear homogeneous equations in the unknowns $A_{mn}$. The approximate natural frequencies of the plate $\omega_1, \omega_2, \ldots$ are obtained from Equation (C33) by setting the determinant of the system equal to zero.

The functions $X_m(x)$ and $Y_n(y)$ inserted in Equation (C33) are the mode shapes of a beam supported by torsional and transverse linear springs along its boundaries. The characteristics of the springs along each side are constant. These spring-type edge conditions allow the effects of edge rotational and edge translational constraints to be analyzed on a quantitative basis. Once the mode shapes are known or determined -- for the clamped-clamped plate, we use the functions given by Equation (C26) and (C26a) as the mode shapes $X(x)$ and $Y(y)$ of a beam with its ends clamped in the Ritz method -- all of the integrals in Equation (C33) can be calculated. Then as explained in Reference (30), the set of integral Equations (C33) can be reduced to a set of linear algebraic equations of the form*

$$\sum_{m=1}^{p} \sum_{n=1}^{q} \left[ c_{mn}^{rs} - \lambda \delta_{mn}^{rs} \right] A_{mn} = 0$$

(C34)

where

$$\delta_{mn}^{rs} = \begin{cases} 1 & \text{for } mn = rs \\ 0 & \text{for } mn \neq rs \end{cases}$$

$$\lambda = \omega^2 \rho h L_x L_y / b \quad (\text{proportional to } \omega^2)$$

In Equation (C34), $r$ assumes all values between 1 and $p$ and $s$ assumes all values between 1 and $q$. The eigenvalues and therefore the natural frequencies $\omega_{rs}$ are found from the condition that the determinant of the system of Equation (C34) must vanish for nontrivial solutions $A_{mn}$. Once the eigenmatrices of Equation (C34) have been determined, the mode shapes of the plate are obtained from Equation (C32).

Reference 29 compares the spectrum of the sound pressure level for a clamped-clamped plate with that of a simply supported plate. The comparison suggests that a simplified and realistic approach to the investigation of plates with nonsimple supports would be to calculate the modal frequencies considering the true (clamped-clamped) end conditions but to use the mode shapes considering the end conditions as simple supports. Comparison runs using this approach, which requires much less computation, and the exact approach (clamped-clamped

---

*The general functional form for $C_{mn}^{rs}$ is given in Reference 30.
In connection with this aspect of the problem, the following equations were developed to obtain the sound radiated from simply supported plates.

For a simply supported plate of dimensions \( L_x, L_y \) and thickness \( h \), the normalized eigenfunctions (mode shapes) and corresponding modal frequencies are \(^2\) (see footnote for paragraph preceding equation (C25))

\[
\phi_{mn} = \sin \frac{m \pi x}{L_x} \sin \frac{n \pi y}{L_y}
\]

\[
\omega_{mn}^2 = \left( \frac{b}{M} \right)^{1/2} \lambda_{mn}^2
\]

where

\[
\lambda_{mn}^2 = \left( \frac{mn}{L_x} \right)^2 + \left( \frac{nn}{L_y} \right)^2
\]

Substituting Equation (C35) in (C28), we obtain for locations in the far field through the center of the plate \( (r_0 - r_0' = 0) \); more generally \( r_0 = r_0' = r \), see Figure C1, and consequently \( r' \approx r \)

\[
\sum_{m} \sum_{n} K_{mn} e^{-a_{mn}r} \left[ C_{mn} \cos \omega_{mn}r + D_{mn} \sin \omega_{mn}r \right] \cdot
\]

\[
\int_{0}^{L_x} \sin \frac{m \pi x}{L_x} dx \int_{0}^{L_x} \sin \frac{m \pi x}{L_x} dx \int_{0}^{L_y} \sin \frac{n \pi y}{L_y} dy \int_{0}^{L_y} \sin \frac{n \pi y}{L_y} dy
\]

Now

\[
\int_{0}^{L_x} \sin \frac{m \pi x}{L_x} dx = \frac{L_x}{m \pi} [\cos m \pi - 1] = \frac{L_x}{m \pi} [-1(-1)^{m-1}]
\]

and similarly for the other three integrals. Hence, the product of the four integrals yields the term \( \frac{L_x}{m^2 \pi^2} \left[ -1(-1)^{m-1} \right]^2 \cdot \frac{L_y^2}{n^2 \pi^2} \left[ -1(-1)^{n-1} \right]^2 \) and the auto correlation of the acoustic pressure at \( r \) is

134
\[ R_p(r, r) = \sum_m \sum_n \frac{K_{mn}}{r^2} \left[ C_{mn} \cos \omega_{mn} r + D_{mn} \sin \omega_{mn} r \right]. \]

\[ e^{-a_{mn} r^2 \frac{L_x^2 + L_y^2}{2}} \frac{1}{\pi^2 n^2} \frac{1}{m^2 n^2} \frac{1}{(-1)^m - 1^2} \frac{1}{(-1)^n - 1^2} \]

The power spectrum of the pressure at \( r \) is

\[ S_p(r, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_p(r, r) e^{-i\omega r} \sum_m \sum_n \frac{K_{mn}}{m^2 n^2} \frac{1}{(-1)^m - 1^2} \frac{1}{(-1)^n - 1^2} \frac{1}{2\pi}. \]

Note that Equation (C27) and what followed held for \( r > 0 \). To obtain \( S_p \) here, we treat \( r \) in the infinite range \( (-\infty, \infty) \) hence \( r \to |r| \) above. Using Table 3.3 of Reference 31 we obtain directly the value of the integral as*

\[ = \left[ \frac{a_{mn} C_{mn} + d_{mn}(\omega + \omega_{mn})}{a_{mn}^2 + (\omega + \omega_{mn})^2} + \frac{a_{mn} C_{mn} - d_{mn}(\omega - \omega_{mn})}{a_{mn}^2 + (\omega - \omega_{mn})^2} \right] \]

and from Equations (C19) and (C27)

\[ a_{mn} = B \omega_{mn} \]

\[ C_{mn} = \omega_{mn}^4 - 6\omega_{mn}^2 a_{mn}^2 + a_{mn}^4 = \omega_{mn}^4 - 6B\omega_{mn}^4 + B^4 \omega_{mn}^4 = \omega_{mn}^4 (1 - 6B^2 + B^4) \]

\[ D_{mn} = 4a_{mn} \omega_{mn} (a_{mn}^2 - \omega_{mn}^2) = 4B \omega_{mn}^2 (B^2 \omega_{mn}^2 - \omega_{mn}^2) \]

Substituting the values of \( a_{mn}, C_{mn}, \) and \( D_{mn} \) in the equation directly above and dropping terms of higher order in \( B \), i.e., \( 0(B^5) \) and \( 0(B^3) \) reduces this equation to

---

*We take one-half the value given in this table since we use \( -\infty \leq \omega \leq \infty \) here whereas Reference 31 gives the one-sided power spectral density function.
Using this value and the value of $K_{mn}$, $K_{mn}$ given by Equations (C28) and (C27), respectively, we get

$$S_p(r, \omega) = \frac{1}{4\pi^2} \left( \frac{L_x L_y}{r^2} \frac{\rho^2 A \theta^2}{M^2} \sum_{m} \sum_{n} \frac{[1 - (-1)^m] [1 - (-1)^n]^2}{1 + \omega_m^2 \theta^2} \right)$$

In accordance with Reference 2, we take

$$A = \frac{2\pi}{\kappa^2} = \frac{2\pi}{(3/\delta^*)^2} = \frac{\pi (\delta^*)^2}{2}, \quad \theta = \frac{30 \delta^*}{U_0}, \quad \rho^2 = (3 \times 10^{-3})^2 \rho_i^2 U_0^4$$

Substituting these values in the equation for $S_p(r, \omega)$, the nondimensional power spectrum at $r$ is then represented by

$$\frac{S_p(r, \omega)}{\rho_i^2 U_0^2 \delta^*} = 33.75 \times 10^{-6} \left( \frac{L_x L_y}{r^2} \right) \left( \frac{\rho \delta^*}{M} \right)^2 \sum_{m} \sum_{n} [1 - (-1)^m] [1 - (-1)^n]^2$$

$$= K$$

$$\cdot \frac{1}{(1 + \omega_m^2 \theta^2) m^2 n^2} \left[ \frac{4 \omega_m - 3}{B^2 + \left( 1 - \frac{\omega_m}{\omega_m} \right)^2} \right. - \left. \frac{3 + 4 \omega_m}{B^2 + \left( 1 + \frac{\omega_m}{\omega_m} \right)^2} \right]$$

*(Note that Equation (C38) does not agree with Equation (6.38) of Reference 26 which appears to contain a typographical error.)*
or

\[
\tilde{S}_p(r, \omega) = \sum_m \sum_n K^r(\omega_{mn}) \Phi\left(\frac{\omega}{\omega_{mn}}\right)
\]  

(C30)

Thus \(\tilde{S}_p(r, \omega)\) is the product of a modal amplitude factor \(K^r(\omega_{mn})\) and a spectrum shape factor \(\Phi\left(\frac{\omega}{\omega_{mn}}\right)\).

**APPENDIX C2 – METHOD FOR DETERMINING INPUT DATA**

The method for determining input data for the Electric Boat Computer Program is the same as the Dyer method (Appendix A), and the system of units is consistent with Dyer.
APPENDIX C3 – PROGRAM IDENTIFICATION

This program computes the space-time cross correlation and cross-spectral density of the acoustic pressures resulting from the vibration of a turbulence-excited finite plate of arbitrary boundary conditions.
APPENDIX C

TABLE 4

Identification for Electric Boat Program – IZZO

This table includes input and output data identification, flow chart, order of input data, and computer running times. Computer program listings are given in Table 5.

Table 4A: Input Data
Table 4B: Output Data
Table 4C: Flow Chart (Electric Boat) for Turbulence – Excited Clamped and Simply Supported Plate Problem
Table 4D: Input Formats
<table>
<thead>
<tr>
<th>Input Data</th>
<th>Description</th>
<th>Type</th>
<th>Program Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>Order of m-mode numbers</td>
<td>Integer</td>
<td>MM</td>
</tr>
<tr>
<td>N</td>
<td>Order of n-mode numbers</td>
<td>Integer</td>
<td>NN</td>
</tr>
</tbody>
</table>
| IM         | 1: m's and n's are read as a vector 
            | 2: MM: m's and n's are read as a 
            |     | matrix such as A(1,1), A(1,2) 
            |     | A(1,3), A(2,1), A(2,2), A(3,3) | Integer | IM |
| IN         | MM: integer | Integer | IN |
| NXYZ       | Number of cases in autocorrelation | Integer | NXYZ |
| KXYZ       | Number of cases in CROSSI-correlation | Integer | KXYZ |
| LXYZ       | Number of cases in CROSS2-correlation | Integer | LXYZ |
| NAUTO      | 1: Access to autocorrelation 
            | 0: No | Integer | NAUTO |
| NCROSS     | 1: Access to CROSSI-correlation 
            | 0: No | Integer | NCROSS |
| NCROSST    | 1: Access to CROSS2-correlation 
            | 0: No | Integer | NCROSST |
| XLI        | Upper-first-integration limit 
            | (plate dimension x) | Decimal | XLI |
| XLO        | Lower-first-integration limit | Decimal | XLO |
| YLI        | Upper-second-integration limit 
            | (plate dimension y) | Decimal | YLI |
| YLO        | Lower-second-integration limit | Decimal | YLO |
| AI         | Speed of sound in fluid | Decimal | AI |
| B          | Damping coefficients | Decimal | B |
| C          | Constants 
            | 1: Simply supported plate 
            | (i.e., clamped) case 
            | 2: Simply supported plate 
            | (i.e., clamped) case 
            | 3: Simply supported plate 
            | (i.e., clamped) case 
        | $\theta$ | Decay constant $= \theta / (2\pi)$ | Decimal | THETA |
| $p^2$      | $(\omega^2 / \rho)^2$ | Decimal | P2 |
| $\omega_{m,n}$ | Frequencies for (m,n) modes (radians) | Decimal | \(\omega_{m,n}\) |
| MAM        | M-mode shape numbers | Integer | MAM |
| NAN        | N-mode shape numbers (MAX-50) | Integer | NAN |
| ALAM       | Normalized eigenfunction 
            | parameters; used only 
            | in the general 
            | case; 
            | for m-mode 
            | (i.e., clamped) case; 
            | for n-mode 
            | Same as above, 
            | only for n-mode | Decimal | ALAM |
| AMM        | Decimal | AMM |
| BMM        | Decimal | BMM |
| CMM        | Decimal | CMM |
| DMM        | Decimal | DMM |
| ALNN       | Decimal | ALNN |
| ANN        | Decimal | ANN |
| BNN        | Decimal | BNN |
| CNN        | Decimal | CNN |
| DNN        | Decimal | DNN |

*For x = lb, hence mass = lb; $p$ is length = ft and time = sec. 
*acceleration = ft/sec²

This system of units is consistent with that of Dye.
### TABLE 4A (Continued)

<table>
<thead>
<tr>
<th>Input Data</th>
<th>Description</th>
<th>Type</th>
<th>Program Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input II</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Type</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Description</strong></td>
<td><strong>for Subroutine Autocorrelation</strong> (Time and Frequency Dependent)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x$</td>
<td>Coordinates of point for correlation</td>
<td>Decimal</td>
<td>X</td>
</tr>
<tr>
<td>$y$</td>
<td></td>
<td>Decimal</td>
<td>Y</td>
</tr>
<tr>
<td>$z$</td>
<td>Measurement</td>
<td>Decimal</td>
<td>Z</td>
</tr>
<tr>
<td>$r_0$</td>
<td>Initial value of $r$</td>
<td>Decimal</td>
<td>TAU</td>
</tr>
<tr>
<td>$\Delta r$</td>
<td>Increment of $r$</td>
<td>Decimal</td>
<td>DTAU</td>
</tr>
<tr>
<td>NTAU</td>
<td>Number of $r$'s</td>
<td>Integer</td>
<td>NTAU</td>
</tr>
<tr>
<td>EPS</td>
<td>Convergence constant (test value)</td>
<td>Decimal</td>
<td>EPS</td>
</tr>
<tr>
<td>NMAX</td>
<td>Maximum number of iterations</td>
<td>Integer</td>
<td>NMAX</td>
</tr>
<tr>
<td>NWA</td>
<td>Number of specified frequencies</td>
<td>Integer</td>
<td>NWA</td>
</tr>
<tr>
<td>WA</td>
<td>Specified frequencies (radians) chosen by user according to his boundary specifications</td>
<td>Decimal</td>
<td>WA</td>
</tr>
</tbody>
</table>

**Comment**: There are NXYZ sets of data cards for this subroutine.

| **Description for Subroutine Cross Correlation** (Space Dependent) | | |
| $x$ | Coordinates of fixed point | Decimal | XX |
| $y$ | | Decimal | YY |
| $z$ | | Decimal | ZZ |
| MXP | Number of variable points | Integer | MXP |
| MTAU | Number of $r$'s | Integer | MTAU |
| EPS | Convergence constant | Decimal | EPS |
| NMAX | Maximum number of iterations | Integer | NMAX |
| $r$ | Value of $r$ | Decimal | TAU |
| $x'$ | Coordinates of variable points | Decimal | XP(I) |
| $y'$ | | Decimal | YP(I) |
| $z'$ | | Decimal | ZP(I) |
| NOR | $= 0$ | Integer | NOR |

**Comment**: There are KXYZ sets of input cards for this subroutine.

| **Description for CROSS 2 Correlation** (Time and Frequency Dependent) | | |
| $x$ | Coordinate of fixed point | Decimal | XX |
| $y$ | | Decimal | YY |
| $z$ | | Decimal | ZZ |
| MXP | Number of variable points | Integer | MXP |
| TAU | Initial value of $r$ | Decimal | TAU |
| DTAU | Increment of $\Delta r$ | Decimal | DTAU |
| NTAU | Number of $r$'s | Integer | NTAU |
| EPS | Convergence constant | Decimal | EPS |
| NMAX | Maximum number of iterations | Integer | NMAX |
| NWA | Number of frequencies | Integer | NWA |
| $x'$ | Coordinates of variable point | Decimal | XP(I) |
| $y'$ | | Decimal | YP(I) |
| $z'$ | | Decimal | ZP(I) |
| NOR | $= 0$ | Integer | NOR |
| WA | Different frequencies (radians) chosen by user | | |

**Comment**: There must be LXYZ sets of input cards.

---

141
### TABLE 4B

#### Output Data

<table>
<thead>
<tr>
<th>Description</th>
<th>Program Label</th>
<th>Output Label</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Description for AUTO 1: Autocorrelation with Two Options</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1. NTAU - no. points of time; 2. NWS - no. frequencies)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of time increments</td>
<td>K</td>
<td>NTAU</td>
</tr>
<tr>
<td>Point of time at which autocorrelation taken</td>
<td>TAU</td>
<td></td>
</tr>
<tr>
<td>Normalized autocorrelation of acoustic pressure over time, i.e., RP(x1 - x2, 0) / RP(x1 - x2, 0)</td>
<td>PRP(K)</td>
<td></td>
</tr>
<tr>
<td>Autocorrelation normalized by rms pressure</td>
<td>PRP(K)</td>
<td></td>
</tr>
<tr>
<td>Normalized factor RP(1) for RPBAR(K), i.e., RP(x1 - x2, 0) or R11(0,0)</td>
<td>RP(1)</td>
<td>NORM. FACTOR</td>
</tr>
</tbody>
</table>

**Comment:** Above not printed out if NTAU = 0

| Number of frequency                                                        | K             | K                   |
| Specified frequencies                                                      | WA(K)         | FREQ                |
| WA(K)/°/s                                                                 | RAD           | RAD/SEC             |
| Cross spectral density                                                     | F(K)          | CROSS SPEC. DENS.   |
| 10 LOG(F(K) + 127.6)                                                      | PHI           | DB(RE0.0002)        |

**Comment:** Above not printed out if NWA = 0

| Indicates time                                                             | I             | I                   |
| Point of time at which cross-correlation computed                         | TAU(I)        | TAU                 |
| Indicates space                                                           | J             | J                   |
| Space coordinates                                                         | XP(J)         | XP(J)               |
| YP(J)                                                                     | YP(J)         |                     |
| ZP(J)                                                                     | ZP(J)         |                     |
| Normalized cross-correlation                                               | CRPBAR(I,J)   | CRPBAR(I,J)         |
| Normalization factor -                                                      | C(NORM)       | C(NORM)             |

**Description for CROSS1 (for Various Times in Space)**

| Description for CROSS2 (Time Variable)                                     |               |                     |
| Number of time point                                                       | K             | NTAU                |
| Point of time                                                              | TAU           |                     |
| Normalized correlation of acoustic pressure, by                            | PRP(K)        | NORM. CORR. OF     |
| RP(x1 - x2, 0) or R11(0,0)                                                 | PRP(K)        | ACC. PRESS.         |
| Correlation normalized by rms pressure                                     | PRP(K)        |                     |
| Normalization factor RP(x1 - x2, 0)                                        | RP(1)         | NORM. FACTOR        |
| Number of frequency                                                        | K             | K                   |
| Specified frequency                                                       | WA(K)         | FREQ                |
| Frequency /°/s                                                            | RAD           | RAD/SEC             |
| Cross spectral density                                                    | F(K)          | CROSS SPEC. DENS.   |
| 10 LOG(F(K) + 127.6)                                                      | PHI           | DB(RE0.0002)        |
Interpretation of Data Output and Computer Running Times

The following information is useful in interpreting the program.

The program generally yields results normalized by \( p^2 \) or by the correlation of two points at \( r = 0 \).

The Electric Boat program consists of two parts which represent the simple case and the general case. The simple case, Equation (C(35)), uses only simply supported boundaries for the plate modal function although frequencies for clamped boundaries may be used. This procedure yielded Figure 12 (see page 154). The general case uses mode shapes represented by Equation (C(26)). In addition the program requires either of the following values for \( C \)

\[
C = \frac{P_0^2 Ap^2}{2\pi^2 M^2 L_x L_y} \quad \text{(simply supported boundaries)}
\]

\[
C = \frac{P_0^2 Ap^2}{8\pi^2 M^2} \quad \text{(clamped-clamped boundaries)}
\]

Figure 12 (Figure 17 in Reference 26), is normalized by \( R_{11}(0,0) \), that is, the auto-correlation function of the fixed point \( x_1 \) at \( r = 0 \). Therefore, the nonnormalized correlation for each \( \Delta x, r \) desired was abstracted from the program and divided by \( R_{11}(0,0) \) to yield this curve. Similarly, calculations were necessary to obtain the data in the form used in Figure 13 (Figure 15 in Reference 26); see page 207 of this report. The program yields a normalized answer in CROSS2, which is not in suitable form for representing the curve as shown. Therefore, the normalized program result is manually multiplied by the NORM FACTOR to give a non-normalized quantity \( R_{12}(\Delta X, r) \); see equation below. More precisely, this is accomplished by first multiplying the number in the upper right corner labeled "NORM FACTOR -". by the corresponding quantity in the column labeled "NORM. CORR. OF. ACC. PRESS.". Only \( r = 0 \) was used for this curve so that the corresponding quantity would appear in the line for TAU = 0. The second step is to get the normalization factors from AUTO 1, which must have as many cases as there are variable points in CROSS2. \( R_{11}(0,0) \) refers to \( R(x_1 - x_1, r = 0) \) for all cases where 11 is a fixed reference point in the longitudinal direction, but \( R_{22}(0,0) \) refers to \( R(x_2 - x_2, r = 0) \) where 22 is any other point in the longitudinal direction. The corresponding cross points are denoted by 12 in the correlation function, i.e., \( R_{12}(\Delta x, r) \). The quantity to be used from AUTO 1 is labeled "NORM.FACTOR -".

The total function plotted then becomes

\[
\sqrt{\text{NORM.FACTOR (AUTO1 with } X_1) \times \text{NORM.FACTOR (AUTO1 with } X_2)}.
\]
The major subroutines results and running times on the IBM 7090 are:

<table>
<thead>
<tr>
<th>AUTO 1</th>
<th>CROSS 1</th>
<th>CROSS 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Autocorrelation</td>
<td>1. Cross correlation keeping time constant</td>
<td>1. Cross correlation, varying time and/or space</td>
</tr>
<tr>
<td>Approximately 3 min per case (i.e., point)</td>
<td>Approximately 3 min for 1 point, 1 time (see CROSS2)</td>
<td>Approximately 1.5 min per variable point over 120 time increments</td>
</tr>
</tbody>
</table>

The printout yields intermediate results, such as integration sums and nonnormalized, noncumulative results for each mode. For example, 37 modes would imply 37 sets of these results, followed by the normalized answers for each of the major subroutines. For the autospectrum final results, the labels FREQ. and RAD/SEC should be interchanged.
TABLE 4C

Flow-Chart (Electric Boat) Turbulence-Excited, Clamped and Simply Supported Plate Problem

START

READ MODAL DATA, FLUID DATA, INTEGRATION LIMITS

AUT01 - DINT - FFF

1

1 - CROSS1 - DINT - FFF

2

2 - CROSS2 - DINT - FFF

STOP

145
TABLE 4C (Continued)

AUTO1

PLATE DATA: SPECIFIED FREQUENCIES

COMPUTE VARIABLES DEPENDENT ON $\omega_{n}$

DINT $\int_{0}^{1} x(t) y(t) dt$

COMPUTE SPECTRAL DENSITY $SP$

$\phi = 10 \log(FIT) + 127.6$

WRITE $R$, FIT, WALK

RETURN

NTAU: 0

NWA: 0

RETURN

COMPUTE MODAL SPECTRAL DENSITY FIT
WRITE ANSWERS

COMPUTE CORRELATION OF ACOUSTIC PRESSURE RP
WRITE ANSWERS

NORMALIZE RP
WRITE ANSWERS

3
TABLE 4C (Continued)

PLATE DATA COORDINATES SPECIFIED FREQUENCIES

COMPUTE VARIABLES DEPENDENT ON \( \omega_{m,n} \)

COMPUTE SPECTRAL DENSITY \( SP \)

COMPUTE MODAL 'ISPECTRAL DENSITY COMPUTE TOTAL IFIT (ca-DEPENDENT)

COMPUTE TOTAL S.D. \( \text{PHI} = 10 \log(\text{FIT}) + 127.6 \) WRITE ANSWERE

RETURN
TABLE 4C (Continued)

DINT

SET UP GRID FOR $dxdy$

$X(a)Y(y)$ EXPSIN

7

COMPUTE SUM$_2$ - $dxdy$ FFF

$\frac{SUM1 - SUM2}{SUM2}$ EPS

NK : NMAX

NK = NK + 1

8

WRITE NK, SUM1, SUM2

RETURN

149
TABLE 4D
Input Format for Main Program, General Case

Columns 1–40, below, contain respectively, in 4-column blocks: MM; NN; IM; IN; XXYZ; KXYZ; LXYZ;
NAUTO; NCROSS; NCRST

IMx IN sets of 6-column blocks are needed for the W(IM,IN) array: w(1,1); w(1,2); ...; w(1,IN); w(2,1); ...
, w(2,IN); ... , w(IM,IN)

W array 6 12 18 24 30 36 40 48 54 60 66 72 80

(MM + NN)/20 cards are needed to complete arrays MAM(MM) and NAN(NN): MAM(1); MAM(2); ...;
MAM(MM); NAN(1); NAN(2); ... ; NAN(NN)

MM cards are needed to complete the following arrays: (used in general case only)

NN cards are needed to complete the following arrays: (used in general case only)
TABLE 4D (Continued)

INPUT FORMAT FOR SUBROUTINE AUTO

<table>
<thead>
<tr>
<th>X</th>
<th>12</th>
<th>Y</th>
<th>24</th>
<th>Z</th>
<th>36</th>
<th>TAU</th>
<th>48</th>
<th>DTAU</th>
<th>NTAU</th>
<th>EPS</th>
<th>NMAX</th>
<th>NWA</th>
</tr>
</thead>
</table>

NWA/12 cards are needed to complete the WA(NWA) array:

WA(1)  WA(2)  ...  ...  WA(12)

INPUT FORMAT FOR SUBROUTINE CROSS

<table>
<thead>
<tr>
<th>XX</th>
<th>12</th>
<th>YY</th>
<th>24</th>
<th>ZZ</th>
<th>36</th>
<th>MXP</th>
<th>42</th>
<th>48</th>
<th>MTAU</th>
<th>EPS</th>
<th>NMAX</th>
<th>66</th>
</tr>
</thead>
</table>

MTAU/6 cards are needed to complete the TAU(MTAU) array:

TAU(1)  TAU(2)  ...  ...  ...  ...  TAU(6)

MXP/2 cards are needed to complete the following arrays:

XP(1)  XP(2)  ...  ...  ZP(1)  ZP(2)  ...  ...
### TABLE 4D (Continued)

**INPUT FORMAT FOR SUBROUTINE CROSS2**

<table>
<thead>
<tr>
<th>XX</th>
<th>12</th>
<th>YY</th>
<th>24</th>
<th>ZZ</th>
<th>36</th>
<th>39</th>
<th>TAU</th>
<th>51</th>
<th>DTAU</th>
<th>63</th>
<th>66</th>
<th>EPS</th>
<th>72</th>
<th>75</th>
<th>78</th>
<th>80</th>
</tr>
</thead>
</table>

MXP/2 cards are needed to complete the following arrays:

<table>
<thead>
<tr>
<th>XP(1)</th>
<th>12</th>
<th>YP(1)</th>
<th>24</th>
<th>ZP(1)</th>
<th>36</th>
<th>40</th>
<th>XP(2)</th>
<th>52</th>
<th>YP(2)</th>
<th>64</th>
<th>ZP(2)</th>
<th>76</th>
<th>80</th>
</tr>
</thead>
</table>

NWA/12 cards are needed to complete the WA(NWA) array:

<table>
<thead>
<tr>
<th>WA(1)</th>
<th>WA(2)</th>
<th>...</th>
<th>...</th>
<th>...</th>
<th>...</th>
<th>...</th>
<th>...</th>
<th>...</th>
<th>...</th>
<th>...</th>
<th>WA(12)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>12</td>
<td>18</td>
<td>24</td>
<td>30</td>
<td>36</td>
<td>42</td>
<td>48</td>
<td>54</td>
<td>50</td>
<td>66</td>
<td>72</td>
</tr>
</tbody>
</table>
APPENDIX C4 – TEST RUNS

Test runs for the power spectrum, longitudinal correlation function, and longitudinal space-time correlation function of the acoustic pressures are plotted in Figures 12, 13, and 14. The computer programs used to obtain these results have been given in Table 4, and the computer listings are presented in Table 5.
Figure 12a – Clamped-Clamped Steel Plate

This subfigure is based on the use of frequencies obtained for a clamped-clamped plate but modes obtained for a simply supported plate. (see statement following Equation (C34))

Figure 12b – Simply Supported Steel Plate

This subfigure is based on the use of frequencies and mode shapes obtained for a simply supported plate.

Figure 12 – Longitudinal Space-Time Correlation Function for a 2-Foot × 2.33-Foot × 3/8-Inch Steel Plate
Figure 13 - Longitudinal Correlation Function for a 2-Foot × 2.33-Foot × 3/8-Inch Steel Plate

Comparison of the results obtained using the (1,1) mode only and using 37 modes shows the error involved, over the longitudinal range, in neglecting the contributions of the higher modes to the correlation.
TABLE 5

Computer Listings for Electric Boat Program – Izzo

Table 5A – Simply Supported Boundaries

$18FTC TURAD2 .
C
C 1000 FORMAT(12A6)
  1001 FORMAT(10I4)
  1002 FORMAT(4E12.6/5E15.8)
  1003 FORMAT(12F6+0)
  1004 FORMAT(20I4)
  1005 FORMAT(5E15.8)
C
  2000 FORMAT(1H1+10X*6HF,Pa$*15X*12A6/22X*12A6///)
  2001 FORMAT (1H0+29X*45HORDER OF M (MODE NUMBERS) ***************** MM =I 0100
  16/30X*45HORDER OF N (MODE NUMBERS) *************** NN =I6/30X 0110
  245HNUMBER OF CASES (XYZ) IN AUTO CORR*** XXYYZ =I6/30X*45HNUMBER
  3OF CASES (XYZ) IN CROSS CORR** XXYYZ =I6 / 30X*45HAUTO CO 0120
  4RELA TION CONSTANT **************** NAUTO =I6/30X*45HCROSS CORRELATION
  5CONSTANT ************* NCROSS =I6/30X*45HLIMITS OF THE 1-ST INTEGRAL
  6 ************** XLI =*$F11.4/30X*45H 0170
  7 XLO =$F11.4/30X*45HLIMITS OF THE 2-ND INTEGRAL ************ Y 0180
  8LI =$F11.4/30X*45H 0190
  9LI =$F11.4/30X*45HSPEED OF SOUND IN WATER *************** AI =$F13.6/30X
  A*45HDAMPING CONSTANT ****************** B =$F13.6/30X*45HC
  BCONSTANT ********************** C =$F13.6/30X*45HTEMPORAL D 0210
  CDCAY FACTOR OF TURBULENCE THE R=THETAFE *$F11.4/30X*45HR.M.S.A PRESSURE *P 0220
  0D ********************** P2 =$F13.6//30X*21101
  2002 FORMAT(1H0+40X*35HUNDAMPED NATURAL FREQUENCIES W(MN)//* (10X*10F8. 0
  X1))
  2003 FORMAT(1H0+50X*12HMODE NUMBERS/(40X*10I5))
  2005 FORMAT(1H0+40X*35HNORMALIZED EIGENFUNCTION PARAMETERS /(15X*5E18. 18))
  2006 FORMAT(1H0+29X*45HNUMBER OF CASES IN CROSS CORR* (TIME)* LXYZ*,
  116/30X*45HCROSS CORR* (TIME) CONSTANT ********** NCROSS*I6//
  230X*2110)
C
  0300
  0310
  0320
  0330
  0340
  0350
  0360

157
TABLE 5A (Continued)

EQUIVALENCE (A(1),MM) (A(2),NN) (A(3),M) (A(4),IM) (A(5),NXYZ) ...
1(A(6),XYZ) (A(7),NAUTO) (A(8),NCROSS) (A(9),M) (A(10),N) ...
2(A(11),XYZ) (A(12),NCROSS).

EQUIVALENCE (A(21),XL1) (A(22),XLO) (A(23),YL1) (A(24),YLO) ...
1(A(25),A1) (A(26),B1) (A(27),C1) (A(28),THETA) (A(29),P2) ...
2(A(31),A1) (A(32),Y1) (A(33),Z1) (A(34),EPI) (A(35),NMAX) ...
3(A(36),WMM) (A(37),AMN) (A(38),SUM1) (A(39),SUM2) ...

EQUIVALENCE (A(101),MAM) (A(151),NAN) (A(1000),M) ...

DIMENSION TITLE(24),W(20),50,MAM(50),NAN(50)

REAL KMN

READ AND PRINT TITLE

READ (5*1000) (TITLE(I),I=1,24)
WRITE(6,2000) (TITLE(I),I=1,24)

READ AND PRINT GENERAL INPUT CONSTANTS

READ (5*1001) MM,NN,IM,IN,NXYZ,XYZ,NAUTO,NCROSS,NCROST
READ (5*1002) XL1,XLO,YL1,YLO,A1+B+C+THETA+P2
READ (5*1003) ((W(I,J),I=1,IN),J=1,IM)
READ (5*1004) (MAM(I),I=1,M),(NAN(I),I=1,NN)

WRITE(6,2001) MM,NN,NXYZ,XYZ,NAUTO,NCROSS,XYL,XYLO,A1+B+...
WRITE(6,2006) LXYZ,NCROSS,AMN
WRITE(6,2002) ((W(I,J),I=1,IN),J=1,IN)
WRITE(6,2003) (MAM(I),I=1,MM),NAN(I),I=1,NN)

PI=3.14159

158
TABLE 5A (Continued)

C AUTO CORRELATION
C IF (NAUTO.EQ.1) CALL AUTO1
C CROSS CORRELATION
C IF (NCROSS.EQ.1) CALL CROSS1
C CROSS CORRELATION (TIME)
C IF (NCROSS.T.EQ.1) CALL CROSS2
C STOP
END
$IBFTC XAUTO1
SUBROUTINE AUTO1
C AUTO CORRELATION) X*X, Y*Y, Z*Z
C
$000 FORMAT(5E12.6,E14.8,E14.8)
$001 FORMAT(12F6.0)
$002 FORMAT(1H1,50X,15HAUTO CORRELATION///30X,1HX,30X,1HY,30X,1HZ//
122X,E14.8,17X,E14.8,17X,E14.8//10X,5HTAU =E14.8,5X,6HDTAU =E14.8
2S5X,5H=EPS =E14.8,5X,5H=MAX =E14.8,5X,5H=M =E14.8,5X,5H=I)
$003 FORMAT(1H1,50X,15HPNT_CNTL,12X,1HX,12X,1HY,12X,1HZ,12X,1HM,4X,1HN,5X,1H)
$004 FORMAT(1H1,50X,15HFFREQ =S,1H,SPECTRAL DENSITY =X,15.2/)
$005 FORMAT(1H1,50X,15HFM =F9.4,1H,15HDB(RE =0.002) ==//
15X,15X,15X,15X,9HRPMN(TAU) ==//)

150
TABLE 5A (Continued)

4003 FORMAT(1H,30X110X,E16.8) 1110
4004 FORMAT(1H,50X12HINTEGRATIONS // 13X9HINT. NUM*,5X8H MESH /* 1120
15X6HM N,10X4HSUM2,16X4HSUM1) 1130
4005 FORMAT(1H1,30X,44H NORM, CORRELATION OF ACOUSTIC PRESSURE /// 1140
110X, 4HINTAU,10X, 3HTAU+10X, 25H NORM, CORR. OF ACC. PRES**10X, 1150
28HRP(1),P2,12X14H NORM, FACTOR =E14.8/) 1160
4006 FORMAT(1H* 8X14X8AF8.6,10X1E16.8) 1170
4010 FORMAT(1H* 7E16.8) 1180
4011 FORMAT(1H,30X25H AUTO SPECTRAL DENSITY // 12X16H 10X, 4HFREQ, 1190
110X, 7HRAD/SEC+10X, 17H CROSS SPEC+ DENS.+10X, 110DB(RE0002)///) 1200
4012 FORMAT(1H 8X114, 8X16F6.0,8X16F9.4,42(10X1E14,8)) 1210
4013 FORMAT(1H* 30X, 18H CHOSEN FREQUENCIES // 10X, 10F10.4) 1220
4014 FORMAT(1H* 30X, 1HK* 10X, 4HFREQ, 10X, 7HRAD/SEC+10X, 17H MODAL SPEC+ DENS. 1230
15X, 110DB(REO002) ///) 1240
4015 FORMAT(1H* 21X110X8X,0+8X,94+2(8XE16.8)) 1250
4016 FORMAT(1H* 21X110X8X,0+8X,94+2(8XE16.8)) 1260
4017 FORMAT(1H* 21X110X8X,0+8X,94+2(8XE16.8)) 1270
4018 FORMAT(1H* 21X110X8X,0+8X,94+2(8XE16.8)) 1280
4019 COMMON /AAA/A(4000) 1290
4020 EQUIVALENCE (A(1),MM),(A(23),NN),(A(31),IM),(A(4),IN),(A(5),NXYZ) 1300
4021 1(A(6),XXYX),(A(7),NAUTO),(A(8),NCROSS),(A(9),M1),(A(10),N) 1310
4022 EQUIVALENCE (A(21),X1),(A(22),Y1),(A(23),Y2),(A(24),Y0),(A(25),A1),(A(26),B1),(A(27),C1),(A(28),THETA),(A(29),P2),(A(30),PI) 1320
4023 2,(A(31),X2),(A(32),Y2),(A(33),Z2),(A(34),EP5),(A(35),NSUM2),(A(36),WMN),(A(37),ANM),(A(38),SUM1),(A(39),SUM2) 1330
4024 4,(A(40),JAI), (A(41),JB) 1340
4025 EQUIVALENCE (A(101),NAM),(A(151),NAN),(A(1001),W) 1350
4026 DIMENSION MAM(50),NAN(50),W(2050),RP(110),RPBAR(110),RP(110), 1360
1WA(110),W(110) 1370
4027 1W 1400
4028 REAL KMN 1410
4029 CONVT = 60000. 1420
4030 DO 5 II=1,NXYZ 1430
C 1440
C 1450
C 1460
C 1470
TABLE 5A (Continued)

```fortran
C     READ (5,3000) XsYsZsTAUsDTAUuNTAUuEPSuNMAtuNWA 1480
C     WRITE(6,4000) XsYsZsTAUsDTAUuNTAUuEPSuNMAtuNWA 1490
C     IF (NWA) 2*1+2 1500
C     2 READ (5,3001) (WA(K),K=1,NWA) 1510
C     WRITE(6,4013) (WA(K),K=1,NWA) 1520
C     1 ATAU = TAU 1530
C     DO 6 IA=1,NTAU 1540
  6 RP(IA) = 0. 1550
C     7 FI(IA) = 0. 1560
C     DO 10 IM=1,IMAX 1570
      JA = J 1580
      JB = J 1590
      IF (IM.EQ.MM) JA=I 1600
      M=M+JA(I) 1610
      N = N+JA(I) 1620
C     WMN = W(I,J) 1630
C     KMNN = C*THETA / (B**(1.0+W(I,J)**2*THETA**2)) 1640
     AMNN = B*W(I,J) 1650
     CMNN = W(I,J)**2*BB**2 + BB**4) 1660
     DMNN = n-B*W(I,J)**(B**2-1) 1670
C     WBAR1 = B**2 + 2.0) / (B*W(I,J)*B**2 + 4.0*(B**2 + 1)) 1680
     WBAR2 = B*WBAR1 / (B**2 + 2) 1690
     CDBAR = KMNN*W(I,J)**P2 1700
```
TABLE 5A (Continued)

<table>
<thead>
<tr>
<th>Year</th>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1850</td>
<td>C</td>
<td>WRITE(6<em>4010) KMN</em>AMN<em>CHN</em>DMN<em>WBAR1</em>WBAR2*CBAR</td>
</tr>
<tr>
<td>1860</td>
<td>C</td>
<td>WRITE(6*4060)</td>
</tr>
<tr>
<td>1870</td>
<td>C</td>
<td>PHASE=1.5708</td>
</tr>
<tr>
<td>1880</td>
<td>C</td>
<td>KKK=2</td>
</tr>
<tr>
<td>1890</td>
<td>C</td>
<td>CALL DINT (AMN<em>PHASE</em>KKK)</td>
</tr>
<tr>
<td>1900</td>
<td>C</td>
<td>CAP2*SUM2</td>
</tr>
<tr>
<td>1910</td>
<td>C</td>
<td>KKK=1</td>
</tr>
<tr>
<td>1920</td>
<td>C</td>
<td>AMN=-AMN</td>
</tr>
<tr>
<td>1930</td>
<td>C</td>
<td>CALL DINT (AMN<em>PHASE</em>KKK)</td>
</tr>
<tr>
<td>1940</td>
<td>C</td>
<td>CAP1*SUM2</td>
</tr>
<tr>
<td>1950</td>
<td>C</td>
<td>KKK=5</td>
</tr>
<tr>
<td>1960</td>
<td>C</td>
<td>PHASE=0.0</td>
</tr>
<tr>
<td>1970</td>
<td>C</td>
<td>CALL DINT (AMN<em>PHASE</em>KKK)</td>
</tr>
<tr>
<td>1980</td>
<td>C</td>
<td>CAP5*SUM2</td>
</tr>
<tr>
<td>1990</td>
<td>C</td>
<td>KKK=4</td>
</tr>
<tr>
<td>2000</td>
<td>C</td>
<td>AMN=-AMN</td>
</tr>
<tr>
<td>2010</td>
<td>C</td>
<td>CALL DINT (AMN<em>PHASE</em>KKK)</td>
</tr>
<tr>
<td>2020</td>
<td>C</td>
<td>CAP4*SUM2</td>
</tr>
<tr>
<td>2030</td>
<td>C</td>
<td>SP = CBAR* (WBAR1<em>CAP1</em>CAP2 + WBAR2<em>CAP1</em>CAP4 - WBAR2<em>CAP5</em>CAP2 + WBAR1<em>CAP5</em>CAP4)</td>
</tr>
<tr>
<td>2040</td>
<td>C</td>
<td>IF (SP) 11<em>11</em>12</td>
</tr>
<tr>
<td>2050</td>
<td>C</td>
<td>PHI = 0.0</td>
</tr>
<tr>
<td>2060</td>
<td>C</td>
<td>GO TO 13</td>
</tr>
<tr>
<td>2070</td>
<td>C</td>
<td>PHI = 10* ALOG10(SP) + 127*6</td>
</tr>
<tr>
<td>2080</td>
<td>C</td>
<td>WRITE(6<em>4002) I1</em>J1<em>J2</em>S1<em>NTAU</em>SP*PHI</td>
</tr>
<tr>
<td>2090</td>
<td>C</td>
<td>IF (NTAU) 22<em>21</em>22</td>
</tr>
<tr>
<td>2100</td>
<td>C</td>
<td>JAU = JAU</td>
</tr>
<tr>
<td>2110</td>
<td>C</td>
<td>DO 15 K=1*NTAU</td>
</tr>
<tr>
<td>2120</td>
<td>C</td>
<td>END</td>
</tr>
<tr>
<td>2130</td>
<td>C</td>
<td>END</td>
</tr>
<tr>
<td>2140</td>
<td>C</td>
<td>END</td>
</tr>
<tr>
<td>2150</td>
<td>C</td>
<td>END</td>
</tr>
<tr>
<td>2160</td>
<td>C</td>
<td>END</td>
</tr>
<tr>
<td>2170</td>
<td>C</td>
<td>END</td>
</tr>
<tr>
<td>2180</td>
<td>C</td>
<td>END</td>
</tr>
<tr>
<td>2190</td>
<td>C</td>
<td>END</td>
</tr>
<tr>
<td>2200</td>
<td>C</td>
<td>END</td>
</tr>
<tr>
<td>2210</td>
<td>C</td>
<td>END</td>
</tr>
</tbody>
</table>
TABLE 5A (Continued)

C     TH1 = CMN*COS(W(I+J)*TAU) + DMN*SIN(W(I+J)*TAU)
     TH2 = CMN*SIN(W(I+J)*TAU) - DMN*COS(W(I+J)*TAU)
C
C     RPN*KMN = EXP(-AMN*TAU) * (TH1*CAP1*CAP2 + TH2*CAP1*CAP4 - TH2
     1*CAP5*CAP2 + TH1*CAP5*CAP4)
     RP(K) = RP(K) + RPN
C
     WRITE(6,4003) K, TAU, RPN
C
15 TAU = TAU + DTAU
C
21 IF (NWA) 14, 10, 14
14 WRITE(6,4014)
C
DO 20 K=1,NWA
     RAD = WA(K)/(2.*PI)
     DD = (B**2*W(I+J)**2 + W(I+J)**2 - WA(K)**2) / DD
     AAMN = (B**2*W(I+J)**2 + W(I+J)**2 - WA(K)**2) / DD
C
     TW1 = CMN*AAMN + DMN*BBMN
     TW2 = CMN*BBMN - DMN*AAMN
C
     FIT = KMN * (TW1*CAP1*CAP2 + TW2*CAP1*CAP4 - TW2*CAP5*CAP2 +
     1TW1*CAP5*CAP4)
C
     FAT = FIT
     IF (FAT) 19, 19, 17
19 PHI = 0.
     FIT = 0.
     GO TO 18
17 PHI = 10.*ALOG10(FIT) + 127.6
18 WRITE(6,4015) K, WA(K), RAD, FAT, PHI

163
TABLE 5A (Continued)

```
20  F1(K) = F1(K) + FIT
10  CONTINUE
     IF (NTAU) 52 ≥ 51 ≥ 52
52  WRITE(6,4005) RP(1)
     DO 30 K = 1, NTAU
          RPBAR(K) = RP(K) / RP(1)
          PRP(K) = RP(K) / P2
          WRITE(6,4006) K, RPBAR(K), PRP(K)
     30  ATAU = ATAU + DTAU
     IF (NWA) 61 ≥ 51 ≥ 61
61  WRITE(6,4011)
     DO 40 K = 1, NWA
          RAD = WA(K) / (2 * PI)
          IF (F1(K)) 71 ≥ 71 ≥ 41
          PH1 = 0
          GO TO 40
41  PH1 = 10 * ALOG10(F1(K)) + 127.6
     WRITE(6,4012) K, WA(K), RAD, F1(K), PH1
5  CONTINUE
     RETURN
     END

$IBFTC XCR0S1
SUBROUTINE XCR0S1
     CROSS CORRELATION) TIME = CONSTANT
```
TABLE 5A (Continued)

C 2960
C 2970
C 0
FRA(E262E26!/61.)
2980
1000 FORAT(EI26#26#El*6t6/4El28))
2990
1001 FORMAT12(3EI2*6.4))
3000
C 3010
C 3020 C 3030
2000 FORMAT(IHO,1HI a4x3HTAU,4x7HPNT CNT,5X2HXP,10X2HYP,20X5HID,10X6HM
3040 M 4X4HFREQ 5X517HCRoss SPEC DENS 5X171HDB(RE0002)
2)5X5 9HPMN(TAU)//I2x2E11,5x14E14,6E12,6E12,6E2X,2152X4F6x0
3050 36X4E14,64XZE14,0)
C 3060
3070 COMMON /AAA/A(4000)
2001 FORMAT(IHO,1HI a4x3HTAU,4x7HPNT CNT,5X2HXP,10X2HYP,20X5HID,10X6HM
3080 M 4X4HFREQ 5X517HCRoss SPEC DENS 5X171HDB(RE0002)
2)5X5 9HPMN(TAU)//I2x2E11,5x14E14,6E12,6E12,6E2X,2152X4F6x0
3090 36X4E14,64XZE14,0)
3050
4000 FORMAT(IHO,1HI a4x3HTAU,4X5HTAU+12X1HJ,10X5XHPX(J),9X5SHYP(J),9X
3070 15HPX(J),12X1HCRPBAR(11J)10X12HNPXYK. FACTOR//165X5X10,5,
3100 24X150X5X14,6E20,6E22,8125X155X3E14,6E20,8E22,8)
3130 2005 FORMAT(IHO,1HI a4x3HTAU,4X5HTAU+12X1HJ,10X5XHPX(J),9X5SHYP(J),9X
3150 15HPX(J),12X1HCRPBAR(11J)10X12HNPXYK. FACTOR//165X5X10,5,
3120 24X150X5X14,6E20,6E22,8125X155X3E14,6E20,8E22,8)
3130
3140
3150
2010 FORMAT(IHO,1HI a4x3HTAU,4X5HTAU+12X1HJ,10X5XHPX(J),9X5SHYP(J),9X
3170 15HPX(J),12X1HCRPBAR(11J)10X12HNPXYK. FACTOR//165X5X10,5,
3180 24X150X5X14,6E20,6E22,8125X155X3E14,6E20,8E22,8)
3130
3140
3150
2020 COMMON /AAA/A(4000)
3190
C 3200
C 3210
1(A(6)KXYZ) t(A(7)NAUTO) t(A(8)NCROSS) t(A(9)M) t(A(10)N)
3220
EQUIVALENCE (A(21)XL1) t(A(22)XLO) t(A(23)YL1) t(A(24)YLO)
3230
1(A(25)AI) t(A(26)B) t(A(27)C) t(A(28)DETA) t(A(29)P2) t(A(30)P1)
3240
2x(A(31)X) m(A(32)Y) t(A(33)Z) m(A(34)EPS) t(A(35)NMAX)
3250
3(A(36)WMN) t(A(37)ANN) t(A(38)SUM1) t(A(39)SUM2)
3260
4(A(40)JAJ) t(A(41)JB)
3270
EQUIVALENCE (A(101)YAXM), (A(151),YAXM) t(A(1001),W) t(A(2001),D1X1)
3280
1(A(3001)D1X5)
3290
C 3300
C 3310
C 3320
DIMENSION TAV(110), XP(50), YP(50), ZP(50), MQR(50), XAXM(50),
3330 INAX(50), W(20,50), D1X1(20,50), D1X5(20,50), CRPCBAR(50)
3340
3350
### TABLE 5A (Continued)

<table>
<thead>
<tr>
<th>REAL KMN</th>
<th>CONVY = 60000$^*$</th>
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</table>

```fortran
C DO 5 II=1,XYZ
C READ (5,1000)XX,YZ,MTAV,EP$S,NMAX(1),I=1,MATAV
C WRITE(6,2000)XX,YZ,MTAV,EP$S,NMAX(1),I=1,MATAV
C DO 5 II=1,XYZ
C READ (5,1000)XX,YZ,MTAV,EP$S,NMAX(1),I=1,MATAV
C WRITE(6,2000)XX,YZ,MTAV,EP$S,NMAX(1),I=1,MATAV
C DO 10 I=1,MTAV
C DO 20 J=1,MTAV
C CRP = 0,
C DO 30 K=1,IN
C DO 30 L=1,IN
C JA=L
C JB=L
C IF (I.EQ.MM) JA = K
C M=M(A,J)
C N=M(N,K)
C WMN = W(K,L)
C KMN = C*THETA / (B*(1+W(K,L)**2)*THETA**2))
C AMN = B*W(K,L)
C CMN = W(K,L)*(1-6*B**2 + B**4)
C DMN = 4*B*W(K,L)*(B**2-1)
C WBAR1 = (B**2+2a)*(B+W(K,L))*(B**2+4a)
C WBAR2 = B*WBAR1 / (B**2+2a)
```
TABLE 5A (Continued)

\[
\begin{align*}
\text{C} & \quad \text{CBAR} = KMN \times W(K+L) \times P2 \\
\text{C} & \quad \text{WRITE}(6, 2010) \text{ KMN}+\text{AMN}+\text{CMN}+\text{DMN}+\text{WBAR1}+\text{WBAR2}+\text{CBAR} \\
\text{C} & \quad \text{WRITE}(6, 2005) \\
\text{C} & \quad \text{IF} (J \neq NE+1) \text{ GO TO 31} \\
\text{C} & \quad X=XX \\
\text{C} & \quad Y=YY \\
\text{C} & \quad Z=ZZ \\
\text{C} & \quad \text{PHASE} = 1 \times 5708 \\
\text{C} & \quad KKK=1 \text{ AMN} = -\text{AMN} \\
\text{C} & \quad \text{CALL DINT(AMN+PHASE+KKK)} \\
\text{C} & \quad \text{D}1\text{X}1(K+L) = \text{SUM2} \\
\text{C} & \quad \text{PHASE} = 0 \times \\
\text{C} & \quad KKK= 5 \\
\text{C} & \quad \text{CALL DINT(AMN+PHASE+KKK)} \\
\text{C} & \quad \text{D}1\text{X}5(K+L) = \text{SUM2} \\
\text{C} & \quad \text{AMN} = -\text{AMN} \\
\text{C} & \quad 31 \ X=XP(J) \\
\text{C} & \quad Y=YP(J) \\
\text{C} & \quad Z=ZP(J) \\
\text{C} & \quad \text{PHASE} = 1 \times 5708 \\
\text{C} & \quad KKK = 2 \\
\text{C} & \quad \text{CALL DINT(AMN+PHASE+KKK)} \\
\text{C} & \quad \text{CAP2} = \text{SUM2} \\
\text{C} & \quad \text{PHASE} = 0 \times \\
\text{C} & \quad KKK = 4 \\
\text{C} & \quad \text{CALL DINT(AMN+PHASE+KKK)} \\
\text{C} & \quad \text{CAP4} = \text{SUM2}
\end{align*}
\]
TABLE 5A (Continued)

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<th>SP</th>
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\[
\begin{align*}
\text{39} & \quad \text{IF (NOR(I)) 39=32+32} \\
\text{SP} & \quad = \text{CBAR} - (\text{WBAR1*DIX1(K+L)})*\text{CAP2} + (\text{WBAR2*DIX1(K+L)})*\text{CAP4} - \text{WBAR2*} \\
\text{10*DIX5(K+L)}*\text{CAP2} + (\text{WBAR1*DIX5(K+L)})*\text{CAP4} \\
\text{37} & \quad \text{IF (SP) 37=37+37} \\
\text{PHI} & \quad = 0, \\
\text{GO TO} & \quad 32 \\
\text{33} & \quad \text{PHI} = 10^{6}* \text{LOG10(SP)} + 127+6 \\
\text{32} & \quad \text{TH1} = \text{CMN*COS(W(K+L)*TAV(I))} + \text{DHN*SIN(W(K+L)*TAV(I))} \\
\text{TH2} & \quad = \text{CMN*SIN(W(K+L)*TAV(I))} - \text{DHN*COS(W(K+L)*TAV(I))} \\
\text{RPN} & \quad = \text{KMN*EXP(-AMN*TAV(I))}*[\text{TH1*DIX1(K+L)}]*\text{CAP2} + \text{TH2*DIX1(K+L)}*\text{CAP4} - \\
\text{ICAP4} & \quad - \text{TH2*DIX5(K+L)}*\text{CAP2} + \text{TH1*DIX5(K+L)}*\text{CAP4} \\
\text{34} & \quad \text{WRITE6}(6+2002) I,TAV(I),J*XP(J),YP(J),ZP(J),SP,PHI,RPN \\
\text{30} & \quad \text{CONTINUE} \\
\text{20} & \quad \text{IF ((I+J)EQ.2) CNORM =CRP} \\
\text{CRPBAR(J)} & \quad = \text{CRP} / \text{CNORM} \\
\text{40} & \quad \text{WRITE6}(6+2004) I,TAV(I),J*XP(J),YP(J),ZP(J),CRPBAR(J),CNORM, \\
\text{10} & \quad \text{CONTINUE} \\
\text{5} & \quad \text{CONTINUE} \\
\end{align*}
\]
TABLE 5A (Continued)

C 4440
C 4450
C 4460
RETURN 4470
END 4480
$IBFT CXRO3 4490
SUBROUTINE CROSS2 4500
C 4510
C 4520
C 4530
C CROSS CORRELATION (TIME) (AUTO CORRELATION INCLUDED FOR THE FIRST POINT) 4540
C 4550
1000 FORMAT(3E12.6,13.E6,29213) 4560
1001 FORMAT(2(E12*6914)l 4570
1002 FORMAT(12F6#Ol 4580
C 4590
2000 FORMAT(lH1.50X924HCROSS CORRELATION (TIME)///3OXo1HXt30X,1HYt30Xq 4600
2001 FORMAT(lHO,3OX,2HXP,2OX,2HYP,2OXt,1OX,3HNOR//(23XEl4.8,8XE14 4610
3///40X,1HK#SX93HTAU#15X,9HRPMN(TAU)II) 4620
2002 FORMAT(lHO, 9HCASE NUM.5X11INTEG. PNT.5X2XMXP10X2HYP10X, 4630
12H2P10X6HM N.5X4HFREQ 5X22HM5S. SPECTRAL DENSITY5X5 4640
211HDB(RE-0021///15.10X15.3X3.3X59414X215.3X603X220+8 4650
3///40X1MK48X3HTAU15X9HRPMN(TAU)///) 4660
2003 FORMAT(lHO, 9HCASE NUM.5X11INTEG. PNT.5X2XMXP10X2HYP10X, 4670
12H2P10X6HM N.5X4HFREQ 15.10X15.3X3.3X59414X215.3X603X220+8 4680
2F60///40X1MK8X3HTAU15X9HRPMN(TAU)///) 4690
2004 FORMAT(1H1,50X*44HNORMALIZED CORRELATION OF ACCOUSTIC PRESSURE/// 4700
110X 4HNTAU10X3HTAU10X25HMOR. CORR. OF ACC. PRES.10X, 4710
28HRP(KJ/2P12X1KMOR. FACTOR =E14.8/) 4720
2005 FORMAT(1H1,50X*44HNORMALIZED CORRELATION OF ACCOUSTIC PRESSURE/// 4730
110X 4HNTAU10X3HTAU10X25HMOR. CORR. OF ACC. PRES.10X, 4740
28HRP(KJ/2P12X1KMOR. FACTOR =E14.8/) 4750
2006 FORMAT(1H1,50X*44HNORMALIZED CORRELATION OF ACCOUSTIC PRESSURE/// 4760
110X 4HNTAU10X3HTAU10X25HMOR. CORR. OF ACC. PRES.10X, 4770
28HRP(KJ/2P12X1KMOR. FACTOR =E14.8/) 4780
2007 FORMAT(1H1,50X*44HNORMALIZED CORRELATION OF ACCOUSTIC PRESSURE/// 4790
110X 4HNTAU10X3HTAU10X25HMOR. CORR. OF ACC. PRES.10X, 4800
28HRP(KJ/2P12X1KMOR. FACTOR =E14.8/) 4810
2008 FORMAT(1H1,50X*44HNORMALIZED CORRELATION OF ACCOUSTIC PRESSURE/// 4820
110X 4HNTAU10X3HTAU10X25HMOR. CORR. OF ACC. PRES.10X, 4830
28HRP(KJ/2P12X1KMOR. FACTOR =E14.8/) 4840
2009 FORMAT(1H1,50X*44HNORMALIZED CORRELATION OF ACCOUSTIC PRESSURE/// 4850
110X 4HNTAU10X3HTAU10X25HMOR. CORR. OF ACC. PRES.10X, 4860
28HRP(KJ/2P12X1KMOR. FACTOR =E14.8/) 4870
2010 FORMAT(1H1,50X*44HNORMALIZED CORRELATION OF ACCOUSTIC PRESSURE/// 4880
110X 4HNTAU10X3HTAU10X25HMOR. CORR. OF ACC. PRES.10X, 4890
28HRP(KJ/2P12X1KMOR. FACTOR =E14.8/) 4900

169
TABLE 5A (Continued)

2011 FORMAT(1HI,30X,23HCROSS SPECTRAL DENSITY / / 12X*1MK,10X* 4HFREQ* 4810
110X* 7RAD/SEC*10X*17HCROSS SPEC* DENS*10X*11HDB(RE*0002) / / 4820
2012 FORMAT(1H* 8X1+4*8X+6F6+08X+8F9+42(10X+EL+08) 4830
2013 FORMAT(1HI,30X,1MK,10X*4HFREQ*10X*7RAD/SEC*10X*17HMODAL SPEC* DENS 4840
15X*10X*1HDB(RE*0002) / / 4850
2014 FORMAT(1H*21X*110* 8X* F6+08X*F9+42(8X*E16*08) 4860
2015 FORMAT(1H*#21X#1109
8X#F6+a0.8X#F9#492(8X*E16#8fl
2016 COMMON /AAA/A(4000)
2017 COMMON //AAA/A(4000)
2019 1(A(7),M)(A(8),N)(A(9),K)(A(10),I) +
2020 (A(11),L)(A(12),NCROSS)
2021 EQUIVALENCE (A(21),XL1)(A(22),XLO)(A(23),YL1)(A(24),YLO)*
21(A31),X)(A32),Y)(A33),Z)(A34),EPS)(A35),NMAX)
21(A36),A(NN)(A37),AMN)(A38),SUM1)(A39),SUM2)
22(A(40),J)(A(41),B)
2023 EQUIVALENCE (A101),MAM),(A111),NAM),(A1001),WW),(A2001),DI1)
2024 1(A3001),DI5)
2025 COMMON /AAA/A(4000)
2026 COMMON //AAA/A(4000)
2027 REAL KMN
2028 COMMON /AAA/A(4000)
2029 COMMON /AAA/A(4000)
2030 CONVT = 60000
2031 DO 5 II=1 LXZ
2032 READ (5,1000) XX,YY,ZZ,MP,TAU,TAU,TAU,TAU,TAU,TAU,TAU,TAU,TAU,TAU,TAU,TAU,TAU
2033 WRITE(6,2000) XX,YY,ZZ,MP,TAU,TAU,TAU,TAU,TAU,TAU,TAU,TAU,TAU,TAU,TAU,TAU,TAU,TAU

170
TABLE 5A (Continued)

C ATAU = TAU
C READ (5,1001) (XP(I),YP(I),ZP(I),NOR(I), I=1,MXP)
WRITE(6,2001) (XP(I),YP(I),ZP(I),NOR(I), I=1,MXP)
C IF (NWA) B+1,8
8 READ (5,1002) (WA(K),K=1,NWA)
WRITE(6,2010) (WA(K),K=1,NWA)
C 1 DO 10 J=1,MXP
C DO 6 IA=1,NTAU
6 RP(IA) = 0.
C DO 7 IA=1,NWA
7 F1(IA) = 0.
C DO 20 I=1,IM
DO 20 J=1,IN
JA = J
JB = J
IF (IM.EQ.MM) JA = I
MM = MIN(JA)
N = MIN(J)
C WMN = W(I,J)
C KMN = C*THETA / (B*(1+W(I,J)**2*THETA**2))
AMN = B*W(I,J)
CMN = W(I,J)*(1.-6.*B**2 + B**4)
DMN = 4.*B*W(I,J)*(B**2-1.)
C WRITE(6,2005)
1F (JJ.NE.1) GO TO 21

171
| X=XX | 5550 |
| Y=YY | 5560 |
| Z=ZZ | 5570 |
| PHASE = 1.5708 | 5580 |
| KKK = 1 | 5590 |
| AMN = AMN | 5960 |
| CALL DINT(AMN,PHASE,KKK) | 5600 |
| DIX1(I+J) = SUM2 | 5610 |

**C**

| PHASE = 0, | 5620 |
| KKK = 5 | 5630 |
| CALL DINT(AMN,PHASE,KKK) | 5640 |
| DIX5(I+J) = SUM2 | 5650 |

**C**

| PHASE = 0, | 5660 |
| KKK = 4 | 5670 |
| CALL DINT(AMN,PHASE,KKK) | 5680 |
| DIX4(I+J) = SUM2 | 5690 |

| PHASE = 0, | 5700 |
| KKK = 4 | 5710 |
| CALL DINT(AMN,PHASE,KKK) | 5720 |
| DIX4(I+J) = SUM2 | 5730 |

**C**

| PHASE = 0, | 5740 |
| KKK = 5 | 5750 |
| CALL DINT(AMN,PHASE,KKK) | 5760 |
| DIX5(I+J) = SUM2 | 5770 |

**C**

| PHASE = 0, | 5780 |
| KKK = 5 | 5790 |
| CALL DINT(AMN,PHASE,KKK) | 5800 |
| DIX5(I+J) = SUM2 | 5810 |

**C**

| PHASE = 0, | 5820 |
| KKK = 4 | 5830 |
| CALL DINT(AMN,PHASE,KKK) | 5840 |
| DIX4(I+J) = SUM2 | 5850 |

| IF (JJ+NE = 1) GO TO 26 | 5860 |
| WPBAR1 = (B**2+2x) / (B**W(I,J) = 4**2+4x) | 5870 |
| WPBAR2 = B*WPBAR1 / (B**2+2x) | 5880 |
| CBAR = KMNW(I,J)*P2 | 5890 |

**C**

| SP = CBAR = (WPBAR1*DIX1(I+J)*CAP2 + WPBAR2*DIX1(I+J)*CAP4 - WPBAR2* | 5900 |
| WPBAR* | 5910 |
TABLE 5A (Continued)

1DIX5(I,J)*CAP2 + WBARI*DIX5(I,J)*CAP4
IF (SP) 20*20*22
28 PHI = 0
GO TO 29
29 PHI = 10*DLOG10(SF) + 127.6
22 WRITE(6,2002) IJX JP(J) YP(J) ZP(J) SX W(I,J) SP PHI
GO TO 25
24 WRITE(6,2003) IJX JP(J) YP(J) ZP(J) SX W(I,J)
25 IF (NTAU) 39,31,39

39 TAU = ATAU

C
DO 30 K=1,NTAU
C
TH1 = CHNH*COS(W(I,J)*TAU) + DMN*SIN(W(I,J)*TAU)
TH2 = CHNH*SIN(W(I,J)*TAU) - DMN*COS(W(I,J)*TAU)
C
RPN = KM*NEXP(-AHN*TAU)*[TH1*DIX(I,J)*CAP2 + TH1*DIX(I,J)*CAP4 - TH2*
1DIX5(I,J)*CAP2 + TH1*DIX5(I,J)*CAP4)]

C
WRITE(6,2004) K,TAU,RPN

C
30 TAU = TAU + DTAU

C
31 IF (NWA) 20,20,29
29 WRITE(6,2013)
C
DO 19 K=1,NWA
RAD = WA(K)/12*PTI
DD = (B**2*W(I,J)**2 + (W(I,J) - WA(K)/**2)) / (W(I,J) + WA(K)/**2)
AAMN = (BBW(I,J)**2*W(I,J)**2 + W(I,J)**2 + WA(K)**2) / DD
BBMN = (W(I,J)**2*W(I,J)**2 - WA(K)**2) / DD
C
TW1 = CHNH*AAMN + DMN*BBMN
TW2 = CHNH*BBMN - DMN*AAMN

C
173
TABLE 5A (Continued)

C  FIT = KMN * (TW1*DIX1(I+J)*CAP2 + TW2*DIX1(I+J)*CAP4 - TW2*DIX5(I+J)*CAP4)
C  FIT = FAT
C  IF (FIT) 71*71*17
71 PHI = 0,
FIT = 0,
GO TO 18
17 PHI = 10*ALOG10(FIT) + 127*6
18 WRITE(6,2014) K,WA(K),RAD*FAT*PHI
C 19 FI(K) = FI(K) + FIT
C 20 CONTINUE
C  IF (NTAU) 47*45*47
47 WRITE(6,2007) RP(1)
TAU = ATAU
C  DO 40 K=1,NTAU
RPBAR(K) = RP(K)/RP(1)
PRP(K) = RP(K)/P2
C  WRITE(6,2008) K,TAU,RPBAR(K),PRP(K)
40 TAU = TAU + DTAU
C 45 IF (NWA) 74*10*74
74 WRITE(6,2011)
C  DO 50 K=1,NWA
RAD = WA(K)/(2*PI)
IF (FI(K)) 75*75*51
75 PHI = 0,
GO TO 50
C 51 PHI = 10*ALOG10(FI(K)) + 127*6

174
TABLE 5A (Continued)

50 WRITE(6,2012) K,Wa(K),Rad,Fl(K),Phi

C 6660
C 6670
C 6680
C 6690
C 6700
C 6710
C 6720
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C 6990
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C 9990

$IBFCT XDINT
SUBROUTINE XDINT (SAMN,SPHASE,KKS)

DOUBLE INTEGRATION SUBROUTINE

COMMON /AAA/A(4000)

EQUIVALENCE (A(9),M),(A(10),K),(A(21),KL1),(A(22),KL0),(A(23),YL1)
1,(A(24),YL0),(A(34),EPS),(A(35),NMAX),(A(38),SUM1),(A(39),SUM2)

DIMENSION P(5),WW(5)

DATA (P(I)),W(I),SUM1(SUM2)/.453089923,.453089923,.2692346590092*.118463443,2*.239314335,e284444444

2000 FORMAT(1HO,28H***** NO CONVERGENCE *****#5X*149lOX*I4#5XoI49l5#

2002 FORMAT(1HO.33X.14e10X,14,AX.15.I5.2E1B.8)

NK=2
TABLE 5A (Continued)

\[
\begin{align*}
\text{BIN}1 & = YL1 - XLO \\
\text{BIN}2 & = YL1 - YLO \\
\text{C}
\end{align*}
\]

\begin{center}
\begin{tabular}{l}
\text{DO 25 } K = 1, NMAX \\
\text{SUM1} = \text{SUM2} \\
\text{SUM2} \leftarrow 0 \\
\text{CNK} \leftarrow NK \\
\text{DINC1} \leftarrow \text{BIN}1 / \text{CNK} \\
\text{DINC2} \leftarrow \text{BIN}2 / \text{CNK} \\
\text{H1} \leftarrow XLO + \text{DINC1} / 2 \\
\text{CNK} \leftarrow NK \\
\text{SUM2} \leftarrow \text{SUM2} + \text{WW(11)} * \text{WW(jj)} * \text{FFF(ABS1, ABS2, SAMNSPAMASE)} \\
\end{tabular}
\end{center}

\text{DO 26 } I = 1, NK
\text{H2} \leftarrow YLO + \text{DINC2} / 2 \\
\text{C}

\text{DO 27 } J = 1, NK
\text{DO 28 } I = 1, 5
\text{ABS1} \leftarrow \text{DINC1} * \text{P(I1)} + \text{H1} \\
\text{DO 29 } JJ = 1, 5
\text{ABS2} \leftarrow \text{DINC2} * \text{P(JJ)} + \text{H2} \\
\text{SUM2} \leftarrow \text{SUM2} + \text{WW(I1)} * \text{WW(JJ)} * \text{FFF(ABS1, ABS2, SAMNSPAMASE)} \\
\text{CONTINUE}
\text{27 } \text{H2} \leftarrow \text{H2} + \text{DINC2} \\
\text{26 } \text{H1} \leftarrow \text{H1} + \text{DINC1} \\
\text{SUM2} \leftarrow \text{SUM2} * \text{DINC1} * \text{DINC2} \\
\text{C}

\text{IF } (K \leftarrow 1, 10) \text{ GO TO 30}
\text{IF } (\text{ABS1}((\text{SUM1} - \text{SUM2}1) / \text{SUM2}1) - \text{EPS1}) \text{ GO TO 30}
\text{CONTINUE}
\text{30 } \text{NK} \leftarrow \text{NK} + 1 \\
\text{SUM2} \leftarrow 0 \\
\text{GO TO 32}
\text{C}

\text{GO TO 32}
\text{C
31 WRITE (6,2002) XKS+NKS+M+N+SUM2+SUM1
C
32 RETURN
END
$IBFTC XFFF
	FUNCTION FFF(XO,YO,FAMN,FPHASE)

DOUBLE INTEGRATION FUNCTION
C
COMMON /AAA/A(4000)
C
C
COMMON /AAA/A(4000)
C
C
INTEGER W,WMN
C
RO=SQRT((X-XO)**2+(Y-YO)**2+Z**2)
C
CM = M
CN = N
PHX = SIN(CMPI*PI*XLI)
PHY = SIN(CMPI*PI*YLI)
C
FFF = PHX * PHY * EXP((FAMN/AI)*RO) * SIN(WMN/AI)*RO+FPHASE/ RO
RETURN
END

TABLE 5A (Continued)
Table 5B – Clamped-Clamped Boundaries

SIBFTC TURGEN

1000 FORMAT(12A6)
1001 FORMAT(10I4)
1002 FORMAT(15E15.8)
1003 FORMAT(12F6.0)
1004 FORMAT(20I4)
1005 FORMAT(5E15.8)
1006 FORMAT(12F6#0)
1007 FORMAT(20I4)
1008 FORMAT(50)
1009 FORMAT(1H1.1OX.6HF.P.S..ISX,12A6/22X,12A6////)
2000 FORMAT(1H1*40X,4SMOER OF M (MODE NUMBERS)
3000 FORMAT(1HO929X945HORDER OF N (MODE NUMBERS)
4000 FORMAT(6HNUMBER OF CASES (XYZ) IN AUTO CORR= XYZ =16/30X45HNUMBERS
5000 FORMAT(6HNUMBER OF CASES (XYZ) IN CROSS CORR= XYZ =16 /30X45HNUMBERS
6000 FORMAT(6HNUMBER OF CASES (XYZ) IN AUTO CORR= XYZ =16/30X45HNUMBERS
7000 FORMAT(6HNUMBER OF CASES (XYZ) IN CROSS CORR= XYZ =16 /30X45HNUMBERS
8000 FORMAT(6HNUMBER OF CASES (XYZ) IN AUTO CORR= XYZ =16/30X45HNUMBERS
9000 FORMAT(6HNUMBER OF CASES (XYZ) IN CROSS CORR= XYZ =16 /30X45HNUMBERS
10000 FORMAT(6HNUMBER OF CASES (XYZ) IN AUTO CORR= XYZ =16/30X45HNUMBERS
11000 FORMAT(6HNUMBER OF CASES (XYZ) IN CROSS CORR= XYZ =16 /30X45HNUMBERS
12000 FORMAT(6HNUMBER OF CASES (XYZ) IN AUTO CORR= XYZ =16/30X45HNUMBERS
13000 FORMAT(6HNUMBER OF CASES (XYZ) IN CROSS CORR= XYZ =16 /30X45HNUMBERS
14000 FORMAT(6HNUMBER OF CASES (XYZ) IN AUTO CORR= XYZ =16/30X45HNUMBERS
15000 FORMAT(6HNUMBER OF CASES (XYZ) IN CROSS CORR= XYZ =16 /30X45HNUMBERS
16000 FORMAT(6HNUMBER OF CASES (XYZ) IN AUTO CORR= XYZ =16/30X45HNUMBERS
17000 FORMAT(6HNUMBER OF CASES (XYZ) IN CROSS CORR= XYZ =16 /30X45HNUMBERS
18000 FORMAT(6HNUMBER OF CASES (XYZ) IN AUTO CORR= XYZ =16/30X45HNUMBERS
19000 FORMAT(6HNUMBER OF CASES (XYZ) IN CROSS CORR= XYZ =16 /30X45HNUMBERS
20000 FORMAT(6HNUMBER OF CASES (XYZ) IN AUTO CORR= XYZ =16/30X45HNUMBERS
21000 FORMAT(6HNUMBER OF CASES (XYZ) IN CROSS CORR= XYZ =16 /30X45HNUMBERS
22000 FORMAT(6HNUMBER OF CASES (XYZ) IN AUTO CORR= XYZ =16/30X45HNUMBERS
23000 FORMAT(6HNUMBER OF CASES (XYZ) IN CROSS CORR= XYZ =16 /30X45HNUMBERS
24000 FORMAT(6HNUMBER OF CASES (XYZ) IN AUTO CORR= XYZ =16/30X45HNUMBERS
25000 FORMAT(6HNUMBER OF CASES (XYZ) IN CROSS CORR= XYZ =16 /30X45HNUMBERS
26000 FORMAT(6HNUMBER OF CASES (XYZ) IN AUTO CORR= XYZ =16/30X45HNUMBERS
27000 FORMAT(6HNUMBER OF CASES (XYZ) IN CROSS CORR= XYZ =16 /30X45HNUMBERS
28000 FORMAT(6HNUMBER OF CASES (XYZ) IN AUTO CORR= XYZ =16/30X45HNUMBERS
29000 FORMAT(6HNUMBER OF CASES (XYZ) IN CROSS CORR= XYZ =16 /30X45HNUMBERS
30000 FORMAT(6HNUMBER OF CASES (XYZ) IN AUTO CORR= XYZ =16/30X45HNUMBERS
31000 FORMAT(6HNUMBER OF CASES (XYZ) IN CROSS CORR= XYZ =16 /30X45HNUMBERS
32000 FORMAT(6HNUMBER OF CASES (XYZ) IN AUTO CORR= XYZ =16/30X45HNUMBERS
33000 FORMAT(6HNUMBER OF CASES (XYZ) IN CROSS CORR= XYZ =16 /30X45HNUMBERS
34000 FORMAT(6HNUMBER OF CASES (XYZ) IN AUTO CORR= XYZ =16/30X45HNUMBERS
35000 FORMAT(6HNUMBER OF CASES (XYZ) IN CROSS CORR= XYZ =16 /30X45HNUMBERS
36000 FORMAT(6HNUMBER OF CASES (XYZ) IN AUTO CORR= XYZ =16/30X45HNUMBERS

178
TABLE 5B (Continued)

```plaintext
EQUIVALENCE (A(1),MM) *(A(2),NN) *(A(3),IM) *(A(4),IN) *(A(5),NXYZ)
1*(A(6),KXYZ) *(A(7),NAUTO) *(A(8),NCROSS) *(A(9),N) *(A(10),N)
2*(A(11),LXYZ) *(A(12),NCROSS)
```

```plaintext
c  EQUIVALENCE (A(21),XL1) *(A(22),XL0) *(A(23),YL1) *(A(24),YL0)
1*(A(25),A1) *(A(26),A0) *(A(27),C) *(A(28),THETA) *(A(29),P2) *(A(30),P1)
2*(A(31),X) *(A(32),Y) *(A(33),Z) *(A(34),EPS) *(A(35),NMAX)
3*(A(36),WMN) *(A(37),AMN) *(A(38),SUM1) *(A(39),SUM2)
```

```plaintext
c  EQUIVALENCE (A(101),MAM) *(A(151),NAM) *(A(1001),W)
```

```plaintext
c  EQUIVALENCE (A(201),AMM) *(A(251),ANN) *(A(301),BMN) *(A(351),BNN)
1*(A(401),CMM) *(A(451),CNN) *(A(501),DMN) *(A(551),DN) *(A(601),ALMM)
2*(A(651),ALNN)
```

```plaintext
c  REAL KMN
```

```plaintext
c  INTEGER W
```

```plaintext
c  REAL KMN
```

```plaintext
c  CALL STARTR
```

```plaintext
c  IF ACCUMULATOR OVERFLOW +
```

```plaintext
c  READ AND PRINT TITLE
```

```plaintext
c  READ(5,1000) (TITLE(I),I=1,24)
```

```plaintext
c  WRITE(8,2000) (TITLE(I),I=1,24)
```

```plaintext
c  READ AND PRINT GENERAL INPUT CONSTANTS
```

```plaintext
c  READ (5,1001) MM,NN,IM,IN,NXYZ,XYZ,XY,NAUTO,NCROSS,NCROSS
```

```plaintext
c  READ (5,1002) XL1,XL0,YL1,YL0,A1,B,C,THE,THETA,P2
```

179
TABLE 5B (Continued)

| READ (5*1003) ((W(I,J)+J=1*IN)+J=1*IM) | 0740 |
| READ (4*1004) (MAM(I)+I=1*MM) (NAM(I)+I=1*NN) | 0750 |
| WRITE(6,2001) MM+NM+XMYZ+XMYZ+NAUTO+NCROSS+XL0+YL0+YO+AI+BO | 0760 |
| WRITE(6,2002) ((W[I,J]+J=1*IN)+J=1*IM) | 0770 |
| WRITE(6,2003) (MAM(I)+I=1*MM) (NAM(I)+I=1*NN) | 0780 |
| WRITE(6,2004) (MAM(I)+I=1*MM) (NAM(I)+I=1*NN) | 0790 |
| PI=3.14159 | 0800 |
| AUTO CORRELATION | 0810 |
| 13 IF (NAUTO) 13=13 | 0820 |
| READ (5*1005) (ALMN(I)+AMM(I)+BMN(I)+CMN(I)+DMM(I)+I=1*MM)+ | 0830 |
| 1(ALMN(I)+AMM(I)+BMN(I)+CMN(I)+DMM(I)+I=1*NN) | 0840 |
| WRITE(6,2005) (ALMN(I)+AMM(I)+BMN(I)+CMN(I)+DMM(I)+I=1*MM)+ | 0850 |
| 1(ALMN(I)+AMM(I)+BMN(I)+CMN(I)+DMM(I)+I=1*NN) | 0860 |
| CALL AUTO1 | 0870 |
| CROSS CORRELATION (SPACE) | 0880 |
| 3 IF (NCROSS) 10=6+10 | 0890 |
| 10 IF (NAUTO=EQ. 1) GO TO 5 | 0900 |
| READ (5*1005) (ALMN(I)+AMM(I)+BMN(I)+CMN(I)+DMM(I)+I=1*MM)+ | 0910 |
| 1(ALMN(I)+AMM(I)+BMN(I)+CMN(I)+DMM(I)+I=1*NN) | 0920 |
| WRITE(6,2005) (ALMN(I)+AMM(I)+BMN(I)+CMN(I)+DMM(I)+I=1*MM)+ | 0930 |
| 1(ALMN(I)+AMM(I)+BMN(I)+CMN(I)+DMM(I)+I=1*NN) | 0940 |
| CALL CROSS1 | 0950 |
| CROSS CORRELATION (TIME) | 0960 |

180
TABLE 5B (Continued)

C IF (NCROSS) 18*8*18

18 IF ((NAUTO=EO+1) OR (NCROSS=EO+1)) GO TO 7

READ (5,1005) (ALMN(I),ANN(I),BNN(I),CNN(I),DMN(I),I=1,MM)

WRITE (6,2005) (ALMN(I),ANN(I),BNN(I),CNN(I),DMN(I),I=1,MM)

CALL CROSS2

STOP
END

SUBROUTINE AUTO1

C AUTO CORRELATION = X*XP + Y*YP + Z*ZP

3000 FORMAT(5E12*6,149E6) 3001 FORMAT(12F6*O)

4000 FORMAT(1H1,8H PNT CNT,12X,1HX,12X,1HY,12X,1HZ,12X,1HM,4X,1HN,5X,5HFREG,5X,21HM*S
SPECTRAL DENSITY,4X,20H DB(RE=0002) //

4003 FORMAT(1H1,30X944HNORMALIZED CORRELATION OF ACCOUSTIC PRESSURE///

4005 FORMAT(1H1,30X44HNORMALIZED CORRELATION OF ACCOUSTIC PRESSURE///

181
TABLE 5B (Continued)

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<tr>
<th>110X</th>
<th>3HTAU</th>
<th>10X</th>
<th>25HNMNS</th>
<th>CORR. OF ACC. PRES.</th>
<th>10X</th>
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<td>28HRP(K)/F2</td>
<td>12X</td>
<td>14HNORMs</td>
<td>FACTOR *E14s9/</td>
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<td>1MM</td>
<td>(A2)</td>
<td>MN</td>
<td>(A3)</td>
<td>IM</td>
</tr>
<tr>
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<td>NAUTO</td>
<td>(A8)</td>
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</tr>
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<td>(A22)</td>
<td>XLO</td>
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<td>YLI</td>
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<td>(A33)</td>
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<td>ANN</td>
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<td>(A41)</td>
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<td>CALL CLOCK(MTIME)</td>
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<td>DO 5 II=1,NXYZ</td>
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TABLE 5B (Continued)

```
READ (5x3000) X,Y,Z+TAU+DTAU+NTAU+EPS+NM MAX+NWA
1  1850
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  2100
  2110
  2120
  2130
  2140
  2150
  2160
  2170
  2180
  2190
  2200
  2210
```
TABLE 5B (Continued)

```
WRITE(6,4010) KMN, AMN, CHN, DMN, WBAR1, WBAR2, CBAR
WRITE(6,4004)

C
C PHASE=1.5708
KKK=2
CALL DINT (AMN*PHASE*KKK)
CAP2=SUM2
C
C KKK=1
AMN=-AMN
CALL DINT (AMN*PHASE*KKK)
CAP1=SUM2
C
C KKK=5
PHASE=0.5
CALL DINT (AMN*PHASE*KKK)
CAP5=SUM2
C
C KKK=4
AMN=-AMN
CALL DINT (AMN*PHASE*KKK)
CAP4=SUM2
C
C SP = CBAR* (WBAR1*CAP1*CAP2 + WBAR2*CAP1*CAP4 - WBAR2*CAP5*CAP2 + WBAR1*CAP5*CAP4 )
C
13 WRITE(6,4002) I1*, X1*, Y1*, Z1*, N1*, W1*, I2*, J2*, SP, PHI
IF (NTAU) 22*21*22
```

184
TABLE 5B (Continued)

22  TAU = ATAU
    DO 15 K=1,NTAU 2590
    C 2600
    C 2610
    TH1 = CHMN*COS(W(I,J)*TAU) + DMN*SIN(W(I,J)*TAU) 2630
    TH2 = CHMN*SIN(W(I,J)*TAU) - DMN*COS(W(I,J)*TAU) 2640
    C 2650
    C 2660
    RP=KMN*EXP(-AMN*TAU) * (TH1*CAP1*CAP2 + TH2*CAP1*CAP4 - TH2
    I*CAP5*CAP2 + TH1*CAP5*CAP4)
    RP(K)=RP(K)+RPN 2680
    C 2690
    WRITE(6,4003) K,TAU,RPN 2700
    C 2710
    C 2720
15  TAU = TAU + DTAU 2730
    C 2740
    21 IF(NWA) 25+10+25 2750
    25 WRITE(6,4014) 2760
    C 2770
    C 2780
    DO 20 K=1,NWA 2790
    DD = (B**2*W(I,J)**2 + (W(I,J)-WA(K))**2) +
        I(W(I,J) + WA(K))**2) 2800
    C 2810
    C 2820
    AAMN = (B*(W(I,J))*(B**2*W(I,J)**2 + W(I,J)**2 + WA(K)**2)) / DD
    BBMN = (W(I,J)*(B**2*W(I,J)**2 + W(I,J)**2 - WA(K)**2)) / DD 2840
    C 2850
    C 2860
    TW1 = CHMN*AAMN + DMN*BBMN 2870
    TW2 = CHMN*BBMN - DMN*AAMN 2880
    C 2890
    IF (FIT) 19+19+17 2900
    19 IF (FIT) 19+19+17 2910
    PHI = 0. 2920
    FIT = 0. 2930
    C 2940
    C 2950
TABLE 5B (Continued)

GO TO 18
17 PHI = 10*ALOG10(FIT) + 127.6
18 WRITE (6, A015) K, WA(K), RAD, FAT, PHI
C
20 FI(K) = FI(K) + FIT
C
10 CONTINUE
C
IF (NTAU) 52, 51, 52
C
52 WRITE (6, A005) RP(1)
C
DO 30 K = 1, NTAU
RPBAR(K) = RP(K)/RP(1)
PRP(K) = RP(K)/P2
WRITE (6, A006) K, NTAU, RPBAR(K), PRP(K)
30 ATAU = ATAU + DTAU
C
51 IF (NWA) 27, 5, 27
27 WRITE (6, A011)
C
DO 40 K = 1, NWA
RAD = WA(K)/12.8*PI
IF (FI(K)) 45, 45, 41
45 PHI = 0
GO TO 40
C
41 PHI = 10*ALOG10(FI(K)) + 127.6
40 WRITE (6, A012) K, WA(K), RAD, FI(K), PHI
C
5 CONTINUE
C
CALL CLOCK(NTIME)
C
TIME = (NTIME - MTIME)/CONVT
C
WRITE (6, A002) TIME

186
TABLE 5B (Continued)

RETURN
END
SUBROUTINE CROSSTC
C CROSS CORRELATION) TIME = CONSTANT
C
1000 FORMAT(3E12.6,2I6,E12.6,16/(6E12.8)) 1001 FORMAT(3E12.6,14))

C 2000 FORMAT(1H1,40X,17HCROSS CORRELATION///20X,1MX,20X,1HY,20X,1HZ,20X,
13MMXP,10X,4HMTAU,10X,3MHP,10X,4HMNX,//12X,E14.8,27X,E14.8,9
214X,14X,9X,14X,5XE12.6,7X,13//,5X,7MTAU(1),5XE16.8//12X,5E16.8//1
2001 FORMAT(1H0,30X,2HXP,20X,2HYP,20X,2HHP,10X,5MNN,//12X,E14.8,8X,E14
1.8X8X,E14.8X5X13 ))

C 2002 FORMAT(1H0,4X,3MTAU,4X,7HPNT CNT,5X,2HXP,10X,2HHP,10X,2HHP,
110X,6HM N,4X,FREQ,5X,17HCROSS SPEC, DENS,5X,11HDB(RE),10002
215X,5X,5HPMN(TAU),1/12X,E11.5,14X,E14.6,E12.6,6E12.6,2E15.2X,2E6,0
3X,E14.8X4X2E14.8 )

C 2004 FORMAT(1H1,4X,11HJ,8X,3MTAU,12X,1HJ,10X,5HXP(J),9X,5HYP(J),9X,
15H2P(J),11X,11HCRRBAR,1X,J1,10X,12HNN,FACTOR,16X,3E10,5
24X,15X,3E14.6,6E20.8,2E22.8/125X,15X,3E14.6,6E20.8,2E22.8 )

C 2005 FORMAT(1H0,50X,12HINTEGRATIONS //35X,9HINT, NUM,5X,8X MESH,8
15X,5HM N,10X,4HSM,16X,4HSM1)

C 2100 FORMAT(1H1,4X,16.8)

C COMMON /AAA/A(4000)

C EQUIVALENCE (A(1),MM),(A(2),MN),(A(3),IM),(A(4),IN),(A(5),XYZ),
1(A(6),KXYZ),(A(7),NAUTO),(A(8),MCRROSS),(A(9),N),(A(10),N)
EQUIVALENCE (A(21),XL1),(A(22),XLO),(A(23),YLL),(A(24),YLO),

187
TABLE 5B (Continued)

\[ I(A(25), A(26), B), I(A(27), C), I(A(28), \Theta A(29), P(2), A(30), P(1)) \]
\[ 2(A(31), X), I(A(32), Y), I(A(33), Z), I(A(34), \Theta), I(A(35), NMAX) \]
\[ 3(A(36), \Theta MN), I(A(37), \Theta AMN), I(A(38), \Theta SUM1), I(A(39), \Theta SUM2) \]
\[ 4(A(40), J_A), (A(41), J_B) \]
\[ \text{EQUIVALENCE (A(101), M(AM)), (A(151), N(ANH)), (A(1001), W), (A(2001), D(IX1)), 1(A(3001), D(IX5))} \]

\[ \text{DIMENSION TAV(110), X(50), Y(50), Z(50), N(50), M(50), W(50), D(IX1)(50), D(IX5)(50), CR(P)(50)} \]

\[ \text{INTEGER W, WMN, KMN} \]

\[ \text{CALL CLOCK(MTIME)} \]

\[ \text{CONVY = 60000} \]

\[ \text{DIMENSION TAV(110), X(50), Y(50), Z(50), N(50), M(50), W(50), D(IX1)(50), D(IX5)(50), CR(P)(50)} \]

\[ \text{INTEGER W, WMN, KMN} \]

\[ \text{CALL CLOCK(MTIME)} \]

\[ \text{CONVY = 60000} \]

\[ \text{DO 5 II = 1, XX, YY, ZZ, MXP, MTAV, EPS, NMAX} \]
\[ \text{DO 10 I = 1, MTAV} \]
\[ \text{DO 20 J = 1, MXP} \]
\[ \text{CRP = 0} \]
\[ \text{DO 30 K = 1, IM} \]
\[ \text{DO 30 L = 1, IN} \]
\[ \text{J(AM) = L} \]
\[ \text{IF (K(= EQ(AM)), J(AM))} \]

188
TABLE 5B (Continued)

\[
\begin{align*}
W &= \text{WAM}(J) \\
N &= \text{MAN}(L) \\
\text{WNN} &= \text{W}(K,L) \\
& \text{C} \\
\text{KMN} &= C \cdot \text{THETA} / \left( \text{B} \cdot \text{(B}^{2} \cdot \text{W}(K,L)^{2} \cdot \text{THETA}^{2}) \right) \\
\text{AMN} &= \text{B} \cdot \text{W}(K,L) \\
\text{CMN} &= \text{W}(K,L)^{2} / \left( \text{B}^{2} \cdot \text{W}(K,L) + \text{B}^{2} \cdot 1 \right) \\
\text{DMN} &= \text{A} \cdot \text{B} \cdot \text{W}(K,L) / \left( \text{B}^{2} \cdot 2 \cdot 1 \right) \\
\text{WBAR1} &= \text{B} \cdot \text{W}(K,L) / \left( \text{B}^{2} \cdot 2 \cdot 2 \cdot 1 \right) \\
\text{WBAR2} &= \text{B} \cdot \text{WBAR1} / \left( \text{B}^{2} \cdot 2 \cdot 2 \cdot 1 \right) \\
\text{CBAR} &= \text{KMN} \cdot \text{W}(K,L) / \text{P2} \\
& \text{C} \\
\text{WRITE}(6,2010) \text{ KNAN4CMNNDNNWBAR1,WBAR2,CB} \\
& \text{WRITE}(6,2005) \\
& \text{C} \\
\text{IF} \ (J \neq 1) \ \text{GO TO} \ 31 \\
& \text{C} \\
X &= X \cdot X \\
Y &= Y \cdot Y \\
Z &= Z \cdot Z \\
& \text{C} \\
\text{PHASE} &= 1 \cdot 5708 \\
\text{KKK} &= 1 \\
\text{AMN} &= -\text{AMN} \\
\text{CALL DINT}(\text{AMN,PHASE,KKK}) \\
\text{DIX1}(K,L) &= \text{SUM2} \\
& \text{C} \\
\text{PHASE} &= 0.5 \\
\text{KKK} &= 5 \\
\text{CALL DINT}(\text{AMN,PHASE,KKK}) \\
\text{DIX2}(K,L) &= \text{SUM2} \\
& \text{C} \\
\text{AMN} &= -\text{AMN} \\
& \text{C} \\
31 &= \text{XSP}(J) \\
Y &= Y \cdot Y \\
Z &= Z \cdot Z \\
\end{align*}
\]
TABLE 5B (Continued)

C
PHASE = 1, 5708
KKK = 2
CALL DINT(AMN, PHASE, KKK)
CAP2 = SUM2

C
PHASE = 0,
KKK = 4
CALL DINT(AMN, PHASE, KKK)
CAP4 = SUM2

C
SP = +5E+20
PHI = +5E+20

C
IF (NOR(J)) 39, 32, 32

C
39 SP = CBAR + (WBARI*DIX1(K+L)*CAP2 + WBAR2*DIX1(K+L)*CAP4 - WBAR2*
1DIX5(K+L)*CAP2 + WBARI*DIX5(K+L)*CAP4)

C
IF (SP) 38, 38, 33

C
38 PHI = 0,
GO TO 32

C
33 PHI = 10*X ALOG10(SP) + 12746

C
32 TH1 = CMN*COS(W(K+L)*TAV(I)) + DMN*SIN(W(K+L)*TAV(I))
TH2 = CMN*SIN(W(K+L)*TAV(I)) - DMN*COS(W(K+L)*TAV(I))

C
RPN = KMN*EXP(-AMN*TAV(I))*TH1*DIX1(K+L)*CAP2 + TH2*DIX1(K+L)*
1CAP4 - TH2*DIX5(K+L)*CAP2 + TH1*DIX5(K+L)*CAP4

C
CRP = CRP + RPN

C
34 WRITE(6, 2002) I, TAV(I), J, XP(J), YP(J), ZP(J), H, N, W(K+L), SP, PHI, RPN

C
30 CONTINUE
TABLE 5B (Continued)

C IF (I+J) EQ 2) CNORM = CRP
CRPBAR(J) = CRP / CNORM
20 CONTINUE
C
WRITE(6,2004) ITAV(I)(J), XP(J), YP(J), ZP(J), CRPBAR(J), CNORM,
1 J=1 MXP)
C
10 CONTINUE
C
5 CONTINUE
C
CALL CLOCK(NTIME)
TIME = (NTIME - ITIME) / CONVT
WRITE(6,2100) TIME
C
RETURN
END

SUBROUTINE CROSS2
C CROSS CORRELATION (TIME) (AUTO CORRELATION INCLUDED FOR THE FIRST POINT)
C
1000 FORMAT(3E12.6,13.2E12.8,I3,E6.2)
1001 FORMAT(2(E12.6,14))
1002 FORMAT(12F6.0)
C
2000 FORMAT(3E12.6,E13.2E12.8,E6.2Z13)
2001 FORMAT(12H )
2002 FORMAT( 1H0,30X,2HXP,20X,2HYP,20XZHZP.1OX,3HNORM(23XE14.8,SE14.88)
19898X#E14*895X#13 2) 5170

SUBROUTINE CR0552
C CROSS CORRELATION (TIME)
C AUTO CORRELATION INCLUDED FOR THE FIRST POINT
C
1000 FORMAT(3E12.6,13.2E12.8,13X6.2)
1001 FORMAT(2(E12.6,14))
1002 FORMAT(12F6.0)
C
2000 FORMAT(3E12.6,E13.2E12.8,E6.2Z13)
2001 FORMAT(12H )
2002 FORMAT( 1H0,30X,2HXP,20X,2HYP,20XZHZP.1OX,3HNORM(23XE14.8,SE14.88)
19898X#E14*895X#13 2) 5170

101
TABLE 5B (Continued)

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</tr>
</tbody>
</table>

C

COMMON /AAA/A (4000)


1(A(6), XYZ) + (A(7), NAUTO) + (A(8), MCRSS) + (A(9), M1) + (A(10), N) +
2(A(11), XYZ) + (A(12), MCRSS)

EQUVALENCE (A(21), XLI) + (A(22), XLO) + (A(23), YL1) + (A(24), YLO)

1(A(25), A(1), (A(26), B1) + (A(27), C1) + (A(28), THEA) + (A(29), P2) + (A(30), P1)
2(A(31), X1) + (A(32), Y1) + (A(33), Z1) + (A(34), EPS1) + (A(35), NMAX)
3(A(36), WNN, (A(37), AMN) + (A(38), SUM1) + (A(39), SUM2) +
4(A(40), JA) + (A(41), JB)

EQUVALENCE (A(101), MAM) + (A(151), NAM) + (A(1001), W) + (A(2001), DIX1)
1(A(3001), DIX1)

C

DIMENSION MAM(50), NAM(50), W(20,50), RP(110), RPBAR(110), PRP(110)
TABLE 5B (Continued)

1WA(110),FI(110)
DIMENSION XP(50),YP(50),ZP(50),DIX1(20,50),DIX5(20,50),NOR(50)
C INTEGER W#WMN#WA
REAL KMN
C CALL CLOCK(MTIME)
C CONVT = 60000,
C DO 5 II=1,XYZ
C READ (5,1000) XX,YYZZ,MXP,TAU,DTAU,NTAU,EPX,NMAX,NWA
C WRITE(6,2000) XX,YYZZ,MXP,TAU,DTAU,NTAU,EPX,NMAX,NWA
C ATAU = TAU
C READ (5,1001) (XP(I),YP(I),ZP(I),NOR(I), I=1,MXP.
C WRITE(6,2001) (XP(I),YP(I),ZP(I),NOR(I), I=1,MXP)
C IF (NWA) 2*1=2
C 2 READ (5,1002) (WA(K) K=1,NWA)
C WRITE(6,2010) (WA(K) K=1,NWA)
C 1 DO 10 J=1,MXP
C 6 DO IA=1,NTAU
C RP(IA) = 0
C 7 DO IA=1,NWA
C FI(IA) = 0
C
193
TABLE 5B (Continued)

```plaintext
DO 20 I=1:IM
DO 20 J=1:IN
JA = J
JB=J
IF (IM.EQ.MM) JA=I
M=MAM(JA)
N = NATAN(J)
WMN = W(I+J)
KMN = C*THETA / (B*(1+W(I+J)**2*THETA**2))
AMN = B*W(I+J)
CMN = W(I+J)*(1-6*AMN**2 + B**4)
DN = 4*AMN*(I+J)*(B**2-1)
WR:TE(6+2005)
IF (JN.EQ.1) GO TO 21
X=XX
Y=YY
Z=ZZ
PHASE =1*5708
KKK=1
AMN = -AMN
CALL DINT(AMN*PHASE*KKK)
DX1(I+J) = SUM2
PHASE = 0
KKK= 5
CALL DINT(AMN*PHASE*KKK)
DX5(I+J) = SUM2
AMN = -AMN
21 X=XP(JJ)
```
TABLE 5B (Continued)

\begin{verbatim}
Y = YP(JJ)
Z = ZP(JJ)

C PHASE = 1 + 5708
KKK = 2
CALL DIINT(AMN, PHASE, KKK)
CAP2 = SUM2

C PHASE = 0
KKK = 4
CALL DIINT(AMN, PHASE, KKK)
CAP4 = SUM2

C IF JJ NE 1 GO TO 24
WBAR1 = (B*2 + 2a) / (B*W(I,J) + (B*2 + 4a))
WBAR2 = B*WBAR1 / (B*2 + 2a)
CBAR = KMN*W(I,J)*P2

C SP = CBAR * (WBAR1*DIX1(I,J)*CAP2 + WBAR2*DIX1(I,J)*CAP4 - WBAR2*
1DIX5(I,J)*CAP2 + WBAR1*DIX5(I,J)*CAP4)
IF (SP) 11 = 1 + 22

11 PHI = 0

C GO TO 23
C PHI = 10** ALOG10(SP) + 127 + 6
C
23 WRITE(6, 2002) JJ, J, XP(JJ), YP(JJ), ZP(JJ), M, N, W(I,J), SP, PHI
GO TO 25
C
24 WRITE(6, 2003) JJ, J, XP(JJ), YP(JJ), ZP(JJ), M, N, W(I,J)
C
25 IF (NTAU) 32 = 31 + 32
32 TAU = ATAU
C
C DO 20 K = 1, NTAU
C
\end{verbatim}

195
TABLE 5B (Continued)

TH1 = CMN*CO(S(W(I,J)*TAU) + DMN* SIN(W(I,J)*TAU) 6660
TH2 = CMN*SIN(W(I,J)*TAU) - DMN*CO(S(W(I,J)*TAU) 6670
C RPN*KMN*EXP(-AMN*TAU)*(TH1*DIX1(I,J)*CAP2 + TH2*DIX1(I,J)*CAP4 - TH2* 6680
IDIX5(I,J)*CAP2 + TH1*DIX5(I,J)*CAP4) 6690
RP(K) = RP(K) + RPN 6700
WRITE(6,2004) K, TAU, RPN 6710
C 30 TAU = TAU + DTAU 6720
C 31 IF (ITA) 33s20s33 6730
C 33 WRITE(6,2013) 6740
C DO 19 K = 1, NWA 6750
RAD = WA(K) / (2**PI) 6760
DD = (B**2 + W(I,J)*2 + (W(I,J) - WA(K))*2) 6770
1(W(I,J) + WA(K))**2) 6780
AAMN = (B**2 + W(I,J)*2 + W(I,J)*2 + WA(K)*2) / DD 6790
BBMN = (W(I,J)*2 + W(I,J)*2 - WA(K)*2) / DD 6800
C TW1 = CMN*AAMN + DMN*BBMN 6810
TW2 = CMN*BBMN - DMN*AAMN 6820
C FIT = KMN*(TW1*DIX1(I,J)*CAP2 + TW2*DIX1(I,J)*CAP4 - TW2*DIX5(I,J)*CAP4) 6830
C FAT = FIT 6840
IF (FIT) 77s77s17 6850
77 P0 = 0 6860
FIT = 0 6870
GO TO 18 6880
17 PHI = 10s*ALOG10(FIT) + 127s6 6890
18 WRITE(6,2014) K, WA(K), RAD, FAT, PHI 7000
19 FIT(K) = FI(K) + FIT 7010

TABLE 5B (Continued)

20 CONTINUE
   IF (NTAU) 47*45*47
   WRITE(6*2007) RP(1)
   TAU = ATAU
   DO 40 K=1,NTAU
      RPBAR(K) = RP(K)/RP(1)
      PRP(K) = RP(K)/P2
   WRITE(6*2008) K, TAU, RPBAR(K), PRP(K)
   40 TAU = TAU + DTAU
   CONTINUE
   IF (NWA) 49*10*49
   WRITE(6*2011)
   DO 50 K=1,NWA
      RAD = WA(K)/(2.0*PI)
      IF (FI(K)) 53*53*53
      PHI = 0
      GO TO 50
   PHI = 10.0*ALOG10(FI(K)) + 127.6
   WRITE(6*2012) K, WA(K), RAD, FI(K), PHI
   CONTINUE
   CALL CLOCK(MTIME)
   TIME = (NTIME-MTIME)/CONVT
   WRITE(6*2009) TIME
   RETURN
TABLE 5B (Continued)

END
$IBFTC J0INT

SUBROUTINE D1Nf (SAMN*3PHASE*KKS)

DOUBLE INTEGRATION SUBROUTINE

COMMON /AAA/A(4000)

EQUIVALENCE (A(9),M),(A(10),N),(A(21),XL1),(A(22),XLO),(A(23),YL1)
1,(A(24),YLO),(A(34),EPS),(A(35),NMAX),(A(38),SUM1),(A(39),SUM2)

DIMENSION P(519),W(5)

DATA (P(I),I=1,5) /453089923.0,453089923.0,269234655.0,1384633432.0,239314351.0 /

2000 FORMAT (1H0,2E8H ***** NO CONVERGENCE *****,5X,14,1X,5X,14,1X,15,12E18+8)
2002 FORMAT (1H0,2E8H)

NK=2
BINC1=XL1-XLO
BINC2=YL1-YLO

DO 25 K=1,NMAX
SUM1=SUM2
SUM2=0.0
CNK = NK
DINC1 = BINC1/CNK
DINC2 = BINC2/CNK
HI = XLO + DINC1/2.
25 CONTINUE

198
TABLE 5B (Continued)

```
DO 26 I=1+NK
   H2 = YLO + DINC2/2.
C
DO 27 J=1+NK
DO 28 K=1+5
   ABS1=DINC1*P(K)+H1
DO 29 L=1+5
   ABS2=DINC2*P(L)+H2
   SUN2=SUN2+SUM2*WW(W)+W(W)+F(N+ABS2*SUM+SPHASE)
   CONTINUE
27 H2=M+DINC2
26 H1=M+DINC1
   SUM2=SUN2*SUM1*DINC1*DINC2
C
   IF (K.EQ.0) GO TO 30
   IF (ABS((SUM1-SUM2)/SUM2).EQ.EPS) 31+31+30
30 NK=NK + 1
   CONTINUE
C
   WRITE*(6,2000) KKS,SNK1*SUM2*SUM1
   SUM2 = 0
   GO TO 32
C
31 WRITE*(6,2002) KKS,SNK1*SUM2*SUM1
C
32 RETURN
END
$IBFTC XFFF
FUNCTION FFF(XO*YO*FAMN*FPHASE)
C
C
DOUBLE INTEGRATION FUNCTION
C

199
```
TABLE 5B (Continued)

C COMMON /AAA/A(4000)
C EQUIVALENCE (A(7)+AUTO)+(A(8)+NCRROSS)+(A(9)+N)+(A(10)+N),
1(A(21)+X1)+(A(22)+X0)+(A(23)+Y1)+(A(24)+Y0)+(A(25)+AI),
2(A(30)+PI)+(A(31)+X1)+(A(32)+Y)+(A(33)+Z)+(A(36)+WMN)+(A(40)+JA),
3(A(41)+JB)
C EQUIVALENCE (A(20)+AMM)+(A(25)+ANN)+(A(30)+BMM)+(A(35)+BNN),
1(A(40)+CMN)+(A(45)+CMM)+(A(50)+DMM)+(A(55)+DNN)+(A(60)+ALMM),
2(A(65)+ALNN)
C DIMENSION AMM(50),ANN(50),BMM(50),BNN(50),CMN(50),CMM(50),DMM(50),
1DNN(50),ALMM(50),ALNN(50)
C INTEGER WW,WMN
C R0*SQRT((X-X0)**2+(Y-Y0)**2 + Z**2)
C PHX = AMM(JA)*SIN(ALMM(JA)*X0) + BMM(JA)*COS(ALMM(JA)*X0) +
1CMN(JA)*SINH(ALMN(JA)*X0) + DMM(JA)*COSH(ALMM(JA)*X0)
C PHY = ANN(JB)*SIN(ALNN(JB)*Y0) + BNN(JB)*COS(ALNN(JB)*Y0) +
1CMM(JB)*SINH(ALMN(JB)*Y0) + DNN(JB)*COSH(ALMM(JB)*Y0)
C FFF = PHX*PHY*EXP((IFMNH/Al)*R0) * SIN((WMN/AI)*R0+RPHASE)/ R0
C RETURN
C END
APPENDIX D

UNDERWATER SOUND LABORATORY PROGRAM (STRAWDERMAN)

APPENDIX D1 – MATHEMATICAL ANALYSIS
APPENDIX D2 – METHOD FOR DETERMINING INPUT DATA
APPENDIX D3 – PROGRAM IDENTIFICATION
APPENDIX D4 – TEST RUNS
NOTATION

\( A \)
Equal to \( 0.75 \times 10^{-5} \ a^2 \rho_0^2 / \sigma_0^3 \)

\( a \)
Plate and acoustic cavity dimension in \( z \)-coordinate (longitudinal) direction

\( b \)
Plate and acoustic cavity dimension in \( y \)-coordinate (lateral) direction

\( b^+ \)
Dimensionless plate and acoustic cavity dimension defined in Equation (D68)

\( C_i \)
Arbitrary constants

\( c \)
Speed of sound in acoustic medium

\( c^+ \)
Dimensionless speed of sound defined in Equation (D77)

\( D \)
Plate flexural rigidity

\( D^+ \)
Dimensionless plate flexural rigidity

\( d \)
Acoustic cavity dimension in \( z \)-coordinate (depth) direction

\( d^+ \)
Dimensionless cavity dimension defined in Equation (D77)

\( E \)\{\}
Denotes ensemble average

\( F_{qsrt}(x_1, x_2, y_1, y_2) \)
Defined by Equation (D51)

\( f_{qrs}(\Omega) \)
Defined by Equation (D52)

\( G_{ns} \)
Defined subsequent to Equation (D21b)

\( G_{mn}(\omega^+) \)
Defined by Equation (D72)

\( G_{jskmn}(x_1, x_2, y_1, y_2) \)
Defined by Equation (D54)

\( \gamma_{jkmn}(x_1, x_2, \omega) \)
Defined by Equation (D55)

\( H(x,x',y,y',\omega) \)
Complex frequency response of plate

\( h(x,x',y,y',\Theta) \)
Plate displacement response to a unit impulsive force

\( i \)
Square root of \(-1\)

\( K_{jkmnqrs} \)
Defined by Equation (D60)

\( k \)
Acoustic wave number defined in Equation (D83)

\( k_x \)
Acoustic wave number in the \( x \)-coordinate direction

\( k_y \)
Acoustic wave number in the \( y \)-coordinate direction

\( k_z, k_j^i(k) (\omega) \)
Acoustic wave number in the \( z \)-coordinate direction
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{jk}^+ (\omega^+)$</td>
<td>Dimensionless acoustic wave number in the $z$-coordinate direction</td>
</tr>
<tr>
<td>$M$</td>
<td>Dimensionless fluid mass defined by Equation (D68)</td>
</tr>
<tr>
<td>$m, n, \text{etc.}$</td>
<td>Mode numbers</td>
</tr>
<tr>
<td>$P_n$</td>
<td>Defined subsequent to Equation (D21b)</td>
</tr>
<tr>
<td>$P_n^+ (\omega^+)$</td>
<td>Defined by Equation (D73)</td>
</tr>
<tr>
<td>$P_t$</td>
<td>Turbulent boundary layer wall pressure</td>
</tr>
<tr>
<td>$P_a$</td>
<td>Cavity acoustic pressure</td>
</tr>
<tr>
<td>$Q_a(x_1, x_2, y_1, y_2, z_1, z_2)$</td>
<td>Cavity acoustic pressure cross correlation</td>
</tr>
<tr>
<td>$Q_{pp}(x_1, x_2, y_1, y_2, t)$</td>
<td>Plate pressure cross correlation</td>
</tr>
<tr>
<td>$Q_{\phi\phi}(x_1, x_2, y_1, y_2, t_1, t_2)$</td>
<td>Plate velocity cross correlation</td>
</tr>
<tr>
<td>$R_m$</td>
<td>Defined subsequent to Equation (D21b)</td>
</tr>
<tr>
<td>$R_m^+ (\omega^+)$</td>
<td>Defined by Equation (D69)</td>
</tr>
<tr>
<td>$r$</td>
<td>Effective plate damping coefficient per unit area</td>
</tr>
<tr>
<td>$r_{mn}$</td>
<td>Critical plate damping coefficient for the $m-n^{th}$ mode</td>
</tr>
<tr>
<td>$r^+$</td>
<td>Dimensionless plate damping coefficient defined in Equation (D69)</td>
</tr>
<tr>
<td>$r_{mn}^+$</td>
<td>Dimensionless critical plate damping coefficient for the $m-n^{th}$ mode</td>
</tr>
<tr>
<td>$S_a(x_1, x_2, y_1, y_2, z_1, z_2, \omega)$</td>
<td>Cavity acoustic pressure cross spectral density</td>
</tr>
<tr>
<td>$S_{pp}(\xi, \eta, \omega)$</td>
<td>Turbulent wall pressure cross spectral density</td>
</tr>
<tr>
<td>$S_{\phi\phi}(x_1, x_2, y_1, y_2, \omega)$</td>
<td>Plate velocity cross spectral density</td>
</tr>
<tr>
<td>$T_{mn}$</td>
<td>Defined subsequent to Equation (D21b)</td>
</tr>
<tr>
<td>$T_{mn}^+ (\omega^+)$</td>
<td>Defined by Equation (D74)</td>
</tr>
<tr>
<td>$t$</td>
<td>Time coordinate</td>
</tr>
<tr>
<td>$t'$</td>
<td>Time at which impulsive force occurs</td>
</tr>
<tr>
<td>$U_0$</td>
<td>Free stream velocity of flowing fluid in $x$-coordinate direction</td>
</tr>
</tbody>
</table>
\( U_c \) Mean convection velocity of turbulent boundary layer

\( U^+ \) Dimensionless free stream velocity defined by Equation (D68)

\( U_{kmn}^{+(\omega^+)} \) Defined by Equation (D78)

\( \tilde{u} \) Acoustic phase velocity vector

\( u_x \) Acoustic phase velocity in the \( x \)-coordinate direction

\( u_y \) Acoustic phase velocity in the \( y \)-coordinate direction

\( u_z \) Acoustic phase velocity in the \( z \)-coordinate direction

\( V_{mnqs} \) Defined subsequent to Equation (D21b)

\( W_{mnqs} \) Defined by Equation (D23)

\( W_{mnqs}^{+(\omega^+)} \) Defined by Equation (D75)

\( w \) Plate displacement in the \( z \)-coordinate direction

\( X_j(\omega) \) Defined by Equation (D40)

\( z \) Longitudinal spatial coordinate

\( z^+ \) Dimensionless longitudinal spatial coordinate defined by Equation (D88)

\( y \) Lateral spatial coordinate

\( y^+ \) Dimensionless lateral spatial coordinate defined by Equation (D88)

\( s \) Spatial coordinate normal to the plate

\( s^+ \) Dimensionless spatial coordinate defined by Equation (D77)

\( a \) A dimensionless constant

\( a_{mn}(x,y) \) Plate normalized natural mode shapes

\( \delta_\ell \) Kronecker delta

\( \delta(\omega - \Omega) \) Dirac delta function

\( \delta^* \) Turbulent boundary layer displacement thickness

\( \delta^+ \) Dimensionless turbulent boundary layer displacement thickness defined by Equation (D88)

\( \xi(x,x',y,y',\theta) \) Plate velocity response to a suit impulsive force

\( \eta \) Relative lateral coordinate \((y - y')\)

\( \theta \) Relative time coordinate \((t - t')\)
\( \lambda_{mn} \)  
Phase angle defined subsequent to Equation (D21b)

\( \mu \)  
Effective mass of plate per unit area

\( \nu_m \)  
Phase angle defined subsequent to Equation (D21b)

\( \xi \)  
Relative longitudinal coordinate \( (x - x') \)

\( \rho_f \)  
Mass density of flowing fluid

\( \rho_{a_0} \)  
Time average mass density of acoustic medium

\( \rho_a \)  
Instantaneous mass density of acoustic medium

\( r \)  
Time difference \( (t_2 - t_1) \)

\( \Phi(\omega) \)  
Turbulent wall pressure spectral density

\( \phi^*(\omega) \)  
Dimensionless turbulent wall pressure spectral density

\( \Phi_\phi(x, y, \omega) \)  
Plate velocity spectral density

\( \Phi_\phi^*(x^+, y^+, \omega^+) \)  
Dimensionless plate velocity spectral density

\( \Phi_a(x, y, z, \omega) \)  
Cavity acoustic pressure spectral density

\( \Phi_a^*(x^+, y^+, z^+, \omega^+) \)  
Dimensionless cavity acoustic pressure spectral density

\( \phi \)  
Plate velocity

\( \psi \)  
Acoustic velocity potential

\( \omega \)  
Radial frequency

\( \omega^+ \)  
Dimensionless radial frequency

\( \omega_{mn} \)  
Natural frequency of \( m-n \) mode of plate

\( \omega_{mn}^+ \)  
Dimensionless natural frequency of \( m-n \) mode of plate

\( \nabla^{4} \)  
Biharmonic operator

\( X(\omega) \)  
Defined by Equation (D53)

\( \Omega \)  
Radial frequency
APPENDIX D1 – MATHEMATICAL ANALYSIS

The equations for the plate velocity and cavity acoustic pressure spectral densities and cross correlations are now derived for a plate (Figure 15) subject to turbulence excitation.

The differential equation governing the displacement of the plate due to turbulent boundary layer pressure excitation on the plate surface is

$$D \psi^4 w + r \frac{\partial w}{\partial t} + \mu \frac{\partial^2 w}{\partial t^2} = p_t(x, y, z)$$  \hspace{1cm} (D1)

Following Dyer (see Appendix A), the equation for the free undamped plate

$$D \psi^4 w + \mu \frac{\partial^2 w}{\partial t^2} = 0$$  \hspace{1cm} (D2)

has the normal mode solution, satisfying the simply supported edge conditions (Figure 15), given by

$$w(x, y, t) = a_{mn}(x, y) \sin \omega_{mn} t$$  \hspace{1cm} (D3)

where

$$a_{mn}(x, y) = \frac{2}{ab \sqrt{1 - \frac{m^2}{a^2} \sin^2 \frac{mn}{a}} \sin \frac{nx}{a}}$$  \hspace{1cm} (D4)

$$\omega_{mn} = \sqrt{\frac{D}{\mu}} \left[ \left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2 \right]$$  \hspace{1cm} (D5)

and

$$\int_0^b \int_0^c a_{mn}(x, y) a_{qr}(x, y) \, dx \, dy = \delta_{m,q} \delta_{n,r}$$  \hspace{1cm} (D6)

The solution to Equation (D1) for any deterministic pressure is then assumed to be

$$w(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn}(x, y) T_{mn}(t)$$  \hspace{1cm} (D7)
Figure 15 – Illustration of the Theoretical Model
Substituting Equation (D7) in (D1) and using Equation (D6), we find $T_{mn}(t)$ must satisfy

$$
\frac{d^2 T_{mn}}{dt^2} + \frac{r}{\mu} \frac{dT_{mn}}{dt} + \omega_m^2 T_{mn} = \frac{1}{\mu} \int_0^b \int_0^a p_{1}(x,y,t) a_m(x,y) \, dx \, dy. \quad (D8)
$$

For later use, we consider the following two cases.

CASE I: Concentrated load applied at $(x',y')$ varying sinusoidally in time

$$
p_{1}(x,y,t) = \delta(x - x') \delta(y - y') e^{i\omega t} \quad (D9)
$$

Substituting Equation (D9) in (D8) results in the following solution for Equation (D7):

$$
\omega(x,y,t) = \sum_{m = 1}^{\infty} \sum_{n = 1}^{\infty} \frac{a_{mn}(x,y) a_{mn}(x',y')}{\mu \left( \frac{\omega_m^2 - \omega^2}{\mu} + i\mu \right)} e^{i\omega t} = H(x,x',y,y',\omega) e^{i\omega t} \quad (D10)
$$

where $H(x,x',y,y',\omega)$, the complex frequency response, is

$$
H(x,x',y,y',\omega) = \frac{1}{\mu} \sum_{m = 1}^{\infty} \sum_{n = 1}^{\infty} \frac{a_{mn}(x,y) a_{mn}(x',y')}{\mu \left( \frac{\omega_m^2 - \omega^2}{\mu} + i\mu \right)} \quad (D11)
$$

CASE II: Impulsive loading at time $t'$ applied at $(x',y')$

$$
p_{1}(x,y,t) = \delta(x - x') \delta(y - y') \delta(t - t') \quad (D12)
$$

Define

$$
\theta = t - t'
$$

$$
h(x,x',y,y',\theta) = \begin{cases} 
\omega(x,y,t) & \theta > 0 \\
0 & \theta \leq 0
\end{cases}
$$

Substituting Equation (D12) in (D8) results in the following solution for Equation (D7):
By superposition, the response for any deterministic pressure field may be written

\[
\psi(x, y, t) = \int_{-\infty}^{t} \int_{0}^{b} \int_{0}^{a} \rho(x', y', t') h(x, x', y, y', \theta) \, dx' \, dy' \, dt'.
\]

(D14)

Since the velocity of the plate, rather than displacement, is required for the boundary value in the acoustic problem, we define the velocity response of the plate to impulse loading thus

\[
\zeta(x, x', y, y', \theta) = \frac{-i \theta}{\mu} \sum_{m,n=1}^{\infty} \frac{a_{mn}(x, y) a_{mn}(x', y')}{\sqrt{\omega_{mn}^2 - \left(\frac{r}{2\mu}\right)^2}} \sin \theta \sqrt{\omega_{mn}^2 - \left(\frac{r}{2\mu}\right)^2}
\]

(D15)

And we define the velocity field of the plate as

\[
\phi(x, y, t) = \frac{\partial \psi(x, y, t)}{\partial t} = \int_{0}^{b} \int_{0}^{a} \rho(x', y', t-\theta) \delta(x, x', y, y', \theta) \, dx' \, dy' \, d\theta
\]

(D16)

We define the turbulent wall pressure cross correlation by (Ε denotes the ensemble average):

\[
Q_{pp}(x_1, x_2, y_1, y_2, t) = \varepsilon \left[ \rho(x_1, y_1, t) \rho(x_2, y_2, t) \right]
\]

(D17)
The plate velocity cross correlation is then

\[ Q_{\phi}(x_1, x_2, y_1, y_2, t_1, t_2) = E[\phi(x_1, y_1, t_1) \phi(x_2, y_2, t_2)] = E \left[ \int_0^b \int_0^b \int_0^b \int_0^b \int_0^\infty \int_0^\infty \cdot \right. \]

\[ \left. \cdot p_1(x_1', y_1', t_1 - \theta_1) p_2(x_2', y_2', t_2 - \theta_2) \delta(x_1, x_1', y_1, y_1', \theta_1) \right] \]

\[ = \int_0^b \int_0^b \int_0^b \int_0^\infty \int_0^\infty Q_{pp}(x_1', x_2', y_1', y_2', t_1 - \theta_1, t_2 - \theta_2) \zeta(x_1, x_1', y_1, y_1', \theta_1) \]

\[ = \int_0^b \int_0^b \int_0^b \int_0^\infty \int_0^\infty Q_{pp}(x_1', x_2', y_1', y_2', \theta_2) d\theta_2 \ dx_1' \ dy_1' \ dz_2' \ dz_2' \ dy_2' \]

\[ = \int_0^b \int_0^b \int_0^b \int_0^\infty \int_0^\infty Q_{pp}(x_1', x_2', y_1', y_2', \theta_2) \zeta(x_1, x_1', y_1, y_1', \theta_1) \]

\[ \zeta(x_1', x_2', y_1', y_2', \theta_2) d\theta_2 \ dx_1' \ dy_1' \ dz_2' \ dz_2' \ dy_2' \] (D18)

In obtaining Equation (D18), we took account of the fact that since the plate velocity impulse function is not a random quantity, the ensemble average applied only to the turbulent pressure field. Also, since the turbulent boundary layer pressure is assumed to be a homogeneous stationary process, \( Q_{pp} \) is a function of the difference between the spatial and temporal coordinates rather than the coordinates themselves.\(^{33}\)

The plate velocity cross spectral density is then (multiplying and dividing Equation (D18) by \( e^{-i\omega(\theta_1 - \theta_2)} \) to obtain the third member below):

210
\[ S_{\phi \phi}(x_1, x_2, y_1, y_2, \omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} Q_{\phi \phi}(x_1, x_2, y_1, y_2, r) e^{-i\omega r} dr = \]
\[
\int_{0}^{b} \int_{0}^{a} 
\int_{0}^{b} \int_{0}^{a} \int_{0}^{\infty} \left\{ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} Q_{pp}(\xi', \eta', r+\theta_1 - \theta_2) e^{-i\omega(r+\theta_1 - \theta_2)} \right\}
\]
\[
\cdot d(r + \theta_1 - \theta_2) \zeta(x_1, x_1', y_1, y_1', \theta_1) e^{i\omega \theta_1}, \zeta(x_2, x_2', y_2, y_2', \theta_2) \cdot e^{-i\omega \theta_2} d\theta_1 d\theta_2 dx'_1 dy'_1 dx'_2 dy'_2 = \]
\[
\int_{0}^{b} \int_{0}^{a} \int_{0}^{b} \int_{0}^{a} \int_{0}^{\infty} \zeta(x_1, x_1', y_1, y_1', \omega) \int_{0}^{\infty} \zeta(x_2, x_2', y_2, y_2', \theta_1) e^{i\omega \theta_1} d\theta_1 .
\]
\[
\cdot \int_{0}^{\infty} \zeta(x_2, x_2', y_2, y_2', \theta_2) e^{-i\omega \theta_2} d\theta_2 dx'_1 dy'_1 dx'_2 dy'_2.
\]
\[
= \int_{0}^{b} \int_{0}^{a} \int_{0}^{b} \int_{0}^{a} \omega^2 S_{pp}(\xi', \eta', \omega) H(x_1, x_1', y_1, y_1'-\omega) H(x_2, x_2', y_2, y_2'-\omega) dx'_1 dy'_1 dx'_2 dy'_2
\]

(D19)

where, since \( \zeta(x, x', y, y', \omega) \) is zero for \( \theta \leq 0 \), the semi-infinite limits in \( \theta_1 \) and \( \theta_2 \) have been replaced by infinite limits and as can be shown (see Equation 2.11 of Reference 33)

\[
\int_{\infty}^{\infty} \zeta(x_2, x_2', y_2, y_2', \theta_2) e^{-i\omega \theta_2} d\theta_2 = i\omega H(x_2, x_2', y_2, y_2'-\omega)
\]

\[
\int_{-\infty}^{\infty} \zeta(x_1, x_1', y_1, y_1', \theta_1) e^{i\omega \theta_1} d\theta_1 = -i\omega H(x_1, x_1', y_1, y_1'-\omega)
\]

(D20)
The mathematical model for $S_{pp}$ used by Strawderman and discussed in detail in Reference 32 is*

$$\begin{align*}
S_{pp}(\xi, \eta, \omega) &= 0.75 \times 10^{-5} \rho_f^2 \nu_0^3 \delta^3 \left[ e^{-0.115 \left( \frac{\omega \xi}{U_c} \right)} \right. \\
&\quad \left. + 0.7 \left( \frac{\omega \eta}{U_c} \right) - i \left( \frac{\omega \xi}{U_c} \right) \right] e^{-\omega \xi / U_c} ; \omega \leq 1.256 \frac{U_0}{\delta^*} \\
&= 1.5 \times 10^{-5} \rho_f^2 \nu_0^6 \left[ e^{-0.115 \left( \frac{\omega \xi}{U_c} \right)} \right. \\
&\quad \left. + 0.7 \left( \frac{\omega \eta}{U_c} \right) - i \left( \frac{\omega \xi}{U_c} \right) \right] e^{-\omega \xi / U_c} ; \omega > 1.256 \frac{U_0}{\delta^*}
\end{align*}$$

(D21a)

where $\alpha = 1.0$ for water and $\alpha = 3.0$ for air.

Substituting Equation (D21a) in (D19) and using Equations (D11) and (D4), we obtain after extensive, but routine, simplification (see Reference 32 for details):

$$\begin{align*}
S_{\phi\phi}(x_1, x_2, y_1, y_2) &= \frac{16\Delta \omega^2}{\mu^2 \alpha^2 b^2} \sum_{m=1}^{\infty} \sum_{q=1}^{\infty} \frac{\sin(m \alpha x_1)}{a} \frac{\sin(n \beta y_1)}{b} \frac{\sin(q \alpha x_2)}{a} \frac{\sin(s \beta y_2)}{b} \cdot G_{ns} V_{mnqs} ; \omega \leq 1.256 \frac{U_0}{\delta^*} \\
S_{\phi\phi}(x_1, x_2, y_1, y_2) &= \frac{32\Delta \omega^2}{\kappa^2 a^2 b^2} \left( \frac{U_0}{\delta^*} \right)^3 \sum_{m=1}^{\infty} \sum_{q=1}^{\infty} \frac{\sin(m \alpha x_1)}{a} \frac{\sin(n \beta y_1)}{b} \frac{\sin(q \alpha x_2)}{a} \frac{\sin(s \beta y_2)}{b} \cdot G_{ns} V_{mnqs} ; \omega > 1.256 \frac{U_0}{\delta^*} \\
&= \frac{16\Delta \omega^2}{\mu^2 \alpha^2 b^2} \sum_{m=1}^{\infty} \sum_{q=1}^{\infty} \frac{\sin(m \alpha x_1)}{a} \frac{\sin(n \beta y_1)}{b} \frac{\sin(q \alpha x_2)}{a} \frac{\sin(s \beta y_2)}{b} \cdot G_{ns} V_{mnqs} ; \omega \leq 1.256 \frac{U_0}{\delta^*} \\
&= \frac{32\Delta \omega^2}{\kappa^2 a^2 b^2} \left( \frac{U_0}{\delta^*} \right)^3 \sum_{m=1}^{\infty} \sum_{q=1}^{\infty} \frac{\sin(m \alpha x_1)}{a} \frac{\sin(n \beta y_1)}{b} \frac{\sin(q \alpha x_2)}{a} \frac{\sin(s \beta y_2)}{b} \cdot G_{ns} V_{mnqs} ; \omega > 1.256 \frac{U_0}{\delta^*} \\
(D21b)
\end{align*}$$

*Equation (D21a) represents a mathematical fit to the Corcos model which is based on experimental data. The Skudrzyk and Haddle expression for the turbulent wall pressure spectral density $\phi(\omega)$ is incorporated in this model. See Equation (3.1) of Reference 32.
where

\[ A = 0.75 \times 10^{-5} \, a^{2} \rho \, \frac{U_{o}^{3} \delta^{*}}{f} \]

\[ T_{m,n} = \sqrt{\left( \omega_{m,n}^{2} - \omega^{2} \right)^{2} + \left( \frac{r\omega}{\mu} \right)^{2}} \quad \text{; similarly for } T_{q,s} \]

\[ P_{n} = \left( 0.7 \, \frac{\omega}{U_{c}} \right)^{2} + \left( \frac{m_{n} \rho}{b} \right)^{2} \quad \text{; similarly for } P_{s} \]

\[ R_{m} = \sqrt{\left( \frac{m_{n} \rho}{a} \right)^{2} - 0.987 \, \left( \frac{\omega}{U_{c}} \right)^{2} + 0.0529 \, \left( \frac{\omega}{U_{c}} \right)^{4}} \quad \text{; similarly for } R_{q} \]

\[ G_{n,s} = 0.35 \, \frac{\omega b}{U_{c}} \, \delta_{ns} \left[ 2 \left( 0.7 \, \frac{\omega}{U_{c}} \right)^{2} + \left( \frac{m_{n} \rho}{b} \right)^{2} + \left( \frac{s_{n} \rho}{b} \right)^{2} + \frac{n_{s} \rho^{2}}{b^{2}} \left[ 1 - \delta_{n,s} \right] \left[ (-1)^{n} \left(-1\right)^{s} - 1 \right] \right. \]

\[ + \frac{n_{s} \rho^{2}}{b^{2}} \left[ 2 - \left[ (-1)^{n} + (-1)^{s} \right] e^{-0.7 \left( \frac{\omega b}{U_{c}} \right)} \right] \left\{ \right. \]

\[ V_{m,n,q,s} = \delta_{m,q} \quad \frac{\omega a}{U_{c}} \quad R_{m} \left( \nu_{m} - 0.463 \pi \right) \]

\[ + \left( 1 - \delta_{m,q} \right) \frac{m_{n} \rho^{2}}{a^{2}} \left[ (-1)^{m} \left(-1\right)^{q} - 1 \right] \left[ \frac{R_{m} \, e^{i \nu_{q} - R_{q} \, e^{-i \nu_{m}}} + 2m_{n} \rho^{2}}{a^{2}} \right] \cos \left( \nu_{q} + \nu_{m} \right) \]

\[ - \frac{m_{n} \rho^{2}}{a^{2}} e^{-0.115 \left( \frac{\omega a}{U_{c}} \right) \left[ (-1)^{m} \, e^{-i \left( \frac{\omega a}{U_{c}} + \nu_{q} + \nu_{m} \right)} - i \left( \frac{\omega a}{U_{c}} + \nu_{q} + \nu_{m} \right) \right]} e^{i \lambda_{n,q,s}} \]

\[ \nu_{m} = \tan^{-1} \left( \frac{m_{n} \rho^{2}}{a^{2}} \right) \quad \text{; similarly for } \nu_{q} \]

218
\[
\lambda_{mn} = \tan^{-1} \frac{n \omega}{\mu} \left( \frac{2}{\omega_{mn}^2 - \omega^2} \right) \quad \text{; similarly for } \lambda_{qs}
\]

The power spectrum of the plate velocity \( \Phi_{\phi}(x, y, \omega) \) is found as follows. First let \( x_1 = x_2 = x, y_1 = y_2 = y \) in Equations (D21b); since \( \Phi(x, y, \omega) \) must be a real, even function then the summations in the resultant equations must be real if they are to be considered as valid solutions for \( \Phi_{\phi}(x, y, \omega) \). Then substitute \( V_{mnqs} \) from Equation (D21b) in the equation for \( \Phi_{\phi}(x, y, \omega) \). After rearranging this equation and taking \( G_{ns} = G_{sn} \), since \( G_{ns} \) is a symmetrical matrix, we find (see Reference 32 for details):

\[
\Phi_{\phi}(x, y, \omega) = S_{\phi}(x_1, x_1, y_1, y_1(\omega)) = \frac{16 \alpha \omega^2}{\mu^2 a^2 b^2} \sum_{m=1}^{\infty} \sum_{q=1}^{\infty} \sin \frac{mnx}{a} \sin \frac{nmy}{b} \sin \frac{qnx}{a} \sin \frac{qny}{b} \frac{T_{mn} T_{qs} P_n P_s R_m R_q}{G_{ns} W_{mnqs}} \quad \omega \leq \frac{U_0}{\delta^*}
\]

\[
= \frac{32 \alpha^2}{\mu^2 a^2 b^2} \left( \frac{\omega \delta^*}{U_e} \right)^3 \frac{\omega a}{U_e} \delta_{mq} \cos (\nu_m - 0.468 \pi) \cos (\lambda_{mn} - \lambda_{qs})
\]

\[
+ \frac{2mqn^2}{a^2} \cos (\nu_m + \nu_q) (\cos \lambda_{mn} - \lambda_{qs}) + (1 - \delta_{mq}) \frac{mqn^2}{a^2} \left[ \left( \frac{m \pi}{a} \right)^2 - \left( \frac{qn \pi}{a} \right)^2 \right] \frac{(-1)^m (-1)^q - 1}{2}
\]

\[
+ [R_m \cos (\lambda_q - \lambda_{mn} - \lambda_{qs} - R_q \cos (\nu_m + \lambda_{qs} - \lambda_{mn})]
\]

(D22)
We now find the cavity acoustic pressure due to an arbitrary plate velocity distribution. From this we will obtain the cavity acoustic pressure cross correlation and spectral density.

We start with the equations governing acoustic phenomena and the boundary conditions for Figure 12.

Momentum equation: \[ \rho_a \frac{\partial \ddot{u}}{\partial t} + \nabla p_a = 0 \] (D24)

Continuity equation: \[ \frac{\partial \rho_a}{\partial t} + \rho_0 \nabla \cdot \ddot{u} = 0 \] (D25)

Equation of state: \[ p_a = c^2 \rho_a \] (D26)

Boundary conditions:
\[ u_x(0, y, z, t) = 0 \] (D27a)
\[ u_x(a, y, z, t) = 0 \] (D27b)
\[ u_y(x, 0, z, t) = 0 \] (D27c)
\[ u_y(x, b, z, t) = 0 \] (D27d)
\[ u_z(x, y, 0, t) = 0 \] (D27e)
\[ u_z(x, y, -d, t) = \phi(x, y, t) \] (D27f)

Since Equation (D24) was derived for an inviscid fluid, the acoustic field may be assumed to be irrotational. Hence the acoustic phase velocity may be defined in terms of the velocity potential
\[ \ddot{u}(x, y, z, t) = \nabla \psi(x, y, z, t) \] (D28)

Equation (D28) specifies \( \psi \) to within an arbitrary function of time. To uniquely specify \( \psi \) define, in addition
\[ p_a(x, y, z, t) = -\rho_0 \frac{\partial \psi(x, y, z, t)}{\partial t} \] (D29)

215
Substitution of Equations (D28) and (D29) into (D24) satisfies the latter equation. To satisfy Equations (D25) and (D26), we proceed as follows:

Substitute Equation (D26) into (D25) to obtain

$$\frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} + \rho \cdot \nabla \cdot \mathbf{u} = 0$$

(D30)

Substitute Equations (D38) and (D39) into (D30) to obtain the scalar wave equation in $\psi$

$$\nabla^2 \psi - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} = 0$$

(D31)

which, by virtue of the separation of variables technique, has the solution

$$\psi(x,y,z,t) = [C_1 \sin k_x x + C_2 \cos k_x x] \left[ C_3 \sin k_y y + C_4 \cos k_y y \right]$$

$$\left[ C_5 \sin k_z z + C_6 \cos k_z z \right] \psi(t) e^{ikct}$$

(D32)

where

$$k_x^2 + k_y^2 + k_z^2 = k^2$$

(D33)

Substituting the boundary conditions, Equations (D27a–e), into Equation (D32), using Equation (D28), we obtain

$$C_1 = C_3 = C_5 = 0$$

(D34)

$$k_x = \frac{j \pi}{a} \quad j = \text{integer}$$

(D35)

$$k_y = \frac{\ell \pi}{b} \quad \ell = \text{integer}$$

(D36)

Defining $\omega = k C$

(D37)

and using Equations (D34), (D35), and (D36), we have

$$\psi(x,y,z,t) = \sum_{j=0}^{\infty} \sum_{\ell=0}^{\infty} C_{2j} \cos \frac{j \pi x}{a} C_{4j} \cos \frac{\ell \pi y}{b} C_{6j} \cos (k_z \ell z) \psi(t) e^{i\omega t}$$

(D38a)

Combinir the constants, we assume that the solution for $\psi(x,y,z,t)$ has the form
\[ \psi(x, y, z, t) = \sum_{j=0}^{\infty} \cos \frac{j \pi x}{a} \cos \frac{\ell \pi y}{b} \int_{-\infty}^{\infty} Y_{j \ell}(\omega) \cos (k_{j \ell} z) e^{i \omega t} \frac{d\omega}{\sqrt{2\pi}} \]  (D38b)

Equation (D38b) satisfies the wave Equation (D31) as can be seen by substituting Equation (D38b) in (D31). Also, by applying the above arguments, Equations (D27a-e) are satisfied. It remains only to satisfy the boundary condition (D27f), \( u_z(x, y, -d, t) = \phi(x, y, t) \)

\[ = \frac{\partial \psi(x, y, z, t)}{\partial z} \bigg|_{z=-d} \]

Hence

\[ \phi(x, y, t) = \sum_{j=0}^{\infty} \cos \frac{j \pi x}{a} \cos \frac{\ell \pi y}{b} \int_{-\infty}^{\infty} X_{j \ell}(\omega) e^{i \omega t} \frac{d\omega}{\sqrt{2\pi}} \]  (D39)

where

\[ X_{j \ell}(\omega) = K_{j \ell} Y_{j \ell}(\omega) \sin (k_{j \ell} d) \]  (D40)

Multiplying both sides of Equation (D39) by \( \cos \frac{j \pi x}{a} \cos \frac{\ell \pi y}{b} \) and integrating over the area of the plate, we find by virtue of the orthogonality principle that*

\[ \int_{-\infty}^{\infty} X_{j \ell}(\omega) e^{i \omega t} \frac{d\omega}{\sqrt{2\pi}} = \frac{4}{ab(1 + \delta_{0j}) (1 + \delta_{0 \ell})} \int_{0}^{a} \int_{0}^{b} \phi(x, y, t) \cos \frac{j \pi x}{a} \cos \frac{\ell \pi y}{b} \, dx \, dy \]  (D41a)

*By L'Hôpital's rule, \( \lim_{j \to 0} \frac{\sin 2j \pi}{2j \pi} = \frac{2\pi}{2\pi} = 1 \), hence \( \lim_{j \to 0} \frac{\sin 2j \pi}{2j \pi} = \delta_{0j} \). Similarly for \( \delta_{0 \ell} \).
Transformation of Equation (D41a) yields

\[ X_{\ell}(\omega) = \frac{4}{ab(1+\delta_0)(1+\delta_0)} \int_{-\infty}^{\infty} \int_{0}^{b} \int_{0}^{a} \phi(x,y,t) \cos \frac{j_{nx}}{a} \cos \frac{j_{ny}}{b} \cdot e^{-i\omega t} dx dy dt \frac{d}{\sqrt{2\pi}} \]

(D41b)

Thus, from Equation (D40)

\[ Y_{\ell}(\omega) = \frac{4}{ab(1+\delta_0)(1+\delta_0)} k_{x_{1}\ell} \sin k_{y_{1}\ell} \int_{-\infty}^{\infty} \int_{0}^{b} \int_{0}^{a} \phi(x,y,t) \cos \frac{j_{nx}}{a} \cos \frac{j_{ny}}{b} e^{-i\omega t} dx dy dt \frac{d}{\sqrt{2\pi}} \]

(D42)

Substitution of Equation (D42) in (D38b) yields

\[ \psi(x,y,z,t) = \frac{2}{\pi ab} \sum_{j=0}^{\infty} \sum_{\ell=0}^{\infty} \cos \frac{j_{nx}}{a} \cos \frac{j_{ny}}{b} \int_{-\infty}^{\infty} \frac{\cos k_{x_{1}\ell} x}{(1+\delta_0)(1+\delta_0)} k_{y_{1}\ell} \sin k_{y_{1}\ell} d \int_{-\infty}^{\infty} \int_{0}^{b} \int_{0}^{a} \phi(x,y,t) \cos \frac{j_{nx}}{a} \cos \frac{j_{ny}}{b} \cdot e^{-i\omega t} dx dy dt e^{i\omega t} d\omega \]

(D43)

Substitution of Equation (D43) in (D29) yields the cavity acoustic pressure for an arbitrary, deterministic plate velocity

\[ p_{a}(x,y,z,t) = \frac{-2i\rho a_{0}}{\pi ab} \sum_{j=0}^{\infty} \sum_{\ell=0}^{\infty} \cos \frac{j_{nx}}{a} \cos \frac{j_{ny}}{b} \int_{-\infty}^{\infty} \frac{\omega \cos k_{x_{1}\ell} x}{(1+\delta_0)(1+\delta_0)} k_{y_{1}\ell} \sin k_{y_{1}\ell} d \int_{-\infty}^{\infty} \int_{0}^{b} \int_{0}^{a} \phi(x,y,t) \cos \frac{j_{nx}}{a} \cos \frac{j_{ny}}{b} e^{-i\omega t} dx dy dt e^{-i\omega t} d\omega \]

(D44)

If now \( \phi(x,y,t) \) is considered to be the only random quantity in \( p_{a} \), then the cavity acoustic pressure cross correlation is
\[ Q_{aa}(x_1, x_2, y_1, y_2, z_1, z_2, t_1, t_2) = E[p_a(x_1, y_1, z_1, t_1) p_a(x_2, y_2, z_2, t_2)] \]

\[ = \frac{-4\pi^2}{a_0^2 b^2} \sum_{\ell = 0}^{\infty} \sum_{r = 0}^{\infty} \frac{j \pi x_1}{a} \frac{\ell \pi y_1}{b} \frac{r \pi z_1}{a} \frac{t \pi y_2}{b} \left( \frac{2}{a} \right)^{j \pi x_2} \left( \frac{2}{b} \right)^{\ell \pi y_2} \left( \frac{2}{a} \right)^{r \pi z_2} \left( \frac{2}{b} \right)^{t \pi y_2} \]

\[ \left\{ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\cos k_{\ell} z_1 \cos k_{r} z_2}{(1+\delta_0)(1+\delta_0\ell)(1+\delta_0)(1+\delta_0)k_{r} \sin k_{\ell} \sin k_{r} d \, d \Omega} \right\} \]

\[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{0}^{b} \int_{0}^{a} \int_{0}^{b} \int_{0}^{a} E\left[ \phi(x_1, y_1, t_1) \phi(x_2, y_2, t_2) \right] \frac{j \pi x_1}{a} \frac{\ell \pi y_1}{b} \frac{r \pi z_1}{a} \frac{t \pi y_2}{b} \left( \frac{2}{a} \right)^{j \pi x_2} \left( \frac{2}{b} \right)^{\ell \pi y_2} \left( \frac{2}{a} \right)^{r \pi z_2} \left( \frac{2}{b} \right)^{t \pi y_2} \cos \frac{\ell \pi y_1}{b} \cos \frac{t \pi y_2}{b} e^{-i \omega t_1} e^{-i \Omega t_2} d \Omega \right\}_{\phi(x_1, y_1, t_1) \phi(x_2, y_2, t_2)} \]

(D45)

In Equation (D45), we note that from Equation (D18)

\[ E[\phi(x_1, y_1, t_1) \phi(x_2, y_2, t_2)] = Q_{\phi\phi}(x_1, x_2, y_1, y_2, r) \]  

(D46)

Also, since \( r = t_2 - t_1 \), then using Equation (D19) and noting that \( \int_{-\infty}^{\infty} e^{-i(\omega + \Omega) t_1} dt_1 = 2\pi \delta(\omega + \Omega) \)

\[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} Q_{\phi\phi}(x_1, x_2, y_1, y_2, r) e^{-i \omega t_1} e^{-i \Omega t_2} dt_1 dt_2 \]

\[ = \int_{-\infty}^{\infty} Q_{\phi\phi}(x_1, x_2, y_1, y_2, r) e^{-i(\omega + \Omega) t_1} dt_1 \]

\[ \cdot \sqrt{2\pi} S_{\phi\phi}(x_1, x_2, y_1, y_2, \omega) \int_{-\infty}^{\infty} e^{-i(\omega + \Omega) t_1} dt_1 = (2\pi)^{3/2} S_{\phi\phi}(x_1, x_2, y_1, y_2, \omega) \delta(\omega + \Omega) \]

(D47)
Substituting Equation (D47) in (D45), integrating over $\omega$, and using the identity 

\[ r = t_2 - t_1 \]

yields

\[ Q_{aa}(x_1, x_2; y_1, y_2; z_1, z_2, r) = \frac{-8\sqrt{2} \mu_0^2}{V n a^2 b^2} \sum_{j=0}^{\infty} \sum_{r=0}^{\infty} \frac{jnx_1}{a} \frac{\ell ny_1}{b} \frac{rnx_2}{a} \frac{tmy_2}{b} \]

\[ \int_{-\infty}^{\infty} \frac{\Omega^2 \cos k_{j\ell} r_1 \cos k_{z_{rt}} r_2}{(1 + \delta_{j\ell}) (1 + \delta_{r,t})} \cos k_{r,t} (-\Omega) \sin k_{j\ell} d \Omega \sin k_{z_{rt}} d \Omega \]

\[ = \int_{0}^{b} \int_{0}^{a} \int_{0}^{b} \int_{0}^{a} S_{\phi\phi}(x_1, x_2; y_1, y_2; \Omega) \cos \frac{jnx_1}{a} \cos \frac{\ell ny_1}{b} \cos \frac{rnx_2}{a} \cos \frac{tmy_2}{b} \]

\[ dx_1 \ dy_1 \ dx_2 \ dy_2 \{ e^{i \Omega r} d \Omega \} \quad (D48a) \]

where from Equations (D33), (D35), (D36), and (D37)

\[ k_{z\ell}(\omega) = \left[ \frac{a^2}{c^2} - \left( \frac{jx}{a} \right)^2 - \left( \frac{\ell y}{b} \right)^2 \right]^{1/2} \quad (D48b) \]

Hence

\[ k_{z\ell}(\omega) = k_{z\ell}(-\omega) \quad (D48c) \]

The cavity acoustic pressure cross spectral density is

\[ S_{aa}(x_1, x_2; y_1, y_2; z_1, z_2, \omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} Q_{aa}(x_1, x_2; y_1, y_2; z_1, z_2, r) e^{-i \omega r} dr \]

\[ \quad (D49) \]

Substitution of Equation (D48a) in (D49) with $S_{\phi\phi}$ given by Equation (D21b) yields (after reassigning the subscripts such that $j,k,m,n$ apply to the acoustics problem and $g,r,s$ and $t$ to the plate)

220
\[ S_{0a}(x_1, x_2; y_1, y_2, s_1, s_2, \omega) = \frac{8\rho^2 a_0}{\pi^2 b^2} \sum_{j=0}^{\infty} \sum_{m=0}^{\infty} G_{jkmn}(a_1, s_2, y_1, y_2) \]

\[ \int_{-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} \frac{g_{jkmn}(x_1, s_2, \Omega)}{(1+\delta_{0j})(1+\delta_{0k})(1+\delta_{0m})(1+\delta_{0n})} X(\Omega) \right\} \]

\[ \sum_{q=1}^{\infty} \sum_{s=1}^{\infty} \sum_{r=1}^{\infty} \sum_{t=1}^{\infty} \left[ \int_{0}^{b} \int_{0}^{a} \int_{0}^{b} \int_{0}^{a} F_{qrs}(x_1, s_2, y_1, y_2) g_{jkmn}(x_1, s_2, y_1, y_2) dx_1 dy_1 dx_2 dy_2 \right] \]

\[ \int_{-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} g_{jkmn}(x_1, s_2, \Omega) X(\Omega) e^{i\Omega r} d\Omega \right\} e^{-i\omega r} dr \]

\[ \frac{8\rho^2 a_0}{\pi^2 b^2} \sum_{j=0}^{\infty} \sum_{m=0}^{\infty} \sum_{q=1}^{\infty} \sum_{s=1}^{\infty} \sum_{r=1}^{\infty} \sum_{t=1}^{\infty} G_{jkmn}(x_1, s_2, y_1, y_2) \]

\[ \int_{0}^{b} \int_{0}^{a} \int_{0}^{b} \int_{0}^{a} F_{qrs}(x_1, s_2, y_1, y_2) g_{jkmn}(x_1, s_2, y_1, y_2) dx_1 dy_1 dx_2 dy_2 \]

\[ \left( D50 \right) \]

where

\[ F_{qrs}(x_1, s_2, y_1, y_2) = \frac{a}{\sin \frac{q\pi x_1}{a} \sin \frac{r\pi y_1}{b} \sin \frac{s\pi x_2}{a} \sin \frac{t\pi y_2}{b}} \]

\[ \left( D51 \right) \]

\[ f_{qrs}(\Omega) = \frac{G_{rj}(\Omega) V_{qrs}(\Omega)}{T_{qr}(\Omega) T_{st}(\Omega) P_r(\Omega) P_t(\Omega) P_q(\Omega) R_s(\Omega) R_t(\Omega)} \]

\[ \left( D52 \right) \]
\[ X(\Omega) = \begin{cases} 
\frac{16 \pi^2}{\mu^2 a^2 b^2} & \Omega \leq 1.256 \frac{U_0}{\delta^*} \\
\frac{32 \pi^2}{\mu^2 a^2 b^2} \left( \frac{\Omega \delta^* - \Omega}{U_0} \right) & \Omega > 1.256 \frac{U_0}{\delta^*}
\end{cases} \] (D53)

\[ G_{jkmn}(x_1, x_2, y_1, y_2) = \cos \frac{jm_1}{a} \cos \frac{kn_1}{b} \cos \frac{m_2}{a} \cos \frac{n_2}{b} \] (D54)

\[ g_{jkmn}(\xi_1, \xi_2, \Omega) = \frac{\Omega^2 \cos k_{j1} \xi_1 \cos k_{j2} \xi_2}{k_{j1}(\Omega) k_{j2}(\Omega) \sin k_{j1}(\Omega) \sin k_{j2}(\Omega)} \] (D55)

Integration over frequency \( \Omega \) of the bracketed term in the right member of Equation (D50) results in

\[ \sqrt{2\pi} g_{jkmn}(x_1, x_2, r) X(r) f_{qrst}(r) \] (D56)

where

\[ f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\omega) e^{i\omega t} d\omega \] (D57)

Substitution of Equation (D56) in (D50) and integration over \( r \) yields the final expression for the cavity acoustic cross spectral density:

\[ S_{ad}(x_1, x_2, y_1, y_2, \xi_1, \xi_2, \omega) = \frac{16 \rho_0^2}{a^2 b^2} \sum_{j=0}^{\infty} \sum_{m=0}^{\infty} \sum_{q=1}^{\infty} \sum_{s=1}^{\infty} \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} \sum_{r=1}^{\infty} \sum_{t=1}^{\infty} \] (D58)

\[ G_{jkmn}(x_1, x_2, y_1, y_2) g_{jkmn}(\xi_1, \xi_2, \omega) X(\omega) f_{qrst}(\omega) \] (D59)
Integrating over the spatial coordinates, using Equations (D51) and (D54), we obtain by means of standard integration techniques

\[ S_{\alpha\alpha}(x_1, x_2, y_1, y_2, \omega, \omega) = \frac{16\rho_a^2}{\pi^4} \sum_{j=0}^{\infty} \sum_{m=0}^{\infty} \sum_{q=1}^{\infty} \sum_{s=1}^{\infty} \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} \sum_{r=1}^{\infty} \sum_{t=1}^{\infty} X(\omega) \]

where \( G_{jkkm}(x_1, x_2, y_1, y_2, \omega, \omega) \) and \( X(\omega) \) are defined in Equations (D54), (D55), (D55), and (D53), respectively, and where

\[ K_{jkkm}(x_1, x_2, y_1, y_2, \omega, \omega) = \frac{(1 - \delta_{jk}) (1 - \delta_{km}) (1 - \delta_{lm}) (1 - \delta_{rn}) \Gamma[1 - (1 - 1)^j (1 - 1)^k] \Gamma[1 - (1 - 1)^l (1 - 1)^s] \Gamma[1 - (1 - 1)^p (1 - 1)^q] \Gamma[1 - (1 - 1)^m (1 - 1)^n] \Gamma[1 - (1 - 1)^p (1 - 1)^q]}{(a^2 - m^2) (t^2 - n^2)} \]

The cavity acoustic pressure spectral density obtained from Equation (D58) is

\[ \Phi_\omega(x_1, y_1, \omega, \omega) = S_{\alpha\alpha}(x_1, x_2, y_1, y_2, \omega, \omega) = \frac{16\rho_a^2}{\pi^4} X(\omega) \]
Setting $x_1 = x_2$, $y_1 = y_2$ in Equation (D54) and $a_1 = a_2$ in Equation (D55), we have

$$G_{jkmn}(x_1, y_1, y_1) = G_{mnj}(x_1, x_1, y_1, y_1) = G_{jkmn}(x_1, y_1)$$  \hspace{1cm} (D62)$$

$$g_{jkmn}(x_1, z_1, \omega) = g_{mnj}(x_1, z_1, \omega) = g_{jkmn}(x_1, \omega)$$  \hspace{1cm} (D63)$$

Also

$$K_{jkmnqrst} = K_{mnjksqrt}$$  \hspace{1cm} (D64)$$

Hence using (D52), (D53), (D62), and (D63) in Equation (D61) and dropping the subscripts on spatial coordinates, we obtain

$$\Phi_0(x, y, z, \omega) = \frac{256 \rho_0^2 \omega^2}{\pi^4 \mu^2 a^2 b^2} \sum_{j=0}^{\omega \omega} \sum_{m=0}^{\omega \omega} \sum_{q=1}^{\omega \omega} \sum_{s=1}^{\omega \omega} \frac{G_{jkmn}(x, y) g_{jkmn}(x, \omega) G_{r}(\omega)}{T_{q}(\omega) T_{s}(\omega) P_{t}(\omega) P_{r}(\omega) R_{s}(\omega) R_{t}(\omega)}$$

$$\cdot K_{jkmnqrst} V_{qrst}(\omega) \quad \omega \leq 1.256 \frac{U_0}{\delta^*}$$  \hspace{1cm} (D65)$$

$$= \frac{512 \rho_0^2 \omega^2}{\pi^4 \mu^2 a^2 b^2} \left( \frac{\omega \delta^*}{U_0} \right)^{-3} \sum_{j=0}^{\omega \omega} \sum_{m=0}^{\omega \omega} \sum_{q=1}^{\omega \omega} \sum_{s=1}^{\omega \omega} \frac{G_{jkmn}(x, y) g_{jkmn}(x, \omega) G_{r}(\omega)}{T_{q}(\omega) T_{s}(\omega) P_{t}(\omega) P_{r}(\omega) R_{s}(\omega) R_{t}(\omega)}$$

$$\cdot K_{jkmnqrst} V_{qrst}(\omega) \quad \omega > 1.256 \frac{U_0}{\delta^*}$$  \hspace{1cm} (D65)$$

As for the case of the plate velocity spectral density, the cavity acoustic pressure spectral density must be a real, even function of frequency. Thus by substituting $V_{qrst}$ (as given below Equation (D21b)) in Equation (D61), rearranging Equation (D65) first for the frequency range $\omega \leq 1.256 \frac{U_0}{\delta^*}$, (see Reference 32 for details) and then for $\omega > 1.256 \frac{U_0}{\delta^*}$, and using Equations (D62), (D63), and (D64), we finally obtain
\[ \Phi_\alpha(x, y, z, \omega) = \frac{256 \rho^2 \omega^2}{\pi^2 \mu^2 \omega^2} \sum_{j=0}^{\infty} \sum_{m=0}^{\infty} \sum_{q=1}^{\infty} \sum_{s=1}^{\infty} \frac{G_{j,kmn}(x,y) G_{kmn}(z,\omega)}{T_{q,\omega}(\omega) T_{s,\omega}(\omega) P_q(\omega) P_s(\omega) R_q(\omega) R_s(\omega)} \times w_{qrs}(\omega) K_{jkmnqrs} \]

\[ \Phi_\phi(x, y, z, \omega) = \frac{512 \rho^2 \omega^2}{\pi^2 \mu^2 \omega^2} \frac{(\omega \delta)^3}{U_0} \sum_{j=0}^{\infty} \sum_{m=0}^{\infty} \sum_{q=1}^{\infty} \sum_{s=1}^{\infty} \frac{G_{j,kmn}(x,y) G_{kmn}(z,\omega)}{T_{q,\omega}(\omega) T_{s,\omega}(\omega) P_q(\omega) P_s(\omega) R_q(\omega) R_s(\omega)} \times w_{qrs}(\omega) K_{jkmnqrs} \]

For practical utility, the plate velocities and cavity acoustic pressures are expressed in nondimensional form. The nondimensional expressions given below are the working expressions used in the computer program.

The nondimensional form of the plate spectral velocity is defined as

\[ \Phi_\phi^+ (x^+, y^+, z^+, \omega^+) = \frac{\Phi_\phi(x, y, z, \omega)}{a^2 U_c} = 4.38 \times 10^{-4} M^2 \delta^+ \omega^+^{-2} \]

\[ \sin m \pi \delta^+ \sin q \pi \delta^+ \sin \frac{ny^+}{b^+} \sin \frac{sy^+}{b^+} G_{n,s}^+ (\omega^+) W_{n,s}^+ (\omega^+) \frac{T_{mn}^+ (\omega^+) T_{qs}^+ (\omega^+) P_n^+ (\omega^+) P_s^+ (\omega^+) R_m^+ (\omega^+) R_s^+ (\omega^+)}{\text{; \quad } \omega^+ \delta^+ \leq 1.932} \]

\[ = 3.2 \times 10^{-3} M^2 \delta^+ \omega^{-5} \sum_{m=1}^{\infty} \sum_{q=1}^{\infty} \sum_{n=1}^{\infty} \sum_{s=1}^{\infty} \frac{G_{n,s}^+ (\omega^+) W_{n,s}^+ (\omega^+)}{T_{mn}^+ (\omega^+) T_{qs}^+ (\omega^+) P_n^+ (\omega^+) P_s^+ (\omega^+) R_m^+ (\omega^+) R_s^+ (\omega^+)} \text{; \quad } \omega^+ \delta^+ > 1.932 \]
where the dimensionless input parameters and spatial and frequency variables are defined as follows

\[ M = \frac{\rho f a}{\mu} \quad z^+ = \frac{z}{a} \]

\[ U_1^+ = \frac{U_0}{U_c} \quad y^+ = \frac{y}{a} \]

\[ \delta^+ = \frac{\delta^*}{a} \quad \omega^+ = \frac{\omega a}{U_c} \]

\[ \xi^+ = \frac{b}{a} \]

\[ \omega^+_{mn} = \frac{\omega_{mn} a}{U_c} \]

\[ r^+ = \frac{r a}{\mu U_c} \]

\[ r_{mn}^+ = \frac{r_{mn} a}{\mu U_c} = \frac{2\omega_{mn} a}{U_c} \]

\[ D^+ = \frac{D}{\mu U_c^2 a^2} \quad (D68) \]

The quantities defined below Equation (D21) for \( R_m, R_q, \nu_m, \nu_q, \lambda_{mn}, \lambda_{q}, G_{mn}, P_n, P_s, T_{mn}, T_q, \) and Equation (D23) have been rewritten in dimensionless terms as follows:

\[ R_m^+(\omega^+) = \left[ \left( \frac{\omega}{\omega^+} \right)^2 - 0.987 \omega^+ + 0.0529 \omega^+ \right]^{1/2} \quad (D69) \]

\[ \nu_m = \tan^{-1}\left[ \frac{0.23 \omega^+}{(mn)^2 - 0.987 \omega^+} \right] \quad (D70) \]

\[ \lambda_{mn} = \tan^{-1}\left[ \frac{r^+ \omega^+}{(\omega_{mn}^+)^2 - (\omega^+)^2} \right] \quad (D71) \]
\[ G_{mn}^+(\omega^+) = 0.35 \omega^+ b^+ \delta_{mn} \left[ 2(0.7 \omega^+ b^+)^2 + (mn)^2 + (n \eta)^2 \right] \]

\[ + mn^2 \left[ (-1)^m (-1)^n - 1 \right] \left[ 1 - \delta_{mn} \right] + mn^2 \left( 2 - \left[ (-1)^m + (-1)^n \right] e^{-0.7 \omega^+ b^+} \right) \]

(D72)

\[ P_m^+ (\omega^+) = (mn)^2 + (0.7 \omega^+ b^+)^2 \]

(D73)

\[ T_{mn}^+ (\omega^+) = \left\{ \left[ \frac{\omega_{mn}}{\omega^+} \right]^2 - 1 \right\}^2 \left( 2 \frac{r^+}{r_{mn}^+} \frac{\omega_{mn}^+}{\omega^+} \right)^2 \right\}^{1/2} \]

(D74)

\[ W_{jkmn}^+ (\omega^+) = 1.0066 \omega + \delta_{jm} \cos (\nu_j - 0.463 \pi) \cos (\lambda_{jk} - \lambda_{mn}) \]

\[ + 2jm^2 \cos (\nu_j + \nu_m) \cos (\lambda_{jk} - \lambda_{mn}) + (1 - \delta_{jm}) \cdot \frac{jmn^2 \left[ (-1)^j (-1)^n - 1 \right]}{\left( (jn)^2 - (mn)^2 \right)} \]

\[ \cdot \left[ R_j^+ \cos (\nu_j + \lambda_{jk} - \lambda_{mn}) - R_m^+ \cos (\nu_j + \lambda_{mn} - \lambda_{jk}) \right] - jmn^2 e^{-0.115 \omega^+} \]

\[ \cdot \left[ (-1)^j \cos (\omega^+ + \nu_j + \nu_m + \lambda_{jk} - \lambda_{mn}) + (-1)^m \cos (\omega^+ + \nu_j + \nu_m + \lambda_{mn} - \lambda_{jk}) \right] \]

(D75)

The nondimensional form of the cavity acoustic pressure spectral density is defined as

\[ \Phi^+_a (x^+_1 y^+_2 z^+_3, \omega^+) = \frac{\Phi_a (x^+_1 y^+_2 z^+_3, \omega^+)}{a^2 \mu^2 \nu_b^3} = \frac{7.008 \times 10^{-3}}{\pi^4} \rho^+ b^+ \gamma^+ \]

\[ \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{q=0}^{\infty} \sum_{s=0}^{\infty} \sum_{t=0}^{\infty} \left\{ G_{rs}^+ (\omega^+) W_{qst}^+ (\omega^+) K_{jkmnqrs} \right\} \]

\[ \frac{U_{jkmn}^+ (\omega^+)}{U_{jkmn}^+ (\omega^+)} \]

927
\[
\cos j\pi^+ \cos m\pi^+ \cos \frac{k\pi y^+}{b^+} \cos \frac{n\pi y^+}{b^+} \cos k^+_{jk} \cdot \cos k^+_{mn} z^+ \cos k^+_{mn} z^+
\]
\[
\frac{T^+_{qj} (\omega^+) T^+_{iz} (\omega^+)}{T^+_{qj} (\omega^+) P^+_{rj} (\omega^+) P^+_{iq} (\omega^+) R^+_{qj} (\omega^+) R^+_{iz} (\omega^+)} ; \omega^+ b^+ \leq 1.932
\]
\[
= 5.112 \times 10^{-2} \frac{\rho^+ \Sigma^+}{\pi^4} M^2 \delta^+-2 \omega^+3 \quad \Sigma^+ \Sigma^+ \Sigma^+ \Sigma^+
\]
\[
j = 0 \quad m = 0 \quad q = 1 \quad z = 1
\]
\[
k = 0 \quad n = 0 \quad r = 1 \quad t = 1
\]
\[
\left\{ \begin{array}{c}
G^+_{rt} (\omega^+) W^+_{qrst} (\omega^+) K_{kmnqrst} \\
U^+_{kmn} (\omega^+)
\end{array} \right\}
\]
\[
\cos j\pi^+ \cos m\pi^+ \cos \frac{k\pi y^+}{b^+} \cos \frac{n\pi y^+}{b^+} \cos k^+_{jk} \cdot \cos k^+_{mn} z^+ \cos k^+_{mn} z^+
\]
\[
\frac{T^+_{qj} (\omega^+) T^+_{iz} (\omega^+)}{T^+_{qj} (\omega^+) P^+_{rj} (\omega^+) P^+_{iq} (\omega^+) R^+_{qj} (\omega^+) R^+_{iz} (\omega^+)} ; \omega^+ b^+ > 1.932
\]

Equation (D76) is obtained from Equations (D66), (D69) through (D75), and the following definitions for the dimensionless input parameters and spatial variable:

\[
c^+ = \frac{c}{U_c} \quad \rho^+ = \frac{\rho a_o a}{\mu} \quad d^+ = \frac{d}{a} \quad z^+ = \frac{z}{a}
\]

which are used to form the following dimensionless quantities

\[
k^+_{jk} (\omega^+) = \left[ \left( \frac{\omega^+}{b^+} \right)^2 - \left( \frac{k\pi y^+}{b^+} \right)^2 \right]^{1/2}
\]

\[
U^+_{kmn} (\omega^+) = k^+_{jk} (\omega^+) k^+_{mn} (\omega^+) \sin k^+_{jk} d^+ \sin k^+_{mn} d^+
\]

(D78)
APPENDIX D2 - METHOD FOR DETERMINING INPUT DATA

The following data are furnished to the computer:

1. Dimensionless input parameters to determine the dimensionless plate velocity spectral density:

\[ \frac{M}{\mu} = \frac{\rho \beta^2}{U_c} \]

\[ U^+ = \frac{U_0}{U_c} \] (See Figure 4 of Reference 32; \( U^+ \) was taken to be constant, equal to 1.54 in Reference 32.)

\[ \delta^+ = \frac{\delta^*}{\mu} \]

\[ \beta^+ = \frac{\beta}{a} \]

\[ D^+ = \frac{D}{\mu U_c \beta^2 a^2} \]

\[ r^+ = \frac{\rho a}{\mu U_c} \]

The values for the data used in determining the input parameters may be either arbitrarily prescribed or measured by methods similar to those presented in Appendix C.*

The range must also be specified for the spacial and frequency variables \( x^+ = \frac{x}{a} \), \( y^+ = \frac{y}{a} \), and \( \omega^+ = \omega a / U_c \)

2. Dimensionless input parameters to determine the dimensionless cavity acoustic pressure spectral density.

In addition to the foregoing parameters, it is necessary to specify the following additions:

\[ c^+ = \frac{c}{U_c} \]

*Additional input parameters \( \omega^+_{mn} = \frac{\omega_{mn} a}{U_c} \) and \( r^+_{mn} = \frac{r_{mn} a}{\mu U_c} = \frac{2 \omega_{mn} a}{U_c} \) are functions of \( D^+ \) and \( b^+ \) only, hence need not be independently specified.
\[ d^+ = \frac{d}{a} \]

\[ \rho^+ = \frac{\rho_0 a}{\mu} \]

and the range for \( z^+ = \frac{z}{a} \).

The values for these data are either known from the geometry and properties of the actual structure and fluid or are arbitrarily specified.

Reference 32 gives dimensionless input parameters which fall in the range of interest for submarine sonar applications.
APPENDIX D3 – PROGRAM IDENTIFICATION

This program computes the plate velocity power spectrum and cavity acoustic pressure power spectrum resulting from the vibrations of a turbulence excited finite plate with simply supported boundaries. The program is designated as TURB3. It consists of two subprograms, I and II. Subprogram I computes the plate velocity power spectrum and Subprogram II the cavity acoustic pressure power spectrum. Both use similar notation. There are slight differences in their inputs and in the interpretation of their output. See identification below.

A time estimate of the computer running times on the IBM 7090 is given below.

<table>
<thead>
<tr>
<th>Figure No.</th>
<th>Subprogram</th>
<th>Frequency Range</th>
<th>Running Time min</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>I</td>
<td>$1 \leq \omega^+ \leq 1000$</td>
<td>9.13</td>
</tr>
<tr>
<td>17</td>
<td>I</td>
<td>$1 \leq \omega^+ \leq 1000$</td>
<td>12.0</td>
</tr>
<tr>
<td>18</td>
<td>II</td>
<td>$1 \leq \omega^+ \leq 1500$</td>
<td>442.0</td>
</tr>
</tbody>
</table>
APPENDIX D

TABLE 6
Identification for Subprograms I and II — Strawderman

This table includes input and output data identification, flow chart, and order of input data. Computer running times have been given on the previous page. Computer program listings are presented in Table 7.

TABLE 6A: Input Data
TABLE 6B: Output Data
TABLE 6C: Flow Charts
TABLE 6D: Input Format
TABLE 6A
Input Data
(Dimensionless Units)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Identification</th>
<th>Program</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^+$</td>
<td>Dimensionless longitudinal spatial coordinate $x/a$</td>
<td>X</td>
</tr>
<tr>
<td>$y^+$</td>
<td>Dimensionless lateral spatial coordinate $y/a$</td>
<td>Y</td>
</tr>
<tr>
<td>$z^+$</td>
<td>Dimensionless spatial coordinate $z/a$</td>
<td>ZP</td>
</tr>
<tr>
<td>$b^+$</td>
<td>Dimensionless plate and acoustic cavity dimension $b/a$</td>
<td>BP</td>
</tr>
<tr>
<td>$d^+$</td>
<td>Dimensionless cavity dimension $d/a$ where $d$ is acoustic cavity dimension in $z$-direction</td>
<td>DEPTH</td>
</tr>
<tr>
<td>(\frac{r^+}{r_{cm,n}^+})</td>
<td>Dimensionless plate damping coefficient divided by dimensionless critical plate damping for the $m$-$n$th mode</td>
<td>DAMP</td>
</tr>
<tr>
<td>$\omega^+$</td>
<td>Dimensionless radial frequency</td>
<td>OMEGA</td>
</tr>
<tr>
<td>$c^+ = \frac{c}{U_c} = \frac{cU^+}{U_0}$</td>
<td>Dimensionless speed of sound</td>
<td>CP</td>
</tr>
<tr>
<td>$D^+$</td>
<td>Dimensionless plate flexural rigidity</td>
<td>DP</td>
</tr>
<tr>
<td>$M$</td>
<td>Dimensionless fluid mass</td>
<td>CAPM</td>
</tr>
<tr>
<td>$\delta^+$</td>
<td>Dimensionless turbulent boundary layer displacement thickness</td>
<td>DELTA</td>
</tr>
<tr>
<td>$a$</td>
<td>Constant mult. factor; changes for different fluids; $a = 1.0$ for water $a = 3.0$ for air (See Interpretation of Data Output)</td>
<td>ALFA</td>
</tr>
<tr>
<td>$\rho^+$</td>
<td>Dimensionless fluid density equal to $\rho a_0 a / \mu$</td>
<td>RHO</td>
</tr>
</tbody>
</table>

Largest frequency of interest, i.e., cutoff frequency at which program is to stop

Convergence criterion in Equation (D76) with $TOLH > 1.0$ (Case I only)

Convergence criterion in Equation (D76) with $TOLL < 1.0$ (Case I only)
**TABLE 6B**

Output Data

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Identification</th>
<th>Program</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_{mn}$</td>
<td>Modal natural frequencies</td>
<td>PNome (I,J)</td>
</tr>
<tr>
<td> </td>
<td>$\omega_{mn} = \sqrt{\frac{D}{\mu} \left[ \left( \frac{m_n}{a} \right)^2 + \left( \frac{n_n}{b} \right)^2 \right]}$</td>
<td> </td>
</tr>
<tr>
<td>$\phi^+_a (x^+, y^+, z^+, \omega^+)$</td>
<td>Subprogram II: Equation (D76) Dimensionless form of the cavity acoustic spectral density. NOTE: PHIP was multiplied by $(\rho^+)^2 = \left( \frac{\rho \sigma_0}{\mu} \right)^2$ to agree with (D76); see Interpretation of Data Output below</td>
<td> </td>
</tr>
<tr>
<td> </td>
<td> </td>
<td> </td>
</tr>
<tr>
<td> </td>
<td> </td>
<td> </td>
</tr>
</tbody>
</table>

Interpretation of Data Output

Comment: Subprogram I involves eight nested do-loops which means that the inner operations are done a minimum of $2^8$ times; the next index on the loop would be $4^8$, $6^8$, . . . etc. until convergence. Case I involves only four nested loops.

Special Instruction: Sense Switch 4 is turned on by operator at beginning of program.

Curves – Three Examples

Example 1 (Figure 16): Subprogram I, Equation (D67) is used for this curve. The computer results for the plate velocity spectrum were then changed to dimensional form and finally converted to the ratio of displacement spectral density to turbulent pressure spectral...
density. The subprogram uses Bull data. The form of the final response is 10 \log_{10} [(\Phi_d(\omega) / \Phi_t(\omega)) \phi_\phi^+ (\omega)] plotted against f. The following conversions were made manually.

a. Use the program to compute \phi_\phi^+ (\omega) Equation (D67) multiplied by \alpha^2

b. Convert \phi_\phi^+ (\omega) = \frac{\phi_\phi(\omega)}{U_x a}, where \alpha^2 \phi_\phi^+ (\omega) is the quantity appearing in the program. This quantity is then multiplied by \U_c a to yield \phi_\phi(\omega).

c. \phi_d(\omega) = \frac{\phi_\phi(\omega)}{\omega^2} (from velocity to displacement).

d. \phi_4(\omega) = \phi_p, Equations 3.1 of Reference 32. Use first of Equations 3.1 of Reference 32 since the Bull data yield maximum frequency \omega = 12,566 cps but \omega = \frac{1.258 U_0}{\delta^*} = 47,272 is cutoff point in this equation.

e. Other unit conversions:

1. \omega^+ to \omega^+ by \omega^+ = \frac{2\pi f a}{U_c}

2. \delta^* = a \delta^+

3. \U_c = \frac{U_0}{U^+}

f. \frac{\Phi_d(\omega)}{\Phi_t(\omega)} = \frac{\phi_\phi(\omega)}{\omega^2 \phi_p(\omega)} \Rightarrow 10 \log \frac{\Phi_d(\omega)}{\Phi_t(\omega)} = 10 \log \frac{\phi_\phi(\omega)}{\omega^2 \phi_p(\omega)} =

\begin{align*}
10 \log (\phi_\phi(\omega)) - 10 \log (\alpha^2 \phi_p(\omega)) &= \\
10 \log (\U_c \alpha \phi_\phi^+ (\omega)) - 10 \log (\omega^2 \phi_p(\omega)) &= \\
[10 \log (\U_c \alpha + 10 \log (\phi_\phi^+ (\omega)) - 10 \log (\omega^2 \phi_p(\omega))] &= \\
[10 \log (\U_c \alpha + 10 \log (\phi_\phi^+ (\omega)) - 20 \log \omega - 10 \log \phi_p(\omega)]
\end{align*}

Plot this versus f.

As an alternative, the foregoing results may be obtained by a simplified procedure. This option eliminates most but not all of the manual computations.
For subprogram use, the Bull data are nondimensionalized (see Figure 16). Note that for this frequency range in the figure, the value of $\phi_p$ is a constant. This makes the correction simpler. The following FORTRAN lines, in which the constants are dependent on the Bull data, are inserted into the Strawderman program to yield direct results for plotting Figure 16. These lines are inserted immediately after Statement 212 in the program. The curve was plotted with \( \text{RESP (PHISUBD/PHISUBT)} \) against \( \text{FREQ} \). It should be noted that the curve is about 6 dB lower than Strawderman's, since he originally used 116 (an arithmetic error which he corrected following publication of Reference 32) instead of 110.392; see subroutine above.

Col. 7

\[
\begin{align*}
\text{PHIC} & = \text{PHIDB} + 110.392 \\
\text{OF} & = \text{OMEGA} \times 190.986 \\
\text{OFW} & = \text{OF} \times 6.2318 \\
\text{WRITE}(6,240) & \text{PHIC, OF, OFW} \\
240 & \text{FORMAT}(1X,6HAPHIC=F17.8,6HAFREO=F17.8,10HA20LOG(W)=E17.8) \\
\text{RESP} & = \text{PHIC} - \text{OFW} \\
\text{WRITE}(6,241) & \text{RESP} \\
241 & \text{FORMAT}(1X, 17HPHISUBD/PHISUBT=E17.8)
\end{align*}
\]

Example 2 (Figure 17): Subprogram I is used. Computer result PHIDB is plotted against OMEGA. This gives representation of Equation (D67) directly, i.e., PHIDB represents the dimensionless plate velocity spectral density corresponding to values of $\omega^+$. Example 3 (Figure 18): Subprogram I is used. Computer result PHIDB is plotted against OMEGA. Note: for agreement with Equation (D76) (cavity acoustic spectral density), the result PHIP must be multiplied by $(\rho^+)^2 = (\rho_0a/\mu)^2$ or $20 \log_{10} \rho^+$ must be added to $10.0 \log_{10} (\text{PHIP}) / 2.302589 = \text{PHIDB}$. In Figure 14 the convergence criterion was a tolerance of 20 percent, with TOLL = 0.8 and TOLH = 1.2.

It is important to note that the original subprograms gave the following results:

\[
\begin{align*}
a^2 \Phi_\phi^+ (x^+, y^+, \omega^+) & \text{ and } a^2 \Phi_\phi^+ (x^+, y^+, s^+, \omega^+) \text{ rather than } \Phi_\phi^+ (x^+, y^+, \omega^+) \text{ and } \Phi_\phi^+ (x^+, y^+, s^+, \omega^+). \\
\text{By setting } \alpha = 1, \text{ the subprograms have now been modified to yield the normalized results } \Phi_\phi^+ (x^+, y^+, \omega^+) \text{ and } \Phi_\phi^+ (x^+, y^+, s^+, \omega^+). \text{ The unnormalized results } \Phi_\phi (x^+, y^+, \omega^+) = a^2 U_c a \Phi_\phi^+ (x^+, y^+, \omega^+) \text{ and } \Phi_\phi (x^+, y^+, s^+, \omega^+) = a^2 \mu^2 U_0^2 \Phi_\phi^+ (x^+, y^+, s^+, \omega^+) \text{ are then obtained manually for water } (\alpha = 1) \text{ or air } (\alpha = 3), \text{ i.e., set } \alpha = 1 \text{ or } 3, \text{ accordingly.}
\end{align*}
\]

*This operation was performed in order to manually compensate for the inadvertent omission of $(\rho^+)^2$ from the program. Subsequent to the computation of Figure 18, the program was corrected to include $(\rho^+)^2$. Hence manual compensation is no longer necessary since the true results are obtained directly from the computer.
Flow Chart for Subprogram I — Plate Velocity Power Spectrum

START

READ DATA (DIMENSIONLESS)
X, Y, BP, DAMP, DEPTH, OMEGA,
CP, DP, CAPM, DELTA, AL, FA, TIP

CALCULATE: CONSTANTS, BOUNDARY
CONDITIONS, AND NATURAL FREQUENCY
PNOME(I,J)

STATEMENT 160
INITIALIZE VARIABLES, ORDER
FREQUENCY, SET LIMITS ITOP; MU

CALCULATE $G_{ij}$, $T_{ij}$, $P_1$, $CAPR_{ij}$

ARCTAN $\left( \frac{0.23u^2 + 2}{v^2 - 0.987u^2 + 2} \right)$
ARCTAN $\left( \frac{2 \cdot DAMP \cdot PNOME_{ij} - OMEGA}{PNOME(I,J)^2 - OMEGA^2} \right)$

DO 714 J = 1, ITOP
DO 713 K = 1, ITOP
DO 712 M = 1, ITOP
DO 711 N = 1, ITOP

CALCULATE $W_{JKMN}$

$PNUM_{JKMN} = SNX(J)SNY(K)SNX(M)SNY(N)G(K,N)W_{JKMN}$
$DEN_{JKMN} = T(J,K)T(M,N)P(K)P(N)CAPR(J)CAPR(M)$

SUM = $\sum_{J,K} \sum_{M,N} \frac{PNUM}{DEN}$

DO 711 CONTINUE
DO 712 CONTINUE
DO 713 CONTINUE
DO 714 CONTINUE

2
\[ \Phi (N) = (4.38 \times 10^{-4}) \text{ m}^2 \]
\[ \Phi (M) = (3.2 \times 10^{-3}) \text{ m}^2 \]

WRITE:
INPUT DATA
\( \Phi (N), \text{TOP, MU} \)

CALCULATE RATIO = \( \frac{\Phi (M)}{\Phi (N) - 1} \)

\( \text{ITOP} = \text{ITOP} + 2 \)

RATIO: 0.97

\( \Phi (P) = \Phi (M) \)
\( \Phi (DB) = 10 \log_6 (\Phi (P)) \)

RATIO: 1.03

WRITE:
\( \Phi (P), \Phi (DB), \Omega, \text{ITOP, MU} \)

\( \text{ITOP} = \text{ITOP} + 2 \)

\( \Omega: \text{TOP} \)

\( \text{ITOP, MU} \)

STOP
START

READ DATA (DIMENSIONLESS) X, Y, BP, DAMP, ZP, DEPTH, OMEGA, CP, DP, CAPM, DELTA, ALFA, TIP, TOLH, TOLL, RHO

CALCULATE: CONSTANTS, BOUNDARY CONDITIONS, AND NATURAL FREQUENCY PNOME(1,J)

STATEMENT 160
INITIALIZE VARIABLES, ORDER FREQUENCY, SET LIMITS ITOP, MU, CALCULATE ACRES(I,J)

CALCULATE G_{ij}, T_{ij}, P_{ij}, \text{CAPR}_{ij},
\text{ARCTAN}
\left(\frac{0.23\omega^2}{\sqrt{\omega^4 - 0.987\omega^2}}\right)
\text{ARCTAN}
\left(\frac{2 \cdot \text{DAMP} \cdot \text{PNOME}_{ij} \cdot \omega}{\text{PNOME}_{ij}^2 - \omega^2}\right)

4
DO 714 J = 1, ITOP
DO 713 K = 1, ITOP
DO 712 M = 1, ITOP
DO 711 N = 1, ITOP
CALCULATE WJKMN

DO 824 JQ = 1, ITOP
DO 823 KR = 1, ITOP
DO 822 MS = 1, ITOP
DO 821 NT = 1, ITOP
CALCULATE PHNUMJQ, KR, MS, NT, J, K, M, N
CALCULATE DENJ, K, M, N, JQ, KR, MS, NT

ITOP ITOP ITOP ITOP
\[ \text{SUM} = \sum_{J,K} \sum_{N,M} \sum_{JQ,MS} \sum_{KR,NT} \text{PHNUM} \]
\[ \text{DEN} \]

CONTINUE
CONTINUE
CONTINUE
CONTINUE
CONTINUE
CONTINUE
CONTINUE
CONTINUE
CONTINUE
CONTINUE

\( \omega^* \delta^* > 1.92 \)

PHIFY(MN) = \((7.008 \times 10^{-5})^2 \delta^* \)
\[ \rho^2 \times \text{SUM} / \pi^4 \]

PHIFY(MU) = \((5.112 \times 10^{-5})^2 \delta^* \)
\[ 8 + 2 \omega^* - 3 \rho^2 \times \text{SUM} / \pi^4 \]

3
3

240
3

WRITE: INPUT DATA
PHIFY (MU), TOP, MU

CALCULATE RATIO =
PHIFY (MU)/PHIFY (MU-1)

< 2

RATIO: TOLL

ITOP = ITOP + 2

4

PHIP = PHIFY (MU)
PHIDB = 10 LOG10 (PHIP)

≤ 2

RATIO: TOLL

ITOP = ITOP + 2

4

WRITE:
PHIP, PHIDB, OMEGA, ITOP, MU

< 2

OMEGA: TIP

1

STOP
### TABLE 6D

**Input Format**

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242
APPENDIX D4 – TEST RUNS

Results obtained from the computer programs of Table 6 are given in Figures 16-18. A test run for the dimensionless plate velocity spectral density \( \phi_d^+ (x^+, y^+, \omega^+) \) converted to the ratio of displacement spectral density to turbulent pressure spectral density \( \phi_d(\omega)/\phi_f(\omega) \) is plotted logarithmically in Figure 16. Test runs for the plate velocity power spectrum and the cavity acoustic pressure power spectrum are plotted in Figures 17 and 18, respectively. Computer listings are given in Table 7.
Figure 16 – Computed Response of a 3.5 x 3.5 x 0.1-Inch Steel Plate to Turbulent Boundary Layer Excitation

See pages 56–57 of Reference 32 for source of data used here.
Figure 17 – Computed Dimensionless Plate Velocity Power Spectrum at Dimensionless Coordinates (1/5, 1/3); 10 Percent Critical Damping

See pages 60, 61 and 75 (Table 2, Case 1) of Reference 32 for source of data used here.

Figure 18 – Computed Dimensionless Cavity Acoustic Pressure Power Spectrum at Dimensionless Cavity Coordinates (1/2, 1/3, 0)

See page 75 (Table 2, Case 1) of Reference 32 for source of data used here.
TABLE 7

Computer Listings for USL Subprograms I and II – Strawderman

Table 7A – Plate Velocity Power Spectrum

```
$SFTC STRTR2
  DIMENSION PNOME(20,20),FREQ(20,20),TMINO(20),PHIFY(100),SNX(20),
  ISNY(20),CSX(20),CSY(20),G(20,20),T(20,20),P(20),CAPR(20)
  DIMENSION PNOD(20,20),GNU(20),PLDA(20,20)
  DIMENSION IDUMP(18)
@02 READ (5) X,Y,BP,DAMP,DEPTH,OMEGA,CP,DP,CAPM,DELTA
@03 FORMAT(5F10.8/5F10.4/2F10.4)
@06 Q=O*O
  PI=3.14159265
  PL=O=PI**2+O
  E=2.71828183
  IPEN=+80000
  SINEX=SIN(PI* X)
  COSEX=COS(PI*X)
  SINEY=SIN(PI* Y/BPI)
  COSEY=COS(PI* Y/BPI)
  IF (ABS(SINEY)-10*0**(-7))=800.0000
    SNX(1)=SINEX
    SNY(1)=SINEY
    CSX(1)=COSEX
    CSY(1)=COSEY
    DO 70 I=2,20
      IF (ABS(SNY(1))=800.0000)
        SNX(I)=SNX(J)*COSEX+CSX(J)*SINEX
        SNY(I)=SNY(J)*COSEY+CSY(J)*SINEY
        CSX(I)=CSX(J)*COSEX-SNX(J)*SINEX
        CSY(I)=CSY(J)*COSEY-SNY(J)*SINEY
        IF (ABS(ISNY(I))=800.0000)
          SNX(I)=SNX(J)*COSEX+CSX(J)*SINEX
          SNY(I)=SNY(J)*COSEY+CSY(J)*SINEY
          CSX(I)=CSX(J)*COSEX-SNX(J)*SINEX
          CSY(I)=CSY(J)*COSEY-SNY(J)*SINEY
          IF (ABS(ISNY(1))=800.0000)
            SNX(1)=SNX(J)*COSEX+CSX(J)*SINEX
            SNY(1)=SNY(J)*COSEY+CSY(J)*SINEY
            CSX(1)=CSX(J)*COSEX-SNX(J)*SINEX
            CSY(1)=CSY(J)*COSEY-SNY(J)*SINEY
```
### TABLE 7A (Continued)

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<th>Line No.</th>
<th>Code</th>
<th>Description</th>
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<tr>
<td>80</td>
<td>SNY(I)=0.0</td>
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<td>81</td>
<td>IF (ABS(SNX(I))-10.0**(-7.0)) &lt;= 82.82.83</td>
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<td>82</td>
<td>SNX(I)=0.0</td>
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<td>83</td>
<td>IF (ABS(CS(I))-10.0**(-7.0)) &lt;= 84.84.85</td>
<td></td>
</tr>
<tr>
<td>84</td>
<td>CS(I)=0.0</td>
<td></td>
</tr>
<tr>
<td>85</td>
<td>IF (ABS(CSX(I))-10.0**(-7.0)) &lt;= 86.86.87</td>
<td></td>
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<tr>
<td>86</td>
<td>CSX(I)=0.0</td>
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<td>87</td>
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<td>701</td>
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<td>7</td>
<td>WRITE(6,7)</td>
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<td>8</td>
<td>FORMAT(11/26HW, STRANDERMAN JOB NO 0775)</td>
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<td>100</td>
<td>CONTINUE</td>
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<tr>
<td>520</td>
<td>WRITE(6,522)X, Y, BP, DAMP, DEPT, OMEG, CP, DP, CAPM, DELTA</td>
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<td>521</td>
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<tr>
<td>522</td>
<td>WRITE(6,522)X, Y, BP, DAMP, DEPT, OMEG, CP, DP, CAPM, DELTA</td>
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</table>

eworthy.
GO TO 102
103 CONTINUE
102 CONTINUE
DO 103 I=1,20
TMINO(I)=0.0
700 CONTINUE
M=0
DO 109 I=1,20
DO 108 J=1,20
IF(FREG(I,J))=99,110,111
111 M=M+1
TMINO(M)=FREG(I,J)
110 CONTINUE
108 CONTINUE
109 CONTINUE
S=TMINO(1)
DO 112 L=1,19
112 CONTINUE
DO 130 L=1,20
IF(S-TMINO(L))=130,132,130
132 N=L
GO TO 180
130 CONTINUE
180 DOMEG=(S-OMEGA)/4.0
Z=0
ITOP=Z+1.5
MUJ=0
300 CONTINUE
DO 217 I=1,100
PHIFY(I)=0.0
217 CONTINUE
217 CONTINUE
SUM=0.0
TOP=0.0
EXPB=EXP(-0.7*OMEGA*BP)
TABLE 7A (Continued)

```
    DO 702 I=1,20
    DO 703 J=1,20
    B=I
    D=J
    IF(B=0)goto 12
    11 G(I,J)=B*D+PI*SQ*(((1-0)**2)+((-1-0)**2)+(-1.0)+((1-0)**2)
    I*1((-1-0)**2)+EXP(B)
    GO TO 703
    12 G(I,J)=(0.5*OMEGA*BP)*((1+0.5*OMEGA*BP)**2)+PI*SQ*(((8**2.0)+
    1(D**2.0(j)))B*PI*SQ/(2.0+((-1.0)**2)+((-1.0)**2)+EXP(B))
    703 CONTINUE
    702 CONTINUE
    CALL OVERFL(J)
    IF(J.EQ.2) GO TO 751
    750 WRITE(6,752)EXP(B)
    752 FORMAT(121
    751 CONTINUE
    DO 705 I=1,20
    DO 704 J=1,20
    PNOND(I,J)=PNOME(I,J)/OMEGA
    T(I,J)=M4(PNOND(I,J)**2 +1.0)**2+(2.0*DAMP*PNOND(I,J)**2)
    1**0.5
    704 CONTINUE
    705 CONTINUE
    DO 706 I=1,20
    A=I
    PI(I)=(A**2)*PI*SQ*(0.5*OMEGA*BP)**2
    706 CONTINUE
    707 CONTINUE
    DO 708 I=1,20
    A=I
    CAPR(I)=((A**2)*PI*SQ-0.987*(OMEGA**2))**2+0.0529*(OMEGA**2)
    14)**2+0.5
    707 CONTINUE
    PNGN=(0.23*(OMEGA**2)
    DO 708 I=1,20
    A=I
    PNGN=(A**2)*PI*SQ-0.987*(OMEGA**2)
    GO TO 7
```
TABLE 7A (Continued)

CALL ACTN(DGNU, PNGNU+GNU(I), IOTA)
13 GO TO 216
14 CONTINUE
708 CONTINUE
DO 710 J = 1, 20
DO 709 J = 1, 20
PNLDA=2.0*DAMP*PNOME(I)*OMEGA
DLDA=(PNOME(I)*OMEGA**2 - (OMEGA**2 - 1))
CALL ACTN(DLDA, PNLDA+DLDA(I), ILDA)
19 GO TO 216
20 CONTINUE
709 CONTINUE
710 CONTINUE
740 CONTINUE
DO 714 J = 1, ITOP
DO 713 K = 1, ITOP
DO 712 M = 1, ITOP
DO 711 N = 1, ITOP
A = J
B = K
C = M
D = N
TMAX = AMAX1(A, B, C, D)
IF (TMAX = TOP) 711 711 737
737 CONTINUE
15 PNUN = 0.0
20 GC TO 310
715 IF (SNY(N)) 716 715 716
716 IF (SNX(M)) 718 717 718
718 IF (SNY(K)) 720 719 720
717 PNUN = 0.0
719 DEN = 1.0
718 GO TO 310
720 PNUN = 0.0
310 IF (SNY(I)) 722 721 722
722 PNUN = 0.0
721 IF (SNX(I)) 724 723 724
724 PNUN = 0.0
723 GO TO 310
724 PNUN = 0.0

250
TABLE 7A (Continued)

DEN=1.0
GO TO 310
1680
720 IF(SNX(J))=22,1870
721 PNUM=0.0
GO TO 310
722 CONTINUE
1890
IF(J-M)=-1
1900
21 W=A*C*PISQ*(2.0*(COS(GNU(J)+GNU(M)))*(COS
X(PLDA(JK)+PLDA(MN)))+
1.0)**J)*(-1.0)**M-1.0)*(CAPR(J)*(COS(GNU(M)+PLDA(JK))-PLDA(MN))-
1910
2PLDA(MN)))*(CAPR(M)*(COS(GNU(M)+PLDA(MN)-PLDA(JK)))))
1920
X/(PISQ*(
1930
3(A**2.0)-(C**2.0)))-(E**(-0.115*OMEGA))*
1940
X((-1.0)**J)*(COS(OMEGA
1950
4+GNU(M)+PLDA(MN)-PLDA(JK)))+((-1.0)**
1960
X(H)*(COS(OMEGA+GNU(J)
1970
5+GNU(M)+PLDA(JK))-PLDA(JK)))
1980
GO TO 33
1990
22 W = 1.0066*OMEGA*CAPR(J)*(COS(GNU(J)+0.463*PI))
2000
X*(COS(PLDA(JK))-
2010
1PLDA(MN)))*A*C*PISQ*(2.0*(COS(GNU(J)+
2020
XGNU(M))+*(COS(PLDA(JK))-2PLDA(MN)))-((E**(-0.115*OMEGA))*(((-1.0)**J)*
2030
X(COS(OMEGA+gnu(J))-
2040
3GNU(M)+PLDA(JK)-PLDA(MN)))+((-1.0)**M)*(COS
2050
XOMEGA+GNU(J)+GNU(M)
2060
4+PLDA(MN)+PLDA(JK))))
33 CONTINUE
2070
310 SUM=SUM+(PNUM/DEN)
2080
CALL SWTCH(2*K00DFX)
2090
IF(K00DFX=EQ.2) GO TO 501
2100
500 WRITE(6,23) SUM,PNUM,DEN+G(K,N)+T(J,K)+T(M,N)+P(K)
2110
1P(N)+CAPR(J)+CAPR(M)+A*B*C*D
2120
23 FORMAT(5E17.8/5E17.8/5E17.8//)
2130
251
TABLE 7A (Continued)

501 CONTINUE
   CALL SSWTCH(6, KOOOFX)
   IF(KOOOFX EQ 2) GO TO 51
   WRITE(5, 52) SNX(J), SNY(K), SNM(M), SHY(N)
   52 FORMAT (E17.8)
   51 CONTINUE
   711 CONTINUE
   712 CONTINUE
   713 CONTINUE
   714 CONTINUE
   IF(OMEGA*DELTA - 1.932) 202, 203
   202 PHICY(MU) = 3.2*(10.0**(-3.0))*CAPM**2.0*(DELTA**(-2.0))
   1.0*OMEGA**(-5.0)*SUM
   GO TO 204
   203 PHICY(MU) = 3.2*(10.0**(-3.0))*CAPM**2.0*(DELTA**(-2.0))
   1.0*OMEGA**(-5.0)*SUM
   GO TO 204
   204 CONTINUE
   CALL SSWTCH(4, KOOOFX)
   IF(KOOOFX EQ 2) GO TO 505
   WRITE(6, 53) X# Y# B# P# D# T# O# C# P# D# A# E#
   53 FORMAT (5F10.8)
   531 CONTINUE
   IF(TOP) 205, 205, 205
   205 CONTINUE
   TOP = TOP + 1
   GO TO 740
   206 CONTINUE
   K = MU + 1
   RATIO = ABS (PHICY(MU)/PHICY(K))

252
CALL $SWITCH(4,KOOOFX)
IF(KOOOFX#EQ*2) GO TO 511
510 WRITE(6,512) RATIO,PHIFY(MU),PHIFY(K),MU,K
512 FORMAT(5E17.8,212)
WRITE(6,515) (PHIFY(J),J=1,10)
515 FORMAT(5E17.8)
511 CONTINUE
IF(RATIO=0.971207*208*209)
207 CONTINUE
TOP=TOP+2*0
ITOP=ITOP+2
MU=MU+1
GO TO 740
208 CONTINUE
IF(RATIO=1.031209*209*210)
209 CONTINUE
GO TO 211
210 CONTINUE
TOP=TOP+2*0
ITOP=ITOP+2
MU=MU+1
GO TO 740
211 CONTINUE
PHIP=PHIFY(MU)
CALL $SWITCH(1,KOOOFX)
IF(KOOOFX=EQ*2) GO TO 4002
4001 WRITE(6*4000)PHIP,OMEGA
4000 FORMAT(2E17.8)
4002 CONTINUE
PHIDB= (10.0*ALOG(PHIP))/2*30258509
WRITE(6,212)PHIP,PHIDB,OMEGA,ITOP,MU
212 FORMAT(5E17.8,212)
IF(PHIDB=300,3000,3000)
3000 PHIDB=0.0
3001 CONTINUE
3000 IF(OMEGA=TIM*215*216*216
215 CONTINUE

TABLE 7A (Continued)
TABLE 7A (Continued)

```
100 IF0=3.01213*214+214 1060
213 OMEGA=OMEGA+DOMEG 1070
0=0+1.0 1080
ITOP=IT+15 1090
MU=1 1100
GO TO 300 1110
214 OMEGA=S+0.001*DAMP 1120
0=0.0 1130
GO TO 160 1140
216 END FILE 4 1150
WRITE(6*3009IPEN*IPEN 1160
3009 FORMAT(216) 1170
END 'ILE 5 1180
END FILE 5 1190
STOP 5 1200
99 STOP 7 1210
END 1220
SIBFTC ACTN 1230
REF
CActN
SUBROUTINE ACTN (A#B#THETA#I)
I=0 1240
ACTN0020
IF (ABS(A)+ABS(B)l 27.28,27 ACTN0030
ACTN0040
22 IF (A) 2221,23 ACTN0050
ACTN0060
23 IF (B) 32.24.31 ACTN0070
ACTN0080
24 THETA=3.1415927 ACTN0090
ACTN0100
GO TO 34 1250
ACTN0110
25 THETA=0.0 1260
GO TO 34 1270
ACTN0120
21 IF (B) 26.28+29 ACTN0130
ACTN0140
26 THETA=4*7123890 ACTN0150
GO TO 34 1280
ACTN0160
29 THETA=1.85707963 ACTN0170
GO TO 34 1290
ACTN0180
31 THETA=3.1415927-THETA ACTN0190
GO TO 34 1300
ACTN0200
32 THETA=THETA+3.1415927 ACTN0210
GO TO 34 1310
ACTN0220
33 THETA=6.283154-THETA ACTN0230
GO TO 34 1320
ACTN0240
28 WRITE (6*315) ACTN0250
ACTN0260
315 FORMAT (40HPROGRAM=CANNOT=CONTINUE;ARCTAN OF (0/0) ) ACTN0270
I=1 1330
ACTN0280
34 RETURN ACTN0290
END 1340
```

254
SIBFTCORITY
DIMENSION PHONE(20,20), FREG(20,20), TMIND(20), PHIF(100), SNX(20)
1SNY(20), CX(20), CSY(20), T(20,20), P(20), CAPR(20)
2DIMENSION PHOND(20,20), GNU(20), PLDA(20,20), CKAYZ(20,20), SINKD(20)
120), COSX(20,20), ACRES(10,10), COKAP(20,20), COKNU(20,20)
DIMENSION IDUMP(18) * 50
402 READ(5,403) X, Y, Z, BP, DEPT, DAMP, OMEGA, CP, DP, CPAM, DELTA
1ALFA, TIP, TOLH, TOLL * 70
403 FORMAT(5F10.4/5F10.4/5F10.4) 0080
READ(5,410) RHO 0 85
410 FORMAT(F10.4) 86
406 O=0.0
PI=3.14159265
PI5=PI**2.0
1=2*71828183
EXP={-0.115*OMEGA)
IPEN=80000
SINEX=SIN(PIDX)
COSEX = COS(PIDX)
SINEY = SIN(PIDY)/BP
COSEY = COS(PIDY)/BP
IF (ABS(SINEX)-10.0**(-7.0)) 800,800,801 0190
800 SINEX=0.0 0200
801 IF (ABS(SINEX)-10.0**(-7.0)) 802,802,803 0210
802 SINEY=0.0 0220
803 IF (ABS(COSEX)-10.0**(-7.0)) 804,804,805 0230
804 COSEX=0.0 0240
805 IF (ABS(COSEX)-10.0**(-7.0)) 806,806,807 0250
806 COSEY=0.0 0260
807 CONTINUE
SNX(1)=0.0
SNY(1)=0.0
CSX(1)=1.0
CSY(1)=1.0
DO 701 I=1,20
701 J=I
SNX(J)=SNX(J)*COSEX+CSX(J)*SINEX
320

Table 7B — Subprogram II — Cavity Acoustic Pressure Power Spectrum
TABLE 7B (Continued)

SNY(I)*SNY(J)*COSEY*CSY(J)*SINEY
CSX(I)*CSX(J)*COSEX*SNX(J)*SINEX
CSY(I)*CSY(J)*COSEY*SNY(J)*SINEY
IF (ABS(SNY(I))=10.0**(7.0)) 80,80,81
80 SNY(I)=0.0
81 IF (ABS(SNX(I))=10.0**(7.0)) 82,82,83
82 SNX(I)=0.0
83 IF (ABS(CSY(I))=10.0**(7.0)) 84,84,85
84 CSY(I)=0.0
85 IF (ABS(CSX(I))=10.0**(7.0)) 86,86,87
86 CSX(I)=0.0
87 CONTINUE
701 CONTINUE
WRITE(6,7)
7 FORMAT(1HI/26HWoSTRAWDERNAN
JOB NO 0775)
DO 100 I=1,20
DO 101 J=1,20
F=I
H=J
PNOME(I,J)=(DP*e0.5)*(((F*PI)**260)*UH*M/PI/BP)*0.20)
101 CONTINUE
100 CONTINUE
CALL SSWTCH(4*JSSTCH)
IF(JSSTCH=EQ.21) GO TO 521
520 WRITE(6,522) XYZPBPDEPTHDAMP.OMEGACPDPCAP4,
IDELIATALFA#TIP
522 FORMAT(5F10.4/3F10.4/3F10.4)
WRITE(6,590) RHO
590 FORMAT(1X*F10.6)
521 CONTINUE
WRITE(6,142)( (PNOME(I,J),Ja1,10).Is1,103
142 FORMAT(I10E12*4) 04
160 DO 106 K=1,20
106 CONTINUE
107 CONTINUE
TABLE 7B (Continued)

```plaintext
DO 102 I=1,20
DO 103 J=1,20
IF(OMEGA=PNOME(I*J))104,103,103
104 K=I
L=J
FREQ(K+L)=PNOME(K+L)
GO TO 102
103 CONTINUE
102 CONTINUE
DO 700 J=1,20
TMINO(J)=0.0
700 CONTINUE
N=0
DO 109 I=1,20
DO 108 J=1,20
IF(FREQ(I*J))99,110,111
111 M=M+1
TMINO(M)=FREQ(I*J)
110 CONTINUE
108 CONTINUE
109 CONTINUE
S=TMINO(1)
DO 112 L=1,19
IF(S=TMINO(L+1))112,121,121
121 S=TMINO(L+1)
112 CONTINUE
DO 130 L=1,20
IF(S=TMINO(L))130,132,130
132 N=N+L
GO TO 180
130 CONTINUE
180 DMEG=(S-OMEGA)/3.0
Z=N
ITOP=Z+1.5
NU=1
DO 901 I=1,10
DO 900 J=1,10
```

257
TABLE 7B (Continued)

\[ \frac{A+1}{B+1} \]

ACRES(I,J)=PI*CP*(A+2)+(B+2)*0.5

900 CONTINUE

901 CONTINUE

WRITE(6,902) ((ACRES(I,J)+J=1,10)+I=1,10)

902 FORMAT(10E12.4)

300 CONTINUE

DO 2171 I=1,100

PHIFE[I]=0.0

217 CONTINUE

2171 CONTINUE

SUM=0.0

TOP=0.0

EXPB=E**(0.7*OMEGA*BP)

DO 702 J=1,20

DO 703 I=1,20

B=I

D=J

IF (I=111,12,11)

11 G(I,J)=3*PI*SOS(((-1.0)**I)*((-1.0)**J)-1.0)+(2.0*(1-1.0)**I)

12 G(I,J)=3.5*PI*SOS((2.0*(0.7*OMEGA*BP)**2)+SOS((B**2 )+

13 I**2 ))+SOS*(2.0-((-1.0)**I)+((-1.0)**J)*EXPB)

702 CONTINUE

703 CONTINUE

CALL OVERFL(J)

704 IF(J=EQ*2) GO TO 751

750 WRITE(6,752)EXPB

752 FORMAT(12)

751 CONTINUE

DO 705 J=1,20

704 DO 704 J=1,20

PHNOND(I,J)=PHOME(I,J)/OMEGA

T(I,J)=((PHNOND(I,J)**2 )+1.0)**2 +5.0**DAMP*PHNOND(I,J)**2

1**0.5

258
TABLE 7B (Continued)

704 CONTINUE
705 CONTINUE
DO 706 I=1,20
A=I
P(I)=(A**2)*PISQ+0.7*OMEGA*BP**2
706 CONTINUE
DO 707 I=1,20
A=I
CAPR(I)=(((A**2)*PISQ-0.987*(OMEGA**2))**2 +0.0529*(OMEGA**2))**0.5
707 CONTINUE
PNGNU=0.25*(OMEGA**2)
DO 708 I=1,20
A=1
DGNU=(A**2)*PISQ-0.987*(OMEGA**2)
CALL ACTN(DGNU,PNGNU,GNU(I),IOTA)
IF(IOTA)13,14,15
12 GO TO 216
14 CONTINUE
708 CONTINUE
DO 709 J=1,20
COGNU(I,J) = COS(GNU(I)+GNU(J))
709 CONTINUE
710 CONTINUE
DO 710 I=1,20
DO 709 J=1,20
PNLDAn2*O*DAMP*PNOME(I,J)*OMEGA
DLDA*(PNOME(I,J)**2-(OMEGA**2)
CALL ACTN(DLDA,PNLDA,PLDA(I,J),ILDA)
IF(ILDA)19,20,21
19 GO TO 216
20 CONTINUE
709 CONTINUE
710 CONTINUE
CQR=(OMEGA/CP)**2
DO 811 I=1,20
**TABLE 7B (Continued)**

```plaintext
DO 810 J=1+20
  A=I-1
  B=J-1
  EOR=P(S0*(((A**2 )+((B/BP)**2 )))
  IF(EOR-COR)812 812 813
812 CKAYZ(I,J)=CKAYZ(COR-EOR)**0.5
    ARGD=CKAYZ(I,J)*DEPTH
    ARGZ=CKAYZ(I,J)*2P
    SINK=(I,J)*SIN(ARGD)
    COSKZ(I,J)=COSI(ARGZ)
  GO TO 810
813 CKAYZ(I,J)=CKAYZ(I,J)*DEPM
    ARGD=CKAYZ(I,J)*DEPTH
    ARGZ=CKAYZ(I,J)*2P
    SINK=(I,J)*COSI(ARGD)
    COSKZ(I,J)=COSI(ARGZ)
  CONTINUE
810 CONTINUE
811 CONTINUE
DO 5002 I=1+20
  A=I
  B=J-1
  L=J-1
  IF(A<B)5003 5004 5003
  5004 COKAPIJ=0.0
  GO TO 5001
  5003 ANAF=(A+B)/2.0
  IHAF=AHAF
  IHAF=AHAF
  IF(AHAF-AHAF)<0.02
  4002 COKAPIJ=0.0
  GO TO 5001
  4001 IF(AHAF-AHAF)>0.02
  5005 DJ=1+0
  GO TO 5007
  5006 DJ=2+0
```

---

280
TABLE 7B (Continued)

5007 COKAP(I,J) = A*1.0 - (-1.0)**K*(-1.0)**L)/(DIJ*(-1.0)**L))  
5001 CONTINUE  
5002 CONTINUE  
740 CONTINUE  
DO 714 J = 1:ITOP  
DO 713 K = 1:ITOP  
DO 712 M = 1:ITOP  
DO 711 N = 1:ITOP  
A = J  
B = K  
C = M  
D = N  
IF(J = M) 21, 22, 22  
21 W = A*C*PISQ*(2*OMEGA(J)*COS(PLDA(J,K) - PLDA(M,N))) +  
1((-1.0)**M)*(-1.0)**J)*CAPR(J)*(COSIGNU(J)*PLDA(J,K) -  
2PLDA(M,N))*CAPR(J)*COSIGNU(J)*PLDA(J,K))/(PISQ*  
3((-1.0)**J)*(1.0)**J)*COS(OMEGA*  
4*GNUM(J)+GNUM(M)+PLDA(J,K)-PLDA(M,N))*(PLDA(J,K)+(-1.0)**M)*COS(OMEGA+GNUM(J  
5)*GNUM(M)+PLDA(J,K)-PLDA(M,N)) + CAPR(J)*(COS(GNUM(J)-0.0463*P  
6*GNUM(J)+GNUM(M)+PLDA(J,K)-PLDA(M,N))*(PLDA(J,K)+(-1.0)**M)*COS(OMEGA+GNUM(J)  
7)*GNUM(M)+PLDA(J,K)-PLDA(M,N)) + CAPR(J)*(COS(GNUM(J)-0.0463*P  
8*GNUM(J)+GNUM(M)+PLDA(J,K)-PLDA(M,N))*(PLDA(J,K)+(-1.0)**M)*COS(OMEGA+GNUM(J) +  
9*GNUM(M)+PLDA(J,K)-PLDA(M,N)) + CAPR(J)*(COS(GNUM(J)-0.0463*P  
10*GNUM(J)+GNUM(M)+PLDA(J,K)-PLDA(M,N)) + CAPR(J)*(COS(GNUM(J)-0.0463*P  
11*GNUM(J)+GNUM(M)+PLDA(J,K)-PLDA(M,N))*(PLDA(J,K)+(-1.0)**M)*COS(OMEGA+GNUM(J  
12)*GNUM(M)+PLDA(J,K)-PLDA(M,N)) + CAPR(J)*(COS(GNUM(J)-0.0463*P  
13*GNUM(J)+GNUM(M)+PLDA(J,K)-PLDA(M,N)) + CAPR(J)*(COS(GNUM(J)-0.0463*P  
14*GNUM(J)+GNUM(M)+PLDA(J,K)-PLDA(M,N)) + CAPR(J)*(COS(GNUM(J)-0.0463*P  
15*GNUM(J)+GNUM(M)+PLDA(J,K)-PLDA(M,N)) + CAPR(J)*(COS(GNUM(J)-0.0463*P  
16*GNUM(J)+GNUM(M)+PLDA(J,K)-PLDA(M,N)) + CAPR(J)*(COS(GNUM(J)-0.0463*P  
17*GNUM(J)+GNUM(M)+PLDA(J,K)-PLDA(M,N)) + CAPR(J)*(COS(GNUM(J)-0.0463*P  
18*GNUM(J)+GNUM(M)+PLDA(J,K)-PLDA(M,N)) + CAPR(J)*(COS(GNUM(J)-0.0463*P  
19*GNUM(J)+GNUM(M)+PLDA(J,K)-PLDA(M,N)) + CAPR(J)*(COS(GNUM(J)-0.0463*P  
20*GNUM(J)+GNUM(M)+PLDA(J,K)-PLDA(M,N)) + CAPR(J)*(COS(GNUM(J)-0.0463*P  
21*GNUM(J)+GNUM(M)+PLDA(J,K)-PLDA(M,N)) + CAPR(J)*(COS(GNUM(J)-0.0463*P  
22*GNUM(J)+GNUM(M)+PLDA(J,K)-PLDA(M,N)) + CAPR(J)*(COS(GNUM(J)-0.0463*P  
23*GNUM(J)+GNUM(M)+PLDA(J,K)-PLDA(M,N)) + CAPR(J)*(COS(GNUM(J)-0.0463*P  
24*GNUM(J)+GNUM(M)+PLDA(J,K)-PLDA(M,N)) + CAPR(J)*(COS(GNUM(J)-0.0463*P  
25*GNUM(J)+GNUM(M)+PLDA(J,K)-PLDA(M,N)) + CAPR(J)*(COS(GNUM(J)-0.0463*P  
26*GNUM(J)+GNUM(M)+PLDA(J,K)-PLDA(M,N)) + CAPR(J)*(COS(GNUM(J)-0.0463*P  
27*GNUM(J)+GNUM(M)+PLDA(J,K)-PLDA(M,N)) + CAPR(J)*(COS(GNUM(J)-0.0463*P  
28*GNUM(J)+GNUM(M)+PLDA(J,K)-PLDA(M,N)) + CAPR(J)*(COS(GNUM(J)-0.0463*P  
29*GNUM(J)+GNUM(M)+PLDA(J,K)-PLDA(M,N)) + CAPR(J)*(COS(GNUM(J)-0.0463*P  
30*GNUM(J)+GNUM(M)+PLDA(J,K)-PLDA(M,N)) + CAPR(J)*(COS(GNUM(J)-0.0463*P  
31*GNUM(J)+GNUM(M)+PLDA(J,K)-PLDA(M,N)) + CAPR(J)*(COS(GNUM(J)-0.0463*P  
32*GNUM(J)+GNUM(M)+PLDA(J,K)-PLDA(M,N)) + CAPR(J)*(COS(GNUM(J)-0.0463*P  
33 CONTINUE  
DO 824 J = 1:ITOP  
DO 823 K = 1:ITOP  
DO 822 M = 1:ITOP  
DO 821 N = 1:ITOP  
AQ = J  
BR = K  
CS = M  
DT = T  
TM = MAX1(A+D+B+C*D+A*BR+C*CS+DT)  
IF(TM = TOP) 821, 821, 737  
737 CONTINUE
### TABLE 7B (Continued)

<table>
<thead>
<tr>
<th>Line</th>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>715</td>
<td>IF(CSY(NT))(\times)716 + 715 = 716</td>
<td>2250</td>
</tr>
<tr>
<td>716</td>
<td>PNUM = 0.0</td>
<td>2560</td>
</tr>
<tr>
<td>717</td>
<td>DEN = 1.0</td>
<td>2570</td>
</tr>
<tr>
<td>718</td>
<td>GO TO 310</td>
<td>2590</td>
</tr>
<tr>
<td>719</td>
<td>IF(CSX(MS))(\times)718 + 717 = 718</td>
<td>2600</td>
</tr>
<tr>
<td>720</td>
<td>PNUM = 0.0</td>
<td>2610</td>
</tr>
<tr>
<td>721</td>
<td>DEN = 1.0</td>
<td>2620</td>
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<tr>
<td>722</td>
<td>GO TO 310</td>
<td>2630</td>
</tr>
<tr>
<td>723</td>
<td>IF(CSX(KR))(\times)720 + 719 = 720</td>
<td>2640</td>
</tr>
<tr>
<td>724</td>
<td>PNUM = 0.0</td>
<td>2650</td>
</tr>
<tr>
<td>725</td>
<td>DEN = 1.0</td>
<td>2660</td>
</tr>
<tr>
<td>726</td>
<td>GO TO 310</td>
<td>2670</td>
</tr>
<tr>
<td>727</td>
<td>CONTINUE</td>
<td>2680</td>
</tr>
<tr>
<td>728</td>
<td>IF(COKAP(JJO))(\times)602 + 601 = 602</td>
<td>2690</td>
</tr>
<tr>
<td>730</td>
<td>CKAPA = 0.0</td>
<td>2700</td>
</tr>
<tr>
<td>731</td>
<td>PNUM = 0.0</td>
<td>2710</td>
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<tr>
<td>732</td>
<td>DEN = 1.0</td>
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</tr>
<tr>
<td>733</td>
<td>GO TO 310</td>
<td>2730</td>
</tr>
<tr>
<td>735</td>
<td>IF(COKAP(KKR))(\times)604 + 603 = 604</td>
<td>2740</td>
</tr>
<tr>
<td>737</td>
<td>CKAPA = 0.0</td>
<td>2750</td>
</tr>
<tr>
<td>738</td>
<td>PNUM = 0.0</td>
<td>2760</td>
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<tr>
<td>739</td>
<td>DEN = 1.0</td>
<td>2770</td>
</tr>
<tr>
<td>740</td>
<td>GO TO 310</td>
<td>2780</td>
</tr>
<tr>
<td>742</td>
<td>IF(COKAP(MS))(\times)606 + 605 = 606</td>
<td>2790</td>
</tr>
<tr>
<td>744</td>
<td>CKAPA = 0.0</td>
<td>2800</td>
</tr>
<tr>
<td>745</td>
<td>PNUM = 0.0</td>
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<tr>
<td>747</td>
<td>DEN = 1.0</td>
<td>2820</td>
</tr>
<tr>
<td>748</td>
<td>GO TO 310</td>
<td>2830</td>
</tr>
<tr>
<td>749</td>
<td>IF(COKAP(NNT))(\times)608 + 607 = 608</td>
<td>2840</td>
</tr>
<tr>
<td>750</td>
<td>CKAPA = 0.0</td>
<td>2850</td>
</tr>
<tr>
<td>751</td>
<td>PNUM = 0.0</td>
<td>2860</td>
</tr>
<tr>
<td>752</td>
<td>DEN = 1.0</td>
<td>2870</td>
</tr>
<tr>
<td>753</td>
<td>GO TO 310</td>
<td>2880</td>
</tr>
<tr>
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TABLE 7B (Continued)

608 CONTINUE
CKAPA(CKAP(J+J0)*CKAP(K+K0)*CKAP(M+MS)*CKAP(N+NT)
UQRST=CAYZ(JO+K*KR)*CAYZ(MS+NT)*SINKD(JO+K*KR)*SINKD(MS+NT)
PNUM=CX(J0+K*KR)*CSY(KR)*CSX(MS)*CSY(NT)*COSKZ(JO+K*KR)*COSKZ(MS+NT)
1*G(K)+N)*W=CKAPA
DEN=TI(J+K)*T(M+K)*P(K)*D*N)*CAPR(J)*CAPR(M)*UQRST
310 SUM=SUNP/DEN
CALL SSWTCH(2#JSSTCH)
IF(JSSTCH=EQ.2) GO TO 501
500 WRITE(6,531) SUM+PNUM+DEN(K+N)+P(J+K),
IP(N)+CAPR(J)+CAPR(M)+CKAPA+UQRST+J0+KR+MS+NT+J0+MS+N
23 FORMAT(5E17.8/5E17.8/3E17.8/8110//)
501 CONTINUE
821 CONTINUE
822 CONTINUE
823 CONTINUE
824 CONTINUE
711 CONTINUE
712 CONTINUE
713 CONTINUE
714 CONTINUE
IF(OMEGA*DELTA)-1.932)202,202,203
202 PHIFY(MU)=7.08(10.0**(-3.00)*((ALFA*2.0)*(CAPM*2.0)*DELTA*SUM
1*(RO**)/(PSQ**2))2
GO TO 204
203 PHIFY(MU)=5.112(10.0**(-2.00)*((ALFA**2.0)*(CAPM**2.0)*(DELTA**
1-2.00)*OMEGA**(-3.00)*SUM*/*(RO**2)/(/PSQ**2)
204 CONTINUE
CALL SSWTCH(4#JSSTCH)
3200 WRITE(6,531) X+Y+Z+BP+DEPHT+DAMP+OMEGA+CP+DP+CAPM
1DELTAA+ALFA+TIP
532 FORMAT(5F10.8/5F10.4/3F10.4)
531 CONTINUE
CALL SSWTCH(4#JSSTCH)
IF(JSSTCH=EQ.2) GO TO 505
504 WRITE(6,502) PHIFY(MU),TOP+MU
3280
TABLE 7B (Continued)

502 FORMAT(2E17.8*112) 3290
CALL SSWTCH(4,JSSTCH) 3300
IF(JSSTCH=EO+2) GO TO 99 3310
505 CONTINUE 3320
IF(TOP=206+205+206 3330
205 CONTINUE 3340
TOP=ITOP 3350
MU=MU+1 3360
ITOP=ITOP+2 3370
GO TO 740 3380
206 CONTINUE 3390
K=MU-1 3400
RATIO = ABS((PHIFY(MU))/(PHIFY(K))) 3410
CALL SSWTCH(4,JSSTCH) 3420
IF(JSSTCH=EO+2) GO TO 511 3430
510 WRITE(6,512) RATIO,PHIFY(MU),PHIFY(K),MU,K 3440
512 FORMAT(3E17.8*212) 3450
WRITE(6,515) (PHIFY(J),J=1,10) 3460
515 FORMAT(5E17.8) 3470
CALL SSWTCH(4,JSSTCH) 3480
IF(JSSTCH=EO+2) GO TO 99 3490
511 CONTINUE 3500
IF(RATIO-TGLL)207=207 3510
207 CONTINUE 3520
TOP=TOP+2 3530
ITOP=ITOP+2 3540
MU=MU+1 3550
GO TO 740 3560
208 CONTINUE 3570
IF(RATIO-TOLM)209=209 3580
209 CONTINUE 3590
GO TO 211 3600
210 CONTINUE 3610
TOP=TOP+2 3620
ITOP=ITOP+2 3630
MU=MU+1 3640
GO TO 740 3650

264
TABLE 7B (Continued)

211 CONTINUE
  PHI=PHI*PHI
  PHIDB = (10.0*ALOG(3.2))/2*30258509
  WRITE(6,212) PHI*PHIDB*OMEGA*ITOP*MU
212 FORMAT(3E17.12)
  CALL SSWTCH(4,JSSTCH)
  IF(JSSSTCH.EQ.2) GO TO 99
  IF(PHIDB)300!#OMEGA*ITOP*MU
3000 PHIDB=0.0
3001 CONTINUE
  IF(OMEGA-TIP1213*216*216
215 CONTINUE
  IFO=2+01213*214
213 OMEGA=OMEGA*DOMEQ
  G=Q+1.0
  ITOP=Z+1.5
  NUM=1
  GO TO 300
214 OMEGA=5*0.001*DAMP
  GO TO 160
216 END FILE 4
  WRITE(6,3009) IPEN*IPEN
3009 FORMAT(0)
  END FILE 5
  STOP 7
99 STOP 7
END
$1BFTC ACTN REF
C A C T N
SUBROUTINE ACTN (A,B,THETA+1)
  I=0
  IF (ABS(A)+ABS(B)) 27,28#27
27 THETA= ATAN(ABS(B/A))
  IF (A) 22#24,31
22 IF (B) 33,25,34
  IF (B) 26,28#29
26 THETA=4#7123890
  GO TO 34
28 WRITE (6,315)
315 FORMAT (4OHOPROGRAM-CANNOT-CONTINUE+ARCTAN OF 0/0)
  I=1
  RETURN
END

23 IF (B) 33,25,34
24 THETA=3*1415927
  GO TO 34
25 THETA=0.0
  GO TO 34
21 IF (B) 26,28#29
26 THETA=4#7123890
  GO TO 34
29 THETA=1.5707963
  GO TO 34
31 THETA=3*1415927-THETA
  GO TO 34
32 THETA=THETA+3*1415927
  GO TO 34
33 THETA=6*283154-THETA
  GO TO 34
28 WRITE (6,315)
315 FORMAT (4OHOPROGRAM-CANNOT-CONTINUE+ARCTAN OF 0/0)
  I=1
34 RETURN
END
APPENDIX E

BOEING PROGRAM II (JACOBS AND LAGERQUIST)

APPENDIX E1 – MATHEMATICAL ANALYSIS
APPENDIX E2 – METHOD FOR DETERMINING INPUT DATA
APPENDIX E3 – PROGRAM IDENTIFICATION
APPENDIX E4 – TEST RUNS
NOTATION

\[ A \]
Diagonal matrix of elemental areas on structure associated with nodal points

\[ A_i, A_j, A_k \]
Elemental area on structure associated with \( i, j, \) and \( k \) node points (in.\(^2\))

\[ A_n \]
A constant, \( A_1 = 28.5, A_2 = 7.30, A_3 = 0.5 \)

\[ a \]
\( 1/U_c \theta (\text{in}^{-1}) \)

\[ B \]
\( 1/0.88^* (\text{in}^{-1}) \)

\[ b \]
\( \omega/U_c (\text{in}^{-1}) \)

\[ [ C] \]
Damping matrix

\[ [ C_F(\omega)] \]
Force co-power spectral density matrix (lb\(^2\) • sec)

\[ C_{Fi}(\omega) \]
Force co-power spectral density (co-PSD) acting on plate pairs \( i \) and \( j \) (lb\(^2\) • sec)

\[ C_f \]
Local coefficient of frictions \( r_w/q \)

\[ C_p(\xi, \eta; \omega) \]
Pressure co-power spectral density function (psi\(^2\) • sec)

\[ [ C_p(\omega)] \]
Pressure co-power spectral density matrix (psi\(^2\) • sec)

\[ [ C_p(\omega)] \]
Deflection co-power spectral density matrix (in.\(^2\) • sec)

\[ [ D] \]
Diagonal matrix, real factor in admittance matrix

\[ [ E] \]
Diagonal matrix, imaginary factor in admittance matrix

\[ [ F(0)] \]
Column force matrix (lb)

\[ g \]
Structural damping coefficient

\[ H_i^{(i)}(i\omega), H_j^{(k)}(i\omega) \]
Complex frequency response function defining deflection at \( j \) due to unit harmonc forcing at \( i \) and \( k \), respectively

\[ [ I] \]
Column matrix, Laplace transform of force column matrix with unit impulse at particular point, zero forces at other points

\[ [ H(\omega)] \]
Complex frequency response matrix

\[ i \]
\( \sqrt{-1} \)

\[ i, j \]
Finite node points

\[ [ K] \]
Stiffness matrix

\[ K^2 \]
Equals \( \frac{p^2}{r_w^2} \); also equals \( \sum_{n=1}^{3} \frac{A_n}{K_n} = 9.56 \) (a normalization constant for power spectral density)
\( k_n \) A constant; \( K_1 = 6.1, K_2 = 0.91, K_3 = 0.26 \)

\( k(t) \) Impulse response function defined at time \( t \) due to a unit impulse applied \( r \) time units earlier

\( \{ F(\delta) \} \) Matrix of the Laplace transform of the deflection

\( M \) Mach number

\( \{ M \} \) Mass matrix

\( m_{ij} \) Generalized mass

\( n_{ij} \) Integers used to denote separation distance between the \( i \) and \( j \) node in \( x \) and \( y \)-directions respectively

\( N \) Number of frequencies used to define the pressure cross power spectral density

\( n \) Integer denoting spectral component; allowable values are 1, 2, and 3

\( p_j(t), p_1(t), p_2(t) \) Pressure at positions \( j, 1, \) and \( 2, \) respectively

\( p^2 \) or \( < p^2 > \) Mean square fluctuating pressure at the wall in turbulent boundary layer (psi²)

\( \{ Q_F(\omega) \} \) Force quad power spectral density matrix (lb²/sec)

\( Q_{F_{ij}}(\omega) \) Force quad-power spectral density acting on plate pairs \( i \) and \( j \) (lb²/sec)

\( Q_p(\xi, \eta; \omega) \) Pressure quad-power spectral density function

\( \{ Q_p(\omega) \} \) Pressure quad-power spectral density matrix (psi²/sec)

\( \{ Q_\delta(\omega) \} \) Deflection quad-power spectral density matrix (in.²/sec)

\( q(t) \) Dynamic pressure (psi)

\( \{ q(t) \} \) Column matrix of principal coordinates

\( R_{p_{jk}}(r) \) Cross correlation of pressures at points \( j \) and \( k \) (psi²)

\( R_{\delta_{j} \delta_{r}}(\xi, \eta; \omega) \) Cross correlation of deflections at points \( q \) and \( r \) resulting from loads at points \( j \) and \( k \) respectively (in.²)

\( S \) \( \omega S^*/U \), Scrouhal number, dimensionless frequency

\( s \) Laplace dummy variable

\( t \) Time (sec)

\( U \) Free-stream air flow velocity or aircraft speed (in./sec)

\( U_c \) Convection velocity (in./sec)

\( x, y \) Cartesian coordinates (in.)
$x_i, y_i$ Cartesian coordinates of $i$th node point (in.)

$Y_n(K_nS), Y_n\left(\frac{\omega K_n}{BU_c}\right)$ A correction factor (equals 1 unless otherwise defined)

$\gamma$ $C_p/C_v$, ratio of specific heats of air, 1.41 when $p_a = 14.7$ psi

$\zeta$ Critical damping ratio

$\{\zeta(\theta)\}$ Column matrix of deflection distance of structure normal to the surface of the structural plate (in.)

$\delta^*$ Boundary layer displacement thickness (in.)

$n$ Separation distance in $y$-direction (in.)

$\hat{\eta}$ $\eta/0.8K_n\delta^*$, normalized separation distance

$\eta'$ $\eta + \eta_j - \eta_i$ (in.)

$\eta_i, \eta_j$ Distance in $y$-direction between node and dummy variable on $i$th and $j$th nodal areas respectively (in.)

$\hat{\eta}_i, \hat{\eta}_j$ $\eta_i/0.8K_n\delta^*, \eta_j/0.8K_n\delta^*$, normalized separation distance

$\eta_0$ Smallest basic separation distance in $y$-direction (in.)

$\hat{\eta}_0$ $\eta_0/0.8K_n\delta^*$, normalized separation distance

$\theta$ Eddy lifetime (sec$^{-1}$)

$\kappa_0(K_nS), \kappa_0\left(\frac{\omega K_n}{BU_c}\right)$ Modified Bessel function of order zero with argument $K_nS$ and $\omega K_n / BU_c$ respectively

$\lambda, \mu$ Proportionality factor between damping and stiffness and inertias respectively

$\xi$ Separation distance in $x$-direction (in.)

$\xi'$ $\xi + \xi_j - \xi_i$ (in.)

$\xi_i, \xi_j$ Distance in $x$-direction between node and dummy variable on $i$th and $j$th nodal areas respectively (in.)

$\xi_0$ Smallest basic separation distance in $x$-direction

$\Pi_{F,ij}(i\omega)$ Normalized cross power spectral density of forces acting on plate pair $i$ and $j$, a complex function of $\omega$ (sec)

$\Pi_{F,(n),ij}(i\omega)$ $n$th component of normalized cross power spectral density of forces acting on plate pair $i$ and $j$, a complex function of $\omega$ (sec)

$\Pi_n(\xi, \eta; i\omega)$ $n$th component of the normalized pressure cross power spectral density, a complex function of $\omega$ (sec)

$\Pi(\xi, \eta; i\omega)$ Normalized pressure cross power spectral density, a complex function of $\omega$ (sec)

$\Pi(\omega), \Pi_n(\omega)$ Normalized pressure power spectral density and the $n$th component of normalized pressure power spectral density respectively
\( \rho_n(\xi, \eta; r) \)  
\( n \)th component of pressure cross-correlation coefficient (psi^2)

\( \rho(\xi, \eta; r) \)  
Cross-correlation coefficient, \(-1 \leq \rho(\xi, \eta; r) \leq 1\)

\( r \)  
Time delay (sec)

\( r_u \)  
Local fluid shearing stress of air measured at wall (psi)

\( [\Phi_F(i\omega)] \)  
Force cross power spectral density matrix (lb^2 \cdot sec)

\( \Phi_F(i\omega) \)  
Force power spectral density acting on the \( i \)th structural plate (lb^2 \cdot sec)

\( \Phi_{F_{ij}}(i\omega) \)  
Cross power spectral density of forces acting on plate pair \( i \) and \( j \), a complex function of \( \omega \) (lb^2 \cdot sec)

\( \Phi_p(\xi, \eta; i\omega) \)  
Pressure cross power spectral density function (psi^2 \cdot sec)

\( \Phi_p(\omega) \)  
Pressure power spectral density (psi^2 \cdot sec)

\( \{\phi\} \)  
Matrix of eigenvectors

\( \{\phi(\sigma)\} \)  
Eigenvector column matrix (\( r \)th normal mode shape)

\( \psi \)  
Phase angle (radians)

\( \omega \)  
Angular frequency, \( 2\pi f \) (radians/sec)

\( \omega_i \)  
Eigenfrequency of \( i \)th mode of structure, modal frequency (radians/sec)

\( \omega_k, \omega_k^* \)  
Frequency and a set of frequencies respectively at which pressure cross power spectral density is defined (radians/sec)

\( \omega_r \)  
Angular eigenfrequency or eigenvalue (radians/sec)

\( [\cdot]^T \)  
Transpose of matrix

\( (\cdot)^* \)  
Complex conjugate

\( (\cdot)' \)  
First derivative with respect to time

\( (\cdot)'' \)  
Second derivative with respect to time

\( (\cdot)\overline{\cdot} \)  
Time average

\( (\cdot) \)  
Vector

\( [\cdot(\cdot\cdot)] \) or [\( (\cdot)\cdot\cdot\cdot \)]  
Denotes diagonal matrix
APPENDIX E1 – MATHEMATICAL ANALYSIS

This section presents equations developed by means of a finite element analysis\textsuperscript{34–37} for the deflection cross power spectral density response of a simple clamped panel to a turbulent boundary layer.

Consider a plate idealized into a finite number of discrete structural elements connected at node points having prescribed freedoms (Figure 19). The physical properties of the plate are assumed to be lumped into individual elements. The equations of motion of each panel element is written in the form of a matrix equation\textsuperscript{18}

$$\begin{align*}
\{M\} \ddot{\delta}(t) + \{C\} \dot{\delta}(t) + \{K\} \delta(t) = \{F(t)\}
\end{align*}
$$

(E1)

where $\delta(t)$ and $F(t)$ are column matrices of time dependent nodal displacements and applied forces, respectively.

The square matrices $\{M\}$, $\{C\}$, and $\{K\}$ are inertia, viscous damping, and stiffness coefficients, respectively.

Elements of the inertia matrix $\{M\}$ correspond to inertia forces associated with the freedoms at each node. For small panel deflections, rotary and inplane inertia forces are small in comparison to the forces corresponding to translational freedoms. Hence the inertia of the elements are treated by assuming their masses to be concentrated at their respective nodes, thereby diagonalizing the inertia matrix. The accuracy of the concentrated mass assumption depends primarily on the number of elements used to represent the panel; accuracy increases with the number of elements used (for some quantitative data on accuracy, see pages 22, 23 and 34 of Reference 34).

The viscous damping is assumed to be proportional to inertia and stiffness; the significance of this assumption is discussed below.* Hence

$$\{C\} = \mu \{M\} + \lambda \{K\}$$

(E2)

where $\mu$ and $\lambda$ are proportionality factors.

For the $j$th mode:

$$C_j = \mu M_j + \lambda K_j$$

It is convenient to represent the damping factor $\zeta_j$ which represents the fraction of critical viscous damping for the $j$th mode

$$2\zeta_j = \frac{C_j}{\sqrt{K_j M_j}} = \frac{\mu M_j + \lambda K_j}{M_j \omega_j} = \frac{\mu}{\omega_j} + \lambda \omega_j$$

(E3)

or

$$2\zeta_j \omega_j = \mu + \lambda \omega_j^2$$

*Viscous and structural damping are forms of damping which allow the equations of motion to be uncoupled when displacements are expressed in terms of normal mode shapes.
PRESSURE ON AREA $A_j$
(ASSOCIATED WITH NODE POINT $j$)

PRESSURE ON AREA $A_k$
(ASSOCIATED WITH NODE POINT $k$)

DISPLACEMENT AT POINT $r$

DISPLACEMENT AT POINT $s$

Figure 19 – Random Pressure Loads
Alternatively, when structural damping is used, the equations of motion for this damping are (see pages 83–86 of Reference 18)

$$[M] \ddot{\delta}(t) + (1 + ig) [K] \dot{\delta}(t) = \dot{F}(t)$$ \hspace{1cm} (E4)

When the structural damping coefficient is small ($g << 1$), then (see page 88, Equation XI and page 16, Equation (27) of Reference 18) the coefficient $g$ is related to an equivalent viscous damping factor

$$g = 2 \zeta_f$$ \hspace{1cm} (E5)

Total panel damping includes both acoustic radiation and structural damping. This total damping based on experimental panel-displacement power spectral density measurements is assumed to have the following mass-proportional viscous-damping representation as in Equations (E2) and (E3) (see pages 24–25 and 34 of Reference 34)

$$\zeta = \frac{\Delta f}{2 f} = \frac{7.5}{f}$$

The symmetric stiffness matrix $[K]$ for the plate is generated by a computer program based on the displacement or stiffness method of static matrix structural analysis. Both applied loading and inertia forces of the panel correspond only to translational freedoms, but the stiffness matrix is formulated with all freedoms included. Through matrix manipulation, the stiffness matrix can also be expressed solely in terms of translational deflections of the panel. Obtaining this "reduced" stiffness matrix in no way restrains the displacements of the unloaded freedoms; thus the accuracy of the stiffness coefficients is unaffected. In the displacement method the panel is idealized as a system of finite plate elements connected at node points. For any point on the plate

$$F_r = K_{r1} \delta_1 + K_{r2} \delta_2 + \ldots + K_{rn} \delta_n$$

where the $K_{rs}$ are force influence coefficients that relate the external force at one point on the plate to deflections at that and other points. The collection of these force equations is represented as

$$\begin{bmatrix}
F_1 \\
F_2 \\
\vdots \\
F_n
\end{bmatrix} = \begin{bmatrix}
K_{11} & K_{12} & \cdots & K_{1n} \\
K_{21} & K_{22} & \cdots & K_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
K_{n1} & K_{n2} & \cdots & K_{nn}
\end{bmatrix} \begin{bmatrix}
\delta_1 \\
\delta_2 \\
\vdots \\
\delta_n
\end{bmatrix}$$ \hspace{1cm} (E6)
where the matrix of force influence coefficients is called the stiffness matrix and the individual terms in the matrix are called the stiffness coefficients. The coefficient (or element) \( K_{rs} \) of \( [K] \) is the static force at nodal point \( i \) corresponding to a unit displacement at point \( j \), all other points held fixed. The foregoing is more fully discussed in Reference 37.

We determine the system response from the equations of motion, (E1) or (E4), for an excitation random in time using the frequency response method of analysis. The method specifies the characteristics of the systems behavior by a matrix of complex frequency response functions, i.e., the admittance matrix now derived.

Let \( \delta(t) \), the response at time \( t \) due to a unit impulse applied at an earlier time \( t' \), be expressed by a unit impulse function \( h(t-t') \). Then the response to an arbitrary input \( F(t') \) is, by use of the superposition or convolution integral

\[
\delta(t) = \int_0^t F(t') h(t-t') \, dt'
\]

since \( h(t-t') = 0 \) for \( t' > t \) and since \( F(t') \) can be defined for all negative \( t' \), the limits of integration can be extended (see Equation 2.8 of Reference 33) so that

\[
\delta(t) = \int_{-\infty}^{\infty} F(t') h(t-t') \, dt'
\]

If \( F(t') = e^{i\omega t'} \), then

\[
\delta(t) = \int_{-\infty}^{\infty} h(t-t') e^{i\omega t'} \, dt'
\]

Let \( r = t-t' \), \( -dr = dt' \). Then since \( -\int_{-\infty}^{\infty} \cdots \, dr = \int_{-\infty}^{\infty} \cdots \, dr \),

\[
\delta(t) = e^{i\omega t} \int_{-\infty}^{\infty} h(r) e^{-i\omega r} \, dr = e^{i\omega t} H(i\omega)
\]

where \( H(i\omega) = \int_{-\infty}^{\infty} h(r) e^{-i\omega r} \, dr \) is defined as the complex frequency response function = 2\( \pi \) times the Fourier transform of the unit impulse response function.
The Laplace transform of Equation (E1) is
\[ \mathcal{L}\left[ (M) \dot{\delta}(t) + \{C\} \ddot{\delta}(t) + \{K\} \delta(t) \right] = \mathcal{L}[F(t)] \]
where \( \mathcal{L}[F(t)] = \int_{0}^{\infty} F(t) e^{-st} dt \), \( s \) being the Laplace dummy variable.

For a system at rest at time \( t = 0 \), this may be written
\[ \left[ s^2[M] + s[C] + [K] \right] \mathcal{L}[\delta(t)] = \mathcal{L}[F(t)] \]
If a unit impulse is applied at time \( t = 0 \) to one load point of the structure, then since the Laplace function of a unit impulse function is unity
\[ \left[ s^2[M] + s[C] + [K] \right] \mathcal{L}[\delta(t)] = \mathbf{1} \mathbf{1} \]
where \( \mathbf{1} \mathbf{1} \) is a null column matrix except for a unit entry corresponding to the excitation point.
If a unit impulse is consecutively applied to each load point and the resulting displacement column matrices are arranged in a square matrix, then
\[ \left[ \mathcal{L}(\delta) \right] = \left[ s^2[M] + s[C] + [K] \right]^{-1} \]
Let \( s = i \omega \). Then \( \mathcal{L}(\delta) = \int_{0}^{\infty} \delta(t) e^{-i\omega t} dt = \int_{0}^{\infty} \delta(\tau) e^{-i\omega \tau} d\tau \) and \( H(i\omega) = \int_{-\infty}^{\infty} h(\tau) e^{i\omega \tau} d\tau = \int_{-\infty}^{\infty} \delta(\tau) e^{-i\omega \tau} d\tau \) since as previously stated \( h(\tau) = \delta(\tau) = 0 \) for \( \tau > 0 \) or \( \tau < 0 \). Thus,
\[ \left[ H(i\omega) \right] = \lim_{s \to i\omega} \left[ \mathcal{L}(\delta) \right] = \left[ -s^2[M] + i\omega[C] + [K] \right]^{-1} \]
is the admittance matrix which is a square complex matrix dependent on frequency \( \omega \). In general (for a large number of elements), the evaluation of this matrix inversion involves extensive computer time. The inversion can be avoided if we assume the damping to be proportional to inertia, to stiffness, or to both (see Equation (E2)). This assumption permits the displacements to be expressed in terms of the normal modes, thereby uncoupling the equations of motion. The decoupling of the equations of motion results in the diagonalization of the admittance matrix, thus eliminating matrix inversion.

The following procedure is used to find an admittance matrix \( H(i\omega) \) which will not involve matrix inversion when the equations of motion are decoupled, the displacements being expressed in terms of the normal mode shapes. The mode shapes are determined from the undamped, unforced equations of motion.\(^8\)
\[ |M| \ddot{\delta}(t) | + |K| \delta(t) | = 0 \]

The solutions to this equation are expressed in terms of normal modes

\[ |\delta(t)| = |\phi(t)| e^{i(\omega_j t + \psi)} \]

Substitution of the latter into the former equation yields

\[ \left[ |K| - \omega_j^2 |M| \right] |\phi(t)| e^{i(\omega_j t + \psi)} = 0 \]

Equating the determinant of the square matrix to zero yields the nontrivial solutions to the classic eigenvalue equation, i.e., set

\[ \left| |K| - \omega_j^2 |M| \right| = 0 \]

Corresponding to each degree of freedom of the system, we can solve for an eigenvalue \( \omega_j^2 \). Associated with each eigenfrequency \( \omega_j \) is an eigenvector \( |\phi(t)| \).

By use of a coordinate transformation, we write

\[ |\delta(t)| = [\phi] |g(t)| \]

where each column of \( [\phi] \) is a normal mode \( |\phi(t)| \) and \( |g(t)| \) is a column matrix of coordinates called principal (or generalized) coordinates. Substituting this equation into Equation (E1) and premultiplying by the transpose of \( [\phi] \) results in

\[ [\phi]^T |M| [\phi] |\dot{g}(t)| + [\phi]^T |C| [\phi] |\dot{g}(t)| + [\phi]^T |K| [\phi] |g(t)| = [\phi]^T |F(t)| \]

Since the modes are assumed orthogonal with respect to inertia and stiffness, the generalized inertia and stiffness matrices become diagonal (see page 132 of Reference 18). Since the damping matrix \( |C| \) is proportional to \( |M|, |K| \), or both (Equation (E2)), it also results in a diagonal matrix, i.e.,

\[ [\phi]^T |M| [\phi] = \begin{bmatrix} -M_j \end{bmatrix} \]
\[ [\phi]^T |K| [\phi] = \begin{bmatrix} -K_j \end{bmatrix} = \begin{bmatrix} -\omega_j^2 M_j \end{bmatrix} \]
\[ [\phi]^T |C| [\phi] = \begin{bmatrix} -C_j \end{bmatrix} \]

where \( M_j, K_j = \omega_j^2 M_j \), and \( C_j \) are the generalized mass, stiffness, and damping, respectively.

Hence, substituting these quantities into the previous equation of motion, we get the decoupled equation of motion

*Recall that Equation (E1) includes viscous damping.
Thus, if *viscous damping* is used, then by means of the unit impulse excitations and the Laplace transform as in the derivation of Equation (E7), the admittance matrix is found from the foregoing equation to be

\[
[H(i\omega)] = [\phi] \begin{bmatrix}
\frac{1}{-\omega^2 M_j + i\omega C_j + \omega_j^2 M_j}
\end{bmatrix} [\phi]^T
\]  

(viscous damping)

\[
= [\phi] \begin{bmatrix}
\frac{1}{M_j(-\omega_j^2 + i\omega(\mu + \lambda \omega_j^2)) + \omega_j^2}
\end{bmatrix} [\phi]^T
\]  

(E8)

where \(\mu\) and \(\lambda\) are constants, \(\omega_j\) is the \(j\)th natural circular frequency of the plate, and each column of \([\phi]\) is a normal mode \(\phi^{(j)}\). The term \(M_j\) is the \(j\)th generalized mass defined by

\[
M_j = |\phi^{(j)}|^2 [M][\phi^{(j)}]
\]  

(E9)

Since \(\omega_j\) and \(\phi_j\) (and therefore \([\phi]\) and \([\phi]^T\)) are solutions to the classic eigenvalue problem and \([M]\) is a known quantity, then for each value \(j\), \(M_j\) is computed from Equation (E9) as a number lying along a diagonal and \(H(i\omega)\) is obtained from Equation (E8). Note that in Equation (E8) the quotient of a scalar quantity, i.e., \(1/[M_j(-\omega_j^2 + i\omega(\mu + \lambda \omega_j^2) + \omega_j^2)]\), for a given value of \(j\) is quite easily determined by a computer for a range of \(\omega\)'s. Hence evaluation of the product of the three matrices in Equation (E8) is generally much simpler (requires less computer time) than the evaluation of the inverse matrix, Equation (E7).

If *structural damping* is used as in Equation (E4), then since \(\omega_j^2 M_j = K_j\), the quotient in the first of Equations (E8) becomes

\[
\frac{1}{-\omega^2 M_j + \omega_j^2 M_j(1+ig)} = \frac{1}{M_j(-\omega_j^2 + ig \omega_j^2 + \omega_j^2)}
\]

so that

\[
[H(i\omega)] = [\phi] \begin{bmatrix}
\frac{1}{M_j(-\omega_j^2 + ig \omega_j^2 + \omega_j^2)}
\end{bmatrix} [\phi]^T
\]  

(structural damping)

(E10)

Equations (E8) or (E10) are *decoupled* admittance matrices and do not involve matrix inversions.
The admittance function will now be used in determining the response to turbulence excitation which is treated as an ergodic stationary random process. For such processes the relationship between response cross spectral density and pressure-loading cross spectral density of the turbulent boundary layer pressures is obtained as follows:

As in the derivation of \( H(i\omega) \), the displacement of point \( q \) of a plate to a pressure \( p_j \) applied over an area \( A_j \) at point \( j \) is (see Figure 19)

\[
\delta_q^j(t) = A_j \int_{-\infty}^{\infty} p_j(t^*_j) h_q^j(t-t^*_j) \, dt.
\]

Similarly the displacement at point \( r \) due to pressure at point \( k \) is

\[
\delta_r^k(t) = A_k \int_{-\infty}^{\infty} p_k(t^*_k) h_r^k(t-t^*_k) \, dt.
\]

The cross correlation of the two responses is

\[
R_{\delta_q^j \delta_r^k}(t) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \delta_q^j(t) \delta_r^k(t + r) \, dt.
\]

\[
= A_j A_k \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} p_j(t-\xi_1) p_k(t+r-\xi_2) \, dt \right] h_q^j(\xi_1) h_r^k(\xi_2) \, d\xi_1 \, d\xi_2
\]

\[
= A_j A_k \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R_{pqjk} (r-\xi_2, \xi_1) h_q^j(\xi_1) h_r^k(\xi_2) \, d\xi_1 \, d\xi_2
\]

where the order of the integrations has been interchanged and \( \xi_1 = t - t^*_1, \xi_2 = t - t^*_2 + r \) and \( R_{pqjk} \) is the cross correlation of the pressures at points \( j \) and \( k \).

The cross spectral density of the two displacements is the Fourier transform of this quantity

\[
\Phi_{\delta_q^j \delta_r^k}(i\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{\delta_q^j \delta_r^k}(r) e^{-i\omega r} \, dr.
\]

\[
= A_j A_k \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{pqjk} (r-\xi_2+\xi_1) d\tau \, e^{-i\omega \tau} \int_{-\infty}^{\infty} h_q^j(\xi_1) e^{i\omega \xi_1} d\xi_1 \int_{-\infty}^{\infty} h_r^k(\xi_2) e^{-i\omega \xi_2} d\xi_2
\]

278
\[ A_j A_k \Phi_{jk}(i\omega) H_j^*(i\omega) \cdot H_k(i\omega) \]

where \( \Phi_{jk} \) is the cross spectral density of pressures \( j \) and \( k \) and \( H_j^* \) is the complex conjugate of \( H_j \).

When all points of the structure are loaded, the displacement at point \( q \) is the sum of the components resulting from each load

\[ \delta_q(t) = \sum_{j=1}^{n} \delta_j(t) \]

where \( n \) is the number of load points. The cross correlation of two displacements when all points are loaded is therefore

\[ R_{\delta_q}(r) = \sum_{j=1}^{n} \sum_{k=1}^{n} R_{\delta_j \delta_k}(r) \]

and the corresponding cross spectral density function for displacements at \( q \) and \( r \) is

\[ \Phi_{\delta_q}(i\omega) = \sum_{j=1}^{n} \sum_{k=1}^{n} A_j A_k \Phi_{jk}(i\omega) H_j^*(i\omega) H_k(i\omega) \]

which can be expressed conveniently in matrix form for all pairs of node displacements*

\[ \begin{bmatrix} \Phi_{\delta}(i\omega) \end{bmatrix} = \begin{bmatrix} H^*(i\omega) \end{bmatrix} \begin{bmatrix} \Phi_p(i\omega) \end{bmatrix} \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} H(i\omega) \end{bmatrix}^T \]

(E11)

where \( \begin{bmatrix} \Phi_p(i\omega) \end{bmatrix} \) and \( \begin{bmatrix} \Phi_p(i\omega) \end{bmatrix} \) are cross power spectral density matrices of displacement and pressure, respectively. The diagonal elements of the resulting matrix in Equation (E11) are power spectral density functions of the displacements. The off-diagonal terms are displacement cross power spectral density terms.

Now substituting Equation (E3) into (E8) we get

\[ H(i\omega) = \begin{bmatrix} \Phi \end{bmatrix} \begin{bmatrix} 1 \\ M_j(\omega^2 - \omega^2) + i\omega(2\zeta_j\omega_j) \end{bmatrix} \begin{bmatrix} \Phi \end{bmatrix}^T \]

\[ = \begin{bmatrix} \Phi \end{bmatrix} \begin{bmatrix} (\omega^2 - \omega^2) - 2\zeta_j\omega_j \omega \\ M_j(\omega^2 - \omega^2)^2 + (2\zeta_j\omega_j \omega)^2 \end{bmatrix} \begin{bmatrix} \Phi \end{bmatrix}^T \]

*The matrices in Equation (E11) are easily expanded to yield the foregoing summation.
\[
\begin{align*}
\Phi_F(\omega) &= [C_F(\omega)] + i[Q_F(\omega)] \\
\Phi_\delta(\delta(\omega)) &= [C_\delta(\omega)] + i[Q_\delta(\omega)] \\
&= [\phi] \begin{bmatrix} -D - i E & 0 \end{bmatrix} [C_F(\omega) + iQ_F(\omega)] \cdot \begin{bmatrix} \phi \end{bmatrix} \begin{bmatrix} 1 & D & i & E \end{bmatrix} \cdot [\phi]^T \\
&= \begin{bmatrix} \phi \end{bmatrix} \begin{bmatrix} 1 & D & i & E \end{bmatrix} \cdot [\phi]^T [C_F + iQ_F] [\phi] \begin{bmatrix} 1 & D & i & E \end{bmatrix} \cdot [\phi]^T
\end{align*}
\]

Both pressure and deflection cross power spectral density matrices are Hermitian matrices which can be decomposed into a real symmetric matrix (co-power spectral density) \([C(\omega)]\) and a skew symmetric imaginary matrix (quad-power spectral density) \([Q(\omega)]\)

\[
[\phi_p(\omega)] = [C_p(\omega)] + i[Q_p(\omega)] \\
[\phi_\delta(\omega)] = [C_\delta(\omega)] + i[Q_\delta(\omega)]
\]

Pre-and post-multiplying by \([-\hat{A}^{-1} \hat{A}^{-1}\]), we have

\[
\begin{align*}
[\Phi_F(\omega)] &= [C_F(\omega)] + i[Q_F(\omega)] \\
[\Phi_\delta(\omega)] &= [C_\delta(\omega)] + i[Q_\delta(\omega)]
\end{align*}
\]

Substituting Equations (E12) and (E15) into Equation (E11), we get (noting that \([\phi],[\phi]^T\) are real quantities)
The equation consists of the sum of eight terms. Consider one of these terms $[\phi] \left[ D \right] \left[ C_p \right] [\phi] \left[ -D \right] [\phi]^T$. For convenience we treat the term as a $3 \times 3$ matrix. The results can then be extended to an $m \times m$ matrix.

Thus

$$[\phi] \left[ D \right] \left[ C_p \right] [\phi] \left[ -D \right] [\phi]^T = \phi \begin{pmatrix} D_1 & 0 & 0 \\ 0 & D_2 & 0 \\ 0 & 0 & D_3 \end{pmatrix} \phi^T = [\phi] \begin{pmatrix} D_1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} [\phi]^T +$$

$$[\phi] \begin{pmatrix} 0 & 0 & 0 \\ 0 & D_2 & 0 \\ 0 & 0 & 0 \end{pmatrix} [\phi]^T + [\phi] \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & D_3 \end{pmatrix} [\phi]^T$$

$$= D_1 [\phi] \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} [\phi]^T + D_2 [\phi] \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} [\phi]^T +$$

$$D_3 [\phi] \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} [\phi]^T +$$

If

$$[\phi] = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{bmatrix} = \begin{pmatrix} \phi^{(1)} & \phi^{(2)} & \phi^{(3)} \end{pmatrix}$$

where

$$\begin{bmatrix} \phi^{(1)} \\ \phi^{(2)} \\ \phi^{(3)} \end{bmatrix} = \begin{pmatrix} \phi_{11} \\ \phi_{12} \\ \phi_{13} \\ \phi_{21} \\ \phi_{22} \\ \phi_{23} \\ \phi_{31} \\ \phi_{32} \\ \phi_{33} \end{pmatrix}$$

then

$$[\phi]^T = \begin{bmatrix} \phi_{11} & \phi_{21} & \phi_{31} \\ \phi_{12} & \phi_{22} & \phi_{32} \\ \phi_{13} & \phi_{23} & \phi_{33} \end{bmatrix} = \begin{pmatrix} [\phi^{(1)}]^T \\ [\phi^{(2)}]^T \\ [\phi^{(3)}]^T \end{pmatrix} \text{; } \phi_{jk} = \phi_{kj}$$

281
Hence

\[ |\phi| \cap |\psi| \cdot |\phi| T = D_1 \begin{pmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \phi_{11} & \phi_{21} & \phi_{31} \\ \phi_{12} & \phi_{22} & \phi_{32} \\ \phi_{13} & \phi_{23} & \phi_{33} \end{pmatrix} + \ldots \]

\[ = D_1 \begin{pmatrix} \phi_{11} & 0 & 0 \\ \phi_{21} & 0 & 0 \\ \phi_{31} & 0 & 0 \end{pmatrix} \begin{pmatrix} \phi_{11} & \phi_{21} & \phi_{31} \\ \phi_{12} & \phi_{22} & \phi_{32} \\ \phi_{13} & \phi_{23} & \phi_{33} \end{pmatrix} + \ldots \]

\[ = D_1 \begin{pmatrix} \phi_{21} & \phi_{11} & \phi_{21} & \phi_{11} & \phi_{31} \\ \phi_{11} & \phi_{21} & \phi_{11} & \phi_{21} & \phi_{31} \\ \phi_{31} & \phi_{11} & \phi_{21} & \phi_{11} & \phi_{31} \end{pmatrix} + \ldots \]

\[ = D_1 \begin{pmatrix} \phi_{11} \\ \phi_{12} \\ \phi_{13} \end{pmatrix} \begin{pmatrix} \phi_{11} & \phi_{12} & \phi_{13} \end{pmatrix} + \ldots \]

\[ = D_1 \begin{pmatrix} \phi^{(1)} \\ \phi^{(2)} \\ \phi^{(3)} \end{pmatrix} T + D_2 \begin{pmatrix} \phi^{(2)} \end{pmatrix} T + D_3 \begin{pmatrix} \phi^{(3)} \end{pmatrix} T \]

which is the sum of dyadic products. Hence

\[ (ABC) \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix} = aAD^T + bBE^T + cCF^T. \]

And the dyadic product \( p \cdot q T \) of two vectors

\[ p = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}, \quad q = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} \]

\[ p \cdot q T = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix} = \begin{pmatrix} p_1 q_1 + p_2 q_2 + p_3 q_3 \end{pmatrix}. \]

The dyadic product should not be confused with the inner product of two vectors

\[ p \cdot q = p_1 q_1 + p_2 q_2 + p_3 q_3 = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = p^T \cdot q. \]
\[
\begin{align*}
\{ \phi \}^T & \left[ C_F \right] \{ \phi \} \\
& = \sum_{j=1}^{3} \sum_{k=1}^{3} D_j D_k C_F^{(jk)}(\omega) \\
& + D_j E_k \left( \{ Q_F^{(jk)}(\omega) \} + \{ Q_F^{(jk)}(\omega) \}^T \right) \\
& + i \left[ D_k E_j \left( \{ Q_F^{(jk)}(\omega) \} - \{ Q_F^{(jk)}(\omega) \}^T \right) \\
& + (D_j D_k + E_j E_k) \{ Q_F^{(jk)}(\omega) \} \right]
\end{align*}
\]

where
\[
\begin{align*}
C_F^{(jk)}(\omega) &= \{ \phi^{(j)} \} \{ \phi^{(j)} \}^T \left[ C_F(\omega) \right] \{ \phi^{(k)} \} \{ \phi^{(k)} \}^T \\
Q_F^{(jk)}(\omega) &= \{ \phi^{(j)} \} \{ \phi^{(j)} \}^T \left[ Q_F(\omega) \right] \{ \phi^{(k)} \} \{ \phi^{(k)} \}^T
\end{align*}
\]

The summation in Equation (E16) is over \( m \) normal modes.

Equation (E16) can be approximated for lightly damped systems. The cross product terms \((j \neq k) D_j D_k, E_j E_k, \) and \( D_j E_k, \) which involve coupling between modes, are considered insignificant for small damping. Neglecting these terms and since \( Q_F^{(i0)} = 0 \) due to the skew symmetry of \( Q_F \), whereas \( C_F^{(i0)} = [C_F^{(ij)}]^T \) due to the real symmetry of \( C_F \) (this will be shown later), we have
The average value of the product of two distinct responses at locations \( q \) and \( r \) is

\[
\overline{\delta_q \delta_r} = \int_0^\infty \Phi_{\delta_q \delta_r}(\omega) \, d\omega
\]

where \( \overline{\delta_q \delta_r} \) is called the joint deflection moment of the two responses \( q \) and \( r \) and denotes the space cross correlation (zero time delay) of the two responses. The joint deflection moments for all responses can be considered at once and written as a matrix integration

\[
\overline{\delta_q \delta_r} = \int_0^\infty \Phi_{\delta_q \delta_r}(\omega) \, d\omega
\]

where the elements of \( \overline{\delta_q \delta_r} \) are joint deflection moments for all pairs of structural node points. The diagonal elements are mean square values of the deflections, and the off-diagonal terms are time averages of products of deflections at different node points. For small un-coupled damping, \( \Phi_{\delta_q \delta_r} \) is given by Equation (E17) and the response is predominantly narrow-band occurring in the regions of the natural frequencies. If broad-band excitation is assumed, the variation of the excitation cross spectral density is small compared to the response variation near the natural frequencies and the force cross power spectral density can be treated as a constant near each natural frequency. Thus we have

\[
\overline{\delta_q \delta_r} = \int_0^\infty \Phi_{\delta_q \delta_r}(\omega) \, d\omega
\]

For small damping, and from the definitions of \( D_j \) and \( E_j \), this equation is evaluated as (see page 63, Equation (2.14) and page 72, Equation (ii) of Reference 18).

\[
\overline{\delta_q \delta_r} = \sum_{j=1}^m |\phi^{(j)}| \, |\phi^{(j')}| \, T \left[ C_F(\omega_j) \right] \left[ |\phi^{(j)}| \, |\phi^{(j')}| \right] T \int_0^\infty (D_j^2 + E_j^2) \, d\omega
\]

The Maestrello mathematical model for the space-time cross correlation of the fluctuating turbulence boundary layer pressures measured in broad frequency bands for Mach numbers ranging from 0.52 to 0.57 is (see Equation (B9) and the corresponding notation as well as Equation (E23) below)

*For \( j \neq K \), evaluation of the joint responses is given in Appendix II of Reference 37.
\[
\rho(\xi, \eta; \tau) = e^{-\frac{|\tau|}{\theta}} \left\{ \sum_{n=1}^{3} \frac{A_n \Phi_n}{\sum_{n=1}^{3} \Phi_n} \right\} 
\]

(E21)

where

\[-1 \leq \rho(\xi, \eta; \tau) \leq 1\]

\[\sum_{n=1}^{3} \frac{A_n}{\Phi_n} = K^2 = 9.56\]

and the mean (broad band) values of \( \theta \) as a function of Mach number \( M \) is given in Figure 16 of Reference 34; see also the method given later in this Appendix for determining computer program output.

The corresponding normalized pressure cross power spectral density is

\[
\pi(\xi, \eta; i \omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} \rho(\xi, \eta; \tau) e^{-i\omega \tau} d\tau; (0 \leq \omega \leq \infty) 
\]

(E22)

(multiplying the second of Equations (E22) by 2 yields the cross power spectral density in the positive frequency domain).

It has not been possible to solve Equation (E22) using \( \rho \) as defined by Equation (E21). However, a solution is possible if we take an alternate approach which makes use of the frozen turbulence model known as the Taylor hypothesis, i.e., assume space and time variations are interrelated according to

\[
\frac{|\tau|}{\theta} = \frac{|\xi|}{U' \delta} 
\]

(E23)

Thus if the time decay is described as spatial decay, the Maestrello space-time correlation function \( \rho \) has the form

*Measured values of \( \theta \) for frequency band width centered at 1200 and 4800 cps are also plotted in Figure 16 of Reference (34).
\[
\rho(\xi, \eta; r) = \frac{e^{-|\xi|/U_c \theta}}{9.56} \left\{ \sum_{n=1}^{3} \frac{A_n K_n}{K_n^2 + B^2 \left[(\xi - U_c r)^2 + \eta^2\right]} \right\}^{3} \rho_n(\xi, \eta; r) \tag{E24}
\]

where

\[
\rho_n(\xi, \eta; r) = \frac{A_n K_n e^{-|\xi|/U_c \theta}}{9.56 \left[K_n^2 + B^2 \left[(\xi - U_c r)^2 + \eta^2\right]\right]} \tag{E25}
\]

Then

\[
\Pi(\xi, \eta; i\omega) = \frac{1}{\pi} \sum_{n=1}^{3} \int_{-\infty}^{\infty} \rho_n(\xi, \eta; r) e^{-i\omega r} dr \quad (0 \leq \omega \leq +\infty) \tag{E26}
\]

and the \(n\)th component of the cross power spectral density is

\[
\Pi_n(\xi, \eta; i\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} \rho_n(\xi, \eta; r) e^{-i\omega r} dr \quad (0 \leq \omega \leq +\infty) \tag{E27}
\]

Substituting Equation (E25) in (E27) and putting Equation (E25) in the form

\[
\rho_n = \frac{A_n K_n e^{-|\xi|/U_c \theta}}{9.56 B^2 U_c^2} \cdot \left[ \frac{1}{K_n^2 + B^2 \eta^2 + \left(r - \frac{\xi}{U_c}\right)^2} \right] \tag{E30}
\]

yields upon taking the Fourier transform

\[
\Pi_n(\xi, \eta; i\omega) = \frac{A_n K_n e^{-|\xi|/U_c \theta}}{9.56 B U_c \left[K_n^2 + B^2 \eta^2\right]^{1/2}} \frac{i\omega \xi}{U_c} \tag{E28}
\]

Dimensionless forms of \(\eta_n\) are plotted in Figures 19 and 20 of Reference 34. The three components defined in Equation (E28) are used in Equation (E26) to define the fluctuating pressure loading function.
From Equation (B7) we have

\[ \Pi(\omega) = \frac{\Phi_p(\omega) U}{r_w^2 \delta^*} \]

and by definition

\[ \int_0^\infty \Pi(\omega) d\omega = \frac{U}{r_w^2 \delta^*} \int_0^\infty \Phi_p(\omega) d\omega = \frac{U}{r_w^2 \delta^*} \bar{p}^2 \]

Hence

\[ \frac{r_w^2 \delta^*}{U} = \bar{p}^2 \]

or

\[ \Pi(\omega) = \frac{\Phi_p(\omega)}{\bar{p}^2} \quad \text{or} \quad \Phi_p(\omega) = \bar{p}^2 \Pi(\omega) \]

but in accordance with the experimental results shown in Figure 14 of Reference 34, \( \{ \bar{p}^2 \}^{1/2} / r_w \) is a function of the Mach \( M \) so that \( \bar{p}^2 = K^2_m r_w^2 \). Thus, the power spectrum in nondimensional form is described by:

\[ \frac{\Phi_p(\omega) U}{r_w^2 \delta^*} = \frac{\bar{p}^2 \Pi(\omega) U}{r_w^2 \delta^*} = \frac{\Pi(\omega) U K^2}{\delta^*} = \sum_{n=1}^{3} A_n e^{-K_n(\omega \delta^*/U)} \]

(see Equation (B7) and note that Figure 2 of Reference 15 and Figure 2 of Reference 39 are equivalent); however, see Appendix E2 with respect to the value \( K \) used in practice.

Hence, using Equation (E29) and noting that \( 1/9.56 \neq 0.105 \)

\[ \Phi_p(\xi,\eta;i\omega) = C_p(\xi,\eta;i\omega) + iQ_p(\xi,\eta;i\omega) = \bar{p}^2 \Pi(\xi,\eta;i\omega) \]

\[ = \left[ \frac{\left( K(M) r_w \right)^2}{BU_c} \right] e^{-\frac{|\xi|}{U_c} - \frac{i\omega \xi}{U_c}} \left[ \sum_{n=1}^{3} A_n K_n^2 e^{-B U_c \left( K_n^2 + B \eta^2 \right)^{1/2}} \right] \]

\[ \left( 0 \leq \omega \leq \infty \right) \quad \text{(E29)} \]

Figure 3 of Reference 34 shows graphs of this function.
The complex pressure cross power spectral density describes pressure loading as a continuous function of separation distances in the x- and y-directions. Pressure loads from boundary layers are small in spatial scale. The load can change appreciably within a few inches or even less. Figure 21 of Reference (34) shows the variation of cross power spectral density over an element. These rapid variations cause problems in using $\Phi_p(\xi, \eta; \omega)$ to define a loading matrix for use with finite element structural methods. Matrix finite structural analysis methods generally assume that the pressure loads vary slowly over the distance of one element, i.e., that approximately constant pressure acts over the element. When this is true, pressure at the node points can be multiplied by area to approximate forces on the elements. But when the loads vary rapidly over an element, as do boundary layer pressure loads, the method is invalid since it results in large overestimates of the total forces on the element.

This rapid variation causes problems in trying to define a matrix of cross power spectral densities acting on pairs of finite element node points; proper application of the matrix would require a very fine grid of elements. Hence an alternate method has been developed to correctly calculate the cross power spectral density of net forces acting on pairs of finite structural elements. Terms of this type are gathered into a cross power spectral density matrix that is compatible with the structural idealization; the number of elements chosen is such that the desired number of modes can be adequately resolved. The infinitesimal forces are then summed to calculate the net force cross power spectral density on finite element pairs, i.e., the elemental loads are determined by summing the contribution of each infinitesimal area within an elemental pair.

We now evaluate the force cross power spectral density and construct matrices for its real and imaginary coefficients.

Consider the geometry of a pair of finite plate elements (Figure 20) and the more detailed drawing of $\xi$-direction separation distances (Figure 21); similar relations hold for $\eta$-direction separation distances. Let $\xi_0$ and $\eta_0$ be the dimensions of the finite elements in the x- and y-directions, respectively. The area of the structure represented at each node is $(\xi_0 \eta_0)$. The separation distances between the nodes are

\[
\xi = (x_j - x_i) = n_{ij}\xi_0 \\
\eta = (y_j - y_i) = m_{ij}\eta_0
\]

The infinitesimal normalized cross power spectral density of the net forces acting on the infinitesimal areas $dA_i$ and $dA_j$ (which are on the finite areas $A_i$ and $A_j$ associated with the node points, i.e., nodal areas) is

*In Mach 0.52 ($\hat{d} = 0.155$ in.) flow in the Boeing boundary layer facility, the 990-cps frequency cross power spectral density varies by a factor of more than ten between pairs of points on an 0.75-in.-long finite element.
Figure 20 - Geometry of a Pair of Finite Plate Elements

Figure 21 - Coordinates and Separation Distances in the x-Direction
\[ d(\Pi_{ij}) = \Pi((x'_i - x'_j), (y'_i - y'_j), i\omega) (dx'_i \ dy'_i) (dx'_j \ dy'_j) \]

where \( \Pi((x'_i - x'_j), (y'_i - y'_j)) \) is the normalized pressure cross-spectral density; \((x'_i, y'_i)\) and \((x'_j, y'_j)\) are points on the \(i\)th and \(j\)th elements, respectively, whose elemental areas are

\[
\begin{align*}
    dA_i &= dx'_i \ dy'_i = d\xi_i \ d\eta_i \\
    dA_j &= dx'_j \ dy'_j = d\xi_j \ d\eta_j
\end{align*}
\]

The net force cross power spectral density on the \((ij)\) pair of nodes is then

\[ \Pi_{F_{ij}}(i\omega) = \int_{\Delta_i} \int_{\Delta_j} \Pi((x'_i - x'_j), (y'_i - y'_j)i\omega) \ dA_j \ dA_i \] (E30)

where \(A_i\) and \(A_j\) are areas associated with the \(i\) and \(j\) nodes.

Figures 19 and 20 show that

\[
\begin{align*}
    \xi' &= (x'_i - x'_j) = n_{ij}\xi_0 + (\xi_j - \xi_i); \quad \xi = n_{ij}\xi_0 \\
    \eta' &= (y'_i - y'_j) = m_{ij}\eta_0 + (\eta_j - \eta_i); \quad \eta = m_{ij}\eta_0
\end{align*}
\]

where \(n_{ij}\) and \(m_{ij}\) are the integral numbers of incremental separation distances in the \(x\)- and \(y\)-directions, respectively.

Hence the force cross power spectral density of the net forces acting on the \((ij)\) pair of nodes is

\[
\Pi_{F_{ij}}(i\omega) = \int_{-\xi_0/2}^{\xi_0/2} \int_{-\eta_0/2}^{\eta_0/2} \int_{-\xi_0/2}^{\xi_0/2} \int_{-\eta_0/2}^{\eta_0/2} \Pi((\xi + \xi_j - \xi_i), (\eta + \eta_j - \eta_i); i\omega) \ d\xi_j \ d\eta_j \ d\xi_i \ d\eta_i \] (E31)

Now rewrite Equation (E28) using the variables of Equation (E31), i.e., replace the separation between nodes \(\xi, \eta\) by the separation between infinitesimal areas \(\xi + \xi_j - \xi_i\), \(\eta + \eta_j - \eta_i\), respectively. Then
\[ \Pi_n[(\xi + \xi_j - \xi_i), (\eta + \eta_j - \eta_i); i\omega] = \frac{A_n K_n}{9.56 Bu_c} e^{\frac{-\left|\frac{1}{2} (\xi + \xi_j - \xi_i)\right| + \frac{i}{2} \left(\omega \frac{\xi + \xi_j - \xi_i}{u_c}\right)}{2}} \]

\[ \int_{-\eta_0}^{\eta_0} \int_{-\eta_0}^{\eta_0} \frac{e^{-\frac{\omega}{Bu_c} \left[K_n^2 + B^2 (\eta + \eta_j - \eta_i)^2\right]^{1/2}}}{\left[K_n^2 + B^2 (\eta + \eta_j - \eta_i)^2\right]^{1/2}} d\eta_j \, d\eta_i \quad (0 \leq \omega \leq \infty) \quad \text{E32} \]

is the \textit{n}th component of the pressure cross spectral density. Equation (E32) shows that the integrand of Equation (E31) is divided into parts so that the integration is over \( \eta \) only, \( \xi \) only, and constant. The first two parts are now discussed.

The integral of \( \eta \) in Equation (E32), which involves \( \eta \) integration only, is

\[ \int_{-\eta_0}^{\eta_0} \int_{-\eta_0}^{\eta_0} \frac{e^{-\frac{\omega}{Bu_c} \left[K_n^2 + B^2 (\eta + \eta_j - \eta_i)^2\right]^{1/2}}}{\left[K_n^2 + B^2 (\eta + \eta_j - \eta_i)^2\right]^{1/2}} d\eta_j \, d\eta_i \quad \text{E33} \]

To generalize the analysis, the following dimensionless separation distances are defined

\[ \hat{\eta} = \frac{\eta}{0.8 K_n \delta^*} = \frac{m_{ij} \eta_0}{0.8 K_n \delta^*} \]

\[ \hat{\eta}_0 = \frac{\eta_0}{0.8 K_n \delta^*} \]

\[ \hat{\eta}_i = \frac{\eta_i}{0.8 K_n \delta^*} \]

\[ \hat{\eta}_j = \frac{\eta_j}{0.8 K_n \delta^*} \]

\[ y = \hat{\eta}_j - \hat{\eta}_i \]

291
Since $\frac{\omega}{BU_c} - \frac{1}{0.85U_c} = S$, Equation (E33) may be rewritten as

$$B'U, I = 0.85*Uc + B_2 \eta_2 + \frac{\eta_0}{2} - \frac{\eta_1}{2} \left[ 1 + \frac{B^2}{K^2} \eta^2 \left( 1 + \frac{\eta_j - \eta_i}{\eta} \right) \right]^{1/2}$$

but

$$\frac{B^2}{K^2} \eta^2 = \frac{\left( 0.85* \right)^2}{K^2} \left[ 0.8K_a \delta^* \right]^2 = \hat{G}^2$$

$$\frac{\eta_j - \eta_i}{\eta} = \frac{\hat{r}_j - \hat{r}_i}{\hat{r}} = \frac{y}{\hat{r}}$$

$$d\eta \, d\eta_i = (0.8K_a\delta^*)^2 \, d\hat{r}_j \, d\hat{r}_i$$

and the variables of integration are transformed by use of the Jacobian

$$d\hat{r}_i \, d\hat{r}_j = \left| \frac{\partial (\hat{r}_j, \hat{r}_i)}{\partial (\hat{r}_i, \hat{r}_j)} \right| dy \, d\hat{r}_i = \left| \begin{array}{cc} \frac{\partial \hat{r}_i}{\partial \hat{r}_i} & \frac{\partial \hat{r}_j}{\partial \hat{r}_i} \\ \frac{\partial \hat{r}_i}{\partial \hat{r}_j} & \frac{\partial \hat{r}_j}{\partial \hat{r}_j} \end{array} \right| \, dy \, d\hat{r}_i$$

$$= \left| \begin{array}{cc} 1 & 0 \\ -1 & 1 \end{array} \right| \, dy \, d\hat{r}_i = dy \, d\hat{r}_i$$

Hence $d\eta \, d\eta_i = (0.8K_a\delta^*)^2 \, dy \, d\hat{r}_i$.

For the limits when $\eta_i = \frac{\eta_0}{2}$, $\hat{r}_i = \frac{\eta_i}{0.8K_a\delta^*} = \frac{\eta_0}{2(0.8K_a\delta^*)} = \frac{\hat{r}_0}{2}$. Similarly
the upper and lower limits of \( \eta_j \) are \( \pm \frac{\delta_0}{2} \) respectively. Hence for the \( y = \frac{\hat{\eta}_i}{\eta_j} \) variable of integration, since \( \eta_j \) remains as a variable in the inner limits, the inner limits are \( y = \pm \frac{\delta_0}{2} - \hat{\eta}_i \).

Hence the previous integral in terms of the new coordinates \( y, \hat{\eta}_i \) is

\[
(0.8K_n \delta^*)^2 \int_{-\delta_0/2}^{+\delta_0/2} \frac{-K_n S[1 + (\hat{\eta} + y)^2]^{1/2}}{K_n[1 + (\hat{\eta} + y)^2]^{1/2}} \, dy \, d\hat{\eta}_i
\]

Now let

\[
y^* = \hat{\eta} + y
\]
\[
dy^* = dy
\]

when \( y = \pm \frac{\delta_0}{2} - \hat{\eta}_i, \ y^* = \pm \frac{\delta_0}{2} + \eta - \hat{\eta}_i \), hence the equation becomes

\[
(0.8K_n \delta^*)^2 \int_{-\delta_0/2}^{+\delta_0/2} \frac{-K_n S[1 + (y^*)^2]^{1/2}}{K_n[1 + (y^*)^2]^{1/2}} \, dy^* \, d\hat{\eta}_i
\]

Since \( y^* \) is a dummy variable, we let \( y^* \rightarrow y \) so that the equation and symbols conform to those in Reference 34. The integral is then:

\[
(0.8K_n \delta^*)^2 \int_{-\delta_0/2}^{+\delta_0/2} \frac{-K_n S[1 + y^2]^{1/2}}{K_n[1 + y^2]^{1/2}} \, dy \, d\hat{\eta}_i \tag{E34}
\]

Equation (E33) or its equivalent Equation (E34) cannot be integrated in closed form. For the general case, a numerical approximation of this integral is necessary. For the special case of the integrand approaching small values within one incremental separation, a closed-form approximation is possible. Thus for \( S \) and \( \delta_0 \) not too small and assuming \( \hat{\eta} = 0 \), the integrand of Equation (E34) is a rapidly decaying function. That is, we assume that the sphere of influence of all pressure points on the \( i \)th finite element does not extend beyond the element; the approximation is good when the boundary layer thickness is small compared
to the element size. We can then extend the limits of integration from $-\infty$ to $+\infty$ and take
\[ \int_{-\infty}^{+\infty} = 2 \int_{0}^{+\infty} \] since the integrand is an even function. With these approximations, Equation (E34) is equal to
\[ f \cdot 2f \quad (E34) \]
\[ \frac{+\hat{f}_{0}}{2} \int_{0}^{+\infty} \int_{0}^{+\infty} - \frac{K_{n} S[1 + y^2]^{1/2}}{K_{n}[1 + y^2]^{1/2}} \frac{dy}{d\hat{f}_{1}}. \quad (E35) \]

Equation (E35) may then be written as
\[ 1.28 K_{n}(\delta^*)^{2} \int_{-\hat{f}_{0}}^{+\hat{f}_{0}} \int_{0}^{+\infty} \frac{K_{n} S[1 + y^2]^{1/2}}{e} \frac{dy}{d\hat{f}_{1}}. \]

The value of the first integral is $\hat{\eta}_{0}$ and the value of the second integral obtained from Reference 40 (page 342, item 3.479.1 with $z = y^2$, $\beta = K_{n} S$ and $\nu = \frac{1}{2}$) is $\kappa_{0}(K_{n} S)$ the modified Bessel function of order zero with argument $K_{n} S$. Hence the solution to Equation (E35) is
\[ 1.28 K_{n}(\delta^*)^{2} \hat{\eta}_{0} \kappa_{0}(K_{n} S) \quad (E36) \]

When the Bessel function approximation is used, it is assumed that the force cross power spectral density is approximately zero for all $(i,i)$ pairs of finite elements except pairs $(i,j)$.

The integral of $\Theta$ in Equation (E32) which involves $\xi$ integration only, must be handled as two special cases

Case I: $\xi = 0$

Case II: $\xi > 0$, $\xi < 0$

When $\xi = 0$, the region of integration is divided into two regions to properly represent the function with absolute value sign. The $\xi > 0$ and $\xi < 0$ cases have been reduced to a single expression which describes both cases.

Now rewriting Equation (E29)
\[ \Phi_{p}(\xi;\eta;\omega) = p^{2} \Pi(\xi,\eta;\omega) \quad (E29) \]
Similarly, using \( \Phi_{Fi_j}(i\omega) \) as defined in Equation (E31)

\[
\Phi_{Fi_j}(i\omega) = \overline{p^2} \Pi_{Fi_j}(i\omega)
\]

where \( i \) and \( j \) are node points with incremental separation distance in the \( x \) and \( y \)-directions such that \( \xi = n_{ij}\xi_0, \eta = m_{ij}\eta_0 \), where \( n_{ij} = \frac{x_j - x_i}{\xi_0} \) and \( m_{ij} = \frac{y_j - y_i}{\eta_0} \); \( n_{ij} \) and \( m_{ij} \) are integers so that all nodes are separated by increments of length \( \xi_0 \) and \( \eta_0 \) in the \( x \)-directions, respectively. But the force power spectral density is the sum of three spectral components as defined by Equations (E31) and (E32)

\[
\Phi_{Fi_j}(i\omega) = \overline{p^2} \sum_{n=1}^{3} \Pi_{Fi_j}(i\omega)
\]

where

\[
\Pi_{Fi_j}(n\omega) = \frac{A_nK_n}{9.56B\upsilon_c} \int \int \int \int \frac{e^{\frac{i\omega}{\upsilon_c}n_{ij}\xi_0 + m_{ij}\eta_0}}{K_n^2 + B^2(m_{ij}\eta_0 + \eta_j - \eta_i)^2} \frac{d\eta\delta d\xi d\eta d\delta}{2}.
\]

The integral of Equation (E38) is determined as discussed above. Final equations for the power and cross power spectral density of force acting on node points of a structure are summarized below (because of the presence of the absolute value signs, evaluation of the integrals requires separate consideration of the various domains; see statement preceding Equation (E29)).

\*Correction of the boundary layer load equations originally presented in Reference 34 were made by the authors of the present report. These corrections as well as certain other minor modifications were adopted in Reference 41 and the correct final results are given here.
FORCE POWER SPECTRAL DENSITY \( (n_{ij} = m_{ij} = 0) \)

\[
\Phi_{F_{ij}}(\omega) = \Phi(\omega) = \hat{K} \cdot \hat{P}(\omega) \hat{\Phi}
\]

\[
C_{F_{ij}}(\omega) = \hat{K} \cdot \hat{P}(\omega) \hat{\Phi} ; \quad Q_{F_{ij}}(\omega) = 0 \tag{E39}
\]

FORCE CROSS POWER SPECTRAL DENSITY \( (n_{ij} \neq 0, m_{ij} \neq 0) \)

\[
\Phi_{F_{ij}}(i\omega) = \Phi(n_{ij}, m_{ij}; i\omega) = C_{F_{ij}}(\omega) + i Q_{F_{ij}}(\omega) \tag{E40}
\]

FORCE CO-POWER SPECTRAL DENSITY \( (n_{ij} \neq 0, m_{ij} \neq 0) \)

\[
C_{F_{ij}}(\omega) = C_p(n_{ij}, m_{ij}; \omega) = \hat{K} \cdot \hat{C} \hat{\Phi} e^{-|n_{ij} a \xi_0|} \tag{E41}
\]

FORCE QUAD-POWER SPECTRAL DENSITY \( (n_{ij} \neq 0, m_{ij} \neq 0) \)

\[
Q_{F_{ij}}(\omega) = Q_p(n_{ij}, m_{ij}; \omega) = \hat{K} \cdot \hat{Q} \hat{\Phi} e^{-|n_{ij} a \xi_0|} \tag{E42}
\]

where \( \hat{K}^* = \text{constant} = \frac{p^2 \eta_0}{4.78 U_c B^2} \).

PRESSURE POWER SPECTRAL DENSITY

Setting \( \xi = \eta = 0 \) and letting \( \{K(m) r(\omega)\}^2 = \frac{p^2}{\eta_0} \) in Equation (E29) and using the foregoing equation for \( \hat{K}^* \), we obtain

\[
\Phi_p(\omega) = \frac{p^2}{9.56 BU_c} \sum_{n=1}^{K_n \omega} A_n e^{-\frac{K_n \omega}{BU_c}} = \frac{K_n \omega}{2\eta_0} \sum_{n=1}^{K_n \omega} e
\]

\( P(\omega) \) (power spectral density, dependent on \( \xi \))

\[
= \frac{2}{(a^2 + \delta^2)^2} \left[ a \xi_0 (a^2 + \delta^2) + (\delta^2 - a^2) \left( 1 - e^{-a^2 \xi_0 \cos(b \xi_0)} + 2 ab e^{-a^2 \xi_0 \sin(b \xi_0)} \right) \right]
\]

\( \Phi = \text{that part of Equation (E37) with dependence on } \eta. \)

*If \( n_{ij} = 0, m_{ij} \neq 0 \), then \( C_{F_{ij}}(\omega) = \hat{K} \cdot \hat{P} \hat{\Phi} \) and \( Q_{F_{ij}} = 0. \)
There are two options for $\omega$ thus:

1. $\Phi(\omega)$ (uses Bessel function approximation)

\[
\Phi(\omega) = \sum_{n=1}^{N} A_n K_n Y_n \left( \frac{\omega K_n}{BU_c} \right) \kappa_0 \left( \frac{\omega K_n}{BU_c} \right) \tag{E44}
\]

(Phi Hat Option 1)

2. $\hat{\Phi}(\omega)$ (uses numerical methods)

\[
\hat{\Phi}(\omega) = \frac{B}{2n_0} \sum_{n=1}^{N} A_n K_n Z_n(m_{ij}, \omega) \tag{E45}
\]

(Phi Hat Option 2)

\[
Z_n(m_{ij}, \omega) = \sqrt{\frac{\omega}{BU_c}} \frac{[K_n^2 + B^2(m_{ij} \eta_0 + \eta_j - \eta_i)]^{1/2}}{[K_n^2 + B^2(m_{ij} \eta_0 + \eta_j - \eta_i)]^{1/2}} \frac{\eta_j}{d\eta_j} \tag{E46}
\]

\[
\hat{\Phi}, \hat{\Phi} = \text{those parts of Equation (E37) with dependence on } \xi
\]

\[
\hat{\Phi} = \frac{4}{(a^2 + b^2)^2} \left[ V \cos (n_{ij} b \xi_0) + W \sin (n_{ij} b \xi_0) \right] \tag{E47}
\]

\[
\hat{\Phi} = \frac{4}{(a^2 + b^2)^2} \left[ W \cos (n_{ij} b \xi_0) - V \sin (n_{ij} b \xi_0) \right] \tag{E48}
\]

or

\[
\begin{align*}
\hat{\Phi} &= \frac{4}{(a^2 + b^2)^2} \left[ V \cos (n_{ij} b \xi_0) \right] \\
\hat{\Phi} &= \frac{4}{(a^2 + b^2)^2} \left[ W \sin (n_{ij} b \xi_0) \right]
\end{align*}
\]

\[
V = ab \sin (b \xi_0) \sinh (a \xi_0) + \frac{(b^2 - a^2)}{2} \left[ 1 - \cos (b \xi_0) \cosh (a \xi_0) \right]
\]

\[
\cdot \frac{(a^2 - b^2)}{2} \sin (b \xi_0) \sinh (a \xi_0) + ab \left[ 1 - \cos (b \xi_0) \cosh (a \xi_0) \right] \tag{E49}
\]

\[
\frac{1}{U_c \theta} = \frac{a}{U_c} \tag{E50}
\]

297
The $C_F$ and $Q_F$ matrices are each constructed separately by ordering terms to match the ordering of terms in the structural stiffness matrix, thus making the final expressions for the turbulent boundary layer loading function compatible with the finite element methods of structural analysis. If $\{\omega_k\}$ is the set of frequencies with $N_\omega$ total frequencies, then there are $N_\omega$ matrices $[C_F(\omega_k)]$ and $[Q_F(\omega_k)]$ as follows (for simplicity, the $F$ subscript is dropped in the terms in the matrices so that $C_{Fij}(\omega_k) = C_{ij}(\omega_k)$ etc.).

$$[C_F(\omega_k)] = \begin{bmatrix}
C_{11}(\omega_k) & C_{12}(\omega_k) & C_{13}(\omega_k) & \cdots & C_{1m}(\omega_k) \\
C_{21}(\omega_k) & C_{22}(\omega_k) & C_{23}(\omega_k) & \cdots & C_{2m}(\omega_k) \\
C_{31}(\omega_k) & C_{32}(\omega_k) & C_{33}(\omega_k) & \cdots & C_{3m}(\omega_k) \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
C_{m1}(\omega_k) & C_{m2}(\omega_k) & C_{m3}(\omega_k) & \cdots & C_{mm}(\omega_k)
\end{bmatrix} \quad (E51)$$

$$[Q_F(\omega_k)] = \begin{bmatrix}
Q_{11}(\omega_k) & Q_{12}(\omega_k) & Q_{13}(\omega_k) & \cdots & Q_{1m}(\omega_k) \\
Q_{21}(\omega_k) & Q_{22}(\omega_k) & Q_{23}(\omega_k) & \cdots & Q_{2m}(\omega_k) \\
Q_{31}(\omega_k) & Q_{32}(\omega_k) & Q_{33}(\omega_k) & \cdots & Q_{3m}(\omega_k) \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
Q_{m1}(\omega_k) & Q_{m2}(\omega_k) & Q_{m3}(\omega_k) & \cdots & Q_{mm}(\omega_k)
\end{bmatrix} \quad (E52)$$

Since $[\Phi_F(i\omega)]$ is Hermitian, then $[\Phi_F(i\omega)] = [\Phi_F^*(i\omega)]^T$ or $[C_F(\omega_k)] = [C_F(\omega_k)]^T$; $[Q_F(\omega)] = -[Q_F(\omega)]^T$ and terms in the matrices have the following properties

$$C_{F_{ij}}(\omega_k) = C_{F_{ji}}(\omega_k) = \Phi_{F}(\omega_k) \quad (E53)$$

$$C_{F_{ij}}(\omega_k) = C_{F_{ji}}(\omega_k) = C(n_{ij}, m_{ij}, \omega_k) = C(n_{ij}, -n_{ji}, m_{ij} = -m_{ji}; \omega_k) \quad (E54)$$

$$Q_{F_{ij}}(\omega_k) = Q_{F_{ji}}(\omega_k) = 0 \quad (E55)$$

$$Q_{F_{ij}}(\omega_k) = -Q_{F_{ji}}(\omega_k) \quad (E56)$$
Hence

\[
[C_F(i\omega_k)] = \begin{bmatrix}
\Phi(\omega_k) & C_{12}(\omega_k) & C_{13}(\omega_k) & \cdots & C_{1m}(\omega_k) \\
C_{12}^*(\omega_k) & \Phi(\omega_k) & C_{23}(\omega_k) & \cdots & C_{2m}(\omega_k) \\
C_{13}^*(\omega_k) & C_{23}^*(\omega_k) & \Phi(\omega_k) & \cdots & C_{3m}(\omega_k) \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
C_{1m}^*(\omega_k) & C_{2m}^*(\omega_k) & C_{3m}^*(\omega_k) & \cdots & \Phi(\omega_k)
\end{bmatrix}
\] (E57)

\[
[Q_F(i\omega_k)] = \begin{bmatrix}
0 & Q_{12}(\omega_k) & Q_{13}(\omega_k) & \cdots & Q_{1m}(\omega_k) \\
-Q_{12}^*(\omega_k) & 0 & Q_{23}(\omega_k) & \cdots & Q_{2m}(\omega_k) \\
-Q_{13}^*(\omega_k) & -Q_{23}^*(\omega_k) & 0 & \cdots & Q_{3m}(\omega_k) \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
-Q_{1m}^*(\omega_k) & -Q_{2m}^*(\omega_k) & -Q_{3m}^*(\omega_k) & \cdots & 0
\end{bmatrix}
\] (E58)

Diagonal terms of \(C_F\) are the collections of power spectral densities at all node points. Because the turbulent boundary layer is approximately a homogeneous random process and its thickness changes very slowly within a panel length, these diagonal terms are all equal.

The \([C_F(\omega)]\) and \([Q_F(\omega)]\) matrices are symmetric and skew symmetric, respectively. The diagonal terms of the \([\Phi_F(i\omega)]\) matrix are the diagonal terms of the \([C_F(\omega)]\) matrix, Equation (E57); hence the diagonal terms of the \([Q_F(\omega)]\) matrix are zero.
APPENDIX E2 – METHOD FOR DETERMINING INPUT DATA

In using the program described herein, it is necessary to calculate numerical values for boundary layer parameters $\delta^*$, $\theta$, $<p^2>$, and $U_c$. These parameters and the associated quantities required for their calculation are defined below. The definitions assume incompressible flow and, therefore, are restricted to subsonic conditions. The Maestrello method (Appendix B2) may be used for computing additional input data.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>Free-stream speed of sound (in./sec)</td>
</tr>
<tr>
<td>$C_f$</td>
<td>Local coefficient of skin friction (dimensionless), equal to $0.059 R_x^{-1/5}$</td>
</tr>
<tr>
<td>$K$</td>
<td>Ratio of rms fluctuating pressure to local wall shearing stress (i.e., $&lt;p^2&gt;^{1/2}/r_w = 3.1$)</td>
</tr>
<tr>
<td>$M$</td>
<td>Mach number of the free stream</td>
</tr>
<tr>
<td>$P_A$</td>
<td>Ambient pressure (psi)</td>
</tr>
<tr>
<td>$&lt;p^2&gt;$</td>
<td>Mean square fluctuating pressure (psi)$^2$, equal to $K^2 r_w^2$</td>
</tr>
<tr>
<td>$q$</td>
<td>Dynamic pressure (psi), equal to $pU^2/2$</td>
</tr>
<tr>
<td>$\Re_x$</td>
<td>Reynolds number based on $x$ (dimensionless), equal to $xU/\nu$</td>
</tr>
<tr>
<td>$R_e \delta^*$</td>
<td>Reynolds number based on $\delta^<em>$ (dimensionless), equal to $\delta^</em> U/\nu$</td>
</tr>
<tr>
<td>$U_c$</td>
<td>Mean convection velocity of pressure fluctuations in the boundary layer (in./sec)</td>
</tr>
<tr>
<td>$x$</td>
<td>Distance from leading edge, or nose, of body (in.)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Ratio of specific heats, 1.4 at sea level; equal to $c_p/c_v$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Boundary layer thickness (in.), equal to $0.37 x R_x^{-1/5}$</td>
</tr>
<tr>
<td>$\delta^*$</td>
<td>Boundary layer displacement thickness (in.), equal to $\delta/8$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Mean eddy lifetime (sec)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Viscosity (lb force sec/in.$^2$)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Kinematic viscosity (in.$^2$/sec), equal to $\mu/\rho$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density of free stream (lb force sec$^2$/in.$^4$)</td>
</tr>
<tr>
<td>$r_w$</td>
<td>Local wall shearing stress (psi), equal to $C_f q = \frac{Y}{2} C_f P_A H^2$</td>
</tr>
</tbody>
</table>

$$\sum_{n=1}^{3} \frac{A_n}{K_n}$$

Equal to 9.56

The system of units is in inch-pound-seconds.
Measurements of the rms fluctuating pressure $<p^2>$ $1/2$ nondimensionalized with respect to the wall shear stress $r_w$ show considerable scatter (Figure 22). Different investigations show different relationships between $<p^2> 1/2/r_w$ and Mach number $M$, so it is difficult to predict the influence of Mach number on the rms pressure. It is assumed here that $<p^2> 1/2/r_w$ has the value 3.1 for all subsonic and low supersonic Mach numbers, and the model for the pressure power spectral density function, as shown in Figure 23, has been chosen accordingly. The value of $(<p^2>)^{1/2}/r_w$ was chosen to be close to the values measured in the majority of the investigations.

In the frequency domain, the convection velocity $U_c$ and the eddy lifetime $\theta$ are functions of frequency. When the overall values of $U_c$ and $\theta$ are considered, it is found that the effective values change with distance from the reference point. Complete representations of $U_c$ and $\theta$, which describe the spatial variation, greatly complicate the boundary layer model and, as a simple alternative, values of $U_c$ and $\theta$ are chosen for a particular separation distance. Corrections to $U_c$ and $\theta$ are proposed, depending on the frequency range of interest for the particular structure under consideration.

The mean convection velocity ratio $U_c/U$, taken as the asymptotic value for large separation distances, is not very sensitive to Mach number in subsonic and low supersonic flow. A value of $U_c/U = 0.82$ can be assumed for subsonic conditions as shown in Figure 24. This figure also indicates variation of subsonic convection velocity with frequency. The mean value is much closer to the low frequency value than it is to the high frequency value.

The mean eddy lifetime is shown in nondimensional form $U_c/\theta/\delta^*$ in Figure 25 as a function of Reynolds number based on displacement thickness $\delta^*$. The variation of $\theta$ with frequency is shown in Figure 26 where the results refer to measurements by Maestrello in fully developed turbulent pipe flow. The data in Figure 26 show that the low-frequency eddy lifetime will be longer by factors of 1.5 or greater than the mean lifetimes predicted in Figure 25.
Figure 22 – Summary of Boundary Layer RMS Pressure Fluctuations

This figure is reproduced from Reference 41. The reference numbers indicated on this figure are those given in Reference 41.
Fitted curve: \( \Phi_p(S) = \frac{3}{r_a^2 \omega} \sum_{n=1}^{3} A_n e^{-K_n S} \)

- \( A_1 = 26.5 \quad A_2 = 3.0 \quad A_3 = 0.5 \) (Subsonic model)
- \( K_1 = 6.1 \quad K_2 = 0.91 \quad K_3 = 0.26 \)
- \( \sum_{n=1}^{3} A_n = 9.56 \)

- \( A_1 = 12.0 \quad A_2 = 7.2 \quad A_3 = 1.58 \) (Maestrello, Reference 14)
- \( K_1 = 13.9 \quad K_2 = 2.94 \quad K_3 = 0.471 \)
- \( \sum_{n=1}^{3} A_n = 6.67 \)

Figure 23 – Dimensionless Pressure Power Spectra
Figure 24 – Convection Speed Ratio

This figure is reproduced from Reference 41. The reference numbers indicated on the figure are those given in Reference 41.

Figure 25 – Characteristic Length

This figure is reproduced from Reference 41. The reference numbers indicated on the figure are those given in Reference 41.
Figure 26 – Eddy Lifetime

Taken from Reference 14.
APPENDIX E3 - PROGRAM IDENTIFICATION

The reader is referred to References 36, 37, and 41 which present an extensive and detailed document of the computer program.
APPENDIX E4 — TEST RUNS

The computer printout of boundary layer input data is shown in Table 8a. Corresponding sample printouts of the computed force co-power spectral density matrix generated by these data are shown in Table 8b and 8c; note that the frequency associated with one sample differs from that of the other sample. The mean square displacement and displacement power spectral density of a 2024 aluminum alloy rectangular plate subject to this excitation are shown in Figures 27 and 28, respectively.*

*The set of force cross-PSD matrices defines loads on a structure with certain geometric characteristics. Compatibility between the terms of both the loading and structural flexibility matrices can be achieved by using some of the structural geometry information and by specifying flow direction and the direction of cyclic structural node numbering (see page 25 of Reference 41 and References 36 and 37). For the convenience of the reader, a description of the numbering schemes used to achieve compatibility is given in Figure 29.
TABLE 5

Sample Printouts of Boundary Layer Input Data and Computed Force
Co-Power Spectral Density Matrix – Jacobs

INPUT DATA

JACOBS BOUNDARY LAYER PLATE – AIAA PAPER 69-20 (SEE REFERENCE 42)

BOUNDARY LAYER FLOW MODEL OPTION - OPTION= 1
NUMBER OF FREQUENCIES FOR WHICH FORCE-CROSS-PSD MATRICES ARE TO BE PRINTED - NPRINT= 33
CYCLIC NODE NUMBERING DIRECTION - NCYCCL= 2
FLOW DIRECTION - NFLOW= 1
ROW PRINT OPTION - NPRINT= -0
DISPLACEMENT THICKNESS - DELTA= 0.1550000E-00 IN.
MEAN EDDY CONVECTION VELOCITY - UC= 0.6200000E 04 IN./SEC
INCREMENTAL SEPARATION DISTANCE T' FLOW DIRECTION - XI= 0.1000000E 01 IN.
INCREMENTAL SEPARATION DISTANCE IN DIRECTION PERPENDICULAR TO FLOW - ETA= 0.1167000E 01 IN.
MEAN EDDY LIFETIME - THETA= 0.9100000E-03 SEC
NUMBER OF STRUCTURAL COORDINATES IN DIRECTION OF CYCLIC NODE NUMBERING -N1= 7
NUMBER OF STRUCTURAL COORDINATES IN DIRECTION PERPENDICULAR TO CYCLIC NODE NUMBERING DIRECTION - N2= 13
MEAN SQUARE FLUCTUATING PRESSURE RATIO - FPR= 0.4070000E-03 SQ. LBS

***** END OF INPUT DATA PRINTOUT *****

(Table 8A is used in Phase II of the RAVNB program with the ABLN load module. The force matrix CF(I,J) for this case is a 55x55 matrix, determined by the number of retained freedoms in Phase I, computed for frequencies specified by the user 36, 37, 41. For any frequency, the number of rows of the matrix that are to be printed is either the first row or the total matrix.)

Table 8A – Turbulent Boundary Layer Pressure Loading on Finite Structural Elements
TABLE 8 (Continued)

FREQUENCY NO. 1 CF(I,J) MATRIX \( \Omega = 0.27371982E \ 04 \)

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Table 8B — First Row 75S x 55 Matrix of the Forcing Function for the Frequency \( w = 0.27371982E04 \)

FREQUENCY NO. 1 CF(I,J) MATRIX \( \Omega = 0.20000000E \ 04 \)

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Table 8C — First Row of 55 x 55 Matrix of the Forcing Function for the Frequency \( w = 0.20000000E04 \)
THE BOUNDARY LAYER AND PLATE PARAMETERS USED IN OBTAINING FIGURE 27 ARE

FLOW DATA
\( \psi \) = \( 4.07 \times 10^{-4} \) (PSI)
\( \theta^* \) = 0.155 IN
\( U_0 \) = 0.99 \( U \) = 6200 IN. SEC
\( N \) = 0.52
\( Q \) = 9.1 \times 10^{-4} SEC
\( \delta_0 \) = 1.00 IN

FLOW IN \( z \)-DIRECTION

PLATE DATA
- 12.0 \times 7.0 \times 0.06-IN. PLATE
- \( \delta_0 \) = 1.00 IN
- \( \delta_0 \) = 1.07 IN
- \( \text{MASS}^* \) = 0.000024 LB-SEC\(^2\)/IN.
- \( E \) (YOUNG'S MODULUS) = 1.05 \times 10^7 PSI
- \( \nu \) OR \( \gamma \) (POISSON'S RATIO) = 0.33
- \( I_z \) (MOMENT OF INERTIA) = 4.287 \times 10^{-5} IN\(^2\)

\( t_x = 0 \)
\( t_y = 0 \)
\( t_z = 0 \)

\( t_z = \frac{t_0}{12} \times 2(0.06)^3 / 12 \text{ IN.}^3\)

SEE PAGE 12 OF REFERENCE 34

\*ELEMENT WEIGHT = (1.000) IN. \times (1.167) IN. \times (0.080) IN. \times (5.1 LB/IN.\(^3\))
= 0.009736 LB
= 0.009736 LB
= 0.000024 LB-SEC\(^2\)/IN.

Figure 27 - Computed Displacement Power Spectral Density for a 12.0- \times 7.0- \times 0.06-Inch Aluminum Alloy Panel
 Obtained at 38th renumbered node—see Figure 29.
Figure 28 — Computed Mean Square Displacement for a 12.0- x 7.0- x 0.08-Inch Aluminum Alloy Panel
Note to Figure 29:

The area \((12 \times 7 \text{ in.})^2\) of the rectangular plate used in obtaining the results in Figures 27 and 28 was divided into 72 equal rectangles \((1 \times 1.167 \text{ in.})\) numbered consecutively in the \(y\)-direction as shown \((1\text{ through }72)\). The constraint conditions for each of the six degrees of freedom \((\theta_x, \theta_y, \delta_x, \delta_y, \delta_x', \delta_y')\) are given for each node (free, fixed or attached to a spring). The present example originally had 91 nodal points as shown \((1\text{ through }91)\), but only 55 of these nodes (inner nodes) have deflection \(\delta_y\) free, which represent the retained freedoms. These (retained) nodes are renumbered in the \(y\)-direction as shown \((1\text{ through }55)\). The sizes of the solution matrices are determined by the number of retained freedoms, 55 in this example. Thus we use a 55 by 55 matrix. Note that the heavy line (centerline) in Figure 29 is the line along which the responses shown in Figures 27 and 28 were obtained.

The grid size \((\xi_0 \times \eta_0)\) and the number of retained freedoms must be identical in Phases I and II of the computer program for compatibility to exist between the terms of both the loading and structural flexibility matrices. In the absence of compatibility, only the force co-power spectral matrices (Table 8) are generated, i.e., the computer will not generate response data, e.g., Figures 27 and 28.

Figure 29 – Cyclic Ordering of Nodes
\((\text{Area} = \xi_0 \times \eta_0 = 1.167 \text{ in.}^2)\)
REFERENCES


*Reference 1 is available upon request to Dr. Don Ross, Head, Acoustics and Vibration Laboratory, Naval Ship Research and Development, Washington, D.C. 20007.


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ENGINEERING GUIDE AND COMPUTER PROGRAMS FOR DETERMINING TURBULENCE-INDUCED VIBRATION AND RADIATION OF PLATES

Final Report 6/67 - 9/69

Ralph C. Leibowitz and Dolores R. Wallace

This report is an engineering guide to the use of the Dyer method of manual computation and to several computer programs for determining turbulence-induced vibration and radiation of finite plates in air and in water. Both simple and clamped boundary conditions are treated. The Dyer method and the computer programs are presented in a series of appendixes:

A. Bolt Beranek and Newman Manual Method (Dyer)
B. Boeing Program I (Maestrello)
C. Electric Boat Program (Isso)
D. Underwater Sound Laboratory Program (Strawderman)
E. Boeing Program II - Finite Element (Jacobs and Lagerquist)

The documentation is intended to facilitate the performance of flow-induced vibroacoustic computations as well as to furnish the groundwork for future research. It should also act as a theoretical guide for experimentalists. In the broader view, the documentation represents the initial steps of an effort to use computer programs to bridge the gap between vibroacoustic research results and design needs for structures that are subject to excitation by turbulence. Research tending to improve and extend the present program is recommended.
Turbulence-induced Vibration and Radiation of Plates
1. Simply Supported and Clamped Boundaries
2. Air and Water Fluid Media
Manual Computation
Digital Computer Programs
Continuum and Finite Element Approach
Theoretical and Experimental Recommendation
Engineering Guide